Depreciation and Taxes

CH402 Lesson 18

FE Reference Manual, pp. 230 “sinking fund”

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n - 1}$
Capital Recovery			$i(1+i)^n$
Uniform Series Compound Amount			
Uniform Series Present Worth			
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\frac{1}{i} - \frac{n}{(1+i)^n - 1}$

“Sinking Fund”

A sinking fund is used by companies that have floated debt in the form of bonds to gradually save money and avoid a large lump-sum payment at maturity. Some bonds are issued with the attachment of a sinking fund feature. A sinking fund is essentially a savings account.

https://www.investopedia.com/terms/s/sinkingfund.asp#toc-understanding-a-sinking-fund

Example 7-3

A loan of \$100,000 at a nominal interest rate of 10 percent per year is made for a repayment period of 10 years. Determine the constant payment per period (monthly loan payment) assuming 12 equal-length months per year.

Lesson 17 – Review – Cash Flow

The cash flow pattern given below were projected for a proposed project. With interest compounded monthly at a rate of 10% per year, calculate the total amount of the cash flow, the present worth at time zero, and the future worth at 12 months.

sum at month 0

(P/F,i,n)

each of these happens in the future

Present worth is at month zero (j=0)

(FEE, p. 230)

end of month j	cash flow, $	DSPPWF	DSPPWF	DSPPWF
1	10000	(P/F,.083,1)	$(1+i)^{-j}$	$(1+.0833)^{-1}$
2	10000	(P/F,.083,2)	$(1+i)^{-j}$	$(1+.0833)^{-2}$
3	10000	(P/F,.083,3)	$(1+i)^{-j}$	$(1+.0833)^{-3}$
4	10000	(P/F,.083,4)	$(1+i)^{-j}$	$(1+.0833)^{-4}$
5	10000	(P/F,.083,5)	$(1+i)^{-j}$	$(1+.0833)^{-5}$
6	10000	(P/F,.083,6)	$(1+i)^{-j}$	$(1+.0833)^{-6}$
7	10000	(P/F,.083,7)	$(1+i)^{-j}$	$(1+.0833)^{-7}$
8	10000	(P/F,.083,8)	$(1+i)^{-j}$	$(1+.0833)^{-8}$
9	10000	(P/F,.083,9)	$(1+i)^{-j}$	$(1+.0833)^{-9}$
10	10000	(P/F,.083,10)	$(1+i)^{-j}$	$(1+.0833)^{-10}$
11	10000	(P/F,.083,11)	$(1+i)^{-j}$	$(1+.0833)^{-11}$
12	10000	(P/F,.083,12)	$(1+i)^{-j}$	$(1+.0833)^{-12}$

PWF's

Implementation in Excel

=(1+0.08333)^-G5

	E	G	H	I	J	K
3						
4		mo., j	cash flow, $	DSPPWF	PW	
5		1	10000	0.923080	9230.80	
6		2	10000	0.852076	8520.76	
7		3	10000	0.786534	7865.34	
8		4	10000	0.726034	7260.34	
9		5	10000	0.670187	6701.87	
10		6	10000	0.618636	6186.36	
11		7	10000	0.571051	5710.51	
12		8	10000	0.527125	5271.25	
13		9	10000	0.486579	4865.79	
14		10	10000	0.449151	4491.51	
15		11	10000	0.414602	4146.02	
16		12	10000	0.382711	3827.11	
17						
18						
19					NPW	
20					74077.66	
21						

added up at j=0 after all cash flows are referenced to the same time

NPW "net present worth"

