Problem Set 1 - Solutions

Problem 1.4

At what absolute temperature do the Celsius and Fahrenheit temperature scales give the same numerical value? What is the value?

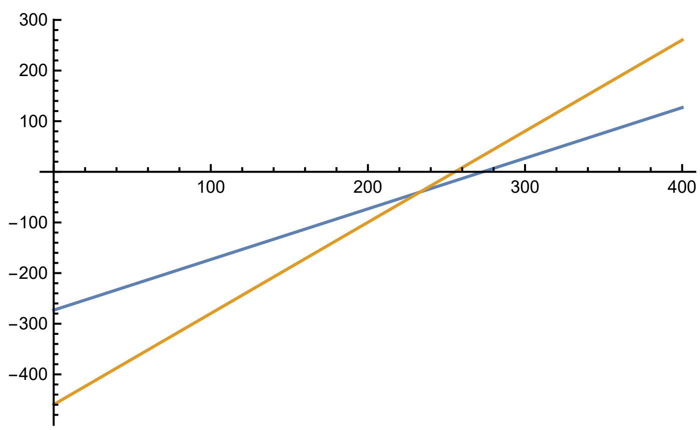
SOLUTION

Write a function for converting Kelvins into deg C. Then write a second function converting Kelvins into deg F. Plot the two functions with Kelvins as the independent variable. The plot shows the intersection. Setting the two functions equal to each other and solving for the intersection gives -40 deg C or deg F at 233.15 K.

```
(*Calculate deg C and F from Kelvins *)
(*Variable x is a dummy variable representing absolute temperature*)
c[x_] = x - 273.15;
f[x_] = (x - 273.15) * 9/5 + 32;
```

In[∘]:=
```
Plot[{c[x], f[x]}, {x, 0, 400}]
```

Out[∘]=



In[∘]:=
```
Solve[c[x] == f[x]]
```

Out[∘]=

{{x → 233.15}}

In[∘]:=
```
c[233.15]
```

Out[∘]=

-40.

In[ ]:=
```
f[233.15]
```
Out[ ]=
```
-40.
```

The Celsius and Fahrenheit temperature scales give the same numerical value of -40 °C or -40 °F at 233.15K. //ANS

Problem 1.6

Pressures up to 3,000 bar are measured with a dead-weight gauge. The piston diameter is 4 mm. What is the approximate mass in kg of the weights required?

SOLUTION

Calculate the area of the piston:

$$\frac{\pi}{4}\left(4.\ \text{mm} * \frac{1\ \text{m}}{1000\ \text{mm}}\right)^2$$

$0.0000125664\ \text{m}^2$

Calculate the force from the given pressure and the calculated area, knowing that pressure is force per unit area:

$$3000\ \text{bar}\ \frac{\frac{10^5\ \text{N}}{\text{m}^2}}{\text{bar}} * .0000125664\ \text{m}^2$$

$3769.92\ \text{N}$

Equate the force to mass*g and solve for mass. (We retain units for illustrative purposes only.)

In[ ]:=
$$\text{eq1} = 3769.92\ \text{N} * \frac{\frac{1\ \text{kg}*\text{m}}{\text{s}^2}}{1\ \text{N}} == \text{mass} * \frac{9.80665\ \text{m}}{\text{s}^2}$$

Out[ ]=
$$\frac{3769.92\ \text{kg m}}{\text{s}^2} == \frac{9.80665\ \text{m mass}}{\text{s}^2}$$

In[ ]:=
```
Solve[eq1, mass]
```
Out[ ]=
```
{{mass → 384.425 kg}}
```

Therefore, the mass is 384.425 kg //ANS.

It is a good idea to check the answer. This can be done quickly by converting the mass back to a force (in Newtons) and then into a pressure (in Pa) and then converting to pressure in bar.

$$\frac{384 * 9.8}{\pi * .004^2 / 4} \bigg/ 10^5$$

2994.66

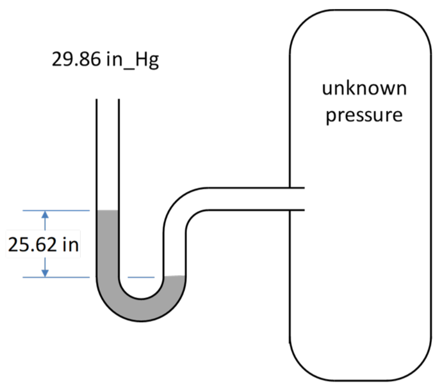
Note that 2994.66 bar is about 3,000 bar, as expected.

