Problem 2.24

A stream of warm water is produced in a steady-flow mixing process by combining 1.0 kg s^{-1} of cool water at 25 degC with 0.8 kg s^{-1} of hot water at 75 deg C. During mixing, heat is lost to the surroundings at the rate of 30 kJ s^{-1}. what is the temperature of the warm water stream? Assume the specific heat of water is constant at 4.18 kJ kg^{-1} K^{-1}.

Solution

The enthalpy balance on the mixer looks like:

$$H_1^t + H_2^t - H_3^t + Q = 0$$

$$\dot{m}_1 C_{P1} \Delta T_1 + \dot{m}_2 C_{P2} \Delta T_2 - \dot{m}_3 C_{P3} \Delta T_3 - 30 = 0$$

$$1.0 \cdot C_P \cdot (25 - T_{ref}) + 0.8 \cdot C_P \cdot (75 - T_{ref}) - 1.8 \cdot C_P \cdot (T - T_{ref}) = 30$$

```
eq1 = 1.0 * 4.18 * (25 - Tref) + 0.8 * 4.18 * (75 - Tref) - 1.8 * 4.18 * (T - Tref) == 30;
```

Assign a value to Tref. It can be anything because it cancels.

```
Tref = 0;
Solve[eq1, T]
{{T → 43.235}}
(*//ANS, degC*)
```

Simple Solution:

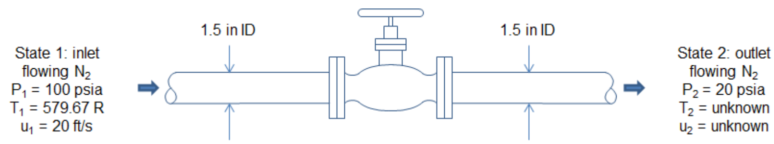
You can just ignore the Tref terms and write eq1 without them like in CH485:

```
eq2 = 1.0 * 4.18 * 25 + 0.8 * 4.18 * 75 - 1.8 * 4.18 * T == 30;
Solve[eq2, T]
{{T → 43.235}}
(*//ANS, degC*)
```

Problem 2.28

Nitrogen flows at steady state through a horizontal, insulated pipe with inside diameter of 1.5 (in). A pressure drop results from flow through a partially opened valve. Just upstream from the valve the pressure is 100 (psia), the temperature is 120 (degF), and the average velocity is 20 (ft)(s)$^{-1}$. If the pressure just downstream from the valve is 20 (psia), what is the temperature? Assume for nitrogen that PV/T is constant, Cv=(5/2)R, and Cp=(7/2)R. (Values of R are given in App. A.)

Solution



use lower-case v for volumetric flow rate, to distinguish it from upper-case V, which is used to designate molar volume

The value of v1 (volumetric flow rate to inlet) was calculated from the given velocity (u1) and the cross-sectional area (below). Once v1 is known, vw (volumetric flow rate from outlet) can be calculated from Pv/T=constant (from PV/T=constant). The velocity in the outlet (u2) can then be calculated from the volumetric flow rate and the area.

Since state 2 has two unknowns, T2 and u2, another way of summarizing this is that we must recognize that we need two independent equations, those being PV/T=constant and the enthalpy balance. The idea is to use Pv/T to get the enthalpy balance reduced to an equation with only one unknown, and then solve for the unknown. The enthalpy balance is:

$$\Delta H + \frac{\Delta u^2}{2g_c} + \frac{g\Delta z}{g_c} = Q + W_s \Rightarrow \Delta H + \frac{\Delta u^2}{2g_c} = 0 \Rightarrow C_P\Delta T = -\frac{\Delta u^2}{2g_c}$$

The heat capacity term is a function only of T2. As will be shown below, the velocity term is also a function of T2.

```
(*MECHANICAL ENERGY BALANCE*)
```

In[∘]:= $eq4 = \frac{7}{2} * R * (T2 - T1) == -\frac{1}{2\,gc}(u2^2 - u1^2) * MW;$

```
(*INLET*)
```

In[∘]:= $T1 = (120 + 459.67);$ (*degR*)

$P1 = 100;$ (*psia*)

$u1 = 20;$ (* $\frac{ft}{s}$ *)

$area = \frac{\pi}{4} * \left(\frac{1.5}{12}\right)^2;$ (* =0.0122718 ft^2*)

$v1 = u1 * area$ (* v1=u1*area=20 $\frac{ft}{s}$ *0.0122718 ft^2 =0.245437 $\frac{ft^3}{s}$ *)

