Precision-Recall Curves and Evaluation Metrics

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Motivation: Real-World Application

Brick Kiln Detection from Satellite Imagery

Problem: Identify illegal brick kilns using satellite imagery

Key Points: Why This Matters

- Environmental monitoring and air quality
- Thousands of square kilometers to survey
- Manual inspection is infeasible

The Challenge: Scale of the Problem

Dataset Scale

- **Images to scan:** 10,000 satellite images
- **Manual inspection time:** 30 seconds per image
- **Total manual effort:** $10{,}000 \times 30s$
- **That's 83 hours of continuous work!**

Can we automate this with machine learning?

Why Not Just Use Accuracy?

Three Models to Choose From

- Model A: 95% accuracy
- Model B: 92% accuracy
- Model C: 89% accuracy

Why Not Just Use Accuracy?

Three Models to Choose From

- Model A: 95% accuracy
- Model B: 92% accuracy
- Model C: 89% accuracy

Key Points: The Problem

Accuracy doesn't tell us about the **types of errors**!

Types of Errors Matter

Example: False Positive (Type I Error)

Model says "brick kiln detected" but there isn't one

- Wastes inspector's time
- Reduces trust in the system

Example: False Negative (Type II Error)

Model misses an actual brick kiln

- Environmental violation goes undetected
- Defeats the purpose of monitoring

Scenario 1: High Precision Model

Example: Conservative Classifier

Model behavior: Only flags when very confident

Results

- Flags 100 images as "has brick kiln"
- Inspector time: $100 \times 30s = 50$ minutes

Key Points: Trade-offs

✓ Few false alarms

✓ Inspector time well-spent

Scenario 2: High Recall Model

Example: Aggressive Classifier

Model behavior: Flags anything suspicious

Results

- Flags 2,000 images as "has brick kiln"
- Inspector time: $2{,}000 \times 30s = 16.7$ hours

Key Points: Trade-offs

✓ Catches almost all kilns

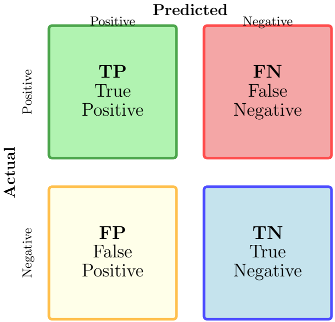
× Many false alarms

Classification Metrics Fundamentals

The Confusion Matrix

Definition: Confusion Matrix

		Predicted	
		Pos	Neg
Actual	Pos	TP	FN
	Neg	FP	TN



- **TP**: Correct positive
- **FP**: Type I error
- **TN**: Correct negative
- **FN**: Type II error

Precision: Reliability of Positive Predictions

Definition: Precision

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

Question it answers:
Of all instances we predicted as positive, what fraction was actually positive?

Precision: Example

Example: Brick Kiln Detection

- Model flags 100 images as having brick kilns
- 80 actually have brick kilns (TP)
- 20 are false alarms (FP)

$$\text{Precision} = \frac{80}{100} = 0.80 \text{ or } 80\%$$

Key Points: Interpretation

When the model says "brick kiln detected," it's correct 80% of the time

Recall: Completeness of Detection

Definition: Recall (Sensitivity, TPR)

$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

Question it answers:
Of all actual positive instances,
what fraction did we correctly identify?

Recall: Example

Example: Brick Kiln Detection

- 150 images actually contain brick kilns
- Model correctly identifies 80 (TP)
- Model misses 70 of them (FN)

$$\text{Recall} = \frac{80}{150} = 0.533 \text{ or } 53.3\%$$

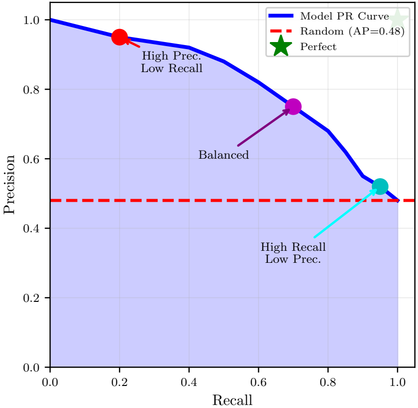
Key Points: Interpretation

The model finds only about half of all brick kilns

The Precision-Recall Trade-off

Key Points: Fundamental Tension

Improving one metric often hurts the other!



