Problem Set 3 - Solutions

Problem 2.24

A stream of warm water is produced in a steady-flow mixing process by combining 1.0 $\frac{\text{kg}}{s}$ of cool water at 25 °C with 0.8 $\frac{\text{kg}}{s}$ of hot water at 75 °C. During mixing, heat is lost to the surroundings at a rate of 30 $\frac{\text{kJ}}{s}$. Assume the specific heat of water is constant at 4.18 $\frac{\text{kJ}}{\text{kg}\cdot K}$.

What is the temperature of the warm water stream?

SOLUTION

Mechanical Energy Balance:

Out[∘]//TraditionalForm=

$$\Delta H^t + \frac{\Delta u^2}{2} + g\,\Delta z = \dot{Q} + \dot{W}_s \Rightarrow \Delta H^t = \dot{Q}$$

Out[∘]=

$$\Delta(\dot{m}\,H) = Q$$

$$\dot{m}_3\,H_3 - \dot{m}_1\,H_1 - \dot{m}_2\,H_2 = Q$$

$$\dot{m}_3\,C_{P3}\,\Delta T_3 - \dot{m}_1\,C_{P1}\,\Delta T_1 - \dot{m}_2\,C_{P2}\,\Delta T_2 = -30$$

$$1.8\cdot 4.18\cdot(T - T_{ref}) - 1.0\cdot 4.18\cdot(25 - T_{ref}) - 0.8\cdot 4.18\cdot(75 - T_{ref}) = -30$$

$$T_{ref} = 25\ °C$$

Note: For brevity, cadets can assume constant pressure and go straight to ΔH=Q without invoking the mechanical energy balance.

Solve:

