CS 225A: Experimental Robotics
Homework 1

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Problem 1

(a) We implement the joint space control law:

$$\Gamma = -k_p(q - q_d) - k_v\dot{q} \tag{1}$$

We have the relations:

$$k_p = m\omega^2 \tag{2}$$

$$k_v = m(2\xi\omega) \tag{3}$$

To achieve critical damping on joint 1 ($\xi = 1$), we get:

$$k_p = m_{11}\omega_1^2 \tag{4}$$

$$k_v = 2m_{11}\omega_1 \tag{5}$$

where m_{11} is the element at the first row and first column of the mass matrix, and $m_{11} \approx 1.69$. Eliminating ω_1 from 4 and 5 gives:

$$k_v = 2\sqrt{m_{11}k_p} = 2\sqrt{1.69 \times 400} = 2 \times 1.3 \times 20 = 52 \tag{6}$$

(b) The plots of the joint trajectory (q and $\dot{q}$) for joints 1, 3 and 4 are shown below.

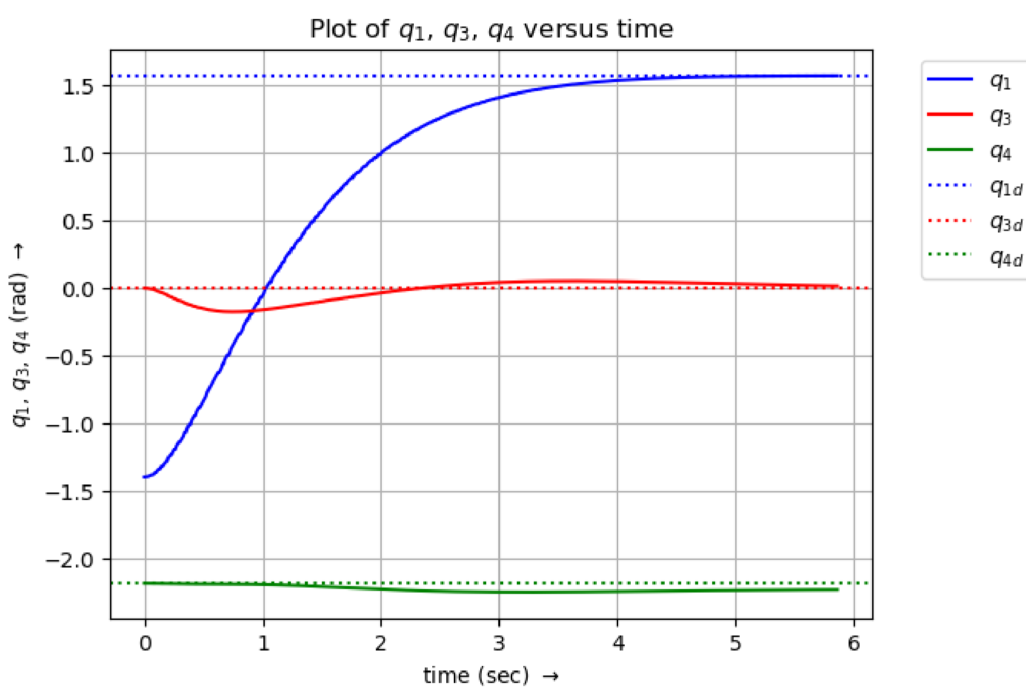

Figure 1: Plot of q_1, q_3 and q_4 versus time.