Problem 1.9

The reading on a mercury manometer at 70 degF (open to the atmosphere at one end) is 25.62 inches. The local acceleration of gravity is 32.243 $\frac{\text{ft}}{s^2}$. Atmospheric pressure is 29.86 inches of mercury (in_Hg). What is the absolute pressure in psia being measured? The density of mercury at 70 degF is 13.543 $\frac{\text{gm}}{\text{cm}^3}$.

SOLUTION

A sketch of the manometer is shown below. The sketch shows how the different pressures and mercury height are related.



The absolute pressure in the vessel is the sum of the manometer pressure and the atmospheric pressure.

The manometer pressure is given by ρgh, where h is the difference in height between the two mercury levels. English units require the conversion factor g_c, so the pressure difference is $\frac{1}{g_c}\rho$gh. Also, density must be in English units:

Density was given in $\frac{g}{\text{cm}^3}$. Convert to English units ($\frac{\text{lb}_m}{\text{ft}^3}$):

$$\left(\frac{13.543\ \text{gm}}{\text{cm}^3}\right) * \left(\frac{2.20462\ \text{lb}_m}{1000\ \text{gm}}\right) * \left(\frac{10^6\ \text{cm}^3}{35.3147\ \text{ft}^3}\right)$$

$$\frac{845.46\ \text{lb}_m}{\text{ft}^3}$$

To get absolute pressure, add the guage pressure to the atmospheric pressure (25.62in+29.86in), then apply the manometer equation:

In[ ]:=
$$\frac{\left(\left(\frac{845.46\ \mathrm{lb_m}}{\mathrm{ft}^3}\right) * \left(\frac{32.243\ \mathrm{ft}}{\mathrm{s}^2}\right) * (25.62\ \mathrm{in} + 29.86\ \mathrm{in}) * \left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right)\right)}{\left(\frac{\frac{32.1740\ \mathrm{ft}*\mathrm{lb_m}}{\mathrm{s}^2}}{1\ \mathrm{lb_f}}\right)} * \left(\frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right)^2$$

Out[ ]=
$$\frac{27.203\ \mathrm{lb_f}}{\mathrm{in}^2}$$

The absolute pressure in the vessel is 27.203 psia //ANS.

Problem 1.11

Liquids that boil at relatively low temperatures are often stored as liquids under their vapor pressures, which at ambient temperature can be quite large. Thus, n-butane stored as a liquid/vapor system is at a pressure of 2.581 bar for a temperature of 300 K. Large-scale storage $\left(> 50\ m^3\right)$ of this kind is sometimes done in spherical tanks. Suggest two reasons why.

SOLUTION

Reason 1:

For a given volume, the surface area to volume ratio is minimized in a sphere. See examples of area and volume calculations at the end of this document. What does this mean? For a unit volume, the surface area for a sphere is lower than for other shapes. Since heat transfer is proportional to surface area, the same volume of fluid experiences less heat transfer when contained inside a sphere as opposed to a cube or rectangle. Less heat flux means less evaporation, which means the losses should be lower. Engineers have to weigh the costs and benefits of using a more expensive spherical vessel lower fluid losses versus versus less expensive vessels with slightly higher fluid losses. //ANS

Reason 2:

A sphere is inherently a very strong structure. The reason for this is the symmetrical and even distribution of stresses on the sphere's surfaces. Generally, this means that there are fewer weak points than in cylindrical vessels. Of course, this reasoning is true only for a "theoretical" sphere. In an actual spherical vessel, there will be imperfections and stress points at the welds between the plates used to fabricate the vessel, but all things being equal, the spherical vessel should be somewhat stronger. //ANS

Problem 1.12

The first accurate measurements of the properties of high-pressure gases were made by E.H. Amagat in France between 1869 and 1893. Before developing the dead-weight gauge, he worked in a mine shaft, and used a mercury manometer for measurements of pressure to more than 400 bar. Estimate the height of the manometer required.

SOLUTION

Since this is an estimate, we do not need highly precise constants. The acceleration of gravity is about 9.8 $\frac{m}{s^2}$, and we can assume a density of mercury of about 13.5 $\frac{\text{gm}}{\text{cm}^3}$, which corresponds to a temperature near 25 °C.

