CS 225A: Experimental Robotics
Homework 2

Name: Abhyudit Singh Manhas
SUID: 06645995

Problem 1

(a) The closed loop equation of motion for joint 7 is:

$$m_{77}\ddot{q}_7 + (K_{v7} + d)\dot{q}_7 + K_{p7}(q_7 - q_{d7}) + M_{[7,1:6]}\ddot{q}_{1:6} = 0 \tag{1}$$

From the information, we can conclude that

$$m_{77} = I_{zz} = 0.25 \text{ kg m}^2 \tag{2}$$

(b) If dynamic coupling terms are negligible, the closed loop equation of motion for joint 7 reduces to:

$$m_{77}\ddot{q}_7 + (K_{v7} + d)\dot{q}_7 + K_{p7}(q_7 - q_{d7}) = 0 \tag{3}$$

To get an oscillatory system with no damping, we set:

$$\begin{aligned} K_{v7} + d &= 0 \\ \implies K_{v7} &= -d \end{aligned} \tag{4}$$

The chosen K_{v7} is negative. I would not recommend doing this on a real system. This is because a manipulator is designed to perform specific tasks, and the accuracy and precision of those tasks depend on the control of the joint movement. An oscillatory system with no damping can result in uncontrolled and unpredictable movements, making it difficult to achieve accurate and precise positioning. Also, these oscillations can cause wear and tear on the joint and reduce the lifespan of the manipulator.

(c) The natural oscillation frequency of the above system is:

$$\omega_n = \sqrt{\frac{K_{p7}}{m_{77}}} = \sqrt{\frac{50}{0.25}} = 10\sqrt{2} = 14.14 \text{ rad/s} \tag{5}$$

(d) The controller gains are:

$$K_p = \begin{bmatrix} 400 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 400 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 400 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 400 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 400 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix}, \; K_v = \begin{bmatrix} 50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -d \end{bmatrix} \tag{6}$$

where $d = 0.31486499$.

(e) The plot of the joint trajectory for joint 7 is shown below.

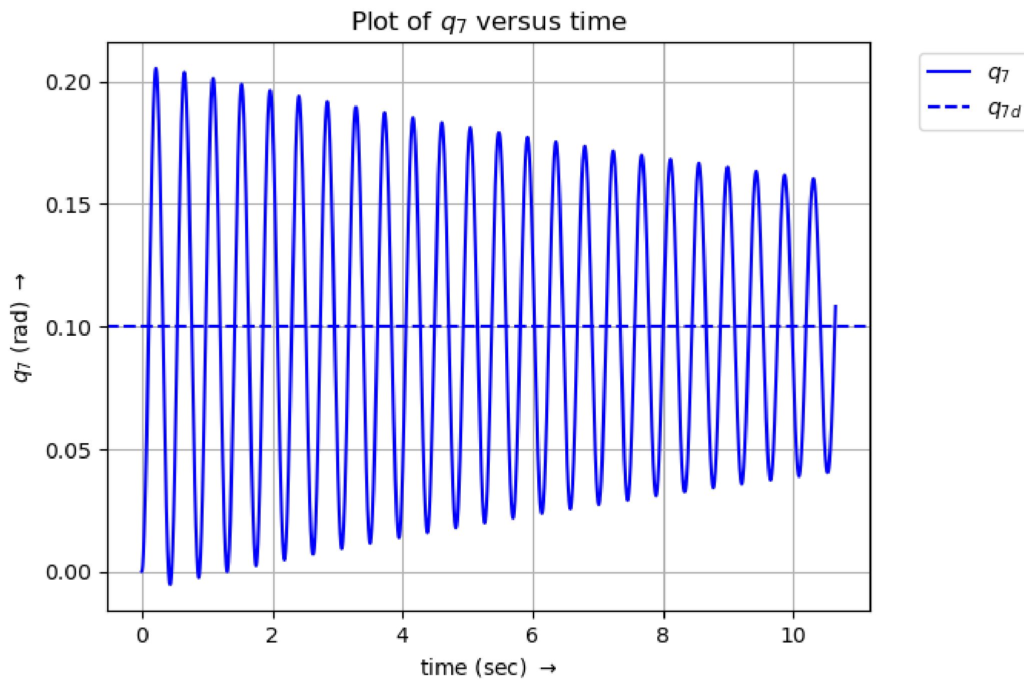

Figure 1: Plot of q_7 versus time.

We observe that we have nearly 4.5 oscillations every 2 seconds. Hence the oscillation frequency (in rad/s) is:

$$\omega_n = 2\pi\left(\frac{4.5}{2}\right) = 2 \cdot \pi \cdot 2.25 = 14.137 \text{ rad/s} \tag{7}$$

This value nearly matches the one from 5.
We also observe from the figure that there is some small non-zero damping as the amplitudes gradually decrease with time. This is because the value of d is not exactly known, and this makes selecting a value of K_{v7} (that makes $K_{v7} + d$ vanish) difficult. As a result, owing to the uncertainty in the value of d, it is difficult to obtain pure oscillatory behaviour with no damping.

Problem 2

(a) We implement the operational space control law with joint space gravity:

$$F = \Lambda(-k_p(x - x_d) - k_v\dot{x})$$
$$\Gamma = J_v^T F + g \tag{8}$$

To achieve critical damping, k_v is given by:

$$k_v = 2\sqrt{k_p} = 2\sqrt{200} = 20\sqrt{2} = 28.28 \tag{9}$$

Since there would be some damping at the joints due to transmission, we choose a value of k_v that is smaller than 28.28. We select $k_v = 24$.
The plots of the joint trajectories and operational point trajectory are shown below:

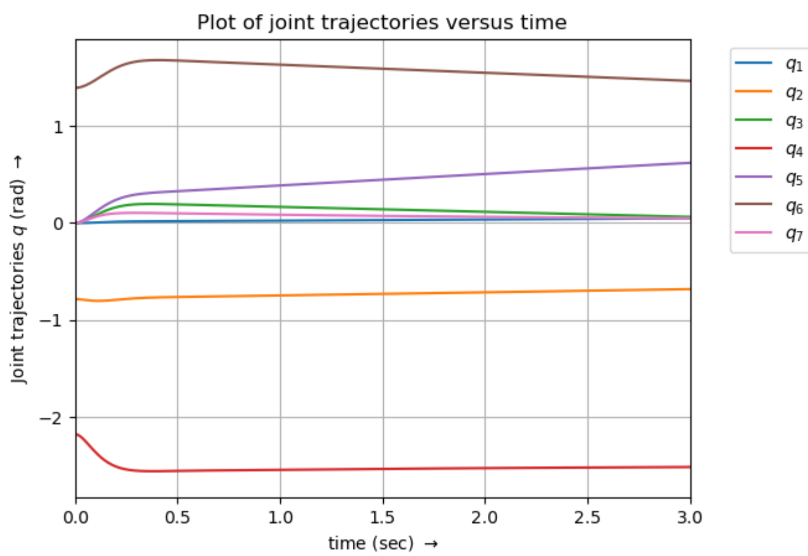

Figure 2: Plot of joint trajectories.