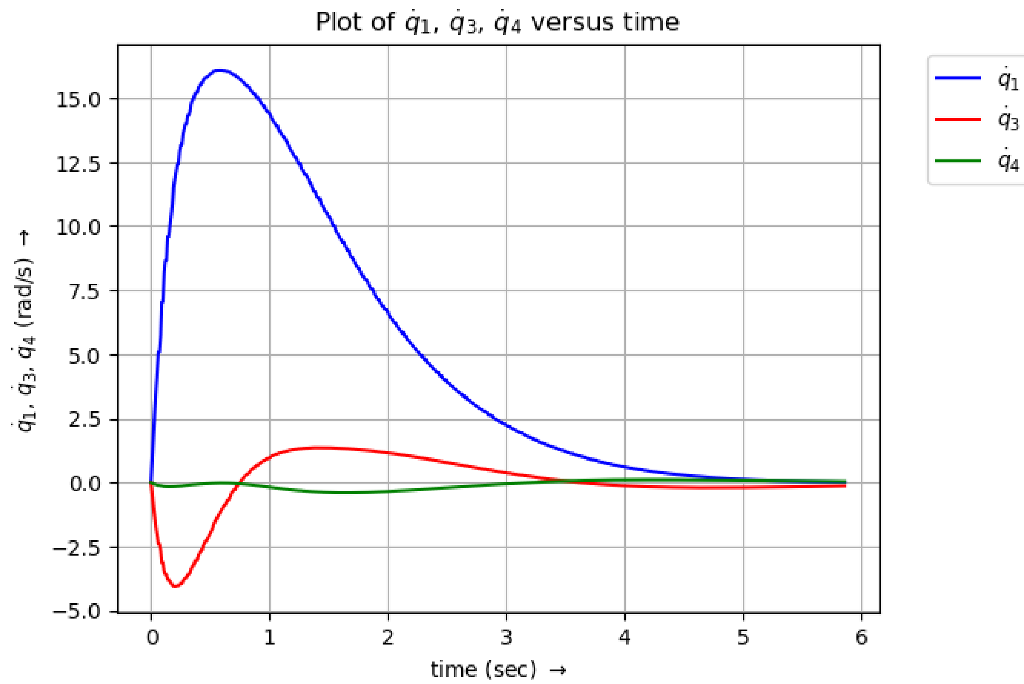

Figure 2: Plot of $\dot{q}_1$, $\dot{q}_3$ and $\dot{q}_4$ versus time.

These values are plotted until $||q - q_d||_2 < 0.05$. We observe that joint 1 converges the fastest as it is critically damped, followed by joint 3. There is a small steady-state error for joint 4, which is due to the uncompensated gravity vector.

Problem 2

(a) We implement the joint space control law:

$$\Gamma = -k_p(q - q_d) - k_v\dot{q} + g(q) \tag{7}$$

Just like in Problem 1, to achieve critical damping on joint 1, k_v is given by:

$$k_v = 2\sqrt{m_{11}k_p} = 2\sqrt{1.69 \times 400} = 2 \times 1.3 \times 20 = 52 \tag{8}$$

(b) The plots of the joint trajectory (q and $\dot{q}$) for joints 1, 3 and 4 are shown below.

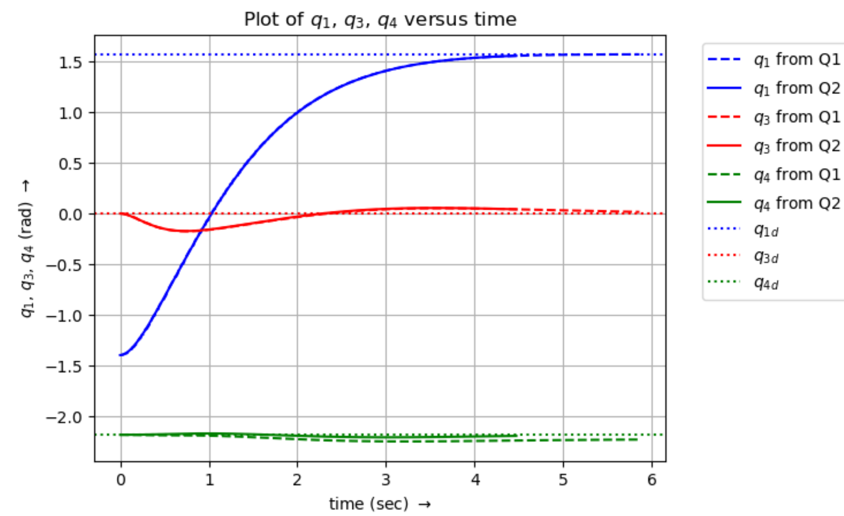

Figure 3: Plot of q_1, q_3 and q_4 versus time.

