
Problem Set 4 - Solutions

Problem 3.2

A renowned laboratory reports quadruple-point coordinates of 10.2 Mbar and 24.1 deg C for the four-phase equilibrium of allotropic solid forms of the exotic chemical β -maiasmone. Examine the claim using the Gibbs phase rule and provide a plausible explanation for your results.

Solution

The report is incorrect because it violates the phase rule. The phase rule states that $F = 2 - \pi + N$. Since the number of phases is 4 and the number of chemically distinct species is 1, then $F = 2 - 4 + 1 = -1$. So either the number of chemically distinct species is greater than 1 or the number of phases is less than 4.

Problem 3.4

A system of propane and n-butane exists in two-phase vapor/liquid equilibrium at 10 bar and 323 K. The mole fraction of propane is about 0.67 in the vapor phase and about 0.40 in the liquid phase. Additional pure propane is added to the system, which is brought again to equilibrium at the same T and P, with both liquid and vapor phases still present. What is the effect of the addition of propane on the mole fractions of propane in the vapor and liquid phases?

Solution

The phase rule states that $F = 2 - \pi + N = 2 - 2 + 2 = 2$, which means that the system has two degrees of freedom. Specifying T and P satisfies the degrees of freedom and fixes the *intensive* state of the system. As long as the system is returned to the same T and P, all intensive variables must also be the same. Since mole fractions are intensive properties, the phase rule predicts that the mole fractions must be unchanged. //ANS

Problem 3.6

Express the volume expansivity (β) and isothermal compressibility (κ) as functions of density and its partial derivatives. For water at 50 degC and 1 bar, $\kappa = 44.18 \times 10^{-6} \text{ bar}^{-1}$. To what pressure must water be compressed at 50 degC to change its density by 1%? Assume κ is independent of P.

Solution - Part 1 - Express β and κ as functions of ρ and its partial derivatives

Express β and κ as functions of density and its partial derivatives. The key to this part of the problem is to use equations 3.2-3.5 as a guide. In those equations, $V=V(T,P)$. Just repeat the four equations using $\rho=\rho(T,P)$:

Out[*]//TraditionalForm=

$$d\rho = \left(\frac{\partial \rho}{\partial P}\right)_T dP + \left(\frac{\partial \rho}{\partial T}\right)_P dT$$

Out[*]//TraditionalForm=

$$\beta \equiv -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T}\right)_P$$

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$$\kappa \equiv +\frac{1}{\rho} \left(\frac{\partial \rho}{\partial P}\right)_T$$

Out[*]//TraditionalForm=

$$\frac{d\rho}{\rho} = -\beta dT + \kappa dP$$

Comments:

β is positive because in the original derivation, $\beta \equiv (\partial V / \partial T)_P / V$. Since molar volume is always positive and molar volume increases as temperature increases, the derivative is also always positive.

To be consistent, we must use a negative sign in front of $\partial \rho / \partial T$, because as temperature goes up, density goes down, making the derivative $\partial \rho / \partial T$ negative.

It is desirable to keep β the same since the property itself does not change. The negative sign compensates for the negative derivative.

Likewise, when derived from density, κ uses a positive sign in front of the derivative because density increases as pressure increases, so $(\partial \rho / \partial P)_T$ is positive.

In the original derivation from molar volume V , we used a negative sign for κ because $(\partial V / \partial P)_T$ is negative. That is, molar volume decreases as pressure increases.

Cadets tend to put the wrong signs on β and κ .

Solution - Part 2 - To what pressure must water be compressed?

We are asked to calculate the pressure required to change the density by 1% at 50 °C. That is, we are changing the pressure at constant T and which to calculate the density response. This relationship is described by κ , so rearrange and integrate as shown:

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$$\kappa \equiv + \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P} \right)_T$$

