Problem Set 8 - Solutions

Problem 5.2

A Carnot engine receives 250 kJ s-1 of heat from a heat source reservoir at 525 °C and rejects heat to a heat-sink reservoir at 50 °C. What are the power developed and the heat ejected?

Solution

Use equation 5.7 to compute efficiency, and then use 5.6 to get the work, based on the heat received.

In[ ]:= $\eta = 1 - \dfrac{50 + 273.15}{525 + 273.15}$

Out[ ]=
0.5951262

In[ ]:= $QH = 250 * \dfrac{kJ}{sec}$;(*heat in is positive*)

$W = -\eta * QH$(*work out is negative*)

Out[ ]=
$-\dfrac{148.7816 \ kJ}{sec}$

The power developed is -148.782 kJ/sec = -148.782 kW. //ANS

To compute the heat ejected, use eq 2.3 (first law) with ΔU=0, that is QC+QH+W=0:

In[ ]:= $QC = -QH - W$

Out[ ]=
$-\dfrac{101.2184 \ kJ}{sec}$

In[ ]:= **Solve[QC + 250 + (-148.782) == 0, QC]**

Out[ ]=
{{QC → -101.218}}

Heat rejected is -101.218 kJ/sec = -101.218 kW. //ANS

Problem 5.6

Which is the more effective way to increase the thermal efficiency of a Carnot engine: to increase T_H with T_C constant, or to decrease T_C with T_H constant? For a real engine, which would be the more practical way?

Solution

In[ ]:= **(∗Extra typsetting function used to add parentheses∗)**
parens[e_] := DisplayForm[RowBox[{"(", MakeBoxes[e], ")"}]]
(∗This is not part of the solution∗)

Use derivatives to answer this question:

Out[ ]//TraditionalForm=

$$\eta = 1 - \frac{T_C}{T_H}$$

Out[ ]//TraditionalForm=

$$\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H} = -\frac{1}{T_H}$$

Out[ ]//TraditionalForm=

$$\left(\frac{\partial \eta}{\partial T_H}\right)_{T_C} = -T_C\left(-\frac{1}{T_H^2}\right) = \frac{1}{T_H}\frac{T_C}{T_H}$$

Taking a look at the derivative of η with respect to T_C, we note that T_H is always positive. Therefore, the derivative with respect to T_C is always negative, meaning that as T_C increases, η decreases. Or, as T_C decreases, η increases. The problem statement specifically asks about decreasing T_C, which increases η.

Now look at the other derivative. Since T_H and T_C are always positive, the derivative of η with respect to T_H is always positive, meaning that as T_H increases, η increases. The problem statement asks about increasing T_H, which increases η.

Now compare the "size" of the derivatives to see which one is larger and therefore causes a larger change. Use absolute values to compare the sizes of the derivatives. By premise, the cold reservoir temperature is less than the warm reservoir temperature, or $T_C < T_H$. Since $T_C < T_H$,

Out[ ]//TraditionalForm=

$$\frac{1}{T_H}\frac{T_C}{T_H} < \left|-\frac{1}{T_H}\right|$$

Therefore,

Out[ ]//TraditionalForm=

$$\left|\left(\frac{\partial \eta}{\partial T_H}\right)_{T_C}\right| < \left|\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H}\right|$$

Therefore, since the derivative with respect to T_C is larger, decreasing T_C with T_H held constant is more

effective at increasing thermal efficiency η than increasing T_H with T_C held constant. This is a general solution and applies at all $T_C > 0$ and $T_H > 0$ as long as $T_H > T_C$.

In a real engine, it is more practical to increase T_H with T_C held constant, because T_C is normally the ambient temperature, which is difficult to change. T_H can be changed by redesigning the engine to use a different fuel, or changing the amount of heat removed from the engine.

Problem 5.8

With respect to 1 kg of liquid water:

(a) Initially at 0 °C, it is heated to 100 °C by contact with a heat reservoir at 100 °C. What is the entropy change of the water? What is the entropy change of the heat reservoir? What is ΔS_{total}?

(b) Initially at 0 °C, it is first heated to 50 °C by contact with a heat reservoir at 50 °C, and then heated to 100 °C by contact with a heat reservoir at 100 °C. What is ΔS_{total}?

(c) Explain how the water might be heated from 0 to 100 °C so that $\Delta S_{total} = 0$.

Solution to Part (a)

In[ ]:= **m = 1; (*kg*)**
Cp = 4.184; (*kJ/kg*degC*)
T2 = 100 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)
ΔT = T2 – T1; (*degC*)
Qhot = m * Cp * ΔT; (*kJ*)
$$\Delta Shot = \frac{-Qhot}{T2} \;(*kJ/K*)$$
$$\Delta Swater = m * \int_{T1}^{T2} \frac{Cp}{T} \; dT \;(*kJ/K*)$$
ΔStotal = ΔShot + ΔSwater (*kJ/K*)

Out[ ]=
    −1.121265

Out[ ]=
    1.305238

Out[ ]=
    0.1839732

The ΔS of the hot reservoir is -1.12123 kJ/K. //ANS
The ΔS of the cold reservoir is 1.30524 kJ/K. //ANS
The total entropy change is is 0.18397 kJ/K. //ANS

