# ME5205 Theory of Vibration - Project

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## 1 Project Statement

Develop a suspension system for a four-wheel automotive, that can take extreme road conditions in a semi-urban setting, where there are potholes and bumps. Your goal is to help minimize the excitation transmitted to a rider in the vehicle.

## 2 Introduction

A seat suspension system can be used to attenuate high amplitude vibration transmitted from the road to the rider in the low frequency range and improve vehicle ride comfort. A linear seat suspension, consisting of a spring and a dashpot, can provide effective isolation when the excitation frequencies are larger than $\sqrt{2}$ times the natural frequency of the system, i.e.

$$\omega > \sqrt{\frac{2k}{m}}$$

This is well documented in the literature [1-3]. From this, it is evident that reducing the stiffness of the system i.e the natural frequency, will give rise to a wider frequency range of isolation. However, a smaller stiffness will also result in a large static displacement between the vehicle floor and seat, and this trade-off between static displacement and isolation is well documented in [2]. This limitation can be overcome by using a Quasi-Zero-Stiffness (QZS) isolator [4-6] as seat suspension. These isolators have a high static stiffness, and hence a small static displacement, alongwith a small dynamic stiffness, which results in a low natural frequency. This is generally achieved by configuring springs so that they act as a negative stiffness in parallel with a positive stiffness. Many QZS isolator mechanisms have been proposed, however for this project, a QZS isolator consisting of two inclined springs, and a vertical spring to stabilize the large displacement behaviour, is used. This vertical spring has a clearance, which is tuned to close exactly when the inclined springs reach a negative stiffness. Without the vertical spring, the isolator exhibits negative stiffness behaviour after the QZS regime, which can possibly cause a snap-through behaviour, leading to some damage to the supported structure.
Hence, a QZS vibration isolator is proposed as seat suspension to improve the vehicle vibration performance, and to minimize the excitation transmitted to the rider. In the next section, the QZS vibration isolator, along with a brief description of its static analysis, is presented. This is then followed by establishing the vehicle seat-human coupled model, with the QZS isolator as seat suspension. The dynamic characteristics of this model subject to shock excitation are then obtained using numerical methods. These results are also compared with the case where a simple linear vibration isolator is used as seat suspension.

## 3 QZS vibration isolator model

The QZS vibration isolator, composed of 2 inclined springs and a vertical spring, is shown in Figure 1.

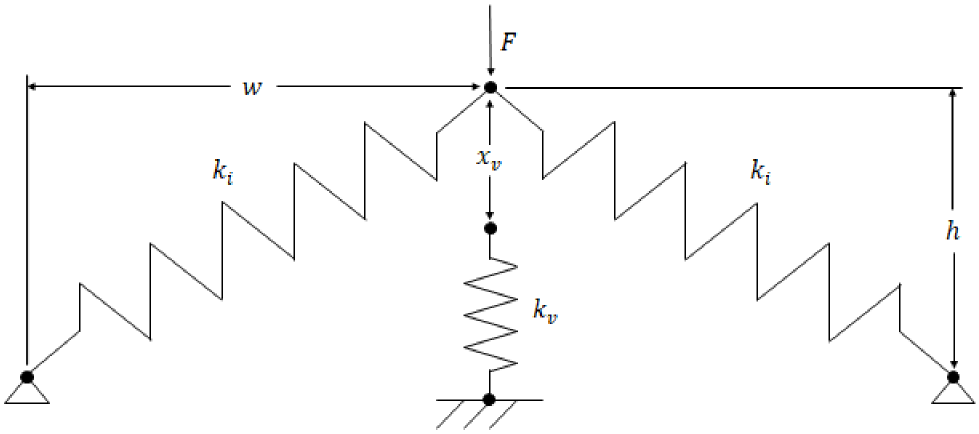


Figure 1: QZS vibration isolator

The width and the height of the isolator are $w$ and $h$ respectively. The stiffness of the inclined springs is $k_i$, and the stiffness of the vertical spring is $k_v$. We now examine the force-deflection characteristics as well as the stiffness of the isolator. The stiffness is determined by first computing the force $F$ needed to deflect the springs vertically by $x$, which is given by

$$F(x) = \begin{cases} \bar{F} = 2k_i(h-x)\left(\sqrt{\dfrac{h^2+w^2}{(h-x)^2+w^2}} - 1\right) & x \leq x_v \\ \bar{F} + k_v(x - x_v) & x > x_v \end{cases}$$

The stiffness of the isolator is then simply given by $\dfrac{dF}{dx}$, that is

$$k_{eff}(x) = \begin{cases} \bar{k} = 2k_i\left(1 - \sqrt{\dfrac{h^2+w^2}{(h-x)^2+w^2}} + \dfrac{(h-x)^2\sqrt{h^2+w^2}}{((h-x)^2+w^2)^{3/2}}\right) & x \leq x_v \\ \bar{k} + k_v & x > x_v \end{cases}$$

The location of the QZS is given by

$$\bar{k} = \frac{d\bar{F}}{dx} = 0$$

Solving this gives

$$x_{QZS} = h\left(1 - \sqrt{(\gamma^6+\gamma^4)^{1/3} - \gamma^2}\right)$$

where $\gamma = \dfrac{w}{h}$. The clearance of the vertical stopper $x_v$ is chosen such that

$$x_v = x_{QZS}$$

This ensures that the vertical spring engages with the inclined springs as soon as the isolator is about to enter the negative stiffness regime.
The structural parameters of this QZS isolator are

$$k_i = k_v = 3.2 \times 10^4 \text{ N/m}$$

$$w = h = 93 \text{ mm.}$$

$$x_v = 45.59 \text{ mm.}$$

For this isolator, the force-deflection characteristics and the plot of the stiffness are shown below.