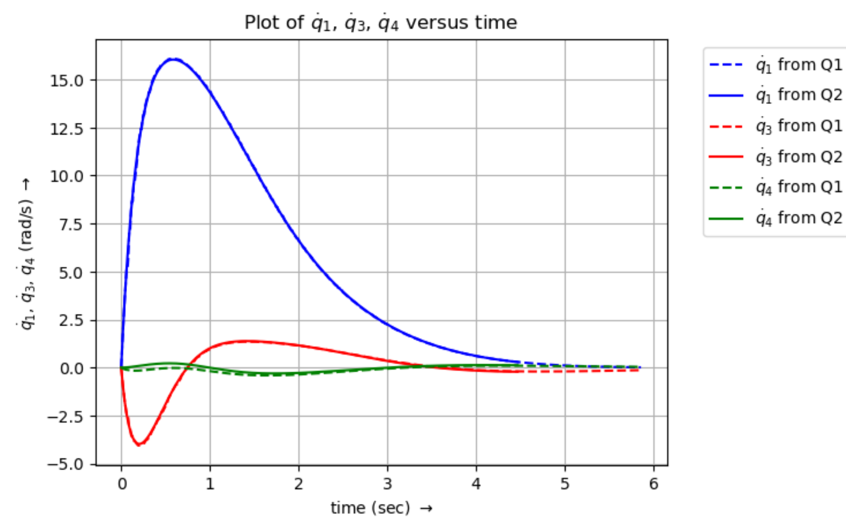

Figure 4: Plot of $\dot{q}_1$, $\dot{q}_3$ and $\dot{q}_4$ versus time.

These values are plotted until $||q - q_d||_2 < 0.05$. Due to gravity compensation (no steady-state error for joint 4), the performance is better and convergence is faster compared to Problem 1, and joints 1, 3 as well as 4 now converge to their desired values.

Problem 3

(a) We implement the joint space control law:

$$\Gamma = A(q)(-k_p(q - q_d) - k_v\dot{q}) + g(q) \tag{9}$$

Since we take into account the dynamics of the robot, the effective mass = 1, and so to achieve critical damping on joint 1, k_v is simply given by:

$$k_v = 2\sqrt{k_p} = 2\sqrt{400} = 2 \times 20 = 40 \tag{10}$$

(b) The plots of the joint trajectory (q and $\dot{q}$) for joints 1, 3 and 4 are shown below.

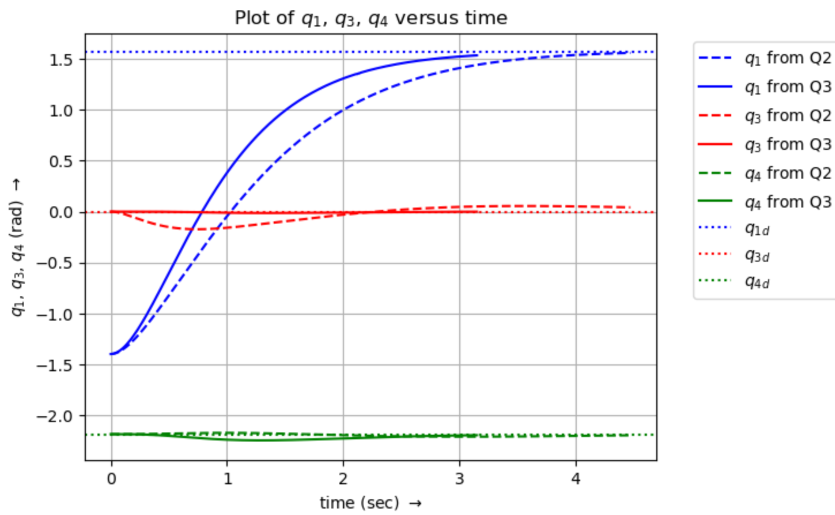

Figure 5: Plot of q_1, q_3 and q_4 versus time.