In[∘]:=
```
Tref = 25;
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In[∘]:=
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1.8 * 4.18 * (T - Tref) - 1.0 * 4.18 * (25 - Tref) - 0.8 * 4.18 * (75 - Tref) == -30 // Solve
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Out[∘]=
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{{T → 43.23498}}
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The temperature of the warm water stream is 43.235 °C. //ANS

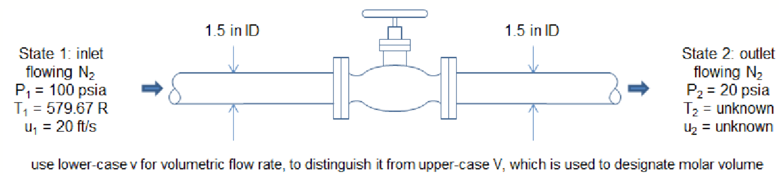
Problem 2.28

Nitrogen flows at steady state through a horizontal, insulated pipe with inside diameter of 1.5 inches. A pressure drop results from flow through a partially opened valve. Just upstream from the valve the pressure is 100 psia, the temperature is 120 °F, and the average velocity is 20 $\frac{\text{ft}}{s}$. Assume for nitrogen that $\frac{PV}{T}$ is constant, $C_V = \frac{5}{2}R$, and $C_P = \frac{7}{2}R$. Values of the gas constant R are given in App. A.

If the pressure just downstream from the valve is 20 psia, what is the temperature?

SOLUTION

Process Sketch:



Solution Outline:

State 2 has two unknowns, T_2 and u_2, requiring two independent equations.

One equation is the mechanical energy balance; the second comes from conservation of moles, or $\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$ = constant.

Since $\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$ = constant, $v_2 = \frac{P_1 v_1}{T_1} \frac{T_2}{P_2}$. With constant area, this leads to $u_2 = u_1 \frac{P_1}{P_2} \frac{T_2}{T_1}$, giving u_2 as a function of T_2.

With u_2 and $\Delta H = C_P \Delta T = C_P(T_2 - T_1)$, the mechanical energy balance leads to a single equation with one unknown (T_2).

Mechanical Energy Balance:

Out[∘]//TraditionalForm=

$$\dot{n}\,\Delta H + \frac{\dot{m}\,\Delta u^2}{2\,g_c} + \frac{\dot{m}\,g\,\Delta z}{g_c} = \dot{Q} + \dot{W}_s \Rightarrow \dot{n}\,\Delta H + \frac{\dot{n}\,MW\,\Delta u^2}{2\,g_c} = 0 \Rightarrow C_P\,\Delta T + \frac{MW\,\Delta u^2}{2\,g_c} = 0$$

Out[∘]=

$$\dot{n} = \text{molar flow } [=] \frac{\text{lbmol}}{\text{sec}}$$

$$H = \text{enthalpy } [=] \frac{\text{ft·lbf}}{\text{lbmol}}$$

$$\dot{m} = \text{mass flow } [=] \frac{\text{lbm}}{\text{sec}}$$

$$u = \text{velocity } [=] \frac{\text{ft}}{\text{sec}}$$

$$g_c = \text{conversion factor} = 32.1740 \frac{\text{ft·lbm/sec}^2}{\text{lbf}}$$

$$g = \text{gravitational acceleration constant} = 32.1740 \frac{\text{ft}}{\text{sec}^2}$$

$$\Delta z = \text{elevation change} = 0 \text{ ft}$$

Out[ ]=

$$\dot{Q} = \text{heat transfer rate} = 0\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}}$$
$$\dot{W}_s = \text{shaft work} = 0\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}}$$
$$\text{MW} = \text{molar mass} = 28.01\ \frac{\text{lbm}}{\text{lbmol}}$$
$$C_P = \text{specific heat capacity} = \frac{7}{2}\text{R}$$
$$R = \text{gas constant} = 1545\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot R}$$
$$\dot{n}\,\Delta\text{H} = \dot{n}\,C_P\,\Delta\text{T}$$

Conservation of Moles:

Out[ ]=

$$\dot{n}_1 = \dot{n}_2$$
$$\frac{P_1\,v_1}{T_1} = \frac{P_2\,v_2}{T_2}$$
$$v_2 = \frac{(P_1\,v_1)\,T_2}{T_1\,P_2}$$
$$\text{area}_1 = \text{area}_2 = \text{area}$$
$$u_2 = \frac{v_2}{\text{area}} = \frac{v_1\,P_1\,T_2}{P_2\,T_1\,\text{area}} = \frac{u_1\,\text{area}\,P_1\,T_2}{P_2\,T_1\,\text{area}} = \frac{u_1\,P_1\,T_2}{P_2\,T_1}$$

Solve:

In[ ]:= $\mathtt{u2 = 20 * \frac{100}{20} * \frac{T2}{(120 + 459.67)};}$

In[ ]:= $\frac{7}{2} * 1545. * (T2 - 579.67) + \frac{28.01}{2 * 32.174}\left(u2^2 - 20^2\right) == 0$

Out[ ]=

$5407.5\,(-579.67 + T2) + 0.4352894\left(-400 + 0.02976037\,T2^2\right) == 0$

In[ ]:= **SolveValues[% && T2 > 0, T2][[1]] - 459.67**

Out[ ]=

119.2294

The outlet temperature is 119.23 °F. //ANS

Solving with units: Note that $\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}}$ appears in each term and cancels:

In[ ]:= $\frac{7}{2} * 1545. * \frac{\text{ft} * \text{lbf}}{\text{lbmol} * \text{Rankine}} * (T2 - 579.67) * \text{Rankine} + \frac{\frac{28.01\,\text{lbm}}{\text{lbmol}}}{2 * \frac{32.174\,\text{ft} * \text{lbm}/\text{sec}^2}{\text{lbf}}}\left(u2^2 - 20^2\right) * \frac{\text{ft}^2}{\text{sec}^2} == 0$

Out[ ]=

$$\frac{5407.5\,\text{ft lbf}\,(-579.67 + T2)}{\text{lbmol}} + \frac{0.4352894\,\text{ft lbf}\left(-400 + 0.02976037\,T2^2\right)}{\text{lbmol}} == 0$$

ABET Notes for Instructor

Problem 2.38

CO_2 gas enters a water-cooled compressor at conditions P_1 = 15 psia and T_1 = 50 °F, and is discharged at conditions P_2 = 520 psia and T_2 = 200 °F. The entering CO_2 flows through a 4-inch-diameter pipe with a velocity of 20 $\frac{ft}{s}$, and is discharged through a 1-inch-diameter pipe. The shaft work supplied to the compressor is 5,360 $\frac{Btu}{lbmol}$.

Additional Information:

$H_1 = 307\ \frac{Btu}{lb_m}$ and $V_1 = 9.25\ \frac{ft^3}{lb_m}$

$H_2 = 330\ \frac{Btu}{lb_m}$ and $V_2 = 0.28\ \frac{ft^3}{lb_m}$

What is the heat-transfer rate from the compressor in $\frac{Btu}{hr}$?

SOLUTION

Solution Outline:

Use the open system energy balance and assume potential energy changes due to changes in elevation are negligible. Under these conditions, the balance simplifies as follows:

Out[∘]//TraditionalForm=

$$\Delta H + \frac{\Delta u^2}{2\, g_c} + \frac{g\, \Delta z}{g_c} = Q + W_s \Rightarrow \Delta H + \frac{\Delta u^2}{2\, g_c} = Q + W_s$$

Recognize that we are given everything except the outlet velocity and the heat duty (u_2 and Q). Also recognize that mass is conserved so that the mass flow rate in is equal to the mass flow rate out ($m_1 = m_2$).

The inlet mass flow rate can be calculated from the given inlet velocity, given inlet specific volume, and calculated inlet pipe cross-sectional area (from given inlet diameter).

This is then equal to the outlet mass flow rate by conservation of mass.

The outlet velocity u_2 can then be calculated from the mass flow rate, the given outlet specific volume and calculated outlet pipeline cross-sectional area (from given outlet diameter).

This leaves only a calculation of Q. So the strategy is to calculate u_2 from the flow rates and areas, and then use the energy balance to calculate Q.

Mechanical Energy Balance:

In[71]:=
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Clear["Global`*"]
```