Cash flow

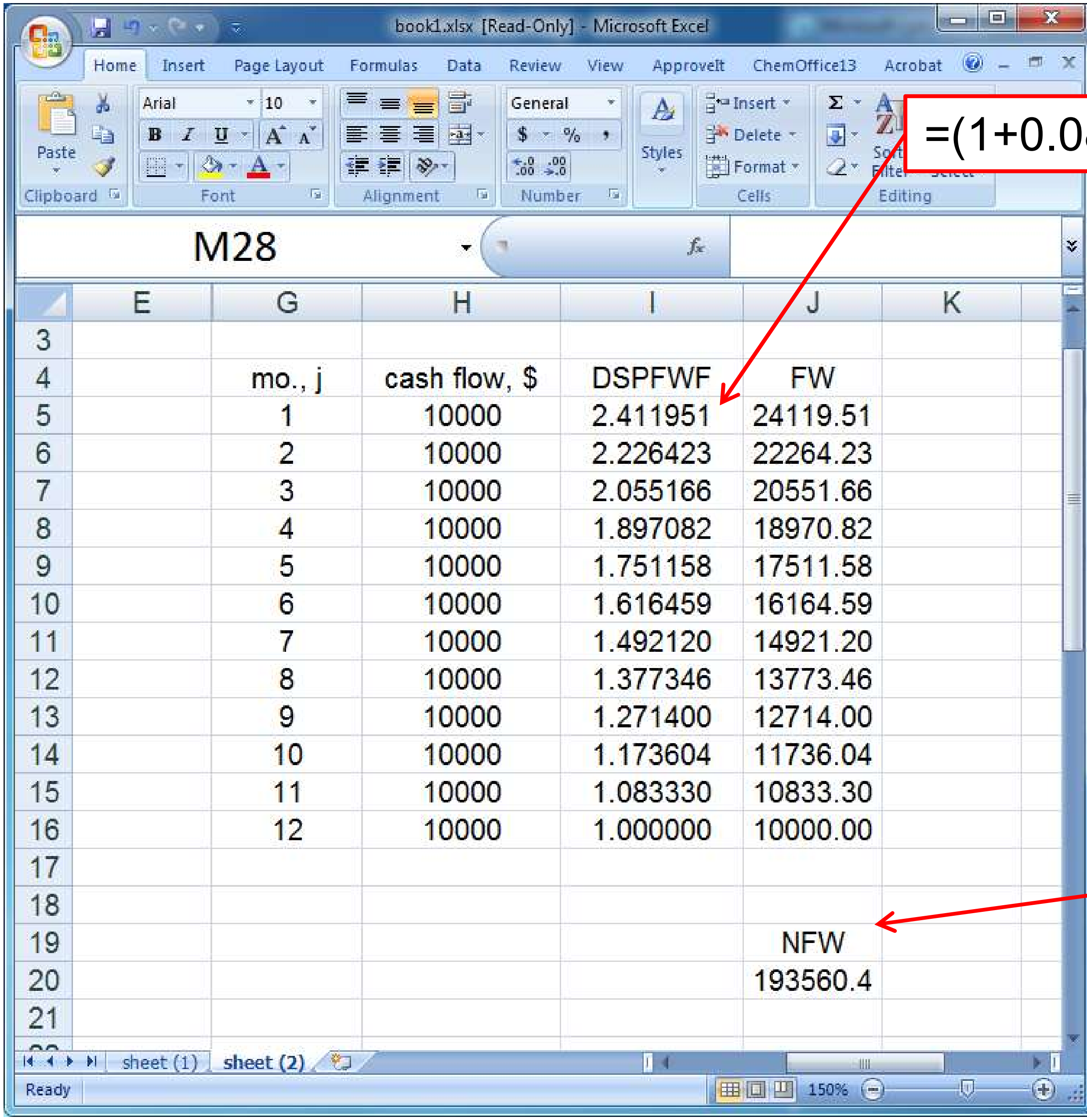
The cash flow pattern given below were projected for a proposed project. With interest compounded monthly at a rate of 10% per year, calculate the total amount of the cash flow, the present worth at time zero, and the future worth at 12 months.

