Problem 12-1
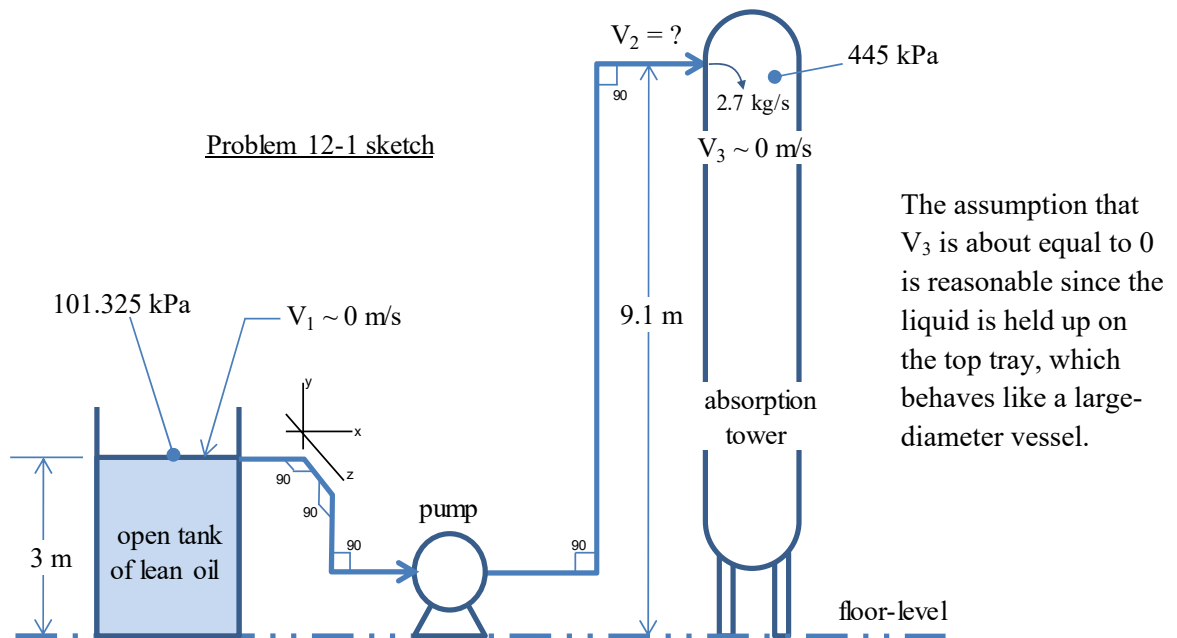
Estimate the size of the motor necessary to pump a lean oil to the top of an absorption tower operating at a pressure of 445 kPa. The oil is to be pumped from an open tank with a liquid level 3 m above the floor through 46 m of pipe with an inside diameter of 0.078 m. There are five 90° elbows in the line, and the top of the tower is 9.1 m above the floor level. A flow of 2.7 kg/s of lean oil is required. The viscosity of the oil is 15 cP, and its density is 857 kg/m³. Assume that the efficiency of the pumping system including the motor is 40%.

Solution:

Start by making a sketch of the system such as the one shown below, and then apply the mechanical energy balance.

Problem 12-1 sketch

The assumption that V_3 is about equal to 0 is reasonable since the liquid is held up on the top tray, which behaves like a large-diameter vessel.

Assuming that the fluid is incompressible, Equation 12-12 on page 492 is the most convenient and appropriate form of the mechanical energy balance:

$$W_o = g\Delta z + \Delta\left(\frac{V^2}{2\alpha}\right) + \Delta(pv) + \Sigma F = g(z_2 - z_1) + \frac{V_2}{2} + \frac{p_2 - p_1}{\rho} + \Sigma F$$

The correction factor α is assumed to be 1 (assuming turbulent flow). Also assume that the velocity of the fluid entering the pipeline is zero, so that $V_1=0$, and that the velocity in the tower is zero, so that $V_3=0$. The velocity of the fluid inside the pipe is given by

$$V_2 = \frac{\left(2.7\,\frac{kg}{s}\right)\big/\left(857\,\frac{kg}{m^3}\right)}{\pi(0.078m)^2\big/4} = 0.659\,\frac{m}{s}$$

The Reynolds number is needed to calculate the frictional losses, and is given by

$$\text{Rn} = \frac{DV_2\rho}{\mu} = \frac{0.078\text{m}\cdot 0.659\frac{\text{m}}{\text{s}}\cdot 857\frac{\text{kg}}{\text{m}^3}}{0.015\frac{\text{kg}}{\text{m}\cdot\text{s}}} = 2938.25$$

Use this value of Reynolds number to read the friction factor from Figure 12-1 on page 487, getting f=0.011.

