Problem 14-2

At an average film temperature of 350 K, what are the individual heat transfer coefficients when the fluid flowing in a 0.0254-m inside diameter tube is air, water or oil? Each fluid in this comparison exhibits a Reynolds number of 5×10^4. How would the pressure drop vary for each fluid? The relevant properties of the three fluids at 350 K are listed in the table below.

	Air	**Water**	**Oil**
Density, kg/m^3	.955	973	854
Viscosity, Pa·s	2×10^{-5}	3.72×10^{-4}	3.56×10^{-2}
Thermal Conductivity, W/m·K	0.030	0.668	0.138
Heat Capacity, J/kg·K	1050	4190	2116

Solution:

This problem will be solved in two parts. First, we will calculate the heat transfer coefficients and then the pressure drops.

Part 1. Heat Transfer Coefficients.

The easiest way to calculate the heat transfer coefficients is to use Equation 14-18 on page 657, assuming $\mu/\mu_w = 1$. However, this equation is only valid for $0.7 < \Pr < 160$, so Pr must be checked. If Pr is outside this range, then we must use Equations 14-19 and 14-19a on page 658.

$$\Pr_{air} = \frac{C_{p,air}\cdot\mu_{air}}{k_{air}} = \frac{1050\,\frac{J}{kg\cdot K}\cdot 2\times10^{-5}\,\frac{kg}{m\cdot s}}{0.030\,\frac{W}{m\cdot K}} = 0.700$$

$$\Pr_{water} = \frac{C_{p,water}\cdot\mu_{water}}{k_{water}} = \frac{4190\,\frac{J}{kg\cdot K}\cdot 3.72\times10^{-4}\,\frac{kg}{m\cdot s}}{0.668\,\frac{W}{m\cdot K}} = 2.333$$

$$\Pr_{oil} = \frac{C_{p,oil}\cdot\mu_{oil}}{k_{oil}} = \frac{2116\,\frac{J}{kg\cdot K}\cdot 3.56\times10^{-2}\,\frac{kg}{m\cdot s}}{0.138\,\frac{W}{m\cdot K}} = 545.867$$

The Prandtl numbers for air and water are in the correct range, but the oil is not, so we must use Equations 14-19 and 14-19a for the oil. For air and water, we have:

$$h_{air} = 0.023\cdot\left(\frac{0.030\,\frac{W}{m\cdot K}}{0.0254\text{ m}}\right)\cdot(50{,}000)^{0.8}\cdot(0.700)^{1/3} = \underline{\underline{138\,\frac{W}{m^2\cdot K}}}$$ ans

$$h_{water} = 0.023\cdot\left(\frac{0.668\,\frac{W}{m\cdot K}}{0.0254\text{ m}}\right)\cdot(50{,}000)^{0.8}\cdot(2.333)^{1/3} = \underline{\underline{4608\,\frac{W}{m^2\cdot K}}}$$ ans

For oil, we have:

$$f_D = \frac{1}{(1.82 \cdot \log_{10} \mathrm{Re} - 1.64)^2} = \frac{1}{(1.82 \cdot \log_{10} 50000 - 1.64)^2} = 0.0209$$

$$h_{oil} = \frac{k \cdot (f_D / 8) \cdot (\mathrm{Re} - 1000) \cdot \mathrm{Pr}}{D_i \cdot \left[1 + 12.7 \cdot (f_D / 8)^{1/2}\right] \cdot \left(\mathrm{Pr}^{2/3} - 1\right)} \cdot \left[1 - \left(\frac{D_i}{L}\right)^{2/3}\right]$$

$$= \frac{0.138 \cdot (0.0209/8) \cdot (50000 - 1000) \cdot 546.867}{0.0254 \cdot \left[1 + 12.7 \cdot (0.0209/8)^{1/2}\right] \cdot \left(546.867^{2/3} - 1\right)} \cdot \left[1 - \left(\frac{0.0254}{1}\right)^{2/3}\right]$$

$$= 3200. \frac{W}{m^2 \cdot K}$$

ans

Cadets may want to apply Equations 14-19 and 14-19a to all three cases for consistency. However, Pr<1 results in a negative heat transfer coefficient.