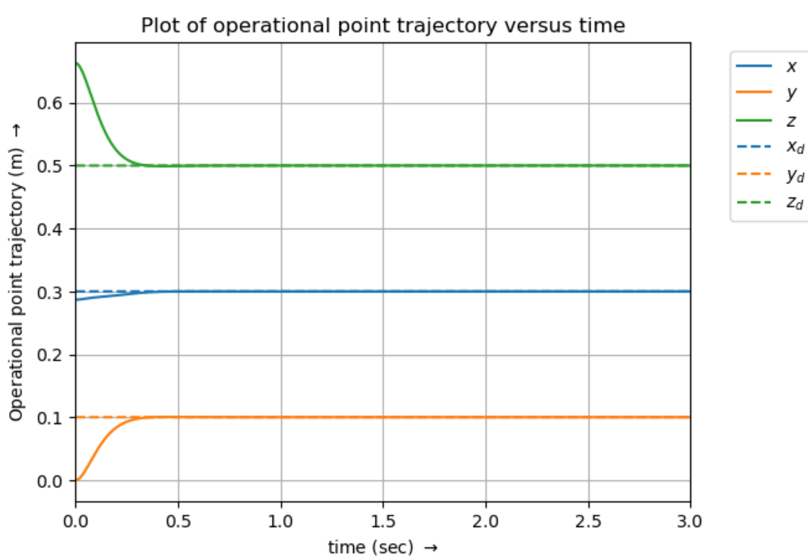

Figure 3: Plot of operational point trajectory.

(b) We observe that the robot reaches the goal position, but some of its joints still keep moving. Since the task is 3 DOF but the Panda is a 7 DOF robot, the additional DOF in the Panda's design cause it to move in unintended ways, such as oscillating or shaking, even if the goal position has been reached.

(c) We now add some joint space damping to our commanded torques. We thus implement:

$$\begin{aligned} F &= \Lambda(-k_p(x - x_d) - k_v\dot{x}) \\ \Gamma &= J_v^T F + g - K_v\dot{q} \end{aligned} \tag{10}$$

where

$$K_v = \begin{bmatrix} 15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix} \tag{11}$$

The plots of the joint trajectories and operational point trajectory are shown below:

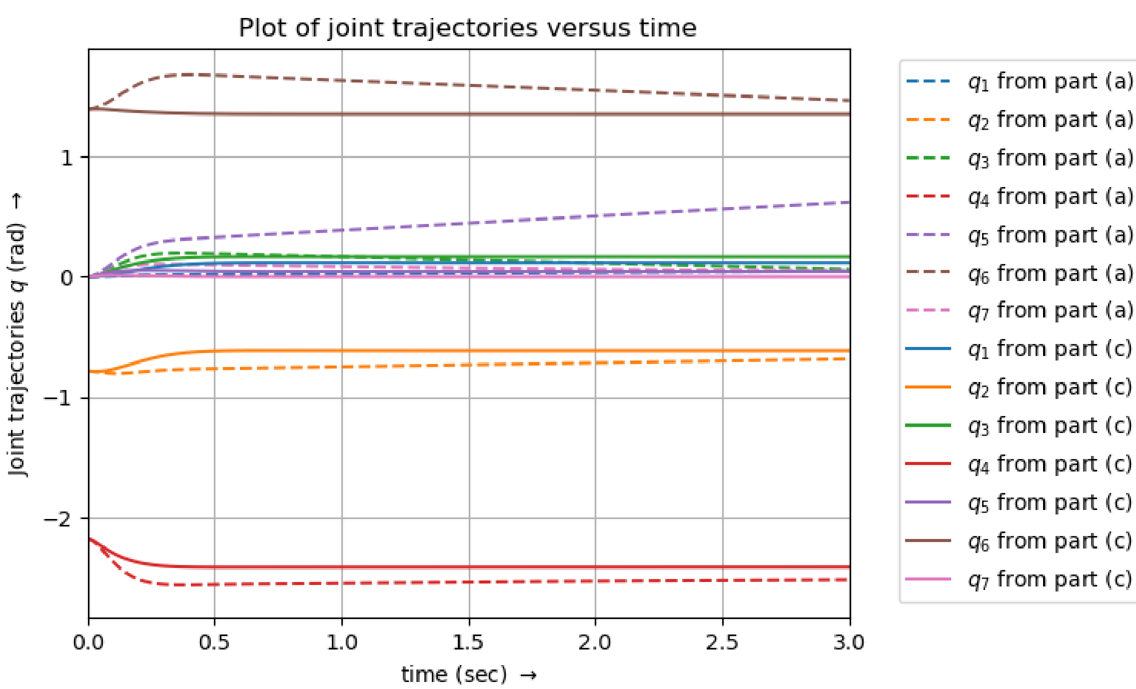

Figure 4: Plot of joint trajectories.

