Multi-Layer Perceptrons

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IIT Gandhinagar

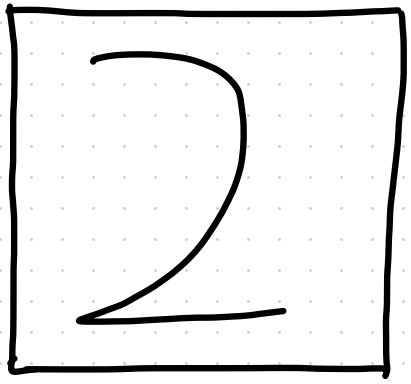
August 30, 2025

RECENT SUCCESSES OF NN

* State-of-the-art (SOTA) in most fields

PARADIGM CHANGE

PARADIGM CHANGE



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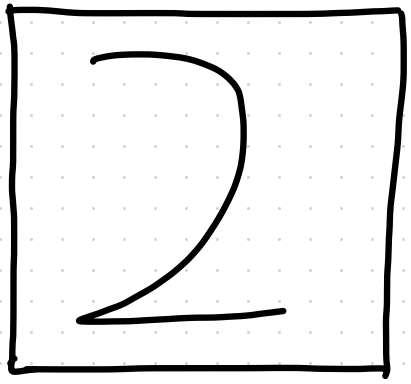
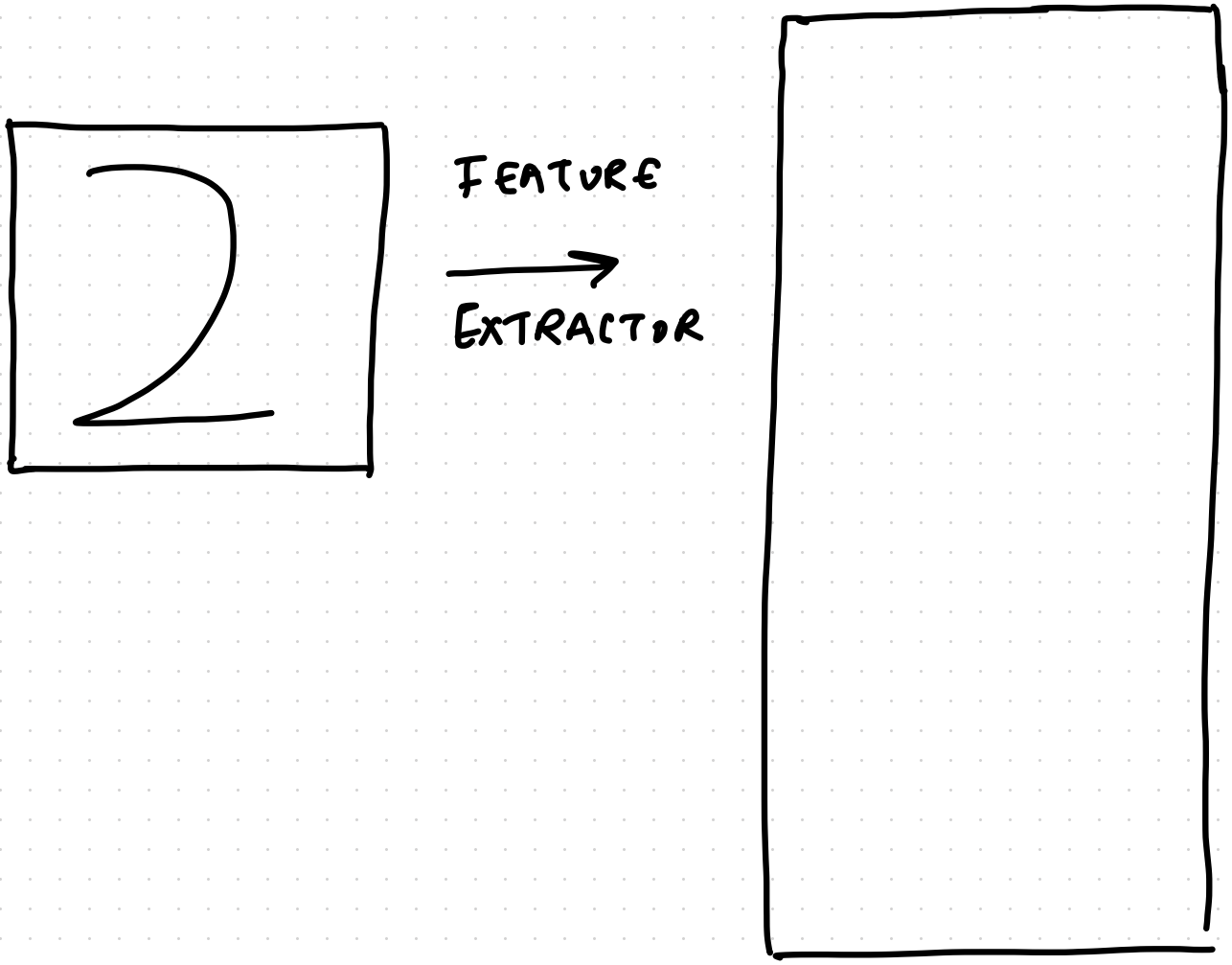


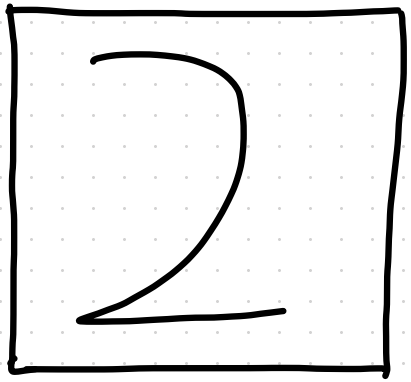
FEATURE

EXTRACTOR

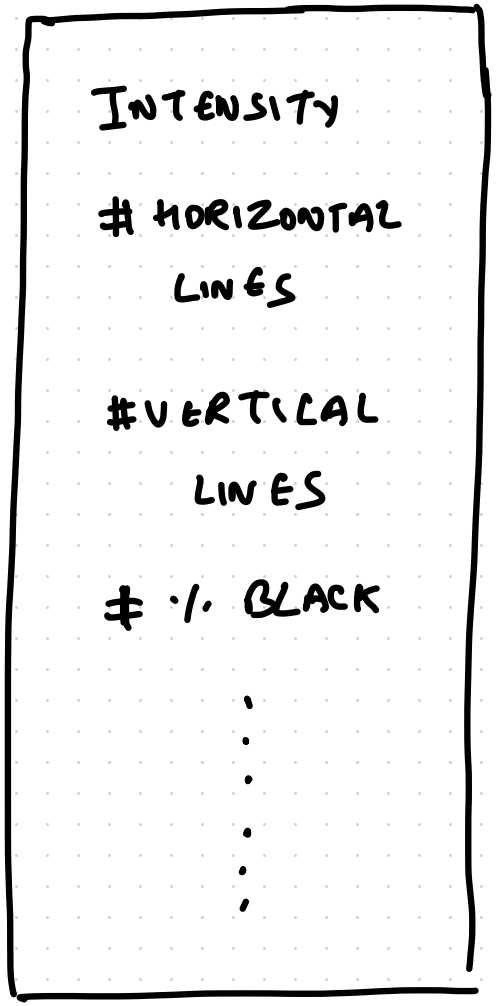
PARADIGM CHANGE



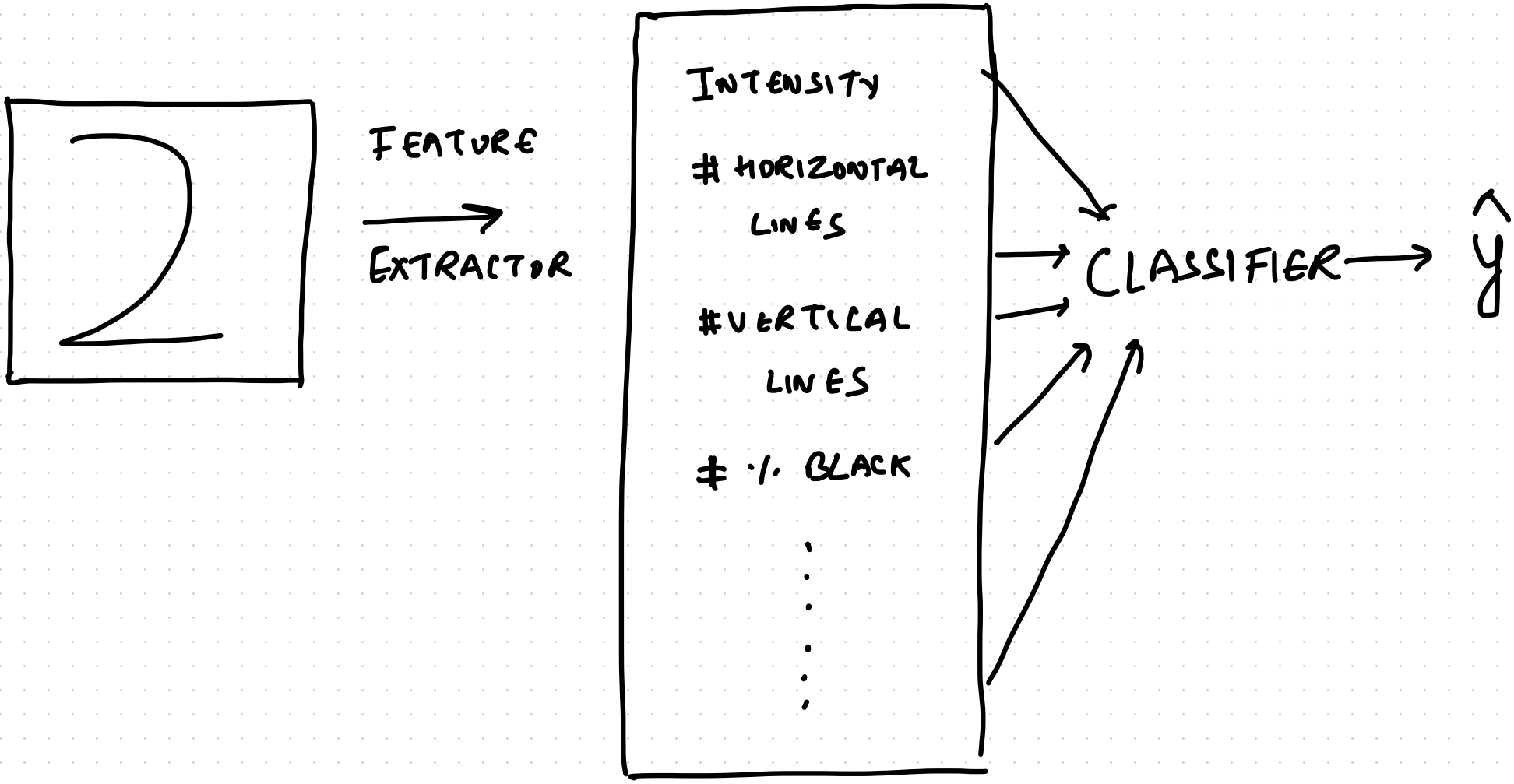
PARADIGM CHANGE



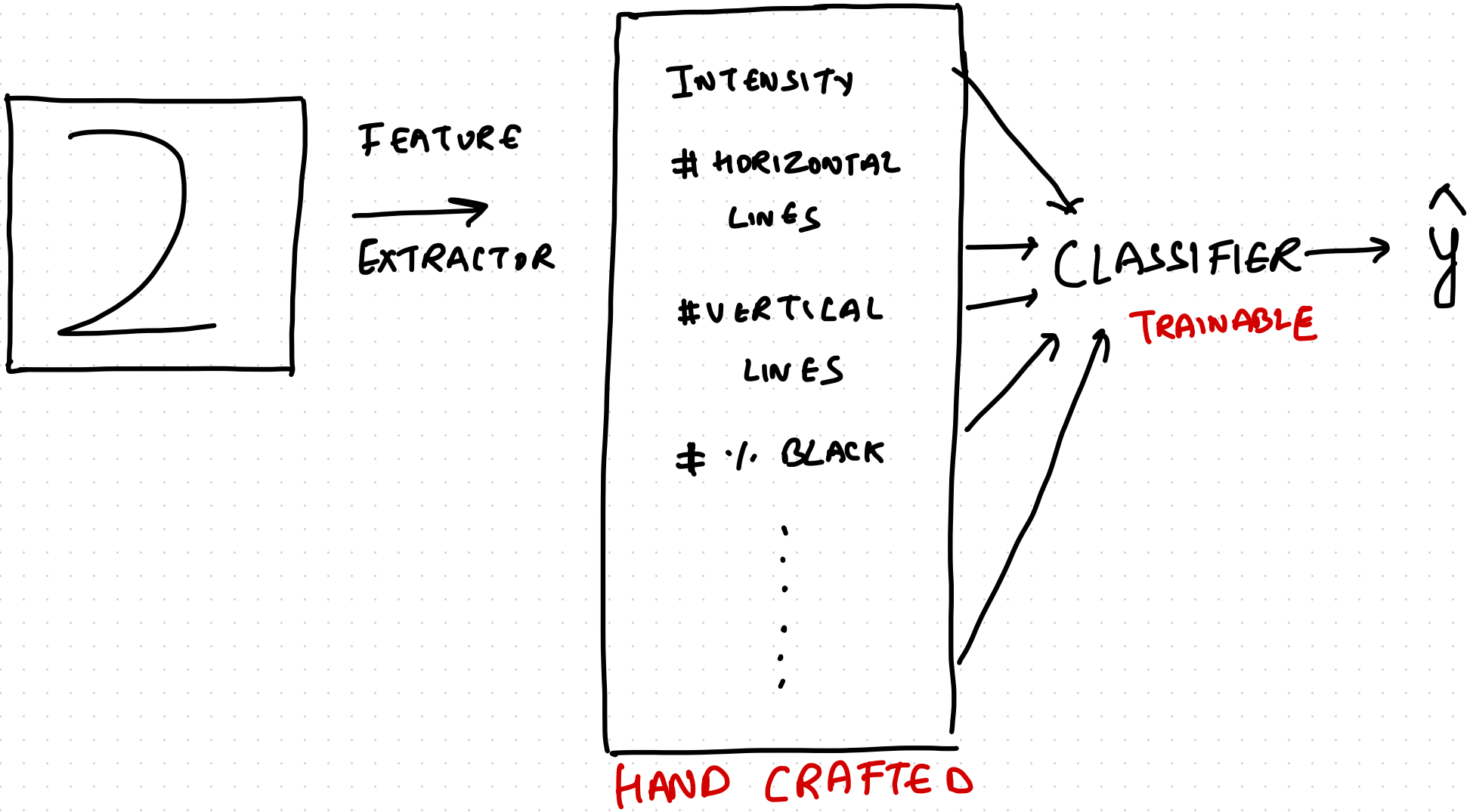
FEATURE EXTRACTOR



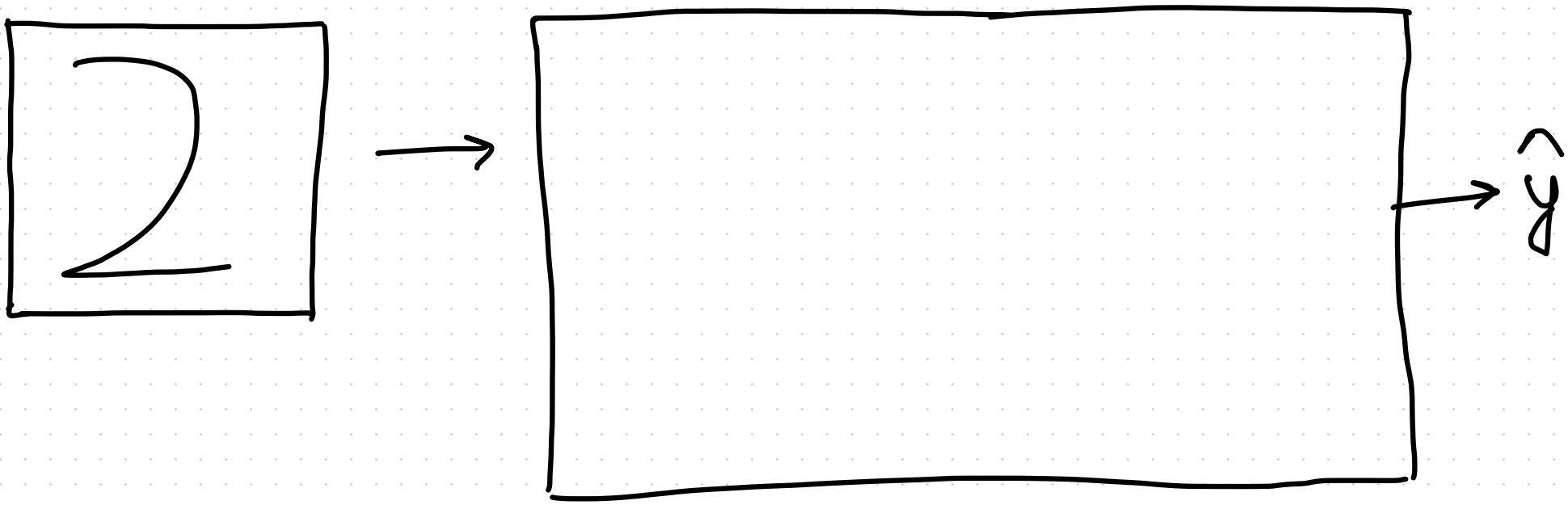
PARADIGM CHANGE



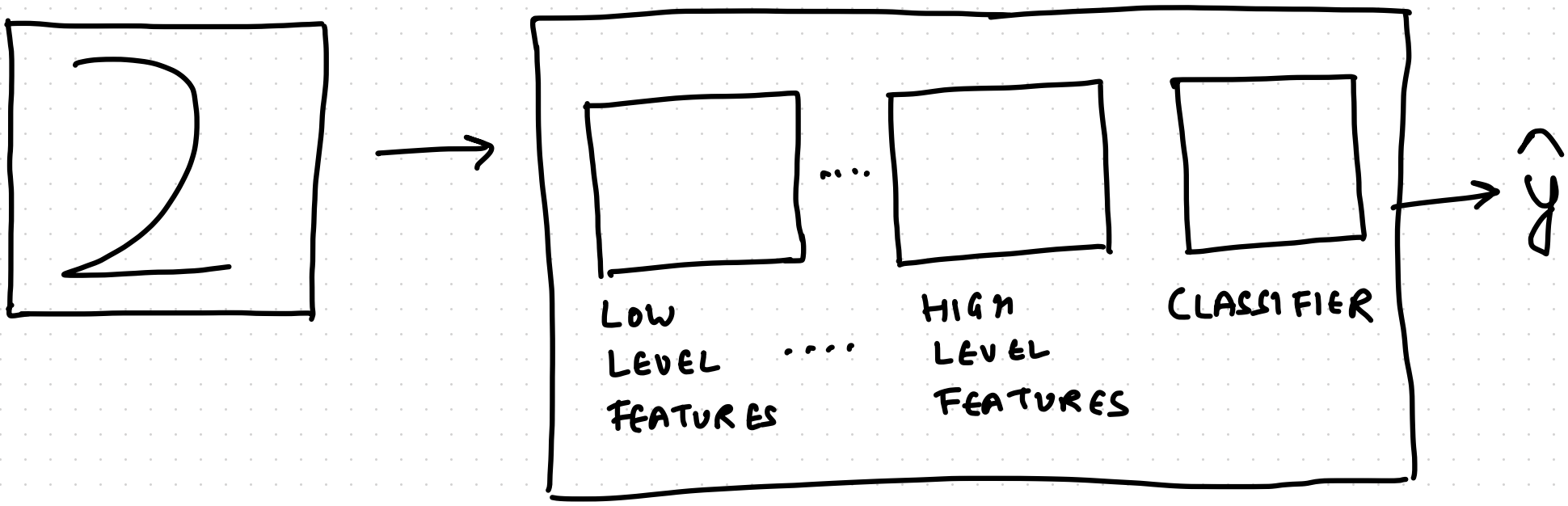
PARADIGM CHANGE



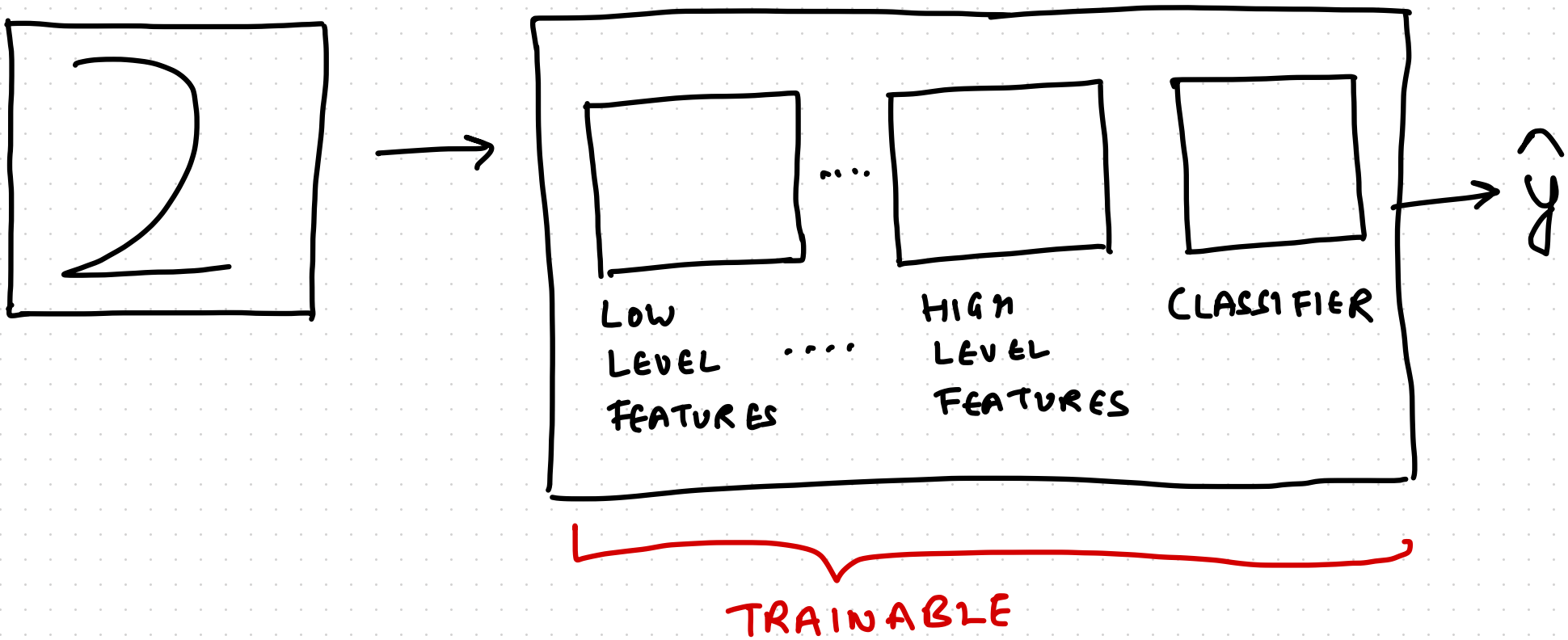
PARADIGM CHANGE (NNs)



PARADIGM CHANGE (NNS)

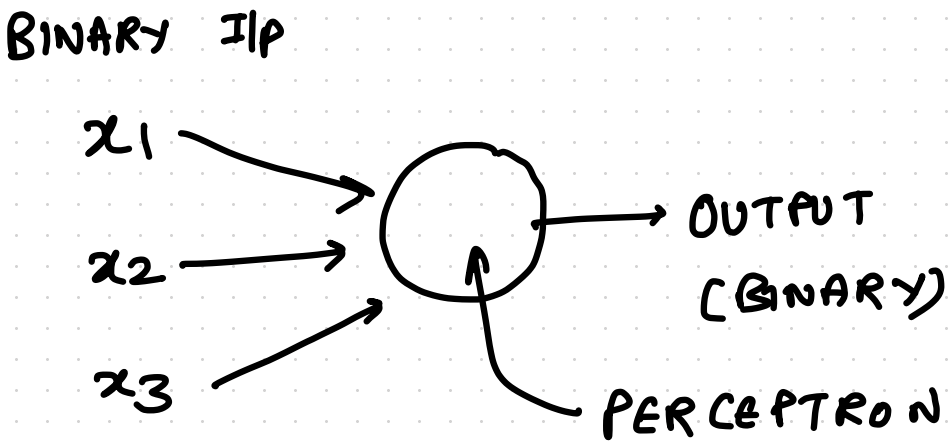


PARADIGM CHANGE (NNS)

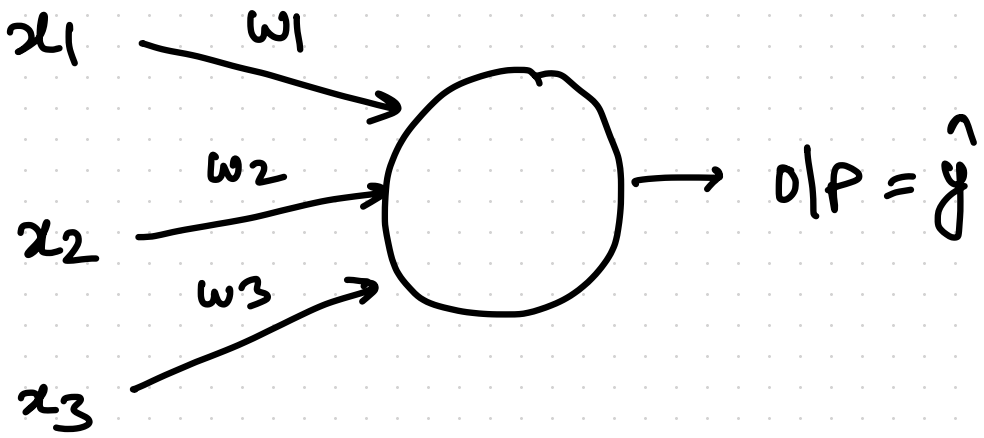


PERCEPTRON

— ARTIFICIAL NEURON DEVELOPED BY ROSENBLATT IN 1960s INSPIRED BY MC CULLOCH & PITTS

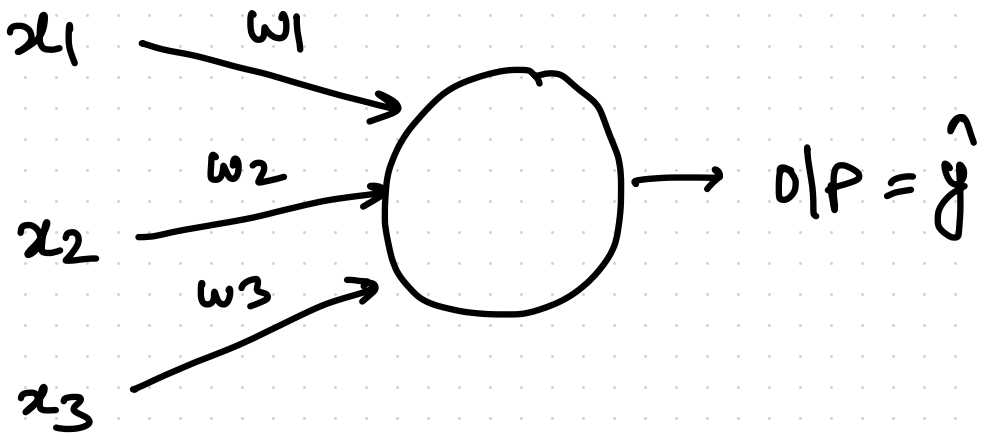


PERCEPTRON



$$o/p = \hat{y} = \begin{cases} 0 \; ; & \sum w_i x_i \leq \text{THRESHOLD} \\ 1 \; ; & \sum w_i x_i > \text{THRESHOLD} \end{cases}$$

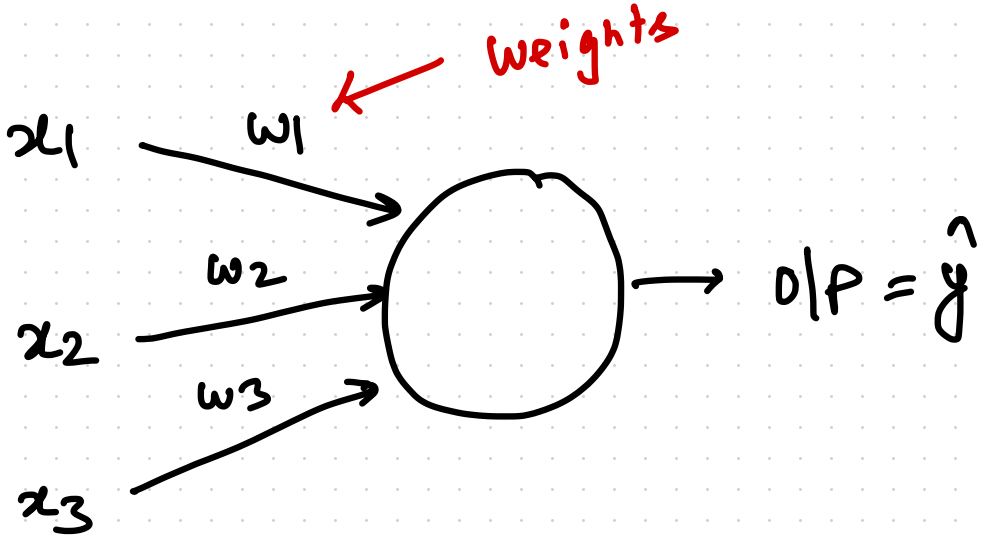
PERCEPTRON



$$o/p = \hat{y} = \begin{cases} 0 \; ; & \sum w_i x_i \leq \text{THRESHOLD} \\ 1 \; ; & \sum w_i x_i > \text{THRESHOLD} \end{cases}$$

NEURONS "FIRE" ABOVE THRESHOLD

PERCEPTRON



$$O/P = \hat{y} = \begin{cases} 0 \; ; & \sum w_i x_i \leq \text{THRESHOLD} \\ 1 \; ; & \sum w_i x_i > \text{THRESHOLD} \end{cases}$$

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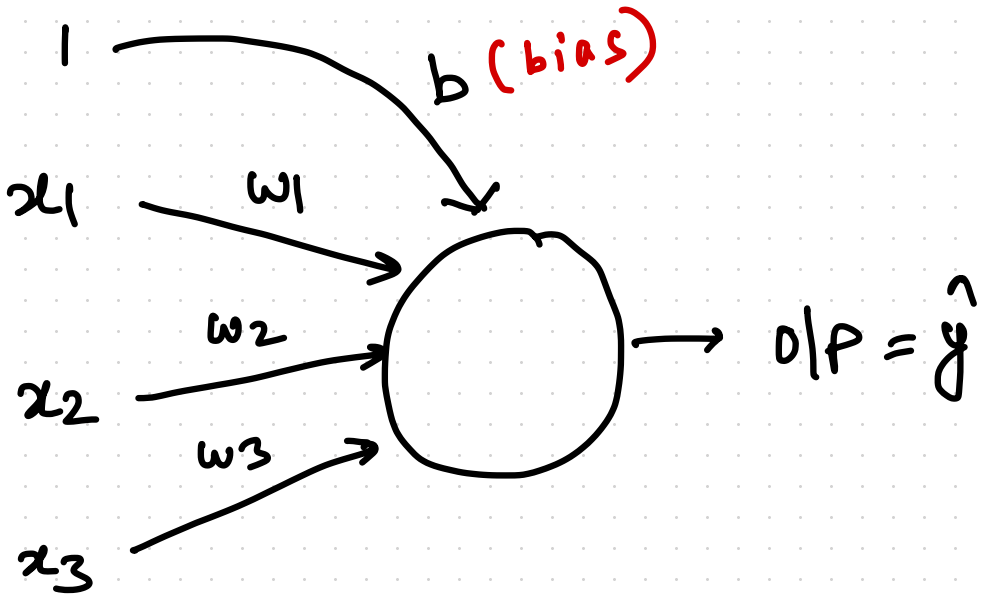
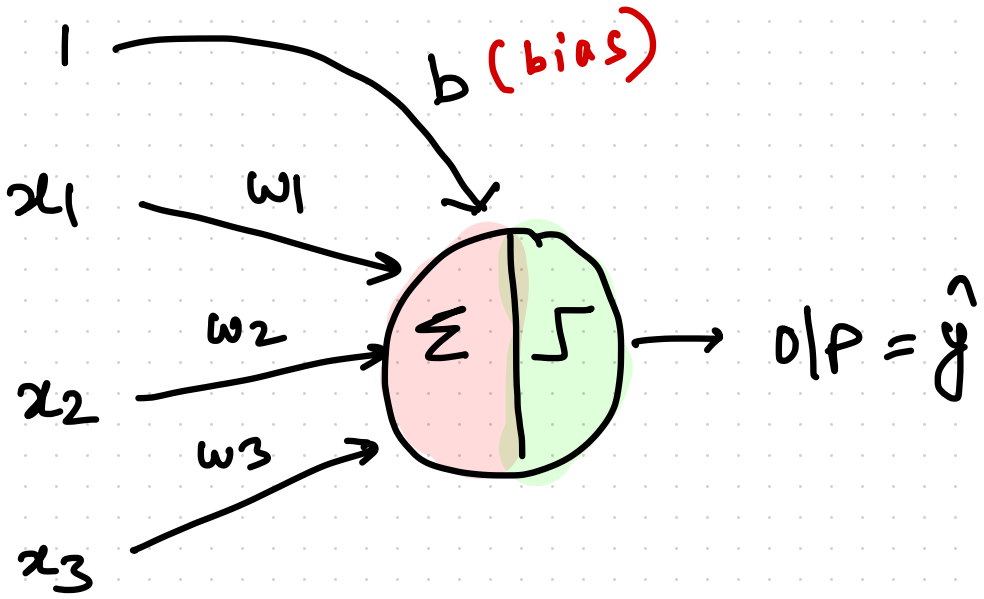


$$o/p = \hat{y} = \begin{cases} 0 \; ; & \sum w_i x_i + b \leq 0 \\ 1 \; ; & \sum w_i x_i + b > 0 \end{cases}$$

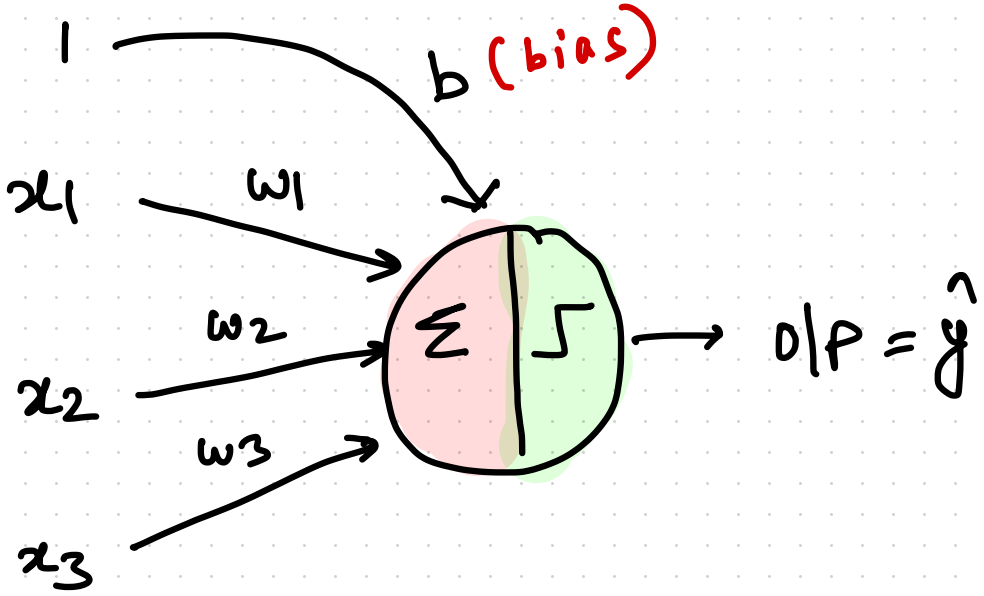
PERCEPTRON



NEURON HAS 2 COMPONENTS

1. SUMMATION
2. ACTIVATION : STEP FUNCTION

PERCEPTRON



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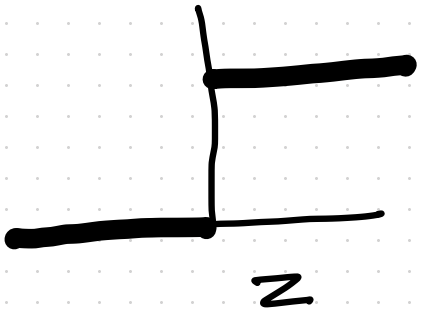
1. SUMMATION
2. ACTIVATION : STEP FUNCTION

(SIGMOID/STEP) Activation



LEARNING BINARY GATES

0) FOR 2 I/Ps x_1 & x_2 learn w's and b for BINARY AND

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

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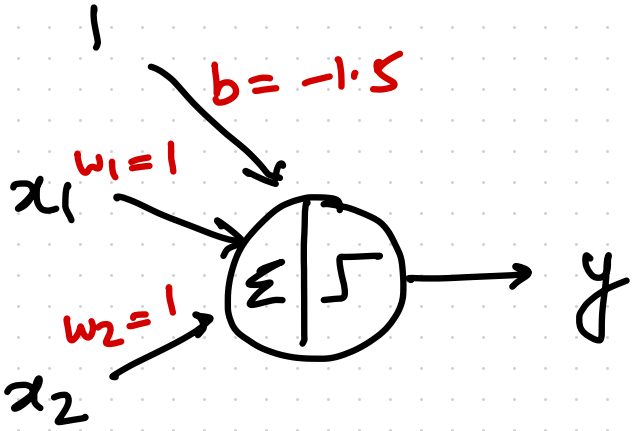

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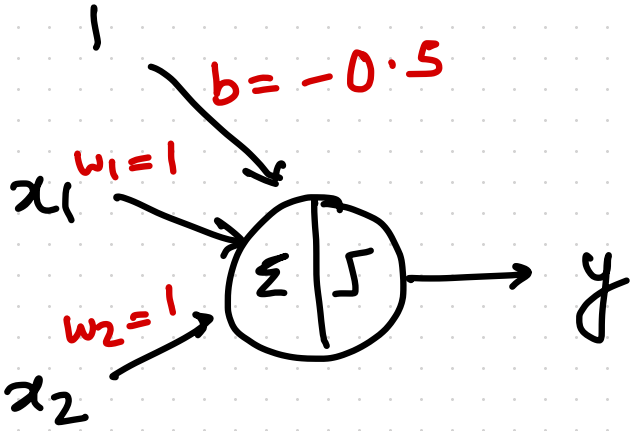
1	1	1



LEARNING BINARY GATES

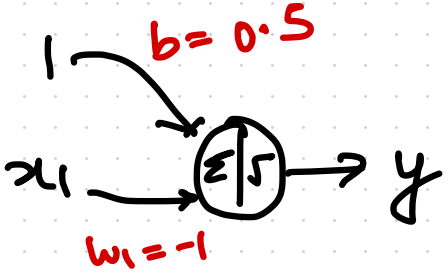
0) FOR 1 I/P s x_1 learn w_i's and b for UNARY NOT

x_1	y
0	1
1	0

LEARNING BINARY GATES

0) FOR 1 I/Ps x_1 learn w_i's and b for UNARY NOT

x_1	y
0	1
1	0



LEARNING BINARY GATES

0) FOR 2 NAND I/Ps x_1 & x_2 learn w's and b for

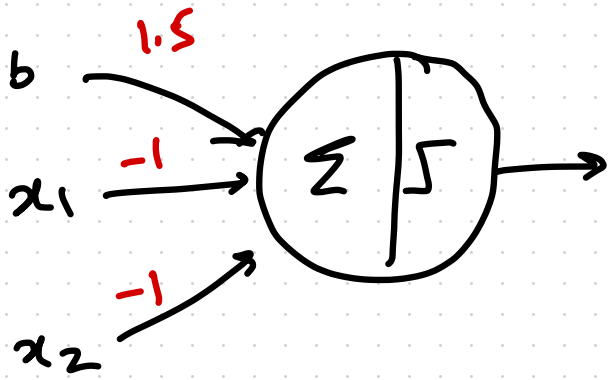
x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

LEARNING BINARY GATES

0) FOR 2 I/Ps x_1 & x_2 learn w_i's and b for NAND

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

APPROACH #1

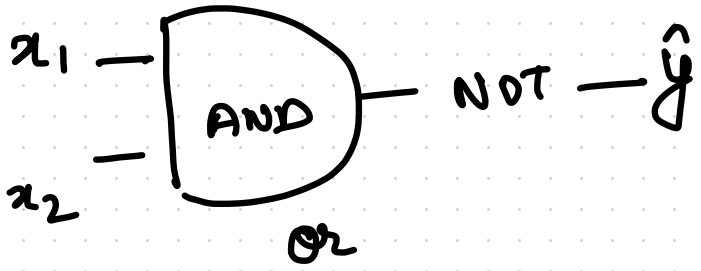


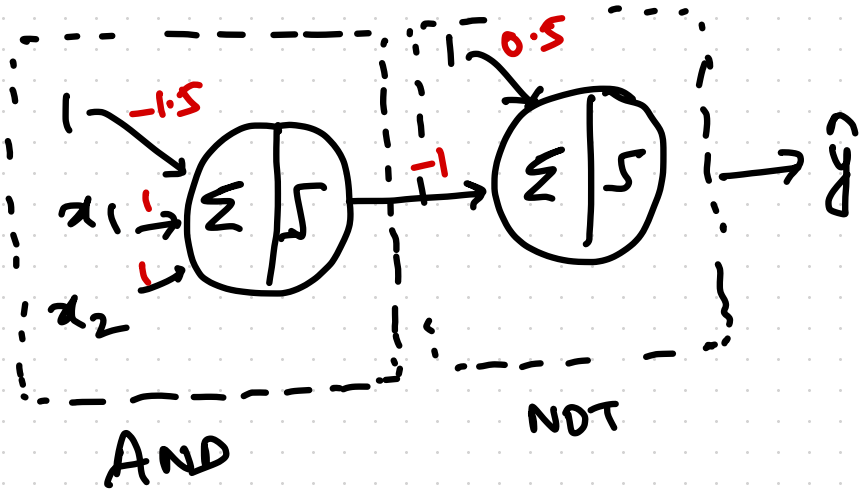
LEARNING BINARY GATES

0) FOR 2 I/Ps x_1 & x_2 learn w's and b for NAND

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

APPROACH #2





PERCEPTRON LEARNING ALGORITHM

Ilp: X → NxD ; y → Nx1 ; lr → learning rate ; it → #iterations

PERCEPTRON LEARNING ALGORITHM

I/p: X → NXD ; y → NX1 ; lr → learning rate ; it → #iterations

S1 AUGMENT x to x' → s.t. $x'[1:5] = x$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