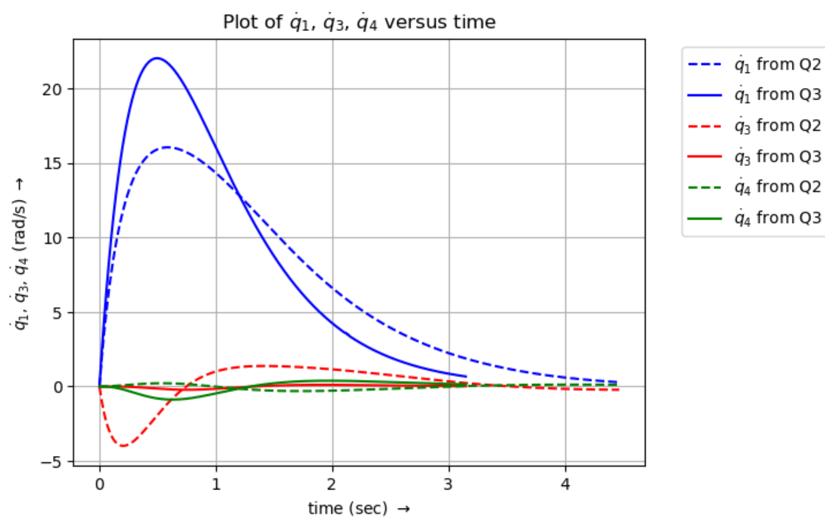

Figure 6: Plot of $\dot{q}_1$, $\dot{q}_3$ and $\dot{q}_4$ versus time.

These values are plotted until $||q - q_d||_2 < 0.05$. We observe the performance is better and convergence is faster compared to Problem 2. This is because we have now accounted for the dynamics of the robot, which leads to dynamic decoupling. Joints 1, 3 and 4 converge to their desired values.

Problem 4

(a) We implement the joint space control law:

$$\Gamma = A(q)(-k_p(q - q_d) - k_v\dot{q}) + b(q, \dot{q}) + g(q) \tag{11}$$

Just like in Problem 3, to achieve critical damping on joint 1, k_v is given by:

$$k_v = 2\sqrt{k_p} = 2\sqrt{400} = 2 \times 20 = 40 \tag{12}$$

(b) The plots of the joint trajectory (q and $\dot{q}$) for joints 1, 3 and 4 are shown below.

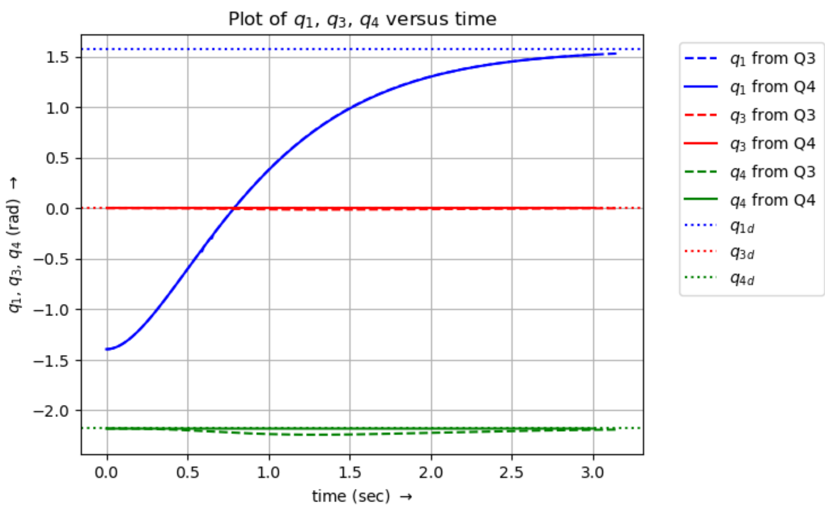

Figure 7: Plot of q_1, q_3 and q_4 versus time.