In[∘]:=
```
eq5 = (H2 - H1) + 1/(2 gc) * (u2^2 - u1^2) == Q + Ws
```

Out[∘]=

$$-H1 + H2 + \frac{-u1^2 + u2^2}{2\, gc} == Q + Ws$$

Givens

In[ ]:= $H2 = 330 \frac{\text{Btu}}{\text{lbm}}$ (*outlet enthalpy*);

$H1 = 307 \frac{\text{Btu}}{\text{lbm}}$ (*inlet enthalpy*);

$V2 = 0.28 \frac{\text{ft}^3}{\text{lbm}}$ (*outlet specific volume*);

$V1 = 9.25 \frac{\text{ft}^3}{\text{lbm}}$ (*inlet specific volume*);

$Ws = 5360 \frac{\text{Btu}}{\text{lbmol}}$ (*shaft work*);

$u1 = 20 \frac{\text{ft}}{\text{sec}}$ (*inlet velocity*);

Lookups

In[ ]:= $gc = 32.174 * \frac{\text{ft} * \text{lbm} / \text{sec}^2}{\text{lbf}}$ (*conversion factor*);

$MW = 44.01 \frac{\text{lbm}}{\text{lbmol}}$ (*molar mass*);

Check eq5 to see how it has changed

In[ ]:= eq5

Out[ ]=

$$\frac{23\ \text{Btu}}{\text{lbm}} + \frac{0.0155405\ \text{lbf}\ \text{sec}^2 \left(-\frac{400\ \text{ft}^2}{\text{sec}^2} + u2^2\right)}{\text{ft lbm}} == \frac{5360\ \text{Btu}}{\text{lbmol}} + Q$$

