

# CH365 Chemical Engineering Thermodynamics

## Lesson 12 Applications of the Virial Equation

Professor Andrew Biaglow

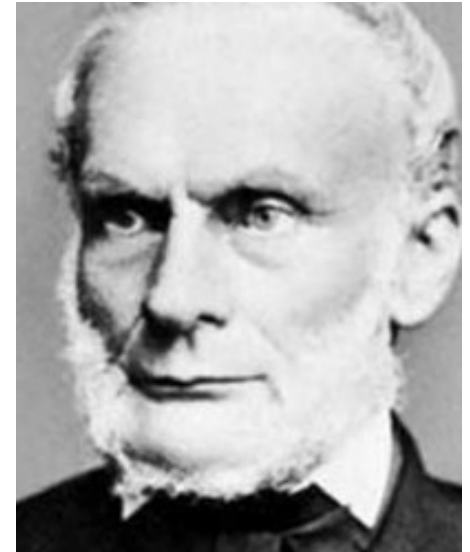
The word “virial” is derived from *vis*, the Latin word for force, strength, power, or energy, and was given its present technical definition by Rudolf Clausius in 1870. The virial theorem relates the total kinetic and potential energies of a system of particles to average of the attractive forces acting on them.

The virial equation of state, the most general function relating PVT properties of fluids, was first proposed by Professor H.K. Onnes. It is a direct application of the virial theorem to chemical systems.

Clausius, RJE (1870), "On a Mechanical Theorem Applicable to Heat". Philosophical Magazine, Series 4, Vol. 40, No. 265, pp. 122–127.

Onnes, HK (1902), "Expression of State of Gases and Liquids by Means of Series," KNAW Proceedings, 4, 1901-1902, Amsterdam, pp. 125-147.

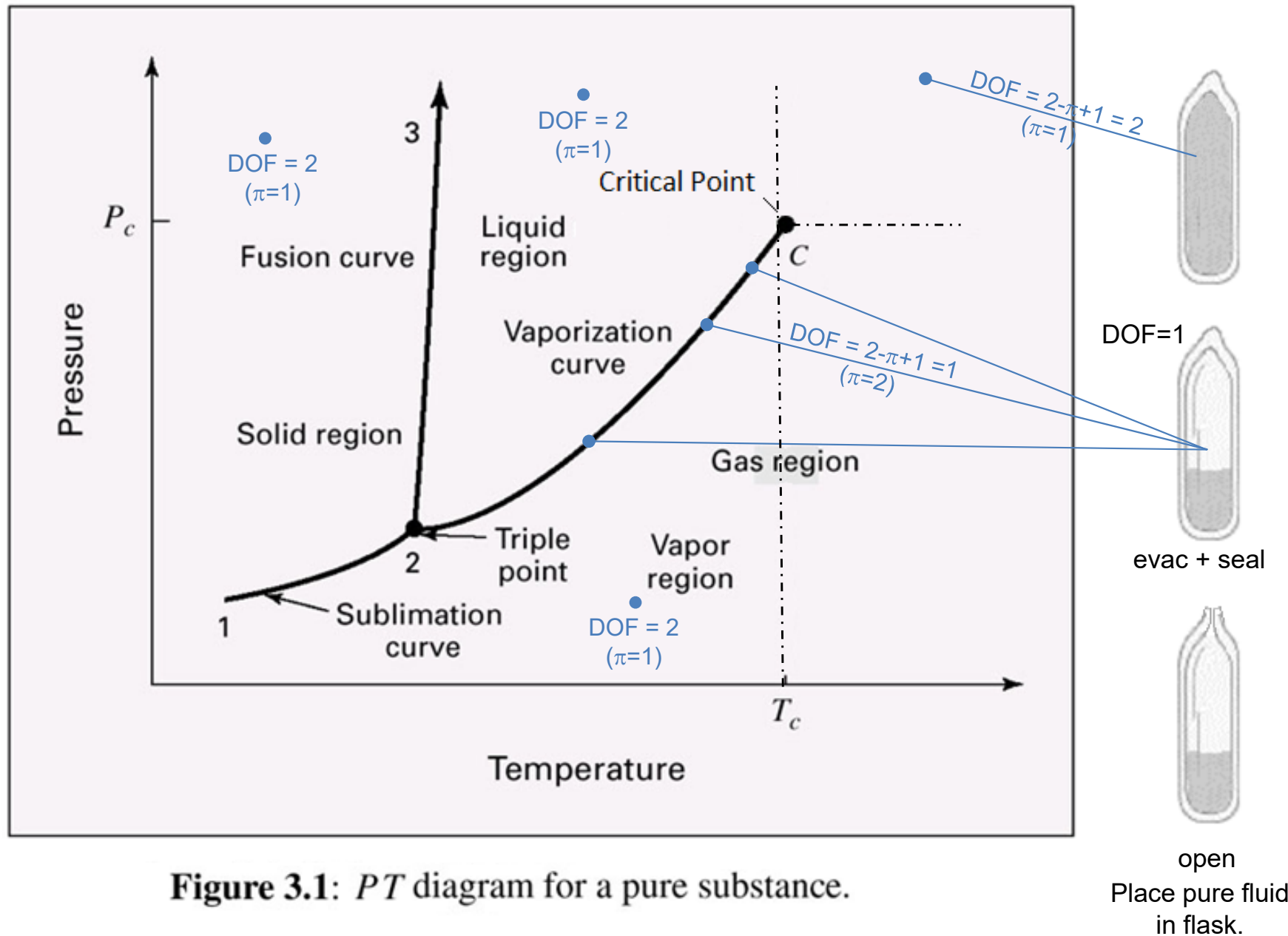
**Rudolph Julius Emanuel Clausius** (1822-1888) was a German physicist and mathematician and is one of the founders of modern thermodynamics. He re-stated Sadi Carnot's "Carnot Cycle" principle in terms of the 2<sup>nd</sup> Law of Thermodynamics.



