# CH365 Chemical Engineering Thermodynamics

Lesson 30
Generalized Property Correlations

#### CDP - IPR3

- Friday 17 November by 1630, 90 points
- Cover Sheet, PDF of Mathematica and PowerPoint slides combined into single signed PDF (with signed cover sheet).
- PDF, MMA and CHEMCAD files in CANVAS.
- Keep it short and simple. Answer Questions 1-4. See published guidance.
  - Schedule complete, all groups
  - Experiments complete, all groups
  - Txy plot in CHEMCAD using NRTL. Guidance provided.
  - Txy plot in MMA for ideal case. This was done in CH362 and CH363. Problem 13.1 is a guide.

Bubble Point:	Dew Point:	Antoine Eq.
x <sub>i</sub> is known	y <sub>i</sub> is known	D <sup>sat</sup>
$y_i = K_i \cdot x_i$	$\mathbf{x}_{i} = \mathbf{y}_{i} \div \mathbf{K}_{i}$	$K_i = f(T,P) = \frac{P_i^{sat}}{P}$
$\sum_{i} y_{i} = 1$	$\sum_{i} x_{i} = 1$	•

#### **WPR3 WARNO**

- WPR3 is Wednesday 15 November
- 3 problems, 200 points, from Chapter 5 and 6
  - Calculation of ideal enthalpy and entropy changes
  - Calculation of residual enthalpy and entropy
  - Combining ideal and residual properties to get real properties
  - Lost work (as in heat exchangers)
  - Derivations property relations, Maxwell relations, generating functions
- 55 minutes, during class hour.
- Open note, open book, open computer. Calculator, Mathematica authorized.
- Upload all exam e-work to SharePoint.

### Today's Agenda

- Complete discussion of residual properties
- Discussion of generalized methods
- Continue discussion of homework

# **Residual Properties**

 $M^R \equiv M - M^{ig}$ 

superscript "R" refers to "residual"

(Eq. 6.41)



$$H^{R} = H - H^{ig}$$

$$S^{R} = S - S^{ig}$$

$$G^{R} = G - G^{ig}$$

$$H = H^{ig} + H^{R}$$

$$S = S^{ig} + S^{R}$$

$$G = G^{ig} + G^{R}$$

$$dH^{ig} = C_P^{ig} dT$$
(Eq. 6.23)

$$dS^{ig} = \frac{C_P^{ig}}{T}dT - R \cdot \frac{1}{P} \cdot dP$$
(Eq. 6.24)

$$H^{ig} = H_0^{ig} + \int_{T_0}^{T_2} C_P^{ig} dT$$

$$S^{ig} = S_0^{ig} + \int_{T_0}^{T_2} \frac{C_P^{ig}}{T} dT - R \cdot In \left(\frac{P}{P_0}\right)$$

$$V^{R} = V - V^{ig}$$

$$V^{ig} = \frac{P}{RT}$$

$$P$$

$$-V = \frac{ZRT}{P}$$

$$V^{R} = \frac{ZRT}{P} - \frac{RT}{P} = \frac{RT}{P}(Z-1)$$
(Eq. 6.40)

- The true worth of the equations for ideal gases is now evident.
- They are important because they provide a convenient base for the calculation of real gas properties.

# Residual Properties from Cubic EOS

Generic cubic equation of state (in Z-Form)

(WPR3)

$$q = q(x) = \frac{\Psi \alpha(x)}{\Omega \cdot x}$$
 (Eq. 3.51)

 $\alpha = \alpha(\mathbf{X})$ 

( $\alpha$  is found in Table 3.1)

These are the red equations from L28 slide 8 (cleanup).

**Important:**  $\alpha$  is written as a function of x where x replaces  $T_r$ .

q derivatives must be evaluated before process conditions ( $T_R$ ,  $P_r$ ) are entered.