Out[*]//TraditionalForm=

$$\kappa \rho = \left(\frac{\partial \rho}{\partial P} \right)_T$$

Out[*]//TraditionalForm=

$$\kappa \rho dP = d\rho$$

Out[*]//TraditionalForm=

$$\kappa dP = \frac{d\rho}{\rho}$$

Out[*]//TraditionalForm=

$$\int_{P_1}^{P_2} \kappa dP = \int_{\rho_1}^{\rho_2} \frac{1}{\rho} d\rho$$

$\rho_1 = 0.9881; (*0.9881 \text{ g/cm}^3 \text{ from engineering toolbox}*)$

$\rho_2 = \rho_1 * 1.01;$

$\kappa = 44.18 * 10^{-6};$

$P_1 = 1;$

In[*]:= $\text{eq1} = \kappa * (P_2 - P_1) == \text{Log}\left[\frac{\rho_2}{\rho_1}\right];$

In[*]:= $\text{Solve}[\text{eq1}, P_2]$

Out[*]=

$\{ \{ P_2 \rightarrow 226.2225 \} \}$

The water must be compressed to 226.223 bar. //ANS

Problem 3.9

For liquid water the isothermal compressibility is given by

$$\kappa = \frac{c}{V(P+b)},$$

where c and b are functions of temperature only. If 1 kg of water is compressed isothermally and reversibly from 1 to 500 bar at 60 °C, how much work is required? At 60 °C, $b=2,700$ bar and $c=0.125 \text{ cm}^3 \text{ g}^{-1}$.

Solution

Start with the two definitions of κ :

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$$\kappa \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

Out[*]//TraditionalForm=

$$\kappa = \frac{c}{V(P+b)}$$

Equate the two definitions:

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$$\frac{c}{V(P+b)} = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

Multiply both sides by V :

Out[*]//TraditionalForm=

$$\frac{c}{P+b} = -\left(\frac{\partial V}{\partial P} \right)_T$$

We are at constant T , so we can multiply both sides by dP :

Out[*]//TraditionalForm=

$$-\frac{c \, dP}{P+b} = dV$$

Write equation 1.3, substitute the new equation for dV , and integrate:

Out[*]//TraditionalForm=

$$W = -\int_{V_1}^{V_2} P \, dV = -\int_{P_1}^{P_2} P \left(-\frac{c}{P+b} \right) dP = \int_{P_1}^{P_2} P \left(\frac{c}{P+b} \right) dP$$

In[*]:= **b = 2700; (*bar*)**

c = 0.125; (* $\frac{\text{cm}^3}{\text{g}}$ *)

(*Solve for absolute work*)

$$\text{In[*]:= } W = 1000 * \int_1^{500} \frac{c * P}{P + b} dP$$

Out[*]=

5159.052

(*Units are bar*cm³ since $g * \frac{\frac{\text{cm}^3}{g} * \text{bar}}{\text{bar}} * \text{bar} = \text{bar} * \text{cm}^3$ *)

Convert units to kJ:

$$W * \text{bar} * \text{cm}^3 * \frac{1 \text{ J}}{10 \text{ bar} * \text{cm}^3} * \frac{1 \text{ kJ}}{1000 \text{ J}} \quad (* // \text{ANS} *)$$

Out[*]=

0.5159052 kJ

The required work is 0.515905 kJ. //ANS

Problem 3.17

One mole of an ideal gas with $C_P = (7/2)R$ and $C_V = (5/2)R$ expands from $P_1 = 8 \text{ bar}$ and $T_1 = 600 \text{ K}$ to $P_2 = 1 \text{ bar}$ by each of the following paths:

- (a) Constant volume,
- (b) Constant temperature,
- (c) Adiabatically

Assuming mechanical reversibility, calculate W , Q , ΔU , and ΔH for each process. Sketch each path on a single PV diagram.

Solution - Common Information

```
R = 8.314; (*J/mol*K*)
CV = 20.785; (*J/mol*K*)
CP = 29.099; (*J/mol*K*)
T1 = 600; (*K*)
P1 = 8.; (*bar*)
P2 = 1.; (*bar*)
```

Solution - Part (a) - Constant Volume

$$T_2 = T_1 * \frac{P_2}{P_1} \quad (*K*)$$

75.