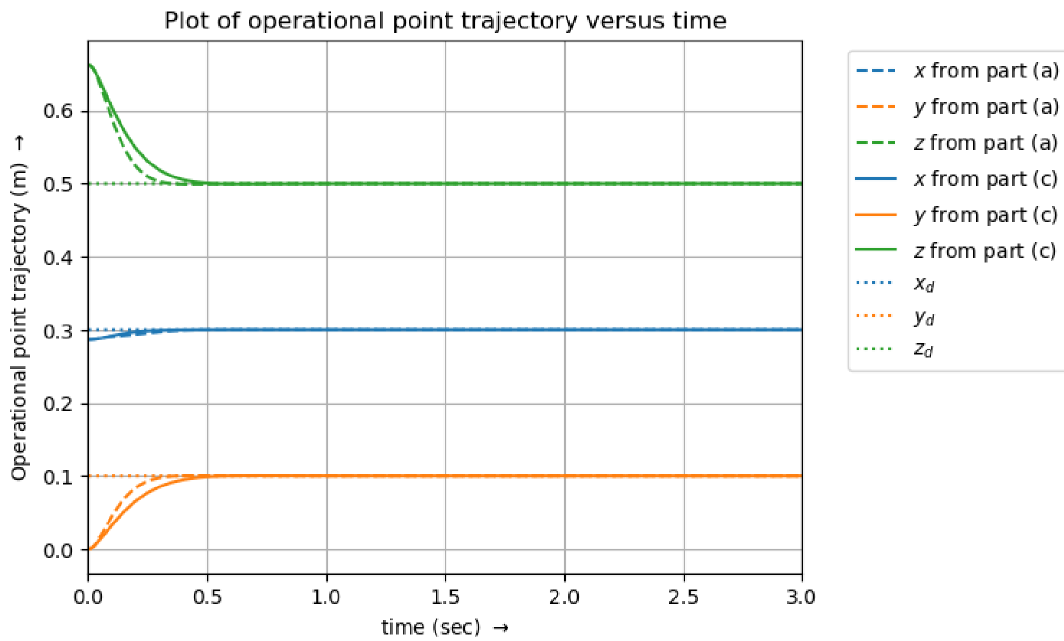

Figure 5: Plot of operational point trajectory.

We observe that there is no residual motion after the robot reaches the goal position. But since now all our joints are damped, the robot moves a lot slower and thus takes longer to reach the goal position.

(d) We now add joint space damping in the null space of the control task. We thus implement:

$$\begin{aligned} F &= \Lambda(-k_p(x - x_d) - k_v \dot{x}) \\ \Gamma &= J_v^T F + g - N^T M (K_v \dot{q}) \end{aligned} \tag{12}$$

The plots of the joint trajectories and operational point trajectory are shown below:

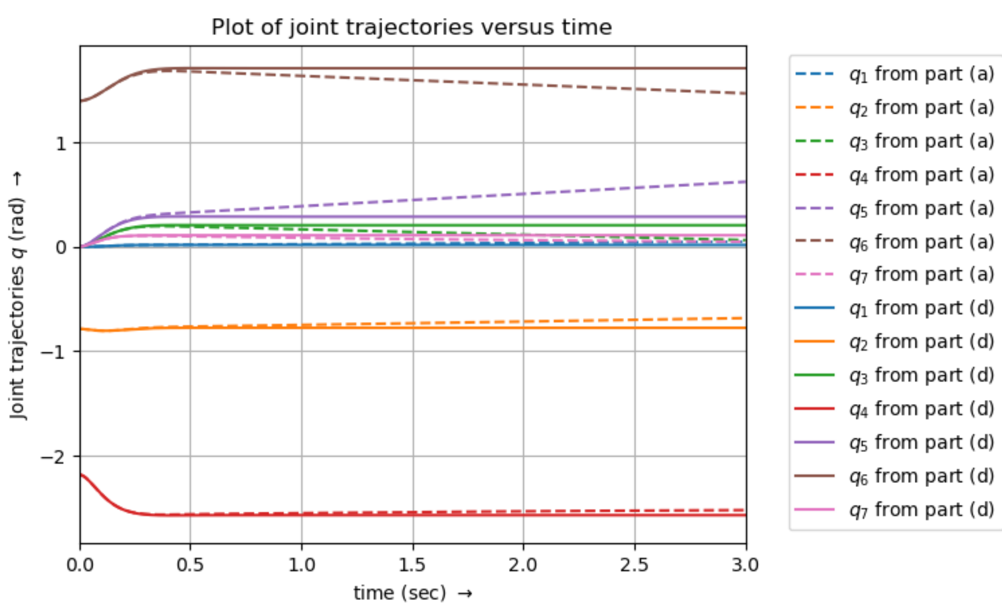

Figure 6: Plot of joint trajectories.

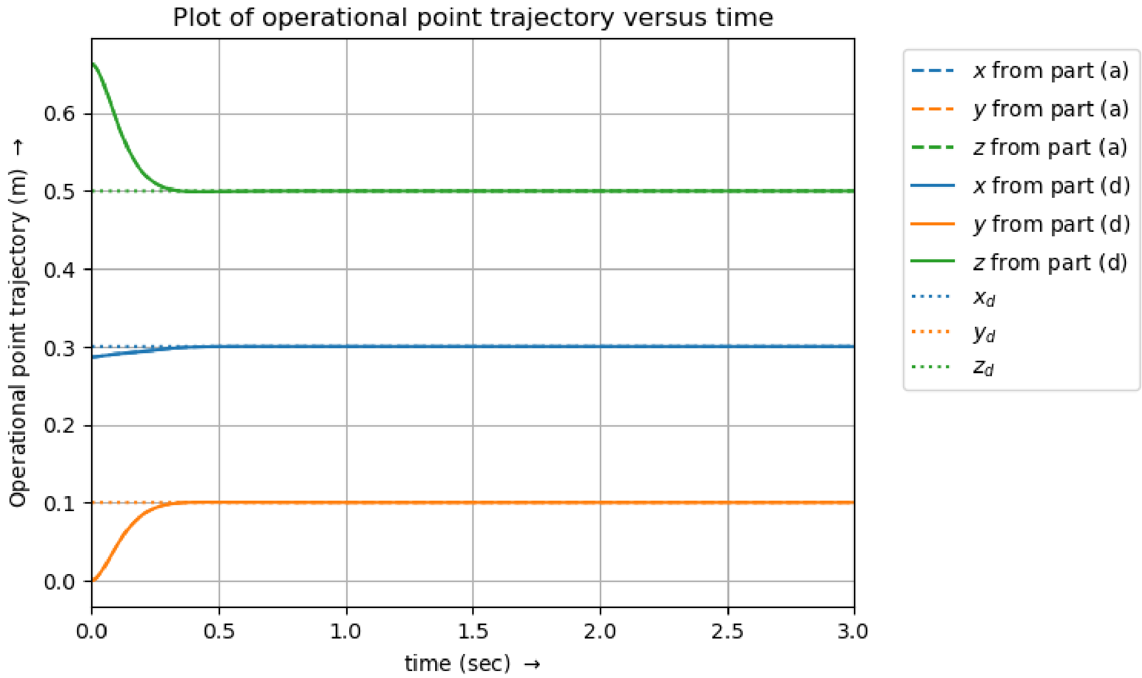

Figure 7: Plot of operational point trajectory.