Convert the pressure of 400 bar to base units:

$$400.\ \text{bar} * \frac{\frac{100\,000\ \text{kg}}{\text{m} * \text{s}^2}}{1\ \text{bar}}$$

$$\frac{4. \times 10^7\ \text{kg}}{\text{m}\ \text{s}^2}$$

The pressure is equal to ρgh:

$$\text{eq1} = \frac{4. * 10^7\ \text{kg}}{\text{m} * \text{s}^2} == \frac{13.5\ \text{gm} * \frac{1\ \text{kg}}{1000\ \text{gm}}}{\text{cm}^3 * \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3} * \frac{9.8\ \text{m}}{\text{s}^2} * (\text{h})$$

$$\frac{4. \times 10^7\ \text{kg}}{\text{m}\ \text{s}^2} == \frac{132\,300.\ \text{h}\ \text{kg}}{\text{m}^2\ \text{s}^2}$$

Solve for h:

```
Solve[eq1, h]
```

```
{{h → 302.343 m}}
```

Therefore the height of the manometer was more than 302 m or 992 ft. That is more than 3 football fields in length! //ANS

Problem 1.18

A gas is confined in a 1.25-ft-diameter cylinder by a piston, on which rests a weight. The mass of the piston and weight together is 250 lb_m. The local acceleration of gravity is 32.169 $\frac{\text{ft}}{s^2}$,and the atmospheric pressure is 30.12 inches of Hg (in_Hg).

(a) What is the force in lb_f exerted on the gas by the atmosphere, the piston, and the weight, assuming no friction between the piston and the cylinder?

(b) What is the pressure of the gas in psia?

(c) If the gas in the cylinder is heated, it expands, pushing the piston and weight upward. If the piston and weight are raised 1.7 ft, what is the work done by the gas in ft $*$ lb_f? What is the change in potential energy of the piston and the weight?

SOLUTION

Part (a)

Calculate the force due to the piston and the weight:

$$250\ \mathrm{lb_m} * (32.169\ \mathrm{ft}/\mathrm{s}^2) * \frac{1\ \mathrm{lb_f}}{\frac{32.174\ \mathrm{ft}*\mathrm{lb_m}}{\mathrm{s}^2}}$$

$249.961\ \mathrm{lb_f}$

Calculate the atmospheric pressure in psia using the conversion factors from Appendix A. Additional conversion factor: 1 Torr = 1 mmHg.

$$30.12\ \mathrm{in\_Hg} * \frac{100\ \mathrm{cm\_Hg}}{39.3701\ \mathrm{in\_Hg}} * \frac{10\ \mathrm{mm\_Hg}}{1\ \mathrm{cm\_Hg}} * \frac{1\ \mathrm{torr}}{1\ \mathrm{mm\_Hg}} * \frac{14.5038\ \mathrm{psia}}{750.061\ \mathrm{torr}}$$

$14.7936\ \mathrm{psia}$

Calculate the piston area:

$$\mathrm{area} = \pi * \left(1.25\ \mathrm{ft} * \frac{12\ \mathrm{in}}{\mathrm{ft}}\right)^2 / 4$$

$176.715\ \mathrm{in}^2$

Now calculate the force due to atmospheric pressure:

$$\frac{14.7936\ \mathrm{lb_f}}{1\ \mathrm{in}^2} * \mathrm{area}$$

$2614.24\ \mathrm{lb_f}$

Now calculate the total force due to weight, piston, air atmosphere:

$$\mathrm{force} = 249.961\ \mathrm{lb_f} + 2614.24\ \mathrm{lb_f}$$

$2864.2\ \mathrm{lb_f}$

The total force is 2864.2 lb_f. //ANS

Part (b)

The pressure of the gas in psia is the force divided by the piston area:

$$\frac{2864.2\ \mathrm{lb_f}}{176.715\ \mathrm{in}^2}$$

$$\frac{16.208\ \mathrm{lb_f}}{\mathrm{in}^2}$$

The pressure of the gas is 16.208 psia. //ANS

Part (c)

The system is defined as the gas in the cylinder. The system (gas) performs work by lifting the piston/weight and by expanding against the constant-pressure of the atmosphere. By equation 1.3

expansion work is given by -Δ(PV) which is -PΔV in this case because the surrounding pressure is constant. The total pressure pushing on the gas was calculated in part (b) which accounts for the atmosphere and the piston/weight. The volume change is given by the displacement multiplied by the area.

$$\frac{-16.2081\ \mathrm{lb_f}}{\mathrm{in}^2} * 176.7\ \mathrm{in}^2 * 1.7\ \mathrm{ft}$$

$-4868.75\ \mathrm{ft\ lb_f}$

Therefore, the system did -4869.16 ft*lbf of work on the surroundings. //ANS Work is **negative** since the total force and displacement are opposite in direction.