$$\Delta U = C_V * (T_2 - T_1) \quad (*J/mol*)$$

$$\Delta H = C_P * (T_2 - T_1) \quad (*J/mol*)$$

-10912.13

-15276.98

$$Q = \Delta U \quad (*\Delta U = Q + W = Q + 0*)$$

-10912.13

$$W = 0;$$

$$(* W = 0 \text{ J/mol} *)$$

$$(* Q = -10912.1 \text{ J/mol} *)$$

$$(* \Delta U = -10912.1 \text{ J/mol} *)$$

$$(* \Delta H = -15277.0 \text{ J/mol} *)$$

$$(* //ANS part (a) *)$$

Solution - Part (b) - Constant Temperature

$$DH = 0;$$

$$DU = 0;$$

$$Q = -R * T1 * \text{Log}\left[\frac{P2}{P1}\right]$$

$$10373.09$$

$$W = -Q$$

$$-10373.09$$

$$(* W = -10373. \text{ J/mol} *)$$

$$(* Q = 10373. \text{ J/mol} *)$$

$$(* \Delta U = 0 \text{ J/mol} *)$$

$$(* \Delta H = 0 \text{ J/mol} *)$$

$$(* //ANS part (b) *)$$

Solution - Part (c) - Adiabatic

$$Q = 0;$$

$$\gamma = CP / CV;$$

$$W = \frac{R * T1}{\gamma - 1} \left(\left(\frac{P2}{P1} \right)^{(\gamma-1)/\gamma} - 1 \right)$$

$$-5586.45$$

$$DU = W$$

$$-5586.45$$

$$T2 = T1 * \left(\frac{P2}{P1} \right)^{R/CP}$$

$$331.2269$$

$$DH = CP * (T2 - T1)$$

$$-7821.03$$

$$(* W = -5586. \text{ J/mol} *)$$

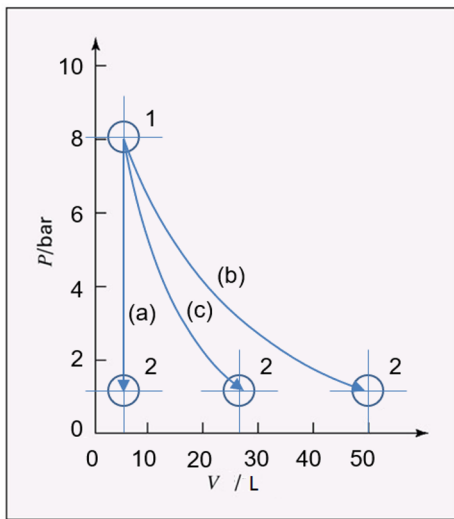
$$(* Q = 0. \text{ J/mol} *)$$

$$(* \Delta U = -5586. \text{ J/mol} *)$$

$$(* \Delta H = -7821. \text{ J/mol} *)$$

$$(* //ANS part (c) *)$$

Solution - Sketches for parts (a) to (c)



Problem 3.21

An ideal gas, $C_p = (5/2)R$ and $C_v = (3/2)R$, is changed from $P_1 = 1 \text{ bar}$ and $V_1^t = 12 \text{ m}^3$ to $P_2 = 12 \text{ bar}$ and $V_2^t = 1 \text{ m}^3$ by the following mechanically reversible processes:

- (a) Isothermal compression.
- (b) Adiabatic compression followed by cooling at constant pressure.
- (c) Adiabatic compression followed by cooling at constant volume.
- (d) Heating at constant volume followed by cooling at constant pressure.
- (e) Cooling at constant pressure followed by heating at constant volume.

Calculate Q , W , ΔU^t , and ΔH^t for each of these processes, and sketch the paths of all processes on a single PV diagram.

Solution - Common Information for Parts (a) to (e)

Since points 1 and 2 are defined on an isotherm in part (a) then $\Delta H=0$ and $\Delta U=0$. These are the same initial and final points for all five parts of the problem. So $\Delta H=0$ and $\Delta U=0$ for parts (a) through (e). Also, carefully note that V , Q , ΔH , ΔU , and W are total (not molar).