**Heike Kamerlingh Onnes** (1853-1926) was a Dutch physicist and Nobel laureate. He used the Hampson-Linde cycle to investigate how materials behave when cooled to nearly absolute zero and liquified helium for the first time in 1908. He is also credited with discovering superconductivity.



The Hampson-Linde cycle used to liquify air and is similar to the common vapor-liquid compression cycle.



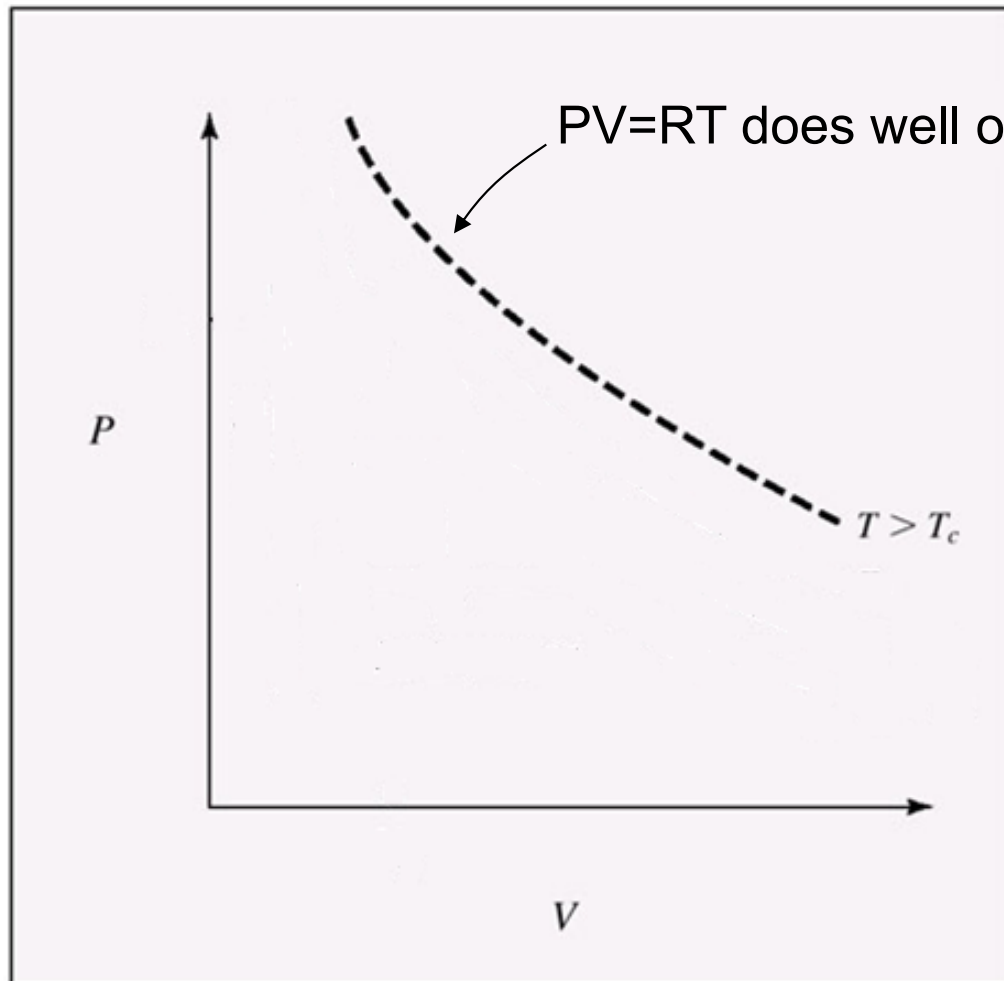
**Figure 3.1:**  $PT$  diagram for a pure substance.

**Critical point** – ( $T_c, P_c$ ) - highest temperature and pressure at which a pure species is observed to exist in vapor/liquid equilibrium.

