Matrix Factorization for Movie Recommendation Systems

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August 30, 2025

Today's Learning Journey

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- **Step-by-Step**: Building intuition with examples
- **Algorithms**: ALS vs Gradient Descent
- **Practice**: Hands-on understanding

Problem Setup

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- Netflix: 200M+ users, 15K+ titles

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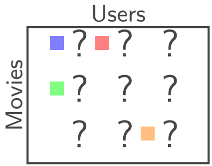
- You've rated 100 movies out of 15,000



Sparse Rating Matrix

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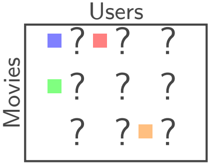

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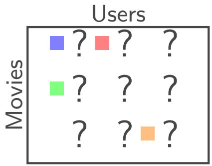

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Answer: $200 \times 10^6 \times 15 \times 10^3 = 3 \times 10^{12}$ possible ratings!
But typical users rate only 20-100 movies. What percentage of the matrix is filled?
Answer: $\frac{100}{15000} = 0.67\%$ - extremely sparse!

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- **Notation**: $\Omega = \{(i, j) : a_{ij} \text{ is observed}\}$

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Key Insight: Latent Features

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- Director (Christopher Nolan, Rajkumar Hirani)
- Language (Hindi, English, Tamil)
- Era (90s classics, modern CGI)

Key Insight:

- Your taste = combination of preferences
- Movie appeal = combination of features
- But we don't know these explicitly!

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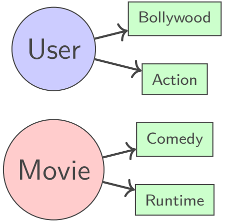

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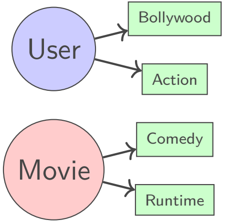

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Movie	**Bollywood**	**Sci-Fi**	**Drama**
Sholay	0.95	0.10	0.85
Swades	1.00	0.20	0.90
Batman	0.05	0.80	0.30
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Movie Feature Matrix $\mathbf{H} \in \mathbb{R}^{3\times 5}$:

$$\mathbf{H} = \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix}$$

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Key Question: How do we learn these w_{ij} values from observed ratings?

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$$\mathbf{A}_{3\times5} = \begin{bmatrix} 5 & 4 & 2 & 3 & 2 \\ ? & 5 & 1 & 4 & ? \\ 4 & ? & 1 & 5 & ? \end{bmatrix} \approx$$

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix} =$$

$$\mathbf{W}_{3\times3}\mathbf{H}_{3\times5}$$

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Sholay's DNA:

- Bollywood-ness: 0.95 (very high!)
- Action-ness: 0.10 (low)
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The Magic Formula:

$$\text{Alice's rating} = \text{Alice's preferences} \cdot \text{Sholay's features}$$

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$$\hat{a}_{11} = \mathbf{w}_1^T \mathbf{h}_1 \quad (1)$$

$$= w_{11} \cdot 0.95 + w_{12} \cdot 0.10 + w_{13} \cdot 0.85 \quad (2)$$

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$$= w_{11} \cdot 0.95 + w_{12} \cdot 0.10 + w_{13} \cdot 0.85 \tag{2}$$

Goal: Find w_{11}, w_{12}, w_{13} such that $\hat{a}_{11} \approx 5$ (Alice's actual rating)

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Answers:

1. $\mathbf{A} \in \mathbb{R}^{N \times M}$
2. $\mathbf{W} \in \mathbb{R}^{N \times r}$
3. $\mathbf{H} \in \mathbb{R}^{r \times M}$
4. Total parameters: $Nr + rM = r(N + M)$

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Key Insight: If $r \ll \min(N, M)$, we have huge parameter

Learning the Factorization

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In Matrix Notation:

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Where:

- $P_\Omega(\cdot)$: projection onto observed entries

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$$\boxed{\text{minimize}_{\mathbf{W},\mathbf{H}} \, \|P_\Omega(\mathbf{A} - \mathbf{W}\mathbf{H})\|_F^2}$$

Where:

- $P_\Omega(\cdot)$: projection onto observed entries
- $\|\cdot\|_F$: Frobenius norm

The Optimization Problem

Objective: Minimize prediction error on observed ratings only

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- Ω: set of observed (i, j) pairs

Why This is Challenging

Problem Characteristics:

- **Non-convex:** Multiple local minima exist

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Key Insight: While non-convex jointly, it's convex in each matrix individually!

Algorithm 1: Alternating Least Squares (ALS)

ALS: The Big Picture

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Least Squares Solution:

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Update Alice's preferences ($\mathbf{w}_1$):
Alice rated: Sholay(5), Swades(4), Batman(2), Interstellar(3), Shawshank(2)

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$$\mathbf{y}_1 = \begin{bmatrix} 5 \\ 4 \\ 2 \\ 3 \\ 2 \end{bmatrix} \tag{5}$$

$$\mathbf{X}_1 = \begin{bmatrix} 0.95 & 0.10 & 0.85 \\ 1.00 & 0.20 & 0.90 \\ 0.05 & 0.80 & 0.30 \\ 0.05 & 0.95 & 0.70 \\ 0.05 & 0.15 & 0.95 \end{bmatrix} \tag{6}$$

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Solution: $\mathbf{w}_1^* = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$
This gives us Alice's feature preferences!