(F/P,i,n)

end of month j	cash flow, $	DSPFWF	DSPFWF	DSPFWF
1	10000	(F/P,.083,11)	$(1+i)^{12-j}$	$(1+.0833)^{11}$
2	10000	(F/P,.083,10)	$(1+i)^{12-j}$	$(1+.0833)^{10}$
3	10000	(F/P,.083,9)	$(1+i)^{12-j}$	$(1+.0833)^{9}$
4	10000	(F/P,.083,8)	$(1+i)^{12-j}$	$(1+.0833)^{8}$
5	10000	(F/P,.083,7)	$(1+i)^{12-j}$	$(1+.0833)^{7}$
6	10000	(F/P,.083,6)	$(1+i)^{12-j}$	$(1+.0833)^{6}$
7	10000	(F/P,.083,5)	$(1+i)^{12-j}$	$(1+.0833)^{5}$
8	10000	(F/P,.083,4)	$(1+i)^{12-j}$	$(1+.0833)^{4}$
9	10000	(F/P,.083,3)	$(1+i)^{12-j}$	$(1+.0833)^{3}$
10	10000	(F/P,.083,2)	$(1+i)^{12-j}$	$(1+.0833)^{2}$
11	10000	(F/P,.083,1)	$(1+i)^{12-j}$	$(1+.0833)^{1}$
12	10000	(F/P,.083,0)	$(1+i)^{12-j}$	$(1+.0833)^{0}$

sum at month 12

Implementation in Excel



	E	G	H	I	J	K
3						
4		mo., j	cash flow, $	DSPFWF	FW	
5		1	10000	2.411951	24119.51	
6		2	10000	2.226423	22264.23	
7		3	10000	2.055166	20551.66	
8		4	10000	1.897082	18970.82	
9		5	10000	1.751158	17511.58	
10		6	10000	1.616459	16164.59	
11		7	10000	1.492120	14921.20	
12		8	10000	1.377346	13773.46	
13		9	10000	1.271400	12714.00	
14		10	10000	1.173604	11736.04	
15		11	10000	1.083330	10833.30	
16		12	10000	1.000000	10000.00	
17						
18						
19					NFW	
20					193560.4	
21						

=(1+0.08333)^(12-G5)

added up at j=12 after all cash flows are referenced to the same time

NFW "net future worth"

Alternate Solution

use (F/A,i,n)

also known as "discount factors"

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n-1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\frac{(1+i)^n-1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\frac{(1+i)^n-1}{i(1+i)^n}$
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2(1+i)^n}-\frac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2}-\frac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\frac{1}{i}-\frac{n}{(1+i)^n-1}$

FEE, p. 230

Alternate Solution

use (F/A,i,n)

$$F = A \cdot (F/A,.08333,12)$$

FEE, p. 230

Depreciation

What is depreciable?