PERCEPTRON LEARNING ALGORITHM

Ilp: X → NXD ; y → NX1 ; lr → learning rate ; it → #iterations

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S2 INIT W^{D+1}

PERCEPTRON LEARNING ALGORITHM

Ilp: X → NXD ; y → NX1 ; lr → learning rate ; it → #iterations

S1 AUGMENT x to x' → s.t. $x'[1:] = x$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

S2 INIT W^{D+1}

S3 FOR I IN IT:

PERCEPTRON LEARNING ALGORITHM

Ilp: X → $N \times D$; y → $N \times 1$; lr → learning rate ; it → #iterations

S1 AUGMENT x to x' → s.t. $x'[1:] = x$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

S2 INIT $W \in R^{D+1}$

S3 FOR I IN IT:

S3.1 FOR d in D:

S3.1.1 $\hat{y} =$ STEP $(x' \cdot W)$

PERCEPTRON LEARNING ALGORITHM

Ilp: X → NXD ; y → NX1 ; lr → learning rate ; it → #iterations

S1 AUGMENT x to x' → s.t. $x'[1:5] = x$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

S2 INIT W^{D+1}

S3 FOR I IN IT:

S3.1 FOR d in D:

S3.1.1 $\hat{y} = \text{STEP}(x' \cdot W)$

S3.1.2 $ERR = y - \hat{y}$

PERCEPTRON LEARNING ALGORITHM

Ilp: X → NXD ; y → NX1 ; lr → learning rate ; it → #iterations

S1 AUGMENT x to x' → s.t. $x'[1:5] = x$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

S2 INIT $W \in R^{D+1}$

S3 FOR I IN IT:

S3.1 FOR n in N:

S3.1.1 $\hat{y} = \text{STEP}(x' \cdot W)$

S3.1.2 $ERR = y - \hat{y}$

S3.1.3 $W \leftarrow W + lr * ERR_n * x'[n]$

PERCEPTRON LEARNING ALGORITHM

Ilp: X → NXD ; y → NX1 ; lr → learning rate ; it → #iterations

S1 AUGMENT x to x' → s.t. $x'[1:5] = x$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

S2 INIT $W \in R^{D+1}$

S3 FOR I IN IT:

S3.1 FOR n in N:

S3.1.1 $\hat{y} = STEP(x' \cdot W)$

S3.1.2 $ERR = y - \hat{y}$

S3.1.3 $W \leftarrow W + lr * ERR_n * x'[n]$

ERROR IN n^{th} sample

n^{th} sample

PERCEPTRON LEARNING ALGORITHM

Ilp: X → $N \times D$; y → $N \times 1$; lr → learning rate ; it → #iterations

S1 AUGMENT x to X' → s.t. $x'[1:] = X$; $x'[1] = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$

S2 INIT $W \in R^{D+1}$

S3 FOR I IN IT:

S3.1 FOR n in N : ← Analogous to S.G.D

S3.1.1 $\hat{y} = STEP(x' \cdot W)$

S3.1.2 $ERR = y - \hat{y}$

S3.1.3 $W \leftarrow W + lr * ERR_n * x'[n]$

Analogous to Gradient

ERROR IN n^{th} sample

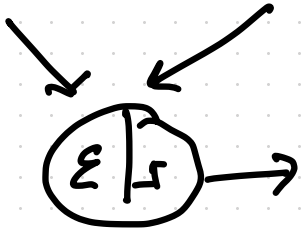
n^{th} sample

PERCEPTRON LEARNING ALGORITHM

S31.3 $w \leftarrow w + lr * ERR_n * x'[n]$

IMAGINE

$$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \; ; \; x'[n] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \; ; \; y = 0$$

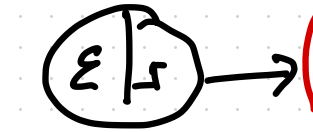


PERCEPTRON LEARNING ALGORITHM

S31.3 $W \leftarrow W + lr * ERR_n * x'[n]$

IMAGINE

$W = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$; $x'[n] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $y = 0$



$\hat{y} = 1$

$ERR_n = 0 - 1 = -1$

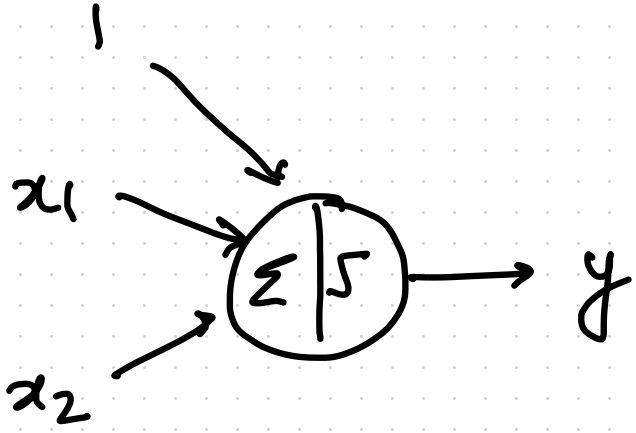
$W \leftarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 0.01 * -1 * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\leftarrow \begin{bmatrix} .99 \\ 1 \\ 1 \end{bmatrix}$

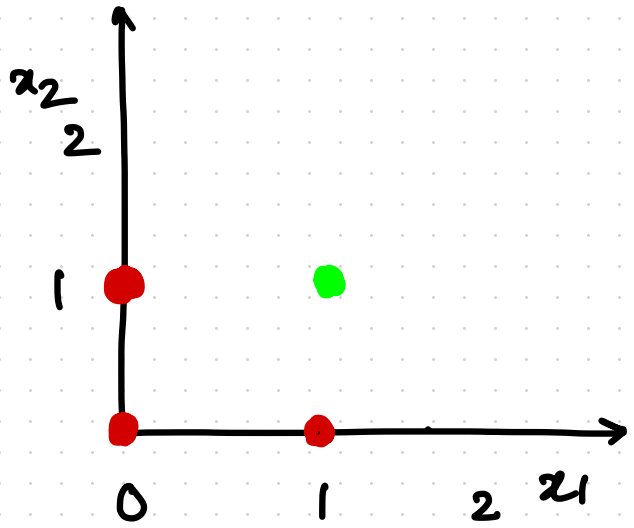
LEARNING BINARY GATES

0) FOR 2 I/Ps x_1 & x_2 learn w's and b for BINARY XOR

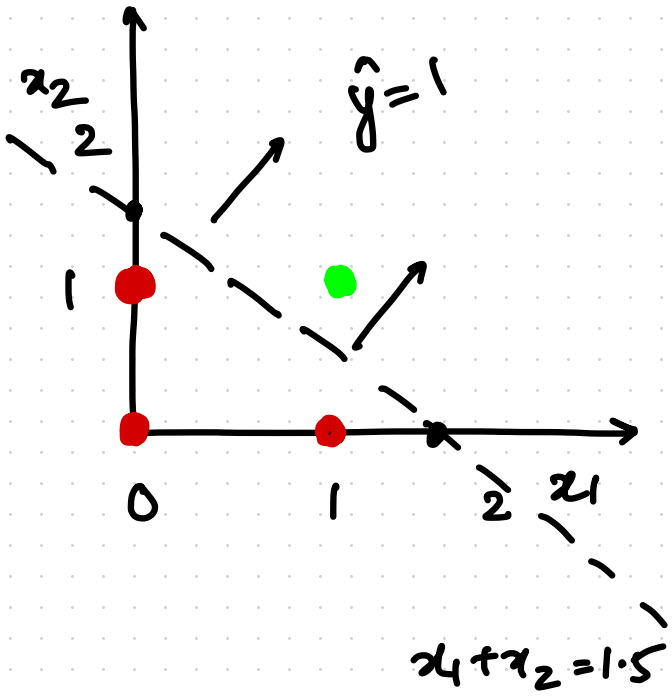
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



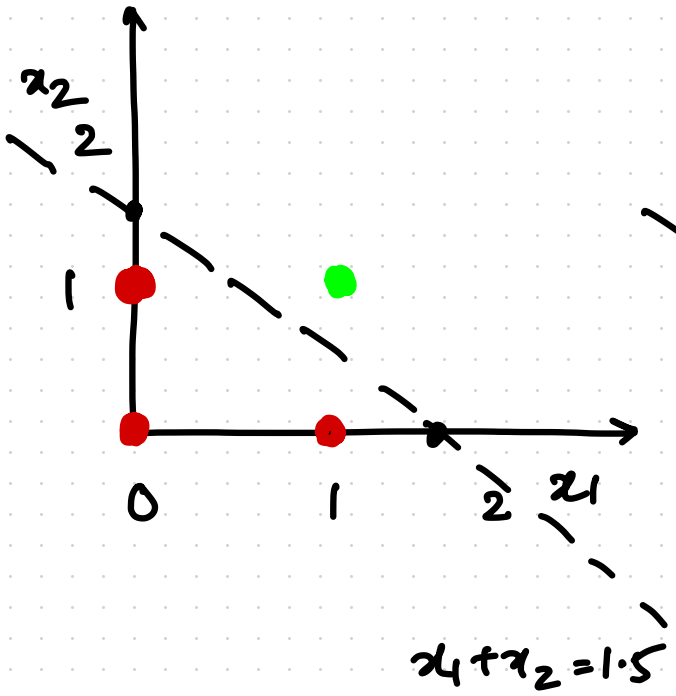
AND



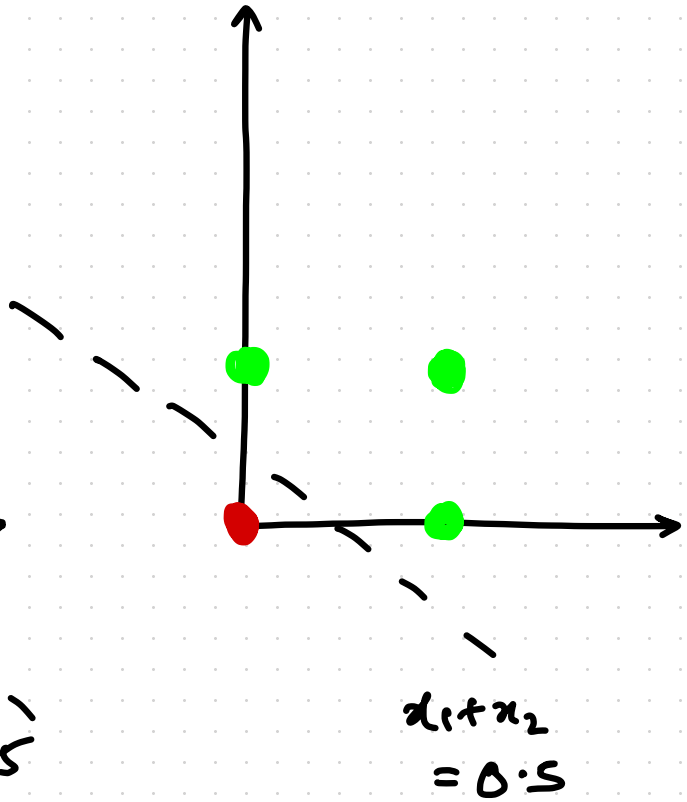
AND



AND



OR



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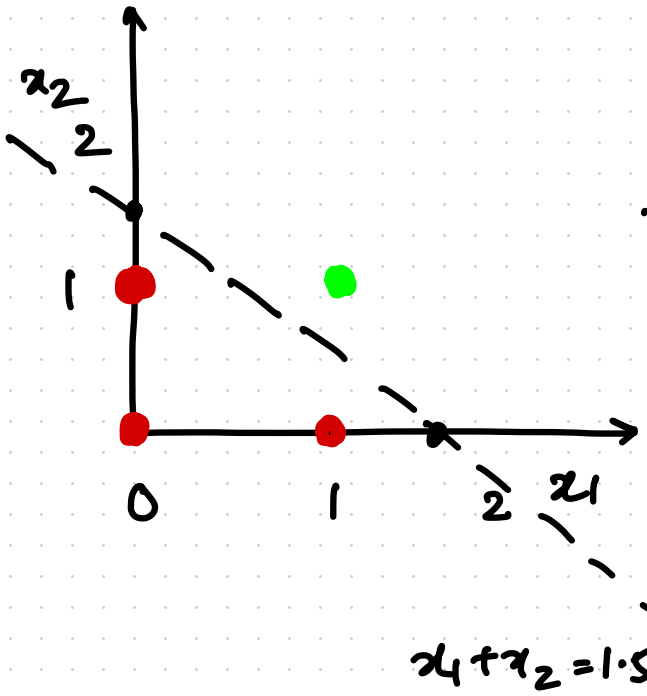

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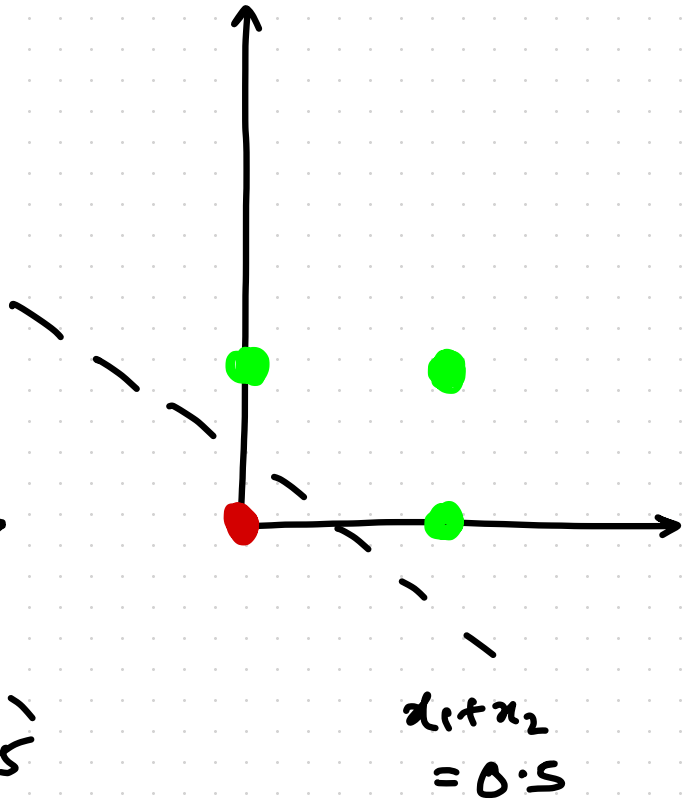
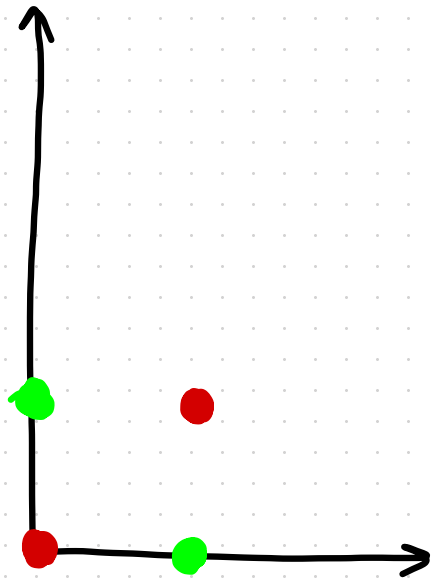


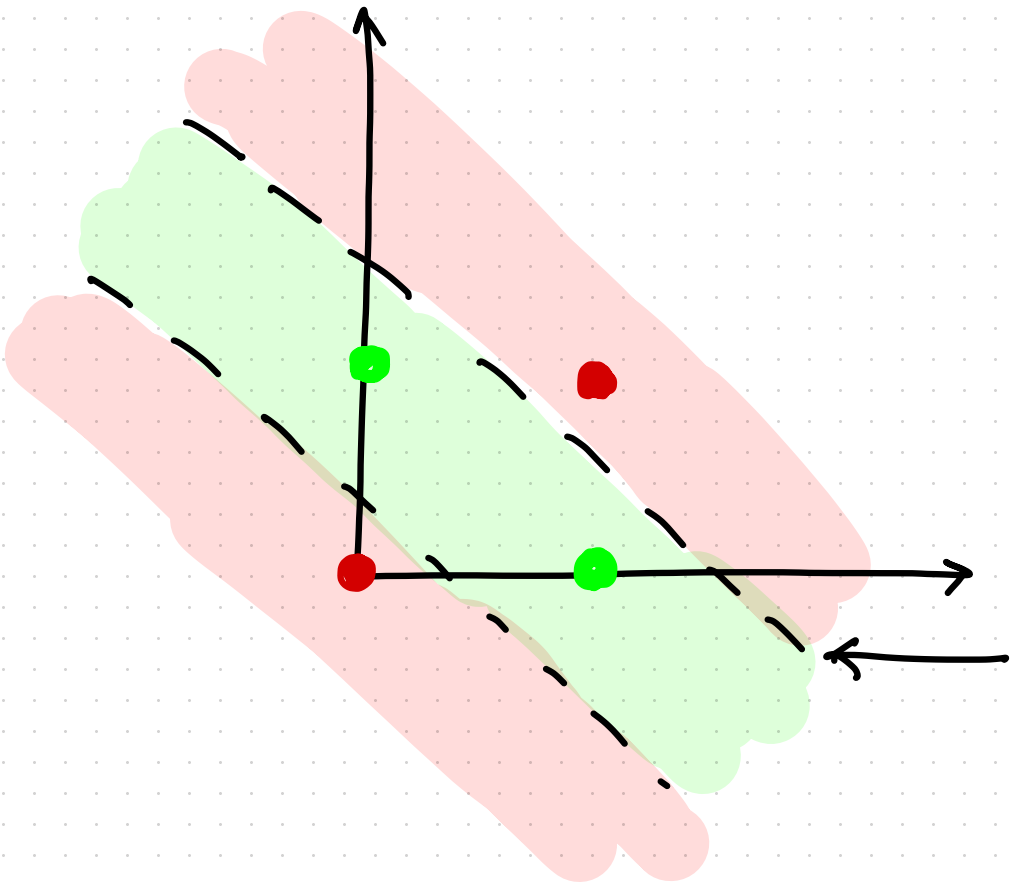
LINEARLY SEPARABLE

XOR



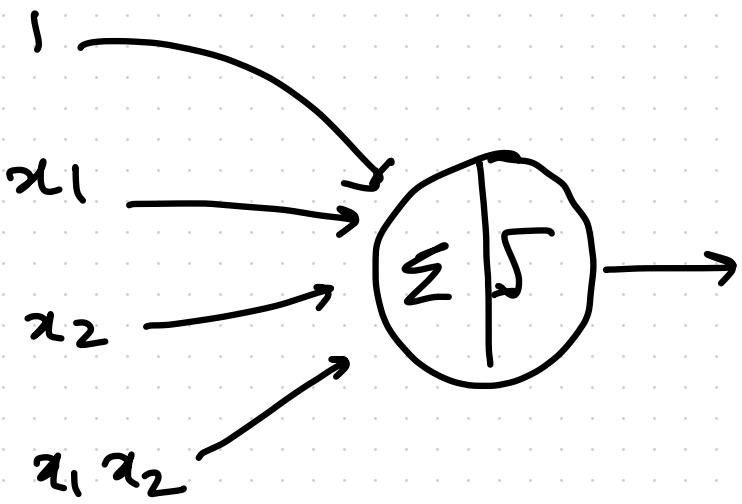
NOT SEPARABLE LINEARLY

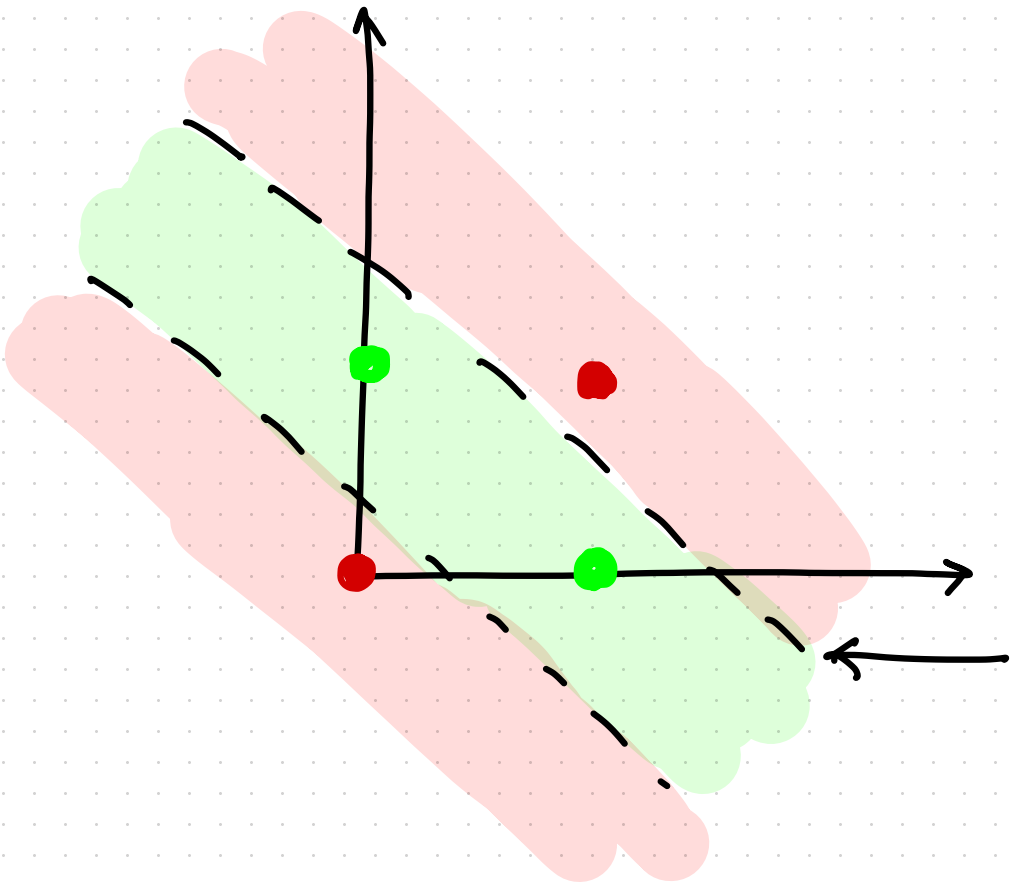
XOR CLASSIFICATION



Decision surface desired

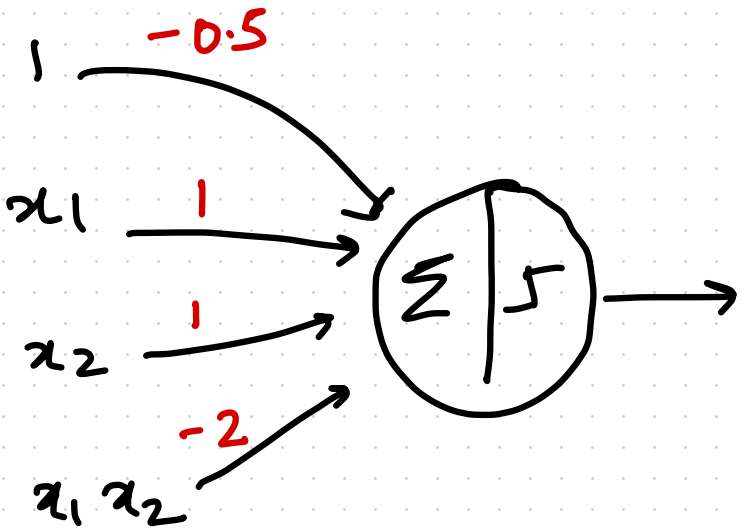
XOR CLASSIFICATION

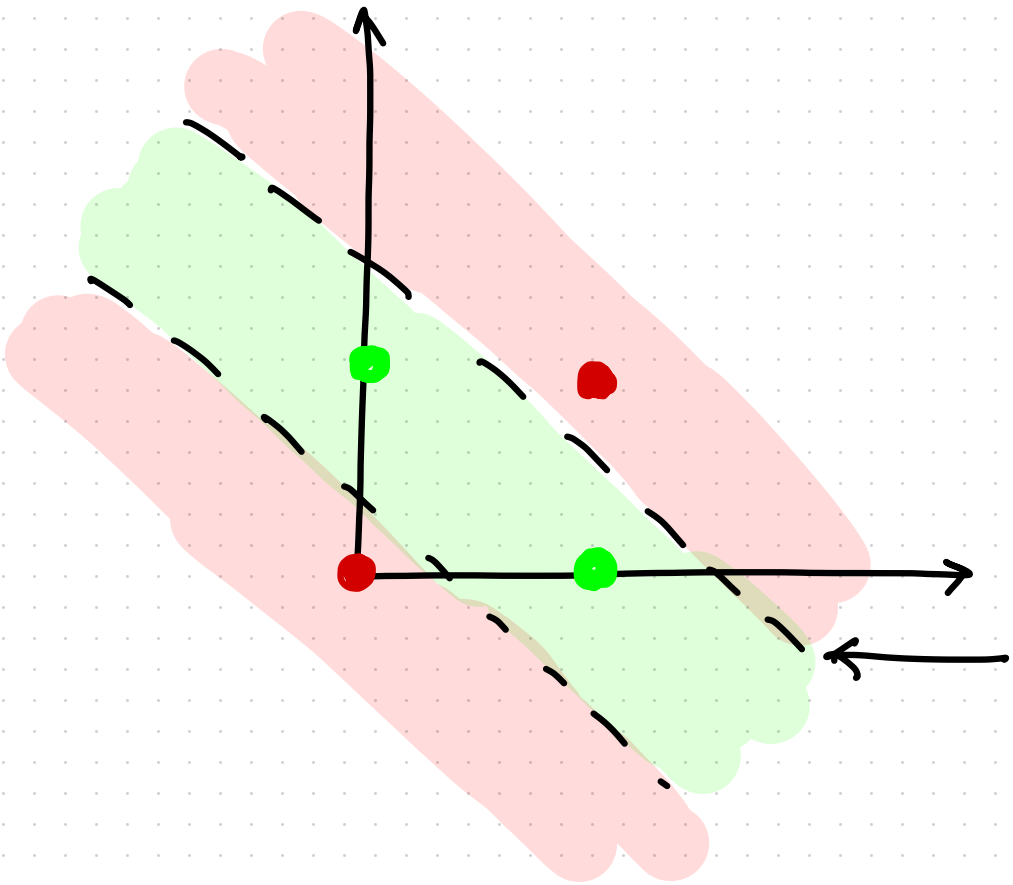




Decision surface desired

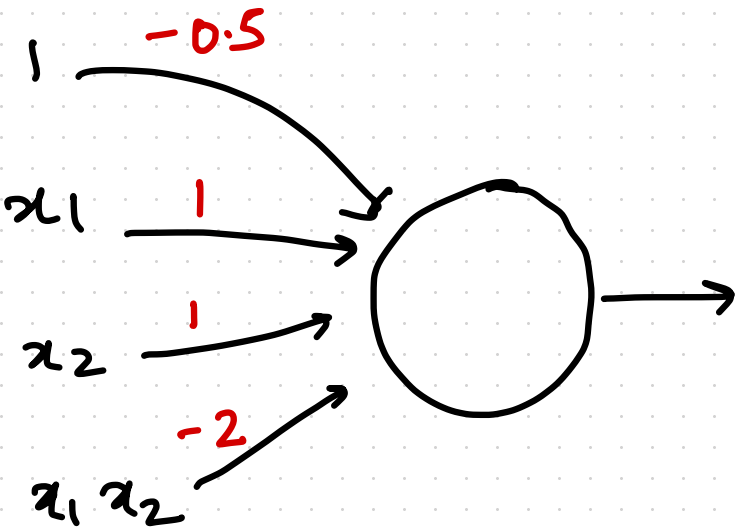
XOR CLASSIFICATION





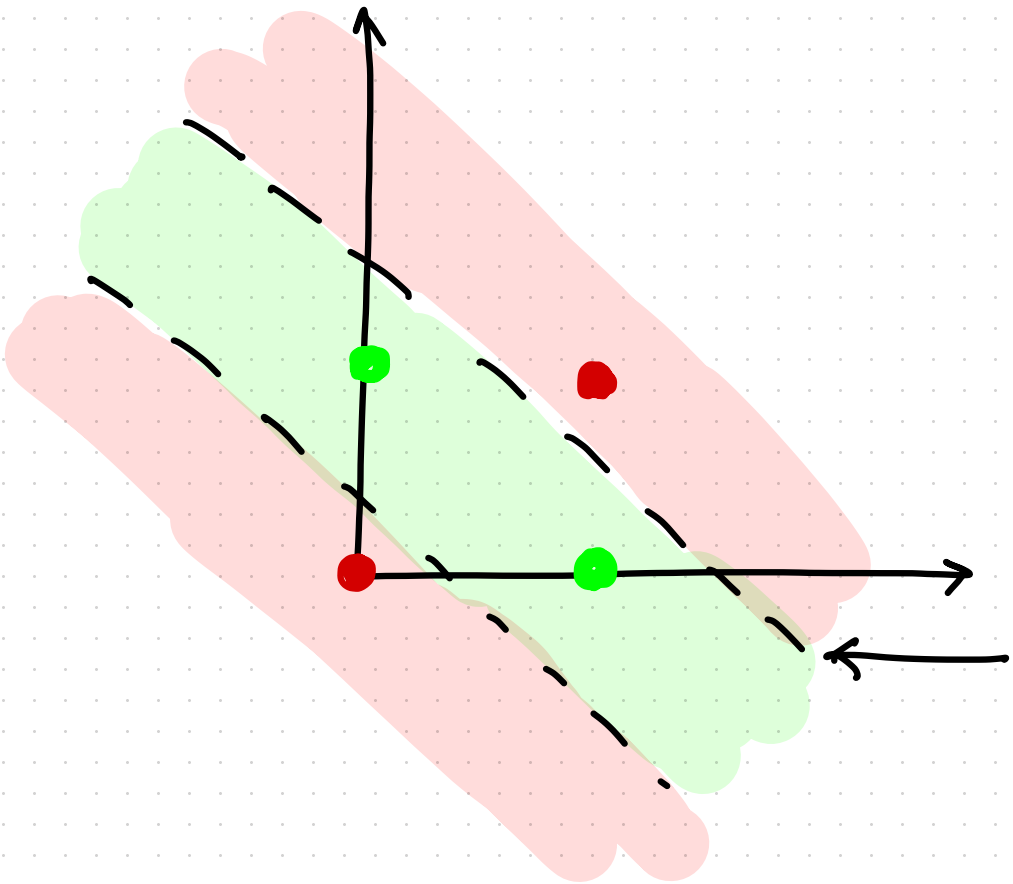
Decision surface desired

XOR CLASSIFICATION



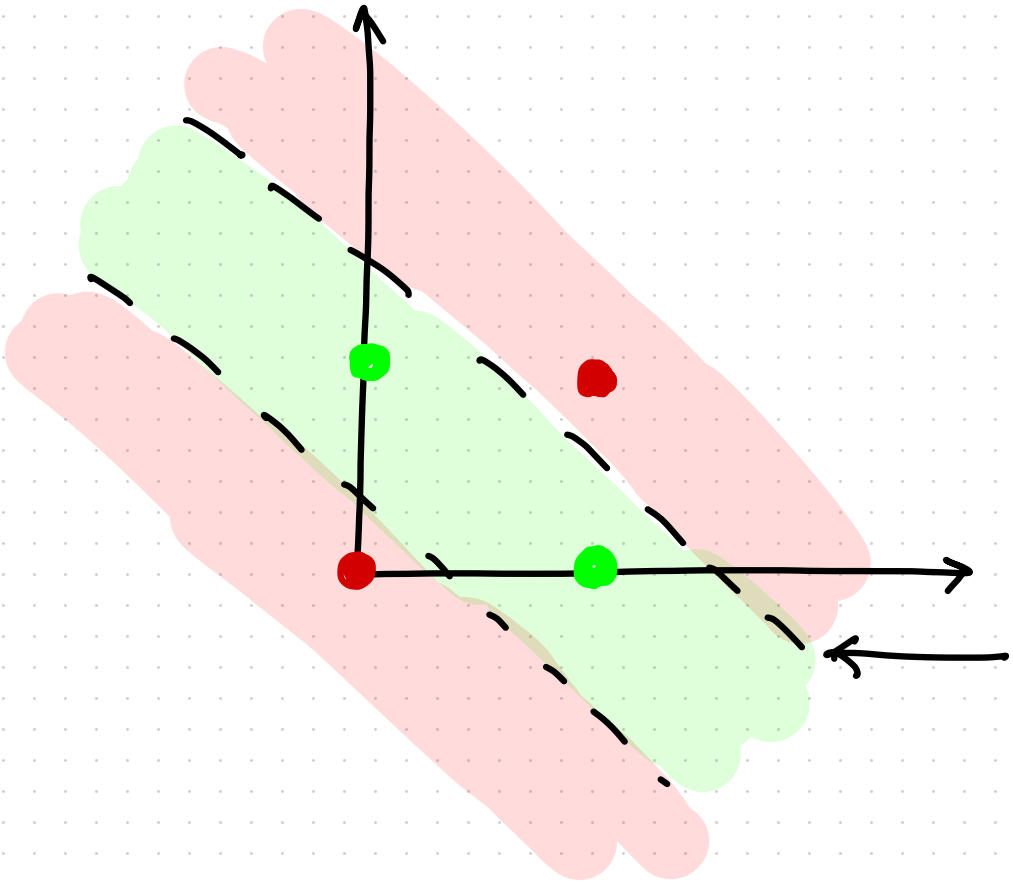
FOR $x_1 = 0 ; x_2 = 0$; we get

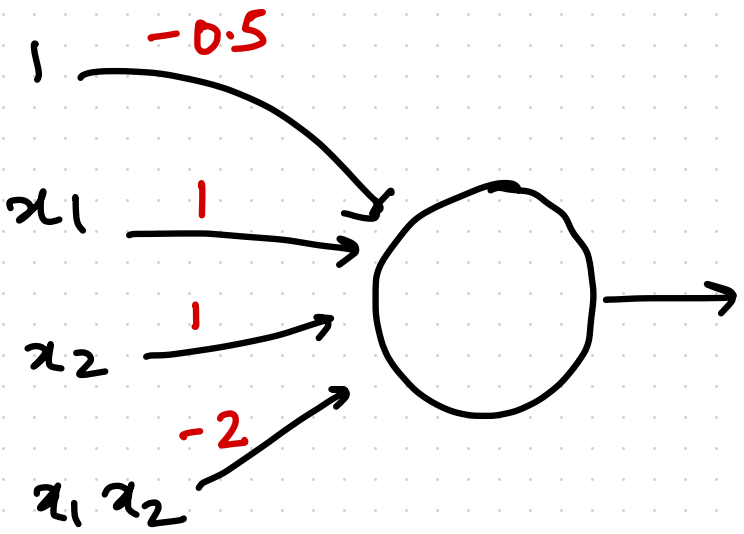
$\hat{y} = \{-0.5 \leq 0\}$ = RED CLASS



Decision surface desired

XOR CLASSIFICATION



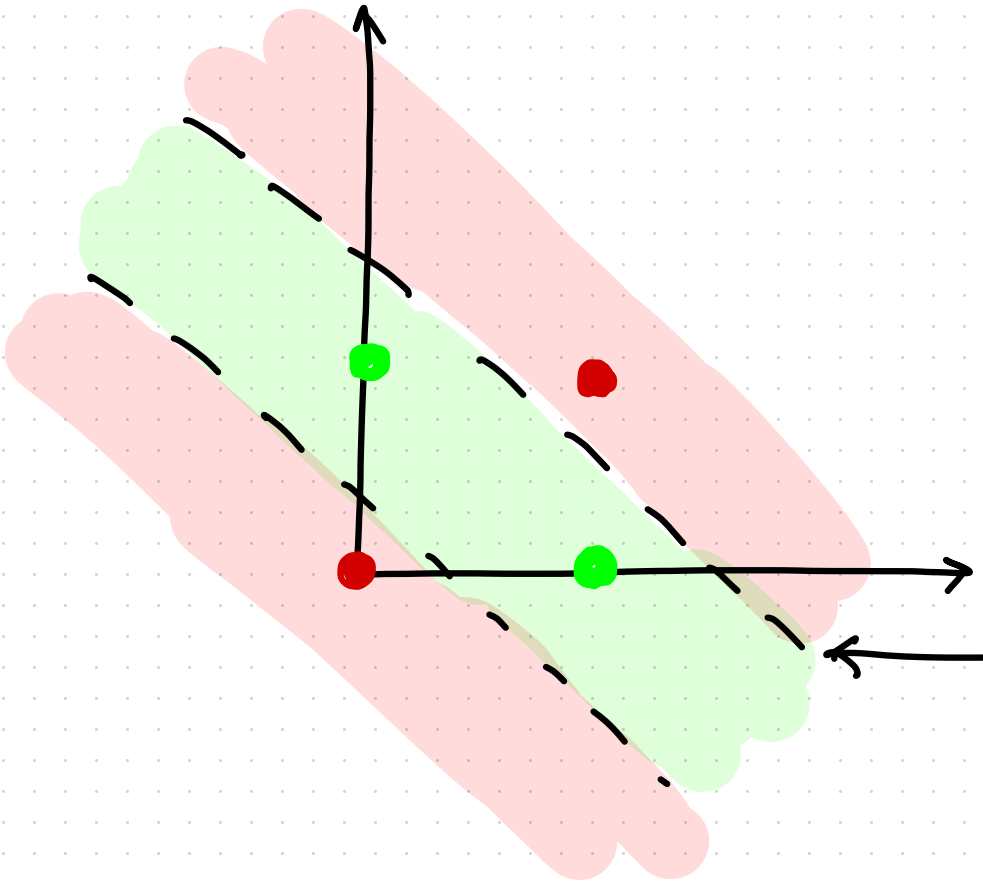


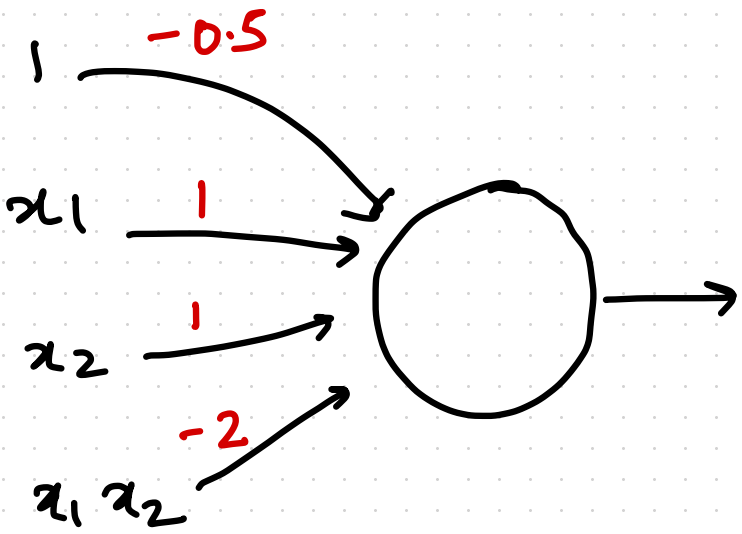
FOR $x_1 = 0 ; x_2 = 1$; we get

$\hat{y} = \{-0.5 + 1 \leq 0\} =$ GREEN CLASS

Decision surface desired

XOR CLASSIFICATION



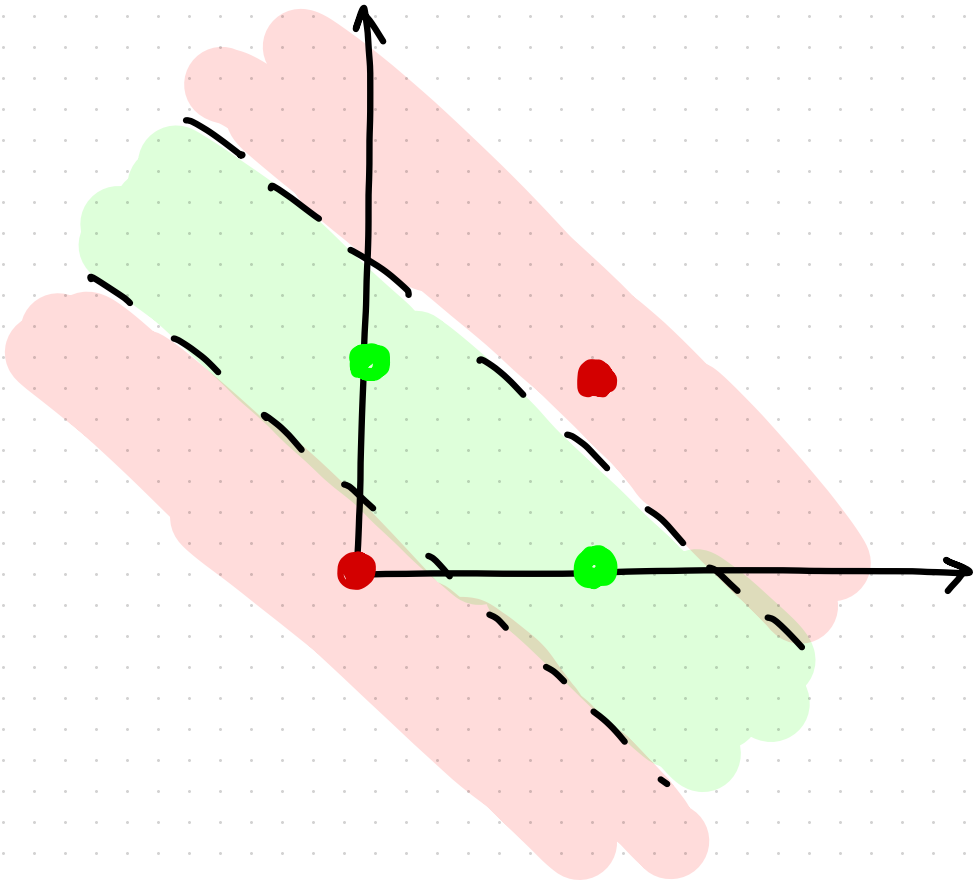


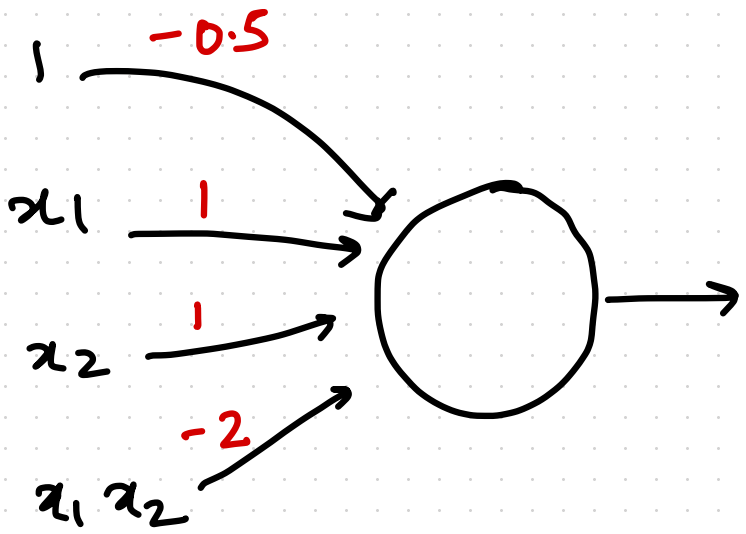
FOR $x_1 = 1$; $x_2 = 0$; we get

$\hat{y} = \{ 0.5 \leq 0 \} =$ GREEN CLASS

Decision surface desired

XOR CLASSIFICATION



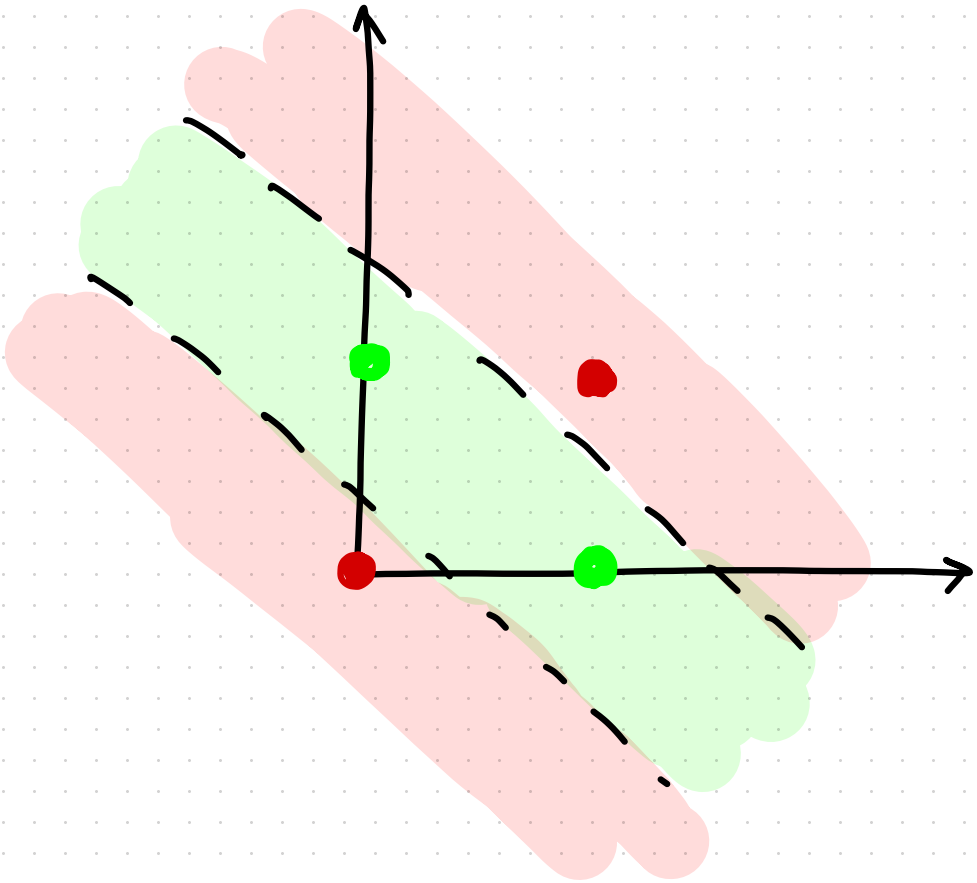


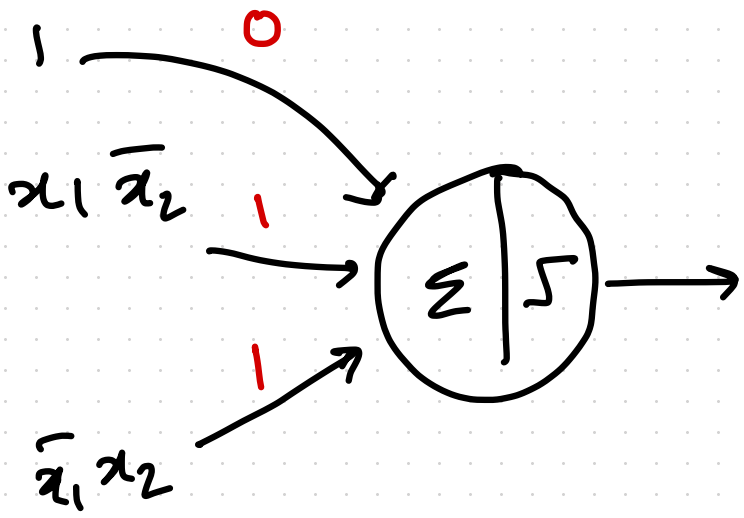
FOR $x_1 = 1$; $x_2 = 1$; we get

$\hat{y} = \{-0.5 \leq 0\}$ = RED CLASS

CAN ADD NON-LINEARITY BY HAND-CRAFTING FEATURES !