$\Delta H = 0$; (*//ANS, parts (a)-(e)*)

$\Delta U = 0$; (*//ANS, parts (a)-(e)*)

$P_1 = 1.$; (*bar*)

$P_2 = 12.$; (*bar*)

$V_1 = 12.$; (*m³*)

$V_2 = 1.$; (*m³*)

$R = 8.314$; (*J/mol*K*)

$C_p = (5/2) * R$;

$C_v = (3/2) * R$;

$\gamma = C_p / C_v$;

Solution - Part (a) - Isothermal compression

(*Q=R*T*Log[$\frac{V_2}{V_1}$] equation 3.20 from L11 slide 6*)

(*PV_{tot}=nRT → RT= $\frac{PV_{tot}}{n}$ *)

(*nQ=Q_{tot}=P*V_{tot}*Log[$\frac{V_2}{V_1}$]*)

(*RT is constant so P₁V₁=P₂V₂*)

$Q = P_1 * V_1 * \text{Log}\left[\frac{V_2}{V_1}\right]$ (* bar*m³ *)

Out[]=

-29.81888

In[]:= $Q * \text{bar} * \text{m}^3 * \frac{1 \text{ J}}{10^{-5} \text{ bar} * \text{m}^3} * \frac{1 \text{ kJ}}{1000 \text{ J}}$ (*//ANS part (a)*)

Out[]=

-2981.888 kJ

(*ΔH=ΔU=0, and since ΔU=0, W=-Q=2981.89 kJ //ANS part (a)*)

Solution - Part (b) - Adiabatic compression followed by cooling at constant pressure

(*STEP 1 - adiabatic compression to 12 bar*)

(*Let V_{int} be the intermediate volume.*)

$\text{eqb} = \frac{P_2}{P_1} == \left(\frac{V_1}{V_{\text{int}}}\right)^{C_p/C_v}$; (*Un-numbered equation from L11 Slide 9*)

V_{int} = V_{int} /. Quiet[Solve[eqb, V_{int}]] [[1]]

Out[]=

2.70192

(*STEP 1 - adiabatic compression*)

$$W1 = \frac{P2 * Vint - P1 * V1}{\gamma - 1} \quad (*\text{bar} * \text{m}^3 *) \quad (*\text{Eq 3.26 L11 slide 10} *)$$

Out[]=

30.63456

(*STEP 2 - isobaric cooling at 12 bar*)

$$\text{In[]:= } W2 = -P2 * (V2 - Vint) \quad (*\text{bar} * \text{m}^3 *)$$

Out[]=

20.42304

$$\text{In[]:= } W = W1 + W2 \quad (*\text{bar} * \text{m}^3 *)$$

Out[]=

51.0576

$$\text{In[]:= } W * \text{bar} * \text{m}^3 * \frac{1 \text{ J}}{10^{-5} \text{ bar} * \text{m}^3} * \frac{1 \text{ kJ}}{1000 \text{ J}} \quad (//\text{ANS part (b)})$$

Out[]=

5105.76 kJ

(*ΔH=ΔU=0, and since ΔU=0, Q=-W=-5105.76 kJ //ANS (part(b)*)

Solution - Part (c) - Adiabatic compression followed by cooling at constant volume

(*STEP 1 - adiabatic compression to 1 m³*)

(*Pint is the intermediate pressure*)

$$\text{In[]:= } Pint = P1 * \left(\frac{V1}{V2} \right)^{Cp/Cv} \quad (*\text{un-numbered equation L11 slide} *)$$

Out[]=

62.89779

(*from slide 10 in lesson 12 eq 3.26 for work*)

$$W1 = \frac{Pint * V2 - P1 * V1}{\gamma - 1} \quad (*\text{bar} * \text{m}^3 *)$$

Out[]=

76.34669

(*STEP 2 - work is zero at const V*)

$$\text{In[]:= } W2 = 0;$$

$$W = W1 + W2 \quad (*\text{bar} * \text{m}^3 *)$$

Out[]=

76.34669

$$\text{In[*]:= } W \cdot \text{bar} \cdot \text{m}^3 * \frac{1 \text{ J}}{10^{-5} \text{ bar} \cdot \text{m}^3} * \frac{1 \text{ kJ}}{1000 \text{ J}} \quad (//\text{ANS part (c)})$$

Out[*]=

7634.669 kJ

(*ΔH=ΔU=0, and since ΔU=0, Q=-W=-7634.67 kJ //ANS part (c)*)

Solution - Part (d) - Heating at constant volume followed by cooling at constant pressure