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ALS: Complete Algorithm

Algorithm 1: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

1. **Initialize:** $\mathbf{W}^{(0)} \in \mathbb{R}^{N \times r}$, $\mathbf{H}^{(0)} \in \mathbb{R}^{r \times M}$ randomly

Output: $\mathbf{W}^{(T)}, \mathbf{H}^{(T)}$

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Algorithm 4: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

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   2) **Update Movies:** For each movie $j = 1, \ldots, M$:

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Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

ALS: Complete Algorithm

Algorithm 5: [

H] **Input:** Rating matrix $\mathbf{A}$, rank r, max iterations T

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   $$\mathbf{h}_j^{(t)} = (\mathbf{X}_j^T \mathbf{X}_j)^{-1} \mathbf{X}_j^T \mathbf{y}_j$$

3. **Check Convergence:** Stop if $\|\mathbf{W}^{(t)}\mathbf{H}^{(t)} - \mathbf{W}^{(t-1)}\mathbf{H}^{(t-1)}\|_F < \epsilon$

Output: $\mathbf{W}^{(T)}$, $\mathbf{H}^{(T)}$

Algorithm 2: Gradient Descent

Gradient Descent Approach

Simultaneous Updates: Update both $\mathbf{W}$ and $\mathbf{H}$ together

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Gradients:

$$\frac{\partial L}{\partial \mathbf{w}_i} = -2 \sum_{j:(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)\mathbf{h}_j \tag{9}$$

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SGD: Learning Like a Human

Imagine you're learning someone's taste in movies...

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Your Process:

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4. Repeat for next movie

SGD does exactly this!

- One rating at a time
- Small adjustments
- Gradually improves

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For each observed rating $(i, j) \in \Omega$:

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$$\mathbf{w}_i \leftarrow \mathbf{w}_i + \alpha \cdot e_{ij} \cdot \mathbf{h}_j \tag{11}$$

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- Learning rate α controls step size

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$$\text{Prediction: } \hat{a}_{11} = 0.4 \times 0.95 + 0.2 \times 0.10 + 0.3 \times 0.85 = 0.655 \tag{14}$$

$$\text{Error: } e_{11} = 5 - 0.655 = 4.345 \tag{15}$$

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Updates with $\alpha = 0.01$:

$$\mathbf{w}_1 \leftarrow [0.4, 0.2, 0.3] + 0.01 \times 4.345 \times [0.95, 0.10, 0.85] \tag{16}$$

$$= [0.4413, 0.2043, 0.3369] \tag{17}$$

$$\mathbf{h}_1 \leftarrow [0.95, 0.10, 0.85] + 0.01 \times 4.345 \times [0.4, 0.2, 0.3] \tag{18}$$

$$= [0.9674, 0.1087, 0.8631] \tag{19}$$

Pop Quiz 3: SGD Understanding

Quick Check

A user gives a rating of 2 to a movie, but our model predicts 4.5.

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Answers:

1. $e_{ij} = 2 - 4.5 = -2.5$
2. Decrease similarity (negative error)
3. $\mathbf{w}_i \leftarrow [0.8, 0.3] + 0.1 \times (-2.5) \times [0.6, 0.9] = [0.65, 0.075]$
4. $\mathbf{h}_j \leftarrow [0.6, 0.9] + 0.1 \times (-2.5) \times [0.8, 0.3] = [0.4, 0.825]$

Algorithm Comparison and Practical Considerations

ALS vs SGD: Head-to-Head Comparison

Aspect	**ALS**	**SGD**
Updates	Alternating	Simultaneous
Convergence	Faster, more stable	Slower, can oscillate
Parallelization	Excellent	Limited
Memory	Higher	Lower
Implementation	Complex	Simple
Hyperparameters	Few (rank r)	Many (α, schedule)
Scalability	Very good	Good

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- **ALS:** Large-scale, production systems (Spark, distributed)
- **SGD:** Online learning, real-time updates, research

Advanced Practical Considerations

Regularization: Prevent overfitting

$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

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Cold Start Problem: New users/items with no ratings

- Content-based features

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- Content-based features
- Demographic information

Advanced Practical Considerations

Regularization: Prevent overfitting

$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

Bias Terms: Account for global, user, and item biases

$$\hat{a}_{ij} = \mu + b_i + b_j + \mathbf{w}_i^T \mathbf{h}_j$$

Implicit Feedback: Binary observations (clicks, views)

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Hands-On Understanding

Let's Build Intuition: Small Example

Our 3×3 rating matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & ? & 2 \\ 4 & 4 & ? \\ ? & 5 & 1 \end{bmatrix}$$

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Constraint: Only minimize error on observed entries!

Step-by-Step ALS Solution

Iteration 1: Initialize randomly

$$\mathbf{W}^{(0)} = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}, \quad \mathbf{H}^{(0)} = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 0.3 & 1.2 & 0.8 \end{bmatrix}$$

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Solve: $\mathbf{w}_1^{(1)} = (\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T \mathbf{y}_1$
Continue for all users and movies...

Pop Quiz 4: Final Challenge

Master Check

You're Netflix's lead ML engineer. You have:

- 200M users, 15K movies
- 20B ratings (0.67% filled)
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Pop Quiz 4: Final Challenge

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Suggested Solution:

Summary and Key Takeaways

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$$\boxed{\text{Collaborative Filtering} = \text{Matrix Factorization} = \text{Dimensionality Reduct}}$$

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Answers:

1. **False** - Works with implicit feedback too (clicks, views)
2. **False** - Converges to local optimum (problem is