Part 2. Pressure Drops.

All of the fluids are flowing inside the 0.0254-m tube, so we are dealing with pressure drop inside the tube only (internal pressure drop). Pressure drop due to changes in elevation are assumed to be zero (assume the tube is horizontal). Pressure drop due to changes in velocity are also assumed to be zero (assume constant cross-sectional area of tube). External pressure increases or decreases do not need to be considered for internal pressure drop calculations. Therefore, the only pressure drop term that we need to be concerned with is frictional, and we may assume no expansions, contractions, or flow reversals.

From Lesson 7, we know that the frictional pressure drop on the tube side of a heat exchanger may be expressed as (Eq. 14-23, page 664):

$$\Delta p_i = \frac{2\,\beta_i\,f_i\,G_i^2\,L\,n_p}{\rho_i\,D_i\,\phi_i}$$

where

$$\beta_i = 1 + \frac{F_e + F_c + F_r}{2 \cdot f_i \cdot G_i^2 \cdot L / \left(\rho_i^2 \cdot D_i \cdot \phi_i\right)} = 1 \quad \left(F_e = F_c = F_r = 0\right)$$

$$f_i = 0.046\,\mathrm{Re}^{-0.2} = 0.046\left(50000\right)^{-0.2} = 0.00528 \quad \left(\mathrm{Re} \ge 2100\right)$$

$$G_i = \text{mass velocity} = \frac{\mu \cdot \mathrm{Re}}{D_i} = \frac{\mu \cdot 50000}{0.0254} = 1.97 \times 10^6 \mu$$

$$L = \text{length} = 1\ \text{m}\left(\text{by assumption}\right)$$

$$n_p = \text{number of tube passes} = 1$$

$$\rho_i = \text{density, given above, in kg/m}^3$$

$$\mu_i = \text{viscosity, given above, in Pa}\cdot\text{s}$$

$$D_i = 0.0254\ \text{m (given)}$$

$$\phi_i = 1.02\cdot\left(\mu_i / \mu_w\right)^{0.14} \approx 1.02$$

Putting everything together, we have:

$$\Delta p_i = \frac{2\,\beta_i\ f_i\ G_i^2\ L\ n_p}{\rho_i\ D_i\ \phi_i} = \frac{2\cdot 1\cdot 0.00528\cdot\left(1.97\times 10^6\mu\right)^2\cdot 1\cdot 1}{\rho_i\cdot 0.0254\cdot 1.02} = 1.58\times 10^{12}\,\frac{\mu^2}{\rho}$$

For air, water, and oil, we have, respectively:

$$\Delta p_{i,air} = 1.58\times 10^{12}\,\frac{\left(2\times 10^{-5}\right)^2}{.955} = \underline{\underline{662\,\tfrac{N}{m^2}}}$$

ans

$$\Delta p_{i,water} = 1.58\times 10^{12}\,\frac{\left(3.72\times 10^{-4}\right)^2}{973} = \underline{\underline{225\,\tfrac{N}{m^2}}}$$

ans

$$\Delta p_{i,oil} = 1.58\times 10^{12}\,\frac{\left(3.56\times 10^{-2}\right)^2}{854} = \underline{\underline{2,345,000\,\tfrac{N}{m^2}}}$$