Trade-off: Model Behavior

Conservative Model

- High threshold
- Few predictions
- High precision
- Low recall

Aggressive Model

- Low threshold
- Many predictions
- Low precision
- High recall

Classification Thresholds

From Probabilities to Predictions

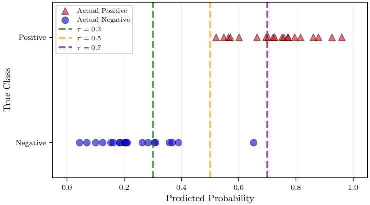
Definition: How Classifiers Work

Most classifiers output **probabilities**, not direct predictions

Classification threshold τ converts probabilities to classes:

$$\hat{y} = \begin{cases} 1 & \text{if } P(y=1|x) \geq \tau \\ 0 & \text{if } P(y=1|x) < \tau \end{cases}$$

Default: $\tau = 0.5$



Threshold Example

Example: Three Images, Different Thresholds

Image	$P(\text{kiln})$	$\tau = 0.5$	$\tau = 0.7$
A	0.85	Positive	Positive
B	0.62	Positive	Negative
C	0.38	Negative	Negative

Key Points: Key Insight

Same model, different thresholds = different predictions!

Low Threshold Effects

Threshold $\tau = 0.3$

Classify as positive if $P(y = 1|x) \geq 0.3$

- More instances classified as positive
- Higher recall (catch more positives)
- Lower precision (more false positives)
- More false alarms

Use when: Missing positives is costly

High Threshold Effects

Threshold $\tau = 0.7$

Classify as positive if $P(y = 1|x) \geq 0.7$

- Fewer instances classified as positive
- Lower recall (miss more positives)
- Higher precision (fewer false positives)
- Fewer false alarms

Use when: False alarms are costly

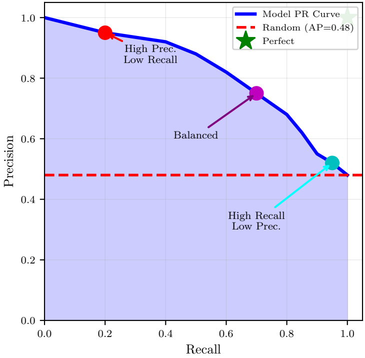
Precision-Recall Curves

What is a PR Curve?

Definition: Precision-Recall Curve

A plot showing precision vs. recall
for all possible threshold values

- **X-axis:** Recall
- **Y-axis:** Precision
- Each point = one threshold value

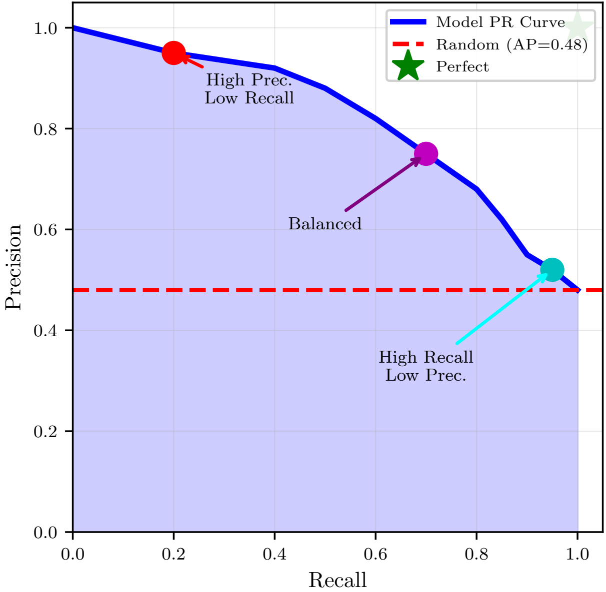


Key Points: What It Shows

Building a PR Curve: Steps

1. Train classifier (e.g., Logistic Regression)
2. Get predicted probabilities for test set
3. For each threshold $\tau \in [0, 1]$:
   - Apply threshold to get predictions
   - Compute confusion matrix
   - Calculate precision and recall
   - Plot (recall, precision) point

Building a PR Curve: Visualization



Implementation in Scikit-learn

Python Code

```
from sklearn.metrics import precision_recall_curve

# Get predicted probabilities
y_scores = model.predict_proba(X_test)[:, 1]

# Compute PR curve
precision, recall, thresholds = \
    precision_recall_curve(y_test, y_scores)
```

