Naive Bayes

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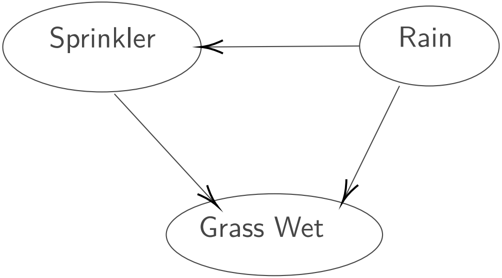
Bayesian Networks



- Nodes are random variables.
- Edges denote direct impact

Example

- Grass can be wet due to multiple reasons:
  - Rain
  - Sprinkler
- Also, if it rains, then sprinkler need not be used.

Bayesian Nets

$\mathbb{P}(X_1, X_2, X_3, \ldots, X_N)$ denotes the joint probability, where X_i are random variables.

$$\mathbb{P}(X_1, X_2, X_3, \ldots, X_N) = \Pi_{k=1}^{N} \mathbb{P}(X_k | \mathit{parents}(X_k))$$

$$\mathbb{P}(S, G, R) = \mathbb{P}(G|S, R)\mathbb{P}(S|R)\mathbb{P}(R)$$

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- Each email corresponds to vector/feature of length N containing zeros or ones.

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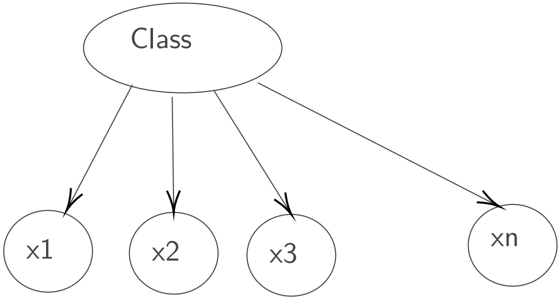
- We want to model $\mathbb{P}(\textit{class}(y) \mid \text{features (x)})$
- We can use Bayes rule as follows:
  $\mathbb{P}(\textit{class}(y) \mid \text{ features (x) }) = \frac{\mathbb{P}(\text{ features (x) } | \textit{class}(y))\mathbb{P}(\textit{class}(y))}{\mathbb{P}(\text{ features (x) })}$

Quick Question



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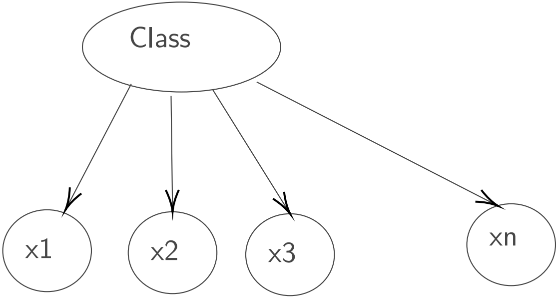

$$\mathbb{P}(x_1, x_2, x_3, \ldots, x_N|y) = \mathbb{P}(x_1|y)\mathbb{P}(x_2|y)\ldots\mathbb{P}(x_N|y)$$

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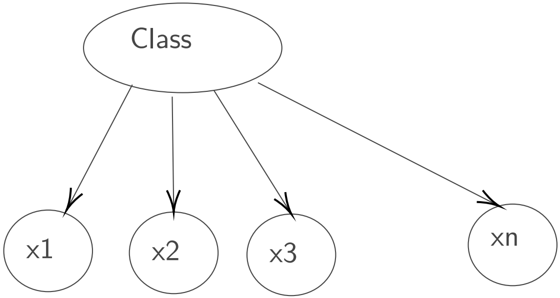

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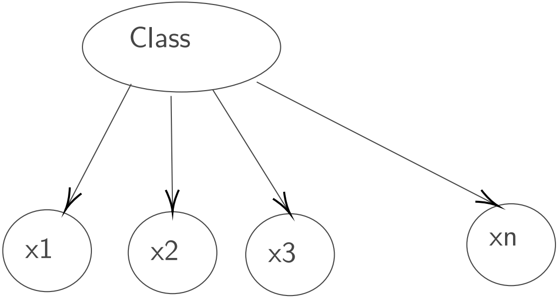

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Why is Naive Bayes model called Naive?
Naive assumption x_i and x_{i+1} are independent given y

$$\text{i.e. } p\left(x_2 \mid x_1, y\right) = p\left(x_2 \mid y\right)$$

Frame Title

It assumes that the features are independent during modelling, which is generally not the case.