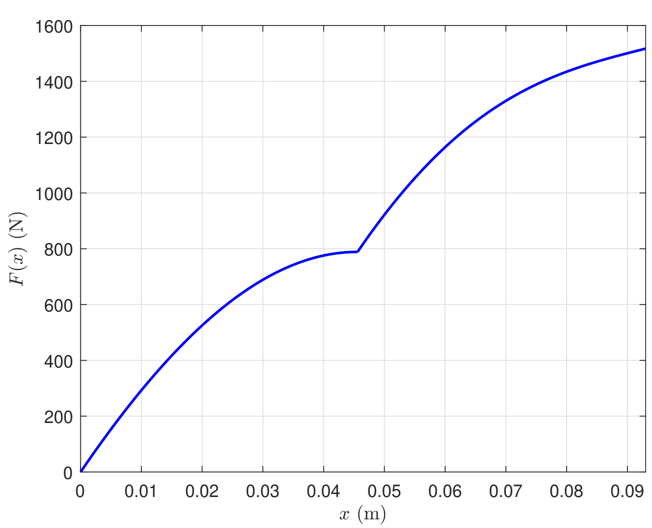


Figure 2: Force-deflection characteristic

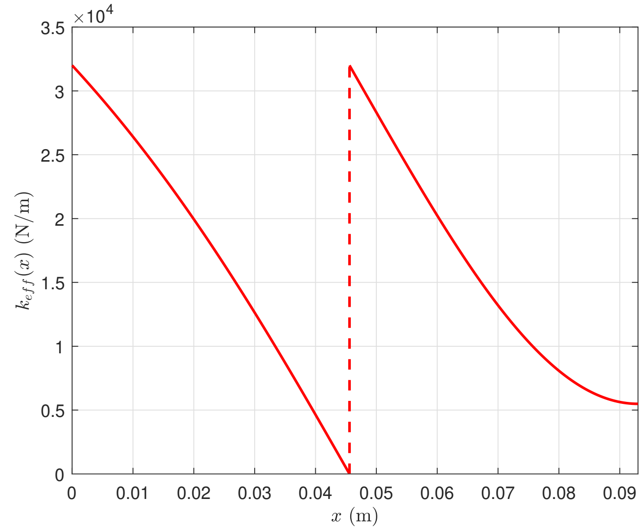


Figure 3: Plot of stiffness

As expected, the stiffness drops to a near zero value at $x = x_v$, and then suddenly increases because of the vertical spring. We would like the isolator to operate in the QZS regime, where the stiffness is nearly zero, just before the vertical spring engages with it. Note that the force is continuous, but not differentiable at $x = x_v$, and the stiffness is discontinuous at $x = x_v$. The mass $m_{QZS}$ that brings the isolator to the QZS regime is given by

$$m_{QZS} = \frac{\bar{F}(x_v)}{g}$$

For this isolator, $m_{QZS} = 80.4$ kg. The isolator has to be designed keeping in mind the total mass of the seat and the rider. As mentioned before, we would want this total mass to be very close to (and less than) $m_{QZS}$, so that we are in a region of near zero stiffness (QZS regime), with very less excitation being transmitted to the rider.

# 4 Vehicle-seat-human coupled model

The vehicle-seat-human coupled model with the QZS vibration isolator as seat suspension is shown in Figure 4.

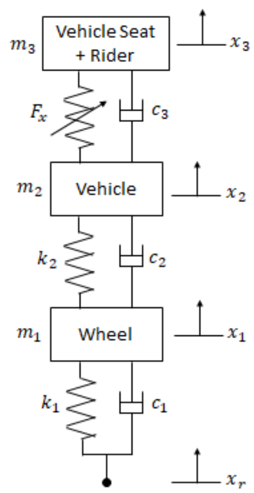


Figure 4: Vehicle-seat-human coupled model

Note that the model is a 3 DOF system. The masses of the wheel and vehicle body are $m_1$ and $m_2$ respectively, and $m_3$ is the total mass of the vehicle seat and the rider. The stiffness and damping of the wheel and the vehicle body are $k_1, k_2$ and $c_1, c_2$ respectively, and $c_3$ is the damping of the vehicle seat. $F_x$ is the force of the isolator in the seat. The displacements of masses $m_1$, $m_2$ and $m_3$ are $x_1$, $x_2$ and $x_3$ respectively, and $x_r$ is the road excitation displacement applied directly to the wheel. The dynamic equations of this model are given by

$$m_1\ddot{x_1} = c_2(\dot{x_2} - \dot{x_1}) - c_1(\dot{x_1} - \dot{x_r}) + k_2(x_2 - x_1) - k_1(x_1 - x_r)$$

$$m_2\ddot{x_2} = c_3(\dot{x_3} - \dot{x_2}) - c_2(\dot{x_2} - \dot{x_1}) - k_2(x_2 - x_1) + F_x$$

$$m_3\ddot{x_3} = -c_3(\dot{x_3} - \dot{x_2}) - F_x$$

To simplify things to an extent, we make an approximation for calculating $F_x$. We first find the statical deflection $\Delta$, which depends on the mass $m_3$. If $m_3 \leq m_{QZS}$, $\Delta$ is obtained by solving the following equation

$$\bar{F}(\Delta) = m_3 g$$

However, if $m_3 > m_{QZS}$, then we instead solve this equation to get $\Delta$

$$\bar{F}(\Delta) + k_v(\Delta - x_v) = m_3 g$$

Finally, after calculating $\Delta$, and knowing the expression of $k_{eff}(x)$, we can approximate $F_x$ by

$$F_x \approx k_{eff}(\Delta)(x_3 - x_2)$$

For the case where the vehicle-seat-human coupled model has a simple linear vibration isolator as seat suspension, $F_x$ is given by

$$F_x = k_v(x_3 - x_2)$$

We now write the above equations in the state-space form, by defining the following state variables

$$x_1 = y_1, \quad \dot{x_1} = y_2, \quad x_2 = y_3, \quad \dot{x_2} = y_4, \quad x_3 = y_5, \quad \dot{x_3} = y_6$$