To calculate the frictional losses, we must use Equation 12-7 on page 488. However, we must first determine the *total equivalent length* to account for the elbow, using Table 12-1:

$$L = 46\text{m} + 5\cdot 32\cdot 0.078\text{m} = 58.48\text{m}$$

$$F = \frac{2\cdot f\cdot V^2\cdot L}{D} = \frac{2\cdot 0.011\cdot\left(0.659\frac{\text{m}}{\text{s}}\right)^2\cdot 58.48\text{m}}{0.078\text{m}} = 7.163\frac{\text{m}^2}{\text{s}^2}$$

There are also frictional losses F_C due to the sudden constriction where the fluid enters the pipe. F_C for sudden constriction is given by the equation in Table 12-1 on page 490:

$$F_C = \frac{K_C\cdot V^2}{2\cdot\alpha} = \frac{0.4\cdot 1.25\cdot\left(0.659\frac{\text{m}}{\text{s}}\right)^2}{2} = 0.109\frac{\text{m}^2}{\text{s}^2}$$

Note that K_C is really a function of the big and small areas of the constriction (A_1 and A_2 in the sketch), but the maximum value is 0.4·1.25.

There are also frictional losses F_E due at the end of the pipe due to the sudden expansion where the fluid enters the column. F_E for sudden expansion is given in Table 12-1 on page 490:

$$F_e = \frac{\left(V_2 - V_3\right)^2}{2\cdot\alpha} = \frac{\left(\sim 0 - 0.659\frac{\text{m}}{\text{s}}\right)^2}{2} = 0.217\frac{\text{m}^2}{\text{s}^2}$$

Now, go back and apply the mechanical energy balance:

$$W_o = 9.8\frac{\text{m}}{\text{s}^2}\cdot\left(9.1\text{m} - 3.0\text{m}\right) + \frac{445000\text{Pa} - 101325\text{Pa}}{857\frac{\text{kg}}{\text{m}^3}} + \frac{\left(0.659\frac{\text{m}}{\text{s}}\right)^2}{2} + \left(0.109 + 7.170 + 0.217\right)\frac{\text{m}^2}{\text{s}^2} = 468.5\frac{\text{m}^2}{\text{s}^2}$$

To convert this answer to Watts, multiply by the mass flow rate:

$$W_o = 468.5\frac{\text{m}^2}{\text{s}^2}\cdot 2.7\frac{\text{kg}}{\text{s}} = 1\underline{2}65.0\text{ Watts}$$

To calculate the power input to the pump-motor system, divide by the efficiency:

$$W_{in} = \frac{1\underline{2}65.0\text{ Watts}}{0.4} = \underline{3\,1\,62.5\text{ Watts}}$$
ANS

PROBLEM SET 1

Design tool for incompressible flow in pipes　　**PTW, Problem 12-1.**
Yellow highlighting designates required **user** input.

QUANTITY	Symbol	Units	Specs	Cell Name	Formulae and notes
viscosity	μ	kg/m·s	0.015	mu	u/s (user specified)
density	ρ	kg/m^3	857	rho	u/s
flow rate, specified	m_v	m^3/s	0.00315	mv	u/s
diameter, specified	D	m	0.07800	D	u/s
pipe cross-sectional area	A	m^2	0.00478	A	s/c (spreadsheet calculation), $\pi D^2/d$
velocity	v	m/s	0.65933	v	s/c, volumetric flow rate divided by cross-sectional area
Reynolds number	Re	dimensionless	2938	Re	s/c, $Dv\rho/\mu$

Optimal Diameter Section

diameter, optimal, SI	D_{opt}	m	0.0589	Dopt	s/c, nested IF statements using eqns 9-76 to 9-79, PTW, p. 404
area, optimal	A_{opt}	m^2	0.0027	Aopt	s/c, $\pi D_{opt}^2/4$
velocity, optimal	v_{opt}	m/s	1.1576	vopt	s/c, volumetric flow divided by A_{opt}
Reynolds number, optimal	Re_{opt}	dimensionless	3893	Reopt	s/c, $D_{opt}v_{opt}\rho/\mu$