Solution to Part (b)

```
In[•]:= m = 1; (*kg*)
       Cp = 4.184; (*kJ/kg*degC*)
       T3 = 100 + 273.15; (*K*)
       T2 = 50 + 273.15; (*K*)
       T1 = 0 + 273.15; (*K*)

       (*First Step*)
       ΔT12 = T2 - T1; (*degC*)
       Qhot12 = m * Cp * ΔT12; (*kJ*)
       ΔShot12 = -Qhot12
                 ─────────
                    T2
                      ⌠T2 Cp
       ΔSwater12 = m  ⎮   ── dT
                      ⌡T1  T
       (*Second Step*)
       ΔT23 = T3 - T2; (*degC*)
       Qhot23 = m * Cp * ΔT23; (*kJ*)
       ΔShot23 = -Qhot23
                 ───────── (*kJ/K*)
                    T3
                     ⌠T3 Cp
       ΔSwater23 = m ⎮   ── dT (*kJ/K*)
                     ⌡T2  T
       (*Total, kJ/K //ANS*)
       ΔStotal = ΔShot12 + ΔSwater12 + ΔShot23 + ΔSwater23
```

Out[•]=
    −0.6473774

Out[•]=
    0.7033116

Out[•]=
    −0.5606325

Out[•]=
    0.6019264

Out[•]=
    0.09722823

ΔS of the hot reservoir in step 1 is -0.64738 kJ/K and in step 2 is -0.56063 kJ/K. //ANS
ΔS of the cold reservoir in step 1 is 0.70331 kJ/K and in step 2 is 0.60192 kJ/K. //ANS
The total entropy change is is 0.097228 kJ/K. //ANS

Solution to Part (c)

The reversible heating of the water requires an infinite number of heat reservoirs covering the range of temperatures from 273.15 to 373.15 K, each one exchanging an infinitesimal quantity of heat with the water and raising its temperature by a differential increment.

Problem 5.10

An ideal gas, $C_P = (7/2)R$, is heated in a steady-flow heat exchanger from 70 to 190 °C by another stream of the same gas, which enters at 320 °C. The flow rates of the two streams are the same, and heat losses from the exchanger are negligible.

(a) Calculate the molar entropy changes of the two gas streams for both parallel and countercurrent flow in the exchanger.

(b) What is ΔS_{total} in each case?

(c) Repeat (a) and (b) for countercurrent flow if the heating stream enters at 200 °C.

Solution Notes

Use the integrated form of Equation 5.10, with constant C_P and assuming constant pressure. The outlet temperature of the hot gas in both co-current and counter-current flow is 200 °C.

```
(*Level 1 enthalpy balance gives mCpΔTc=-mCpΔTh *)
(*Both m and Cp cancel, so ΔTc=-ΔTh  *)
(*190-70==-(Thout-320) *)
```

In[•]:= `Solve[(190 - 70) == -(Thout - 320)]`

Out[•]=

$\{\{Thout \to 200\}\}$

In[•]:= `(*Gas constant and heat capacity*)`
`R = 8.314; (*J/mol*K*)`
`Cp = (7 / 2) R;`

Solution to Part (a)

`(*Cold-side inlet and outlet temperatures in K: *)`
`70 + 273.15`
`190 + 273.15`

Out[•]=

343.15

Out[•]=

463.15

In[•]:= $\Delta Scold = Cp * Log\left[\dfrac{(463.15)}{(343.15)}\right] - Log\left[\dfrac{P}{P}\right]$

Out[•]=

8.726304

$(*\Delta Scold = 8.726 \ \frac{J}{mol*K} \ //ANS*)$

In[ ]:= **(*Hot-side inlet and outlet temperatures, K*)**
320 + 273.15
200 + 273.15

Out[ ]=
593.15

Out[ ]=
473.15

In[ ]:= Δ**Shot = Cp * Log$\left[\dfrac{(473.15)}{(593.15)}\right]$ - Log$\left[\dfrac{P}{P}\right]$**

Out[ ]=
-6.577388

(*ΔShot = -6.577 $\dfrac{J}{mol*K}$ //ANS*)
(*Co-current and counter-current have the same answers*)

Solution to Part (b)

In[ ]:= Δ**Scold + ΔShot**

Out[ ]=
2.148916

(*ΔStotal = 2.149 $\dfrac{J}{mol*K}$ //ANS*)

Solution to Part (c)

In[ ]:= **(*190-70==-(Thout-200) *)**
Solve[(190 - 70) == - (Thout - 200)]

Out[ ]=
{{Thout → 80}}

(*Cold-side temperatures, K*)
(*Cold-side temperatures do not change*)
(*ΔScold = 8.726 $\dfrac{J}{mol*K}$ //ANS*)

In[ ]:= **(*Hot-side temperatures*)**
(80 + 273.15)
(200 + 273.15)

Out[ ]=
353.15

Out[ ]=
473.15

In[ ]:= Δ**Shot = Cp * Log$\left[\dfrac{(353.15)}{(473.15)}\right]$ - Log$\left[\dfrac{P}{P}\right]$**

Out[ ]=
-8.512027

$(*\Delta Shot = -8.512 \; \frac{J}{mol*K} \;\; //ANS*)$

ΔScold + ΔShot

Out[◦]=

0.2142774

$(*\Delta Stotal = 0.214 \; \frac{J}{mol*K} \;\; //ANS*)$