Therefore, the equations in state-space are given by

$$\dot{y_1} = y_2$$

$$\dot{y_2} = \frac{c_2(y_4 - y_2) - c_1(y_2 - \dot{x_r}) + k_2(y_3 - y_1) - k_1(y_1 - x_r)}{m_1}$$

$$\dot{y_3} = y_4$$

$$\dot{y_4} = \frac{c_3(y_6 - y_4) - c_2(y_4 - y_2) - k_2(y_3 - y_1) + F_x}{m_2}$$

$$\dot{y_5} = y_6$$

$$\dot{y_6} = \frac{-c_3(y_6 - y_4) - F_x}{m_3}$$

where $F_x$ is given by

$$F_x = \begin{cases} k_{eff}(\Delta)(y_5 - y_3) & \text{for a QZS vibration isolator} \\ k_v(y_5 - y_3) & \text{for a simple linear vibration isolator} \end{cases}$$

The performance of the suspension system with the QZS isolator and the simple linear isolator should be compared by examining the values of $x_3$, $\dot{x_3}$ and $\ddot{x_3}$, i.e. the rider position, velocity and acceleration. The values of $|x_2 - x_1|$ and $|x_3 - x_2|$ should also be examined, because if these values become large, the masses can hit each other and can destroy the suspension mechanism. These state-space equations are solved numerically by a MATLAB program, and the results obtained are discussed in the next section.

# 5 Numerical results

We now examine the vehicle-seat human coupled model under shock excitation. The parameters for the model are listed below

$$m_1 = 70 \text{ kg},\ m_2 = 400 \text{ kg},\ m_{QZS} = 80.4 \text{ kg},\ m_{3min} = 70 \text{ kg},\ m_{3max} = 90 \text{ kg}$$

$$k_1 = 2 \times 10^5 \text{ N/m},\ k_2 = 1.6 \times 10^4 \text{ N/m},\ k_v = 3.2 \times 10^4 \text{ N/m}$$

$$c_1 = 10 \text{ Ns/m},\ c_2 = 1500 \text{ Ns/m},\ c_3 = 800 \text{ Ns/m}$$

$$|x_2 - x_1|_{max} = 0.12 \text{ m},\ |x_3 - x_2|_{max} = 0.1 \text{ m}$$

The shock excitation is caused by a single irregularity on the road, such as a bump or a pothole. This excitation $x_r$, in the case of a single bump, can be given as

$$x_r(t) = \begin{cases} h_b \sin^2\left(\dfrac{\pi v_s t}{l_b}\right) & 0 \le t \le \dfrac{l_b}{v_s} \\ 0 & t > \dfrac{l_b}{v_s} \end{cases}$$

where $h_b$ is the height of the bump, $v_s$ is the vehicle speed, and $l_b$ is the length of the bump. For this analysis, these parameters were chosen as

$$h_b = 0.1 \text{ m},\ v_s = 15 \text{ m/s and } l_b = 3 \text{ m}.$$

We now compare the performance of the suspension system with the QZS isolator and the simple linear isolator by looking at the plots of $x_3$, $\dot{x_3}$, $\ddot{x_3}$, $(x_2 - x_1)$ and $(x_3 - x_2)$, for $m_3 = m_{QZS}$.