Example: Synthetic Dataset

Example: Dataset from Notebook

- Created using `make_blobs()`
- 100 samples, 2 features, 2 classes
- Training: 40 samples
- Test: 60 samples
- Cluster standard deviation: 8.0
- Classifier: Logistic Regression

Threshold Analysis: Low Values

Example: From Notebook: Threshold = 0.00

- **Precision:** 0.48
- **Recall:** 1.00

Interpretation:

- Classifies almost everything as positive
- Catches all positive cases (perfect recall)
- But only 48% are actually positive

Threshold Analysis: Medium Values

Example: From Notebook: Threshold = 0.50

- **Precision:** 0.74
- **Recall:** 0.69

Interpretation:

- Balanced operating point
- Good precision: 74% of predictions correct
- Good recall: finds 69% of positives
- This is the default threshold

Threshold Analysis: High Values

Example: From Notebook: Threshold = 0.90

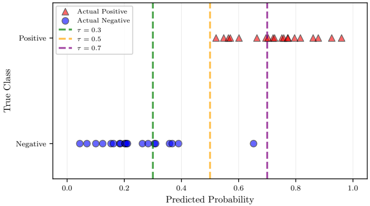
- **Precision:** 1.00
- **Recall:** 0.24

Interpretation:

- Very conservative classification
- Perfect precision: all predictions correct!
- But misses 76% of positive cases
- Only confident predictions are made

Complete Threshold Table

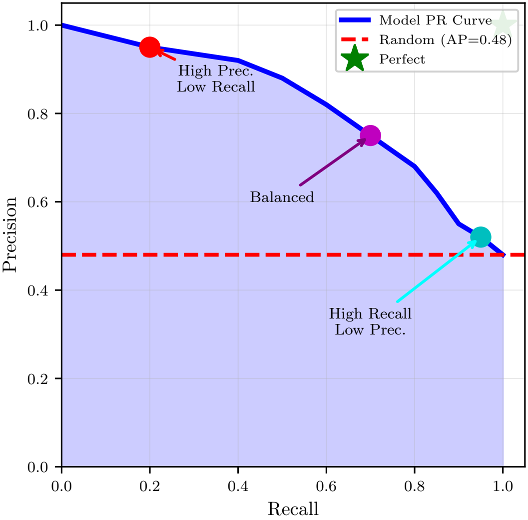
Threshold	Precision	Recall
0.00	0.48	1.00
0.10	0.55	0.98
0.30	0.65	0.85
0.50	0.74	0.69
0.70	0.85	0.45
0.90	1.00	0.24



Key Points: Observation

As threshold increases: Precision ↑, Recall ↓

Interpreting PR Curves



Key Points: What Makes a Good Curve?

Interpreting PR Curves: Baseline

Baseline: Random Classifier

Horizontal line at $y = \frac{\text{\# positives}}{\text{total}}$

For balanced classes: $y = 0.5$

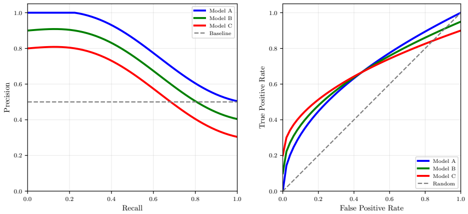
Example: Example

If 48% of data is positive class:
Random classifier has precision ≈ 0.48 at all recall levels

Comparing Models with PR Curves

Model Comparison Rules

1. If one curve dominates (always above), that model is better
2. If curves cross, choice depends on your needs:
   - Need high precision? Use left side of curve
   - Need high recall? Use right side of curve



Application-Specific Decisions

When to Prioritize Precision

Example: High Precision Scenarios

False positives are costly:

- **Spam detection**
  Don't want legitimate emails in spam folder
- **Medical diagnosis**
  Before expensive/risky treatment
- **Fraud detection**
  Don't block legitimate transactions

Strategy: Choose high threshold

When to Prioritize Recall

Example: High Recall Scenarios

False negatives are costly:

- **Cancer screening**
  Can't afford to miss cases
- **Security threats**
  Missing a threat is catastrophic
- **Environmental compliance**
  Must catch all violations

Strategy: Choose low threshold

Decision Analysis: Option A

High Precision Choice: $\tau = 0.7$

Metrics:

- Precision: 0.85
- Recall: 0.55

Example: Implications

- Flags 200 images
- 170 true positives, 30 false positives
- Inspection time: 1.7 hours
- Misses 45% of kilns

Decision Analysis: Option B

High Recall Choice: $\tau = 0.4$

Metrics:

- Precision: 0.65
- Recall: 0.85

Example: Implications

- Flags 500 images
- 325 true positives, 175 false positives
- Inspection time: 4.2 hours
- Only misses 15% of kilns

Which Option to Choose?