Determine outlet velocity

Out[ ]//TraditionalForm=

$$\left(\begin{array}{c}\text{inlet}\\\text{velocity}\end{array}\right) \Rightarrow \left(\begin{array}{c}\text{inlet}\\\text{volumetric}\\\text{flow rate}\end{array}\right) \Rightarrow \left(\begin{array}{c}\text{inlet mass}\\\text{flow rate}\end{array}\right) = \left(\begin{array}{c}\text{outlet mass}\\\text{flow rate}\end{array}\right) \Rightarrow \left(\begin{array}{c}\text{outlet}\\\text{volumetric}\\\text{flow rate}\end{array}\right) \Rightarrow \left(\begin{array}{c}\text{outlet}\\\text{velocity}\end{array}\right)$$

In[ ]:= $a1 = \frac{\pi}{4} * (4. / 12)^2 * \text{ft}^2$ (*inlet area*)

Out[ ]=

$0.08726646\ \text{ft}^2$

In[ ]:= v1 = u1 * a1 (*inlet volumetric flow rate*)

Out[ ]=

$$\frac{1.745329\ \text{ft}^3}{\text{sec}}$$

In[∘]:= m1 = v1 / V1 (*inlet mass flow rate*)

Out[∘]=

$$\frac{0.1886842\ \text{lbm}}{\text{sec}}$$

In[∘]:= m2 = m1 (*oulet mass flow rate from conservation of mass*)

Out[∘]=

$$\frac{0.1886842\ \text{lbm}}{\text{sec}}$$

In[∘]:= v2 = m2 * V2 (*outlet volumetric flow rate*)

Out[∘]=

$$\frac{0.05283159\ \text{ft}^3}{\text{sec}}$$

In[∘]:= $a2 = \frac{\pi}{4} * (1. / 12)^2 * ft^2$ (*outlet area*)

Out[∘]=

$$0.005454154\ \text{ft}^2$$

In[∘]:= u2 = v2 / a2 (*outlet velocity*)

Out[∘]=

$$\frac{9.686486\ \text{ft}}{\text{sec}}$$

Check eq5 to see how it has changed

In[∘]:= eq5

Out[∘]=

$$\frac{23\ \text{Btu}}{\text{lbm}} - \frac{4.758065\ \text{ft lbf}}{\text{lbm}} == \frac{5360\ \text{Btu}}{\text{lbmol}} + Q$$

It is clear that the units are inconsistent.

We need to convert (ft·lbf) to Btu in the kinetic energy term.

We need to convert lbmol to lbm in the shaft work using MW.

The conversion factor is $\left(\frac{.000947831\ \text{Btu}}{.737562\ \text{ft*lbf}}\right)$ from Appendix A.

Re-write mechanical energy balance with conversion factors

In[∘]:= $eq6 = (H2 - H1) + \frac{1}{2\ gc} * \left(u2^2 - u1^2\right) * \frac{0.000947831\ Btu}{0.737562\ ft * lbf} == Q + \frac{Ws}{MW}$

Out[∘]=

$$\frac{22.99389\ \text{Btu}}{\text{lbm}} == \frac{121.7905\ \text{Btu}}{\text{lbm}} + Q$$

The units are now consistent.

Solve for heat Q

In[∘]:=
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Qm = SolveValues[eq6, Q][[1]]
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Out[∘]=

$$-\frac{98.79662\ \mathrm{Btu}}{\mathrm{lbm}}$$

Convert heat to required units

In[∘]:=
$$Q = Qm * m2 * \frac{3600\ \mathrm{sec}}{\mathrm{hr}}$$

Out[∘]=

$$-\frac{67108.91\ \mathrm{Btu}}{\mathrm{hr}}$$

The heat transfer rate from the compressor is -67,108.9 $\frac{\mathrm{Btu}}{\mathrm{hr}}$. //ANS

ABET Notes for Instructor

Problem 2.40

One kilogram of air is heated reversibly at constant pressure from an initial state of 300 K and 1 bar until its volume triples. Assume for air that $\frac{PV}{T} = 83.14 \frac{\text{bar}\cdot\text{cm}^3}{\text{mol}\cdot K}$ and $C_P = 29 \frac{J}{\text{mol}\cdot K}$.

Calculate W, Q, ΔU, and ΔH for the process. Report your answers in kJ.