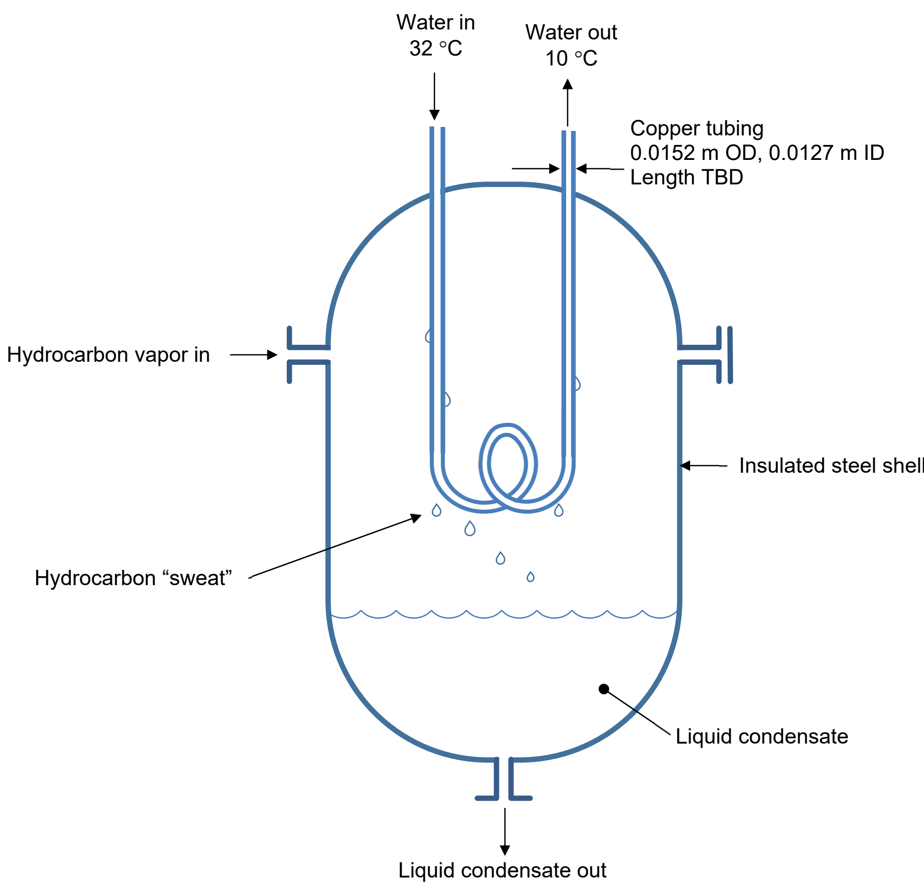
ans

Problem 14-9

A heat exchanger is to be constructed by forming copper tubing into a coil and placing the coil inside an insulated steel shell. In this exchanger, water will flow inside the tubing, and a hydrocarbon vapor will be condensing on the outside surface of the tubing at a rate of 0.126 kg/s. The tubing has an inside diameter of 0.0127 m and an outside diameter of 0.0152 m. The inlet temperature is 10 °C and the exit temperature is 32 °C. The heat of condensation of the hydrocarbon at the condensation temperature of 88 °C is 335 kJ/kg. The heat transfer coefficient for the condensing vapor is 1420 $W/m^2 \cdot K$. Heat losses from the shell may be neglected. What length of copper tubing will be required to accomplish the desired heat transfer?

Solution

It is best practice to make a sketch of the process that includes the relevant details. A sketch is shown below.



To compute the tube length, we need to know the heat transfer area A from $Q=U \cdot A \cdot \Delta T_{LM}$. U is calculated from the wall thickness, the thermal conductivity of the wall, and the h values for the hydrocarbon and water. The h value for the hydrocarbon is given, and the h value for water can be calculated once we know the Reynolds and Prandtl numbers. To get the Reynolds number, we need the velocity, density, and viscosity of the water, as well as the tube inside diameter (given). The velocity is calculated from the mass flow rate of the water, which in turn is calculated from Q. Finally, Q can be calculated from the rate of condensation and the heat of condensation of the hydrocarbon, both given in the problem, as shown below.

$$Q = \Delta\hat{H}_{vap}\dot{m}_{hydrocarbon} = 335\frac{kJ}{kg}\cdot 0.126\frac{kg}{s} = 42.21\frac{kJ}{s}$$

In this solution, assume the properties of water can be taken at the average temperature of (32+10)/2 = 21 °C or 294 K.

$$\dot{m}_{water} = \frac{Q}{\hat{C}_p\Delta T} = \frac{42.21\frac{kJ}{s}}{4.1816\frac{kJ}{kg\cdot K}\cdot(32-10)K} = 0.4\underline{5}88\frac{kg}{s}$$

$$\text{mass velocity} = G = \frac{0.4588\frac{kg}{s}}{\frac{\pi}{4}\cdot 0.0127^2\ m^2} = 3622\frac{kg}{m^2\cdot s}$$

Specific Heat:
The specific heat of water is found in Table D-4 on page 954. To get 4.1816 kJ/kg·K, interpolate to 294K using data at 290 and 295 K.