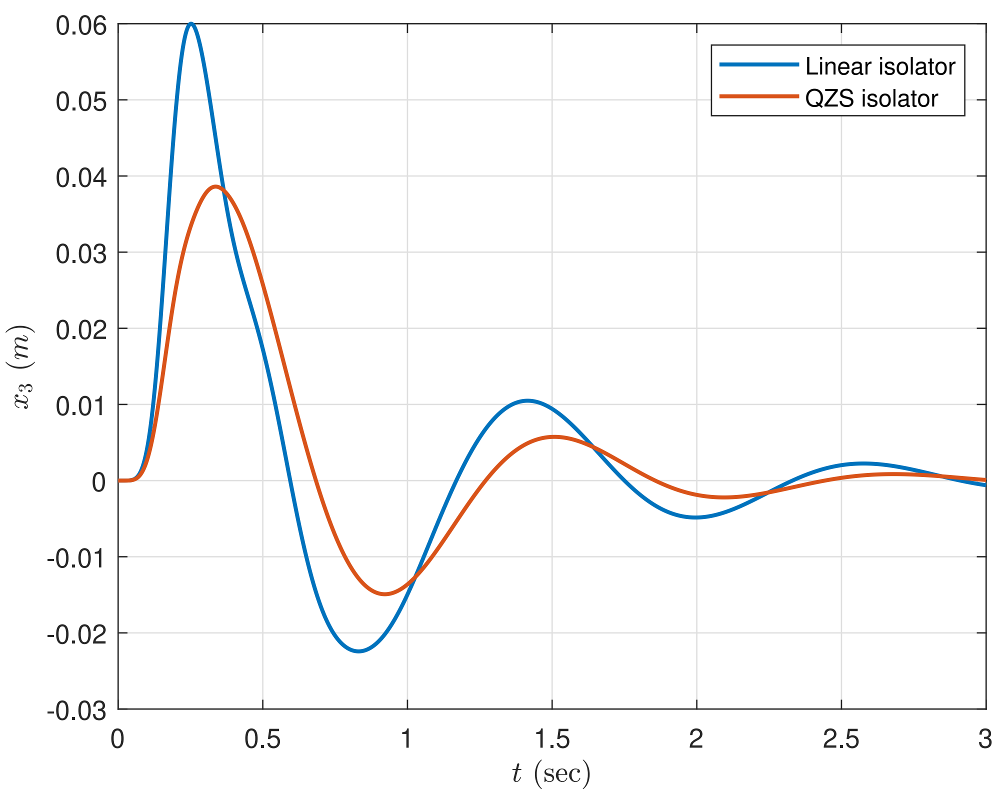


Figure 5: Position $x_3$ of seat and rider

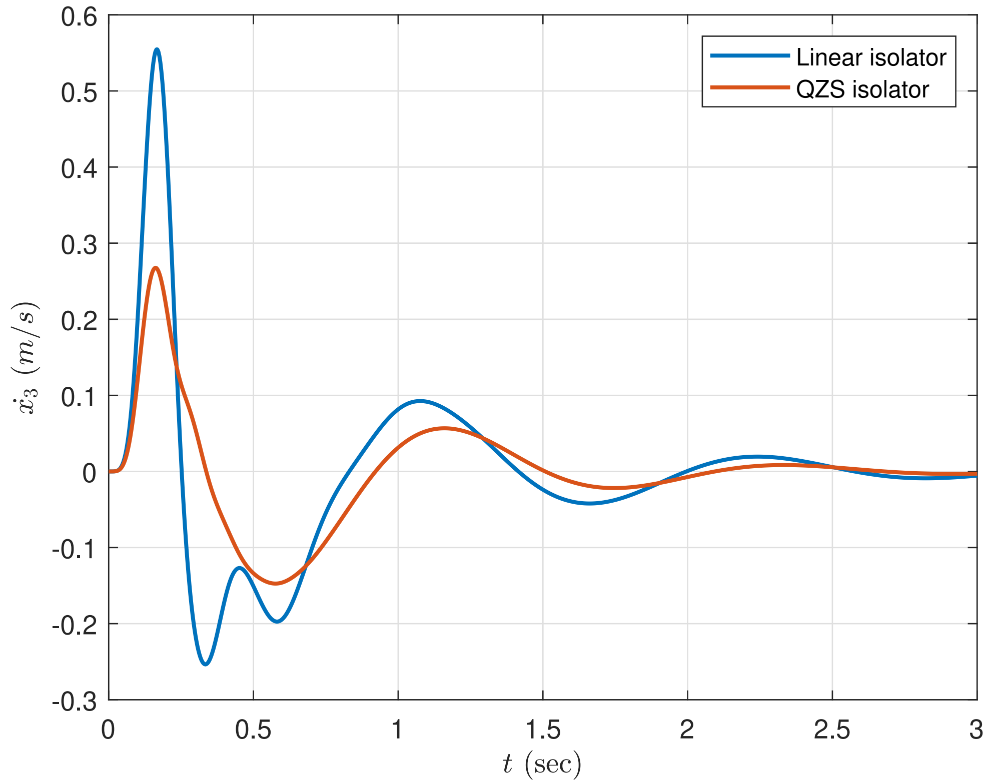


Figure 6: Velocity $v_3$ of seat and rider

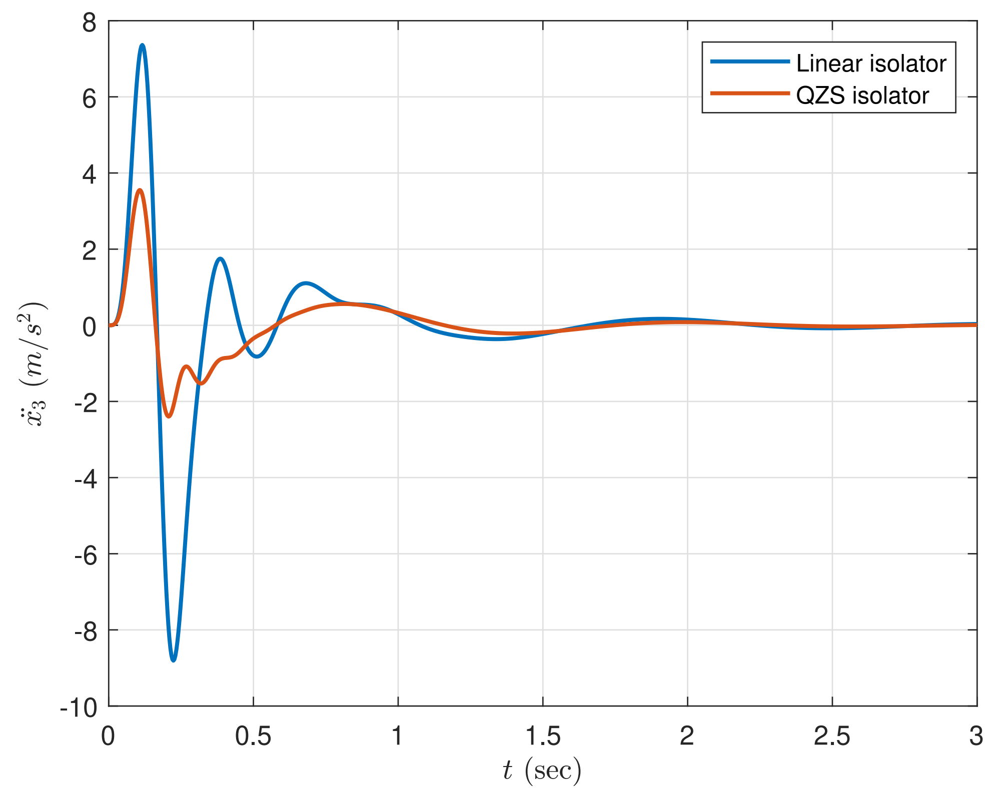


Figure 7: Acceleration $a_3$ of seat and rider