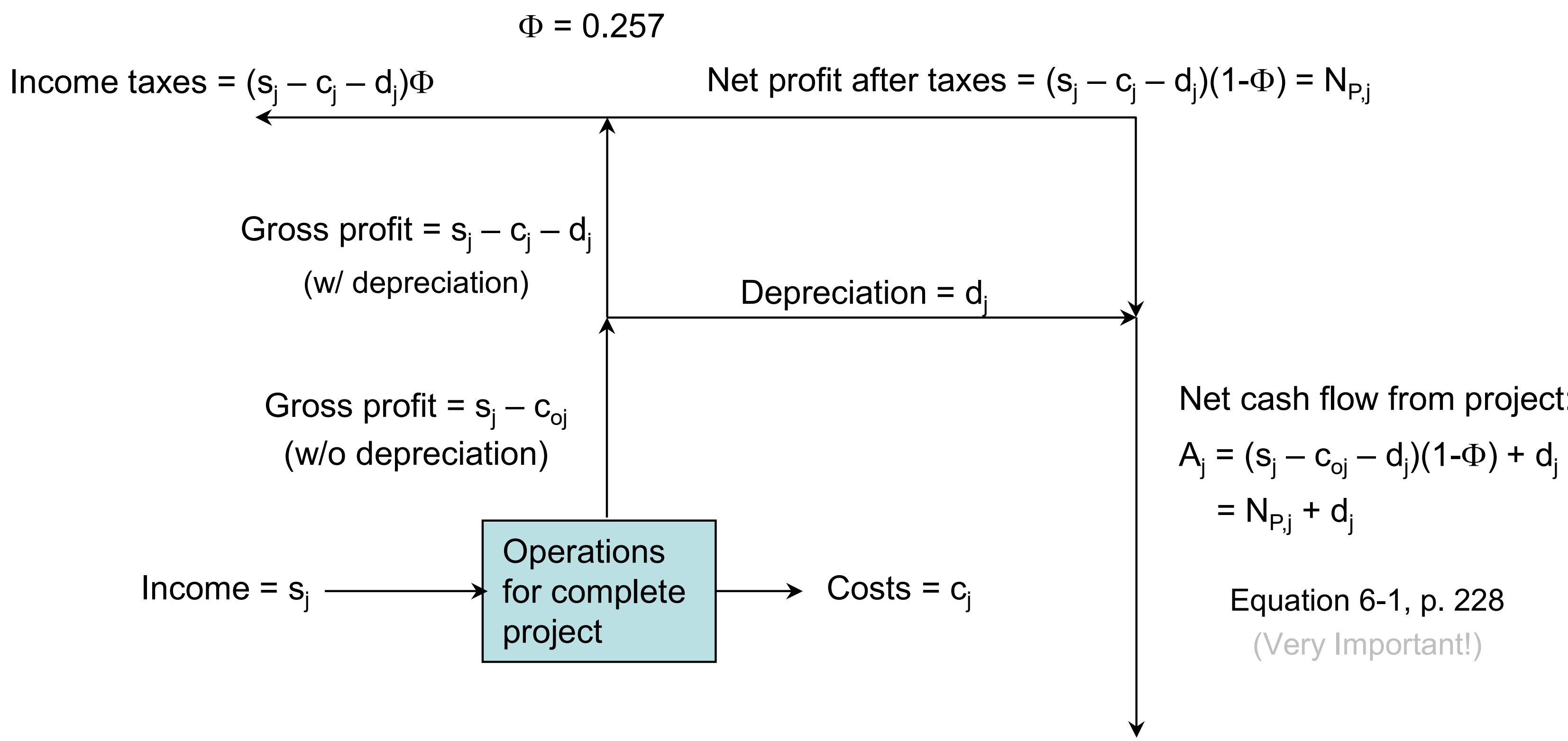
“A deduction for depreciation may be claimed each year for property with a limited useful life that is used in a trade or business or held for the production of income. This deduction allows taxpayers to recover their costs for the property over a number of years”

Prentice-Hall Federal Tax Adviser, PTW page 308

Depreciation as a Cash Flow

Review – Figure 6-1 from Lessons 14, 17

Top Portion of Fig. 6-1



Depreciation

What is depreciable?

Reduced tax for cost associated with wear and tear on equipment.

Mathematical definition in Figure 6-1.

$$A_j = (s_j - c_j - d_j)(1-\phi) + \underbrace{d_j} = (s_j - c_j)(1-\phi) + \underbrace{\phi d_j} = N_{Pj} + d_j$$

Significant impact on corporate cash flow.

Looks like an accounting trick, but operationally **d_j is a tax deduction** and is closely controlled by federal tax law.

What is depreciable?
Any fixed capital investment not including land.

Depreciation

What is depreciable?

"Any equipment or property with a useful life of more than 1 year that is used in a trade or business."

Fixed capital investment not including land.

Associated Concepts:
replacement value
purchase value
current value
salvage value
recovery period

recovery period is the time period over which depreciation is charged.
(not the same as service life)

Recovery periods are established by tax law.