Change in potential energy is given by m*g*h:

$$250\ \mathrm{lb_m} * 32.169\ \frac{\mathrm{ft}}{\mathrm{s}^2} * 1.7\ \mathrm{ft} * \frac{1\ \mathrm{lb_f}}{\frac{32.174\ \mathrm{ft} * \mathrm{lb_m}}{\mathrm{s}^2}}$$

$424.934\ \mathrm{ft\ lb_f}$

The change in potential energy is 424.934 ft*lb_f. //ANS

Problem 1.20

Verify that the SI unit of kinetic and potential energy is the joule.

SOLUTION

This problem deals with converting energy units. Students must know how to convert kinetic and potential energy from base units into Joules.

For kinetic energy, start with $\frac{1}{2}mv^2$:

$$\mathrm{kg} * \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 * \frac{1\ \mathrm{N}}{\frac{1\ \mathrm{kg} * \mathrm{m}}{\mathrm{s}^2}} * \frac{1\ \mathrm{J}}{1\ \mathrm{N} * \mathrm{m}}$$

J

For potential energy, start with m*g*h:

$$\mathrm{kg} * \frac{\mathrm{m}}{\mathrm{s}^2} * \mathrm{m} * \frac{1\ \mathrm{N}}{\frac{1\ \mathrm{kg} * \mathrm{m}}{\mathrm{s}^2}} * \frac{1\ \mathrm{J}}{1\ \mathrm{N} * \mathrm{m}}$$

J

In both cases the units reduce to J. //ANS

Problem 1.22

The turbines in a hydroelectric plant are fed by water falling from a 50-m height. Assuming 91%

efficiency for conversion of potential to electrical energy, and 8% loss of the resulting power in transmission, what is the mass flow rate of water required to power a 200-watt light bulb?

SOLUTION

Students must know how to convert potential energy into *energy flow rate*. The following conversion factors into base energy units are important:

$$\frac{kg * m}{s^2} \; (*N*)$$

$$\frac{kg * m^2}{s^2} \; (*Nm \text{ or } joule*)$$

$$\frac{kg * m^2}{s^3} \; (*Nm/s \text{ or } Watt*)$$

$$\frac{kg}{m * s^2} \; (*N/m^2 \text{ or } pascal*)$$

Use the conversion factor for Watts to convert bulb power to base energy units:

In[ ]:= $$\text{BulbPower} = 200\,W * \frac{\frac{kg*m^2}{s^3}}{W} \quad (*\text{Light bulb power}*)$$

Out[ ]= $$\frac{200\,kg\,m^2}{s^3}$$

Use ρgh to convert the height of the reservoir to base energy units:

$$\text{HydrostaticPressure} = \frac{1000\,kg}{m^3} * \frac{9.8\,m}{s^2} * 50\,m \quad (*\rho*g*h*)$$

Out[ ]= $$\frac{490\,000.\,kg}{m\,s^2}$$

By comparison to the units for Watts, these two terms differ by multiplication of ρgh by volumetric flow in $\frac{m^3}{s}$. To illustrate, divide them:

In[ ]:= $$\frac{\text{BulbPower}}{\text{HydrostaticPressure}}$$

Out[ ]= $$\frac{0.000408163\,m^3}{s}$$

Now add the efficiencies, noting that any losses must increase the flow rate to account for the lost energy. We do this by dividing by the efficiencies.

In[•]:= $\frac{\text{BulbPower}}{0.91 * (1 - .08) * \text{HydrostaticPressure}}$

Out[•]= $\frac{0.000487534\ \text{m}^3}{\text{s}}$

In[•]:= $\frac{0.000487534\ \text{m}^3}{\text{s}} * \frac{1000\ \text{kg}}{\text{m}^3}$

Out[•]= $\frac{0.487534\ \text{kg}}{\text{s}}$

The mass flow rate required is $0.4875\ \frac{kg}{s}$ or $1{,}755\ \frac{kg}{h}$. //ANS

Problem 1.27

Energy costs vary greatly with energy source. For example, coal is about \$35.00/ton, gasoline has a pump price of \$2.75/gallon, and electricity as about \$0.10/kWhr. Conventional practice is to put these on a common basis by expressing them in \$/GJ. For this purpose, assume gross heating values of 29 MJ/kg for coal and 37 GJ$/m^3$ for gasoline.