**Vapor-Liquid Separation Processes** – always operate between triple and critical points.

# Ideal Gas Equation of State

$$PV = RT \Rightarrow P = \frac{\text{const}}{V}$$



Ideal gas equation of state does very well at predicting volumetric properties when  $T \gg T_c$

“Compressibility” form of ideal gas equation:

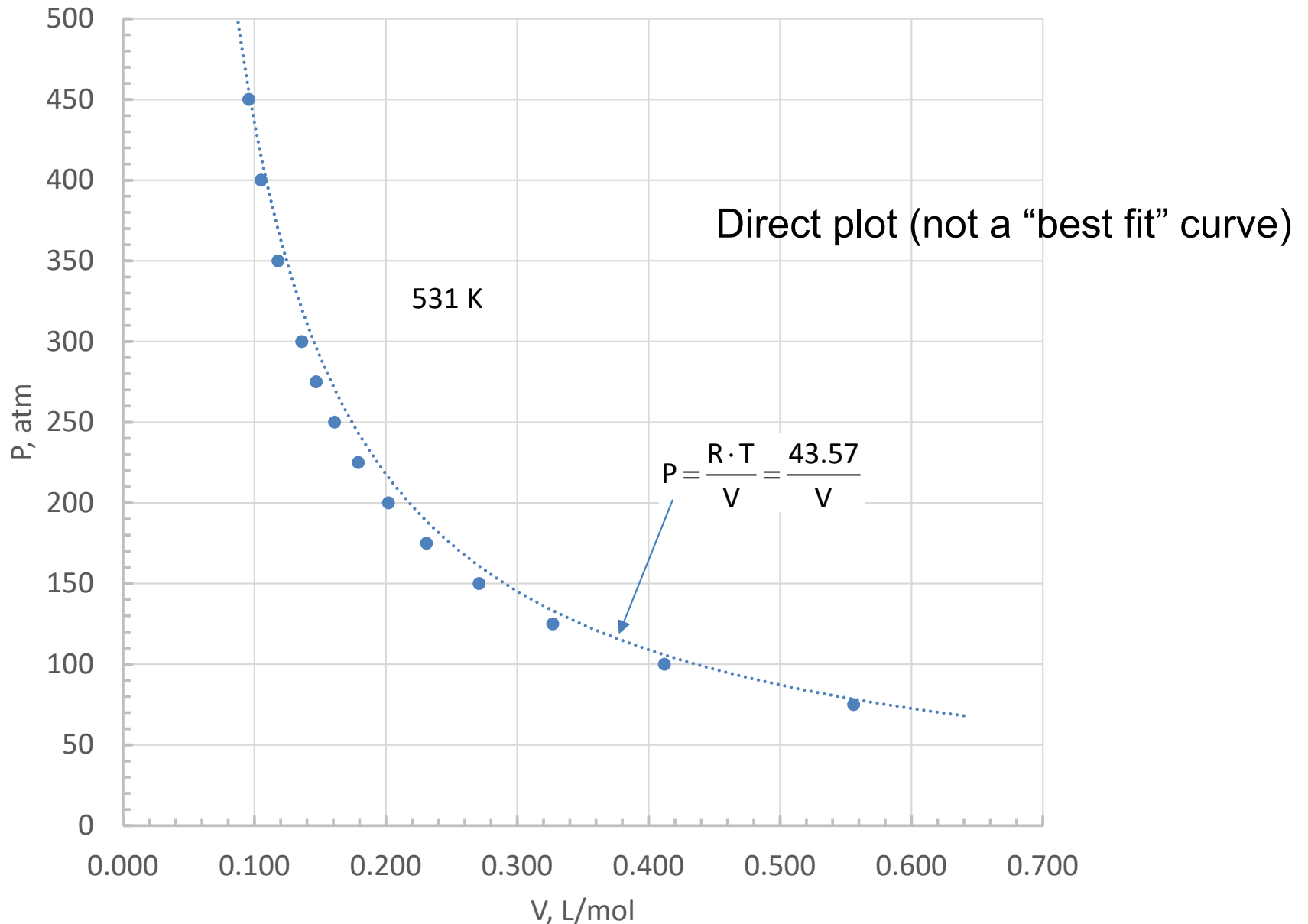
$$Z = \frac{PV}{RT} = 1$$

$Z$  is defined as the compressibility factor