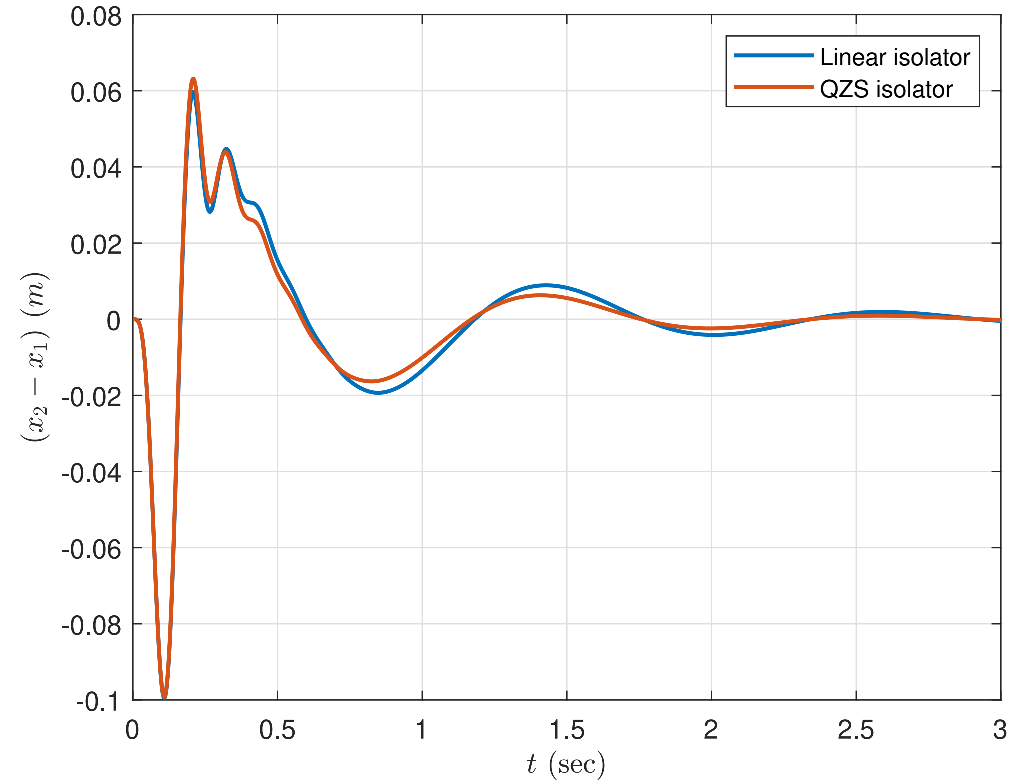


Figure 8: Vehicle suspension stroke $(x_2 - x_1)$

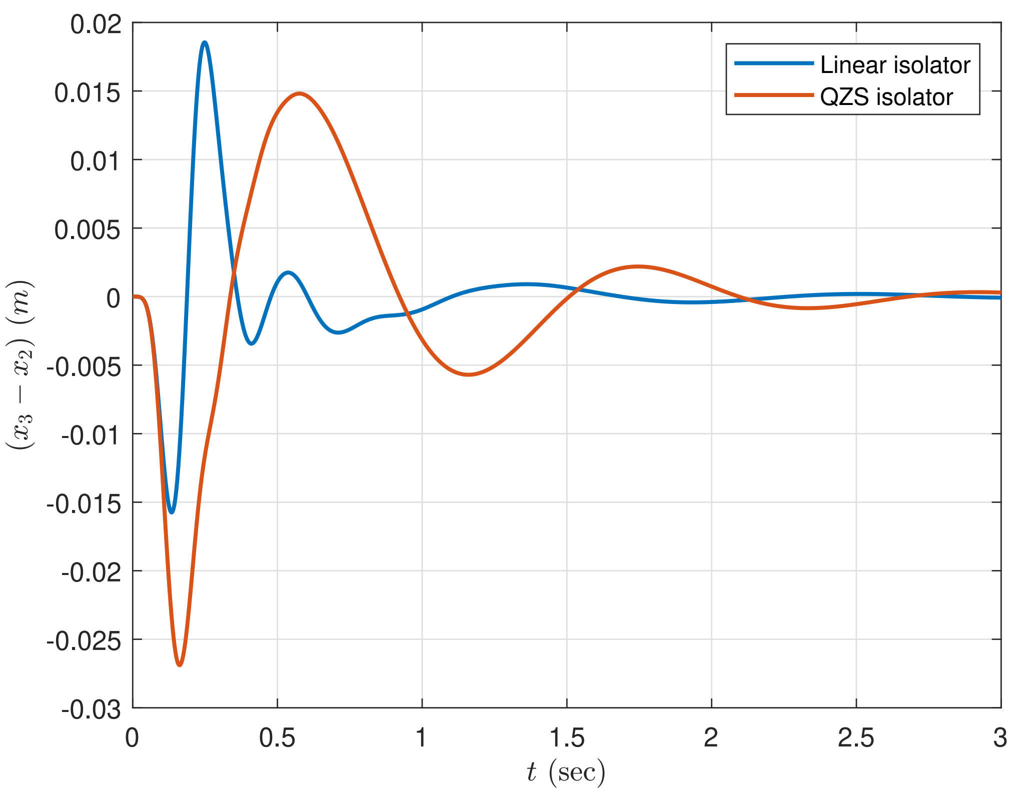


Figure 9: Seat suspension stroke $(x_3 - x_2)$

The maximum values of rider position, velocity, acceleration, and suspension strokes are given in the table below.