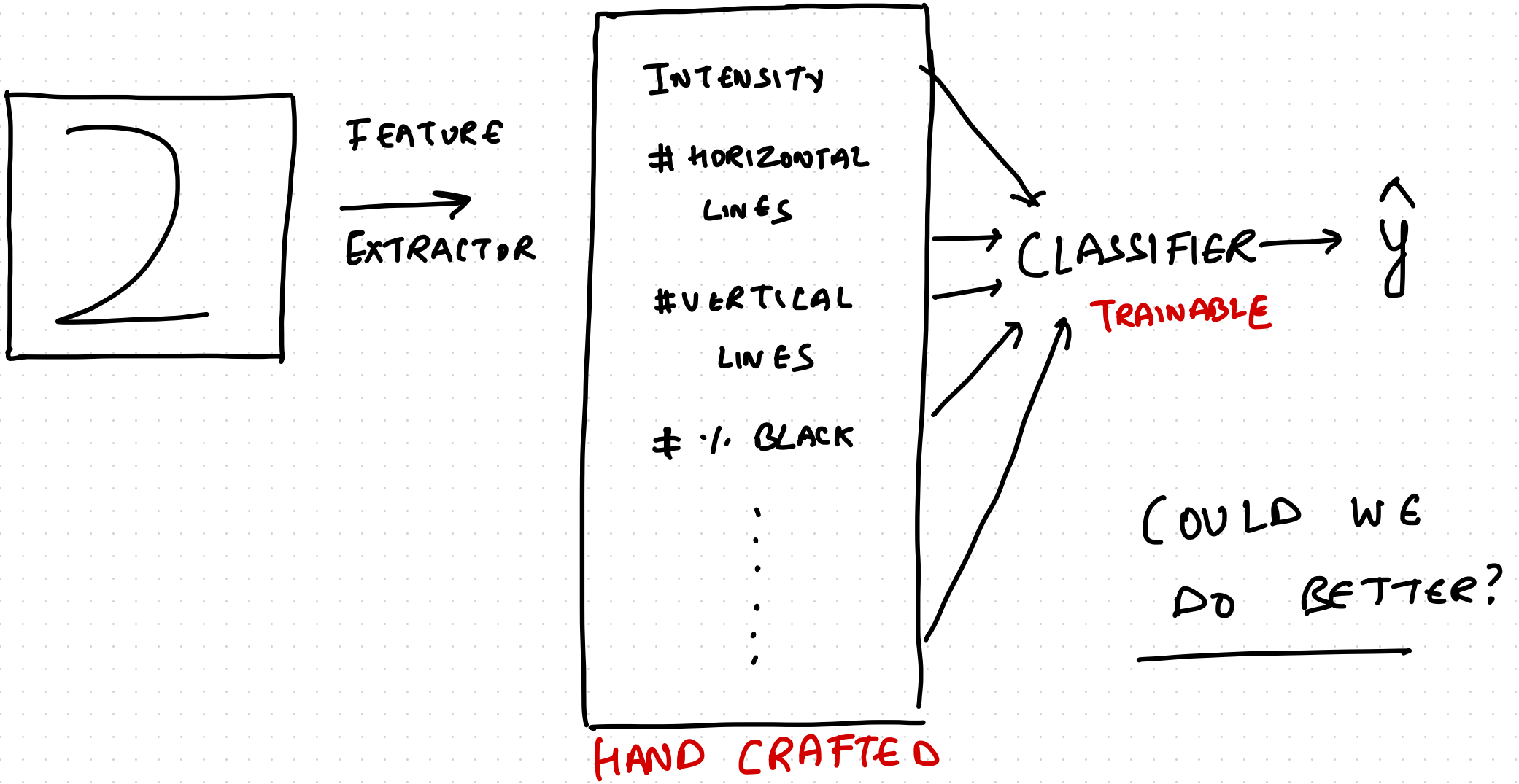
XOR CLASSIFICATION



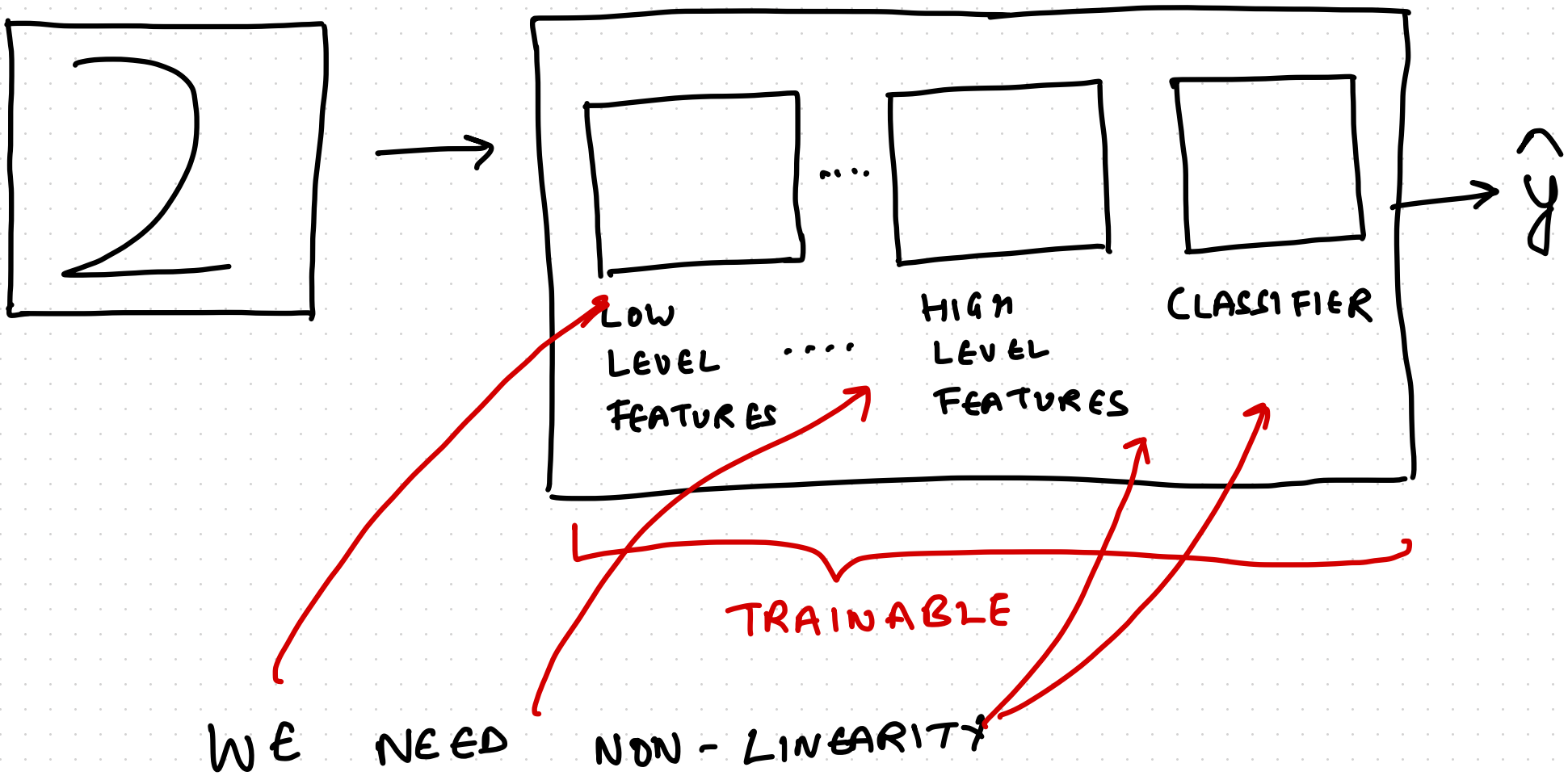


CAN ADD NON-LINEARITY BY HAND-CRAFTING FEATURES!

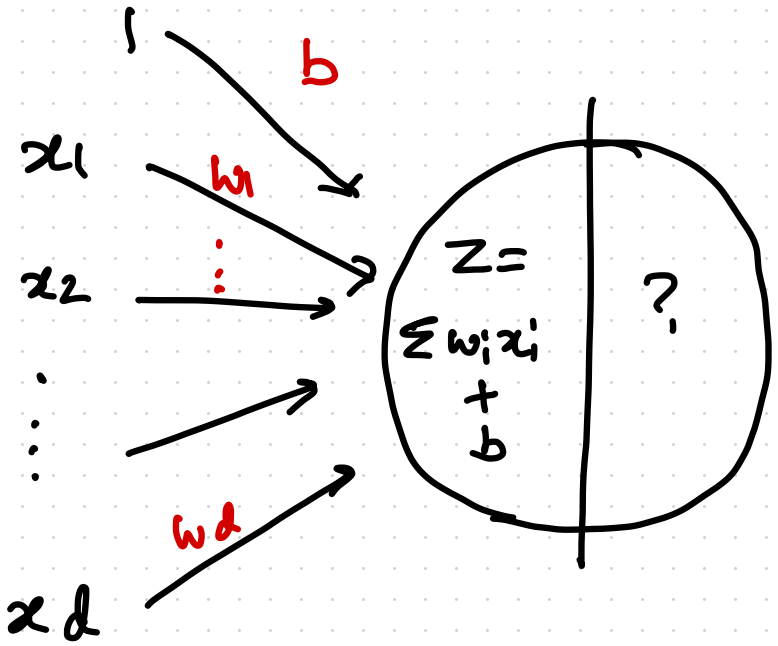
PARADIGM CHANGE



PARADIGM CHANGE (NNs)

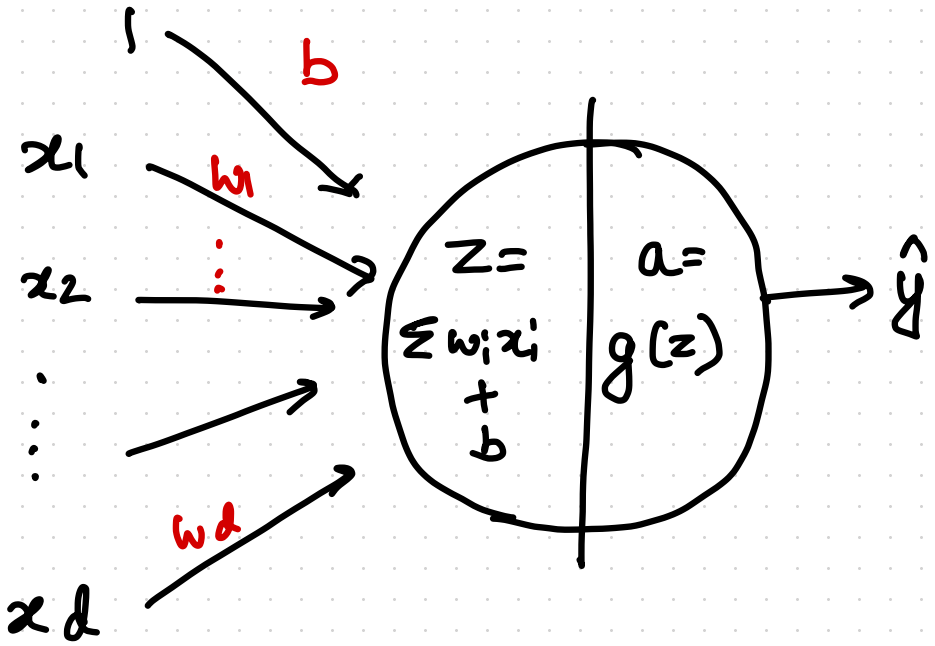


BACK PROPAGATION SUPPORTED ACTIVATIONS



key idea: Use activation similar to ⎍ but differentiable

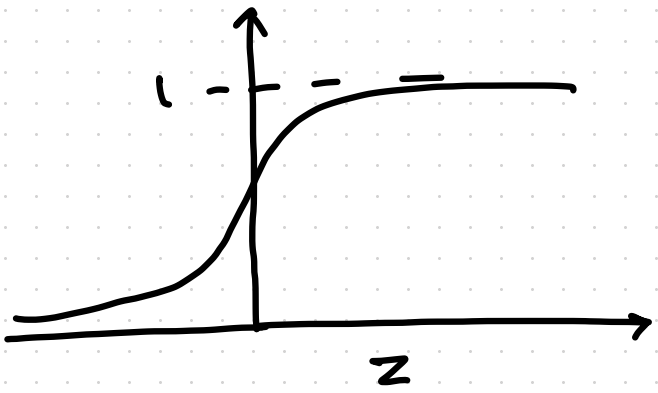
ADDING A NON-LINEARITY



$g(z)$: NON LINEAR TRANSFORMATION

ACTIVATION FUNCTIONS

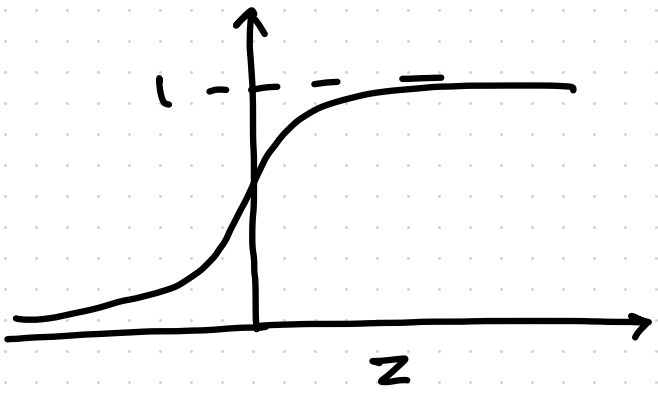
SIGMOID



$$g(z) = \frac{1}{1+e^{-z}}$$

ACTIVATION FUNCTIONS

SIGMOID



$$g(z) = \frac{1}{1+e^{-z}}$$

Q): If we have 1 neuron

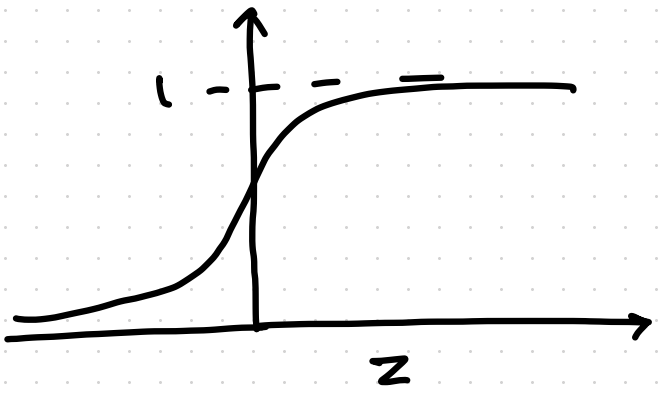
&

$$g(z) = \frac{1}{1+e^{-z}}$$

what do we get?

ACTIVATION FUNCTIONS

SIGMOID



$$g(z) = \frac{1}{1+e^{-z}}$$

Q): If we have 1 Neuron

&

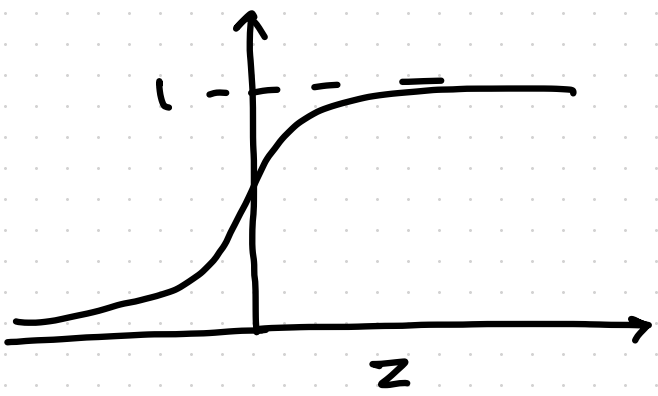
$$g(z) = \frac{1}{1+e^{-z}}$$

what do we get?

Logistic Regression

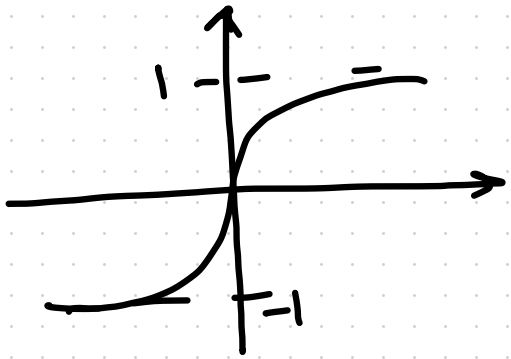
ACTIVATION FUNCTIONS

SIGMOID



$$g(z) = \frac{1}{1+e^{-z}}$$

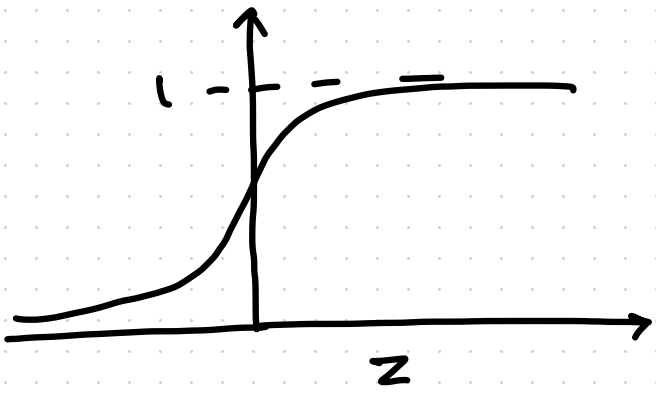
TANH



$$g(z) = \frac{e^{z} - e^{-z}}{e^{z}+e^{-z}}$$

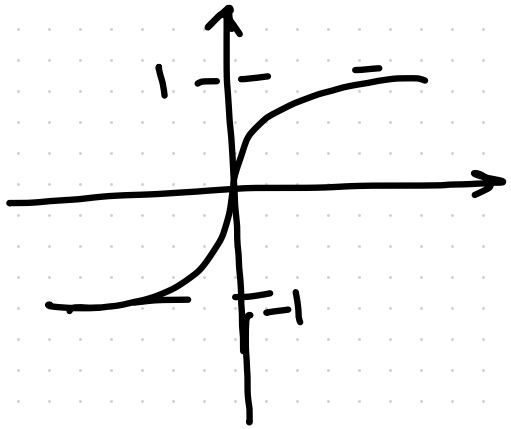
ACTIVATION FUNCTIONS

SIGMOID



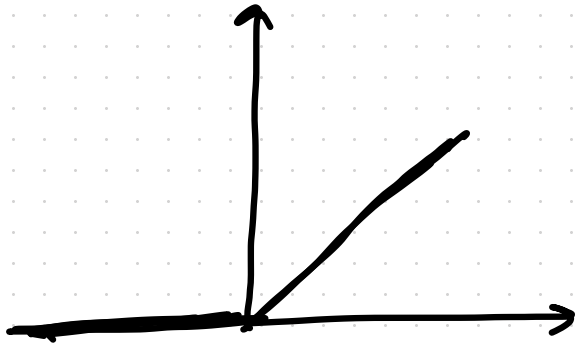
$$g(z) = \frac{1}{1+e^{-z}}$$

TANH



$$g(z) = \frac{e^{z} - e^{-z}}{e^{z}+e^{-z}}$$

ReLU



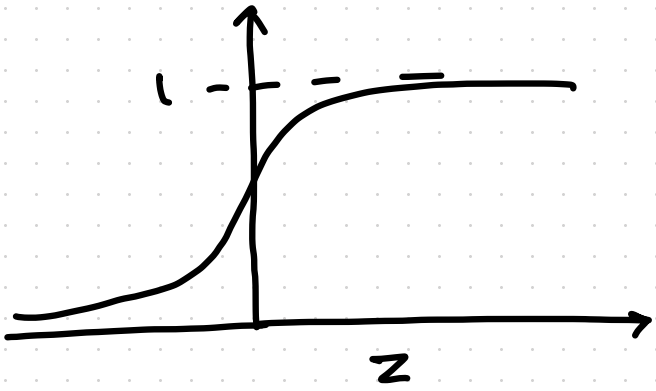
$$g(z) = \begin{cases} z; z \geq 0 \\ 0; \text{o/w} \end{cases}$$

or

$$g(z) = \max(0, z)$$

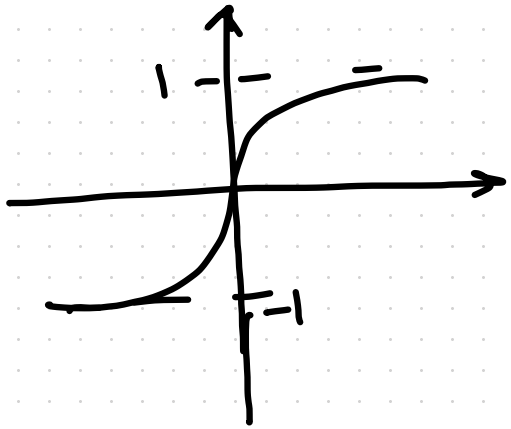
ACTIVATION FUNCTIONS

SIGMOID



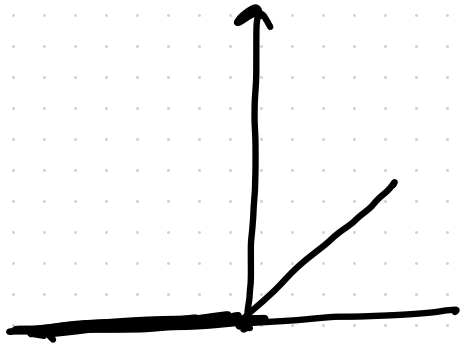
$$g(z) = \frac{1}{1+e^{-z}}$$

TANH



$$g(z) = \frac{e^{z} - e^{-z}}{e^{z}+e^{-z}}$$

ReLU

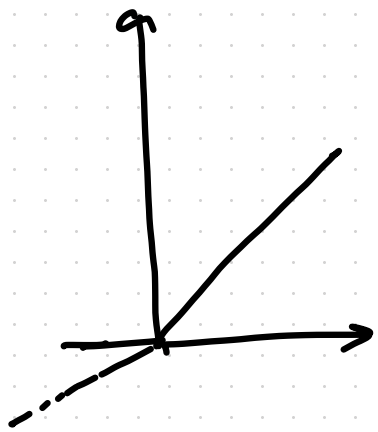


$$g(z) = \begin{cases} z; z \geq 0 \\ 0; \text{o/w} \end{cases}$$

or

$$g(z) = MAX(0, z)$$

Leaky ReLU

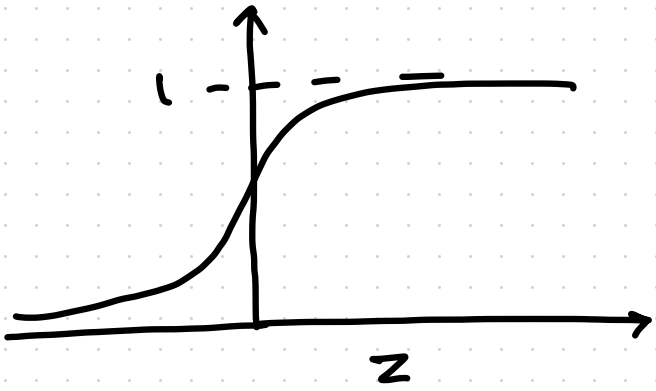


$$g(z) = MAX(\alpha z, z)$$

$$\alpha \rightarrow 0$$

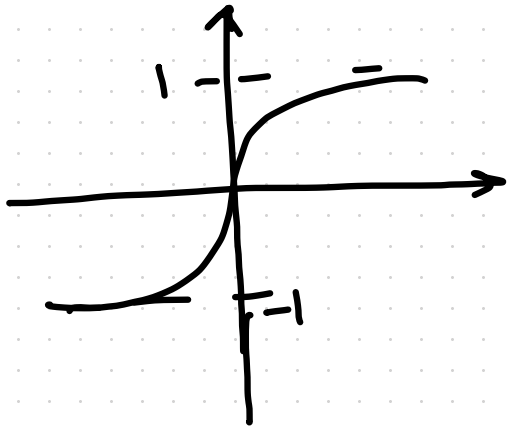
ACTIVATION FUNCTIONS

SIGMOID



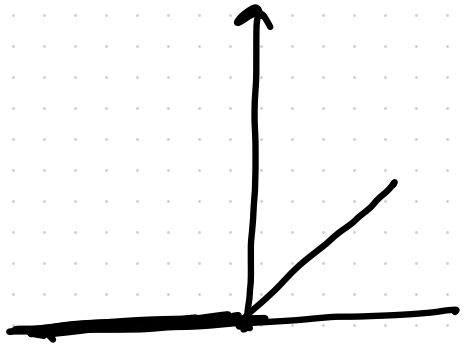
USEFUL FOR PROBABILISTIC ESTIMATES
∵ B/W 0 & 1

TANH



USEFUL IF DATA TRANSFORMED WITH MEAN 0

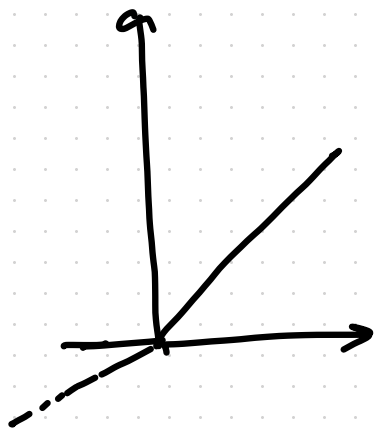
ReLU



GAME CHANGER (Default)

Good learning for |z| = high.

Leaky ReLU



Similar to ReLU

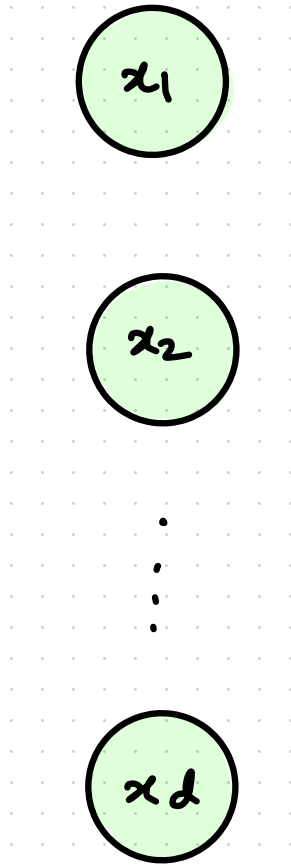
Learns for z < 0 also

DESIRABLE ATTRIBUTES OF ACTIVATION FUNCTIONS

1) NON-LINEAR

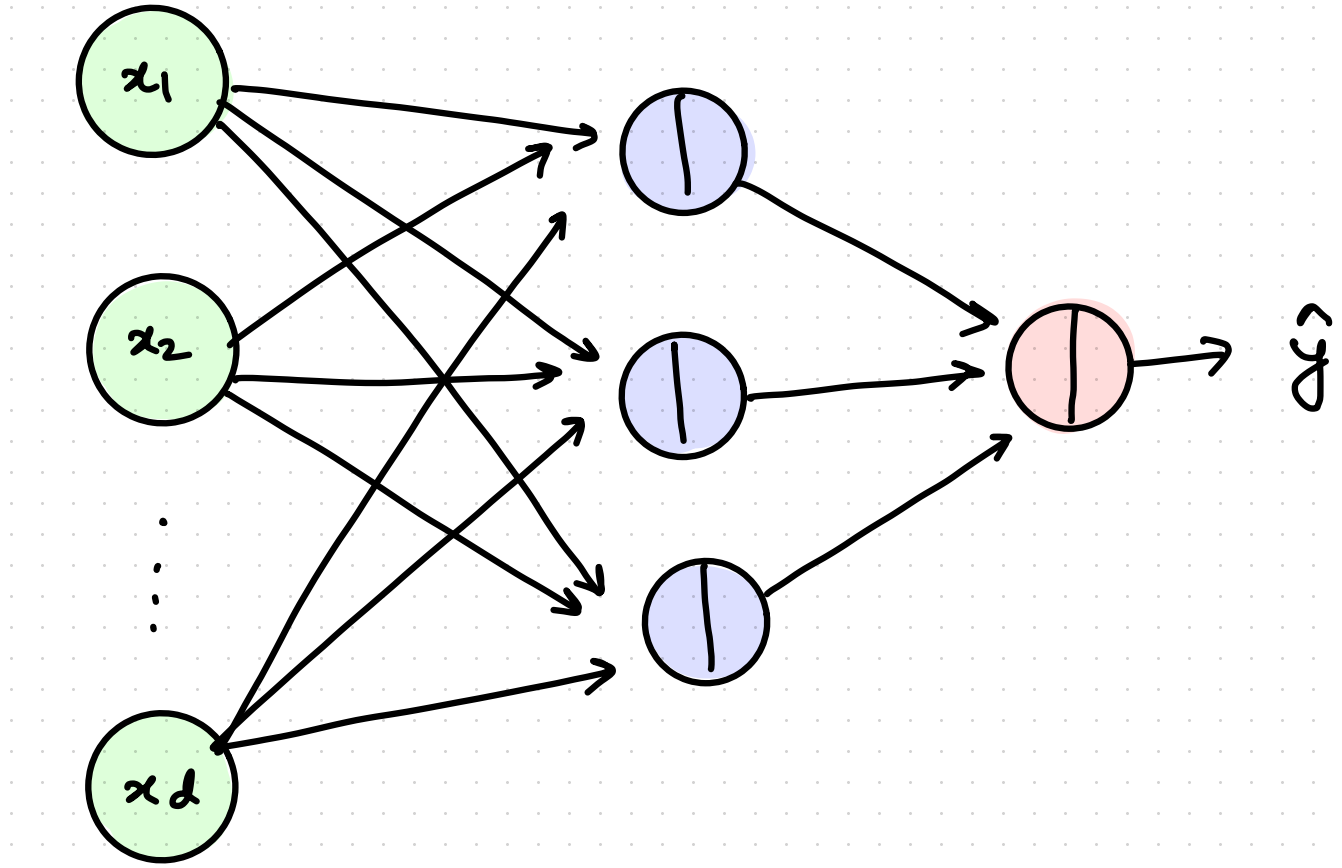
2) (MOSTLY) SMALL CHANGE IN I/P $\Rightarrow$ SMALL CHANGE IN O/P

1 LAYER PERCEPTRON (NN)

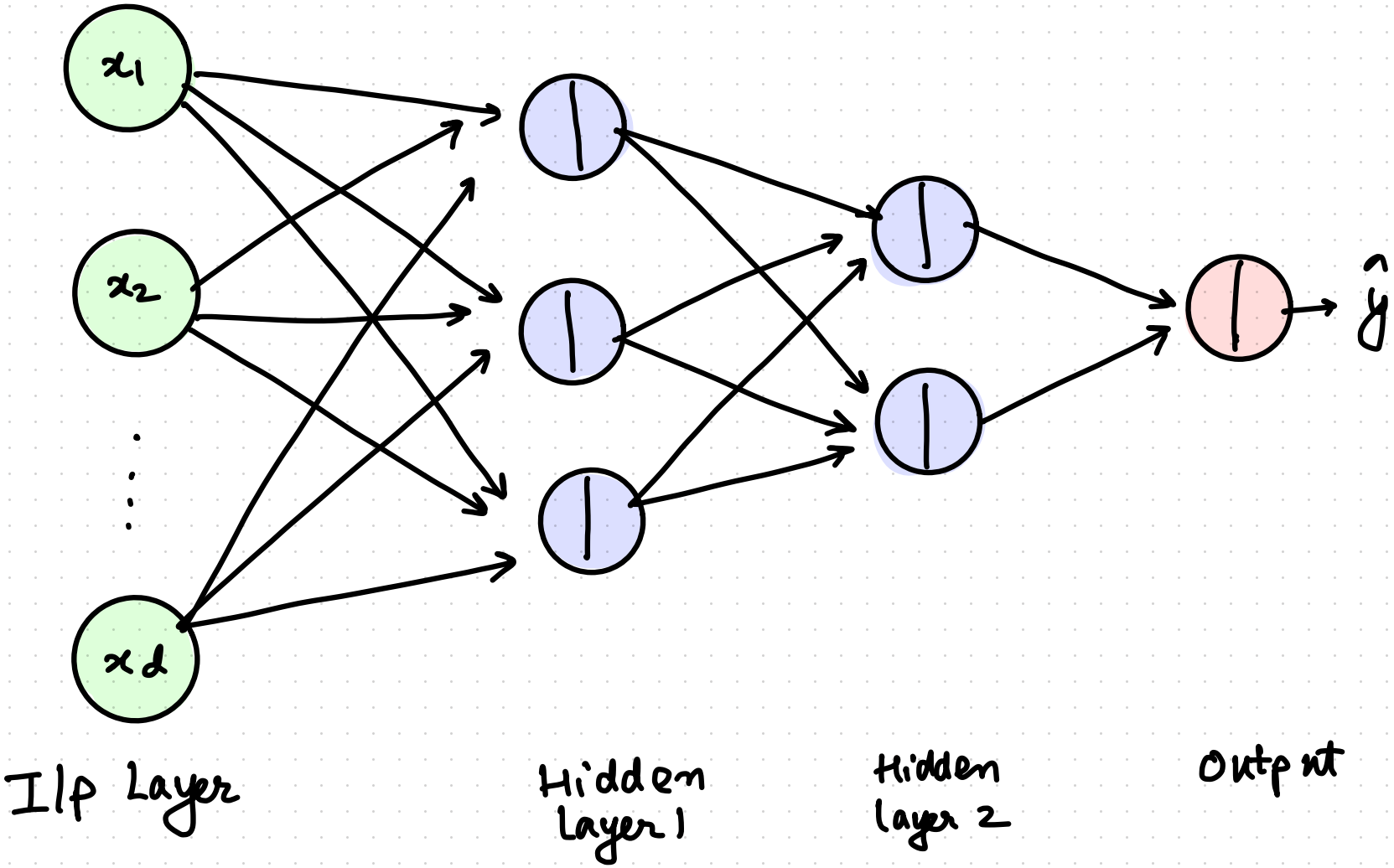


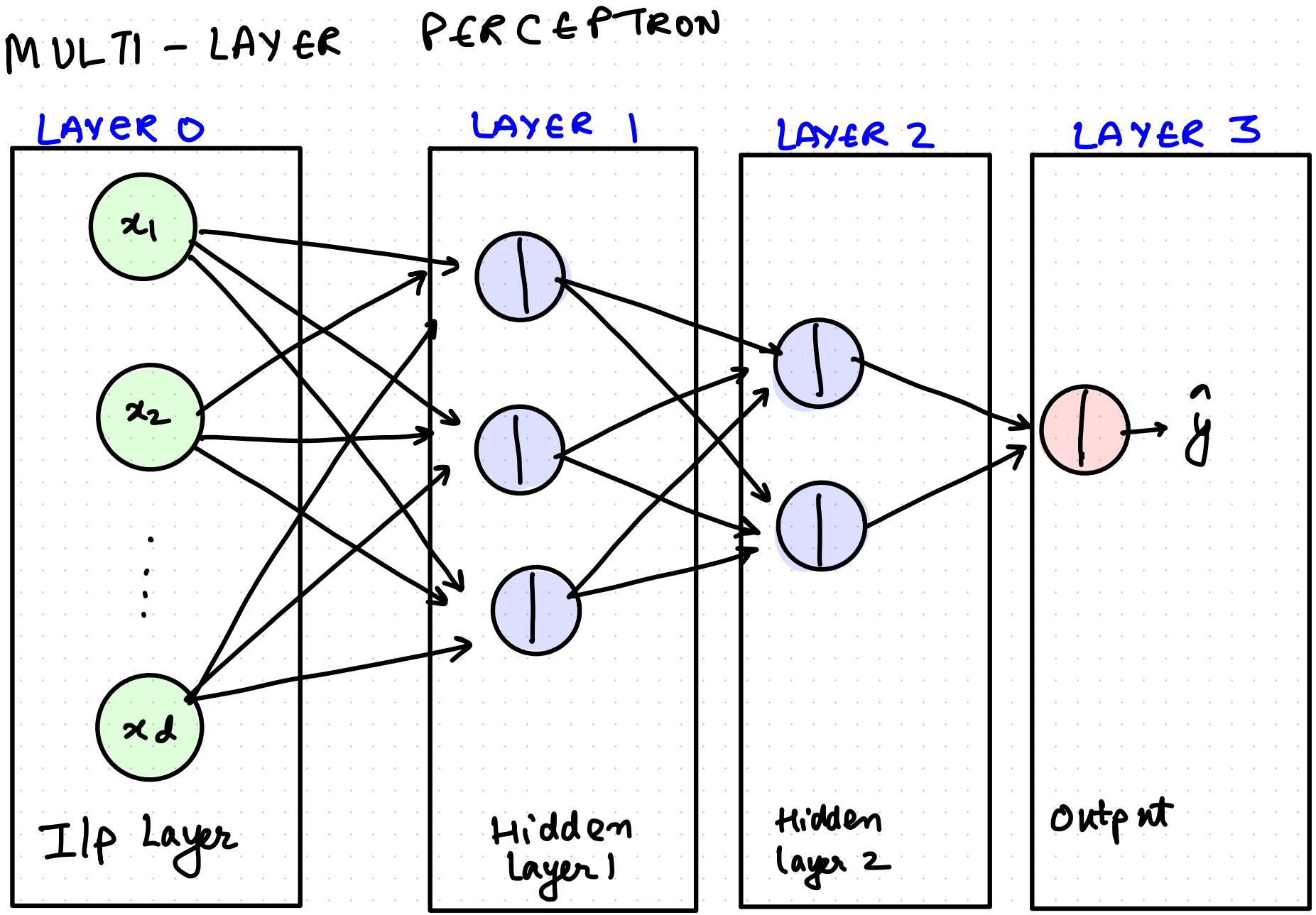
Ilp Layer

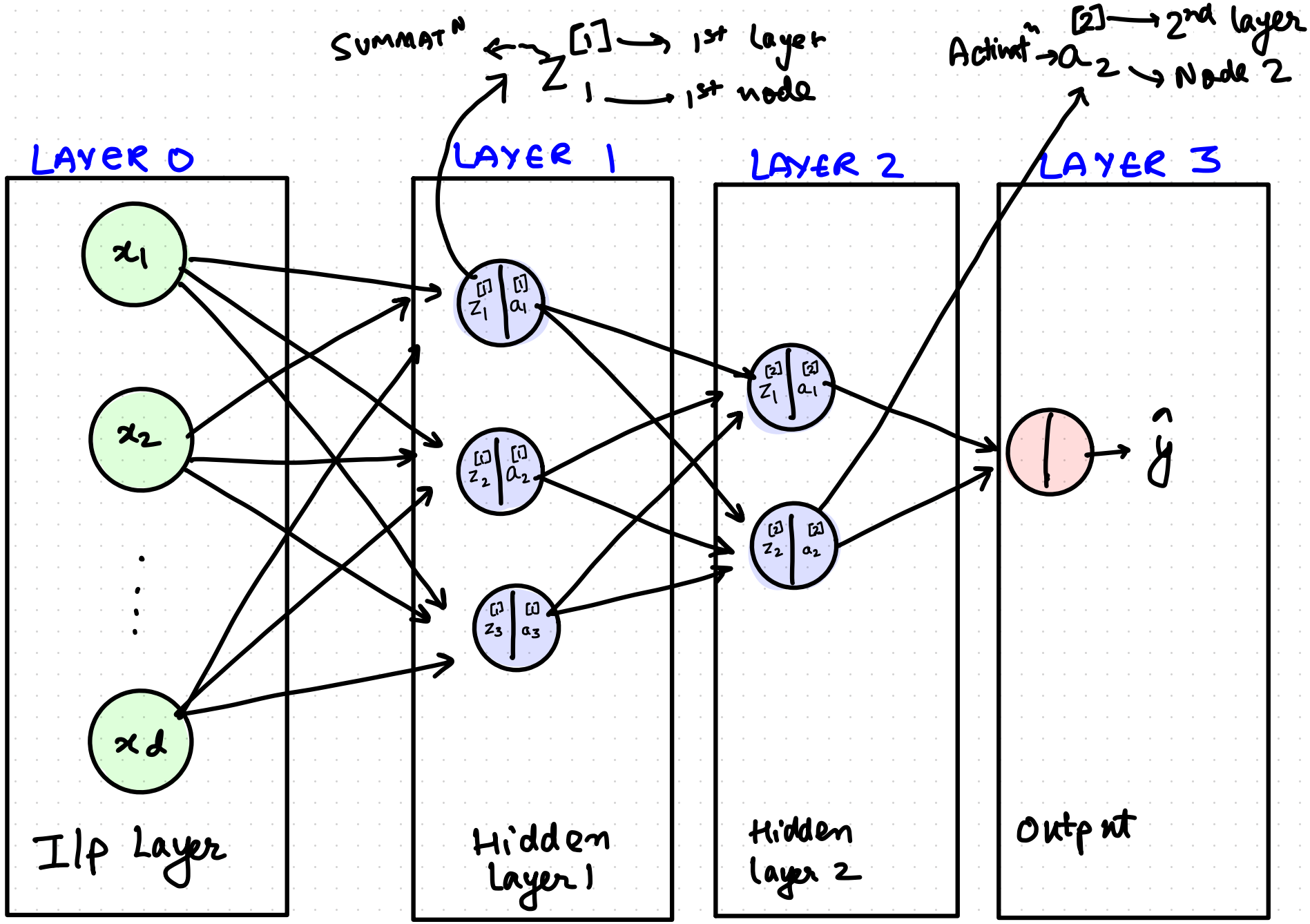
1- LAYER PERCEPTRON (NN)