# Experimental CO<sub>2</sub> Isotherms

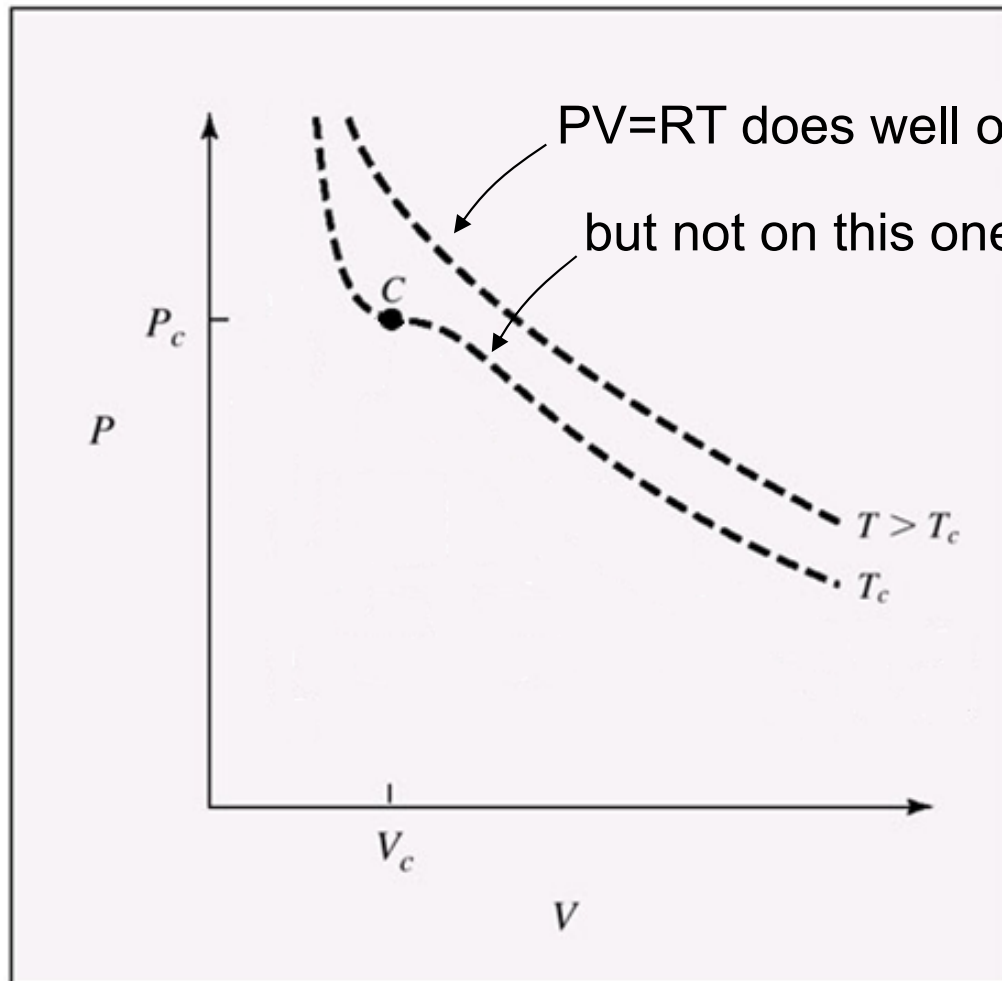
Slide 5

Data from J.L. Pauley and Elwyn H. Davis, "PVT Isotherms of Real Gases,"  
*Journal of Chemical Education*, Volume 63, Number 6, pages 466-469



# Problem with Ideal Gas EOS

$$PV = RT \Rightarrow P = \frac{\text{const}}{V}$$



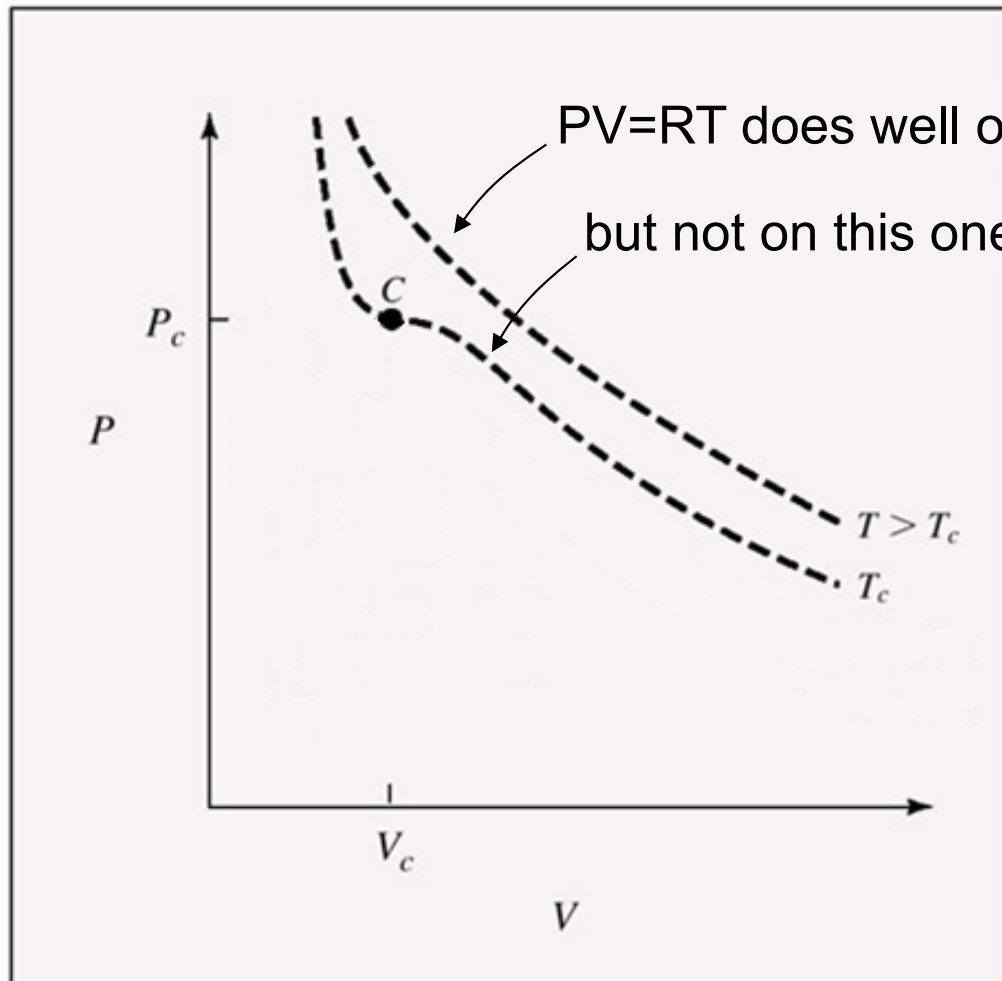
$$Z = \frac{PV}{RT} = \frac{V}{V_{ig}}$$