Decision Factors

- **Budget:** How much inspector time available?
- **Legal:** Required detection rate?
- **Environmental urgency:** Cost of missed kilns?

Key Points: Typical Choice

For environmental compliance:
Option B (high recall) is usually preferred

Missing violations is worse than
spending extra inspection time

Related Metrics

F1 Score: Balancing Both Metrics

Definition: F1 Score

Harmonic mean of precision and recall:

$$F_1 = 2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$$

Alternative form:

$$F_1 = \frac{2 \cdot \text{TP}}{2 \cdot \text{TP} + \text{FP} + \text{FN}}$$

Why Harmonic Mean?

Key Points: Properties of F1

- Range: $[0, 1]$, higher is better
- Heavily penalizes imbalanced metrics
- Both precision and recall must be good

Example: Example Comparison

- $P = 0.80, R = 0.60 \Rightarrow F_1 = 0.686$
- $P = 0.70, R = 0.70 \Rightarrow F_1 = 0.700$

Balanced metrics give better F1!

F_β Score: Weighted Version

Definition: F_β Score

$$F_\beta = (1 + \beta^2) \cdot \frac{\text{Precision} \cdot \text{Recall}}{\beta^2 \cdot \text{Precision} + \text{Recall}}$$

Parameter β:

- $\beta = 1$: Equal weight (F_1 score)
- $\beta < 1$: Favor precision (e.g., $F_{0.5}$)
- $\beta > 1$: Favor recall (e.g., F_2)

F_β Applications: High Recall

Example: F_2 Score

Use when: Recall is $2\times$ more important than precision

Applications:

- **Cancer screening**
  Missing a cancer case is catastrophic
- **Security threat detection**
  Can't afford to miss threats
- **Environmental compliance**
  Our brick kiln detection example

Higher β = More weight on recall

F_β Applications: High Precision

Example: $F_{0.5}$ Score

Use when: Precision is $2\times$ more important than recall

Applications:

- **Search engines**
  Show most relevant results first
- **Spam detection**
  Avoid false positives (legitimate emails in spam)
- **Medical diagnoses**
  Before expensive/invasive treatments

Lower β = More weight on precision

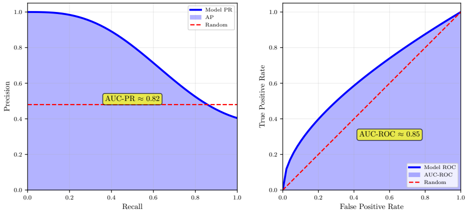
Average Precision (AP)

Definition: Average Precision

Area under the precision-recall curve:

$$\mathrm{AP} = \sum_{n=1}^{N} (R_n - R_{n-1}) \cdot P_n$$

where P_n and R_n are precision and recall at the n-th threshold



Average Precision: Properties

Key Points: Key Properties

- Range: $[0, 1]$, higher is better
- Single number summarizing entire curve
- Perfect classifier: AP $= 1.0$
- Weighted by recall changes

When to Use Average Precision

Key Points: Use Cases

- Comparing models across all thresholds
- When you can't choose single operating point
- Benchmark competitions

Example: Object Detection

mAP (mean Average Precision):

Average of AP across all object classes

Standard metric in COCO, Pascal VOC

Specificity (True Negative Rate)

Definition: Specificity

$$\text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}}$$

Fraction of negatives correctly identified

Example: Example

Out of 100 non-kiln images, if we correctly identify 90:
$\text{Specificity} = 90/100 = 0.90$

False Positive Rate (FPR)

Definition: FPR

$$\text{FPR} = \frac{\text{FP}}{\text{FP} + \text{TN}} = 1 - \text{Specificity}$$

Fraction of negatives wrongly classified

Key Points: Relationship

FPR and Specificity are complements:
$\text{FPR} + \text{Specificity} = 1$

ROC Curves

What is ROC?