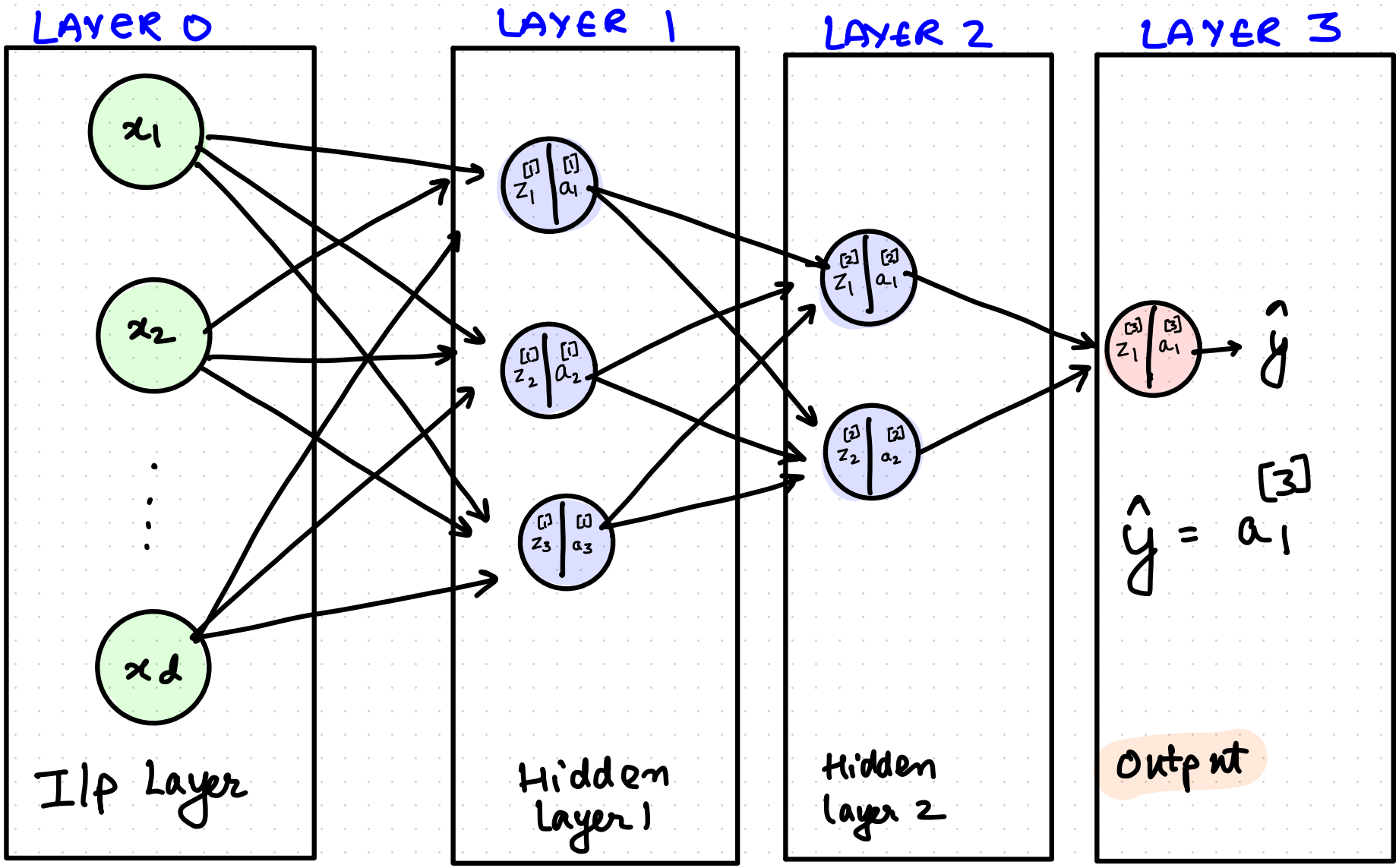


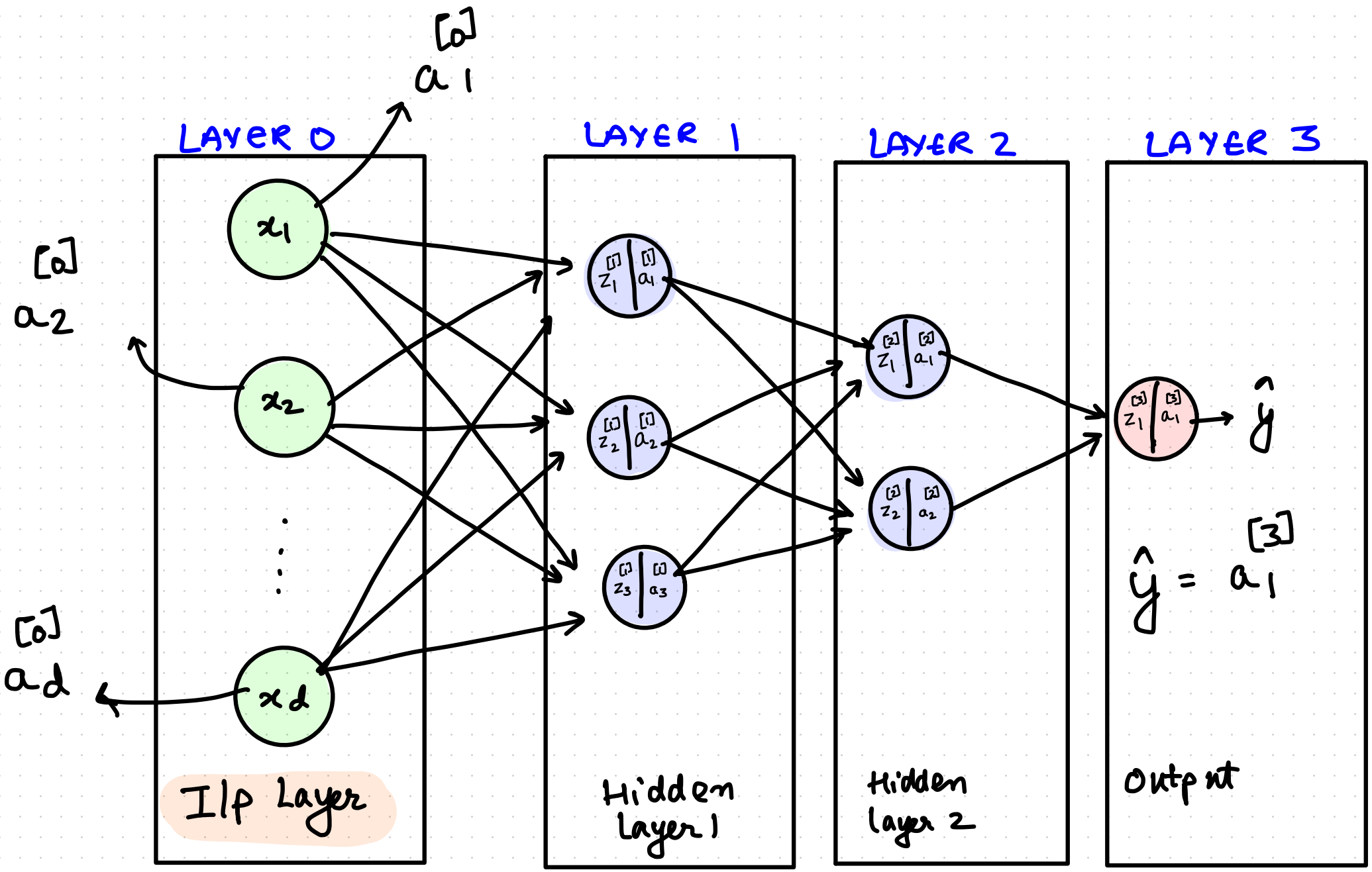
MULTI - LAYER PERCEPTRON

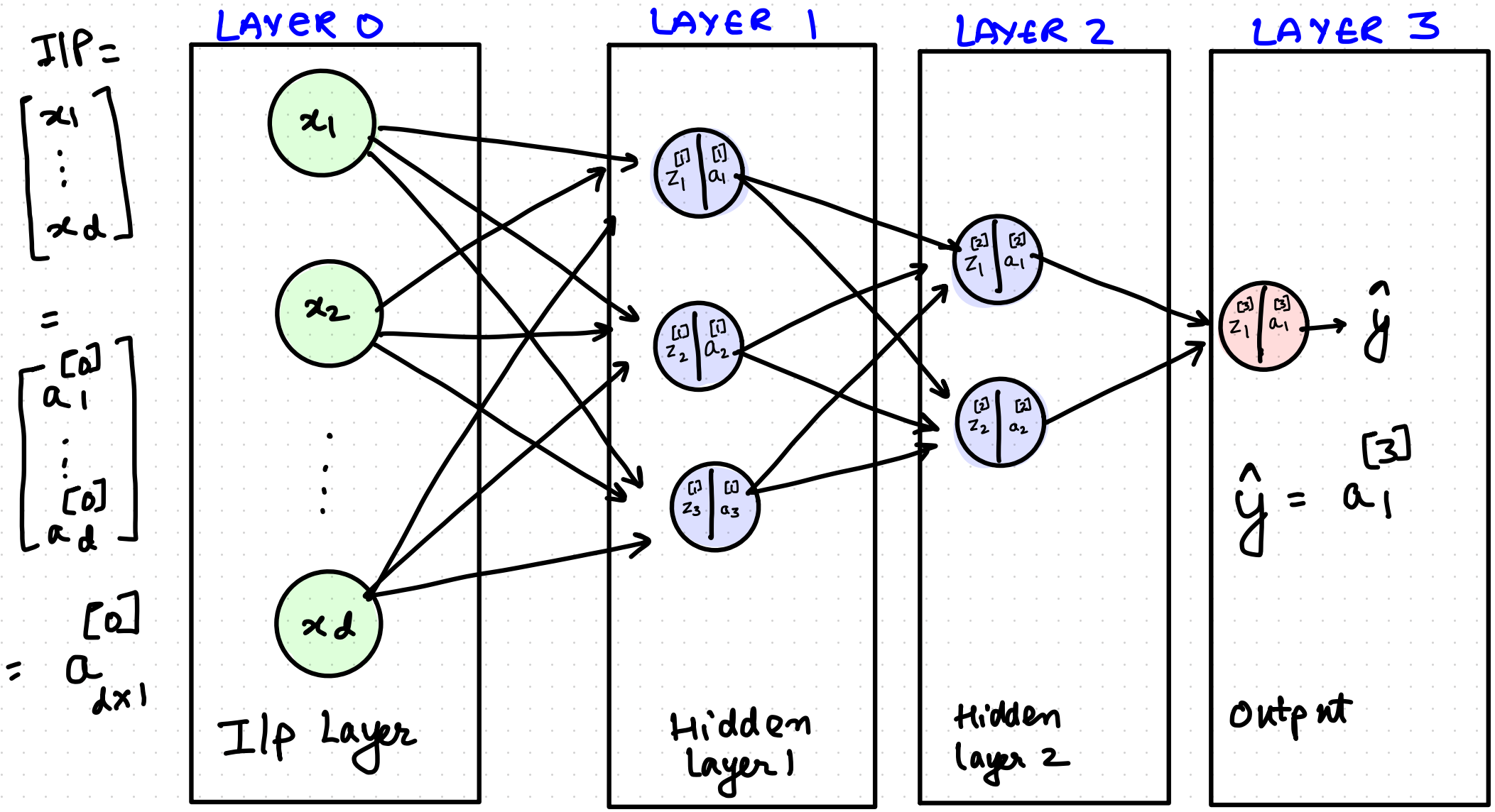



MULTI - LAYER PERCEPTRON
LAYER 0
LAYER 1
LAYER 2
LAYER 3
x_1
x_2
x_d
$\hat{y}$
Ilp Layer
Hidden layer 1
Hidden layer 2
Output

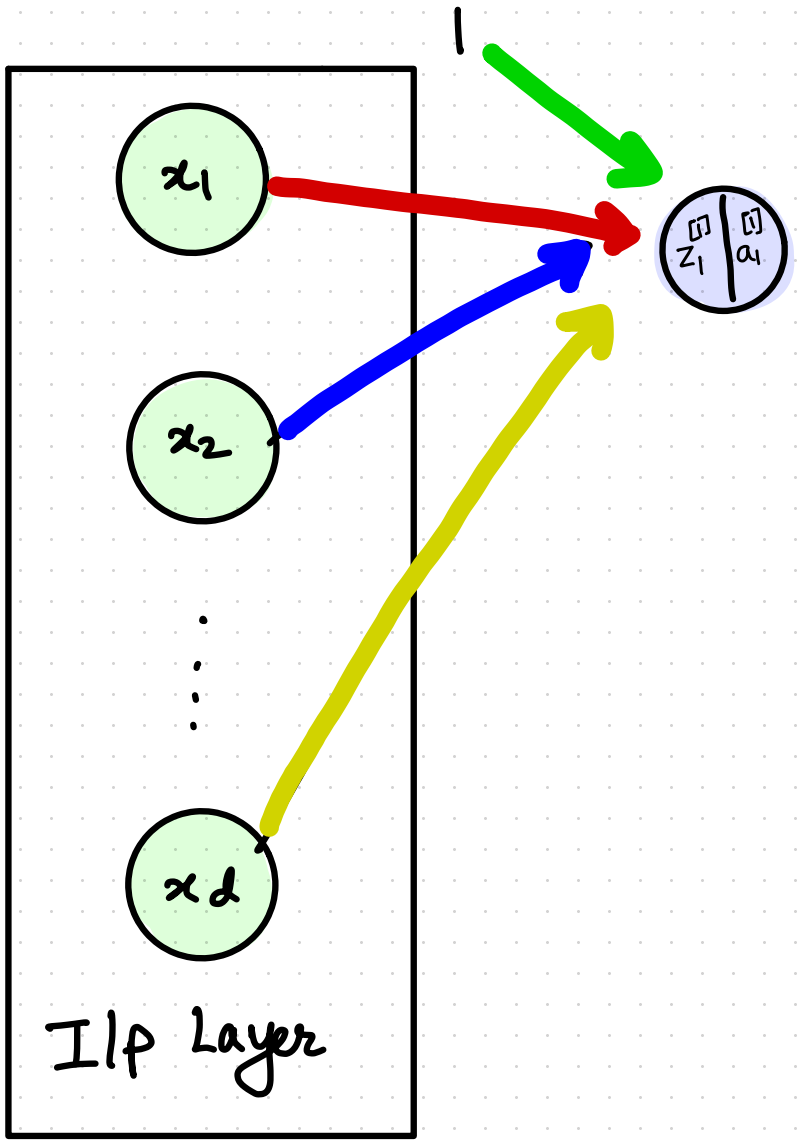

SUMMATN
[1] → 1st layer
1 → 1st node
Activatn
[2] → 2nd layer
2 → Node 2
LAYER 0
LAYER 1
LAYER 2
LAYER 3
x1
x2
xd
Ilp Layer
Hidden layer 1
Hidden layer 2
Output
ŷ


LAYER 0
LAYER 1
LAYER 2
LAYER 3
x_1
x_2
x_d
$z_1^{[1]}$ $a_1^{[1]}$
$z_2^{[1]}$ $a_2^{[1]}$
$z_3^{[1]}$ $a_3^{[1]}$
$z_1^{[2]}$ $a_1^{[2]}$
$z_2^{[2]}$ $a_2^{[2]}$
$z_1^{[3]}$ $a_1^{[3]}$
$\hat{y}$
$\hat{y} = a_1^{[3]}$
Ilp Layer
Hidden layer 1
Hidden layer 2
Output


LAYER 0
LAYER 1
LAYER 2
LAYER 3
$a^{[0]}_1$
$a^{[0]}_2$
$a^{[0]}_d$
x_1
x_2
x_d
$z^{[1]}_1$ $a^{[1]}_1$
$z^{[1]}_2$ $a^{[1]}_2$
$z^{[1]}_3$ $a^{[1]}_3$
$z^{[2]}_1$ $a^{[2]}_1$
$z^{[2]}_2$ $a^{[2]}_2$
$z^{[3]}_1$ $a^{[3]}_1$
$\hat{y}$
$\hat{y} = a^{[3]}_1$
Ilp Layer
Hidden layer 1
Hidden layer 2
Output


LAYER 0
LAYER 1
LAYER 2
LAYER 3
I/P =
$\begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$
=
$\begin{bmatrix} a_1^{[0]} \\ \vdots \\ a_d^{[0]} \end{bmatrix}$
$= a^{[0]}_{d \times 1}$
x_1
x_2
x_d
$z_1^{[1]}$ $a_1^{[1]}$
$z_2^{[1]}$ $a_2^{[1]}$
$z_3^{[1]}$ $a_3^{[1]}$
$z_1^{[2]}$ $a_1^{[2]}$
$z_2^{[2]}$ $a_2^{[2]}$
$z_1^{[3]}$ $a_1^{[3]}$
$\hat{y}$
$\hat{y} = a_1^{[3]}$
I/p Layer
Hidden layer 1
Hidden layer 2
Output

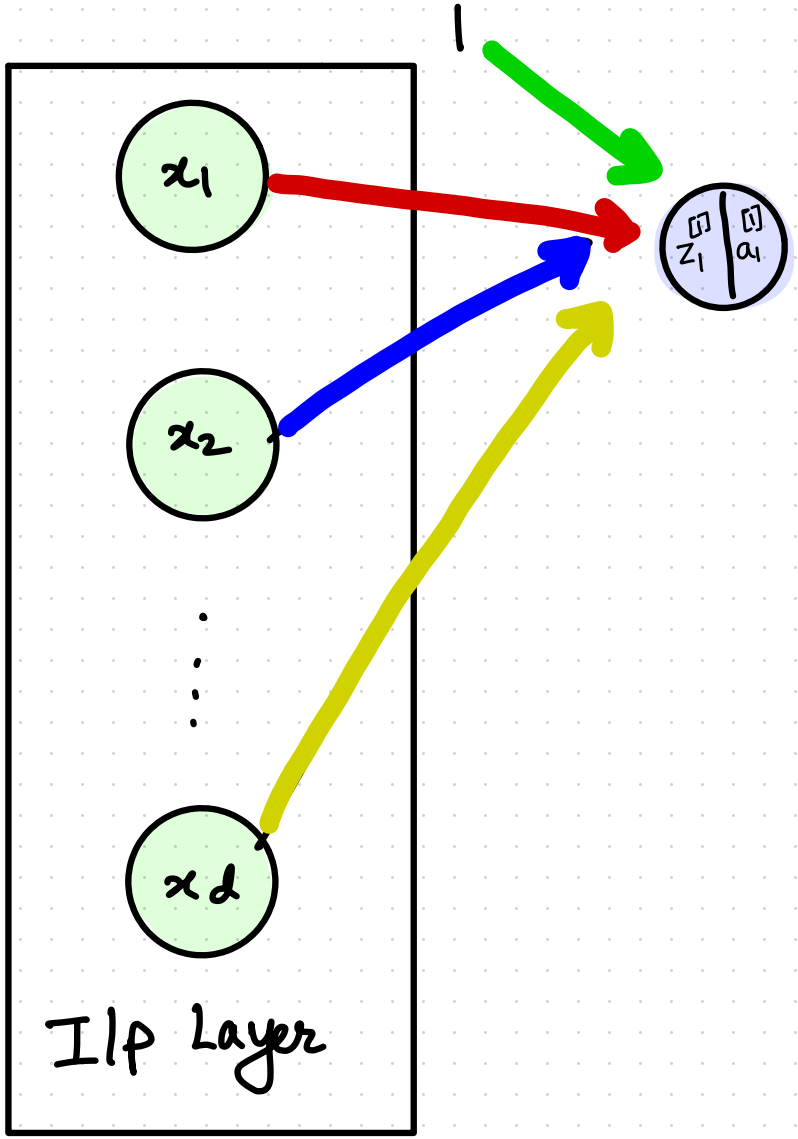
CONSIDER SINGLE NEURON (LAYER 1, NODE 1)



$$z_1^{[1]} = 1 * b_1^{[1]} + x_1 * w_{1,1}^{[1]} + x_2 * w_{1,2}^{[1]} + \cdots + x_d * w_{1,d}^{[1]}$$

bias layer 1 Node 1

CONSIDER SINGLE NEURON (LAYER 1, NODE 1)



bias layer 1 Node 1

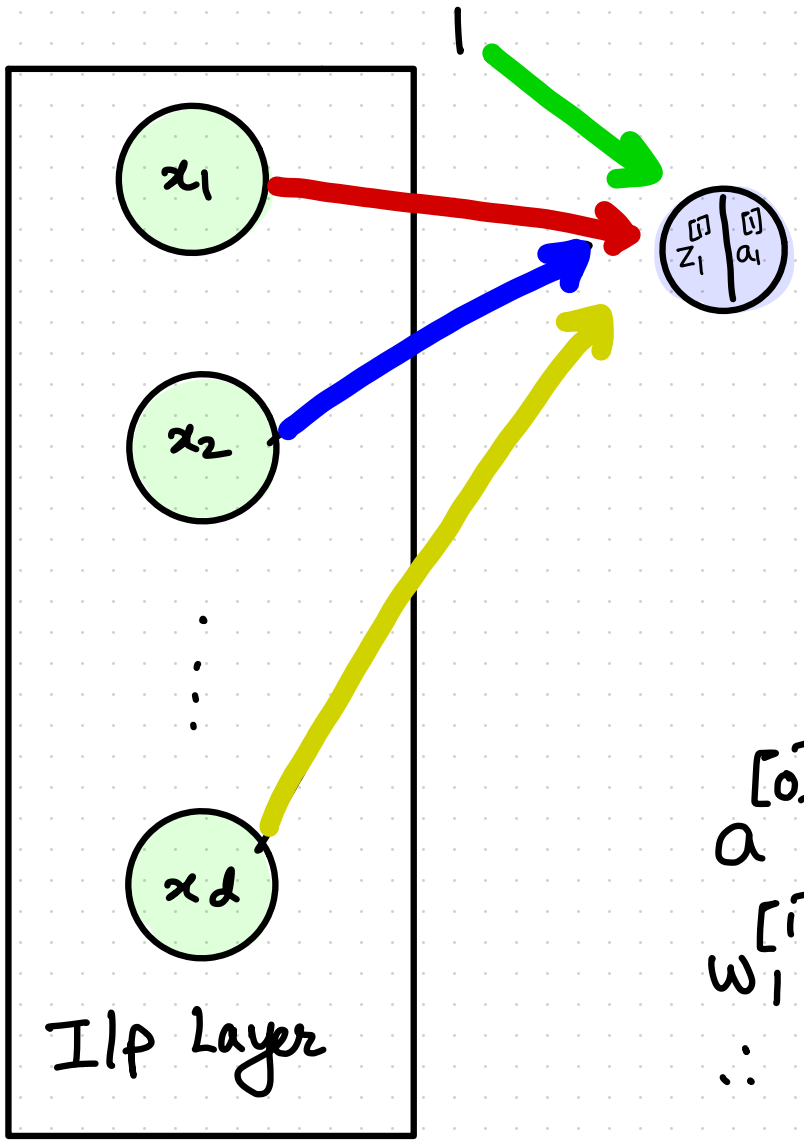
$$z_1^{[1]} = 1 * b_1^{[1]} + x_1 * w_{1,1}^{[1]} + x_2 * w_{1,2}^{[1]} + \cdots + x_d * w_{1,d}^{[1]}$$

$[1]$ ← 1^{th} layer

$w_{a,b}$

- a^{th} node in 1^{th} layer
- b^{th} component of prev. layer activat^n

CONSIDER SINGLE NEURON (LAYER 1, NODE 1)



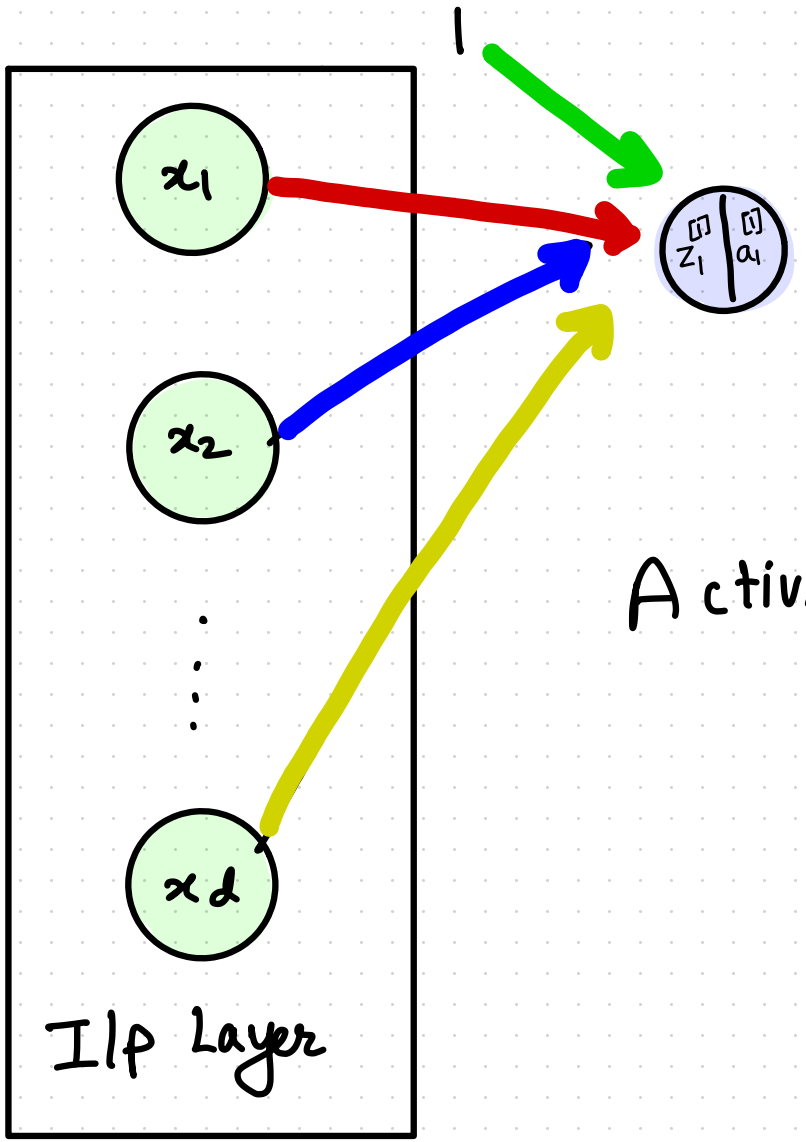
$$z_1^{[1]} = 1 * b_1^{[1]} + a_1^{[0]} * w_{1,1}^{[1]} + a_2^{[0]} * w_{1,2}^{[1]} + \cdots + a_d^{[0]} * w_{1,d}^{[1]}$$

$$a^{[0]} \in R^D$$

$$w_1^{[1]} \in R^D$$

$$\therefore z_1^{[1]} = w_1^{[1]T} a^{[0]} + b_1^{[1]}$$

CONSIDER SINGLE NEURON (LAYER 1, NODE 1)

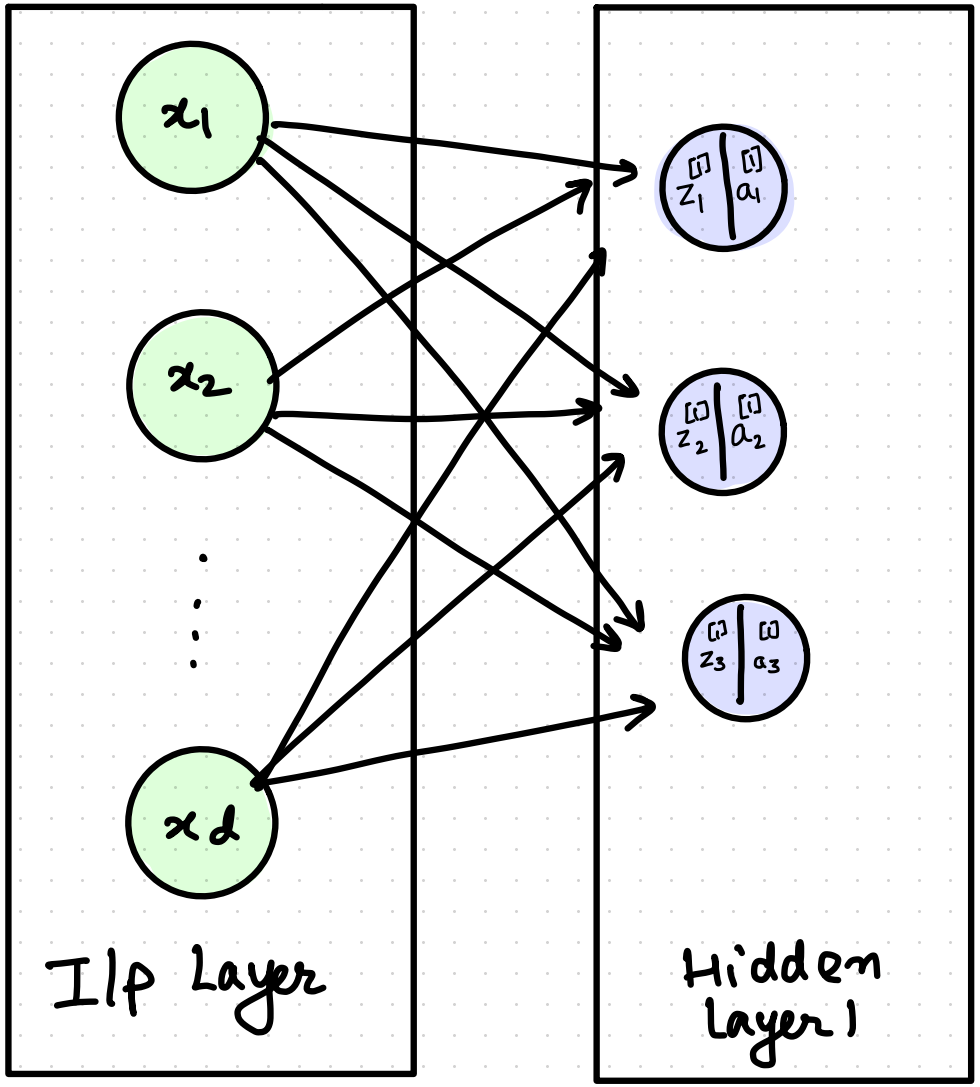


$$z_1^{[1]} = \omega_1^{[1]T} a^{[0]} + b_1^{[1]}$$

Activatⁿ $= a_1^{[1]} = g(z_1^{[1]})$

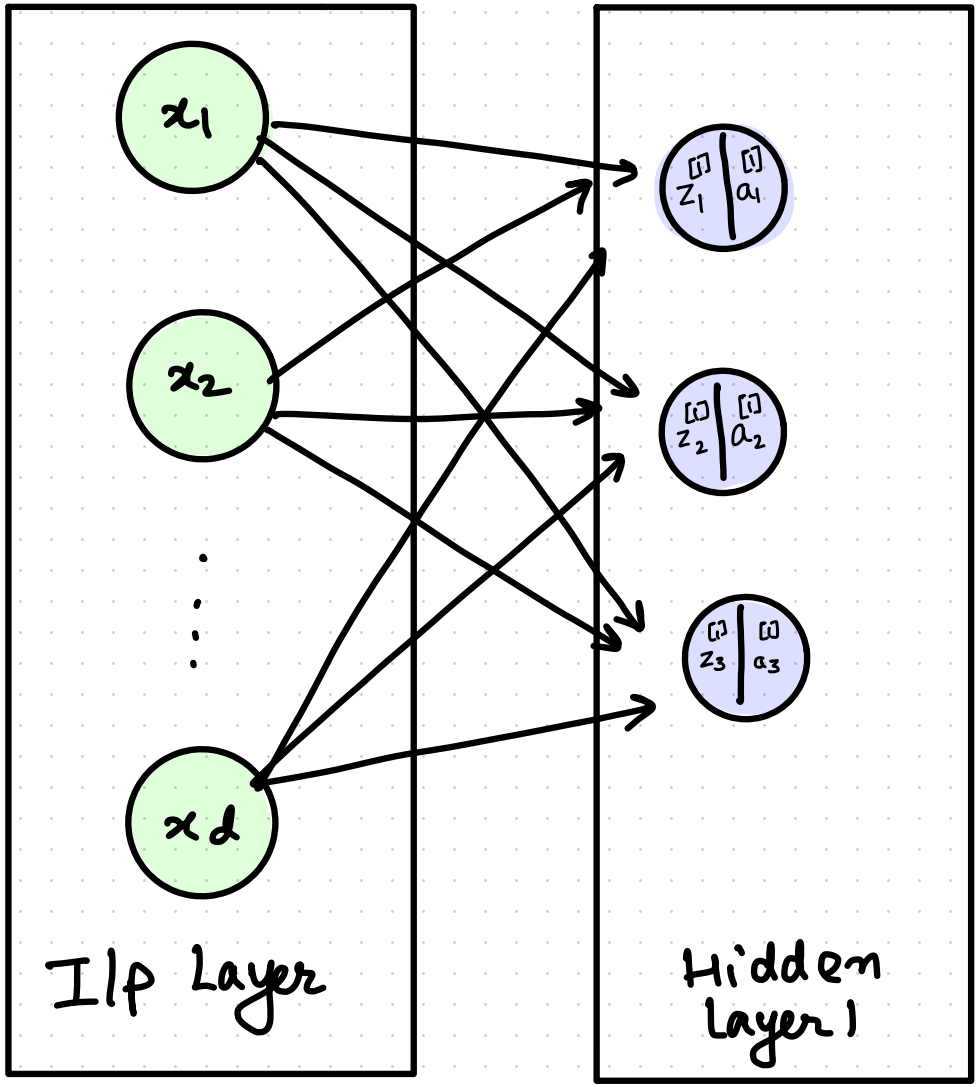
$a_1^{[1]} \in R$

FORWARD PROPAGATION



$$a_1^{[1]} = g\left(w_1^{[1]T} a^{[0]} + b_1^{[1]}\right)$$

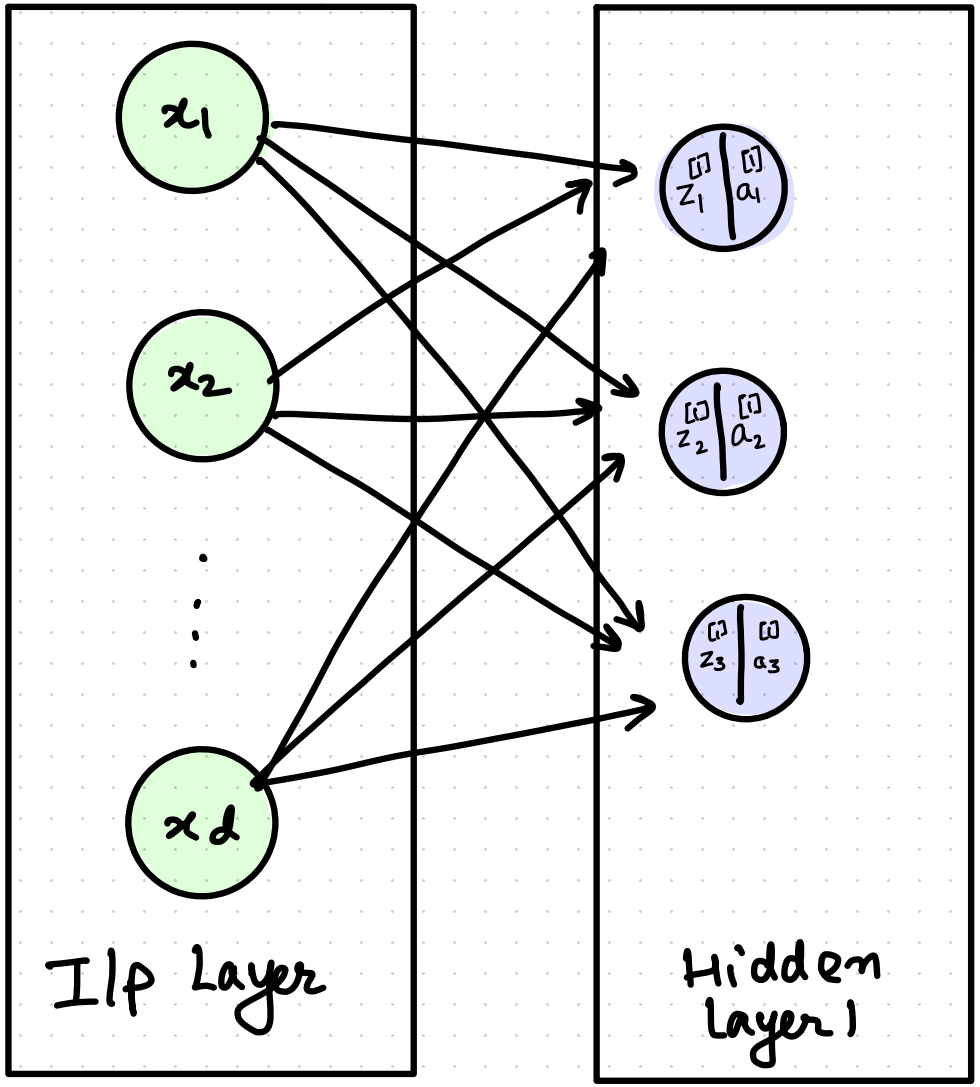
FORWARD PROPAGATION



$$a_1^{[1]} = g\left(w_1^{[1]T} a^{[0]} + b_1^{[1]}\right)$$

$$a_2^{[1]} = g\left(w_2^{[1]T} a^{[0]} + b_2^{[1]}\right)$$

FORWARD PROPAGATION

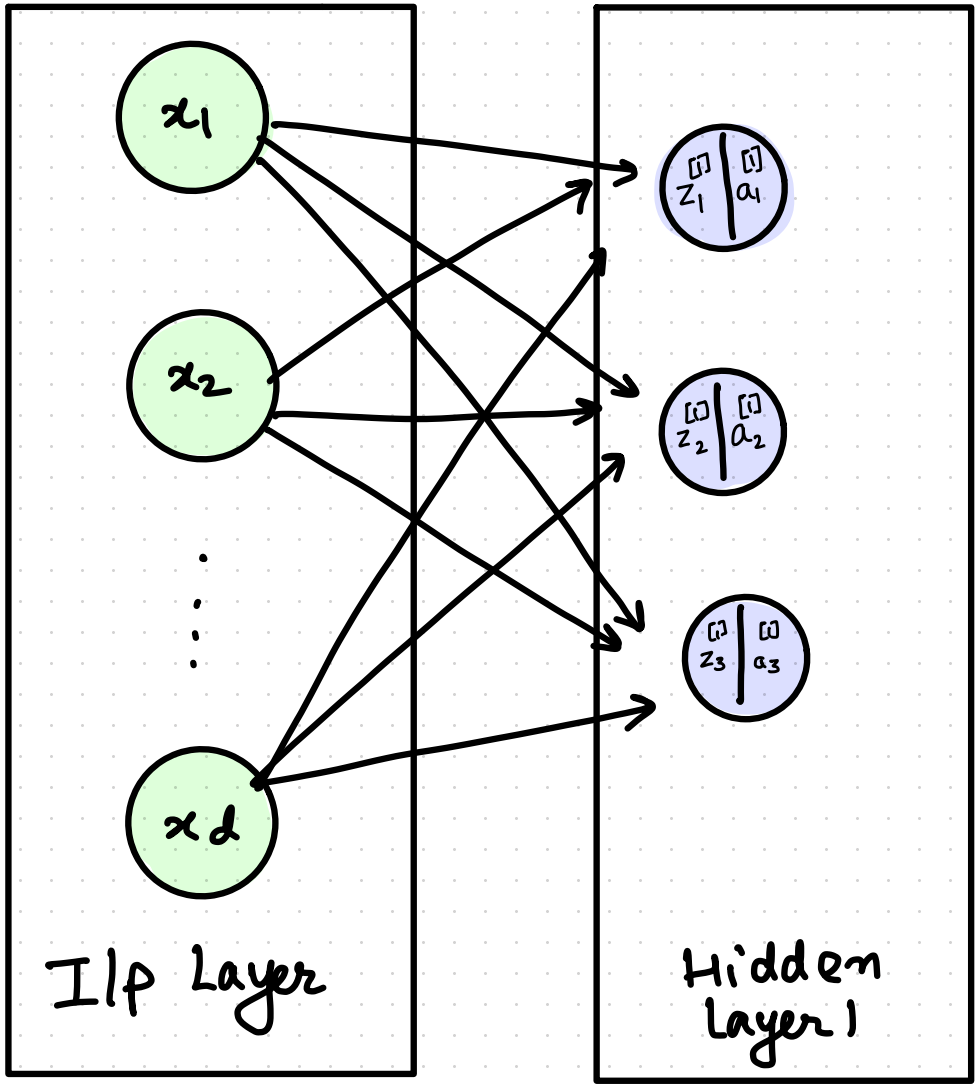


$$a_1^{[1]} = g\left(w_1^{[1]T} a^{[0]} + b_1^{[1]}\right)$$

$$a_2^{[1]} = g\left(w_2^{[1]T} a^{[0]} + b_2^{[1]}\right)$$

$$a_3^{[1]} = g\left(w_3^{[1]T} a^{[0]} + b_3^{[1]}\right)$$

FORWARD PROPAGATION (VECTORISATION)