Potential Energy Section *(Static Head (Gravitation))*

elevation 1	h_1	m	3	hinit	u/s
elevation 2	h_2	m	9.1	hfinal	u/s
gravitational acceleration constant	g	m/s^2	9.8	g	u/s
gravitational potential head	g·Δh	m^2/s^2 or N·m/kg	59.78	not named	s/c

External Pressure Section *(Static Head (Pressure))*

pressure 1	p_1	Pa	101325	pinit	u/s
pressure 2	p_2	Pa	445000	pfinal	u/s
pressure head	$\Delta(pv)$	m^2/s^2 or N·m/kg	401.0210035	not named	s/c

Kinetic Energy Section *(Dynamic Head (Kinetic E))*

initial fluid velocity	v_1	m/s	0.00000	Vinit	s/c, from cell E10
final fluid velocity	v_2	m/s	0.65933	Vfinal	s/c, from cell E10
velocity correction	α	dimensionless	1	alpha	n/a
kinetic energy head	$\Delta(v^2/2\alpha)$	m^2/s^2 or N·m/kg	0.217359032	not named	s/c

Pipe Friction Section *(Dynamic Head (Friction))*

Colebrook relation for f			-0.000278739	not named	s/c, eq. 12-6, p. 487
pipe length	L	m	46	L	u/s
pipe roughness	ε	m	0.0000457	eps	u/s, fig. 12-1, p. 487
fanning friction factor, turbulent	f	dimensionless	0.01108	not named	s/c; run solver to minimize blue cell with respect to cell E40
fanning friction factor, laminar	f	dimensionless	0.00545	not named	s/c, eq. 12-4, p. 487
fanning friction factor	f	dimensionless	0.01108	f	s/c, IF statement to choose laminar or turbulent
frictional loss due to straight pipe	F_{pipe}	m^2/s^2 or N·m/kg	5.7	not named	s/c, eq. 12-7, p. 488
losses due to straight pipe + fittings	$F_{pipe+fittings}$	m^2/s^2 or N·m/kg	7.2	not named	s/c, eq. 12-7, p. 488, with equivalent length L+Le
total friction head	$F_{pipe+fittings+ec}$	m^2/s^2 or N·m/kg	7.5	not named	s/c, eq. 12-7, p. 488, with expansions and constrictions

Pump Work Section

total head (pump work @ 100% efficiency)	P_o	m^2/s^2 or N·m/kg	469	not named	Energy balance; summation of all gray cells
total head in Watts	P_o	W	1265	not named	s/c, m^2/s^2·kg/s
efficiency	η	dimensionless	0.4	eta	u/s
pump work at specified efficiency	P	m^2/s^2 or N·m/kg	1171	not named	s/c, divide by efficiency
pump work at specified efficiency	P	W	3163	not named	s/c, divide by efficiency

Frictional Losses Due to Fittings and Valves

Description of fittings	Le/D		number		
45° elbows	15	dimensionless		not named	u/s, Table 12-1, p. 490, fittings, vavles, etc.
90° elbows, standard radius	32	dimensionless	5	not named	u/s
90° elbows, medium radius	26	dimensionless		not named	u/s
90° elbows, long sweep	20	dimensionless		not named	u/s
90° elbows, square	60	dimensionless		not named	u/s
180° bends, close return	75	dimensionless		not named	u/s
180° bends, medium radius	50	dimensionless		not named	u/s
Tee (used as elbow, entering run)	60	dimensionless		not named	u/s
Tee (used as elbow, entering branch)	90	dimensionless		not named	u/s
Couplings	0	dimensionless		not named	u/s
Unions	0	dimensionless		not named	u/s
Gate valves, open	7	dimensionless		not named	u/s
Globe valves, open	300	dimensionless		not named	u/s
Angle valves, open	170	dimensionless		not named	u/s
Water meters, disk	400	dimensionless		not named	u/s
Water meters, piston	600	dimensionless		not named	u/s
Water meters, impulse-wheel	300	dimensionless		not named	u/s
Equivalent length of fittings	Le	m	12.48	Le	s/c, summation of all losses due to fittings