If you know the mass flow rate and pipe size, DG/μ is more convenient for calculating Re than Dvρ/μ since we do not need to look up density or calculate velocity.

$$Re = \frac{DG}{\mu} = \frac{.0127m\cdot 3622\frac{kg}{m^2\cdot s}}{0.0009832\frac{kg}{m\cdot s}} = 46,785$$

Viscosity:
The viscosity of water is found in Table D-4 on p. 954. To get 0.9832 cP, interpolate to 294K using data at 290 and 295 K.

$$Pr = \frac{\hat{C}_p\cdot\mu}{k} = \frac{4181.6\frac{J}{kg\cdot K}\cdot 0.0009832\frac{kg}{m\cdot s}}{0.6044\frac{W}{m\cdot K}} = 6.80$$

Thermal Conductivity:
The thermal conductivity of water is found in Table D-4 on p. 954. To get 0.6044 W/m·K, interpolate to 294 K using data at 290 and 295 K.

$$h_{water} = 0.023\cdot\left(\frac{k}{D}\right)\cdot Re^{0.8}\cdot Pr^{1/3}\cdot\left(\frac{\mu}{\mu_w}\right)^{0.14} = 0.023\cdot\left(\frac{0.6044}{0.0127}\right)\cdot 46785^{0.8}\cdot 6.80^{1/3}\cdot 1 = 11,294\frac{W}{m^2K}$$

The overall heat transfer coefficient is calculated from Equation 14-4a, assuming the fowling factors are zero and cancelling π and L from the area terms:

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i x_w}{k_w A_{m,w}} + \frac{A_i}{h_o A_o} = \frac{1}{11294} + \frac{0.0127\cdot(0.0152-0.0127)/2}{\frac{0.0152-0.0127}{Log[0.0152/0.0127]}\cdot 378.5} + \frac{0.0127}{0.0152\cdot 1420} = 0.000680$$

$$U_i = \frac{1}{0.000680} = 1471\frac{W}{m^2K}$$

Now determine ΔT_{LM}:

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} = \frac{(88-10)-(88-32)}{\ln\left[(88-10)/(88-32)\right]} = 66.39\text{K}$$

Now determine area:

$$A = \frac{Q}{U\Delta T_{LM}} = \frac{42.21\frac{\text{kJ}}{\text{s}}\cdot\frac{1000\text{J}}{\text{kJ}}}{1471\frac{\text{W}}{\text{m}^2\cdot\text{K}}\cdot 66.4\text{K}} = 0.432\text{m}^2$$

Since area for heat transfer = $\pi D_i L$,

$$L = \frac{\text{area}}{\pi D_i} = \frac{0.432\text{m}^2}{\pi\cdot 0.0127\text{m}} = 10.83\text{m} \approx \underline{\underline{10.8\text{m}}}$$ ans

For increasing speed in an exam situation, consider using properties of water at 295K *without* interpolating. This gives 10.86m as the final answer. That is,

$$\hat{C}_p \approx 4.18\,\text{kJ}/(\text{kg}\cdot\text{K})$$
$$\mu = 0.959\,\text{cP}$$
$$\rho = 997\,\text{kg}/\text{m}^3$$
$$k_{water} = 0.606\,\text{W}/(\text{m}\cdot\text{K})$$
$$k_{copper} = 388\,\text{W}/(\text{m}\cdot\text{K}) \text{ at } 100\,^\circ\text{C}$$

NOTE: Changing to steel tubing with k=46.6 W/m·K gives a length of 11.2 m.