| Parameter | $\lvert x_3 \rvert_{max}$ | $\lvert v_3 \rvert_{max}$ | $\lvert a_3 \rvert_{max}$ | $\lvert x_2 - x_1 \rvert_{max}$ | $\lvert x_3 - x_2 \rvert_{max}$ |
|---|---|---|---|---|---|
| Linear isolator | 0.06 | 0.5547 | 8.8079 | 0.0995 | 0.0186 |
| QZS isolator | 0.0386 | 0.2676 | 3.5528 | 0.0992 | 0.0269 |

We observe that when a QZS vibration isolator is used in the suspension system, the maximum values of the rider position, velocity and acceleration are smaller compared to the case when a linear vibration isolator is used. Hence, the excitation transmitted to the rider decreases, and the vehicle ride comfort improves considerably. The time history of the vehicle suspension stroke is almost identical for the two isolators. However, the seat suspension stroke is larger for the QZS isolator because its dynamic stiffness is much smaller than the linear isolator in the QZS regime.

We now look at the rider position, velocity and acceleration, for different values of mass $m_3$.

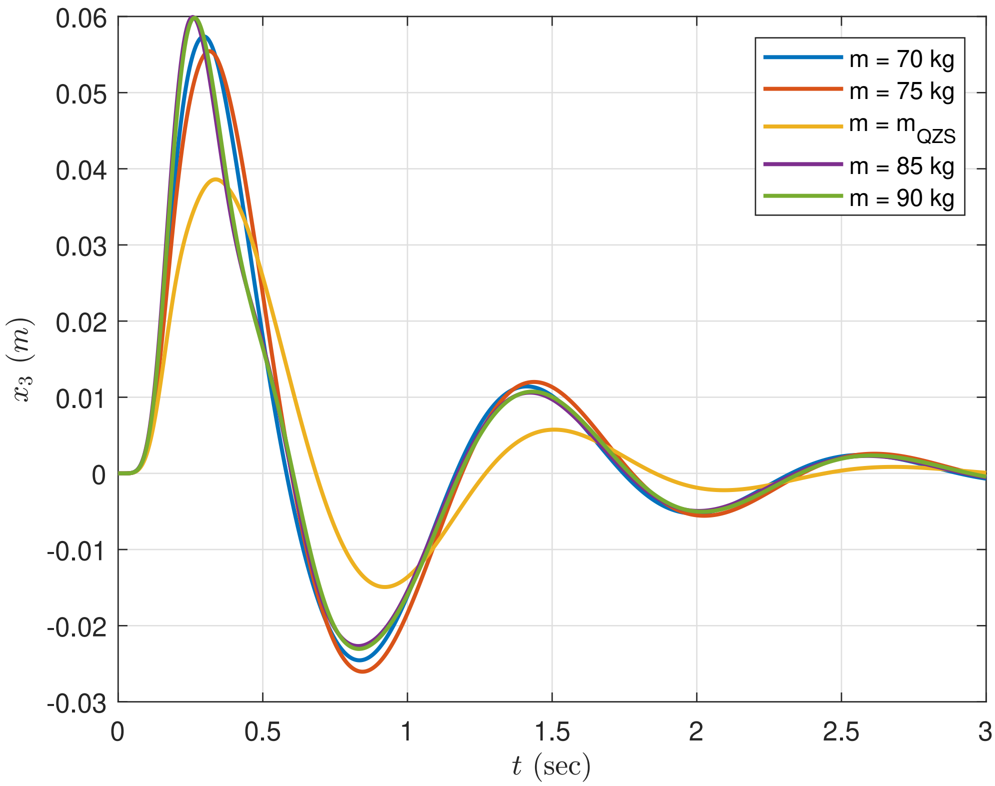


Figure 10: Position $x_3$ of seat and rider