Frictional Losses Due to Enlargements and Constrictions

Area on narrow side of constriction/expansion	A_{narrow}	m^2	0.00478	not named	s/c, calculated earlier in cell E9
Area on wide side of constriction/expansion	A_{wide}	m^2	4.77836	not named	u/s, used a ratio of 1000 in this case
Ratio of narrow area to wide area	A_{narrow}/A_{wide}	dimensionless	0.001	not named	s/c, A_{wide} is not known in this case, so the ratio is made small
Velocity on narrow side of constriction/expansion	v_{narrow}	m/s	0.65933	not named	s/c, volumetric flow divided by area of narrow side
Velocity on wide side of constriction/expansion	v_{wide}	m/s	0.00066	not named	s/c, volumetric flow divided by area of wide side
Losses due to enlargements	F_e	m^2/s^2 or N·m/kg	0.21692	not named	s/c, Table 12-1, p. 490, sudden enlargement
Losses due to constrictions	F_c	m^2/s^2 or N·m/kg	0.10859	not named	s/c, Table 12-1, p. 490, sudden constriction
Number of enlargements	---	dimensionless	1	not named	u/s, typical value is 1 for pipe emptying into a vessel
Number of constrictions	---	dimensionless	1	not named	u/s, typical value is 1 for pipe removing liquid from a vessel
Losses due to enlargements and constrictions	F_e+F_c	m^2/s^2 or N·m/kg	0.32552	not named	s/c

CHEMCAD Solution

Problem 12-1

"Lean oil" modelled with 1-propanol

Equip. No.	1
Name	
Method	2
Diameter m	0.0780
Calculated ID m	0.0780
Pipe length m	46.0000
Roughness factor m	4.5720e-005
Elevation change m	6.1000
Pressure drop Pa	57079.0039
Reynolds # liq	3008.6821
Fric factr liq	0.0450
Avg density kg/m3	853.9491
Calc. velocity m/sec	0.6617
Min. velocity m/sec	0.4158
DP friction Pa	5995.2720
DP elevation Pa	51083.7344
Output press. Pa	445000.0000
DP/100ft, psi	0.4768
Liquid flow kg/sec	2.7000
Liquid density kg/m3	853.9491
Liq viscosity cP	14.6487
Surface tension N/m	0.0285
EL. fittings m	9.5862
Total ELength m	55.5862
small dia./large dia.	0.0010
Standard elbow 90 de	5
Sudden contraction	1
Sudden expansion	1
Friction fac. model	1
Incl. expansion fac.	1
Pipe wall cond. W/m-	51.2818
Inclination angle	7.6204

Stream No.	1	2	3
Name	"lean oil"		
- - Overall - -			
Mass flow kg/sec	2.7000	2.7000	2.7000
Temp K	231.5000	232.1012	232.1012
Pres Pa	101325.0000	502079.0000	445000.0000
Enth kJ/sec	-13982.	-13978.	-13978.
Entropy kJ/K/sec	-21.96	-21.94	-21.94
GibbsEnergy kJ/sec	-8898.2	-8885.8	-8885.9
- - Liquid only - -			
Actual dens kg/m3	854.4092	853.9491	853.9490
Visc cP	14.98	14.65	14.64
Flow rates in kg/sec			
Ethanol	0.0000	0.0000	0.0000
N-Propanol	2.7000	2.7000	2.7000

Equip. No.	2
Name	
Output pressure Pa	502079.0000
Efficiency	0.4000
Calculated power	3168.3052
(J/sec)	
Calculated Pout Pa	502079.0000
Head m	47.8290
Vol. flow rate m3/h	11.3763
Mass flow rate kg/sec	2.7000

This problem would be fairly easy to solve in ChemCAD if the chemical composition of the lean oil were known. However, this was not given. The key to the problem is therefore to identify a reasonable component that matches the density and viscosity of the lean oil. This is sometimes referred to as using a "psuedo-component."

After some searching, I settled on 1-propanol at 231.5 K, which is a pretty close match. The given density was 857 kg/m3 and the viscosity was given as 15 cP. The corresponding values for 1-propanol at 231.5 K are 854.4 and 14.98.

