

Problem 5.2

A Carnot engine receives 250 kJ s⁻¹ of heat from a heat source reservoir at 525 deg C and rejects heat to a heat-sink reservoir at 50 deg C. What are the power developed and the heat rejected?

Solution

Use equation 5.7 to compute efficiency, and then use 5.6 to get the work, based on the heat received.

```
In[*]:= 
$$\eta = 1 - \frac{50 + 273.15}{525 + 273.15}$$

Out[*]= 0.595126229406

In[*]:= 
$$Q_H = 250 \frac{\text{kJ}}{\text{sec}}; (*\text{heat in is positive}*)$$


$$W = -\eta * Q_H (*\text{work out is negative}*)$$

Out[*]= 
$$- \frac{148.781557351 \text{ kJ}}{\text{sec}}$$


(*Power developed is -148.782 kJ/sec = -148.782 kW. //ANS*)

Use eq 2.3 (first law) with ΔU=0 to compute heat ejected, that is QC+QH+W=0:

In[*]:= 
$$Q_C = -Q_H - W$$

Out[*]= 
$$- \frac{101.218442649 \text{ kJ}}{\text{sec}}$$


In[*]:= 
$$\text{Solve}[Q_C + 250 + (-148.782) == 0, Q_C]$$

Out[*]= { { Q_C → -101.218 } }
```

(*Heat rejected is -101.218 kJ/sec = -101.218 kW. //ANS*)

Problem 5.6

Which is the more effective way to increase the thermal efficiency of a Carnot engine: to increase T_H with T_C constant, or to decrease T_C with T_H constant? For a real engine, which would be the more practical way?

Solution

Use derivatives to answer this question.

$$\eta = 1 - \frac{T_C}{T_H}$$

$$\left(\frac{\partial \eta}{\partial T_H} \right)_{T_C} = -T_C \cdot \left(\frac{-1}{T_H^2} \right) = \frac{T_C}{T_H} \cdot \frac{1}{T_H}$$

$$\left(\frac{\partial \eta}{\partial T_C} \right)_{T_H} = -\frac{1}{T_H}$$

The derivative with respect to T_C is negative, meaning that as T_C increases, η decreases. Or as T_C decreases, η increases. The problem statement asks about decreasing T_C , which increases η .

Now look at the other derivative. The derivative with respect to T_H is positive, meaning that as T_H increases, η increases. The problem statement asks about increasing T_H , which increases η .

Now compare the “size” of the derivatives to see which one causes a larger change. By premise $T_C < T_H$.

$$\text{Since } T_C < T_H, \left| \left(\frac{\partial \eta}{\partial T_H} \right)_{T_C} \right| < \left| \left(\frac{\partial \eta}{\partial T_C} \right)_{T_H} \right|, \text{ because } \frac{T_C}{T_H} \cdot \frac{1}{T_H} < \frac{1}{T_H}$$

Therefore, decreasing T_C with T_H constant is more effective at increasing thermal efficiency η than increasing T_H with T_C constant. This is a general solution and applies at all T_C and $T_H > 0$.

In a real engine, it is more practical to increase T_H because T_C is normally the ambient temperature, which is difficult to change, while T_H can be changed by changing fuel or changing the engine operating conditions.

Mathematica operations:

$$\text{In}[1]:= \eta = 1 - \frac{T_C}{T_H};$$

$$\partial_{T_H} \eta$$

$$\partial_{T_C} \eta$$

$$\text{Out}[2]= \frac{T_C}{T_H^2}$$

$$\text{Out}[3]= -\frac{1}{T_H}$$

Problem 5.8

With respect to 1 kg of liquid water:

(a) Initially at 0 deg C, it is heated to 100 deg C by contact with a heat reservoir at 100 deg C. What is the entropy change of the water? What is the entropy change of the heat reservoir? What is ΔS_{total} ?

(b) Initially at 0 deg C, it is first heated to 50 deg C by contact with a heat reservoir at 50 deg C, and then heated to 100 deg C by contact with a heat reservoir at 100 deg C. What is ΔS_{total} ?

(c) Explain how the water might be heated from 0 deg C to 100 deg C so that $\Delta S_{\text{total}} = 0$.

Solution to Part (a)

```
In[11]:= m = 1; (*kg*)
Cp = 4.184; (*kJ/kg*degC*)
T2 = 100 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)
ΔT = T2 - T1; (*degC*)
Qhot = m * Cp * ΔT; (*kJ*)
ΔShot =  $\frac{-Q_{\text{hot}}}{T_2}$  (*kJ/K*)
ΔSwater = m *  $\int_{T_1}^{T_2} \frac{C_p}{T} dT$  (*kJ/K*)
ΔStotal = ΔShot + ΔSwater (*kJ/K*)

Out[17]=
-1.12126490687

Out[18]=
1.30523808247

Out[19]=
0.183973175596

(*The ΔS of the hot reservoir is -1.12123 kJ/K//ANS*)
(*The ΔS of the cold reservoir is 1.30524 kJ/K//ANS*)
(*The total entropy change is is 0.18397 kJ/K//ANS*)
```