Definition: ROC: Receiver Operating Characteristic

Developed during World War II for analyzing radar signals

Breaking down the name:

- **R**eceiver: The detector/classifier receiving signals
- **O**perating: Different operating points (thresholds)
- **C**haracteristic: Performance at each threshold

Key Points: Historical Context

Originally used to analyze radar operators' ability
to correctly detect enemy aircraft from radar signals

ROC Curve Definition

Definition: What ROC Plots

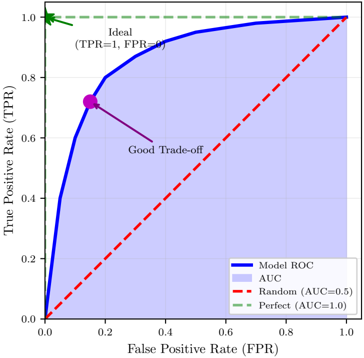
ROC curve plots TPR vs FPR at all thresholds

- **X-axis:** False Positive Rate (FPR)

$$\text{FPR} = \frac{\text{FP}}{\text{FP} + \text{TN}}$$

- **Y-axis:** True Positive Rate (TPR) = Recall

$$\text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$



Intuitive Understanding: TPR

Example: True Positive Rate (TPR)

$$\text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{\text{TP}}{\text{All Actual Positives}}$$

Question it answers: Of all actual brick kilns, what fraction did we detect?

- Same as Recall!
- Measures: Sensitivity of the detector
- High TPR = Catches most positives
- Low TPR = Misses many positives

Intuitive Understanding: FPR

Example: False Positive Rate (FPR)

$$\text{FPR} = \frac{\text{FP}}{\text{FP} + \text{TN}} = \frac{\text{FP}}{\text{All Actual Negatives}}$$

Question it answers: Of all non-kiln images, what fraction did we
incorrectly flag as having kilns?

- Measures: False alarm rate
- High FPR $=$ Many false alarms
- Low FPR $=$ Few false alarms
- FPR $= 1-$ Specificity

The ROC Trade-off

Key Points: Fundamental Trade-off

As we vary the threshold:

- Lower threshold $\rightarrow$ Higher TPR, Higher FPR
- Higher threshold $\rightarrow$ Lower TPR, Lower FPR

Low Threshold

- Catch more positives
- But more false alarms
- Top-right of ROC

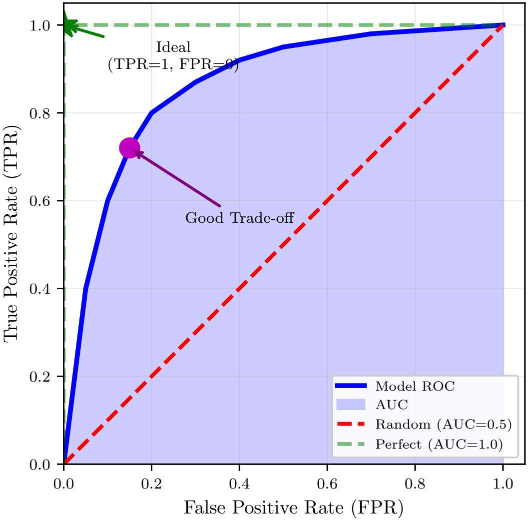
High Threshold

- Fewer false alarms
- But miss more positives
- Bottom-left of ROC

Building a ROC Curve: Steps

1. Train classifier, get predicted probabilities
2. For each threshold $\tau \in [0, 1]$:
   - Apply threshold to get predictions
   - Compute confusion matrix
   - Calculate TPR and FPR
   - Plot point (FPR, TPR)
3. Connect points to form curve

Building a ROC Curve: Interpretation

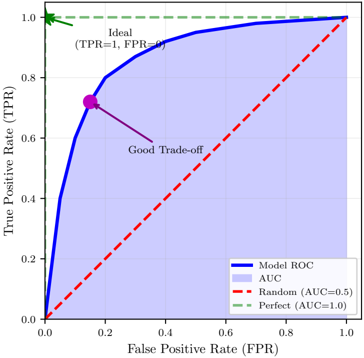


- **Perfect classifier:** Curve hugs top-left corner

Interpreting ROC Curves

Key Points: Good ROC Curve

- Closer to top-left
- Top-left = perfect!
- TPR=1, FPR=0
- High TPR, low FPR



Baselines

- **Perfect:** Top-left
- **Random:** Diagonal
- **Bad:** Below diagonal

Example: Same Dataset

Example: From Notebook

Using our Logistic Regression model

Threshold	TPR (Recall)	FPR
0.00	1.00	1.00
0.30	0.83	0.35
0.50	0.69	0.23
0.70	0.52	0.10
0.90	0.24	0.00