SOLUTION

Inventory of Process Variables:

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \text{State 1} \\ P_1 = 1 \text{ bar} \\ T_1 = 300\ K \\ V_1 = 24\,942 \frac{\text{cm}^3}{\text{mol}} \end{pmatrix} \Longrightarrow \begin{pmatrix} \text{State 2} \\ P_2 = 1 \text{ bar} \\ T_2 = \text{unknown} \\ V_2 = \text{unknown} \end{pmatrix}$$

Identify the key equations:

The key to this problem is reversibility. “Heated reversibly at constant pressure” means internal & external pressures are the same and W=-PΔV.

The second key to this problem is that the pressure is constant. This means Q=ΔH and re know the definition of enthalpy change is $\Delta H = nC_P\,\Delta T$.

We were given that $\frac{PV}{T}$ = constant, which is another way of saying moles are conversed.

We were also given that $V_2 = 3\,V_1$.

Find the new molar volume V_2 and new temperature T_2:

$(*\text{use } \frac{PV}{T} = \text{constant or } \frac{P1\ V1}{T1} = \frac{P2\ V2}{T2} *)$

In[∘]:= $V1 = \frac{83.14\ \text{bar} * \text{cm}^3}{\text{mol} * K} * \frac{300\ K}{1\ \text{bar}}$

Out[∘]= $\frac{24\,942.\ \text{cm}^3}{\text{mol}}$

In[∘]:= V2 = 3 * V1

Out[∘]= $\frac{74\,826.\ \text{cm}^3}{\text{mol}}$

$(* \frac{P1\ V1}{T1} = \frac{P2\ V2}{T2} \Rightarrow \frac{T1}{P1\ V1} = \frac{T2}{P2\ V2} \Rightarrow T2 = P2\ V2 \frac{T1}{P1\ V1} \Rightarrow T2 = V2 \frac{T1}{V1} *)$

In[∘]:= T1 = 300 K;

In[ ]:= $T2 = V2 * \frac{T1}{V1}$

Out[ ]=

900. K

New Inventory of Process Variables:

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \text{State 1} \\ P_1 = 1\text{ bar} \\ T_1 = 300\ K \\ V_1 = 24\,942\ \frac{\text{cm}^3}{\text{mol}} \end{pmatrix} \Longrightarrow \begin{pmatrix} \text{State 2} \\ P_2 = 1\text{ bar} \\ T_2 = 900.\ K \\ V_2 = 74\,826\ \frac{\text{cm}^3}{\text{mol}} \end{pmatrix}$$

Determine Moles:

In[ ]:= $MW = \frac{28.97\text{ g}}{\text{mol}}$; (*molar mass of air*)

In[ ]:= $n = \frac{1\,000\text{ g}}{MW}$

Out[ ]=

34.51847 mol

Calculate the enthalpy change ΔH:

In[ ]:= $Cp = \frac{29\text{ J}}{\text{mol} * \text{K}}$;

In[ ]:= ΔH = n * Cp * (T2 - T1)

Out[ ]=

600 621.3 J

ΔH = 600,621J = 60<u>0</u>.6 kJ. //ANS

Calculate heat transfer, Q:

In[ ]:= Q = ΔH (*constant pressure*)

Out[ ]=

600 621.3 J

Q = ΔH = 600,621J = 60<u>0</u>.6 kJ. //ANS

Calculate the work, W:

```
(*W=-PΔV, constant pressure*)
```

In[ ]:= P = 1 bar;

In[∘]:= $W = -1\,\text{bar} * (V2 - V1) * \frac{1\,\text{J}}{10\,\text{cm}^3 * \text{bar}} * 1000\,\text{g} * \frac{1\,\text{mol}}{28.97\,\text{g}}$

Out[∘]=

−172 191.9 J

W = -172,192 J = -17$\underline{2}$.192 kJ. //ANS

Calculate the internal energy change ΔU:

In[∘]:= ΔU = Q + W

Out[∘]=

428 429.4 J

ΔU = 428,429 J or 42$\underline{8}$.429 kJ. //ANS