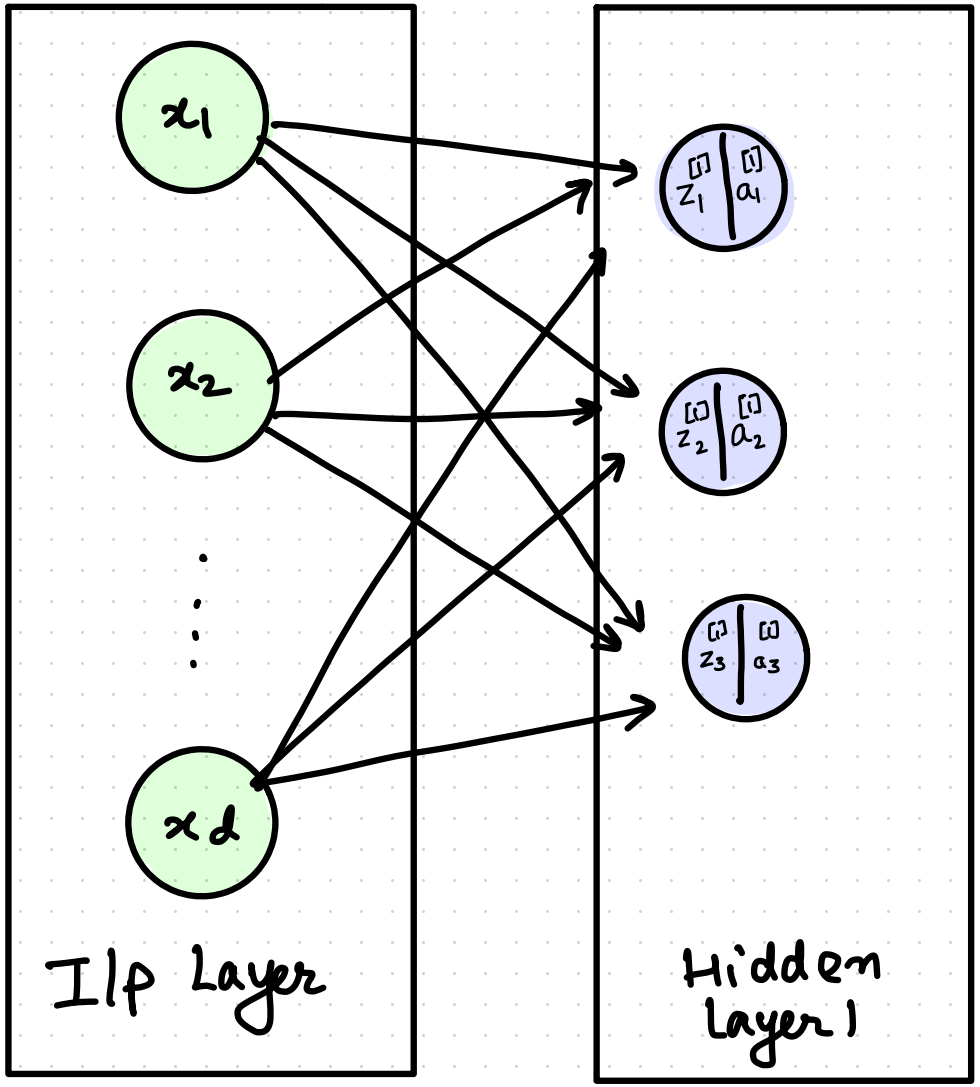


$$z_1^{[1]} = w_1^{[1]T} a^{[0]} + b_1^{[1]}$$

$$z_2^{[1]} = w_2^{[1]T} a^{[0]} + b_2^{[1]}$$

$$z_3^{[1]} = w_3^{[1]T} a^{[0]} + b_3^{[1]}$$

FORWARD PROPAGATION (VECTORISATION)

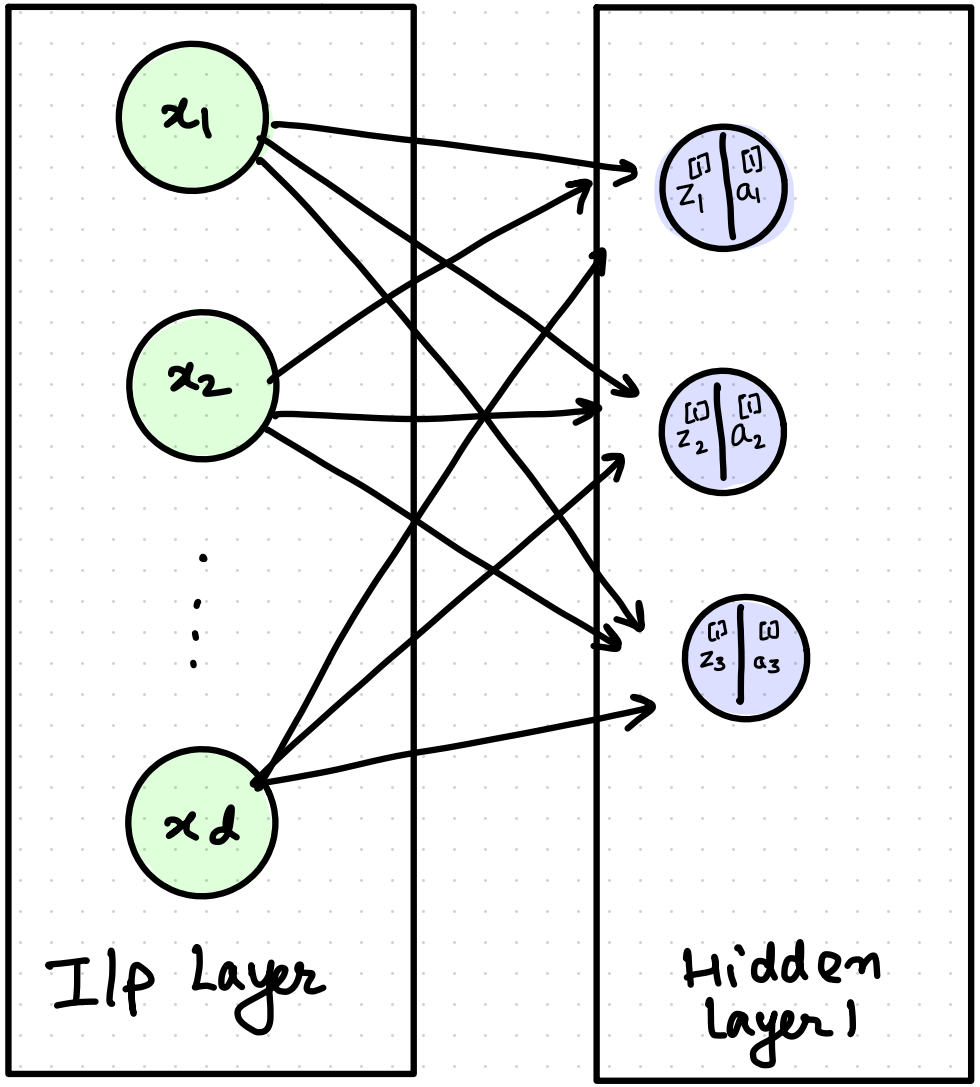


$$z_{1}^{[1]}_{1\times1} = w_{1}^{[1]T}_{1\times3}\, a^{[0]}_{3\times1} + b_{1}^{[1]}_{1\times1}$$

$$z_{2}^{[1]}_{1\times1} = w_{2}^{[1]T}_{1\times3}\, a^{[0]}_{3\times1} + b_{2}^{[1]}_{1\times1}$$

$$z_{3}^{[1]}_{1\times1} = w_{3}^{[1]T}_{1\times3}\, a^{[0]}_{3\times1} + b_{3}^{[1]}_{1\times1}$$

FORWARD PROPAGATION (VECTORISATION)



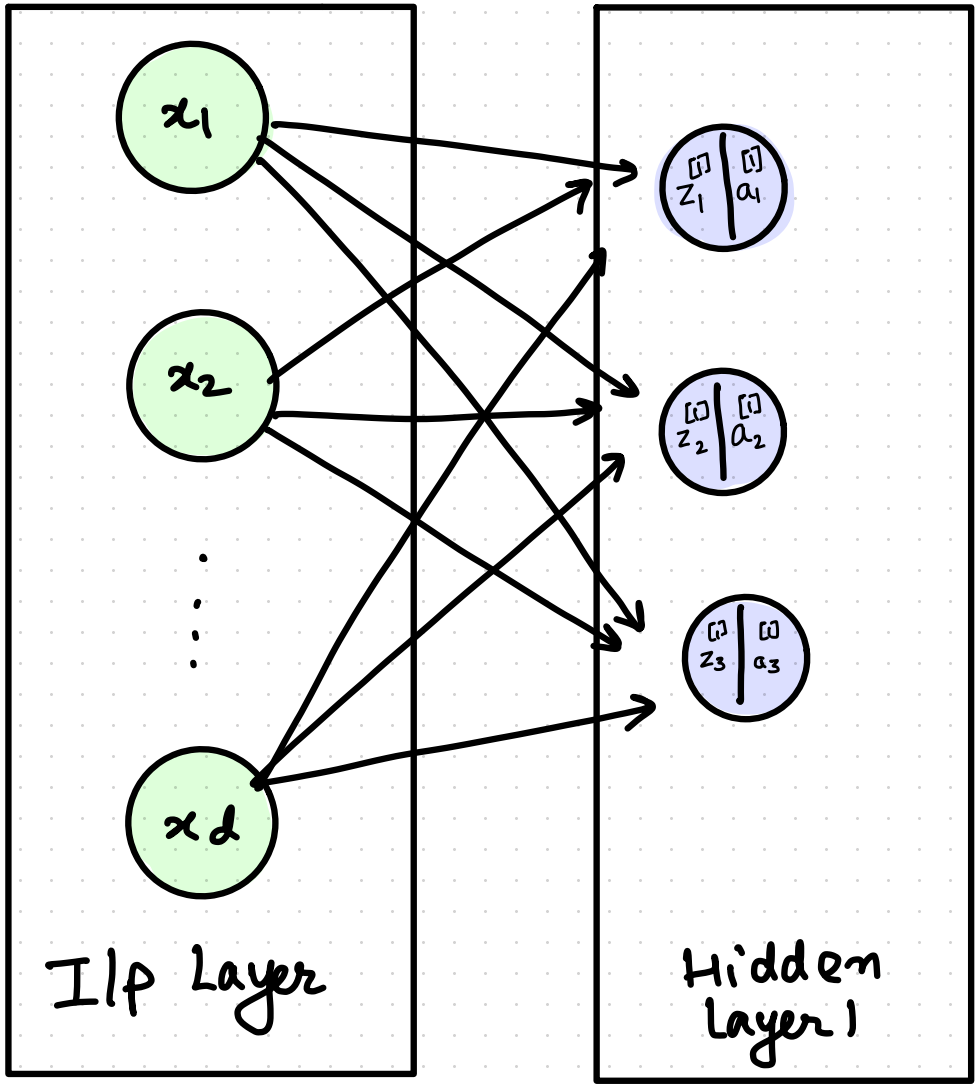
$$z_{1}^{[1]}_{1\times 1} = w_{1}^{[1]T}_{1\times 3}\, a^{[0]}_{3\times 1} + b_{1}^{[1]}_{1\times 1}$$

$$z_{2}^{[1]}_{1\times 1} = w_{2}^{[1]T}_{1\times 3}\, a^{[0]}_{3\times 1} + b_{2}^{[1]}_{1\times 1}$$

$$z_{3}^{[1]}_{1\times 1} = w_{3}^{[1]T}_{1\times 3}\, a^{[0]}_{3\times 1} + b_{3}^{[1]}_{1\times 1}$$

$$z^{[1]}_{3\times 1} = \begin{bmatrix} — & w_1^{[1]T} & — \\ — & w_2^{[1]T} & — \\ — & w_3^{[1]T} & — \end{bmatrix}_{3\times 3} a^{[0]}_{3\times 1} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix}$$

FORWARD PROPAGATION (VECTORISATION)

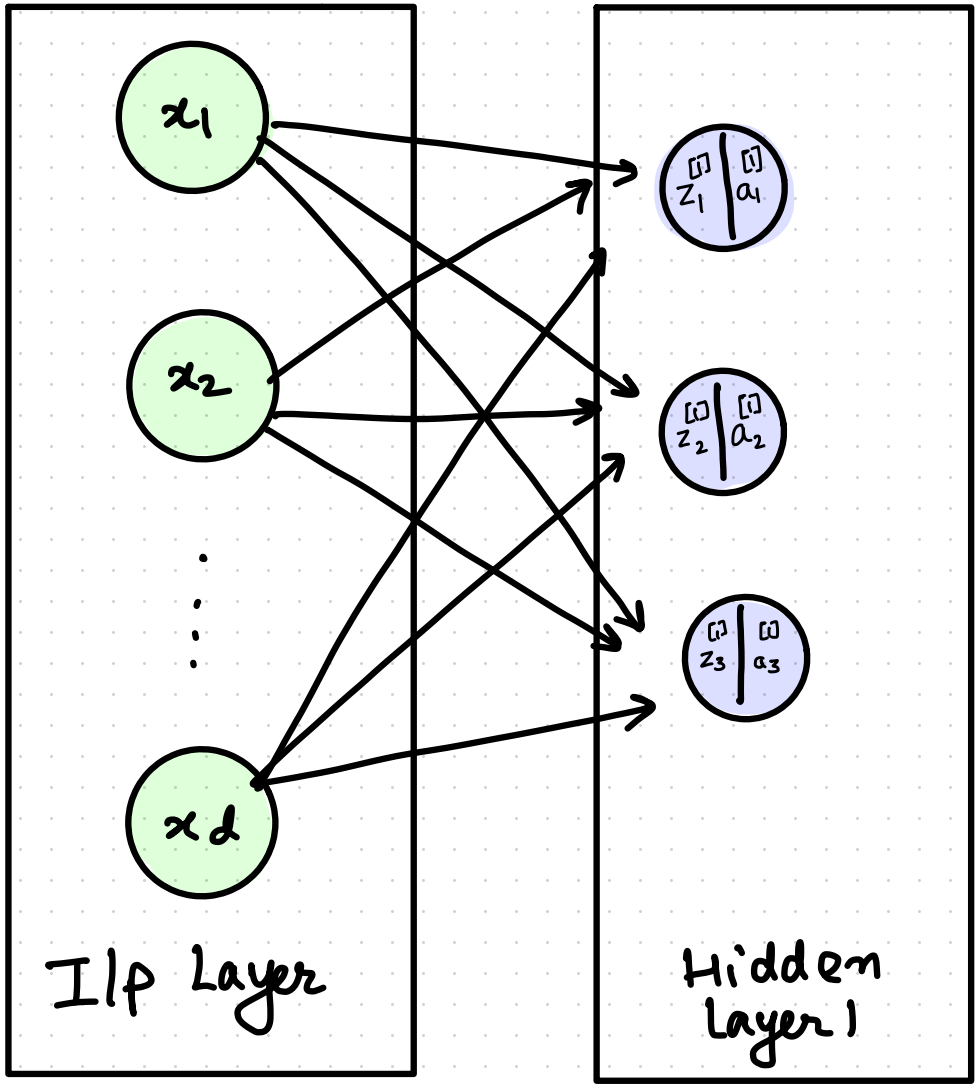


$$z^{[1]}_{3\times 1} = \begin{bmatrix} — & w_1^{[1]T} & — \\ — & w_2^{[1]T} & — \\ — & w_3^{[1]T} & — \end{bmatrix}_{3\times 3} a^{[0]}_{3\times 1} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix}$$

$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]}$$

↖ capitals for matrices

FORWARD PROPAGATION (VECTORISATION)



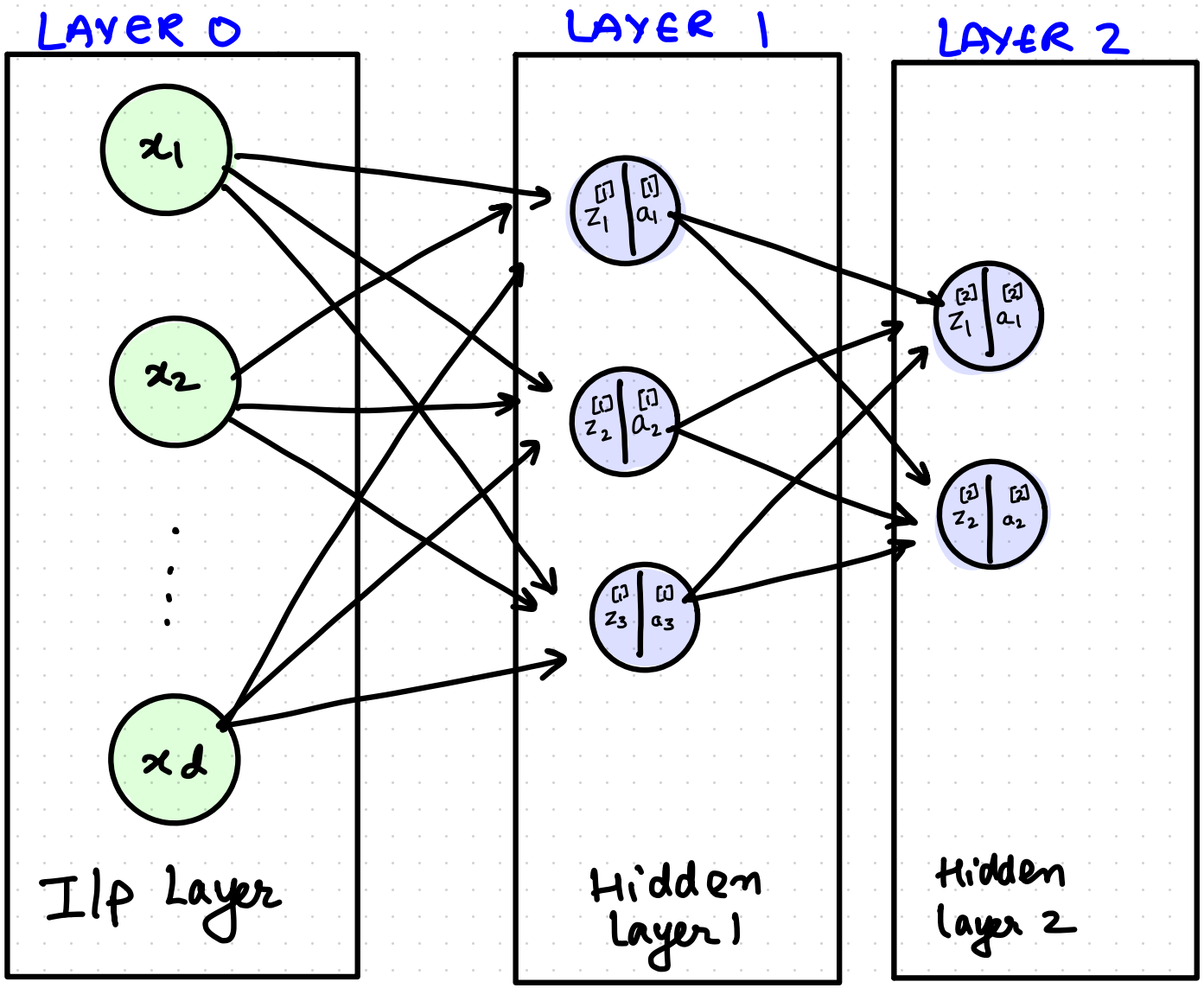
$$z^{[1]}_{3\times 1} = \begin{bmatrix} — & w_1^{[1]T} & — \\ — & w_2^{[1]T} & — \\ — & w_3^{[1]T} & — \end{bmatrix}_{3\times 3} a^{[0]}_{3\times 1} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix}$$

$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]}$$

↖ capitals for matrices

$$a^{[1]} = g(z^{[1]})$$

FORWARD PROPAGATION

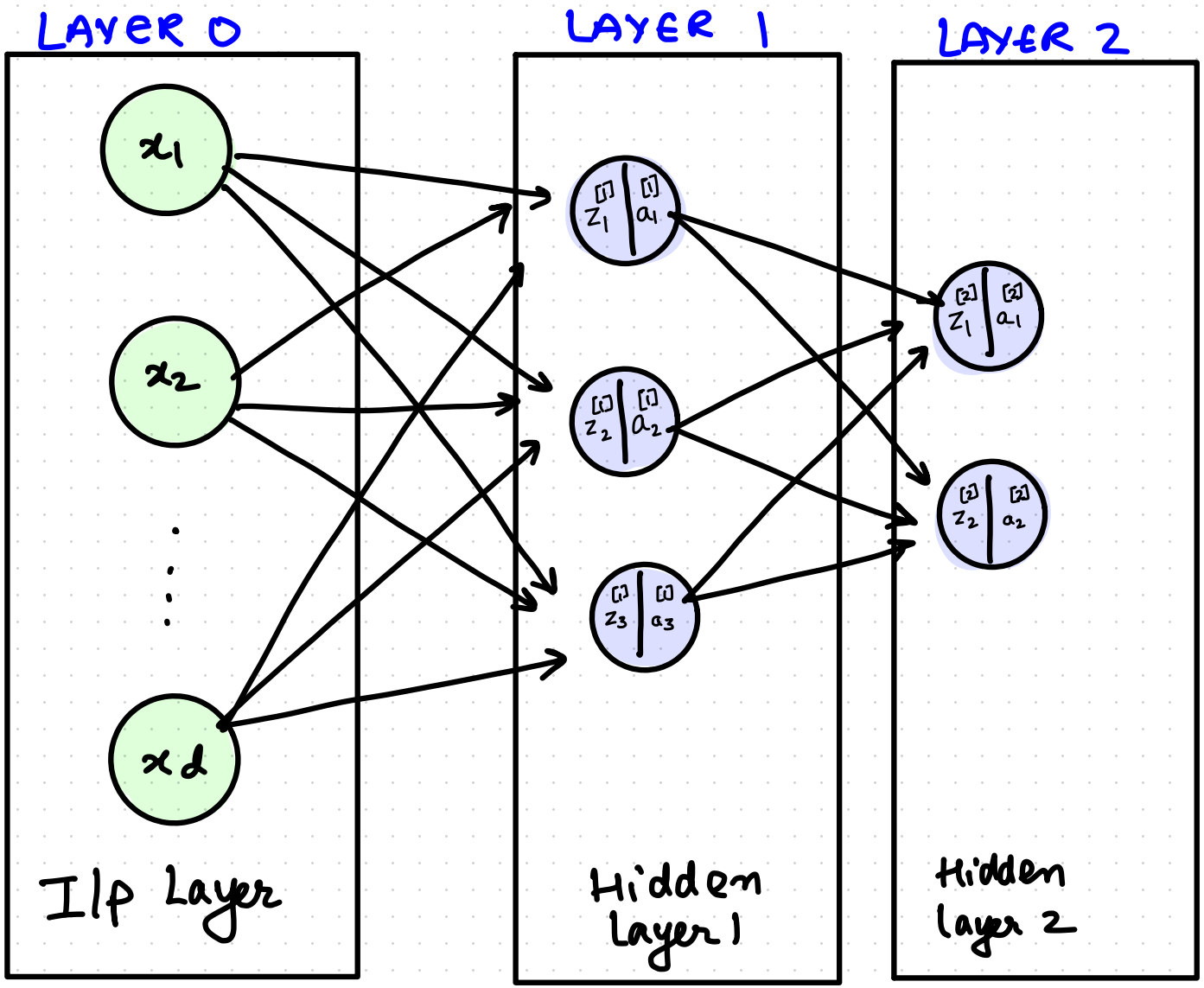


$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$$

$$a^{[2]} = g(z^{[2]})$$

Q. Dim. of $W^{[2]}$?

FORWARD PROPAGATION



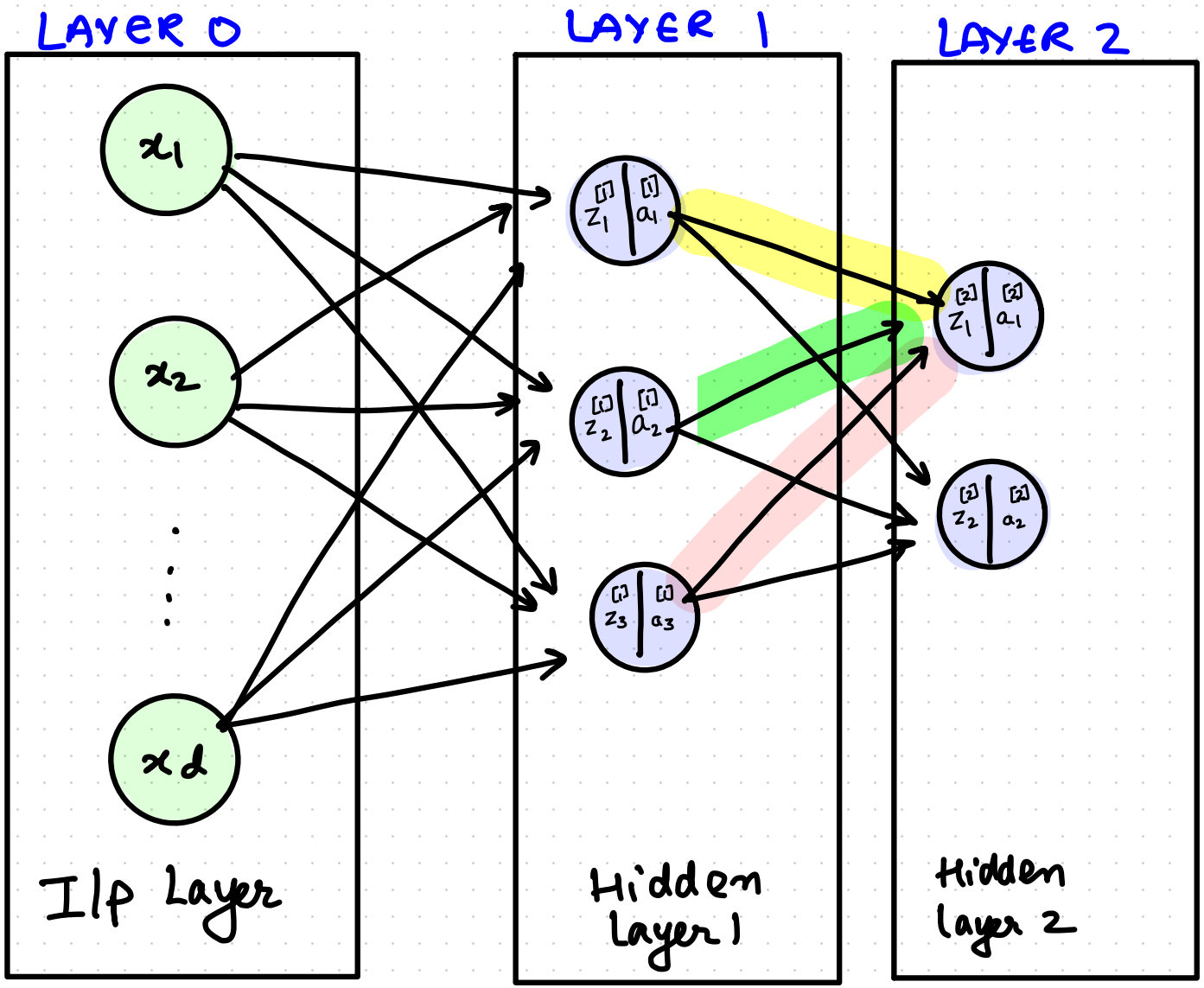
$$z^{[2]} = w^{[2]} a^{[1]} + b^{[2]}$$

$$a^{[2]} = g(z^{[2]})$$

Q. Dim. of $w^{[2]}$?

$$w^{[2]} = \begin{bmatrix} - & w_1^{[2]} & - \\ - & w_2^{[2]} & - \end{bmatrix}$$

FORWARD PROPAGATION



$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$$

$$a^{[2]} = g\left(z^{[2]}\right)$$

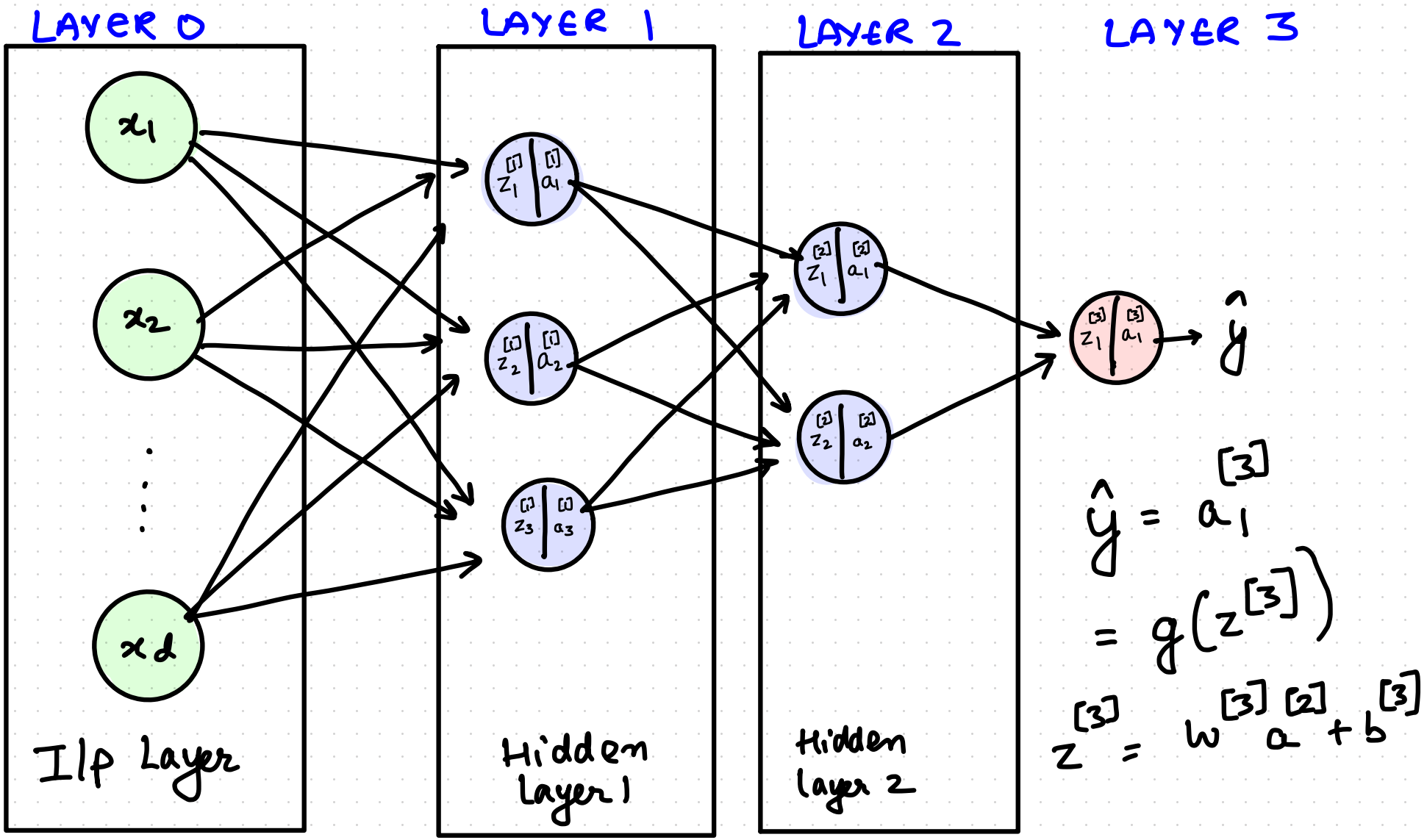
Q. Dim. of $W^{[2]}$?

$$W^{[2]} = \begin{bmatrix} - & w_1^{[2]T} & - \\ - & w_2^{[2]T} & - \end{bmatrix}$$