Lowering the  $T$  to near the critical temperature induces an inflection point in the curve.

Further decreases below critical temperature shows a bifurcation in the data. This situation is best handled with cubic and other more advanced EOS's.

# Problem with Ideal Gas EOS

$$PV = RT \Rightarrow P = \frac{\text{const}}{V}$$



Improved model:

$$PV = a + bP + cP^2 + dP^3 \dots$$

$$PV = a(1 + B'P + C'P^2 + D'P^3 \dots)$$

$$b = aB', \quad c = aC', \quad d = aD', \quad \text{etc.}$$

"a" is found by experiment to be the same function of  $T$  for all species

Constants  $b$ ,  $c$ ,  $d$ , etc., are species-dependent and functions of  $T$ .

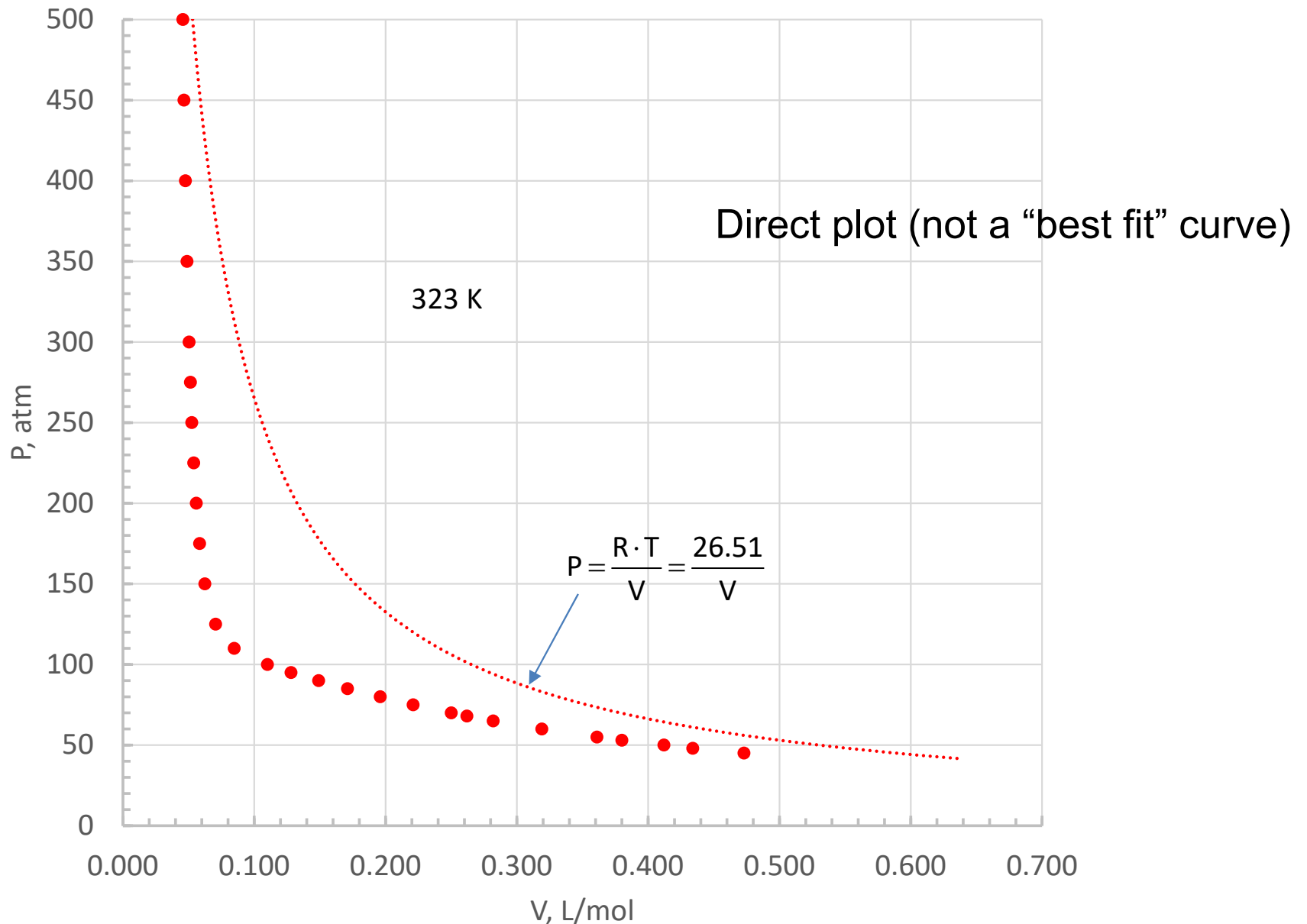
The value of  $b$  is found from high-level theory to depend on IMFs between pairs of molecules.