(*STEP 1 - isochoric step - work is zero at const V*)

$$\text{In[*]:= } W1 = 0;$$

(*STEP 2 - isobaric work at 1 bar is -PΔV*)

$$\text{In[*]:= } W2 = -P2 * (V2 - V1);$$

$$\text{In[*]:= } W = W1 + W2 \quad (*\text{bar} \cdot \text{m}^3*)$$

Out[*]=

132.

$$\text{In[*]:= } W \cdot \text{bar} \cdot \text{m}^3 * \frac{1 \text{ J}}{10^{-5} \text{ bar} \cdot \text{m}^3} * \frac{1 \text{ kJ}}{1000 \text{ J}} \quad (//\text{ANS part (d)})$$

Out[*]=

13200. kJ

(*ΔH=ΔU=0, and since ΔU=0, Q=-W=-13200 kJ //ANS part (d)*)

Solution - Part (e) - Cooling at Constant Pressure followed by heating at constant volume

(*STEP 1 - isobaric work at 1 bar is -PΔV*)

$$\text{In[*]:= } W1 = -P1 * (V2 - V1); \quad (*\text{bar} \cdot \text{m}^3*)$$

(*STEP 2 - isochoric step - work is zero at const V*)

$$\text{In[*]:= } W2 = 0;$$

$$W = W1 + W2; \quad (*\text{bar} \cdot \text{m}^3*)$$

$$\text{In[*]:= } W \cdot \text{bar} \cdot \text{m}^3 * \frac{1 \text{ J}}{10^{-5} \text{ bar} \cdot \text{m}^3} * \frac{1 \text{ kJ}}{1000 \text{ J}} \quad (//\text{ANS})$$

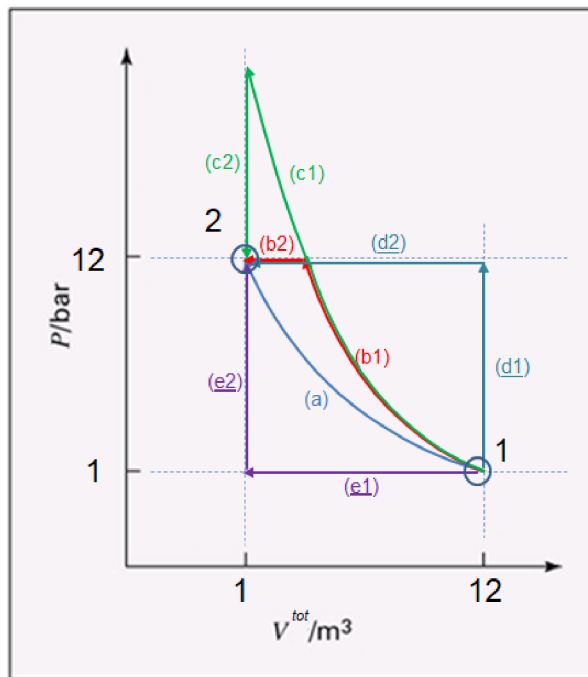
Out[*]=

1100. kJ

(*ΔH=ΔU=0, and since ΔU=0, Q=-W=-1100 kJ //ANS*)

Solution - Sketches for parts (a) through (e)

- (a) isothermal compression
- (b1) adiabatic compression
- (b2) cooling at constant pressure
- (c1) adiabatic compression
- (c2) cooling at constant volume
- (d1) heating at constant volume
- (d2) cooling at constant pressure
- (e1) cooling at constant pressure
- (e2) heating at constant volume



Problem 3.42

For methyl chloride at 100 °C, the second and third virial coefficients are:

$$B = -242.5 \text{ cm}^3 \text{ mol}^{-1} \text{ and } C = 25,200 \text{ cm}^6 \text{ mol}^{-2}$$

Calculate the work of mechanically reversible, isothermal compression of 1 mole of methyl chloride from 1 bar to 55 bar at 100 °C.