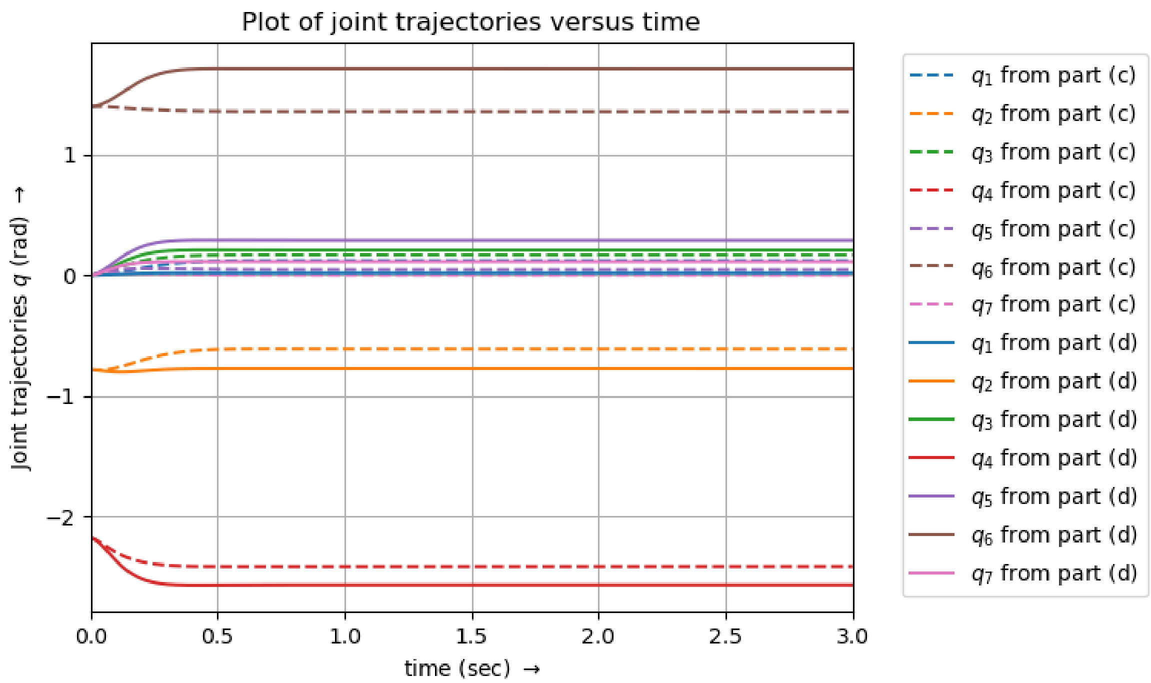

Figure 8: Plot of joint trajectories.

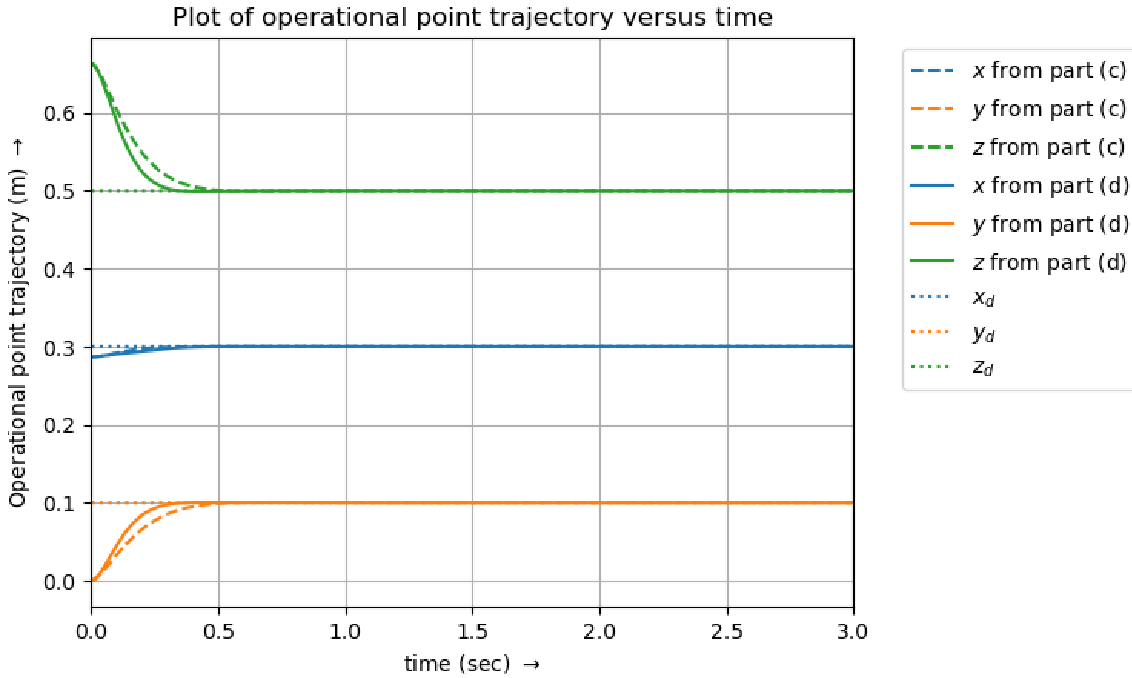

Figure 9: Plot of operational point trajectory.

We observe that there is no residual motion after the robot reaches the goal position. But since now we damped our joints in the null space of the control task only, the robot no longer moves slowly, and reaches the goal position as quickly as in part (a).

Problem 3

(a) We implement the control law:

$$
\begin{aligned}
p &= \bar{J}^T g \\
F &= \Lambda(-k_p(x - x_d) - k_v \dot{x}) + p \\
\Gamma &= J_v^T F - N^T M(K_v \dot{q})
\end{aligned}
\tag{13}
$$

The plots of the joint trajectories and operational point trajectory are shown below:

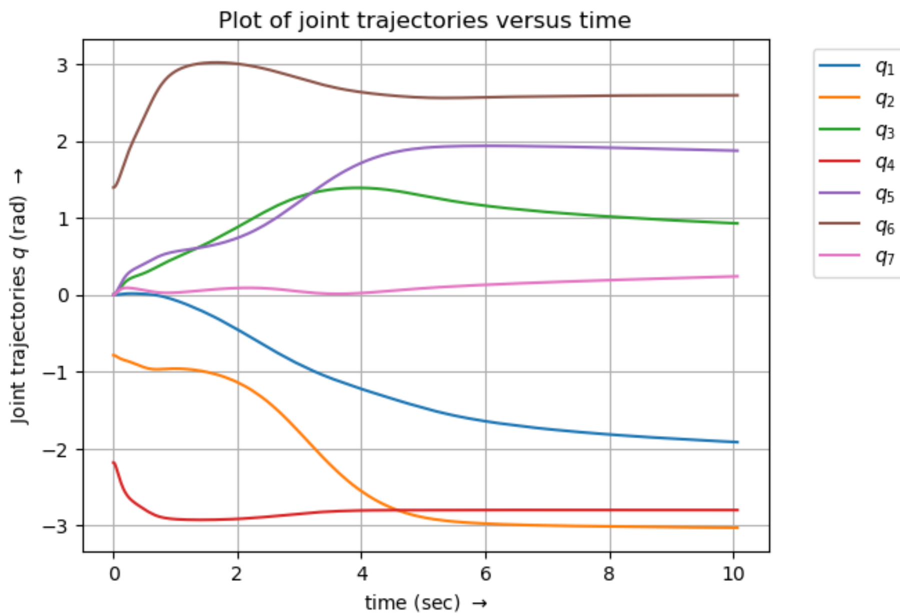

Figure 10: Plot of joint trajectories.