The calculated pump power of 3168 J/s is in good agreement with the value of 3163 W from the by-hand calculations. The small difference is due to small differences in viscosity and density and the Le/D values used by CHEMCAD.

The pipeline tool allows the user to set fluid properties to their own values and then override the CHEMCAD values. Trying this and changing the density and viscosity to the values given in the problem statement, changes the Reynolds number from 3009 to 2938, in excellent agreement with the by-hand calculations. However, this changes the pump power from 3168 W to 3170, or an increase of 2 W. So there is an effect, but it is small and in the wrong direction.

Problem 12-2

What is the pressure loss when 2.14 kg/s of pure benzene at 40 ºC flows through a 21-m length of straight pipe with an inside diameter of 0.0409 m? The pipeline contains six 90º elbows, one tee used as an elbow (equivalent resistance equal to 60 pipe diameters), one globe valve, and one gate valve. The density of benzene is 849 kg/m³, and its viscosity at 40 ºC is 5×10⁻⁴ Pa·s.

Solution:

Start by making a sketch of the system, and then apply the frictional loss Equation 12-7, p. 488.
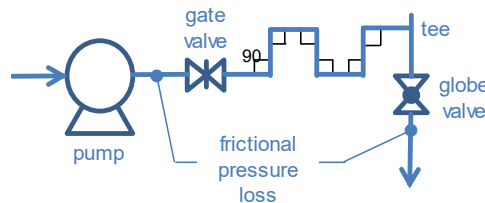
Start with velocity inside the pipe, which is given by

$$V_2 = \frac{\left(2.14 \, \frac{kg}{s}\right) / \left(849 \, \frac{kg}{m^3}\right)}{\pi (0.0409 m)^2 / 4} = 1.919 \, \frac{m}{s}$$

The Reynolds number is given by

$$Rn = \frac{DV\rho}{\mu} = \frac{0.0409 m \cdot 1.919 \, \frac{m}{s} \cdot 849 \, \frac{kg}{m^3}}{0.0005 \, \frac{kg}{m \cdot s}} = 133,271$$

Use the Reynolds number, read the friction factor from Figure 12-1 on page 487, getting f=0.0055. The *total equivalent length* accounts for the elbows, tee, and valves (Table 12-1):

$$L = 21m + (6 \cdot 32 + 60 + 300 + 7) \cdot 0.0409 m = 43.86 m$$

$$F = \frac{2 \cdot f \cdot V^2 \cdot L}{D} = \frac{2 \cdot 0.0055 \cdot \left(1.919 \, \frac{m}{s}\right)^2 \cdot 43.86 m}{0.0409 m} = 43.440 \, \frac{m^2}{s^2}$$

To convert this answer to Pa, multiply by the density:

ANS may vary slightly depending on how you read the "Moody Plot"

$$F = 43.440 \, \frac{m^2}{s^2} \cdot 849 \, \frac{kg}{m^3} = \underline{\underline{36,881 \, Pa}}$$
ANS

CHEMCAD answer is 37.1 kPa (set roughness correctly!)

Excel answer is 43.5 N·m/kg 36.9 kPa when multiplied by ρ (essentially the same)

Design tool for incompressible flow in pipes PTW 12-2

Yellow highlighting designates required **user** input.

QUANTITY	Symbol	Units	Specs	Cell Name	Formulae and notes
viscosity	μ	kg/m·s	0.0005	mu	u/s (user specified)
density	ρ	kg/m^3	849	rho	u/s
flow rate, specified	m_v	m^3/s	0.002521	mv	u/s
diameter, specified	D	m	0.04090	D	u/s
pipe cross-sectional area	A	m^2	0.00131	A	s/c (spreadsheet calculation), $\pi D^2/d$
velocity	v	m/s	1.91883	v	s/c, volumetric flow rate divided by cross-sectional area
Reynolds number	Re	dimensionless	133259	Re	s/c, $Dv\rho/\mu$

Optimal Diameter Section

diameter, optimal, SI	D_{opt}	m	0.0488	Dopt	s/c, nested IF statements using eqns 9-76 to 9-79, PTW, 5th ed, p. 404
area, optimal	A_{opt}	m^2	0.0019	Aopt	s/c, $\pi D_{opt}^2/4$
velocity, optimal	v_{opt}	m/s	1.3452	vopt	s/c, volumetric flow divided by A_{opt}
Reynolds number, optimal	Re_{opt}	dimensionless	111577	Reopt	s/c, $D_{opt}v_{opt}\rho/\mu$