What do we need to predict?

$$\mathbb{P}(y|x_1, x_2, \ldots, x_N) = \frac{\mathbb{P}(x_1, x_2, \ldots, x_N|y)\mathbb{P}(y)}{\mathbb{P}(x_1, x_2, \ldots, x_N)}$$

Spam Mail Classification

Probability of x_i being a spam email

$$\mathbb{P}(x_i = 1 | y = 1) = \frac{\text{Count}(x_i = 1 \text{ and } y = 1)}{\text{Count } (y = 1)}$$

Similarly,

$$\mathbb{P}(x_i = 0 | y = 1) = \frac{\text{Count}(x_i = 0 \text{ and } y = 1)}{\text{Count } (y = 1)}$$

Spam Mail classification

$$\mathbb{P}(y = 1) = \frac{\text{Count } (y = 1)}{\text{Count } (y = 1) + \text{Count } (y = 0)}$$

Similarly,

$$\mathbb{P}(y = 0) = \frac{\text{Count } (y = 0)}{\text{Count } (y = 1) + \text{Count } (y = 0)}$$

Example

lets assume that dictionary is $[w_1, w_2, w_3]$

Index	w_1	w_2	w_3	y
1	0	0	0	1
2	0	0	0	0
3	0	0	0	1
4	1	0	0	0
5	1	0	1	1
6	1	1	1	0
7	1	1	1	1
8	1	1	0	0
9	0	1	1	0
10	0	1	1	1

Spam Classification

if y=0

- $\mathbb{P}(w_1 = 0|y = 0) = \frac{3}{5} = 0.6$
- $\mathbb{P}(w_2 = 0|y = 0) = \frac{2}{5} = 0.4$
- $\mathbb{P}(w_3 = 0|y = 0) = \frac{3}{5} = 0.6$

$\mathbb{P}(y = 0) = 0.5$
Similarly, if y=1

- $\mathbb{P}(w_1 = 1|y = 1) = \frac{2}{5} = 0.4$
- $\mathbb{P}(w_2 = 1|y = 1) = \frac{1}{5} = 0.2$
- $\mathbb{P}(w_3 = 1|y = 1) = \frac{3}{5} = 0.6$

$\mathbb{P}(y = 1) = 0.5$

Spam Classification

Given, test email 0,0,1, classify using naive bayes

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$$\mathbb{P}(y = 1 | w_1 = 0, w_2 = 0, w_3 = 1)$$

$$= \frac{\mathbb{P}(w_1 = 0 | y = 1)\mathbb{P}(w_2 = 0 | y = 1)\mathbb{P}(w_3 = 1 | y = 1)\mathbb{P}(y = 1)}{\mathbb{P}(w_1 = 0, w_2 = 0, w_3 = 1)}$$

$$= \frac{0.6 \times 0.8 \times 0.6 \times 0.5}{Z}$$

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Similarly, we can calculate
$\mathbb{P}(y = 0 | w_1 = 0, w_2 = 0, w_3 = 1) = \frac{0.6 * 0.4 * 0.6 * 0.5}{Z}$

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$$= \frac{0.6 \times 0.8 \times 0.6 \times 0.5}{Z}$$

Similarly, we can calculate $\mathbb{P}(y = 0 | w_1 = 0, w_2 = 0, w_3 = 1) = \frac{0.6 * 0.4 * 0.6 * 0.5}{Z}$ $\frac{P(y=1|w_1=0,w_2=0,w_3=1)}{P(y=0|w_1=0,w_2=0,w_3=1)} = 2 > 1$. Thus, classified as a spam example.

Naive Bayes for email/sentiment analysis

- “This product is pathetic”. We would assume the sentiment of such a sentence to be negative. Why? Presenece of “pathetic”
- Naive bayes would store the probabilities of words belonging to positive or negative sentiment.
- Good is positive, Bad is negative
- What about: This product is not bad. Naive Bayes is very naive and does not account for sequential aspect of data.