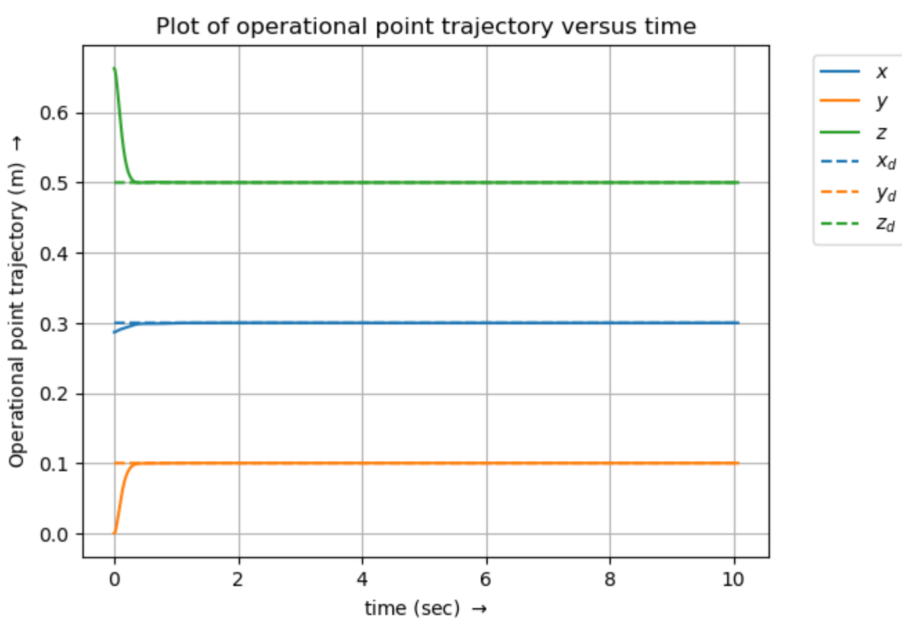

Figure 11: Plot of operational point trajectory.

We observe that the robot reaches the goal position, but its joints still keep moving, before settling down to some value. This results in the arm dropping down.

(b) Due to not using the joint space gravity vector, the effects of gravity are not accounted for, which causes all joints have some steady state error (which further leads to the arm dropping down after some time). We don't observe this behavior with joint space gravity compensation as it cancels gravity in all joints, leading to no steady state error in the joint values.

Problem 4

(i) We implement the control law:

$$\begin{aligned} p &= \bar{J}^T g \\ F &= \Lambda(-k_p(x - x_d) - k_v(\dot{x} - \dot{x}_d)) + p \\ \Gamma &= J_v^T F + N^T M(-K_v \dot{q}) \end{aligned} \tag{14}$$

The plots of the joint trajectories and operational point trajectory are shown below:

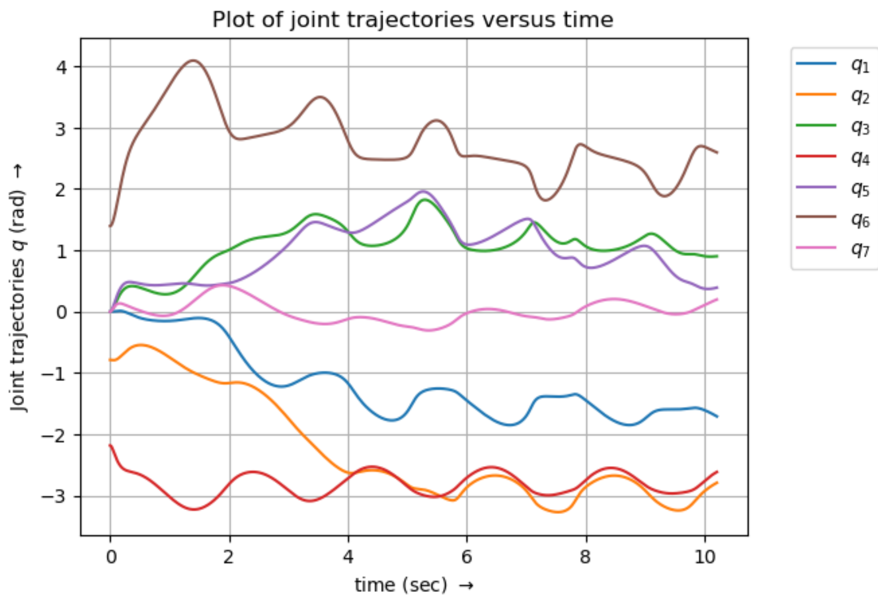

Figure 12: Plot of joint trajectories.

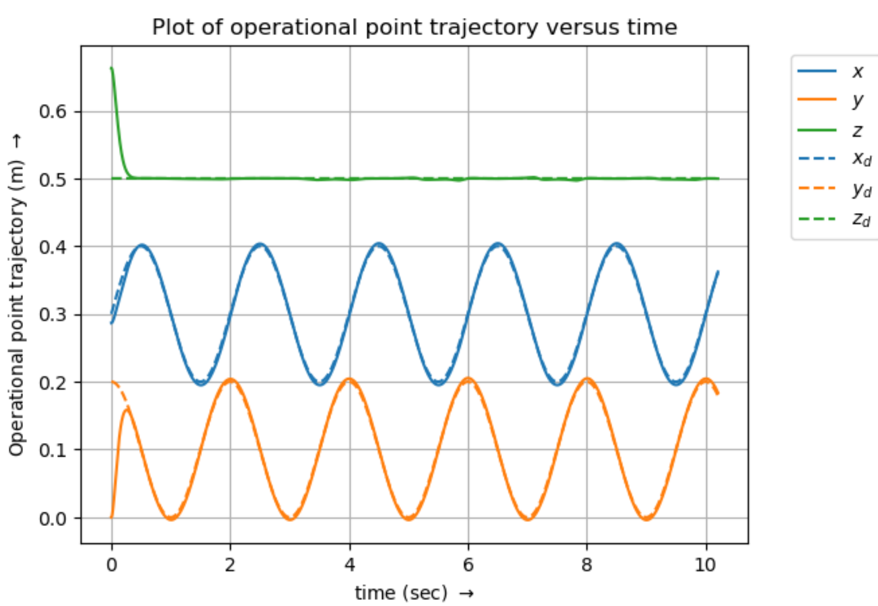

Figure 13: Plot of operational point trajectory.