(a) Rank order the three energy sources with respect to energy cost in \$/GJ.

(b) Explain the large disparity in the numerical results of Part (a).

SOLUTION

Part (a)

Coal:

$\frac{35\ \text{dollars}}{\text{ton}} * \frac{1\ \text{ton}}{2000\ \text{lbs}} * \frac{2.20462\ \text{lbs}}{\text{kg}} * \frac{1\ \text{kg}}{29\ \text{MJ}} * \frac{1000\ \text{MJ}}{\text{GJ}}$

Out[•]= $\frac{1.33037\ \text{dollars}}{\text{GJ}}$

Gasoline:

$\frac{2.75\ \text{dollars}}{\text{gal}} * \frac{264.172\ \text{gal}}{1\ \text{m}^3} * \frac{1\ \text{m}^3}{37\ \text{GJ}}$

Out[•]= $\frac{19.6344\ \text{dollars}}{\text{GJ}}$

Electricity:

$$\frac{0.1000\,\text{dollars}}{\text{kWhr}} * \frac{2.77778 * 10^{-7}\,\text{kWhr}}{1\,\text{J}} * \frac{10^{9}\,\text{J}}{\text{GJ}}$$

$$\frac{27.7778\,\text{dollars}}{\text{GJ}}$$

Coal < Gasoline < Electrcitiy. //ANS

Part (b)

Gasoline and electricity are more expensive because they are more highly refined forms of energy requiring more infrastructure. Converting coal to electricity requires a power plant plus a train line plus and electrical power grid. Burning coal directly to produce heat energy requires only a fire box. To convert coal to mechanical energy requires a fire box and a heat engine such as a steam engine. Steam engines are mechanically very simple. //ANS

Problem 1.29

A laboratory reports the following vapor-pressure (P^{sat}) data for a particular organic chemical:

t/ °C	P^{sat} / kPa
-18.5	3.18
-9.5	5.48
0.2	9.45
11.8	16.9
23.1	28.2
32.7	41.9
44.4	66.6
52.1	89.5
63.3	129
75.5	187

Correlate the data by fitting them to the Antoine equation:

$$\ln P^{sat}/\text{kPa} = A - \frac{B}{T/K + C}$$

That is, find numerical values of parameters A, B, and C by an appropriate regression procedure. Discuss the comparison of correlated and experimental values. What is the predicted normal boiling point of this chemical (i.e., the temperature at which the vapor pressure is 1 atm).

SOLUTION

Find numerical values of A, B, and C:

The data is first entered into Excel because I find that easiest. The data is in a file on Canvas (in Modules→Lesson3). There you will find a link for “Student Excel Data for Problem 1.29.” Download this data to your computer. Or, you can just type it into Excel and save the file:

Insert the complete path and use *SemanticImport*:

In[ ]:=
```
filename =
  "C:\\Users\\andrew.biaglow\\OneDrive - West Point\\Desktop\\Problem_1-29_Data.xlsx";
ds = SemanticImport[filename];
```

Convert the data to "Normal" form:

In[ ]:=
```
data = Normal[ds]
```

Out[ ]=
```
{{-18.5, 3.18}, {-9.5, 5.48}, {0.2, 9.45}, {11.8, 16.9}, {23.1, 28.2},
 {32.7, 41.9}, {44.4, 66.6}, {52.1, 89.5}, {63.3, 129.}, {75.5, 187.}}
```

Define the model:

In[ ]:=
```
model = Exp[a - b/(t + c)]
```

Out[ ]=

$$e^{a-\frac{b}{c+t}}$$

Find the best fit using the *FindFit* function:

In[ ]:=
```
fit = FindFit[data, model, {{a, 14.}, {b, 2800}, {c, 200}}, t]
```

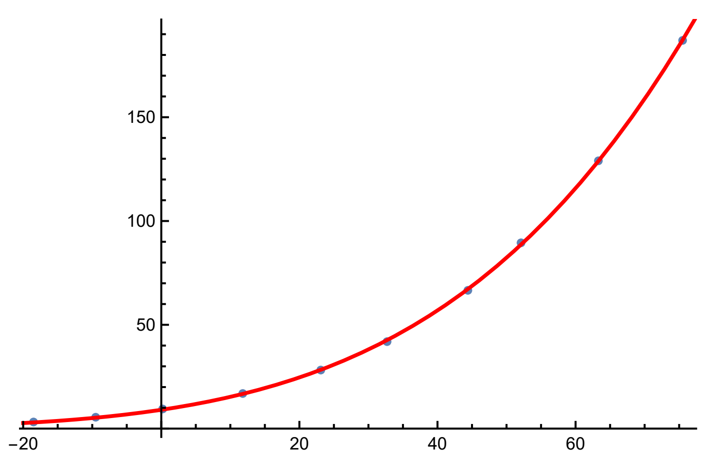
Out[ ]=
```
{a → 13.421, b → 2289.59, c → 204.097}
```

The numerical values of A, B, and C from the regression are A = 12.421, B = 2289.59, and C = 204.097. //ANS

Comparison of correlated and experimental values using plots:

In[ ]:=
```
plot1 = ListPlot[data, PlotRange → All];
plot2 = Plot[Evaluate[model /. fit], {t, -20, 100}, PlotStyle → Directive[Thick, Red]];
Show[plot1, plot2]
```

Out[ ]=



Based on the plot, the fit is very good. //ANS

We could also be more quantitative using sum of squares. This is discussed in the supllement below.

What is the predicted normal boiling point?

Wikipedia: The normal boiling point is the temperature at which the vapor pressure is equal to an atmospheric pressure of 1 atm.

In[∘]:=

```
kilopascals in 1.0 atmosphere
UnitConvert[1. atm, "Kilopascals"]
```

Out[∘]=

101.325 kPa

In[∘]:=
```
NSolveValues[(model /. fit) == 101.325, t, Reals]
```

Out[∘]=

{56.0037}

The predicted boiling point is 56.0037 deg C. //ANS

END OF APPROVED SOLUTION

Supplement - Using Sum of Squares

Create a list of temperatures:

In[∘]:=
```
t = ds[[All, 1]] // Normal
```

Out[∘]=

{-18.5, -9.5, 0.2, 11.8, 23.1, 32.7, 44.4, 52.1, 63.3, 75.5}

Create a list of pressures from the given data:

In[∘]:=
```
p1 = ds[[All, 2]] // Normal
```

Out[∘]=

{3.18, 5.48, 9.45, 16.9, 28.2, 41.9, 66.6, 89.5, 129., 187.}

Create a list of calculated pressures:

In[∘]:=
```
p2 = model /. fit (*calculated pressure*)
```

Out[∘]=

{2.95848, 5.23427, 9.15105, 16.7103, 28.3176, 42.6078, 67.1739, 88.6064, 128.834, 187.194}

Absolute value of difference between data and calculated data:

```
In[ ]:= residuals = Abs[p2 - p1]
Out[ ]=
{0.221517, 0.245728, 0.29895, 0.189728,
 0.117552, 0.707804, 0.573882, 0.893575, 0.165722, 0.194121}
```

Average residual error:

```
In[ ]:= Plus @@ residuals / Length[t]
Out[ ]=
0.360858
```

This means the average residual error is about 0.4 kPa. This is about 2% average error. You can see this by calculating the individual percent errors and then averaging them:

```
In[ ]:= 100 * residuals / p1
Out[ ]=
{6.96593, 4.48408, 3.16349, 1.12265,
 0.416852, 1.68927, 0.861684, 0.998407, 0.128467, 0.103808}
```

```
In[ ]:= Plus @@ (100 * residuals / p1) / Length[t]
Out[ ]=
1.99346
```

We can also look as sum of squares error:

```
In[ ]:= SQ = (p2 - p1)^2
Out[ ]=
{0.0490697, 0.0603821, 0.0893712, 0.0359967,
 0.0138185, 0.500987, 0.32934, 0.798476, 0.0274639, 0.0376828}
```

```
In[ ]:= SSQ = Plus @@ SQ
Out[ ]=
1.94259
```

```
(*This is a tiny little bit better than Excel which gave SSQ=1.950.*)
```

```
In[ ]:= averageError = Sqrt[SSQ / Length[t]]
Out[ ]=
0.440748
```

This percentage 0. 0.44% is about the same as the answer obtained above, 0.36%.