$w_1^{[2]} \in R^3$

$\therefore W^{[2]} \in R^{2\times 3}$

FORWARD PROPAGATION

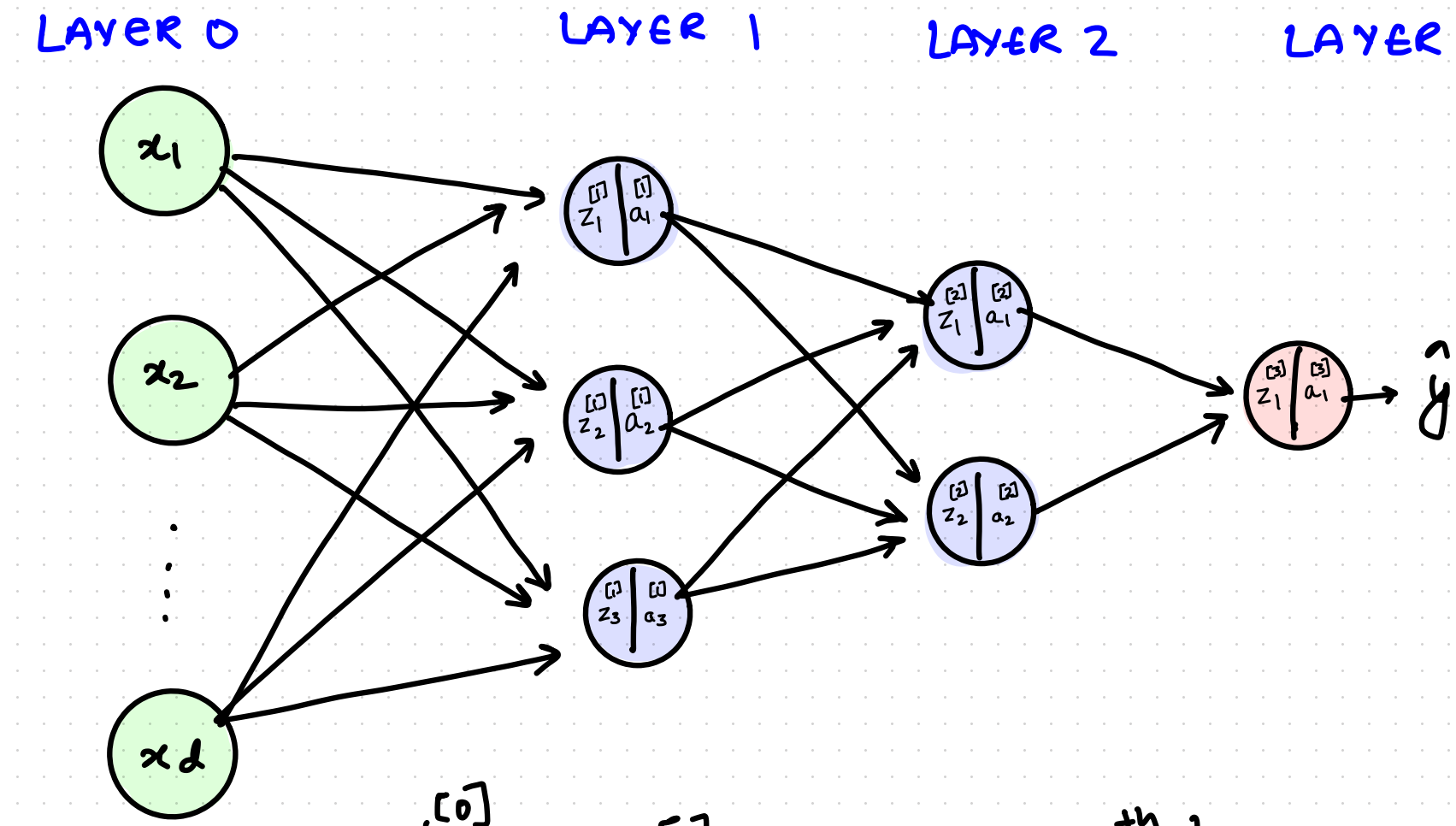


$$\hat{y} = a_1^{[3]}$$

$$= g(z^{[3]})$$

$$z^{[3]} = w^{[3]} a^{[2]} + b^{[3]}$$

WHAT CAN WE SAY ABOUT SHAPES OF a, b, W

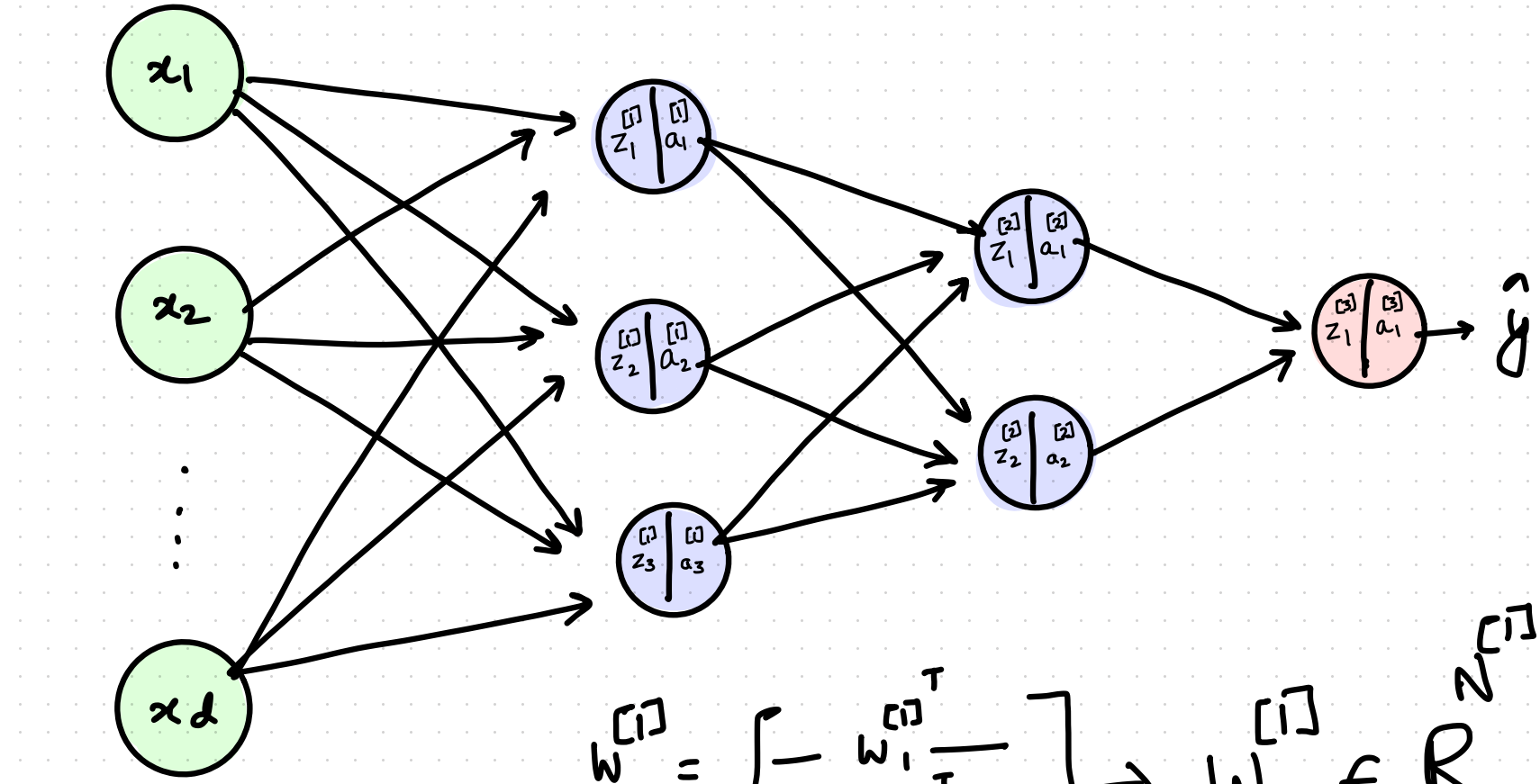


$a^{[0]} \in R^d$ or $R^{N^{[0]}}$

$d \rightarrow$ # i/p features

$N^{[0]}$ = # units in 0^{th} Layer

WHAT CAN WE SAY ABOUT SHAPES OF a, b, W

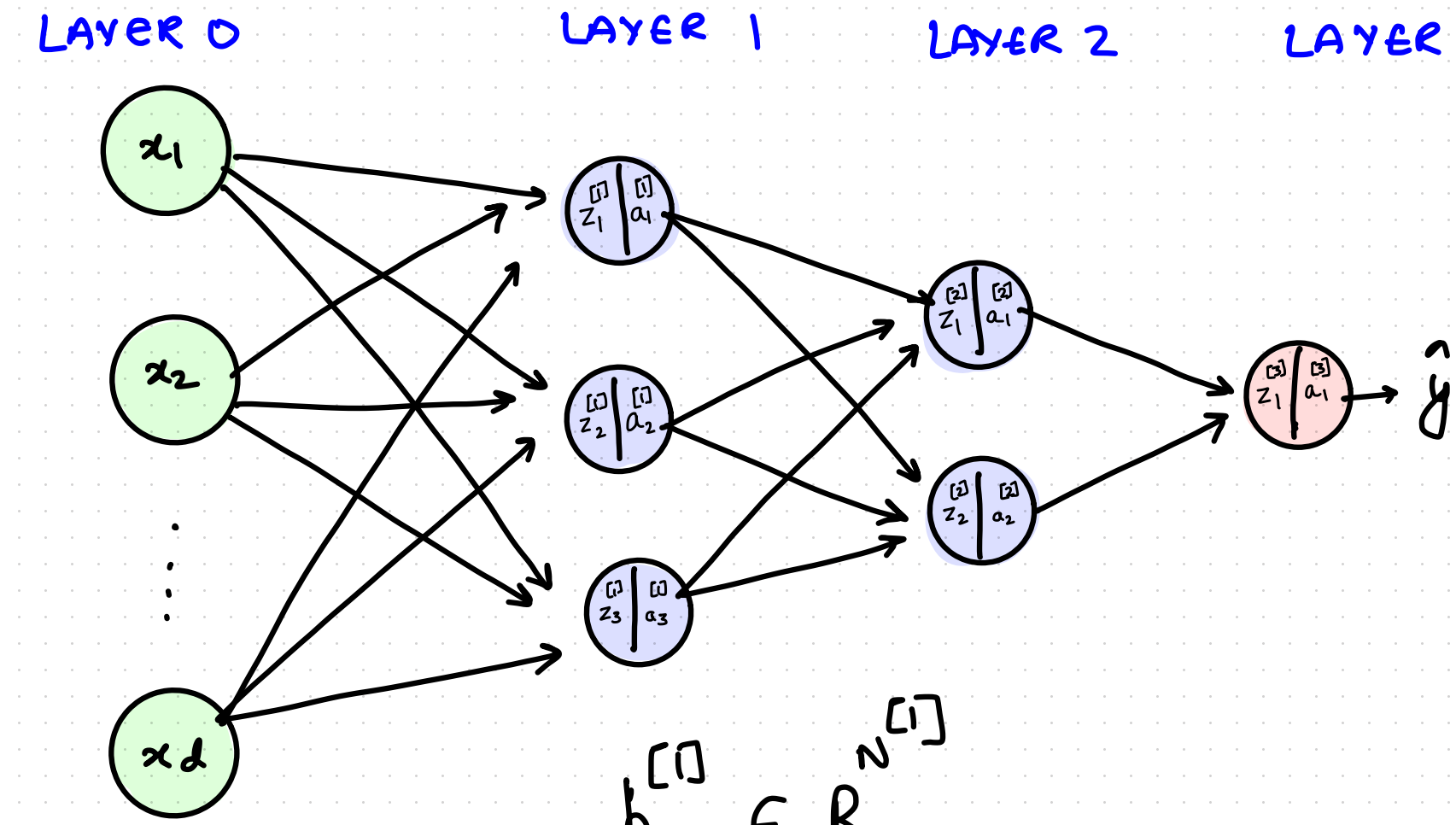


$a^{[0]} \in R^d$

$d \rightarrow$ # i/p features

$$W^{[1]} = \begin{bmatrix} - & w_1^{[1]T} & - \\ - & w_2^{[0]T} & - \\ & \vdots & \\ - & w_{n^{[0]}}^{[0]T} & - \end{bmatrix} \Rightarrow W^{[1]} \in R^{n^{[1]} \times n^{[0]}}$$

WHAT CAN WE SAY ABOUT SHAPES OF a, b, W



$a^{[0]} \in R^{d}$

$d \rightarrow$ # i/p features

$b^{[1]} \in R^{n^{[1]}}$

SUMMARY OF SHAPES

$N^{[l]}$: # NODES IN l^{th} Layer

$a^{[0]} \in R^{N^{[0]}}$

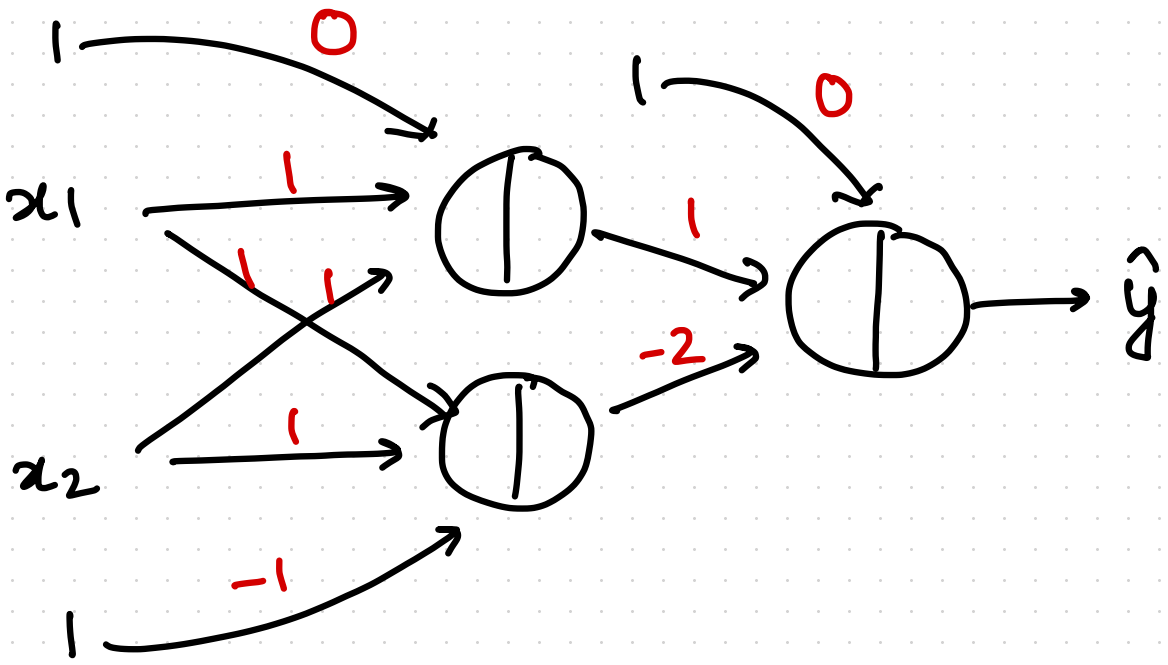
$W^{[l]} \in R^{N^{[l]} \times N^{[l-1]}}$

$b^{[l]} \in R^{N^{[l]}}$

$Z^{[l]} \in R^{N^{[l]}}$

$a^{[l]} \in R^{N^{[l]}}$

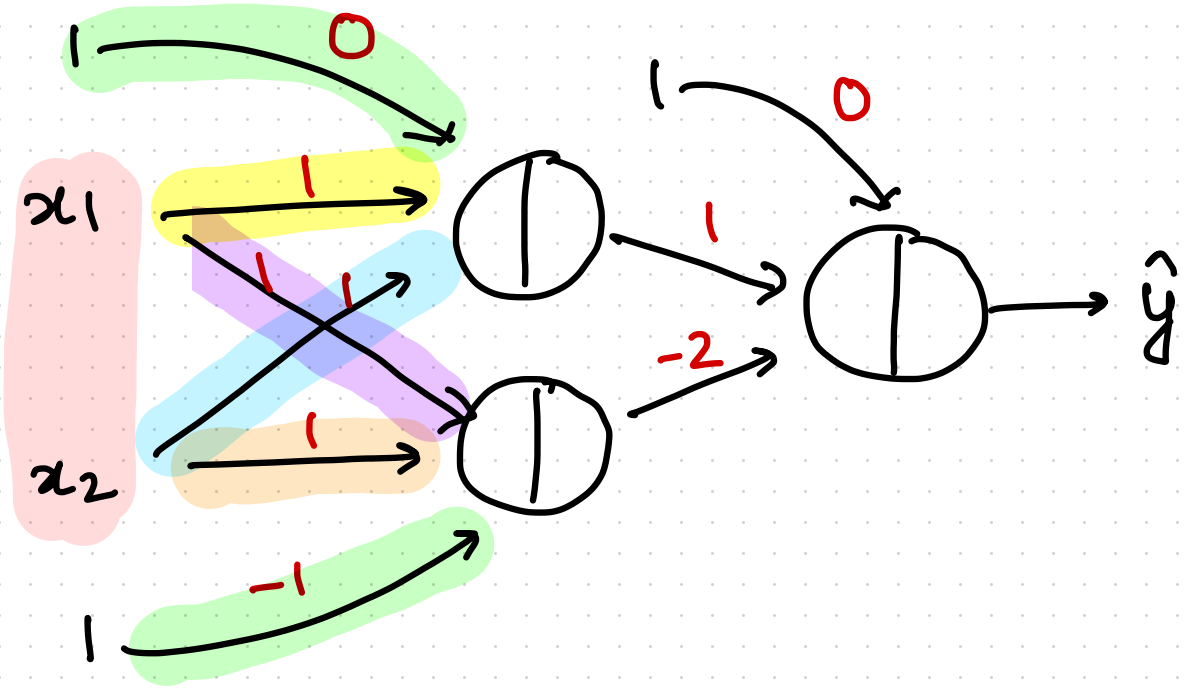
XOR USING "MLP" RELU



CONFIRM IF ABOVE N/W IS CORRECT FOR XOR.

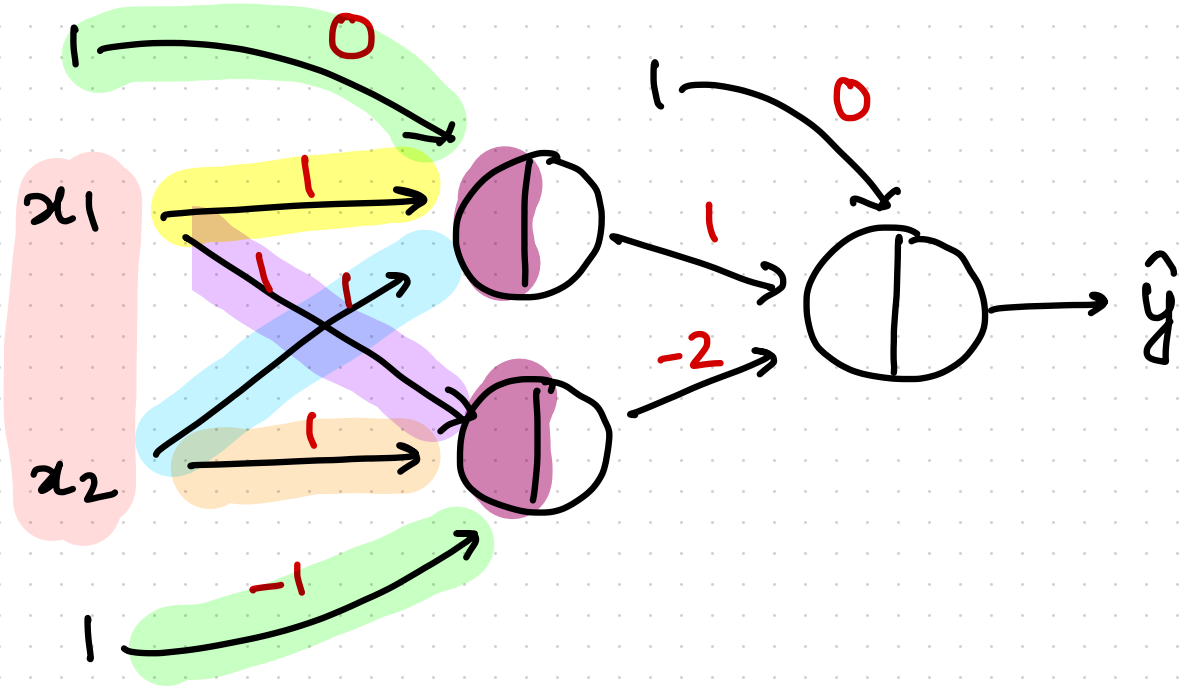
Start with $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y_{TRUE} = 0$

XOR USING "MLP" RELU



$$a^{[0]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad b^{[1]} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; \quad W^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

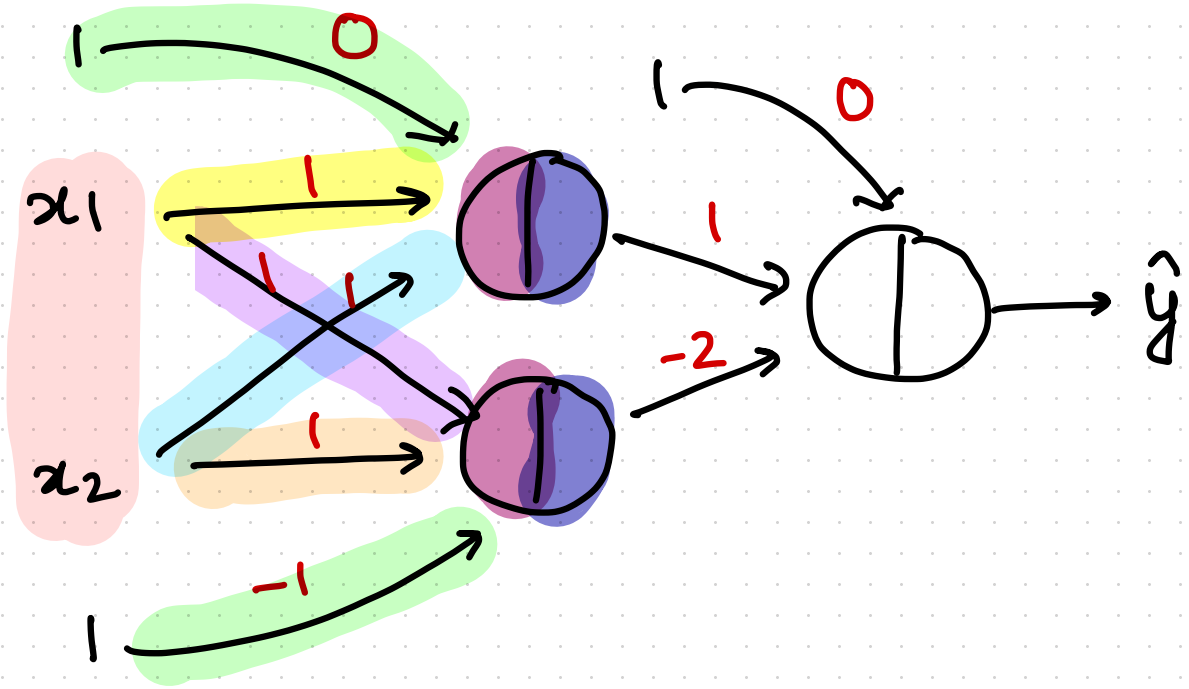
XOR USING "MLP" RELU



$$a^{[0]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad b^{[1]} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; \quad W^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

XOR USING "MLP" RELU

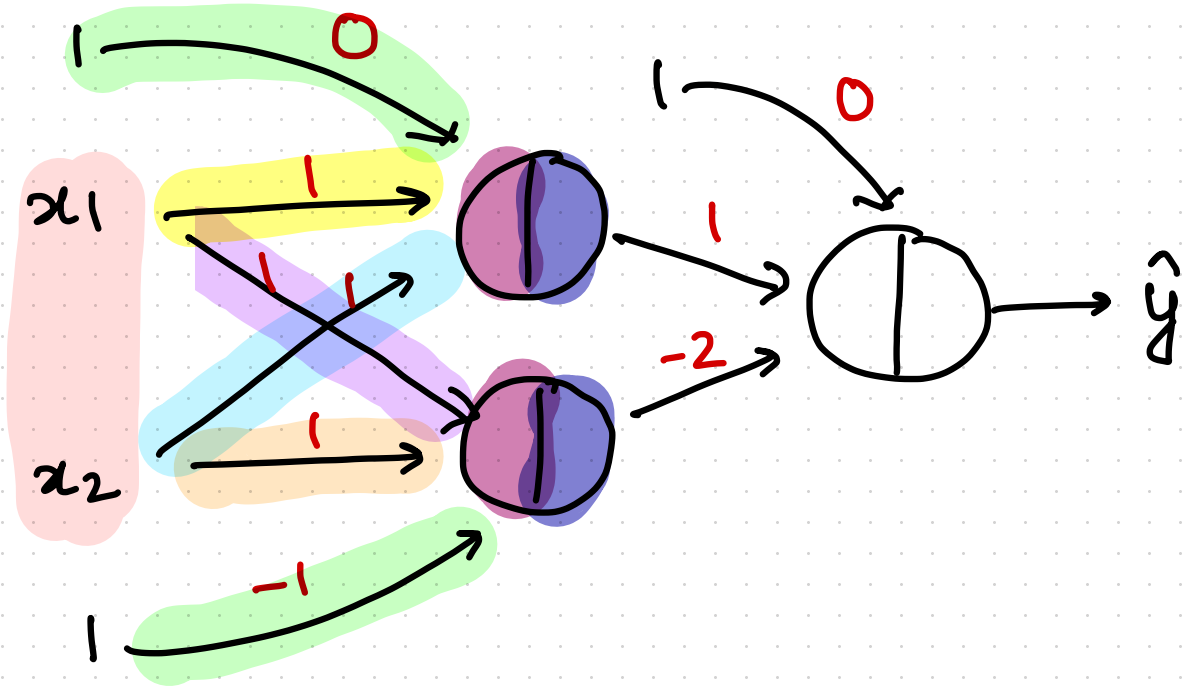


$$a^{[0]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad b^{[1]} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; \quad W^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$a^{[1]} = \text{RELU} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

XOR USING "MLP" RELU

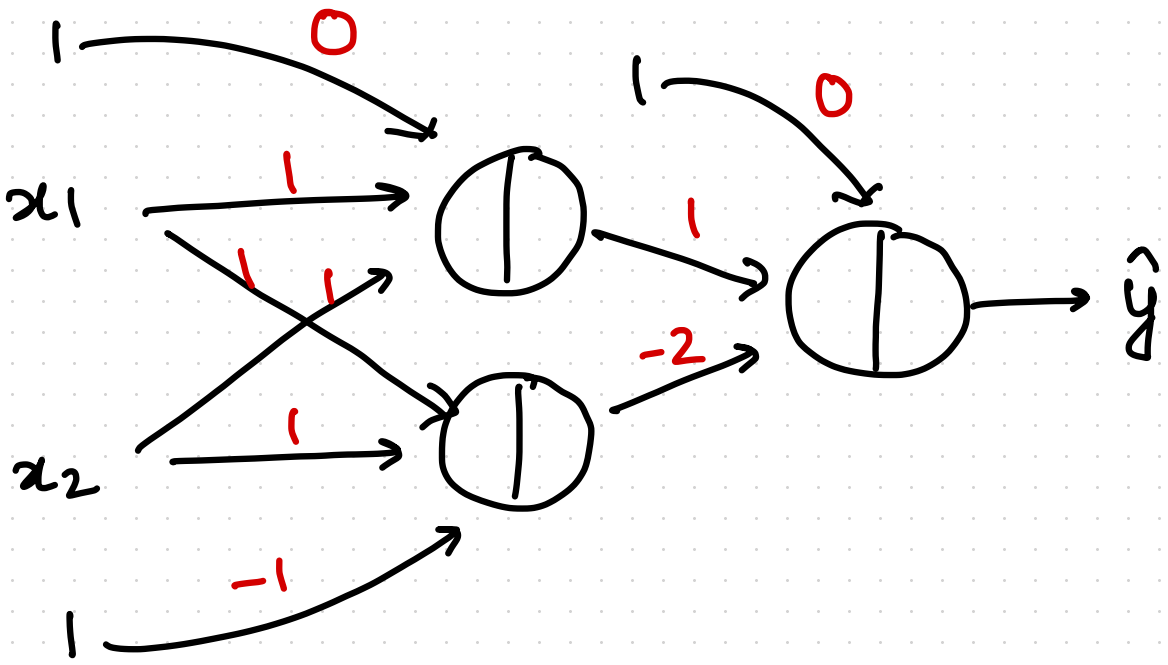


$$a^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad ; \quad W^{[2]} = \begin{bmatrix} 1 & -2 \end{bmatrix} \quad ; \quad b^{[2]} = [0]$$

$$z^{[2]} = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + [0] = 0 \quad ; \quad a^{[2]} = \hat{y} = \text{RELU}([0]) = 0$$

✓

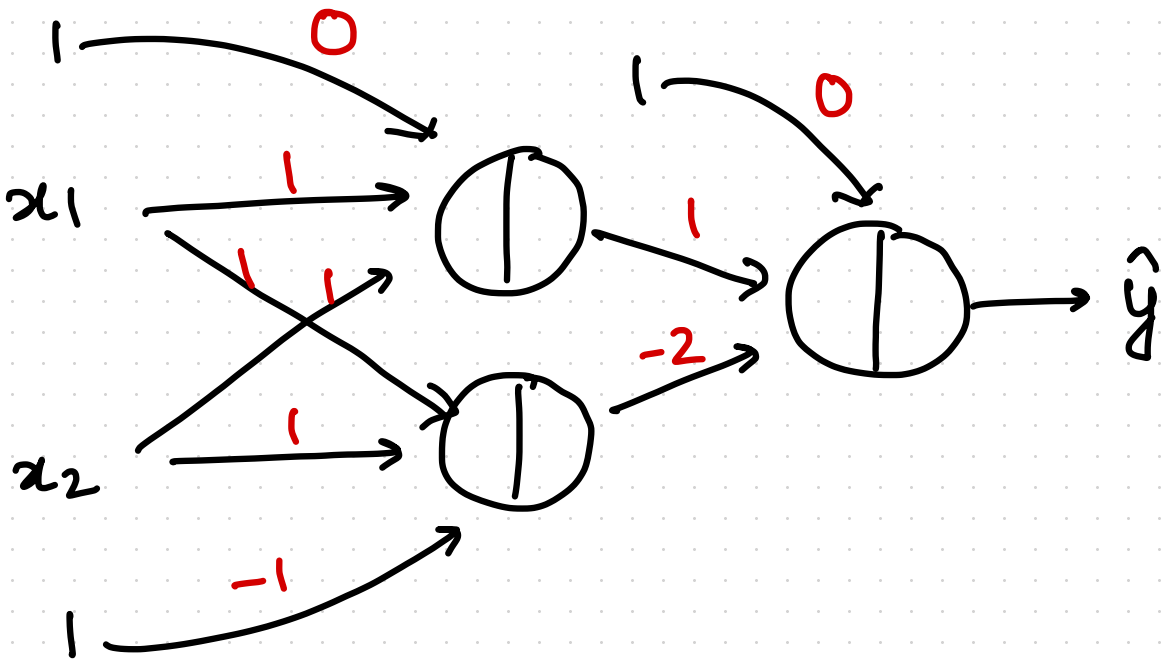
XOR USING "MLP" RELU



CONFIRM IF ABOVE N/W IS CORRECT FOR XOR.

Start with $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $y_{TRUE} = 1$

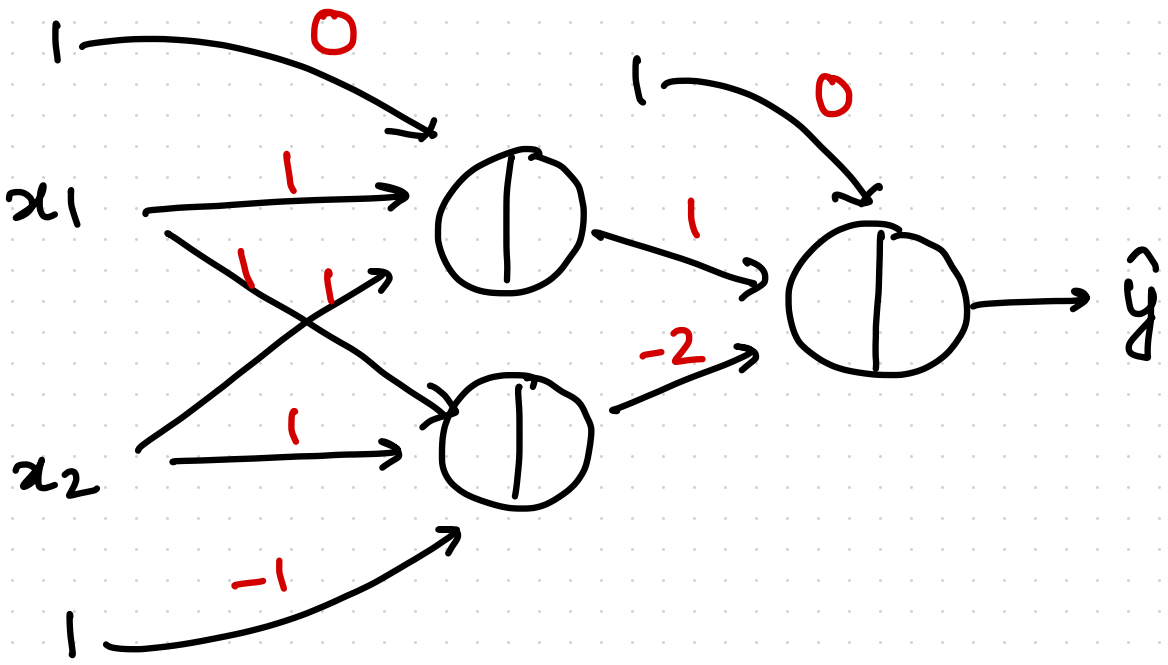
XOR USING "MLP" RELU



$$z^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$a^{[1]} = \text{RELU}(z^{[1]}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

XOR USING "MLP" RELU



$$z^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$a^{[1]} = \text{RELU}(z^{[1]}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$z^{[2]} = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0] = [1]$$

$$a^{[2]} = \text{RELU}(1) = 1$$

$$= \hat{y}$$

COMPUTATION FOR N EXAMPLES

$x^{(i)} \in R^{d}$ or $R^{n^{[0]}}$

$$X = \begin{bmatrix} — & x^{(1)T} & — \\ — & x^{(2)T} & — \\ & \vdots & \\ — & x^{(N)T} & — \end{bmatrix} = \begin{bmatrix} — & a^{[0](1)T} & — \\ & \vdots & \\ — & a^{[0](N)T} & — \end{bmatrix} = A^{[0]} \in R^{N \times n^{[0]}}$$

↑ matrix

COMPUTATION FOR N EXAMPLES

$x_{(i)} \in R^{d}$ or $R^{N^{[0]}}$

$$X = \begin{bmatrix} — & x_{(1)}^{T} & — \\ — & x_{(2)}^{T} & — \\ & \vdots & \\ — & x_{(N)}^{T} & — \end{bmatrix} = \begin{bmatrix} — & a_{(1)}^{[0]T} & — \\ & \vdots & \\ — & a_{(N)}^{[0]T} & — \end{bmatrix} = A^{[0]} \in R^{N \times N^{[0]}}$$

$Z^{[i]}$ ← Layer

$Z^{[i]}_{(i)}$ → Instance/ Sample

$$= W^{[i]} A^{[0]T}_{(i)} + b^{[i]} \in R^{N^{[i]}}$$

Independent of 'i'

$$a^{[0]}_{(i)} \in R^{d} \equiv \begin{bmatrix} \\ \vdots \\ \end{bmatrix}$$

COMPUTATION FOR N EXAMPLES

$x_{(i)} \in R^D$

$$X = \begin{bmatrix} — & x_{(1)}^T & — \\ — & x_{(2)}^T & — \\ & \vdots & \\ — & x_{(N)}^T & — \end{bmatrix} = \begin{bmatrix} — & a_{(1)}^{[0]T} & — \\ & \vdots & \\ — & a_{(N)}^{[0]T} & — \end{bmatrix}$$

$$z_{(1)}^{[1]} = \left\{ W^{[1]} a_{(1)}^{[0]} + b^{[1]} \right\} \in R^{N^{[1]}}$$

$$z_{(2)}^{[1]} = \left\{ W^{[1]} a_{(2)}^{[0]} + b^{[1]} \right\} \in R^{N^{[1]}}$$

$$\vdots$$

$$z_{(N)}^{[1]} = \left\{ W^{[1]} a_{(N)}^{[0]} + b^{[1]} \right\} \in R^{N^{[1]}}$$

COMPUTATION FOR N EXAMPLES

$x_{(i)} \in R^D$

$$X = \begin{bmatrix} — & x_{(1)}^T & — \\ — & x_{(2)}^T & — \\ & \vdots & \\ — & x_{(N)}^T & — \end{bmatrix} = \begin{bmatrix} — & a_{(1)}^{[0]T} & — \\ & \vdots & \\ — & a_{(N)}^{[0]T} & — \end{bmatrix} = A^{[0]} \in R^{N \times N^{[0]}}$$

$$z_{(1)}^{[1]} = W^{[1]} a_{(1)}^{[0]} + b^{[1]} \in R^{N^{[1]}}$$

$$z_{(2)}^{[1]} = W^{[1]} a_{(2)}^{[0]} + b^{[1]} \in R^{N^{[1]}}$$

$$\vdots \quad ! \quad \vdots$$

$$z_{(N)}^{[1]} = W^{[1]} a_{(N)}^{[0]} + b^{[1]} \in R^{N^{[1]}}$$

COMPUTATION FOR N EXAMPLES

$x_{(i)} \in R^D$

$$X = \begin{bmatrix} — & x_{(1)}^T & — \\ — & x_{(2)}^T & — \\ & \vdots & \\ — & x_{(N)}^T & — \end{bmatrix} = \begin{bmatrix} — & a_{(1)}^{[0]T} & — \\ & \vdots & \\ — & a_{(N)}^{[0]T} & — \end{bmatrix} = A^{[0]} \in R^{N \times N^{[0]}}$$

$$z_{(1)}^{[1]} = W^{[1]} a_{(1)}^{[0]} + b^{[1]}$$

$$z_{(2)}^{[1]} = W^{[1]} a_{(2)}^{[0]} + b^{[1]}$$

$$\vdots$$

$$z_{(N)}^{[1]} = W^{[1]} a_{(N)}^{[0]} + b^{[1]}$$

$$\Rightarrow Z^{[1]} = \begin{bmatrix} — & z_{(1)}^{[1]T} & — \\ — & z_{(2)}^{[1]T} & — \\ & \vdots & \\ — & z_{(N)}^{[1]T} & — \end{bmatrix} \in R^{N \times N^{[1]}}$$

COMPUTATION FOR N EXAMPLES

$x_{(i)} \in R^D$

$$X = \begin{bmatrix} — & x_{(1)}^T & — \\ — & x_{(2)}^T & — \\ & \vdots & \\ — & x_{(N)}^T & — \end{bmatrix} = \begin{bmatrix} — & a_{(1)}^{[0]T} & — \\ & \vdots & \\ — & a_{(N)}^{[0]T} & — \end{bmatrix} = A^{[0]} \in R^{N \times N^{[0]}}$$

$$z_{(1)}^{[1]} = W^{[1]} a_{(1)}^{[0]} + b^{[1]}$$

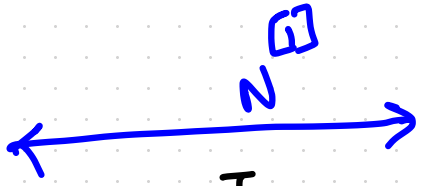
$$z_{(2)}^{[1]} = W^{[1]} a_{(2)}^{[0]} + b^{[1]}$$

$$\vdots$$

$$z_{(N)}^{[1]} = W^{[1]} a_{(N)}^{[0]} + b^{[1]}$$

$\Rightarrow$

$$Z^{[1]} = \begin{bmatrix} — & z_{(1)}^{[1]T} & — \\ — & z_{(2)}^{[1]T} & — \\ & \vdots & \\ — & z_{(N)}^{[1]T} & — \end{bmatrix} \in R^{N \times N^{[1]}}$$



COMPUTATION FOR N EXAMPLES

$$A^{[0]} \in R^{N \times N^{[0]}}$$

$$W^{[1]} \in R^{N^{[1]} \times N^{[0]}}$$

$$b^{[1]} \in R^{N^{[1]}}$$

$$B^{[1]} = \begin{bmatrix} | & | & | \\ b^{[1]} & b^{[1]} & \cdots \\ | & | & | \end{bmatrix} \in R^{N \times N^{[1]}}$$

all same entries

$$Z^{[1]} \in R^{N \times N^{[1]}}$$

COMPUTATION FOR N EXAMPLES

$$A^{[0]} \in R^{N \times N^{[0]}}$$

$$W^{[1]} \in R^{N^{[1]} \times N^{[0]}}$$

$$b^{[1]} \in R^{N^{[1]}}$$

$$B^{[1]} = \begin{bmatrix} — & b^{[1]T} & — \\ & \vdots & \\ — & b^{[1]T} & — \end{bmatrix} \in R^{N \times N^{[1]}}$$

$$Z^{[1]} \in R^{N \times N^{[1]}}$$

$$Z^{[1]} = A^{[0]} W^{[1]T} + B^{[1]}$$

COMPUTATION FOR N EXAMPLES

$$A^{[0]} \in \mathbb{R}^{N \times N^{[0]}}$$

$$W^{[1]} \in \mathbb{R}^{N^{[1]} \times N^{[0]}}$$

$$b^{[1]} \in \mathbb{R}^{N^{[1]}}$$

$$B^{[1]} = \begin{bmatrix} — & b^{[1]^T} & — \\ — & b^{[1]^T} & — \end{bmatrix} \in \mathbb{R}^{N \times N^{[1]}}$$

$$Z^{[1]} \in \mathbb{R}^{N \times N^{[1]}}$$

$$Z^{[1]} = A^{[0]} W^{[1]^T} + B^{[1]} \Rightarrow A^{[1]} = g(Z^{[1]})$$

$$Z^{[l]} = A^{[l-1]} W^{[l]^T} + B^{[l]}$$

$$A^{[l]} = g(Z^{[l]})$$

XOR ALL EXAMPLES

① $X = A^{[0]} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}_{4\times 2}$; $\hat{y} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}_{4\times 1}$

XOR ALL EXAMPLES

① $X = A^{[0]} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}_{4\times 2}$; $\hat{y} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}_{4\times 1}$

$W^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2\times 2}$; $b^{[1]} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \Rightarrow B^{[1]} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}_{4\times 2}$

XOR ALL EXAMPLES

① $X = A^{[0]} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}_{4\times 2}$; $\hat{y} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}_{4\times 1}$

$W^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2\times 2} = W^{[1]^T}$; $b^{[1]} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \Rightarrow B^{[1]} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}_{4\times 2}$

$Z^{[1]} = A^{[0]} W^{[1]^T} + B^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$

XOR ALL EXAMPLES

$$Z^{[1]} = A^{[0]} W^{[1]} + B^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^{[1]} = RELU\left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \right) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}_{4 \times 2}$$

$$W^{[2]} = [1 \ -2]$$

$$b^{[2]} = [0] \Rightarrow B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

XOR ALL EXAMPLES

$$A^{[1]} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}_{4\times 2}$$

$$W^{[2]} = \begin{bmatrix} 1 & -2 \end{bmatrix}_{1\times 2}$$

$$b^{[2]} = [0] \Rightarrow B^{[2]} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4\times 1}$$

$$Z^{[2]} = A^{[1]} W^{[2]^T} + B^{[2]} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

XOR ALL EXAMPLES

$$Z^{[2]} = A^{[1]} W^{[2]T} + B^{[2]} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$A^{[2]} = \hat{y} = RELU\left(\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right) = y \text{ G.T}$$

PARAMETERS

Parameters: $W^{[l]}$; $b^{[l]}$ $\forall\, l$

Q: # parameters for XOR example?

PARAMETERS

Parameters: $W^{[l]}$; $b^{[l]}$ $\forall\, l$

Q: # parameters for XOR example?

$N^{[0]} = 2$; $N^{[1]} = 2$; $N^{[2]} = 1$

$W^{[1]} \in R^{N^{[1]} \times N^{[0]}} = \begin{bmatrix} \\ \end{bmatrix}_{2\times 2}$ = 4 params = $N^{[1]} * N^{[0]}$

$b^{[1]} \in R^{N^{[1]}} = \begin{bmatrix} \\ \end{bmatrix}_{2\times 1}$ = 2 params = $N^{[1]}$

$W^{[2]} \rightarrow$ 2 params = $N^{[2]} * N^{[1]}$ & $b^{[2]} = N^{[2]}$ params

PARAMETERS

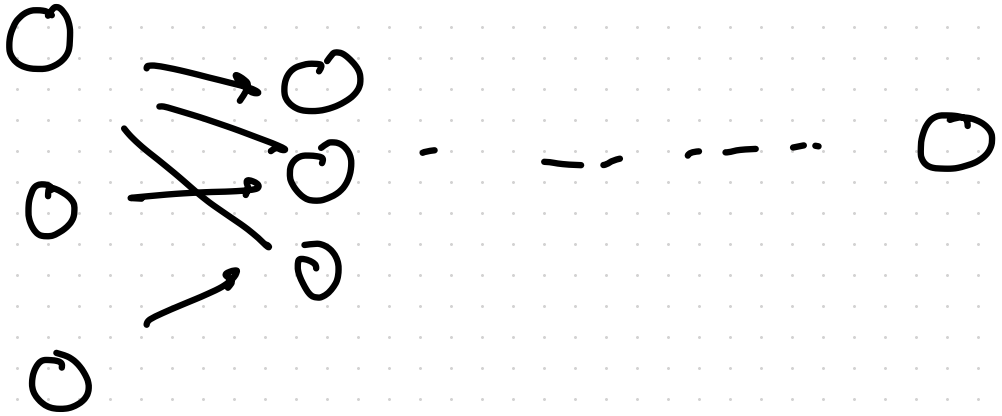
Parameters: $W^{[l]}$; $b^{[l]}$ $\forall\, l$

$$\text{PARAMS} = \sum_{l=1}^{L} N^{[l]} \cdot N^{[l-1]} + N^{[l]}$$

? XOR 4 examples, 9 parameters !!

⇒ NOTEBOOK: XOR demo

LEARNING PARAMETERS



FORWARD PROPAGATION (PREDICT BASED ON CURRENT PARAMS)

BACKWARD PROPAGATION (CHANGE WEIGHTS TO IMPROVE OBJECTIVE)

LEARNING PARAMETERS

Assume i/p : $X \in R^{N \times N^{[0]}}$

o/p : $\hat{y}$; G.T. : y

$$\text{Loss} = \frac{1}{N} \sum_{i=1}^{N} L(\hat{y}^{(i)}, y) \quad \text{or} \quad \sum_{i=1}^{N} L(\hat{y}^{(i)}, y)$$

Params : $W^{[1]}$, $b^{[1]}$, ...

LEARNING PARAMETERS

Assume i/p : $X \in R^{N \times N^{[0]}}$

o/p : $\hat{y}$; G.T. : y

$$\text{Loss} = \frac{1}{N} \sum_{i=1}^{N} L\left(\hat{y}^{(i)}, y\right) = J(\theta)$$

Params : $\theta = \{W^{[1]}, b^{[1]}, \ldots\}$

GRADIENT DESCENT

① INIT Params randomly

② Till convergence :

$$W^{[1]} = W^{[1]} - \alpha \boxed{\frac{\partial J(\theta)}{\partial W^{[1]}}} \ldots$$

→ HOW TO COMPUTE?