The constant  $c$  depends on IMFs between groups of 3 molecules (3-body IMF),  $d$  between 4, etc.

# Experimental CO<sub>2</sub> Isotherms

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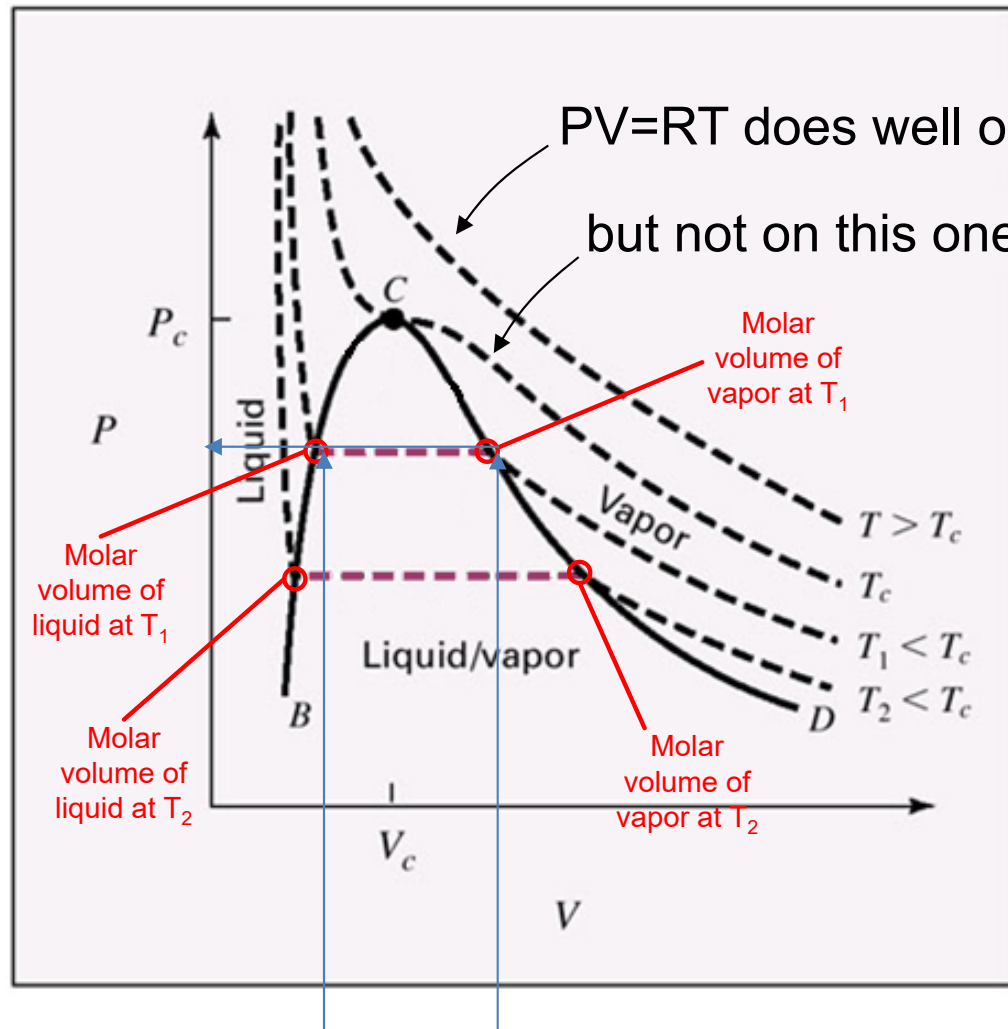
Data from J.L. Pauley and Elwyn H. Davis, "PVT Isotherms of Real Gases,"  
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# Improved Equations of State

## Virial Equation of State – First Improved Model



Improved model:

$$PV = a + bP + cP^2 + dP^3 \dots$$

$$PV = a(1 + B'P + C'P^2 + D'P^3 \dots)$$

$$b = aB', \quad c = aC', \quad d = aD', \quad \text{etc.}$$

“a” is found by experiment to be the same function of T for all species

Constants b, c, d, etc., are species-dependent and functions of T.