Solution to Part (b)

```

In[20]:= m = 1; (*kg*)
Cp = 4.184; (*kJ/kg*degC*)
T2 = 100 + 273.15; (*K*)
Tb = 50 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)

(*First Step*)
ΔT1b = Tb - T1; (*degC*)
Qhot1b = m * Cp * ΔT1b; (*kJ*)
ΔShot1b =  $\frac{-Q_{hot1b}}{T_b}$ 
ΔSwater1b =  $m \int_{T_1}^{T_b} \frac{C_p}{T} dT$ 

(*Second Step*)
ΔTb2 = T2 - Tb; (*degC*)
Qhotb2 = m * Cp * ΔTb2; (*kJ*)
ΔShotb2 =  $\frac{-Q_{hotb2}}{T_2}$  (*kJ/K*)
ΔSwaterb2 =  $m \int_{T_b}^{T_2} \frac{C_p}{T} dT$  (*kJ/K*)

(*Total, kJ/K //ANS*)
ΔStotal = ΔShot1b + ΔSwater1b + ΔShotb2 + ΔSwaterb2

Out[27]=
-0.647377378926

Out[28]=
0.703311642572

Out[31]=
-0.560632453437

Out[32]=
0.601926439898

Out[33]=
0.0972282501067

(*ΔS of the hot reservoir in step 1 is -.64738 kJ/K//ANS*)
(*ΔS of the cold reservoir in step 1 is .70331 kJ/K//ANS*)
(*ΔS of the hot reservoir in step 2 is -.56063 kJ/K//ANS*)
(*ΔS of the cold reservoir in step 2 is .60192 kJ/K//ANS*)
(*The total entropy change is is 0.097228 kJ/K//ANS*)

```

Solution to Part (c)

The reversible heating of the water requires an infinite number of heat reservoirs covering the range of temperatures from 273.15 to 373.15 K, each one exchanging an infinitesimal quantity of heat with the water and raising its temperature by a differential increment.

Problem 5.10

An ideal gas, $C_p = (7/2)R$, is heated in a steady-flow heat exchanger from 70 deg C to 190 deg C by another stream of the same gas, which enters at 320 deg C. The flow rates of the two streams are the same, and heat losses from the exchanger are negligible.

- (a) Calculate the molar entropy changes of the two gas streams for both parallel and countercurrent flow in the exchanger.
- (b) What is ΔS_{total} in each case?
- (c) Repeat (a) and (b) for countercurrent flow if the heating stream enters at 200 deg C.

Solution

Use the integrated form of Equation 5.10, with constant C_p and assuming constant pressure. The outlet temperature of the hot gas in both co-current and counter-current flow is 200 deg C.

(*Level 1 enthalpy balance gives $mC_p\Delta T_c = -mC_p\Delta T_h$ *)
(*Both m and C_p cancel, so $\Delta T_c = -\Delta T_h$ *)
(* $190 - 70 = -(T_{\text{out}} - 320)$ *)

In[1]:= Solve[(190 - 70) == - (Tout - 320)]

Out[1]= { { Tout -> 200 } }

In[2]:= (*Gas constant and heat capacity*)
R = 8.314; (*J/mol*K*)
Cp = (7 / 2) R;

Solution, Part (a)

In[4]:= (*Cold-side temperatures, K*)
190 + 273.15
70 + 273.15

Out[4]= 463.15

Out[5]= 343.15

$$\Delta S_{\text{cold}} = C_p * \text{Log}\left[\frac{(463.15)}{(343.15)}\right] - \text{Log}\left[\frac{P}{P}\right]$$

Out[6]= 8.726304331

$$(*\Delta S_{\text{cold}} = 8.726 \frac{\text{J}}{\text{mol}\cdot\text{K}} // \text{ANS}*)$$

In[7]:= (*Hot-side temperatures, K*)
(200 + 273.15)
(320 + 273.15)

Out[7]= 473.15

Out[8]= 593.15

$$\Delta S_{\text{hot}} = C_p * \text{Log}\left[\frac{(473.15)}{(593.15)}\right] - \text{Log}\left[\frac{P}{P}\right]$$

Out[9]= -6.57738825

```
(*ΔShot = -6.577  $\frac{\text{J}}{\text{mol}\cdot\text{K}}$  //ANS*)
(*Co-current and counter-current have the same answers*)
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Solution, Part (b)

$\Delta S_{\text{cold}} + \Delta S_{\text{shot}}$

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Out[10]=
2.148916081
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(*ΔStotal = 2.149  $\frac{\text{J}}{\text{mol}\cdot\text{K}}$  //ANS*)
```

Solution, Part (c)

```
In[11]:= (*190-70==-(Thout-200) *)
Solve[(190 - 70) == -(Thout - 200)]
```

```
Out[11]=
{{Thout -> 80}}
```

```
(*Cold-side temperatures, K*)
(*Cold-side temperatures do not change*)
(*ΔScold = 8.726  $\frac{\text{J}}{\text{mol}\cdot\text{K}}$  //ANS*)
```

```
In[15]:= (*Hot-side temperatures*)
(80 + 273.15)
(200 + 273.15)
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Out[15]=
353.15
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```
Out[16]=
473.15
```

```
In[17]:= ΔShot = Cp * Log[ $\frac{(353.15)}{(473.15)}$ ] - Log[ $\frac{P}{p}$ ]
```

```
Out[17]=
-8.512026883
```

```
(*ΔShot = -8.512  $\frac{\text{J}}{\text{mol}\cdot\text{K}}$  //ANS*)
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$\Delta S_{\text{cold}} + \Delta S_{\text{shot}}$

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Out[18]=
0.2142774483
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```
(*ΔStotal = 0.214  $\frac{\text{J}}{\text{mol}\cdot\text{K}}$  //ANS*)
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