⋮

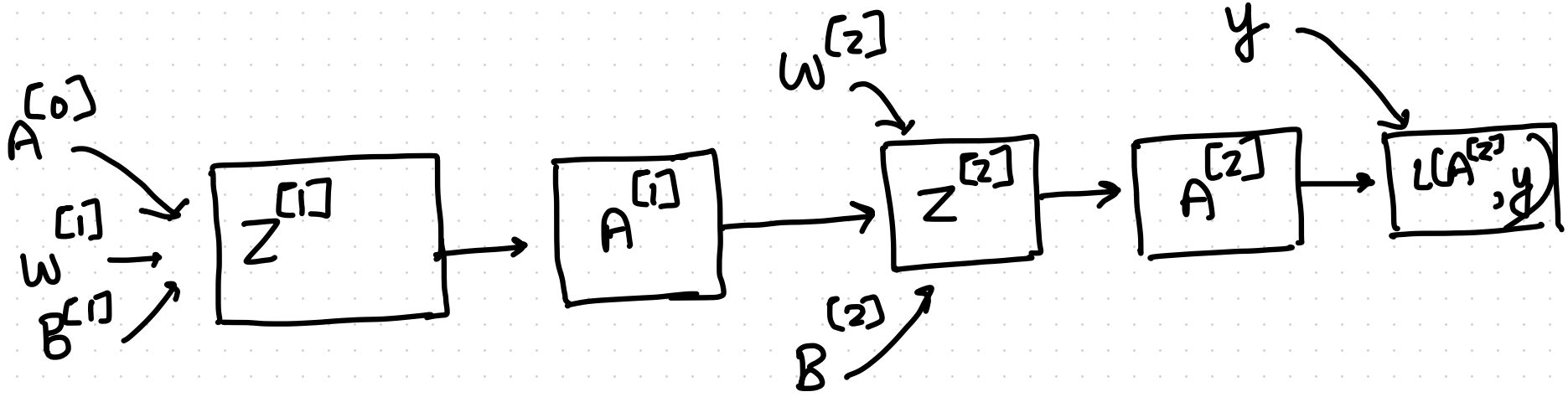
COMPUTATION GRAPH (FOR XOR EXAMPLE)

① $z^{[1]} = A^{[0]} W^{[1]^T} + B^{[1]}$

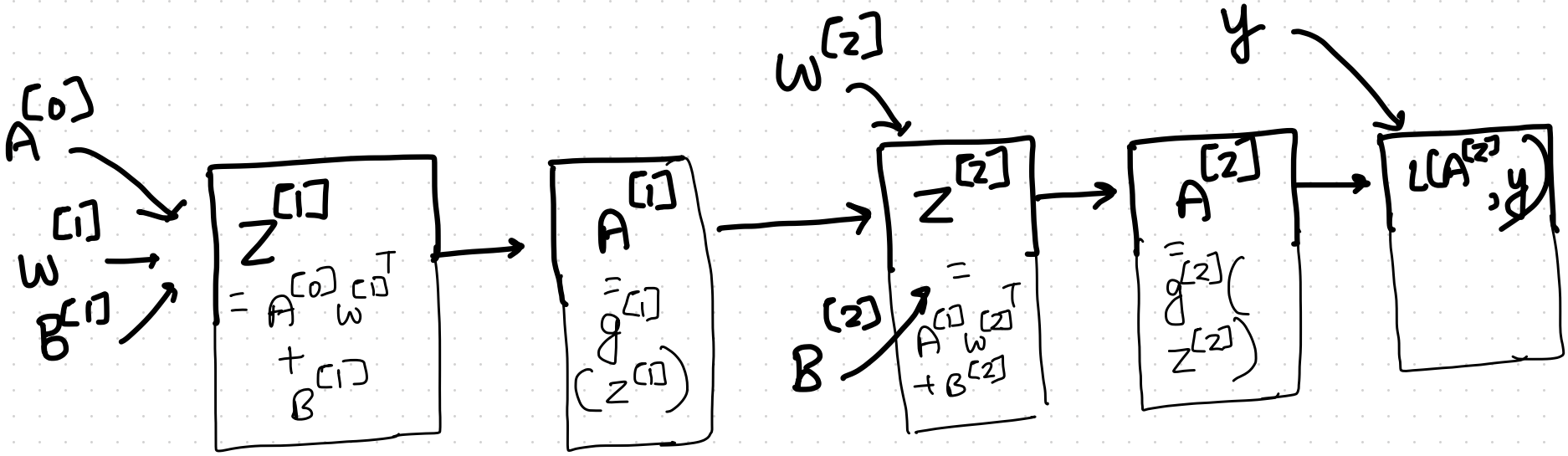
② $A^{[1]} = g^{[1]}(z^{[1]})$

③ $z^{[2]} = A^{[1]} W^{[2]^T} + B^{[2]}$

④ $A^{[2]} = g^{[2]}(z^{[2]}) = \hat{y}$



COMPUTATION GRAPH (FOR XOR EXAMPLE)



Let $g^{[1]}$ = SIGMOID ; ASSUME CROSS ENTROPY LOSS

$g^{[2]}$ = SIGMOID

WHAT IS $\frac{\partial L(\theta)}{\partial W^{[1]}}$? ; WHAT IS $\frac{\partial L(\theta)}{\partial A^{[2]}}$?

DERIVATIVES OF ACTIVATION FUNCTIONS

RELU

$$g(z) = \begin{cases} z; & z > 0 \\ 0; & z < 0 \\ \text{undefined}; & \text{o/w} \end{cases}$$

↓ Assume $z \neq 0$

$$g(z) = \begin{cases} z; & z \geq 0 \\ 0; & z < 0 \end{cases}$$

$$g'(z) = \begin{cases} 1; & z \geq 0 \\ 0; & \text{o/w} \end{cases}$$

DERIVATIVES OF ACTIVATION FUNCTIONS

RELU (LEAKY)

$$g(z) = \begin{cases} z; & z > 0 \\ \alpha z; & z < 0 \\ \text{undefined}; & \text{o/w} \end{cases}$$

$$\downarrow$$

$$g(z) = \begin{cases} z; & z \geq 0 \\ \alpha z; & z < 0 \end{cases}$$

$$g'(z) = \begin{cases} 1; & z \geq 0 \\ \alpha; & z < 0 \end{cases}$$

DERIVATIVES OF ACTIVATION FUNCTIONS

SIGMOID

$$g(z) = \frac{1}{1+e^{-z}}$$

$$g'(z) = \frac{-1}{(1+e^{-z})^2} \frac{d}{dz}(1+e^{-z}) = \frac{-1\,(e^{-z})(-1)}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2}$$

$$g'(z) = \frac{1+e^{-z}}{(1+e^{-z})^2} \quad \frac{-1}{(1+e^{-z})} = g(z)\left(1-g(z)\right)$$

DERIVATIVES OF ACTIVATION FUNCTIONS

TANH

$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}} = \frac{u}{v}$$

$$g'(z) = \frac{v\frac{du}{dz} - u\frac{dv}{dz}}{v^{2}} = \frac{\left(e^{z} + e^{-z}\right)\left(e^{z} + e^{-z}\right) - \left(e^{z} - e^{-z}\right)\left(e^{z} - e^{-z}\right)}{\left(e^{z} + e^{-z}\right)^{2}}$$

$$= 1 - \left(g(z)\right)^{2}$$

ACTIVATION FUNCTIONS (SUMMARY)

RELU

$$g(z) = \begin{cases} z; z \geq 0 \\ 0; z < 0 \end{cases}$$

$$g'(z) = \begin{cases} 1; z \geq 0 \\ 0; z < 0 \end{cases}$$

L-RELU

$$g(z) = \begin{cases} z; z \geq 0 \\ \alpha z; z < 0 \end{cases}$$

$$g'(z) = \begin{cases} 1; z \geq 0 \\ \alpha; z < 0 \end{cases}$$

SIGMOID

$$g(z) = \frac{1}{1+e^{-z}}$$

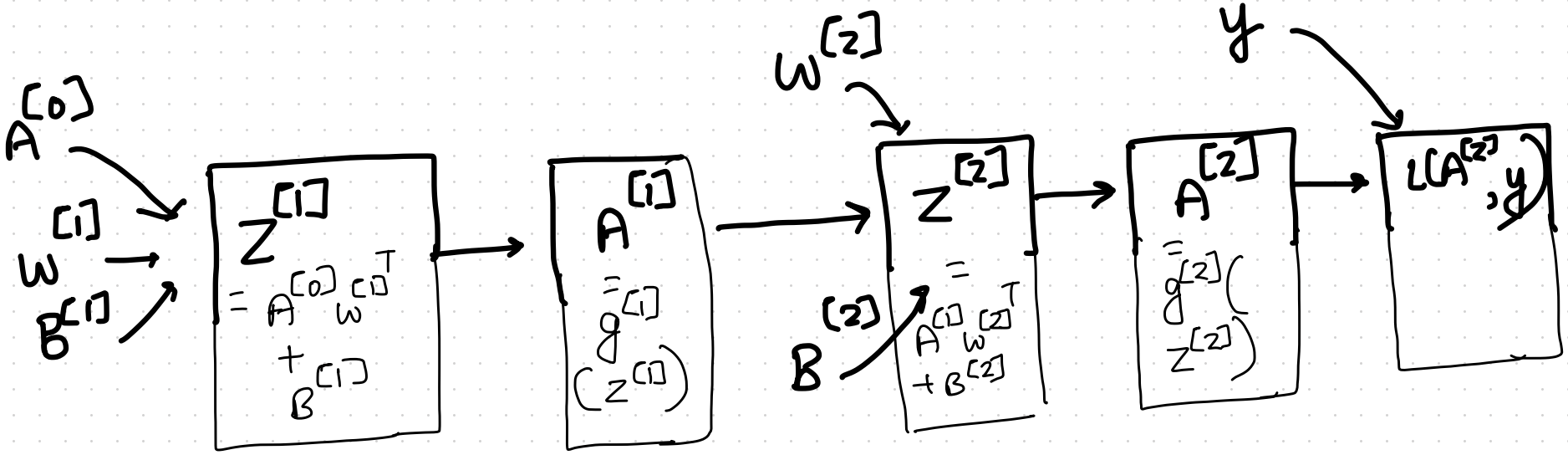
$$g'(z) = g(z) * (1 - g(z))$$

TANH

$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - (g(z))^{2}$$

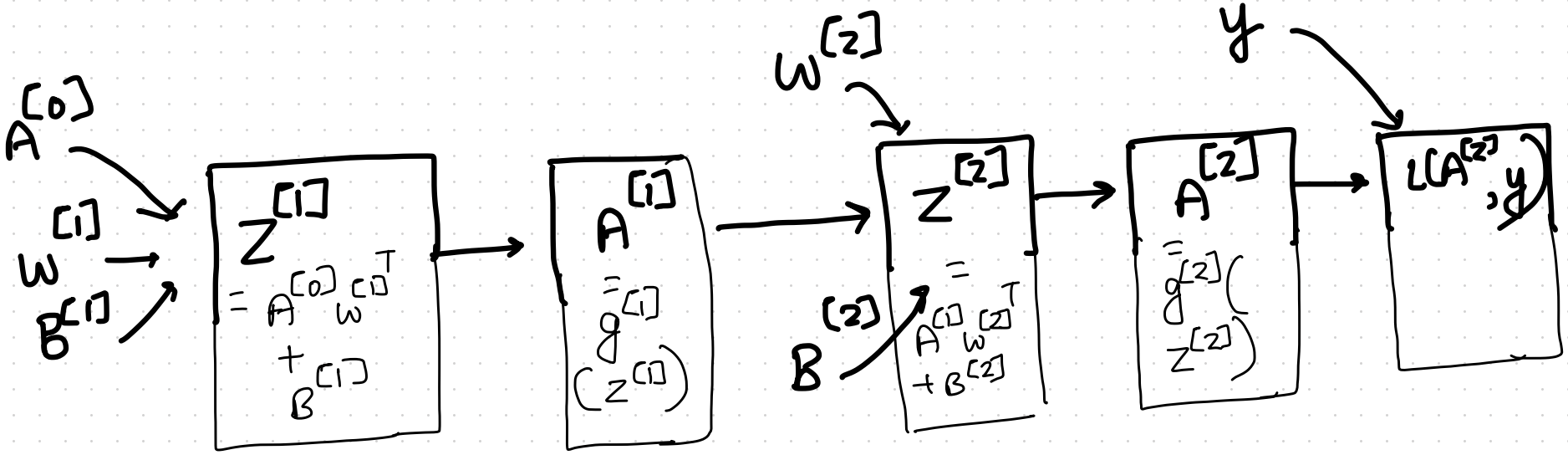
COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$L(A^{[2]}, y) = \sum_{i=1}^{N} -y_{(i)} \log A^{[2]}_{(i)} - (1 - y_{(i)}) \log (1 - A^{[2]}_{(i)})$$

WRITE IN VECTOR FORM

COMPUTATION GRAPH (FOR XOR EXAMPLE)



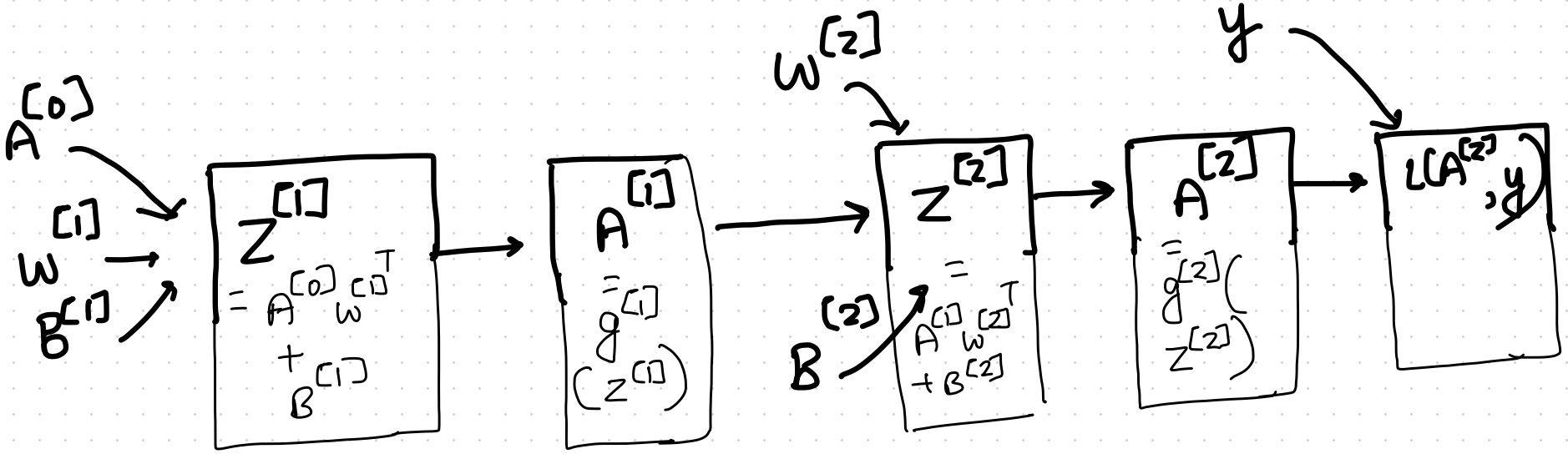
$$L(A^{[2]}, y) = \sum_{i=1}^{N} -y_{(i)} \log A^{[2]}_{(i)} - (1 - y_{(i)}) \log (1 - A^{[2]}_{(i)})$$

$$= -\underbrace{y^T}_{1 \times N} \log \underbrace{\left(A^{[2]}\right)}_{N \times 1} - \underbrace{(1 - y)^T}_{1 \times N} \log \underbrace{\left(1 - A^{[2]}\right)}_{N \times 1}$$

$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}_{N \times 1}$$

APPLIED ELEMENT-WISE

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$L(A^{[2]}, y) = \sum_{i=1}^{N} -y_{(i)} \log A^{[2]}_{(i)} - (1 - y_{(i)}) \log (1 - A^{[2]}_{(i)})$$

$$= -\underbrace{y^T}_{1 \times N} \log \underbrace{\left(A^{[2]}\right)}_{N \times 1} - \underbrace{(1 - y)^T}_{1 \times N} \log \underbrace{\left(1 - A^{[2]}\right)}_{N \times 1} \quad \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{N \times 1}$$

APPLIED ELEMENT-WISE

$$\frac{\partial L(A^{[2]}, y)}{\partial A^{[2]}} = ?$$

Let $q = y^T \log\left(A^{[2]}\right)$

$$q = y^{(1)} \log A^{[2](1)} + \cdots \quad y^{(N)} \quad A^{[2](N)}$$

Let $q = y^T \log\left(A^{[2]}\right)$

$$q = y_{(1)} \log A^{[2]}_{(1)} + \cdots \quad y_{(N)} \log A^{[2]}_{(N)}$$

$$\frac{\partial q}{\partial A^{[2]}} = \begin{bmatrix} \frac{\partial}{\partial A^{[2]}_{(1)}} \left(y_{(1)} \log A^{[2]}_{(1)} + \cdots \right) \\ \vdots \\ \frac{\partial}{\partial A^{[2]}_{(N)}} \left(y_{(1)} \cdots \right) \end{bmatrix} = \begin{bmatrix} \frac{y_{(1)}}{A^{[2]}_{(1)}} \\ \vdots \\ \frac{y_{(N)}}{A^{[2]}_{(N)}} \end{bmatrix}$$

$$\underset{N\times 1}{\frac{\partial q}{\partial A^{[2]}}} = \underset{N\times 1}{y} \oslash \underset{N\times 1}{A^{[2]}}$$

$\oslash$ ← Element-wise division

Let $r = (1-y)^T \log\left(1-A^{[2]}\right)$

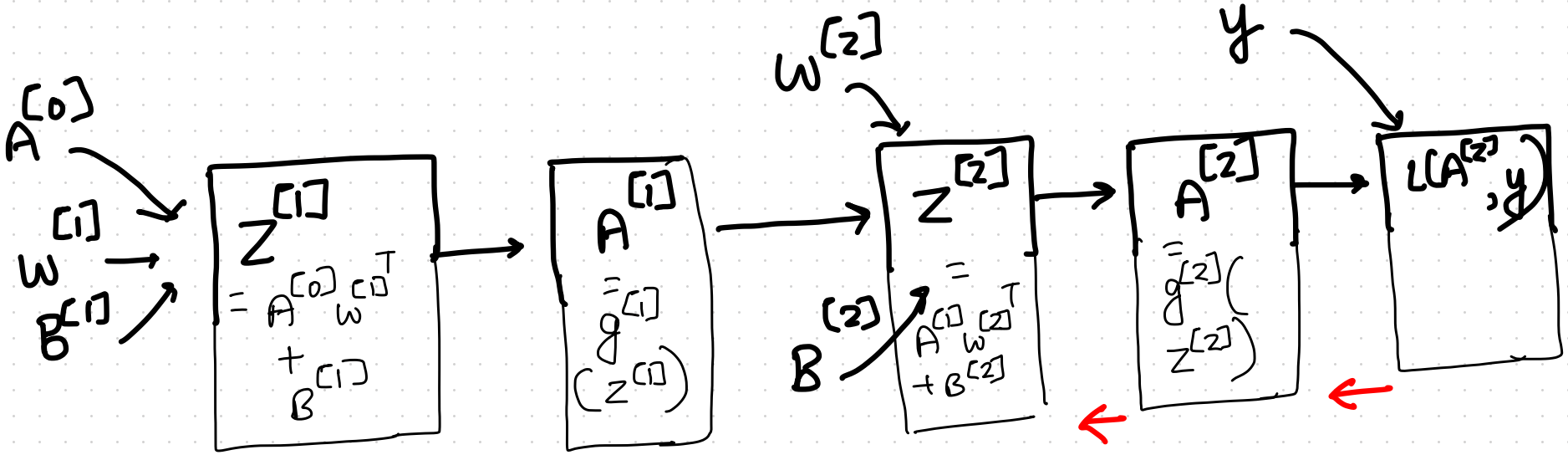
$$r = (1-y^{(1)})\log\left(1-A^{[2]}_{(1)}\right) + \cdots\cdots$$

$$\frac{\partial r}{\partial A^{[2]}} = \begin{bmatrix} \frac{\partial}{\partial A^{[2]}_{(1)}}\left\{(1-y^{(1)})\log\left(1-A^{[2]}_{(1)}\right)+\cdots\right\} \\ \\ \frac{\partial}{\partial A^{[2]}_{(N)}}\left\{(1-y^{(N)})\log\left(1-A^{[2]}_{(N)}\right)+\cdots\right\} \end{bmatrix} = \begin{bmatrix} \frac{(1-y^{(1)})}{(1-A^{[2]}_{(1)})}(-1) \\ \\ \vdots \\ \\ \left(\frac{1-y^{(N)}}{1-A^{[2]}_{(N)}}\right)(-1) \end{bmatrix}$$

$$\underset{N\times 1}{\frac{\partial r}{\partial A^{[2]}}} = -\underset{N\times 1}{(1-y)} \oslash \underset{N\times 1}{(1-A^{[2]})}$$

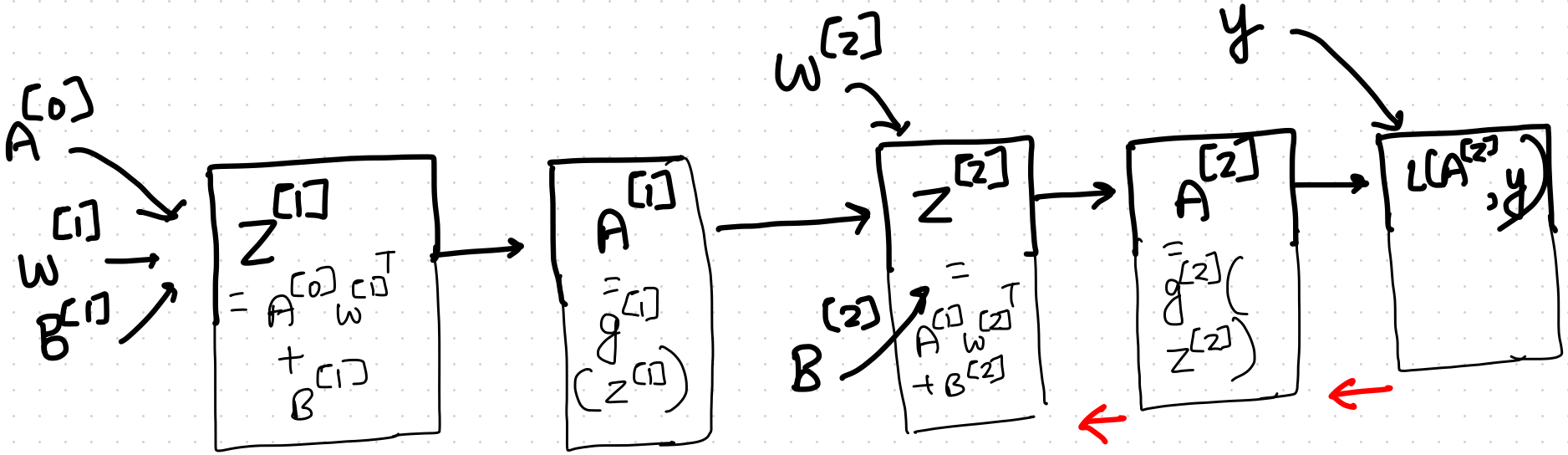
$$\frac{\partial L(A^{[2]}, y)}{\partial A^{[2]}} = -y \oslash A^{[2]} + (1-y) \oslash (1-A^{[2]})$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$\frac{\partial L}{\partial z^{[2]}} = ?$ $\qquad \frac{\partial L}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial z^{[2]}}$ (Chain Rule)

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$\frac{\partial L}{\partial Z^{[2]}} = ?$ $\quad \frac{\partial L}{\partial Z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial A^{[2]}}{\partial Z^{[2]}}$ (Chain Rule)

$= \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial g^{[2]}(Z^{[2]})}{\partial Z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} g(Z^{[2]})(1 - g(Z^{[2]}))$

$\because g^{[2]} = \text{SIGMOID}$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

$$= \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial g^{[2]}(z^{[2]})}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} g(z^{[2]})(1 - g(z^{[2]}))$$

$$= \underset{N\times 1}{\frac{\partial L}{\partial A^{[2]}}} \circ \underset{N\times 1}{A^{[2]}} \circ \underset{N\times 1}{(1 - A^{[2]})}$$

Element-wise multiply

$$= \left(-y \circ A^{[2]} + (1-y) \circ (1 - A^{[2]})\right)\left(A^{[2]} \circ (1 - A^{[2]})\right)$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

$$= \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial g^{[2]}(z^{[2]})}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} g(z^{[2]})(1 - g(z^{[2]}))$$

$$= \frac{\partial L}{\partial A^{[2]}} \underset{N \times 1}{A^{[2]}} \odot \underset{N \times 1}{(1 - A^{[2]})}$$

Element-wise

$$= \left(-y \odot A^{[2]} + (1-y) \odot (1 - A^{[2]})\right)\left(A^{[2]}(1 - A^{[2]})\right)$$

$$= -y \odot A^{[2]} \circ A^{[2]} \circ (1 - A^{[2]}) + (1-y) \odot (1 - A^{[2]}) \circ A^{[2]} \circ (1 - A^{[2]})$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

$$= \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial g^{[2]}(z^{[2]})}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} g(z^{[2]})(1 - g(z^{[2]}))$$

$$= \frac{\partial L}{\partial A^{[2]}} \underset{N\times 1}{A^{[2]}} \odot \underset{N\times 1}{(1 - A^{[2]})}$$

Element-wise

$$= \left(-y \oslash A^{[2]} + (1-y) \oslash (1 - A^{[2]})\right)\left((A^{[2]}(1 - A^{[2]})\right)$$

$$= -y \oslash A^{[2]} \circ A^{[2]} \circ (1 - A^{[2]}) + (1-y) \oslash (1 - A^{[2]}) \circ A^{[2]} \circ (1 - A^{[2]})$$

$$= -y \circ (1 - A^{[2]}) + (1-y) \circ A^{[2]} = \underset{N\times 1}{-y} + \underset{N\times 1}{y} \circ \underset{N\times 1}{A^{[2]}} + \underset{N\times 1}{A^{[2]}} - \underset{N\times 1}{y} \circ \underset{N\times 1}{A^{[2]}}$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

$$= \frac{\partial L}{\partial A^{[2]}} \cdot \frac{\partial g^{[2]}(z^{[2]})}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} g(z^{[2]})(1 - g(z^{[2]}))$$

$$= \frac{\partial L}{\partial A^{[2]}} \underset{N\times 1}{A^{[2]}} \odot \underset{N\times 1}{(1 - A^{[2]})}$$

Element-wise

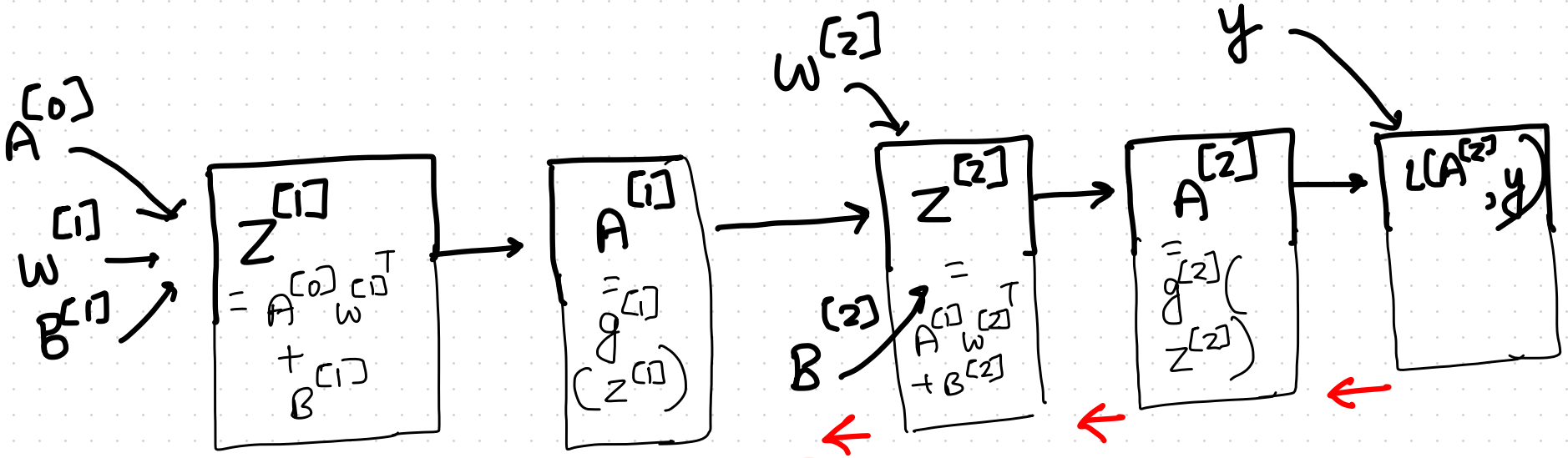
$$= \left(-y \oslash A^{[2]} + (1-y) \oslash (1 - A^{[2]})\right)\left((A^{[2]}(1 - A^{[2]})\right)$$

$$= -y \oslash A^{[2]} \circ A^{[2]} \circ (1 - A^{[2]}) + (1-y) \oslash (1 - A^{[2]}) \circ A^{[2]} \circ (1 - A^{[2]})$$

$$= -y \circ (1 - A^{[2]}) + (1-y) \circ A^{[2]} = \underset{N\times 1}{-y} + \underset{N\times 1}{y} \circ \underset{N\times 1}{A^{[2]}} + \underset{N\times 1}{A^{[2]}} - \underset{N\times 1}{y} \circ \underset{N\times 1}{A^{[2]}}$$

$$\boxed{\frac{\partial L}{\partial z^{[2]}} = A^{[2]} - y}$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial W^{[2]}} = \frac{\partial L}{\partial Z^{[2]}} \frac{\partial Z^{[2]}}{\partial W^{[2]}} \in R^{N^{[2]} \times N^{[1]}}$$

(Chain Rule)

SAME DIMENSION (∵ L is scalar)

ASIDE

* GRADIENT: VECTOR IN, SCALAR OUT
* JACOBIAN : VECTOR IN, VECTOR OUT

$$f: R^N \rightarrow R^M$$

Ilp: R^N

Olp: R^M

$$y = f(x)$$

Derivative of 'f' at 'x' called Jacobian is MxN matrix

$$\frac{\partial y}{\partial x} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial y_M}{\partial x_1} & & \frac{\partial y_M}{\partial x_N} \end{pmatrix}_{M \times N}$$

GENERALISED JACOBIAN : TENSOR IN, TENSOR OUT

$$f: R^{N_1 \times \cdots \times N_{D_x}} \longrightarrow R^{m_1 \times \cdots m_{D_y}}$$

I/p: D_x - dimensional tensor of shape $N_1 \times \cdots N_{D_x}$

o/p: D_y - dimensional tensor of shape $m_1 \times \cdots m_{D_y}$

$$y = f(x)$$

Then; $\frac{\partial y}{\partial x}$ = Gen. Jacobian = $(m_1 \times \cdots m_{D_y}) \times (N_1 \times \cdots N_{D_x})$

shape

BACK PROP. WITH TENSORS

Let $G_1 = \alpha\beta$

α: $N \times D$

β: $D \times M$

G_1: $N \times M$

BACK PROP. WITH TENSORS

Let $G_1 = \alpha\beta$

α: $N \times D$

β: $D \times M$

G_1: $N \times M$

→ We are given Loss L as fun^n of G_1

→ We also know $\frac{\partial L}{\partial G_1}$

BACK PROP. WITH TENSORS

Let $G_1 = \alpha\beta$

α: $N \times D$

β: $D \times M$

G_1: $N \times M$

→ We are given Loss L as fun^n of G_1

→ We also know $\frac{\partial L}{\partial G_1}$

→ Q: $\frac{\partial L}{\partial \alpha} = ?$; $\frac{\partial L}{\partial \beta} = ?$

BACK PROP. WITH TENSORS

Let $G_1 = \alpha\beta$

α: $N \times D$

β: $D \times M$

G_1: $N \times M$

Let's choose: $N=1$; $D=2$; $M=3$

BACK PROP. WITH TENSORS

Let $G_1 = \alpha\beta$

α: N x D

β: D x M

G_1: N x M

Let's choose: N=1; D=2; M=3

$$G_{1x3} = \begin{pmatrix} G_{1,1} & G_{1,2} & G_{1,3} \end{pmatrix}$$

$$\alpha_{1x2} = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} \end{pmatrix}$$

$$\beta_{2x3} = \begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \end{pmatrix}$$

BACK PROP. WITH TENSORS

$$G_{1\times 3} = \begin{pmatrix} G_{1,1} & G_{1,2} & G_{1,3} \end{pmatrix}$$

$$\alpha_{1\times 2} = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} \end{pmatrix}$$

$$\beta_{2\times 3} = \begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \end{pmatrix}$$

$$\alpha\beta = \left[\alpha_{1,1}\,\beta_{1,1} + \alpha_{1,2}\,\beta_{2,1} \;;\; \alpha_{1,1}\,\beta_{1,2} + \alpha_{1,2}\beta_{2,2} \;;\; \alpha_{1,1}\,\beta_{1,3} + \alpha_{1,2}\beta_{2,3} \right]$$

BACK PROP. WITH TENSORS

$$G_{1\times 3} = \begin{pmatrix} G_{1,1} & G_{1,2} & G_{1,3} \end{pmatrix}$$

$$\alpha_{1\times 2} = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} \end{pmatrix}$$

$$\beta_{2\times 3} = \begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \end{pmatrix}$$

$$\alpha\beta = \left[\alpha_{1,1}\,\beta_{1,1} + \alpha_{1,2}\,\beta_{2,1} \;;\; \alpha_{1,1}\,\beta_{1,2} + \alpha_{1,2}\beta_{2,2} \;;\; \alpha_{1,1}\,\beta_{1,3} + \alpha_{1,2}\beta_{2,3}\right]$$

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,1}} = \left[\beta_{1,1} \;;\; \beta_{1,2} \;;\; \beta_{1,3}\right]$$

$$\frac{\partial[\alpha\beta)}{\partial\alpha_{1,2}} = \left[\beta_{2,1} \;;\; \beta_{2,2} \;;\; \beta_{2,3}\right]$$

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,1}} = [\beta_{1,1} ; \beta_{1,2} ; \beta_{1,3}]$$

: Shape = Shape of $\alpha\beta$ x Shape of $\alpha_{1,1}$

= $(N \times M) \times 1$

$$\frac{\partial[\alpha\beta)}{\partial\alpha_{1,2}} = [\beta_{2,1} ; \beta_{2,2} ; \beta_{2,3}]$$

Shape = $(N \times M) \times 1$

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,1}} = [\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}]$$

$$\frac{\partial[\alpha\beta)}{\partial\alpha_{1,2}} = [\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}]$$

: Shape = Shape of $\alpha\beta$ x Shape of $\alpha_{1,1}$

= $(N \times M) \times 1$

Shape = $(N \times M) \times 1$

Generalised Tensor shape

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,1}} = [\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}]$$

$$\frac{\partial[\alpha\beta)}{\partial\alpha_{1,2}} = [\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}]$$

We know $\frac{\partial L}{\partial G} = [dG_{1,1} \quad dG_{1,2} \quad dG_{1,3}]$

Generalised shape = $1 \times (N \times M)$

↳ why?

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,1}} = [\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}]$$

$$\frac{\partial[\alpha\beta)}{\partial\alpha_{1,2}} = [\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}]$$

We know $\frac{\partial L}{\partial G} = [dG_{1,1} \quad dG_{1,2} \quad dG_{1,3}]$

Generalised shape = $1 \times (N \times M)$

$\because$ L is a scalar

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial \alpha_{1,1}} = \left[\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}\right]$$

$$\frac{\partial[\alpha\beta)}{\partial \alpha_{1,2}} = \left[\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}\right]$$

We know $\frac{\partial L}{\partial G} = \left[dG_{1,1} \quad dG_{1,2} \quad dG_{1,3}\right]$

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial L}{\partial \alpha_{1,1}} \quad \frac{\partial L}{\partial \alpha_{1,2}}\right]$$

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial \alpha_{1,1}} = \left[\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}\right] = \frac{\partial G}{\partial \alpha_{1,1}}$$

$$\frac{\partial(\alpha\beta)}{\partial \alpha_{1,2}} = \left[\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}\right] = \frac{\partial G}{\partial \alpha_{1,2}}$$

We know $\frac{\partial L}{\partial G} = \left[dG_{1,1} \quad dG_{1,2} \quad dG_{1,3}\right]$

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial L}{\partial \alpha_{1,1}} \quad \frac{\partial L}{\partial \alpha_{1,2}}\right]$$

$$\frac{\partial L}{\partial \alpha_{1,1}} = \frac{\partial L}{\partial G} \cdot \frac{\partial G}{\partial \alpha_{1,1}}$$

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial \alpha_{1,1}} = \left[\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}\right] = \frac{\partial G}{\partial \alpha_{1,1}}$$

$$\frac{\partial(\alpha\beta)}{\partial \alpha_{1,2}} = \left[\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}\right] = \frac{\partial G}{\partial \alpha_{1,2}}$$

We know $\frac{\partial L}{\partial G} = \left[dG_{1,1} \quad dG_{1,2} \quad dG_{1,3}\right]$

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial L}{\partial \alpha_{1,1}} \quad \frac{\partial L}{\partial \alpha_{1,2}}\right]$$

$$\frac{\partial L}{\partial \alpha_{1,1}} = \frac{\partial L}{\partial G} \cdot \frac{\partial G}{\partial \alpha_{1,1}}$$

$1 \times (N \times m)$

$(N \times m) \times 1$

BACK PROP. WITH TENSORS

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,1}} = [\beta_{1,1} \; ; \; \beta_{1,2} \; ; \; \beta_{1,3}] = \frac{\partial G}{\partial\alpha_{1,1}}$$

$$\frac{\partial(\alpha\beta)}{\partial\alpha_{1,2}} = [\beta_{2,1} \; ; \; \beta_{2,2} \; ; \; \beta_{2,3}] = \frac{\partial G}{\partial\alpha_{1,2}}$$

We know $\frac{\partial L}{\partial G} = [dG_{1,1} \quad dG_{1,2} \quad dG_{1,3}]$

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial L}{\partial\alpha_{1,1}} \quad \frac{\partial L}{\partial\alpha_{1,2}}\right]$$

$$\frac{\partial L}{\partial\alpha_{1,1}} = \frac{\partial L}{\partial G} \cdot \frac{\partial G}{\partial\alpha_{1,1}} = [dG_{1,1}\,\beta_{1,1} + dG_{1,2}\beta_{1,2} + dG_{1,3}\beta_{1,3}]$$

$(1\times(n\times m))\times((n\times m)\times 1)$ → Dot product of two.