The value of b is found from theory to depend on IMFs between pairs of molecules.

The constant c depends on IMFs between groups of 3 molecules (3-body IMF), d between 4, etc.

# Applicability of Truncated Forms

Very useful for cases where convergence is rapid – 2 or 3 terms in the expansion  
(gases and vapors at low to moderate pressures)

All isotherms approach  $Z=1$  as  $P \rightarrow 0$

Limit of  

$$\frac{PV}{RT} = \frac{a + bP + \dots}{RT}$$
 as  $P \rightarrow 0$   
 $a/RT \rightarrow 1$

Relatively little is known about virial coefficients above C.

Use of higher order equations using D, E, etc., are uncommon

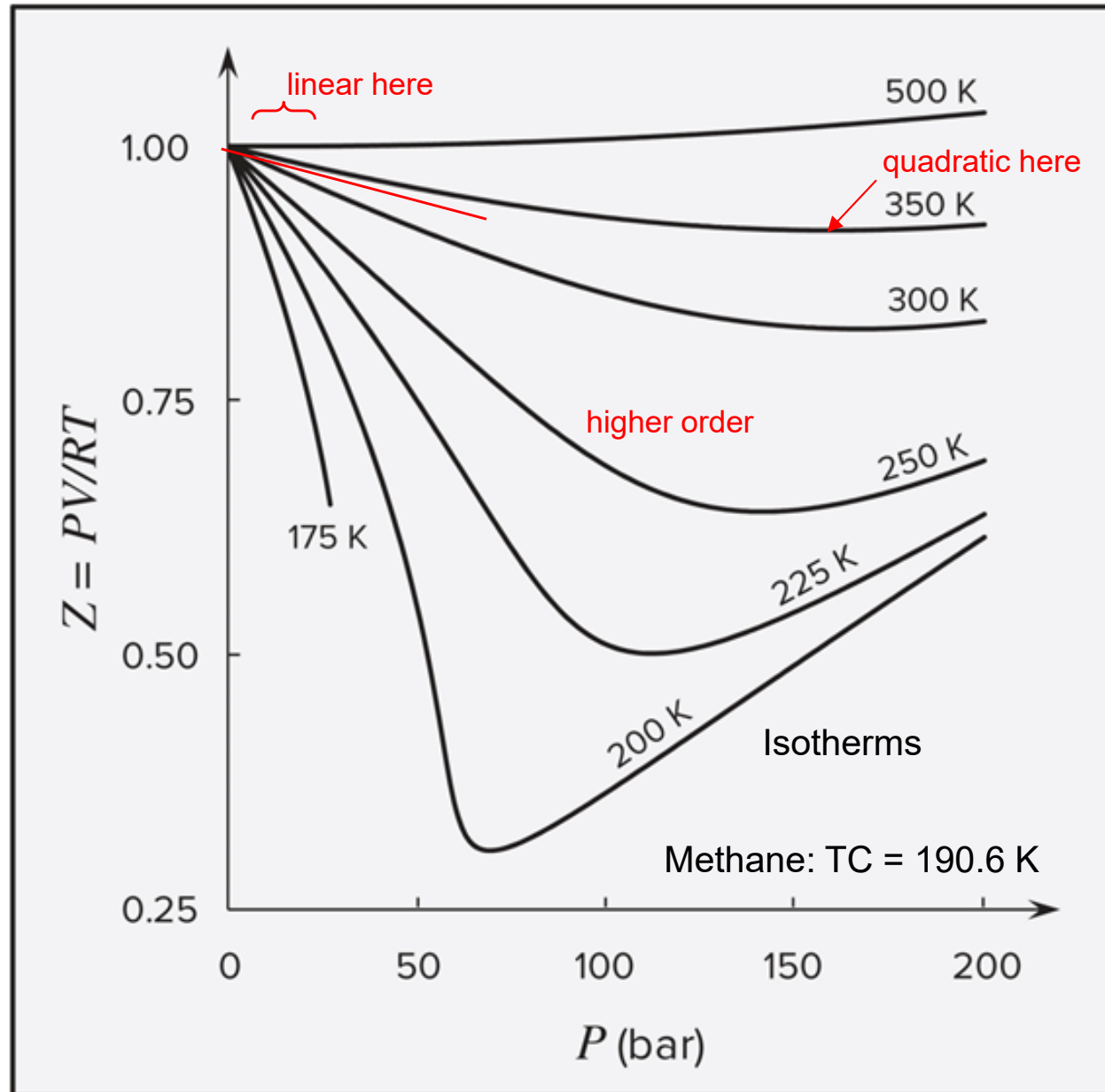
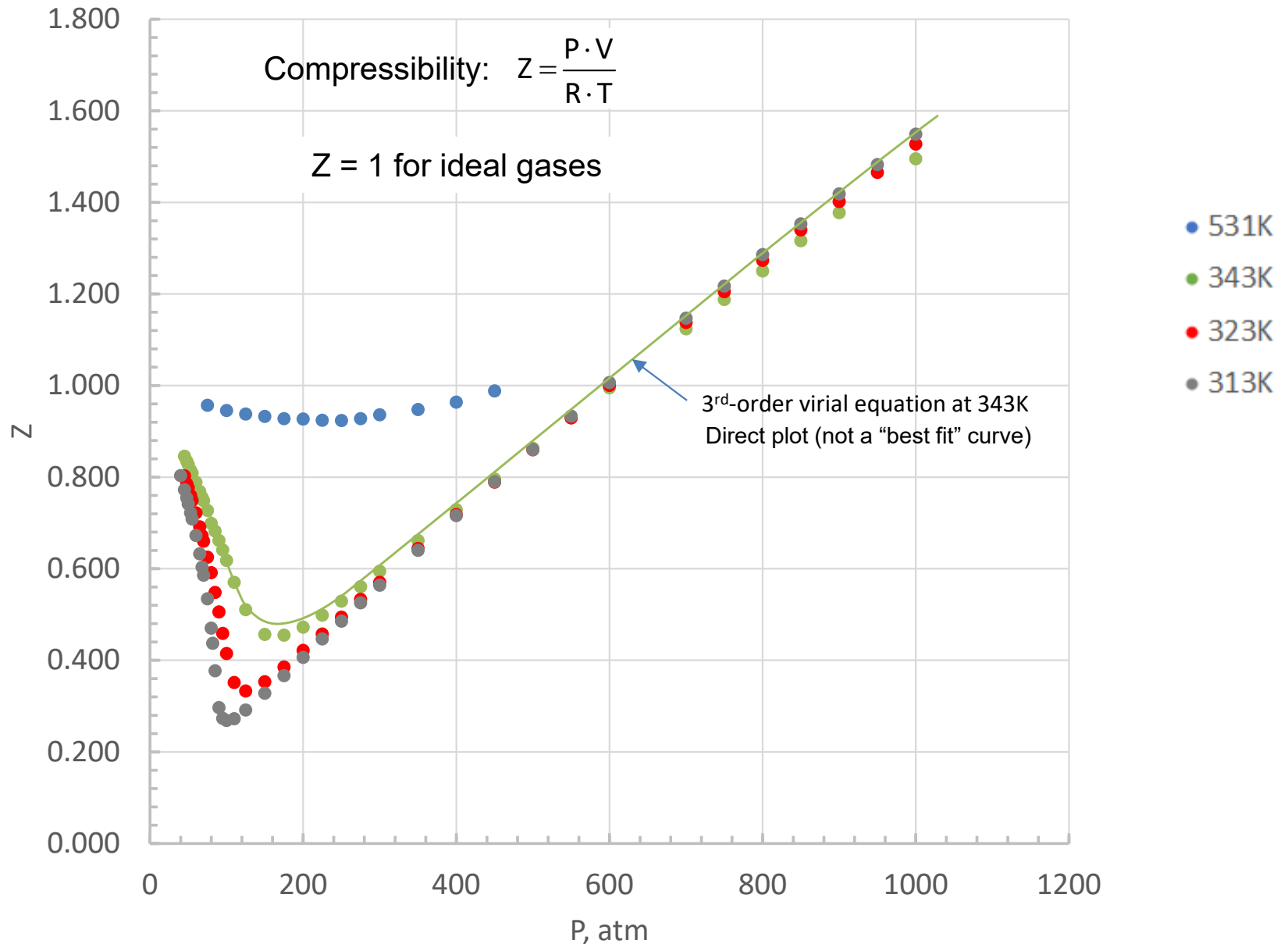