Static Head (Gravitation) — Potential Energy Section

elevation 1	h_1	m	0	hinit	u/s
elevation 2	h_2	m	0	hfinal	u/s
gravitational acceleration constant	g	m/s^2	9.8	g	u/s
gravitational potential head	$g \cdot \Delta h$	m^2/s^2 or N·m/kg	0	not named	s/c

Static Head (Pressure) — External Pressure Section

pressure 1	p_1	Pa	101325	pinit	u/s
pressure 2	p_2	Pa	101325	pfinal	u/s
pressure head	$\Delta(pv)$	m^2/s^2 or N·m/kg	0	not named	s/c

Dynamic Head (Kinetic E) — Kinetic Energy Section

initial fluid velocity	v_1	m/s	1.91883	Vinit	s/c, from cell E10
final fluid velocity	v_2	m/s	1.91883	Vfinal	s/c, from cell E10
velocity correction	α	dimensionless	1	alpha	n/a
kinetic energy head	$\Delta(v^2/2\alpha)$	m^2/s^2 or N·m/kg	0	not named	s/c

Dynamic Head (Friction) — Pipe Friction Section

Colebrook relation for f			-2.1104E-07	not named	s/c, eq. 12-6, p. 487
pipe length	L	m	21	L	u/s
pipe roughness	ε	m	0.0000457	eps	u/s, fig. 12-1, p. 487
fanning friction factor, turbulent	f	dimensionless	0.00551	not named	s/c; run solver to minimize **blue cell** with respect to cell E40
fanning friction factor, laminar	f	dimensionless	0.00012	not named	s/c, eq. 12-4, p. 487
fanning friction factor	f	dimensionless	0.00551	f	s/c, IF statement to choose laminar or turbulent
frictional loss due to straight pipe	F_{pipe}	m^2/s^2 or N·m/kg	20.8	not named	s/c, eq. 12-7, p. 488
losses due to straight pipe + fittings	$F_{pipe+fittings}$	m^2/s^2 or N·m/kg	43.5	not named	s/c, eq. 12-7, p. 488, with equivalent length L+Le
total friction head	$F_{pipe+fittings+ec}$	m^2/s^2 or N·m/kg	43.5	not named	s/c, eq. 12-7, p. 488, with expansions and constrictions

Pump Work Section

total head (pump work @ 100% efficiency)	P_o	m^2/s^2 or N·m/kg	43.5	not named	Energy balance; summation of all gray cells
total head in Watts	P_o	W	93	not named	s/c, m^2/s^2·kg/s
efficiency	η	dimensionless	0.4	eta	u/s
pump work at specified efficiency	P	m^2/s^2 or N·m/kg	109	not named	s/c, divide by efficiency
pump work at specified efficiency	P	W	233	not named	s/c, divide by efficiency

Frictional Losses Due to Fittings and Valves

Description of fittings	Le/D		number		
45° elbows	15	dimensionless		not named	u/s, Table 12-1, p. 490, fittings, vavles, etc.
90° elbows, standard radius	32	dimensionless	6	not named	u/s
90° elbows, medium radius	26	dimensionless		not named	u/s
90° elbows, long sweep	20	dimensionless		not named	u/s
90° elbows, square	60	dimensionless		not named	u/s
180° bends, close return	75	dimensionless		not named	u/s
180° bends, medium radius	50	dimensionless		not named	u/s
Tee (used as elbow, entering run)	60	dimensionless	1	not named	u/s
Tee (used as elbow, entering branch)	90	dimensionless		not named	u/s
Couplings	0	dimensionless		not named	u/s
Unions	0	dimensionless		not named	u/s
Gate valves, open	7	dimensionless	1	not named	u/s
Globe valves, open	300	dimensionless	1	not named	u/s
Angle valves, open	170	dimensionless		not named	u/s
Water meters, disk	400	dimensionless		not named	u/s
Water meters, piston	600	dimensionless		not named	u/s
Water meters, impulse-wheel	300	dimensionless		not named	u/s
Equivalent length of fittings	Le	m	22.86	Le	s/c, summation of all losses due to fittings