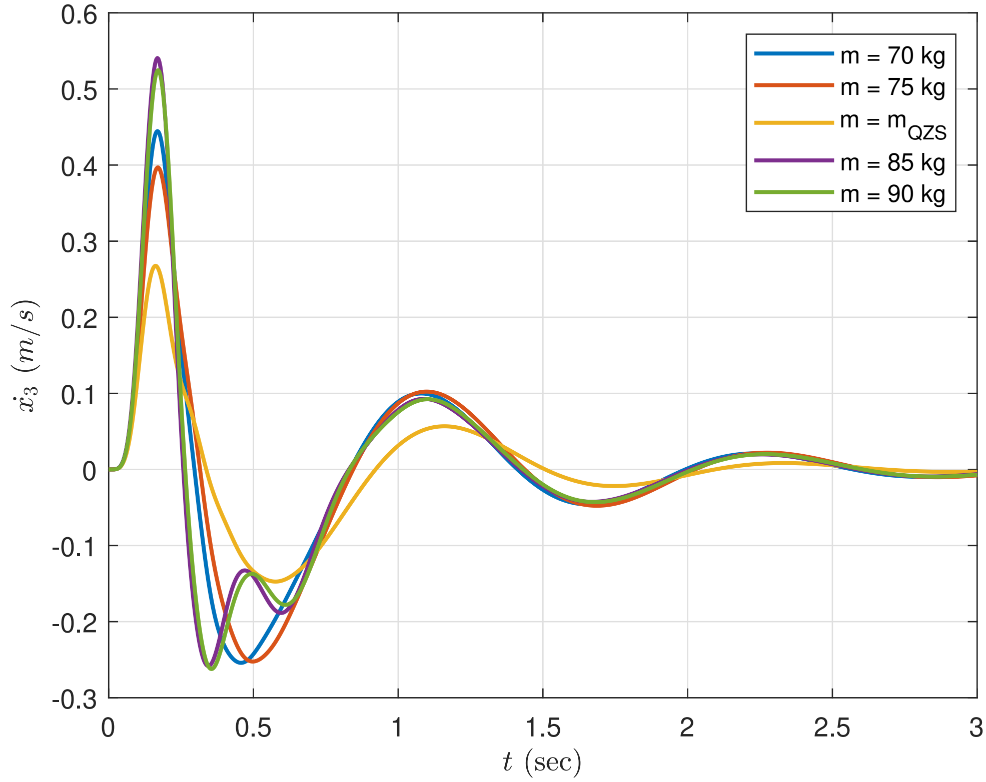


Figure 11: Velocity $v_3$ of seat and rider

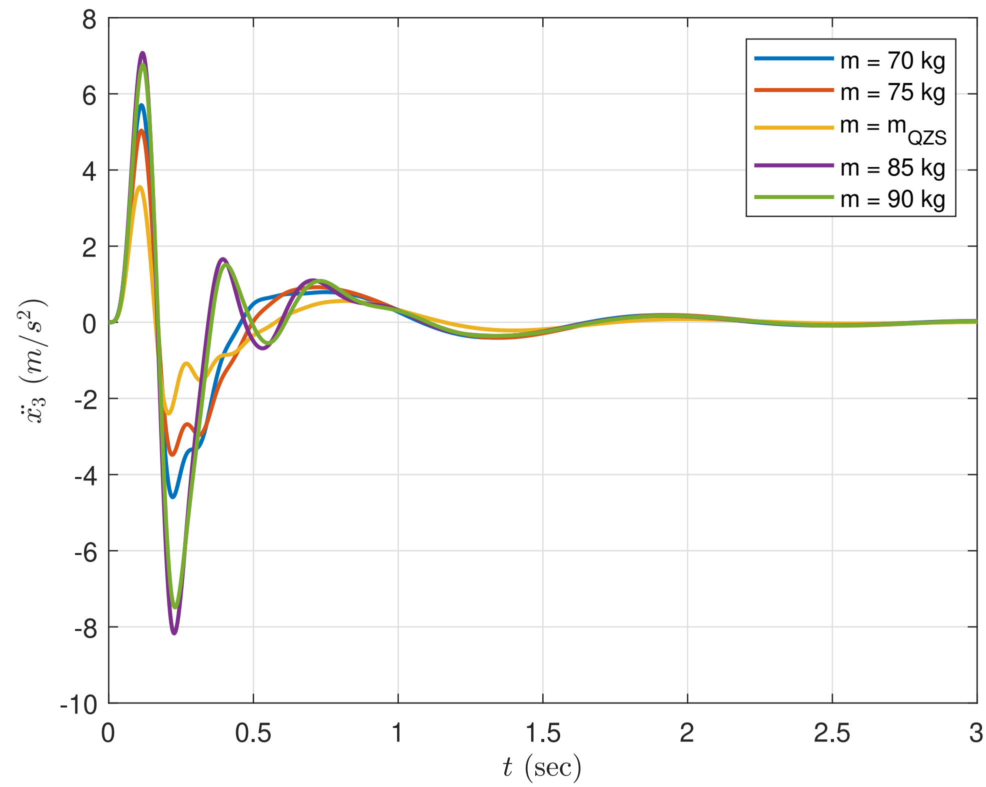


Figure 12: Acceleration $a_3$ of seat and rider

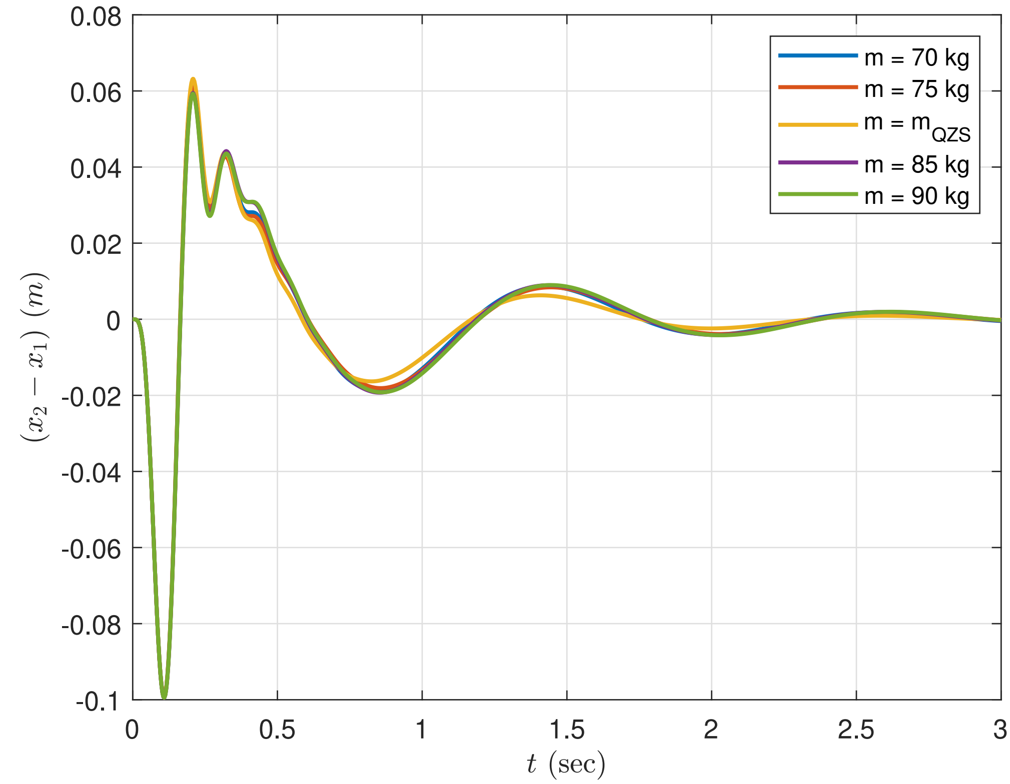


Figure 13: Vehicle suspension stroke $(x_2 - x_1)$

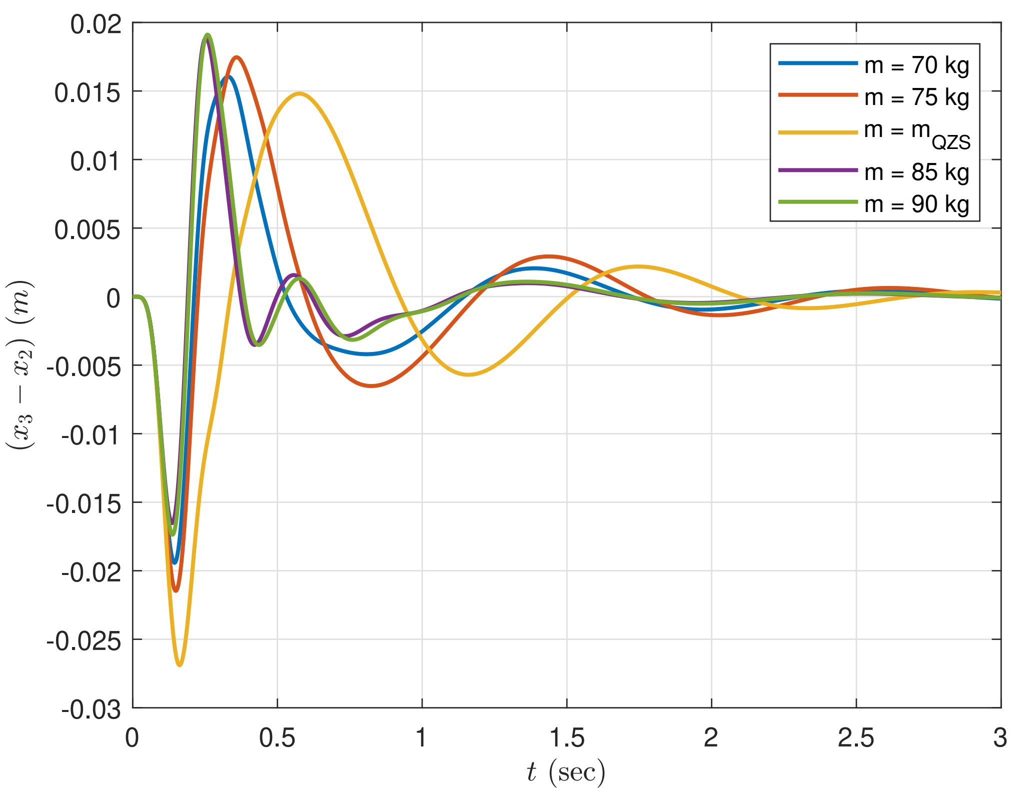


Figure 14: Seat suspension stroke $(x_3 - x_2)$

As is evident from the time histories of the rider position, velocity and acceleration, it is very important that the total mass of the rider and seat, $m_3$, should be as close as possible to $m_{QZS}$. Therefore, the QZS vibration isolator for the suspension system should be designed keeping in mind the value of $m_3$.

# 6 Conclusion

A suspension system with a Quasi-Zero-Stiffness (QZS) isolator is proposed to minimize the excitation transmitted to the rider. The main feature of such an isolator is the use of negative stiffness elements in parallel with a positive stiffness, to achieve low stiffness without having a large static deflection. The isolator model, consisting of two inclined springs and a vertical spring, and its characteristics were discussed. Then the model of the suspension system with the QZS isolator was presented, and its dynamic equations were developed. These equations were solved numerically in a MATLAB program for some specific values of the model parameters. It was found that when the QZS vibration isolator is used as seat suspension, the maximum values of the rider position, velocity and acceleration were smaller than the linear seat suspension. The excitation transmitted to the rider decreases, and the vehicle ride comfort improves significantly. The time histories of the vehicle suspension stroke are almost identical. However, the seat suspension stroke is larger for the QZS isolator, but it stays lower than the maximum allowed value. The important thing is that the QZS isolator should be designed such that the total mass of the seat and rider should be very close to, and less than $m_{QZS}$, which is the mass that brings the isolator to the QZS regime.

# 7 References

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