Base your calculations on the following forms of the virial equation:

- (a) $Z = 1 + \frac{B}{V} + \frac{C}{V^2}$
- (b) $Z = 1 + B'P + C'P^2$, where $B' = \frac{B}{RT}$ and $C' = \frac{C-B^2}{(RT)^2}$
- (c) $Z = 1$
- (d) $Z = 1 + \frac{B}{V}$

Solution

The key to this problem is understanding the reversible work equation (Equation 1.3) and then understanding how to integrate it. That is,

$$dW = -PdV$$

It is very important to realize that $P=P(V)$, and that whether we are using the ideal gas law or the virial

equation, we must substitute the function $P(V)$ into equation 1.3 and integrate from V_1 to V_2 . This means we must also determine consistent values V_1 and V_2 .

What follows below for the solution to Part (a) is (1) a determination of V_1 and V_2 , (2) determination of the function for $P(V)$, then (3) integrating Equation 1.3 from V_1 to V_2 .

Part (a)

```
In[*]:= Quit[];
```

```
In[*]:= R = 83.14 (*  $\frac{\text{cm}^3 \cdot \text{bar}}{\text{mol} \cdot \text{K}}$  *);
P1 = 1. (*bar*);
P2 = 55. (*bar*);
T = 373.15 (*K*);
B = -242.5 (*  $\frac{\text{cm}^3}{\text{mol}}$  *);
c = 25200 (*  $\frac{\text{cm}^6}{\text{mol}^2}$  *);
```

(1) Determination V_1 and V_2 :

```
In[*]:= eq1 =  $\frac{P1 * V1}{R * T} == 1 + \frac{B}{V1} + \frac{c}{V1^2}$ ;
```

```
In[*]:= Quiet[sol1 = Solve[eq1, V1, Reals]]
```

```
Out[*]:= { {V1 -> 30780.1} }
```

```
In[*]:= V1 = V1 /. sol1[[1]] (*units are  $\frac{\text{cm}^3}{\text{mol}}$  *)
```

```
Out[*]:= 30780.1
```

```
In[*]:= eq2 =  $\frac{P2 * V2}{R * T} == 1 + \frac{B}{V2} + \frac{c}{V2^2}$ ;
```

```
In[*]:= Quiet[sol2 = Solve[eq2, V2, Reals]]
```

```
Out[*]:= { {V2 -> 241.3331} }
```

```
In[*]:= V2 = V2 /. sol2[[1]] (*units are  $\frac{\text{cm}^3}{\text{mol}}$  *)
```

```
Out[*]:= 241.3331
```

(2) Determination of the function $P(V)$:

```
In[*]:= P[V_] =  $\left(1 + \frac{B}{V} + \frac{c}{V^2}\right) * \frac{R * T}{V}$ ; (*bar*)
```

(3) Integration of equation 1.3:

$$\text{In[*]:= } W = - \int_{V_1}^{V_2} (P[V]) \, dV \quad \left(\frac{\text{bar} \cdot \text{cm}^3}{\text{mol}} \right)$$

Out[*]=

126198.6

(*Put dimensions of work into kJ:*)

$$\text{In[*]:= } 126199. \cdot \frac{\text{bar} \cdot \text{cm}^3}{\text{mol}} \cdot \frac{10^5 \text{ Pa}}{\text{bar}} \cdot \frac{\frac{1 \text{ N}}{\text{m}^2}}{\text{Pa}} \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 \cdot \frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}} \cdot \frac{1 \text{ kJ}}{1000 \text{ J}} \cdot 1 \text{ mol}$$

Out[*]=

12.6199 kJ

The reversible work is 12.6199 kJ. //ANS

Part (b)

The solution for Part (b) is a little different. In this case, we are asked to use the virial equation in the form of a power series expansion in P. In this equation (given above), V is easy to isolate, so the strategy is to isolate it, then evaluate dV. This produces an expression in terms of dP. Substitute the new dV into equation 1.3, and integrate with respect to pressure (instead of volume). This means the limits of integration are P1 and P2, not V1 and V2 as in Part (a).

What follows below the solution to Part (b) is (1) a determination of V(P), then (2) determination work using Equation 1.3 after substituting in dV.