Frictional Losses Due to Enlargements and Constrictions

Area on narrow side of constriction/expansion	A_{narrow}	m^2	0.00131	not named	s/c, calculated earlier in cell E9
Area on wide side of constriction/expansion	A_{wide}	m^2	1.31382	not named	u/s, used a ratio of 1000 in this case
Ratio of narrow area to wide area	A_{narrow}/A_{wide}	dimensionless	0.001	not named	s/c, A_{wide} is not known in this case, so the ratio is made small
Velocity on narrow side of constriction/expansion	v_{narrow}	m/s	1.91883	not named	s/c, volumetric flow divided by area of narrow side
Velocity on wide side of constriction/expansion	v_{wide}	m/s	0.00192	not named	s/c, volumetric flow divided by area of wide side
Losses due to enlargements	F_e	m^2/s^2 or N·m/kg	1.83727	not named	s/c, Table 12-1, p. 490, sudden enlargement
Losses due to constrictions	F_c	m^2/s^2 or N·m/kg	0.91974	not named	s/c, Table 12-1, p. 490, sudden constriction
Number of enlargements	---	dimensionless	0	not named	u/s, typical value is 1 for pipe emptying into a vessel
Number of constrictions	---	dimensionless	0	not named	u/s, typical value is 1 for pipe removing liquid from a vessel
Losses due to enlargements and constrictions	F_e+F_c	m^2/s^2 or N·m/kg	0.00000	not named	s/c

CHEMCAD Solution
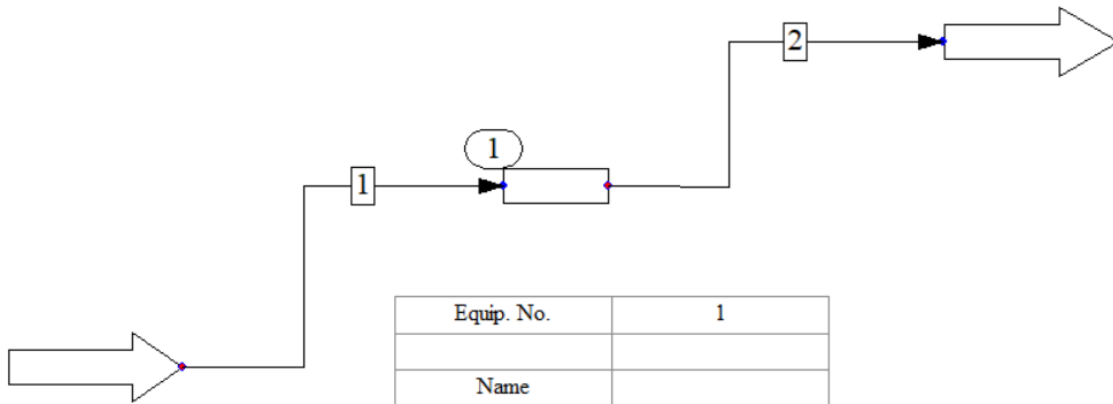
Equip. No.	1
Name	
Method	2
Diameter m	0.0409
Calculated ID m	0.0409
Pipe length m	21.0000
Roughness factor m	4.5720e-05
Pressure drop Pa	37076.4922
Reynolds # liq	133237.8125
Fric factr liq	0.0222
Avg density kg/m3	849.0000
Calc. velocity m/sec	1.9185
DP friction Pa	37076.4922
Output press. Pa	64248.5078
DP/100ft, psi	3.7498
Stream prop. flag	1
Liquid flow kg/sec	2.1400
Liquid density kg/m3	849.0000
Liq viscosity Pa-sec	0.0005
Surface tension N/m	0.0262
EL. fittings m	22.7108
Total ELength m	43.7108
Gate valve	1
Glb seat flatBevelPlug	1
Standard elbow 90 de	6
Std T, flow-thr brnch	1
Friction fac. model	1
Incl. expansion fac.	1
Pipe wall cond. W/m	51.2818

CHEMCAD gives a pressure drop of 37,077 Pa, compared to 36,881 Pa from the by-hand calculation. CHEMCAD allows input of the exact same properties, so the difference is not due to the property estimations in CHEMCAD. There is a difference in friction factor. CHEMCAD uses 0.0222 and in Lesson 2, I showed 0.022, which accounts for the difference.