Out[∘]= 0.245437

```
(*OUTLET*)
```

```
In[]:= P2 = 20; (*psia*)
       v2 = v1 * (P1/P2) (T2/T1) (*same units as v1*)
       u2 = u1 * (v2/v1) (*u2*A2*ρ2=u1*A1*ρ1; u2/v2=u1/v1; same units as u1*)
Out[]= 0.00211704 T2
Out[]= 0.172512 T2
```

Observe that outlet velocity u2 is a function of T2 only. When u2 is added back to the enthalpy balance, the result is one equation with one unknown (T2).

```
(*OTHER INFORMATION*)
In[]:= R = 1545.; (*gas constant with units of ft*lbf/(lbmol*degR)*)
       gc = 32.1740; (*ft*lbm/s²/lbf*)
       MW = 28.; (*lb/lbmol*)

(*MECHANICAL ENERGY BALANCE*)
In[]:= eq4
Out[]= 5407.5 (-579.67 + T2) == -0.435134 (-400 + 0.0297604 T2^2)
In[]:= Solve[eq4 && T2 > 0, T2]
Out[]= {{T2 → 578.9}}
In[]:= 578.9 - 459.67 (*Outlet temperature, degF //ANS*)
Out[]= 119.23

(*The outlet temperature is 119.23 degF //ANS*)
```

Problem 2.38

Carbon dioxide gas enters a water-cooled compressor at conditions P_1 = 15 (psia) and T_1 = 50 (degF), and is discharged at conditions P_2 = 520 (psia) and T_2 = 200 (degF). The entering CO_2 flows through a 4-inch-diameter pipe with a velocity of 20 (ft) $(s)^{-1}$, and is discharged through a 1-inch-diameter pipe. The shaft work supplied to the compressor is 5,360 (Btu) (lb mol)$^{-1}$. What is the heat-transfer rate from the compressor in (Btu) (hr)$^{-1}$?

Additional Information:
H_1 = 307 (Btu) $(lb_m)^{-1}$ and V_1 = 9.25 $(ft)^3$ $(lb_m)^{-1}$
H_2 = 330 (Btu) $(lb_m)^{-1}$ and V_2 = 0.28 $(ft)^3$ $(lb_m)^{-1}$

Solution

Use the open system energy balance assuming potential energy changes due to elevation are negligible. Under these conditions, the balance reduces to:

$$\Delta H + \frac{\Delta u^2}{2g_c} + \frac{g\Delta z}{g_c} = Q + W_s \quad \Rightarrow \quad \Delta H + \frac{\Delta u^2}{2g_c} = Q + W_s$$

Recognize that we are given everything except the outlet velocity and the heat duty. The outlet velocity can be calculated from the specific volume and flow rates that are given. This leaves only a calculation of Q. So the strategy is to calculate u2 from the flow rates and areas, and then use the energy balance to calculate Q.

```
(*MECHANICAL ENERGY BALANCE*)

In[∘]:= eq5 = (H2 - H1) + 1/(2 * gc) * (u2^2 - u1^2) * .000947831/.737562 == Q + Ws/MW
  (* .000947831Btu/.737562ft*lbf conversion factors from Appendix A*);

In[∘]:= 1/((ft*lbm/s^2)/lbf) * (ft/s)^2 * Btu/(ft * lbf)

Out[∘]= Btu/lbm

(*INLET*)

In[∘]:= H1 = 307; (* Btu/lbm, given*)
area1 = π/4 * (4./12)^2 ;(*cross-sectional area in ft^2*)
u1 = 20; (*velocity in ft/s, given*)
v1 = u1 * area1; (*volumetric flow rate in ft^3/s *)
V1 = 9.25;(*specific volume in ft^3/lbm, given*)
m1 = v1/V1 * 3600 (*mass flow rate in lbm/hr*)

Out[∘]= 679.263

(*OUTLET*)
```

In[∘]:=
```
H2 = 330; (*Btu/lbm, given*)
m2 = m1; (*in lbm/hr from conservation of mass*)
area2 = π/4 * (1./12)^2 ;(* =0.00545415 ft^2*)
V2 = 0.28;(*specific volume in ft^3/lbm, given*)
u2 = ((m2/3600) * V2)/area2 (*velocity in ft/s*)
```

Out[∘]= 9.68649

In[∘]:= eq5

Out[∘]= $23 - \frac{0.196729}{gc} == Q + \frac{Ws}{MW}$

```
(*OTHER INFORMATION*)

gc = 32.1740; (*(ft*lbm/s^2)/lbf, lookup on page ix*)
MW = 44.01; (*lbm/lbmol, lookup on page 652*)
Ws = 5360;(*Btu/lbmol, given*)
```