Problem 6.141 in PS10

$$\beta = \Omega \frac{P_{r}}{T_{r}} \qquad (3.50)$$

$$Z = 1 + \beta - q\beta \frac{Z - \beta}{(Z + \epsilon\beta)(Z + \sigma\beta)}$$
(3.48)

$$\frac{H^{R}}{RT} = Z - 1 + T_{r} \left( \frac{dq}{dT_{r}} \right) \cdot I$$

(page 497)

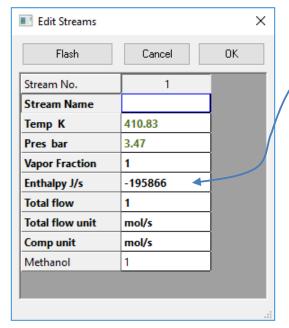
$$\frac{S^{R}}{R} = \ln(Z - \beta) + \left(q + T_{r} \frac{dq}{dT_{r}}\right) \cdot I$$

$$I = \frac{1}{\sigma - \varepsilon} \ln \left( \frac{Z + \sigma \beta}{Z + \varepsilon \beta} \right)$$
 (13.72)

# Connection to Total Properties 6

Add ideal gas and residual

#### CHEMCAD



$$\begin{split} H &= H_0^{ig} + R \int\limits_{T_0}^T C_P^{ig} dT + H^R \\ &= H_0^{ig} + R \int\limits_{T_0}^{T_1} C_P^{ig} dT + H_1^R \\ H_1 &= H_0^{ig} + R \int\limits_{T_0}^{T_2} C_P^{ig} dT + H_1^R \\ &= H_0^{ig} + R \int\limits_{T_0}^{T_2} C_P^{ig} dT + H_2^R \end{split}$$

 $H_0^{ig} = \Delta H_{f298}^{o}$  = standard formation enthalpy, App C, pp 671-672

Difference between streams: residual property at 
$$T_2$$

$$\Delta H = H_2 - H_1 = R \int_{T_1}^{T_2} C_P^{ig} dT + H_2^R - H_1^R \qquad \text{(Eq. 6.72)}$$

For CHEMCAD entropy, use Report -> Stream Reports -> Customize Property Sets -> Overall

$$S = S_0^{ig} + \int\limits_{T_0}^T C_P^{ig} \frac{dT}{T} - R \ln \frac{P}{P_0} + S^R \\ \text{(Eq. 6.51)} \qquad \Delta S = \int\limits_{T_1}^{T_2} C_P^{ig} \frac{dT}{T} - R \ln \frac{P_2}{P_1} + S_2^R - S_1^R \\ \text{(Eq. 6.73)}$$

 $S_0^{ig}$  =  $\Delta S_{f298}^{o}$ , also from App C, pp 660-662, from  $\Delta H_{f298}^{o}$  and  $\Delta G_{f298}^{o}$ 

CHEMCAD standard enthalpy and Gibbs energy are in Component Data -> Basic Data

$$\Delta S_{f298}^{ig} = \frac{\Delta H_{f298}^{ig} - \Delta G_{f298}^{ig}}{298.15}$$

# Three Methods for Residual Properties

Cubic Equations of States (L28 & L29):

•RK, SRK, PR EOS from Table 3.1

## Generalized Methods (New for Today):

- Lee-Kesler Tables
- Virial Equation of State

## Lee-Kesler Table Method

Valid for liquids, vapors and gases

$$Z = Z^0 + \omega Z^1$$
 (Pitzer correlation, Eq. 3.53)

$$\frac{\mathsf{H}^{\mathsf{R}}}{\mathsf{R}\mathsf{T}_{\mathsf{c}}} = \frac{\left(\mathsf{H}^{\mathsf{R}}\right)^{0}}{\mathsf{R}\mathsf{T}_{\mathsf{c}}} + \omega \frac{\left(\mathsf{H}^{\mathsf{R}}\right)^{1}}{\mathsf{R}\mathsf{T}_{\mathsf{c}}} \tag{Eq.6.66}$$

$$\frac{S^{R}}{R} = \frac{\left(S^{R}\right)^{0}}{R} + \omega \frac{\left(S^{R}\right)^{1}}{R}$$
 (Eq. 6.67)

$$Z^{0}$$
,  $Z^{1}$ ,  $\frac{\left(H^{R}\right)^{0}}{RT_{c}}$ ,  $\frac{\left(H^{R}\right)^{1}}{RT_{c}}$ ,  $\frac{\left(S^{R}\right)^{0}}{R}$ ,  $\frac{\left(S^{R}\right)^{1}}{R}$ ,  $\phi^{0}$ , and  $\phi^{1}$  Appendix D pages 676-692

# Virial Equation

Truncated

$$Z = 1 + \frac{BP}{RT} = 1 + \hat{B}\frac{P_r}{T_r}$$
 (Eq.3.57)

$$\hat{B} = \frac{BP_c}{RT_c}$$
 (Eq.3.58)