(1) Determine the function V(P):

$$\text{In[*]:= } B_{\text{prime}} = \frac{B}{R \cdot T}; \quad (*\text{Eq 3-35a slide 13}*)$$

$$C_{\text{prime}} = \frac{C - B^2}{(R \cdot T)^2}; \quad (*\text{Eq 3-35b}*)$$

(*Solve eq 3.34 for V in terms of P*)

$$\text{In[*]:= } V[P_] = \left(1 + B_{\text{prime}} \cdot P + C_{\text{prime}} \cdot P^2 \right) \cdot \frac{R \cdot T}{P};$$

(2) Integrate equation 1.3 (dW = -PdV) with respect to P after substituting dV = V' dP:

$$\text{In[*]:= } W = - \int_{P_1}^{P_2} P \cdot V'[P] \, dP$$

Out[*]=

125960.1

$$\text{In[*]:= } 125960. \cdot \frac{\text{bar} \cdot \text{cm}^3}{\text{mol}} \cdot \frac{10^5 \text{ Pa}}{\text{bar}} \cdot \frac{\frac{1 \text{ N}}{\text{m}^2}}{\text{Pa}} \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 \cdot \frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}} \cdot \frac{1 \text{ kJ}}{1000 \text{ J}} \cdot 1 \text{ mol}$$

Out[*]=

12.596 kJ

The reversible work is 12.596 kJ. //ANS

Part (c)

Proceed as in Part (a) - find V_1 and V_2 , write $P(V)$, substitute into equation 1.3, and integrate:

```
In[*]:= Clear[V1, V2];

In[*]:= eq1 =  $\frac{P1 * V1}{n * R * T} == 1;$ 

In[*]:= Quiet[sol1 = Solve[eq1, V1]]

Out[*]:=
{ {V1 -> 31 023.69 n} }
```

```
In[*]:= V1 = V1 /. sol1[[1]]

Out[*]:=
31 023.69 n
```

```
In[*]:= eq2 =  $\frac{P2 * V2}{n * R * T} == 1;$ 

In[*]:= Quiet[sol2 = Solve[eq2, V2]]

Out[*]:=
{ {V2 -> 564.0671 n} }
```

```
In[*]:= V2 = V2 /. sol2[[1]]

Out[*]:=
564.0671 n
```

```
In[*]:= Pideal[V_] =  $\frac{R * T}{V};$ 

In[*]:= W = -  $\int_{V1}^{V2} Pideal[V] dV$ 

Out[*]:=
124 322.3
```

```
In[*]:= 124 322. *  $\frac{\text{bar} * \text{cm}^3}{\text{mol}}$  *  $\frac{10^5 \text{ Pa}}{\text{bar}}$  *  $\frac{\frac{1 \text{ N}}{\text{m}^2}}{\text{Pa}}$  *  $\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$  *  $\frac{1 \text{ J}}{1 \text{ N} * \text{m}}$  *  $\frac{1 \text{ kJ}}{1000 \text{ J}}$  * 1 mol

Out[*]:=
12.4322 kJ
```

The reversible work is 12.4322 kJ. //ANS

Important Note: The work equation can also be integrated with respect to P as in part (b), resulting in exactly the same answer:

```
In[*]:= V[P_] =  $\frac{R * T}{P};$ 
```

$$\text{In}[*]:= W = - \int_{P_1}^{P_2} P * V'[P] dP$$

Out[*]=
124322.3

Part (d)

This problem is a little tricky because the equation is quadratic and there are no real roots for V_2 . Essentially, the problem cannot be solved as posed. We can come close, though, using a fix. To get a reasonable approximation, use the values of V_1 and V_2 determined in Part (a).

In[*]:= **V1 = 30780.1;**
V2 = 241.33;

$$\text{In}[*]:= P[V_] = \left(1 + \frac{B}{V}\right) * \frac{R * T}{V}; (*bar*)$$

$$\text{In}[*]:= W = - \int_{V_1}^{V_2} P[V] dV$$

Out[*]=
119487.4

$$\text{In}[*]:= 119487. * \frac{\text{bar} * \text{cm}^3}{\text{mol}} * \frac{10^5 \text{ Pa}}{\text{bar}} * \frac{\frac{1 \text{ N}}{\text{m}^2}}{\text{Pa}} * \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 * \frac{1 \text{ J}}{1 \text{ N} * \text{m}} * \frac{1 \text{ kJ}}{1000 \text{ J}} * 1 \text{ mol}$$

Out[*]=
11.9487 kJ

The reversible work is 11.9487 kJ. //ANS