Key Points: Observation

As threshold increases: TPR $\downarrow$, FPR $\downarrow$

AUC-ROC: Area Under ROC Curve

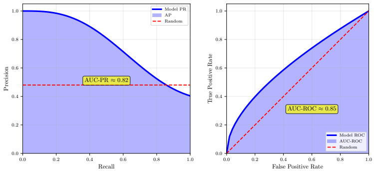
Definition: AUC-ROC

Single number summarizing entire ROC curve

$$\text{AUC-ROC} = \int_0^1 \text{TPR}(\text{FPR})\, d(\text{FPR})$$

Interpretation:

- Range: $[0, 1]$
- Perfect: AUC $= 1.0$
- Random: AUC $= 0.5$
- Higher is better



AUC-ROC Intuition

Key Points: Probabilistic Interpretation

AUC-ROC = Probability that the model ranks
a random positive example higher than
a random negative example

Example: Example

- AUC = 0.95: 95% chance model scores a true kiln higher than a non-kiln
- AUC = 0.50: Model is guessing randomly
- AUC = 0.85: Good discrimination ability

ROC Implementation

Scikit-learn Implementation

```
from sklearn.metrics import (
    roc_curve, roc_auc_score,
    RocCurveDisplay
)

# Get predicted probabilities
y_scores = model.predict_proba(X_test)[:, 1]

# Compute ROC curve
fpr, tpr, thresholds = roc_curve(y_test, y_scores)
auc_roc = roc_auc_score(y_test, y_scores)

# Visualize
display = RocCurveDisplay(fpr=fpr, tpr=tpr,
                          roc_auc=auc_roc)
display.plot()
```

Comparing Multiple Models

Example: From Notebook: 3 Classifiers

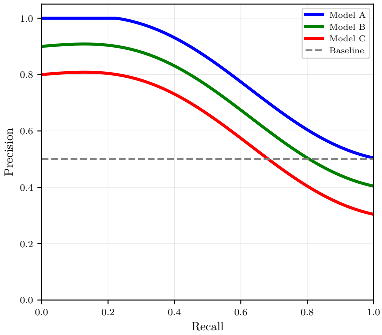
- Logistic Regression (linear boundary)
- Random Forest (non-linear, ensemble)
- SVM with RBF kernel (non-linear)

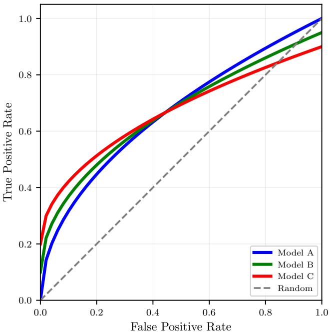
Model	**AUC-ROC**	**AUC-PR**
Random Forest	0.92	0.90
SVM (RBF)	0.89	0.87
Logistic Regression	0.86	0.83

(Values approximate from notebook example)

PR vs ROC: When to Use Each

Comparing PR and ROC Curves





PR Curve

Plots: Precision vs Recall
Focus: Positive class
Sensitive to: Imbalance

ROC Curve

Plots: TPR vs FPR
Focus: Both classes
Robust to: Imbalance

Key Difference: Class Imbalance

Critical Insight

ROC curves can be overly optimistic
on highly imbalanced datasets!

Example: Why?

FPR uses TN in denominator:

$$\text{FPR} = \frac{\text{FP}}{\text{FP} + \text{TN}}$$

With many negatives, even lots of FPs
can give a low FPR

Example: Imbalanced Data Setup

Example: Scenario: Highly Imbalanced Dataset

- **Total images:** 1,000
- **Positive class** (has brick kilns): 50 (5%)
- **Negative class** (no kilns): 950 (95%)

This is a realistic scenario!
Many real-world problems have imbalanced classes

Example: Imbalanced Data Analysis

Model with 100 False Positives

Suppose our model produces 100 false alarms:

Precision impact:

- Many false alarms per true positive
- Precision will be **low** (obvious problem!)