Second virial coefficient

$$\mathbf{\hat{B}} = \mathbf{B}^0 + \omega \mathbf{B}^1$$

 $\hat{B} = B^0 + \omega B^1$  (Pitzer Correlation, Eq. 3.59)

$$\frac{H^{R}}{RT_{c}} = P_{r} \left[ B^{0} - T_{r} \frac{dB^{0}}{dT_{r}} + \omega \left( B^{1} - T_{r} \frac{dB^{1}}{dT_{r}} \right) \right]$$
 (Eq. 6.68)

$$\frac{S^{R}}{R} = -P_{r} \left( \frac{dB^{0}}{dT_{r}} + \omega \frac{dB^{1}}{dT_{r}} \right)$$
 (Eq. 6.69)

$$B^{0} = 0.083 - \frac{0.422}{T_{r}^{1.6}} \qquad (Eq. 3.61) \qquad B^{1} = 0.139 - \frac{0.172}{T_{r}^{4.2}} \qquad (Eq. 3.62)$$
 
$$\frac{dB^{0}}{dT_{r}} = \frac{0.675}{T_{r}^{2.6}} \qquad (Eq. 6.70) \qquad \frac{dB^{1}}{dT_{r}} = \frac{0.722}{T_{r}^{5.2}} \qquad (Eq. 6.71)$$

# Mixtures of Gases I - Textbook 10

The critical temperature of a gas is the maximum temperature at which compression can cause liquefaction. The critical pressure of a gas is the minimum pressure at which cooling causes liquefaction. Mixing of gases results in new critical properties for the mixture, and the mixture behaves as a "pseudocritical" pure component.

#### "pseudocritical" properties

$$\omega \equiv \sum_{i} y_{i} \omega_{i}$$
 (Eq. 6.78)

"pseudocritical acentric factor"

$$T_{pc} \equiv \sum_{i} y_{i} T_{ci}$$
(Eq. 6.79)

"pseudocritical temperature" subscript "p"

$$P_{pc} \equiv \sum_{i} y_{i} P_{ci}$$
(Eq. 6.80)

"pseudocritical pressure"

"pseudoreduced temperature"

$$T_{pr} = \frac{I}{T_{pc}}$$
(Eq. 6.81)

"pseudoreduced pressure"

$$P_{pr} = \frac{P}{P_{pc}}$$
(Eq. 6.82)

## Mixtures of Gases II – API - Best 11

API Technical Data Book, T.T. Daubert, ed., 5<sup>th</sup> Edition, American Petroleum Institute, 1992. Standard method now widely used.

"pseudocritical" properties

worked example is in reference, p. 4-25

"true critical temperature" of mixture 
$$T_{cm} = \sum_{i=1}^{11} (\theta_i \cdot T_{ci})$$
 (Eq. 4B1.1-1)

volume fraction of component i 
$$\theta_i = \frac{x_i \cdot V_{ci}}{V_{pc}}$$
 (Eq. 4B1.1-2)

molar average critical volume 
$$V_{pc} = \sum_{i} x_{i} \cdot V_{ci}$$
 (Eq. 4B1.1-3.79)

"true critical pressure" of mixture

$$P_{cm} = P_{pc} + P_{pc} \left[ 5.808 + 4.93 \cdot \sum_{i=1}^{n} (x_i \cdot \omega_i) \right] \cdot \left[ \frac{T_{cm} - T_{pc}}{T_{pc}} \right]$$

worked example is in reference, pp. 4-28 to 4-29 (Eq. 4B2.1-1)

pseudocritical temperature of mixture 
$$T_{pc} = \sum_{i} (x_i \cdot T_{ci})$$
 (Eq. 4B2.1-2)

pseudocritical pressure of mixture 
$$P_{pc} = \sum_{i=1}^{n} (x_i \cdot P_{ci})$$
 (Eq. 4B2.1-3)

# Homework

# Problem 6.84 (e,f)

Calculate VR, HR, and SR for each of the following using the generalized virial method:

- (e) Ethylbenzene at 620 K and 20 bar
- (f) Methane at 250 K and 90 bar

## Problem 6.101

Use the API mixture rules in slide 11 to estimate the critical temperature and pressure of air at 1 bar total pressure and 50% relative humidity. Check your answers with CHEMCAD.