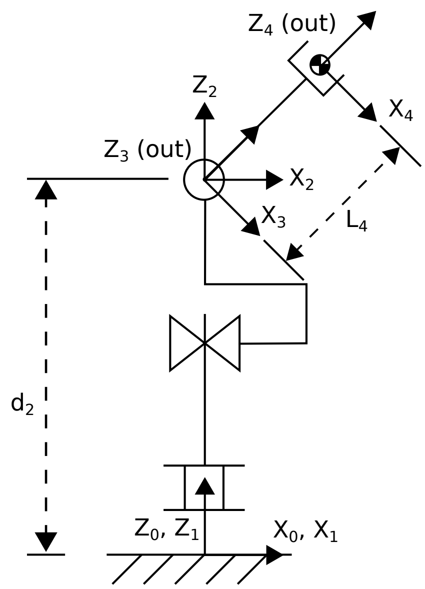
Experimental Robotics (CS225A) Homework #0

(Spring 2022/2023) Due: **Monday, April 17**

Make sure to provide justification for your answers. This includes labeling all of your plots (title, axes, legend, etc.) and explaining what is shown in the plots. Otherwise, you will lose points.

In this homework assignment, we will review kinematics and dynamics concepts from CS223A and then set up the simulation environment (SAI) that will be used for subsequent homework assignments and the final project.

1. Consider the RPR manipulator below. Assume this robot is massless except for the end effector, which is a point mass $m_4 = 2.0\,\text{kg}$. Note that the joint angle θ_3 is negative in the configuration drawn below.



   (a) Find the position ${}^0P_4$ of the end effector expressed in frame $\{0\}$. *Hint:* Avoid DH parameters and use geometric intuition instead. Break up the projections by finding ${}^2P_4$ before ${}^0P_4$.

   (b) Find the orientation ${}^0_4R$ of the end effector frame $\{4\}$ expressed in frame $\{0\}$. *Hint:* Break up the projections by finding ${}^2_4R$ before ${}^0_4R$.

   (c) Find the linear Jacobian ${}^0J_v$ of the end effector expressed in frame $\{0\}$.

   (d) Find the angular Jacobian ${}^0J_\omega$ of the end effector expressed in frame $\{0\}$.

   (e) Find the linear singularities of this robot. For each singularity you find, draw the robot in the singular configuration and specify the singular direction. Avoid taking the determinant and use intuition to identify the singularities.

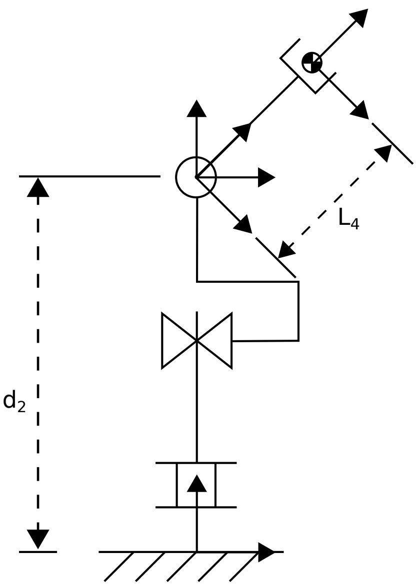
   (f) Write out ${}^0J_v$ for the singular configurations you drew above. For each configuration, explain why these are singularities in terms of joint motion. For each singular configuration, what is the singular direction expressed in frame $\{0\}$ when $\theta_1 = 0°$? When $\theta_1 = 90°$?

   (g) Find the joint space mass matrix M for this manipulator. Recall that all the robot is massless (i.e., no mass or inertia on any links or joints) except for the end effector, which is a point mass $m_4 = 2.0\,\text{kg}$.

   (h) Find the gravity vector G. Assume acceleration due to gravity is $[0, 0, -g]^T$, where $g = 9.81\,\text{m}\,\text{s}^{-2}$.

2. Now, we will run this robot in simulation. We will let the length $L_4 = 2.5\,\text{m}$. Follow the setup instructions from Canvas and the repository (`https://github.com/manips-sai-org/cs225a`) to get SAI installed on your system.

   (a) Often, the urdf specification of robots does not follow the standard DH conventions. This is the case for the rprbot.urdf provided in the folder hw0. Complete the following schematic by placing the frames that correspond to the model described by the urdf file. *Hint:* Keep in mind that, in the urdf file, the kinematic structure is described by the joints only.



   (b) What is the position of the end effector in frame {3}, ${}^3P_4$? Replace the value of "ee pos in link" in the hw0.cpp file with this value.

   (c) Find the end effector position in frame {0}, ${}^0P_4$, for the two configurations below. Are the results consistent with the expression you found in 1 (a)? What is different? Explain.

      i. $\theta_1 = 0°$, $d_2 = 1.5\,\text{m}$, $\theta_3 = -90°$

      ii. $\theta_1 = 90°$, $d_2 = 1.5\,\text{m}$, $\theta_3 = -90°$

   (d) Find the simulated linear Jacobian ${}^0J_v$ for the two configurations considered in 2 (c). Are the results consistent with the expression you found in 1 (c)? What is different? Explain.

   (e) For the following plots, output the desired mass matrix values to a file and use a program like MATLAB or Excel for plotting [1]. For each, compare your results to 1 (g) and explain physically why each configuration sweep produces your results.

      i. Set $\theta_1 = 0°$ and $d_2 = 1.5\,\text{m}$. Plot m_{11}, m_{22}, and m_{33} as θ_3 varies from $-90°$ to $90°$.

      ii. Set $\theta_1 = 0°$ and $\theta_3 = 0°$. Plot m_{11}, m_{22}, and m_{33} as d_2 varies from $0.0\,\text{m}$ to $2.0\,\text{m}$.

   (f) Now, produce the same plots for G. For each, compare your results to 1 (h) and explain physically why each configuration sweep produces your results.

      i. Set $\theta_1 = 0°$ and $d_2 = 1.5\,\text{m}$. Plot G as θ_3 varies from $-90°$ to $90°$.

      ii. Set $\theta_1 = 0°$ and $\theta_3 = 0°$. Plot G as d_2 varies from $0.0\,\text{m}$ to $2.0\,\text{m}$.

   (g) **Extra credit:** Modify the rprbot.urdf file to add a prismatic joint between frame {3} and frame {4}. Plot the gravity vector for this new robot when $\theta_1 = 0°$, $d_2 = 1.5\,\text{m}$, $\theta_3 = 45°$ and d_4 varies from $0\,\text{m}$ to $2.0\,\text{m}$.

3. Submit your SAI code (**required:** hw0.cpp file; **extra credit:** modified rprbot.urdf).

[1] If you're feeling ambitious, you can output the keys to Redis and create a Python script to catch the keys and plot them with Matplotlib. We will be using Redis heavily later in the course, so it'll be good to familiarize yourself with it.