BACK PROP. WITH TENSORS

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial L}{\partial \alpha_{1,1}}, \frac{\partial L}{\partial \alpha_{1,2}} \right] = \left(\frac{\partial L}{\partial G} \right) \beta^T$$

$1 \times (N \times D)$ $\quad 1 \times (N \times M)$ $\quad M \times D$

BACK PROP. WITH TENSORS

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial L}{\partial \alpha_{1,1}}; \frac{\partial L}{\partial \alpha_{1,2}}\right] = \left(\frac{\partial L}{\partial G}\right) \beta^T$$

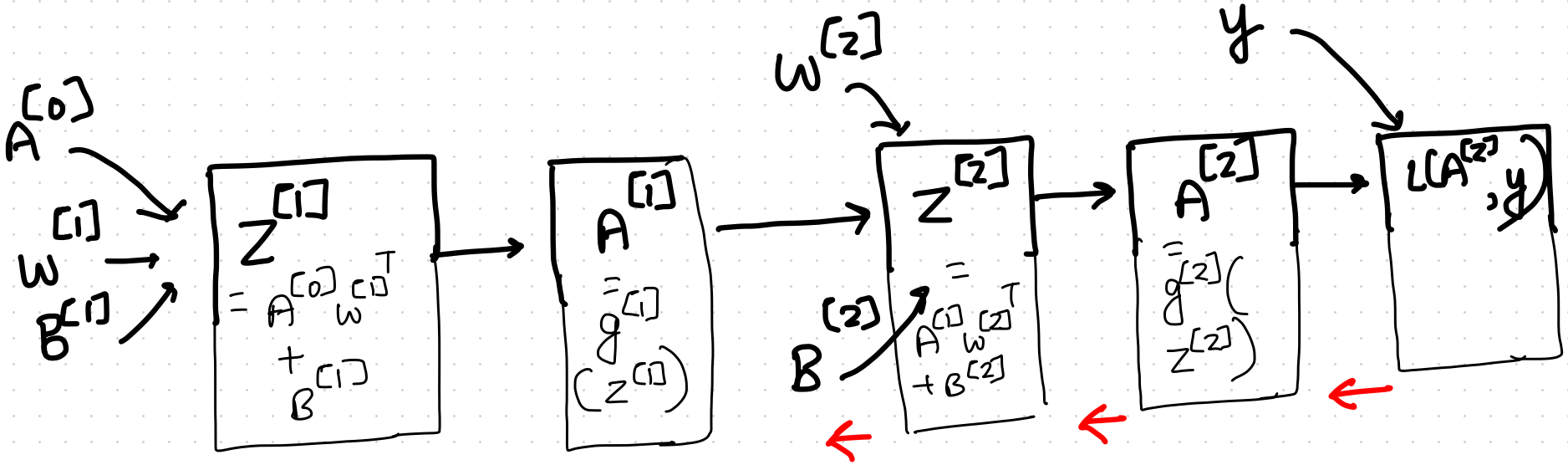
$1 \times (N \times D)$ $\quad 1 \times (N \times M)$ $\quad M \times D$

Similarly;

$$\frac{\partial L}{\partial \beta} = \alpha^T \frac{\partial L}{\partial G}$$

$1 \times (D \times M)$ $\quad D \times N$ $\quad 1 \times (N \times M)$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



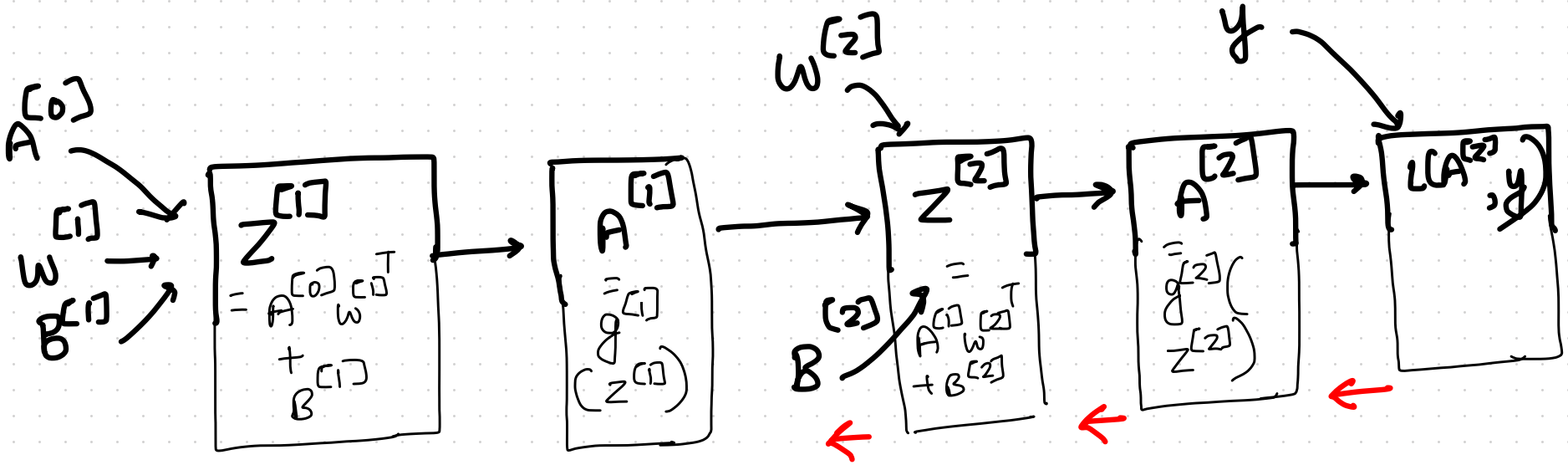
Equivalence from Aside

$G \leftrightarrow Z^{[2]}$

$\alpha \leftrightarrow A^{[1]}$

$\beta \leftrightarrow W^{[2]T}$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



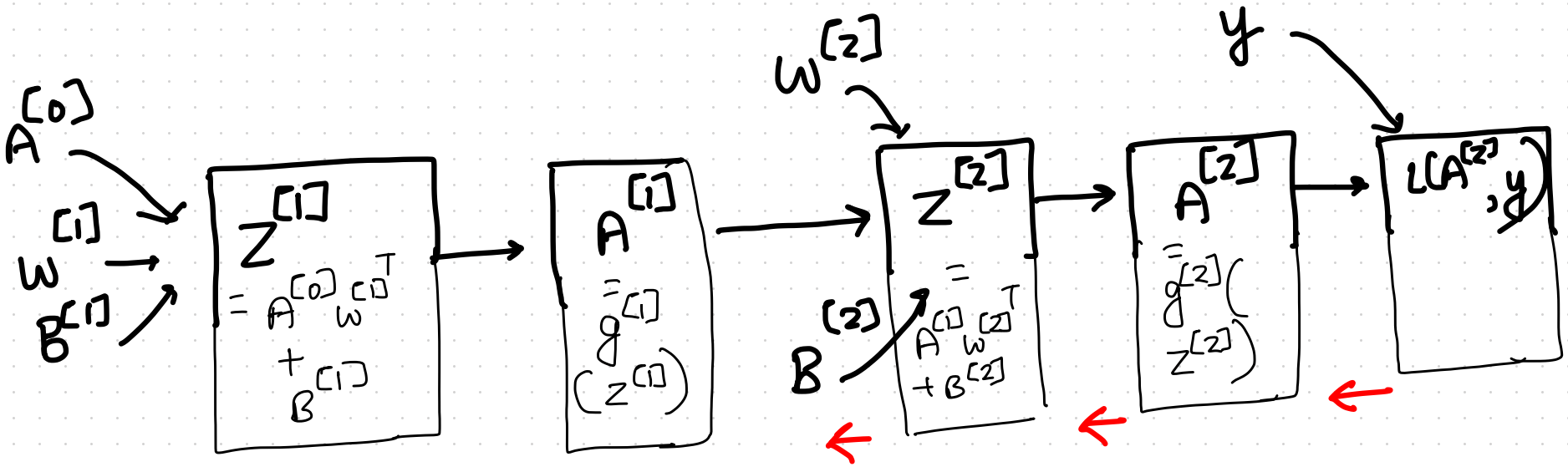
Equivalence from Aside

$$G \leftrightarrow Z^{[2]}$$
$$\alpha \leftrightarrow A^{[1]}$$
$$\beta \leftrightarrow W^{[2]T}$$

$$\frac{\partial L}{\partial W^{[2]T}} = A^{[1]T} \frac{\partial L}{\partial Z^{[2]}}$$

$$= A^{[1]T}(A^{[2]} - y)$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

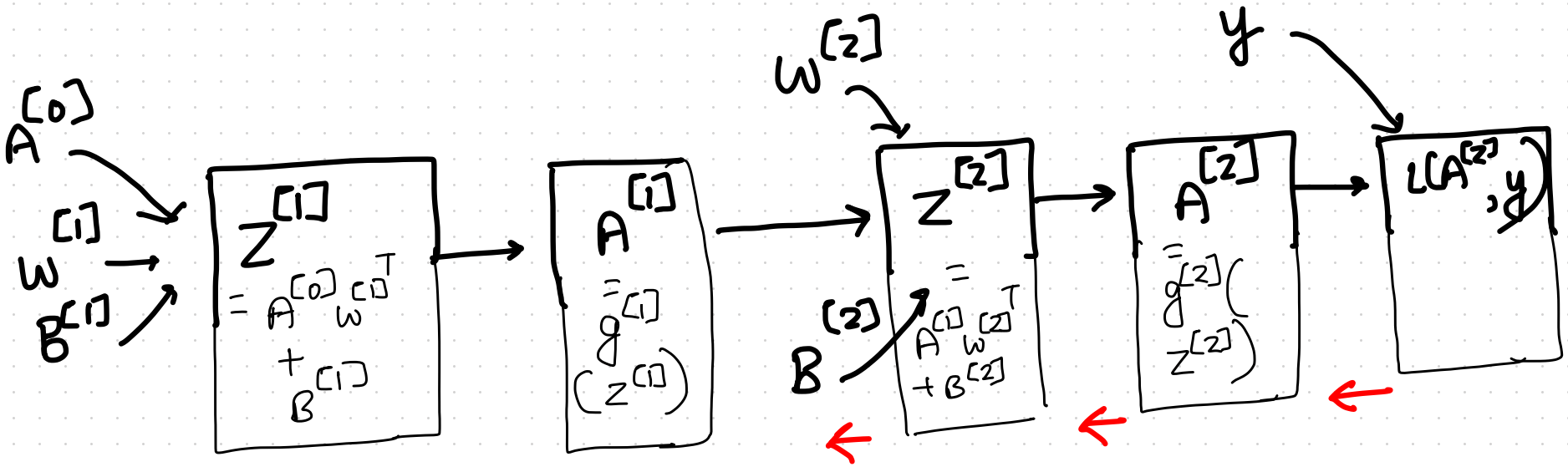


$$\frac{\partial L}{\partial W^{[2]T}} = A^{[1]T}\left(A^{[2]} - y\right)$$

$$\Rightarrow \frac{\partial L}{\partial W^{[2]}} = \left(A^{[2]} - y\right)^T A^{[1]}$$

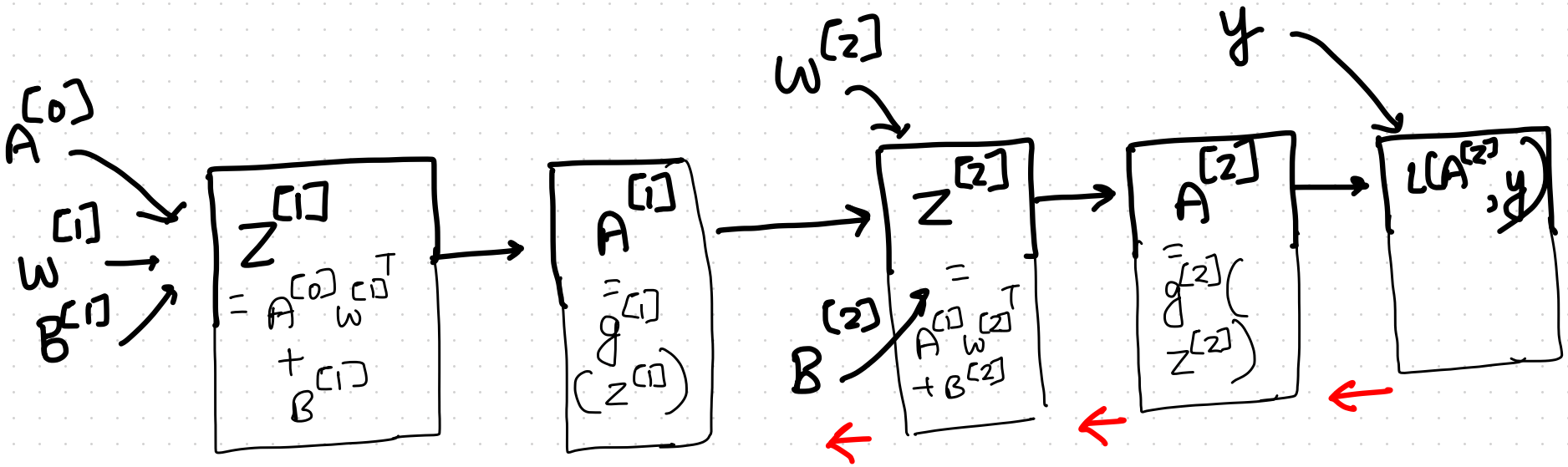
$1 \times (N^{[2]} \times N^{[1]})$ $\quad N^{[2]} \times N \quad N \times N^{[1]}$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial B^{[2]}} = ?$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

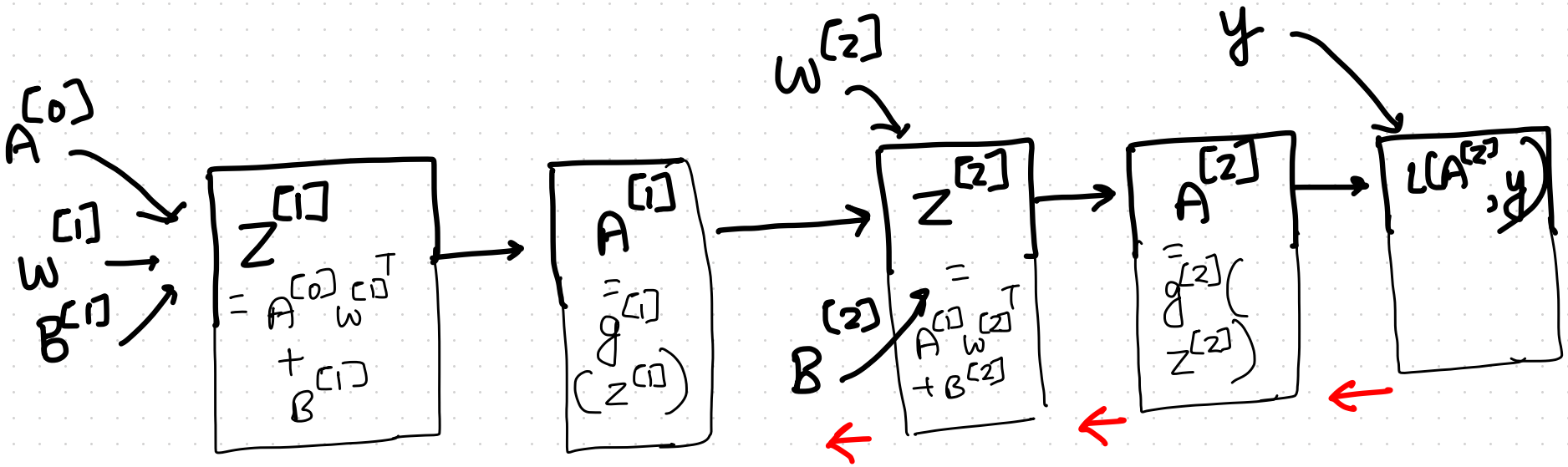


$$\frac{\partial L}{\partial B^{[2]}} = ?$$

Let $c = b^{[2]T}$

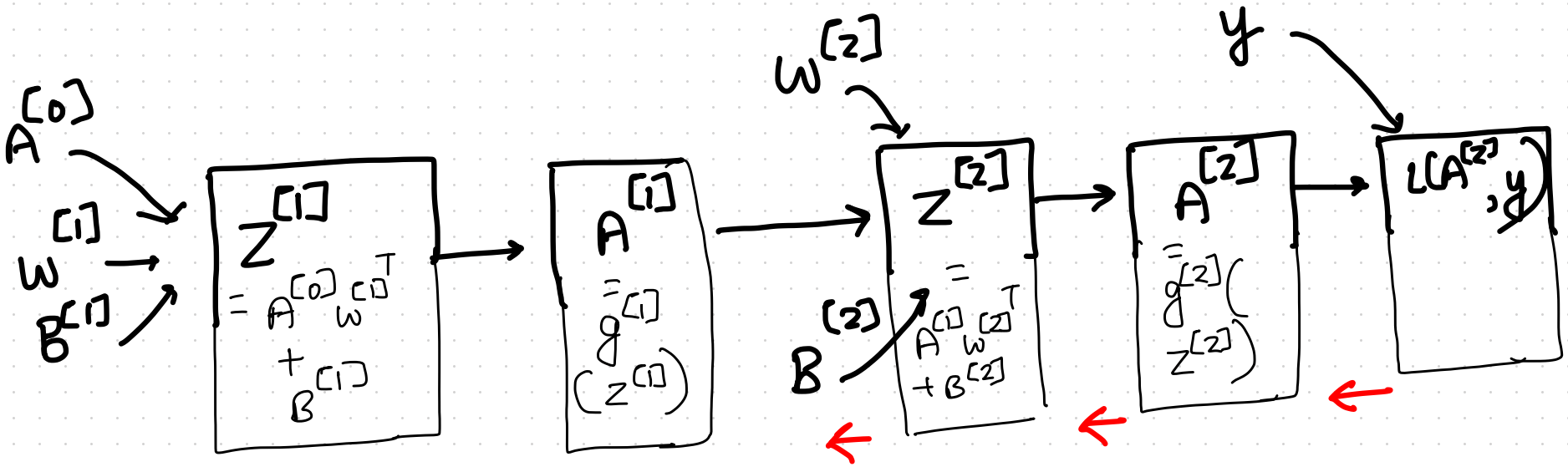
$$B^{[2]} = \begin{bmatrix} - & c & - \\ - & c & - \\ - & \vdots & - \end{bmatrix}$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial c} = ?$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

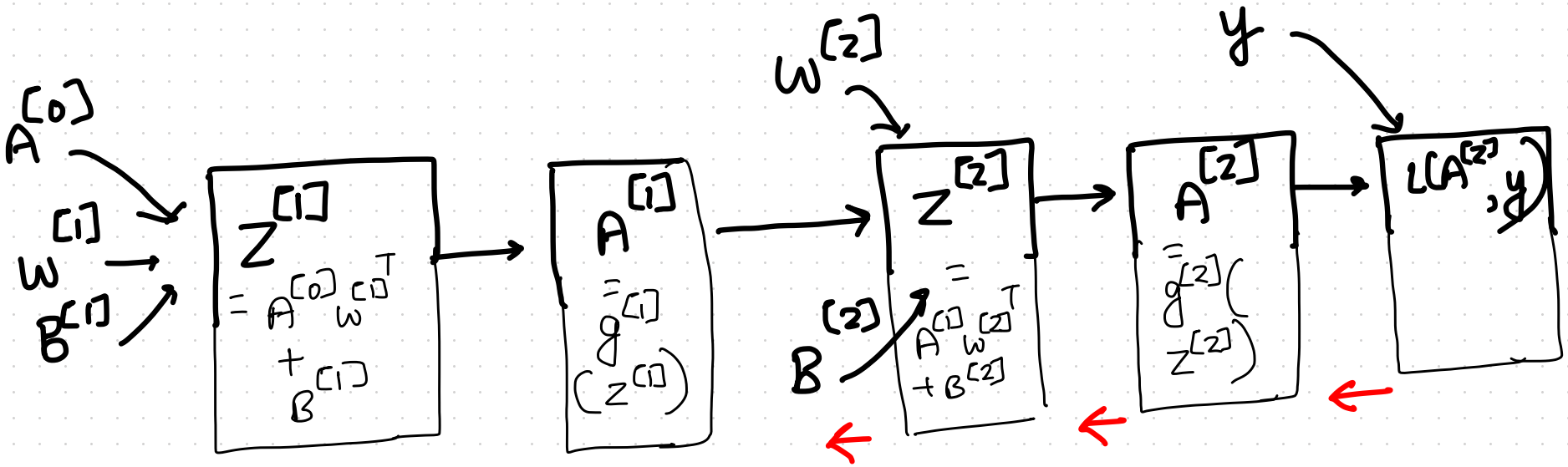


$$\frac{\partial L}{\partial c} = \left[\frac{\partial L}{\partial c_1} \quad \cdots \quad \frac{\partial L}{\partial c_{N^{[2]}}}\right]$$

$1 \times (1 \times N^{[2]})$

$$= \left[\frac{\partial L}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial c_1} \quad \cdots \cdots\right]$$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

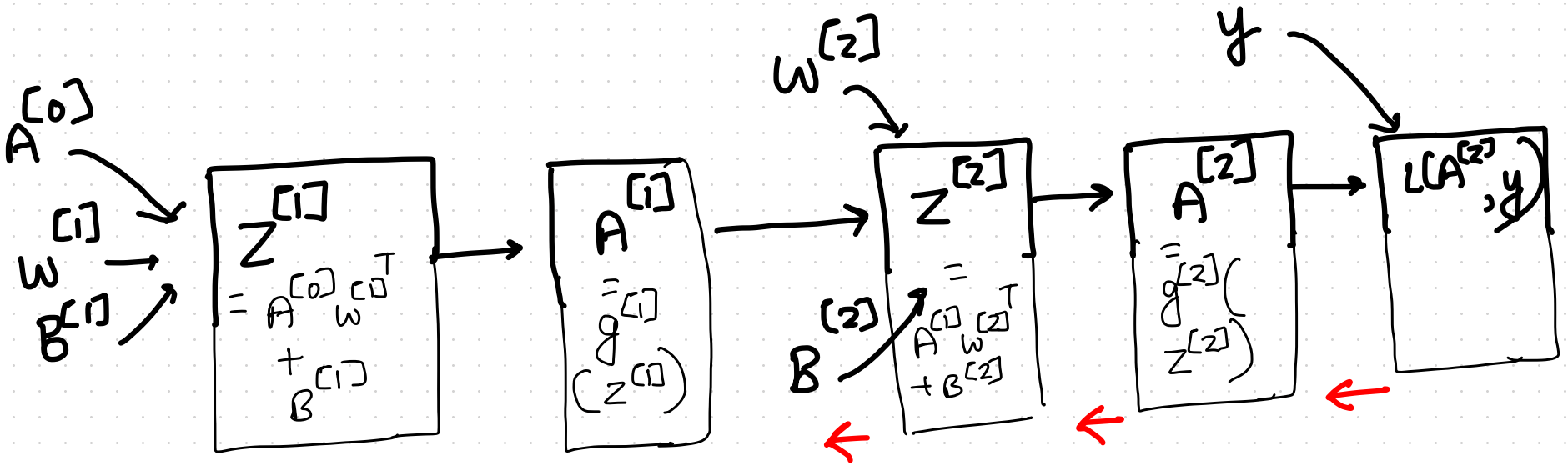


$$\frac{\partial L}{\partial c_1} = \frac{\partial L}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial c_1}$$

$$Z^{[2]} = \begin{bmatrix} z^{[2]}_{1,1} & \cdots & z^{[2]}_{1,N^{[2]}} \\ \vdots & & \\ z^{[2]}_{N,1} & \cdots & z^{[2]}_{N,N^{[2]}} \end{bmatrix} = A^{[2]} W^{[2]T} + \begin{bmatrix} c_1 & c_2 & \cdots & c_{N^{[2]}} \\ c_1 & c_2 & \cdots & c_{N^{[2]}} \\ \vdots & & & \vdots \end{bmatrix}$$

$N \times N^{[2]}$

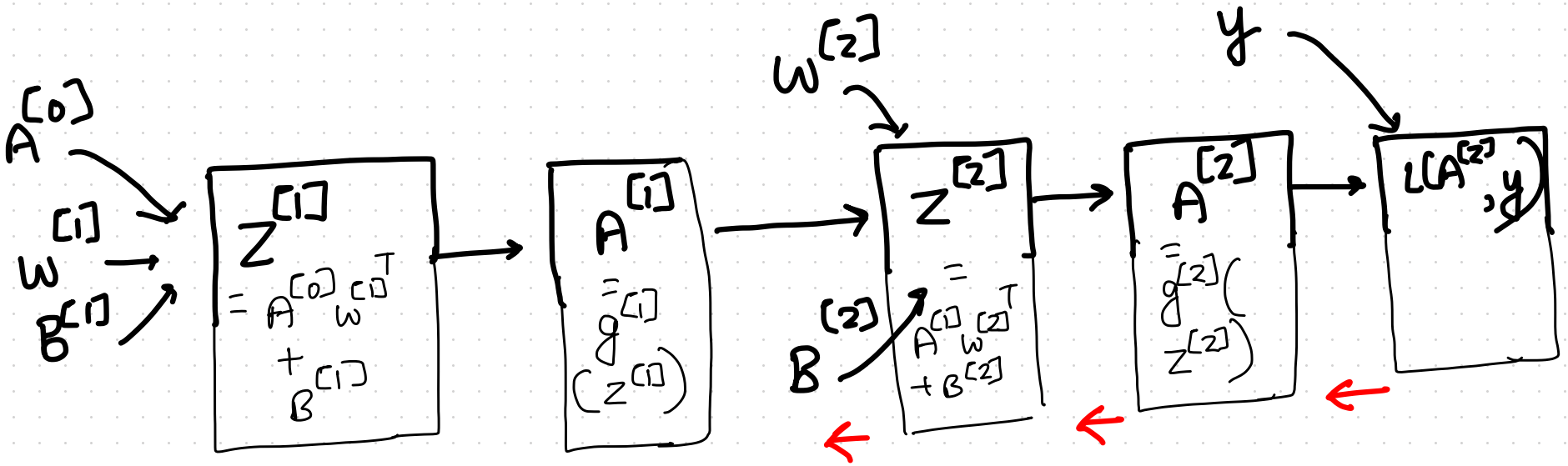
COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial Z^{[2]}}{\partial c_1} = \begin{bmatrix} \frac{\partial c_1}{\partial c_1} & \frac{\partial c_2}{\partial c_1} & \cdots \\ \vdots & & \\ \vdots & \cdots & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & & & & \\ \vdots & & \cdots & & \\ 1 & \cdots & & \cdots & \end{bmatrix}$$

$(N \times N^{[2]}) \times (1)$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



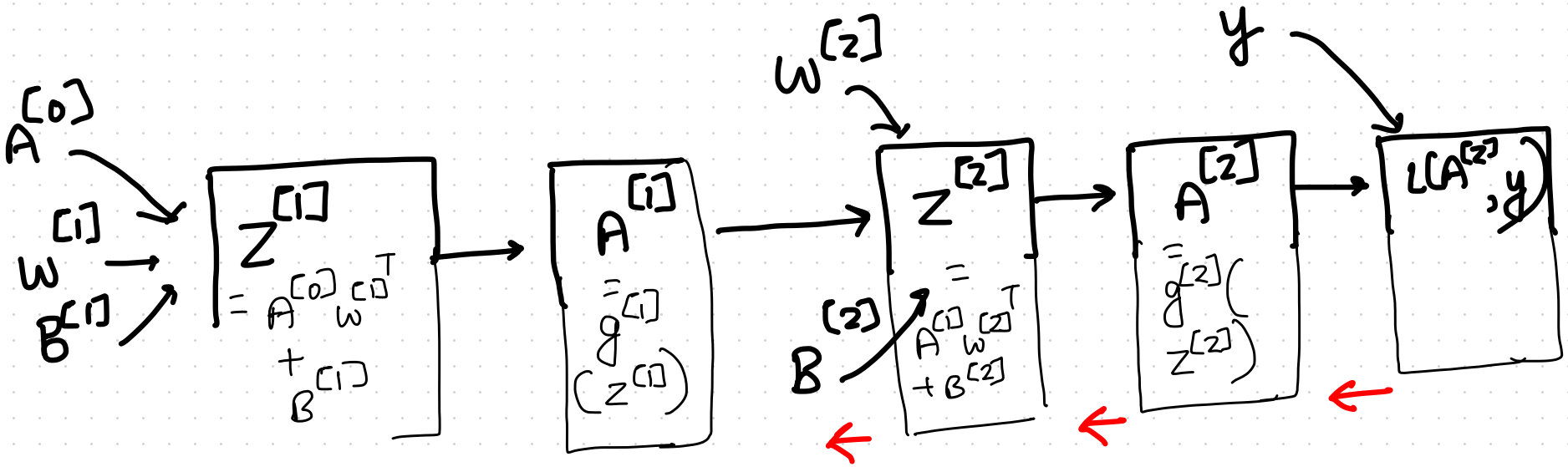
$$\frac{\partial Z}{\partial c_1} = \begin{bmatrix} \frac{\partial c_1}{\partial c_1} & \frac{\partial c_2}{\partial c_1} & \cdots \\ \vdots & & \\ \vdots & \cdots & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & & & & \\ \vdots & & \cdots & & \\ 1 & \cdots & & \cdots & \end{bmatrix}$$

$(N \times N^{[2]}) \times (1)$

$$\frac{\partial L}{\partial c_1} = \frac{\partial L}{\partial Z^{[2]}} \frac{\partial Z^{[2]}}{\partial c_1}$$

$1 \times (N \times N^{[2]})$ $\quad (N \times N^{[2]}) \times 1$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



Let $\frac{\partial L}{\partial Z^{[2]}} = \begin{bmatrix} dz^{[2]}_{1,1} & \cdots & dz^{[2]}_{1,n^{[2]}} \\ dz^{[2]}_{N,1} & \cdots & dz^{[2]}_{N,n^{[2]}} \end{bmatrix}$

Then $\boxed{\frac{\partial L}{\partial c_1} = \sum_{i=1}^{N} dz^{[2]}_{i,1}}$

COMPUTATION GRAPH (FOR XOR EXAMPLE)

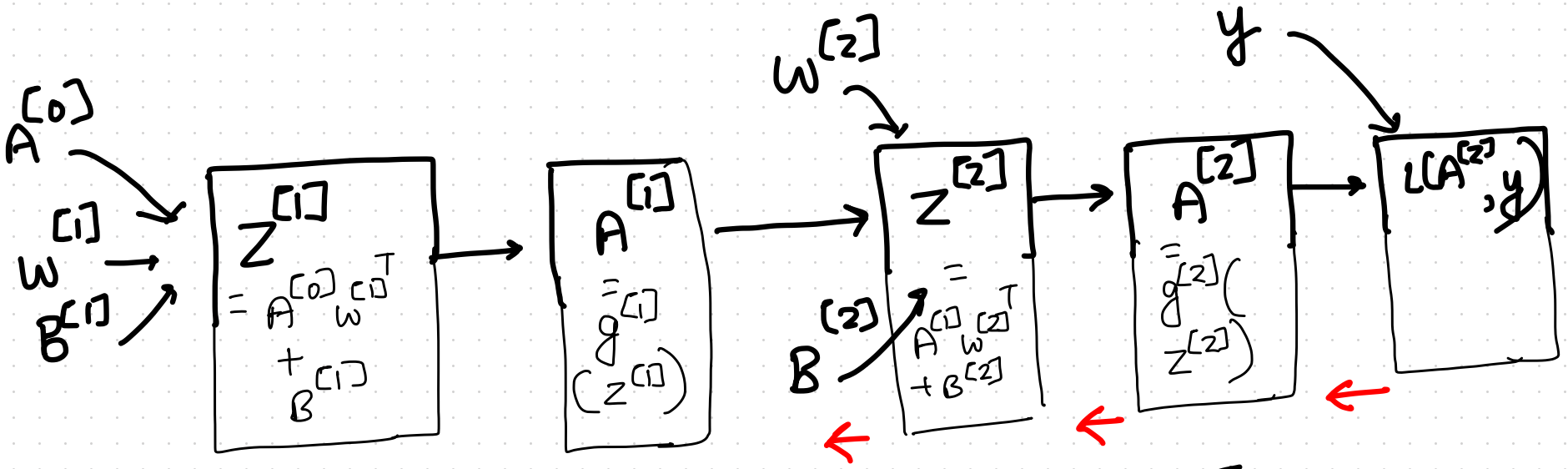
$$\frac{\partial L}{\partial c_1} = \sum_{i=1}^{N} dz_{i,1}^{[2]}$$

$$\Rightarrow \frac{\partial L}{\partial c} = \left[\sum_{i=1}^{N} dz_{i,1}^{[2]} \quad \sum_{i=1}^{N} dz_{i,2}^{[2]} \quad \cdots \quad \sum_{i=1}^{N} dz_{i,N^{[2]}}^{[2]} \right]$$

$$\Rightarrow \frac{\partial L}{\partial b} = \left(\frac{\partial L}{\partial c}\right)^T = \begin{bmatrix} \sum_{i=1}^{N} dz_{i,1}^{[2]} \\ \vdots \\ \sum_{i=1}^{N} dz_{i,N^{[2]}}^{[2]} \end{bmatrix}$$

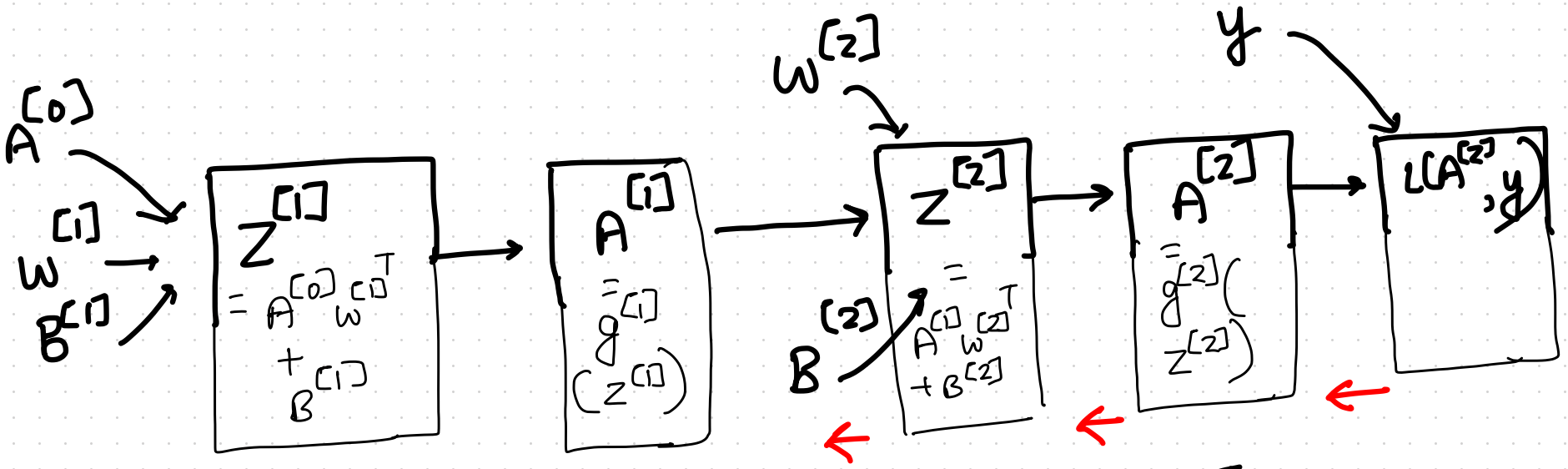
$N^{[2]} \times 1$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial A^{[1]}} = \frac{\partial L}{\partial Z^{[2]}} \frac{\partial Z^{[2]}}{\partial A^{[1]}} = \frac{\partial L}{\partial Z^{[2]}} \left(W^{[2]T}\right) = \frac{\partial L}{\partial Z^{[2]}} W^{[2]}$$

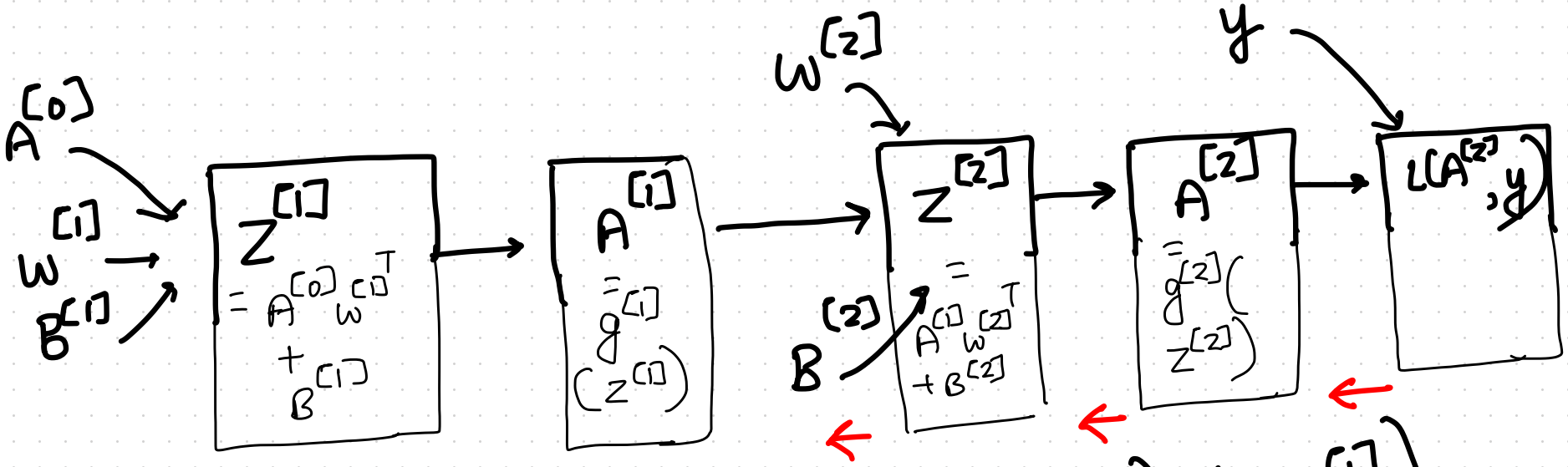
COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial A^{[1]}} = \frac{\partial L}{\partial Z^{[2]}} \frac{\partial Z^{[2]}}{\partial A^{[1]}} = \frac{\partial L}{\partial Z^{[2]}} \left(W^{[2]T}\right) = \frac{\partial L}{\partial Z^{[2]}} W^{[2]}$$

$1 \times (N \times N^{[1]})$ $\quad$ $1 \times (N \times N^{[2]})$ $\quad$ $N^{[2]} \times N^{[1]}$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



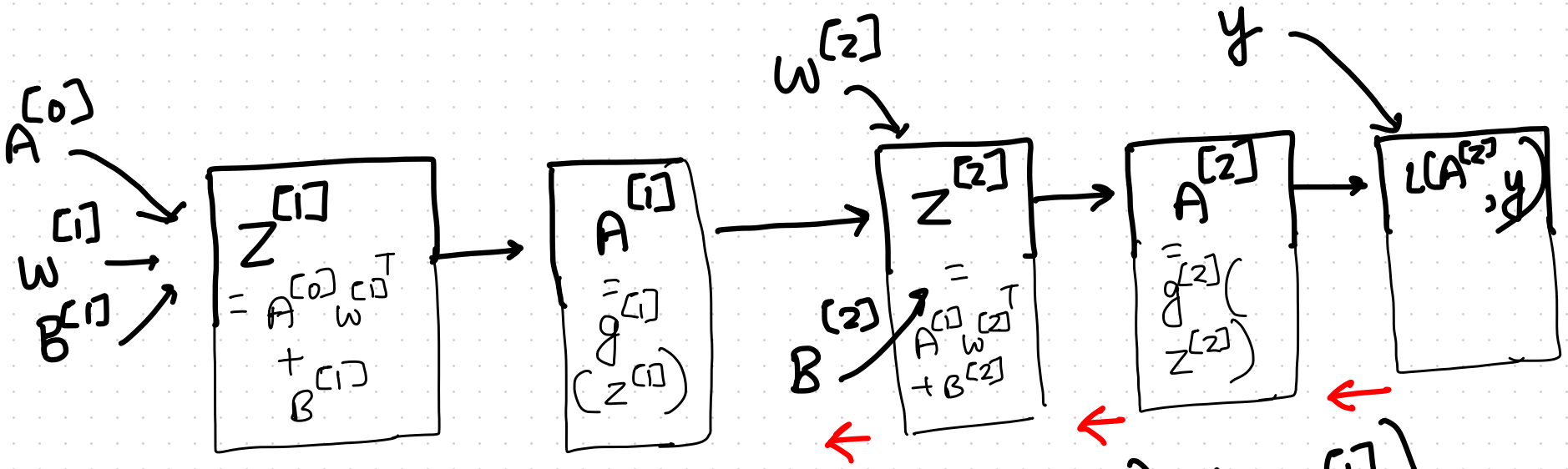
$$\frac{\partial L}{\partial Z^{[1]}} = \frac{\partial L}{\partial A^{[1]}} \frac{\partial A^{[1]}}{\partial Z^{[1]}} = \left(\frac{\partial L}{\partial Z^{[2]}} W^{[2]}\right)\left(\frac{\partial A^{[1]}}{\partial Z^{[1]}}\right)$$

$1 \times (N \times N^{[1]})$

$1 \times (N \times N^{[2]}) \times (N^{[2]} \times N^{[1]})$

$(N \times N^{[1]}) \times (N \times N^{[1]})$

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial Z^{[1]}} = \frac{\partial L}{\partial A^{[1]}} \frac{\partial A^{[1]}}{\partial Z^{[1]}} = \left(\frac{\partial L}{\partial Z^{[2]}} W^{[2]}\right)\left(\frac{\partial A^{[1]}}{\partial Z^{[1]}}\right)$$

$1 \times (N \times N^{[1]})$

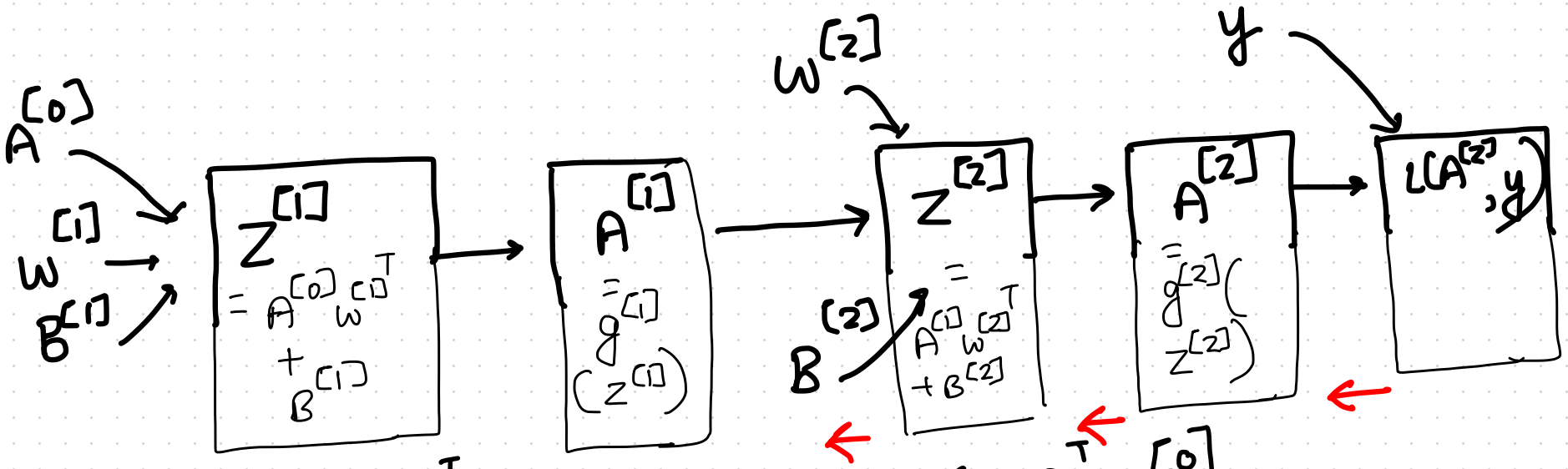
$1 \times (N \times N^{[2]}) \times (N^{[2]} \times N^{[1]})$

$(N \times N^{[1]}) \times (N \times N^{[1]})$

$$= \frac{\partial L}{\partial Z^{[1]}} W^{[2]} \odot g^{[1]'}(Z^{[1]})$$

Element wise or dot product of Jacobian

COMPUTATION GRAPH (FOR XOR EXAMPLE)



$$\frac{\partial L}{\partial W^{[1]T}} = A^{[1]T} \frac{\partial L}{\partial Z^{[1]}} \Rightarrow \frac{\partial L}{\partial W^{[1]}} = \left(\frac{\partial L}{\partial Z^{[1]}}\right)^{T} A^{[0]}$$

$$\frac{\partial L}{\partial b^{[1]}} = \begin{bmatrix} \sum_{i=1}^{N} dz_{i,1}^{[1]} \\ \vdots \\ \sum_{i=1}^{N} dz_{i,N^{[2]}}^{[1]} \end{bmatrix}$$

$N^{[2]} \times 1$