Decision Trees Pruning

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Pruning and Overfitting

The Problem: Overfitting in Decision Trees

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  - Rules that are too specific to training data
- **Solution**: Pruning to control model complexity

Pre-pruning (Early Stopping)

Stop growing tree before it becomes too complex:

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Advantages: Simple, computationally efficient
Disadvantages: May stop too early, miss good splits later

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- **Minimum impurity decrease**: Only split if improvement $>$ threshold

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Grow full tree, then remove unnecessary branches:

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Systematic approach to find optimal tree size:

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    5. Use cross-validation to select optimal α

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- **Domain knowledge**: Consider interpretability requirements