FPR appears good:

$$\text{FPR} = \frac{100}{100 + 850} = \frac{100}{950} = 0.105$$

Even with 100 false positives, FPR is only 10.5%!

Key Points: Conclusion

PR curve: Shows the problem clearly

ROC curve: Can hide issues in imbalanced data

Practical Considerations

PR Curves vs ROC Curves

Key Points: Use PR Curves When:

- Classes are highly imbalanced
- You care primarily about positive class
- False positives and negatives differ in cost

Examples: Rare disease, fraud, information retrieval

Use ROC Curves When:

- Classes are relatively balanced
- Both classes equally important

Why PR for Imbalanced Data?

Example: Brick Kiln Dataset

- Total: 10,000 images
- Positive (has kiln): 150 (1.5%)
- Negative (no kiln): 9,850 (98.5%)

Naive Classifier

Always predict "no kiln":

- Accuracy: 98.5% (looks great!)
- Precision: undefined
- Recall: 0% (useless!)

The Problem with Accuracy

Key Points: Why Accuracy Fails

With extreme imbalance (1.5% positive):

- Accuracy dominated by majority class
- High accuracy doesn't mean good performance
- Need metrics focused on positive class

Use Precision, Recall, and PR curves!

Visualization with Scikit-learn

Complete Implementation

```
from sklearn.metrics import (
    precision_recall_curve,
    average_precision_score,
    PrecisionRecallDisplay
)

# Get scores
y_scores = model.predict_proba(X_test)[:, 1]

# Compute metrics
precision, recall, thresholds = \
    precision_recall_curve(y_test, y_scores)
ap = average_precision_score(y_test, y_scores)

# Visualize
display = PrecisionRecallDisplay(
    precision, recall, average_precision=ap)
```

Pop Quiz 1

Answer this!

A model detects defective products (2% of all products).

Your model achieves:

- Precision: 0.60
- Recall: 0.90

Out of 10,000 products, how many will be flagged?

1. 150
2. 300
3. 600
4. 900

Pop Quiz 1: Answer

Example: Solution

Answer: (b) 300

Step 1: Actual defective products

$$10{,}000 \times 0.02 = 200$$

Step 2: True Positives (Recall $= 0.90$)

$$\text{TP} = 200 \times 0.90 = 180$$

Step 3: Use precision formula

$$0.60 = \frac{180}{\text{Total flagged}}$$

$$\text{Total flagged} = \frac{180}{0.60} = 300$$

Pop Quiz 2

Answer this!

Which scenario needs model with
Precision=0.70, Recall=0.85
over Precision=0.85, Recall=0.70?

1. Email spam detection
   (false positives lose legitimate mail)
2. Airport security screening
   (missing threats is catastrophic)
3. Credit card fraud
   (false positives block legitimate purchases)
4. All equally

Pop Quiz 2: Answer

Example: Solution

Answer: (b) Airport security screening

Reasoning:

- First model has **higher recall (0.85)**
- Catches more true positives
- Missing a threat = catastrophic
- Better to have false alarms than miss threats

Options (a) and (c): False positives are costly
$\Rightarrow$ Need high precision

Summary

Key Takeaways (1/2)

Key Points: Core Concepts

1. **Precision:** Reliability of predictions
2. **Recall:** Completeness of detection
3. **Trade-off:** Can't maximize both
4. **Thresholds:** Control the trade-off

Key Takeaways (2/2)

Key Points: Practical Insights

5. **PR curves:** Show all trade-offs
6. **Application:** Determines best point
7. **Imbalanced data:** PR better than accuracy
8. **Summary metrics:** F1, AP

Workflow Summary

1. Train classifier
2. Generate PR curve on validation set
3. Analyze precision-recall trade-offs
4. Choose threshold based on:
    - Application requirements
    - Cost of errors
    - Available resources
5. Validate on test set
6. Monitor in production
7. Adjust if requirements change

The Right Model for YOUR Application

The best model makes the right trade-offs
for **your specific application**

Not the highest accuracy,
not the highest F1,
but the one that aligns with your goals!

Further Resources

- **Notebook:** `pr-curve.html`
  Running example with visualization code
- **Documentation:**
  Scikit-learn Precision-Recall guide
- **Related topics:**
  - ROC curves and AUC
  - Cost-sensitive learning
  - Threshold optimization
  - Multi-class metrics

Thank you!

Questions?