Three methods: (1) straight line, (2) MACRS, and (3) SOD

Recovery Periods from Table 7.8

(established by tax law)

Type of Asset	MACRS	Straight line
Heavy general purpose trucks	5	5
Industrial steam and electrics	15	22
IT Systems	5	5
Chemical Manufacture	5	9.5~10
Pulp and Paper Manufacturing	7	13
Petroleum Refining	5	12
Pipeline Trans.	15	22

Depreciation – Method 1

"Straight-Line" Method

$$d_j = \text{annual depreciation} = \frac{\text{purchase value - salvage value}}{\text{recovery period}}$$

PTW Eq 7-44, p.311

FEE Manual, p. 231

Depreciation – Method 2

"MACRS" Method

Modified Accelerated Cost Recovery System

Most common method

d_j = annual depreciation

= (purchase value - salvage value) × allowance factor

PTW discussion pp.311-314
No equation given

Eq 7-44, p.311 if factor = 1/n

FEE Manual, p. 231

"MACRS" factors Modified Accelerated Cost Recovery System

PTW Table 7-9, p.313

year	3-y	5-y	7-y	10-y	15-y	20-y
	depreciation rate, %					
1	33.33	20.00	14.29	10.00	5.00	3.750
2						19
3						77
4						77
5						13
6						85
7						88
8						22
9						62
10						61
11						62
12						61
13						62
14						61
15						62
16						61
17						62
18						61
19						62
20						4.461
21						2.231

MACRS FACTORS

Year	Recovery Period (Years)			
	3	5	7	10
	Recovery Rate (Percent)			
1	33.33	20.00	14.29	10.00
2	44.45	32.00	24.49	18.00
3	14.81	19.20	17.49	14.40
4	7.41	11.52	12.49	11.52
5		11.52	8.93	9.22
6		5.76	8.92	7.37
7			8.93	6.55
8			4.46	6.55
9				6.56
10				6.55
11				3.28

3-y to 10-y appear in FE Ref Manual p. 128

FEE Manual, p. 232

Depreciation – Method 3

Needed for Problem 7-18†

"Sum of Digits" Method

$$d_j = \frac{2\cdot(n-j+1)}{n\cdot(n+1)}\cdot(C-S_n)$$

$$d_j = \frac{n+1-j}{\sum_{j=1}^{n} j}\cdot(C-S_n)$$

number of useful service years left divided by sum of years in recovery period

where
n = recovery period
j = year for which depreciation is being determined
C = equipment cost
S_n = salvage value

† Footnote on page 318 directs reader to 4th Edition, page 283

Problem 7-17 (PS9)

A laboratory piece of equipment was purchased for $35,000 and is estimated to be used for 5 years with a salvage value of $5,000.

(a) Tabulate the annual depreciation allowances and year-end book values for the 5 years using (1) straight-line depreciation, (2) MACRS 5-yr recovery period depreciation, and (3) sum-of-digits method. The recovery period is five years.

(b) Compare the net present worth of each of the three depreciation methods assuming an interest rate of 6%.

Note: Part (b) is not in the book.

Problem 7-18 (PS9)

A piece of equipment with an original cost of $10,000 and no salvage value has a depreciation allowance of $2381 during its second year of service when depreciated by the sum-of-the-digits method. What recovery period has been used?

Problem 7-9 (PS9)

The fixed capital investment for an existing chemical plant is $20 million. Annual property taxes amount to 1% of the fixed-capital investment, and state income taxes are 5% of the gross earnings. The net income after all taxes is $2 million, and the federal income taxes amount to 35% of gross earnings.

If the same plant had been constructed for the same fixed capital investment but at a location where property taxes were 4% of the fixed capital investment and the state income taxes were 2% of the gross earnings, what would be the net income per year after taxes, assuming all other cost factors were unchanged?

Questions?