The plot of the y-coordinate versus the x-coordinate of the end-effector is shown below:

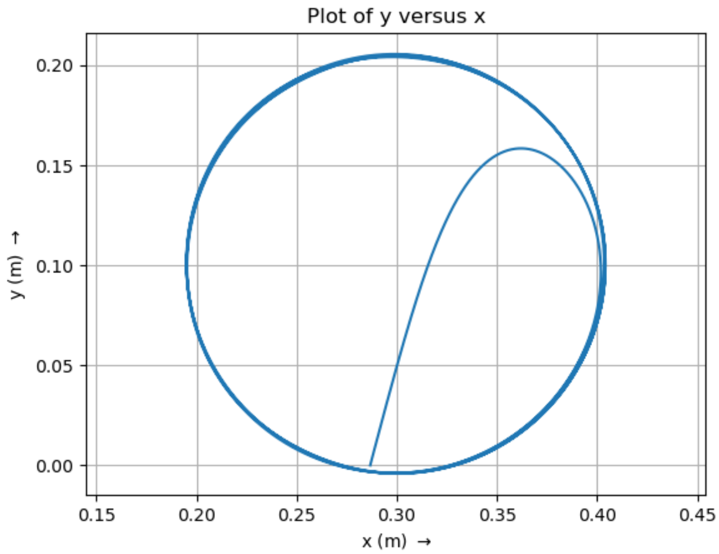

Figure 14: Plot of y versus x for the end-effector.

We observe that the end-effector tracks the circular trajectory properly. But due to not using the joint space gravity vector, the effects of gravity are not accounted for, which leads to the arm dropping down as the trajectory is tracked.

(ii) We implement the control law:

$$\begin{aligned} p &= \bar{J}^T g \\ F &= -k_p(x - x_d) - k_v(\dot{x} - \dot{x}_d) + p \\ \Gamma &= J_v^T F + N^T M(-K_v \dot{q}) \end{aligned} \tag{15}$$

The plots of the joint trajectories and operational point trajectory are shown below:

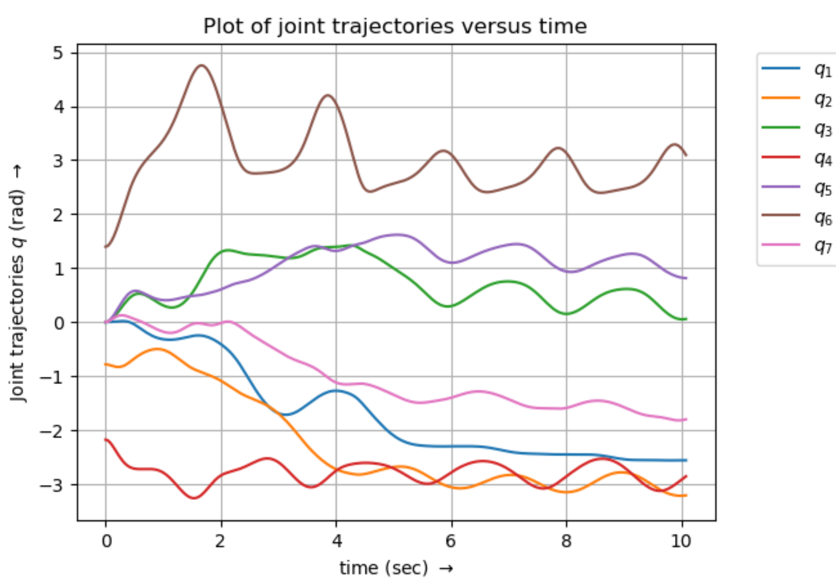

Figure 15: Plot of joint trajectories.

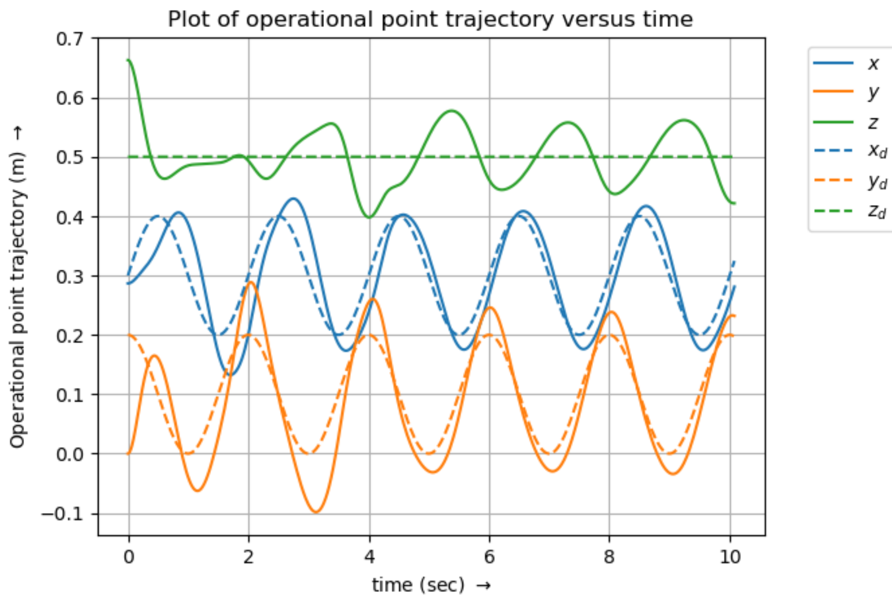

Figure 16: Plot of operational point trajectory.