In[∘]:= eq5

Out[∘]= 22.9939 == 121.791 + Q

In[∘]:= ans = Solve[eq5, Q]

Out[∘]= {{Q → -98.7966}}

In[∘]:= Q = Q /. ans[[1]]

Out[∘]= -98.7966

```
(*DIMENSIONS*)
```

In[∘]:=
```
(*Dimensions of Q are Btu/lbm from dimensions of H*)
(*Dimensions of Q are Btu/lbm also from kinetic energy term*)
```
$$\frac{lbf}{ft * lbm / s^2} * \left(\frac{ft}{s}\right)^2$$

Out[∘]= $\frac{ft\ lbf}{lbm}$

In[∘]:= $\frac{lbf}{ft * lbm / s^2} * \left(\frac{ft}{s}\right)^2 * \frac{Btu}{ft * lbf}$

Out[∘]= $\frac{Btu}{lbm}$

In[ ]:=
```
(*Convert Q from Btu/lbm to Btu/hr using m1 (in lbm/hr) *)
Q * m1
```

Out[ ]= -67108.9

```
(*The heat transfer rate from the compressor is -67108.9 Btu/hr*)
(*//ANS*)
```

Problem 2.40

One kilogram of air is heated reversibly at constant pressure from an initial state of 300 K and 1 bar until its volume triples. Calculate W, Q, ΔU, and ΔH for the process. Assume for air that (PV/T)=83.14 (bar)(cm)^3(mol)$^{-1}$(K)$^{-1}$ and Cp=29(J)(mol)$^{-1}$(K)$^{-1}$. Report your answers in kJ.

Solution

The key to this problem is “reversibility.” “Heated reversibly at constant pressure” means internal & external pressures are the same and W=PΔV.

Make a sketch of the process

State 1:		State 2:
P_1 = 1 bar	$\longrightarrow$	P_2 = 1 bar
T_1 = 300 K		T_2 = ??? K
V_1 = 24,942 cm³/mol		V_2 = ??? cm³/mol

Find the new molar volume V2, the new temperature T2, and the moles, n.

```
(*use PV/T=constant*)
```

In[∘]:= $\mathbf{eq1} = \dfrac{1\,\text{bar} * V}{300\,K} == \dfrac{83.14\,\text{bar} * \text{cm}^3}{\text{mol} * K};$

In[∘]:= **Solve[eq1, V]**

Out[∘]= $\left\{\left\{V \to \dfrac{24942.\ \text{cm}^3}{\text{mol}}\right\}\right\}$

In[∘]:= **V1 = V /. %[[1]][[1]]**

Out[∘]= $\dfrac{24942.\ \text{cm}^3}{\text{mol}}$

In[∘]:= **V2 = 3 * V1**

Out[∘]= $\dfrac{74826.\ \text{cm}^3}{\text{mol}}$

In[∘]:= **T1 = 300 K;**

In[∘]:= $T2 = \dfrac{T1}{V1}(V2)$

Out[∘]= 900. K

State 1:		State 2:
P_1 = 1 bar	$\longrightarrow$	P_2 = 1 bar
T_1 = 300 K		T_2 = 900. K
V_1 = 24,942 cm³/mol		V_2 = 74,826 cm³/mol

In[∘]:= $MW = \dfrac{28.97\,g}{\text{mol}};$ (*molar mass of air*)

In[∘]:= $n = \dfrac{1000\,g}{MW}$

Out[∘]= 34.5185 mol

Calculate enthalpy change, ΔH

In[ ]:= $Cp = \frac{29\ J}{mol * K};$

In[ ]:= $\Delta H = n * Cp * (T2 - T1)$

Out[ ]= 600 621. J

(* ΔH=600,621J = 600.6 kJ //ANS*)

Calculate heat, Q

In[ ]:= $Q = \Delta H$

Out[ ]= 600 621. J

(* Q = ΔH =600,621J = 600.6 kJ //ANS*)

Calculate work, W

(*W=-PΔV*)

$P = 1\ bar * \frac{\frac{10^5\ kg}{m*s^2}}{bar}$ (*Conversion factor in Appendix A*)

Out[ ]= $\frac{100\,000\ kg}{m\ s^2}$

$W = -P * (V2 - V1) * n * \frac{1\ m^3}{10^6\ cm^3} * \frac{1\ J}{\frac{1\ kg*m^2}{s^2}}$ (*factors in Appendix A*)

Out[ ]= −172 192. J

(* W = -172,192 J = -172.192 kJ //ANS*)

Internal energy

In[ ]:= $\Delta U = Q + W$

Out[ ]= 428 429. J

(*ΔU = 428,429 J or 428.429 kJ //ANS*)