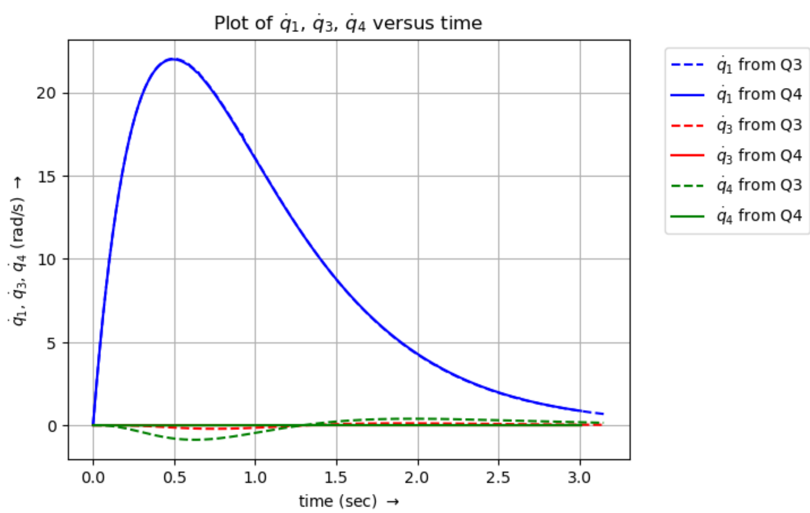

Figure 8: Plot of $\dot{q}_1$, $\dot{q}_3$ and $\dot{q}_4$ versus time.

These values are plotted until $||q - q_d||_2 < 0.05$. We observe the performance is better and convergence is faster (though only slightly) compared to Problem 3. Since we have taken into account the dynamics of the robot (dynamic decoupling) and compensated for both gravity and coriolis force, we have achieved critical damping for all joints. Hence joints 1, 3 and 4 converge the fastest to their desired values.

Problem 5

We implement the joint space control law from Problem 4:

$$\Gamma = A(q)(-k_p(q - q_d) - k_v\dot{q}) + b(q, \dot{q}) + g(q) \tag{13}$$

with a payload of 2.5 kg at the end-effector.
The plots of the joint trajectory (q and $\dot{q}$) for joints 1, 3 and 4 are shown below.

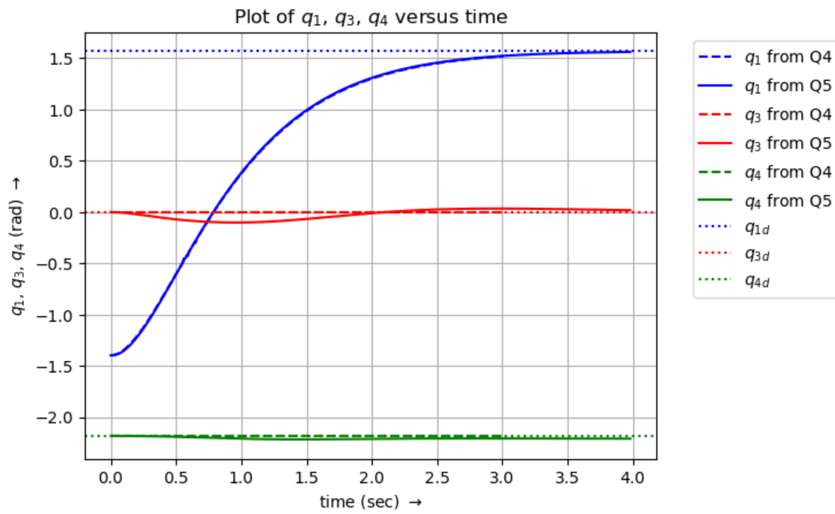

Figure 9: Plot of q_1, q_3 and q_4 versus time.

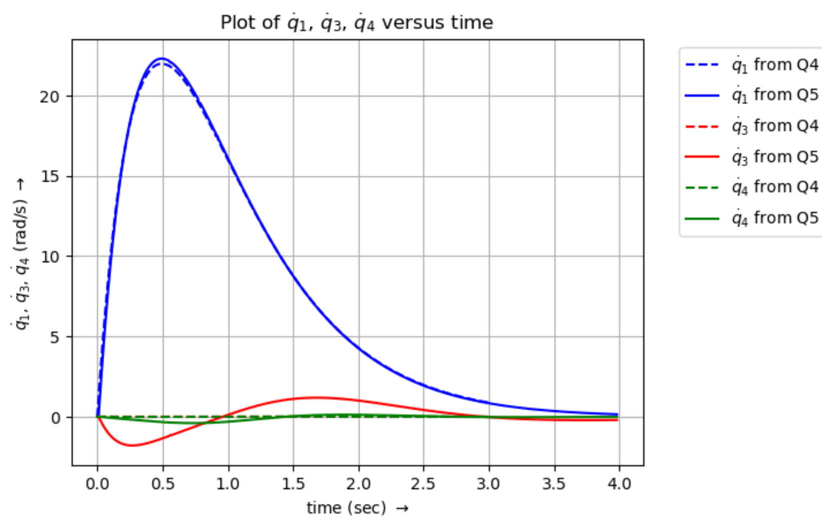

Figure 10: Plot of $\dot{q}_1$, $\dot{q}_3$ and $\dot{q}_4$ versus time.