The plot of the y-coordinate versus the x-coordinate of the end-effector is shown below:

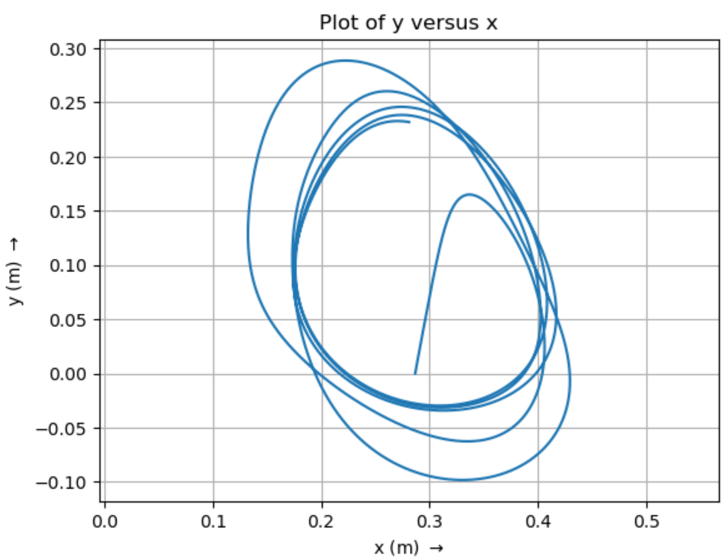

Figure 17: Plot of y versus x for the end-effector.

Since we do not use the Lambda matrix, there is no dynamic decoupling in the operational space, and so we observe that the end-effector does not track the circular trajectory properly. And not using the joint space gravity vector causes the arm to drop down.

(iii) We implement the control law:

$$\begin{aligned} p &= \bar{J}^T g \\ F &= \Lambda(-k_p(x - x_d) - k_v(\dot{x} - \dot{x}_d)) + p \\ \Gamma &= J_v^T F + N^T M(-K_p q - K_v \dot{q}) \end{aligned} \tag{16}$$

The plots of the joint trajectories and operational point trajectory are shown below:

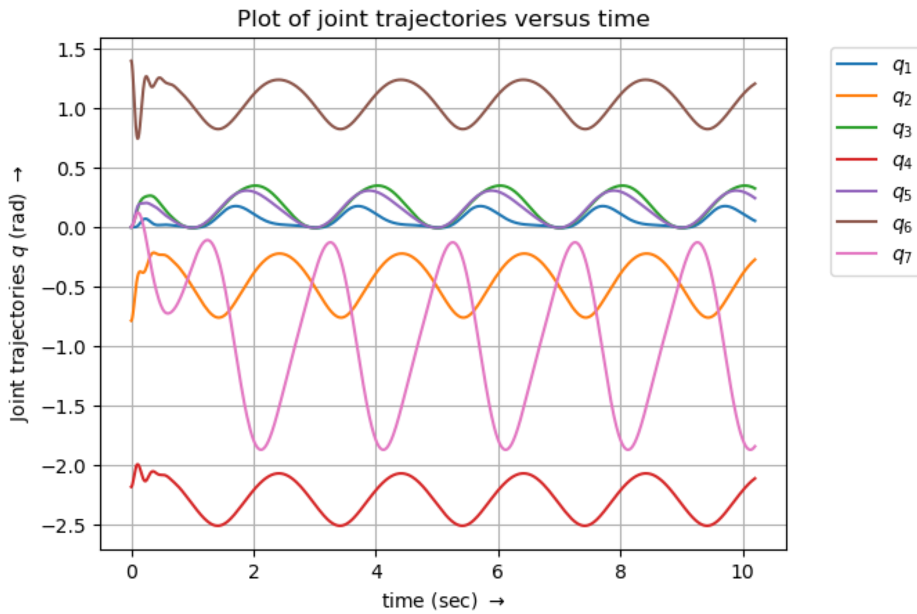

Figure 18: Plot of joint trajectories.

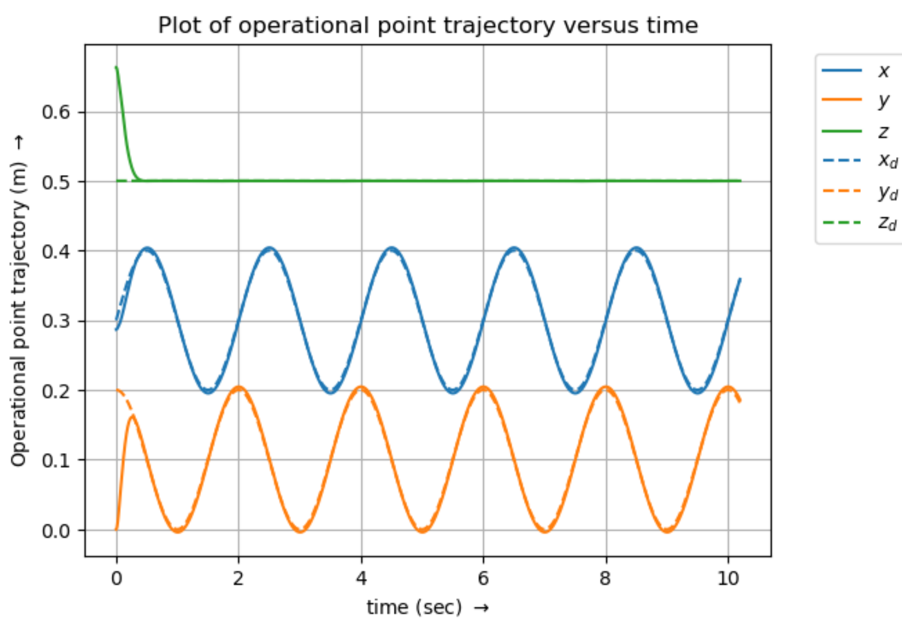

Figure 19: Plot of operational point trajectory.

The plot of the y-coordinate versus the x-coordinate of the end-effector is shown below:

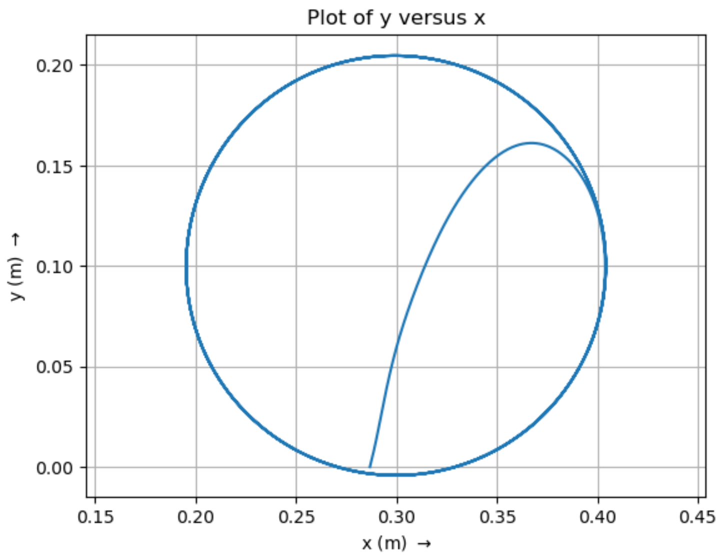

Figure 20: Plot of y versus x for the end-effector.