Figure 3.7: Compressibility-factor graph for methane, p. 93.

# Experimental CO<sub>2</sub> Isotherms

Slide 11

Data from J.L. Pauley and Elwyn H. Davis, "PVT Isotherms of Real Gases,"  
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# Auxiliary Forms of Virial Equations

Infinite Series – Truncate to 2 or 3 terms

$$Z = 1 + B'P + C'P^2 + D'P^3 + \dots \quad (3.33)$$

$$Z = 1 + \frac{B}{V} + \frac{C}{V^2} + \frac{D}{V^3} + \dots \quad (3.34)$$

Gases and vapors at low to moderate pressures

$$Z = 1 + \frac{BP}{RT} = 1 + \frac{B}{V} \quad (3.36, 3.37)$$

2-term truncation preferred form (linear zone on slide 10)

Gases and vapors below critical point

$$Z = 1 + \frac{B}{V} + \frac{C}{V^2} \quad (3.38)$$

3-term truncation preferred form (higher-order forms on slide 10)

(3.12 is far superior to 3.11)  
(for iterative calculations)

$$B' = \frac{B}{RT} \quad (3.35a)$$

$$C' = \frac{C - B^2}{(RT)^2} \quad (3.35b)$$

$$D' = \frac{D - 3BC + 2B^3}{(RT)^3} \quad (3.35c)$$

# Questions