These values are plotted until $||q - q_d||_2 < 0.05$. We now observe that performance has become worse and joints 1, 3 and 4 take longer to converge to their desired values, as compared to Problem 4.

Problem 6

To take the payload into account, we compute the modified mass matrix $\tilde{A}(q)$ and the modified gravity vector $\tilde{g}(q)$:

$$\tilde{A}(q) = A(q) + m_p J_v^T J_v \tag{14}$$

$$\tilde{g}(q) = g(q) - m_p J_v^T g \tag{15}$$

where $m_p = 2.5$ kg is the mass of the payload, and J_v is the linear Jacobian of the end-effector. The following joint space control law is then implemented:

$$\Gamma = \tilde{A}(q)(-k_p(q - q_d) - k_v\dot{q}) + b(q, \dot{q}) + \tilde{g}(q) \tag{16}$$

The plots of the joint trajectory (q and $\dot{q}$) for joints 1, 3 and 4 are shown below.

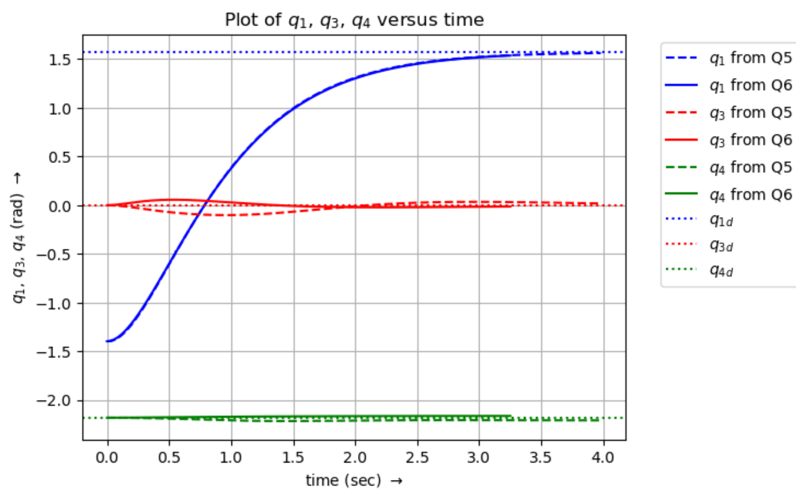

Figure 11: Plot of q_1, q_3 and q_4 versus time.

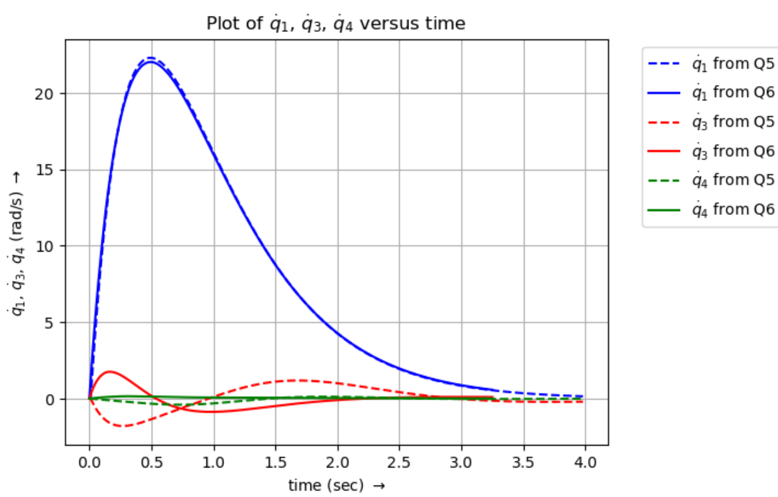

Figure 12: Plot of $\dot{q}_1$, $\dot{q}_3$ and $\dot{q}_4$ versus time.

These values are plotted until $||q - q_d||_2 < 0.05$. We observe the performance is better and convergence is faster compared to Problem 5. Joints 1, 3 and 4 converge to their desired values.