We observe that the end-effector tracks the circular trajectory properly. Since the PD controller tries to bring the posture to $q_d = [0, 0, 0, 0, 0, 0, 0]$ (that is, there is a non-zero K_p term for each joint that significantly reduces the steady-state error), the arm does not drop down like in parts (i) and (ii).

(iv) We implement the control law:

$$\begin{aligned} p &= \bar{J}^T g \\ F &= \Lambda(-k_p(x - x_d) - k_v(\dot{x} - \dot{x}_d)) + p \\ \Gamma &= J_v^T F + N^T(M(-K_p q - K_v \dot{q}) + g) \end{aligned} \tag{17}$$

The plots of the joint trajectories and operational point trajectory are shown below:

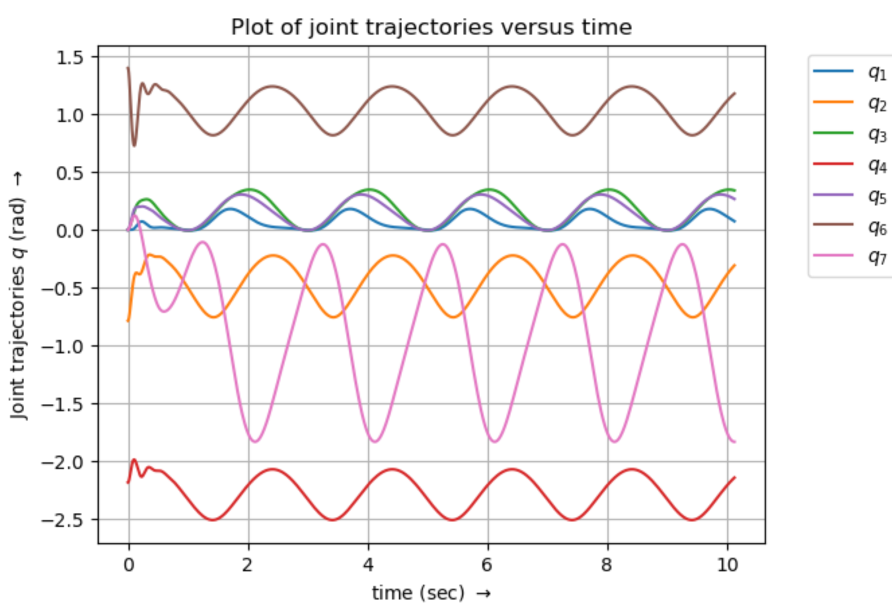

Figure 21: Plot of joint trajectories.

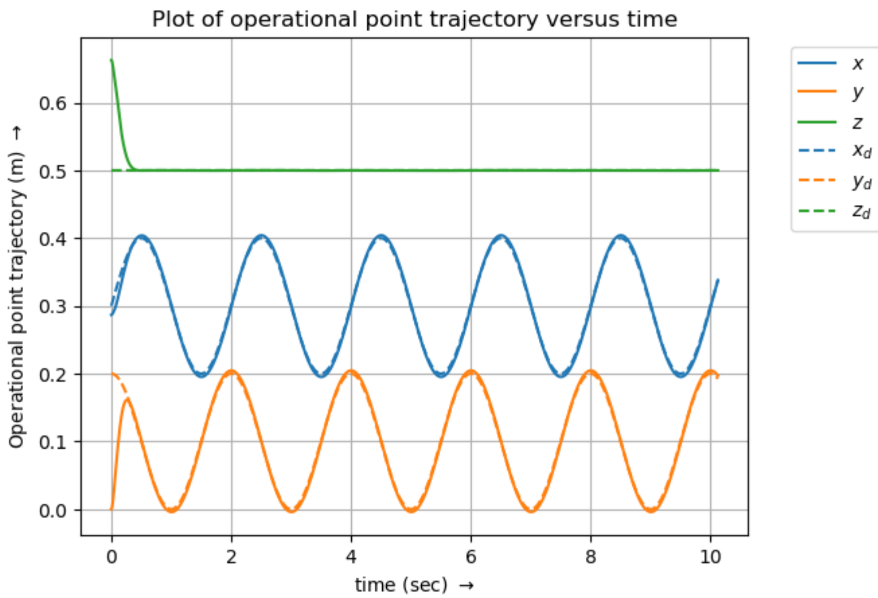

Figure 22: Plot of operational point trajectory.

The plot of the y-coordinate versus the x-coordinate of the end-effector is shown below:

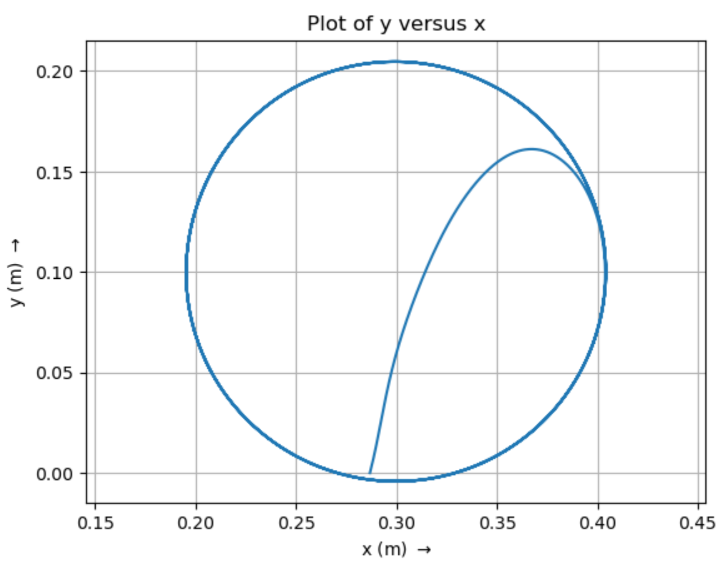

Figure 23: Plot of y versus x for the end-effector.

We observe that the end-effector tracks the circular trajectory properly. With the PD controller trying to bring the posture to $q_d = [0, 0, 0, 0, 0, 0, 0]$ and with joint space gravity compensation, the arm does not drop down like in parts (i) and (ii).

For the controllers above, the gains are:

$$k_p = 200,\ k_v = 24$$

$$K_p = \begin{bmatrix} 400 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 400 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 400 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 400 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 400 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix},\ K_v = \begin{bmatrix} 15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix} \tag{18}$$