
Problem Set 1 - Solutions

Problem 1.4

At what absolute temperature do the Celsius and Fahrenheit temperature scales give the same numerical value? What is the value?

SOLUTION

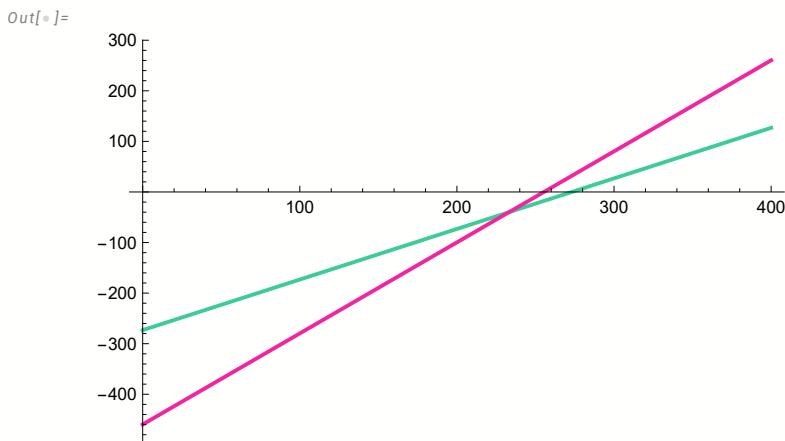
Write a function for converting Kelvins into °C. Then write a second function converting Kelvins into °F.

Plot the two functions with Kelvins as the independent variable. The plot shows the intersection.

Setting the two functions equal to each other and solving for the intersection gives -40 °C or °F at 233.15 K.

```
In[1]:= (*Calculate °C and °F from Kelvins *)
(*Variable x is a dummy variable representing absolute temperature*)
c[x_] = x - 273.15;
f[x_] = (x - 273.15) * 9/5 + 32;
```

```
In[2]:= Plot[{c[x], f[x]}, {x, 0, 400}]
```



```
In[3]:= Solve[c[x] == f[x]]
```

Out[3]=

$$\{ \{ x \rightarrow 233.15 \} \}$$

```
In[4]:= c[233.15]
```

Out[4]=

$$-40.$$

```
In[5]:= f[233.15]
```

Out[5]=

$$-40.$$

The Celsius and Fahrenheit temperature scales give the same numerical value of -40.00 °C or -40.00 °F at 233.15K. //ANS

Problem 1.6

Pressures up to 3,000 bar are measured with a dead-weight gauge. The piston diameter is 4 mm. What is the approximate mass in kg of the weights required?

SOLUTION

Calculate the area of the piston:

$$\text{In[1]:= } \frac{\pi}{4} \left(4 \text{ mm} * \frac{1 \text{ m}}{1000 \text{ mm}}\right)^2$$

Out[1]=

$$0.00001256637 \text{ m}^2$$

Calculate the force from the given pressure and the calculated area, knowing that pressure is force per unit area:

$$\text{In[2]:= } 3000 \text{ bar} \frac{\frac{10^5 \text{ N}}{\text{m}^2}}{\text{bar}} * %$$

Out[2]=

$$3769.911 \text{ N}$$

Equate the force to mass*g and solve for mass. (We retain units for illustrative purposes only.)

$$\text{In[3]:= } \text{eq1} = \% * \frac{\frac{1 \text{ kg*m}}{\text{s}^2}}{1 \text{ N}} == \text{mass} * \frac{9.80665 \text{ m}}{\text{s}^2}$$

Out[3]=

$$\frac{3769.911 \text{ kg m}}{\text{s}^2} == \frac{9.80665 \text{ m mass}}{\text{s}^2}$$

In[4]:= Solve[eq1, mass]

Out[4]=

$$\{\{\text{mass} \rightarrow 384.424 \text{ kg}\}\}$$

Therefore, the mass is 384.425 kg //ANS.

It is a good idea to check the answer. This can be done quickly by converting the mass back to a force (in Newtons) by multiplying by g, and then into a pressure (in Pa) by dividing by area, and then converting pressure units from Pa to bar:

$$\text{In[5]:= } \frac{384 * 9.8}{\pi * .004^2 / 4} / 10^5$$

Out[5]=

$$2994.659$$

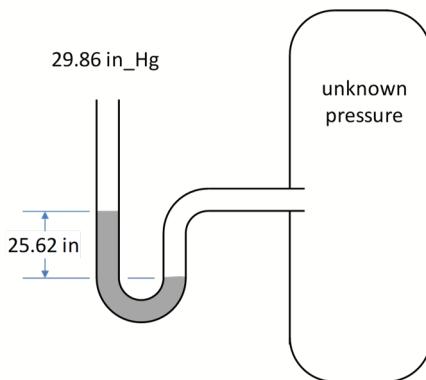
Note that 2994.66 bar is about 3,000 bar, as expected.

Problem 1.9

The reading on a mercury manometer at 70 °F (open to the atmosphere at one end) is 25.62 inches. The local acceleration of gravity is 32.243 $\frac{\text{ft}}{\text{s}^2}$. Atmospheric pressure is 29.86 inches of mercury (in_Hg). What is the absolute pressure in psia being measured? The density of mercury at 70 °F is 13.543 $\frac{\text{gm}}{\text{cm}^3}$.

SOLUTION

A sketch of the manometer is shown below. The sketch shows how the different pressures and mercury height are related.



The absolute pressure in the vessel is the sum of the manometer pressure and the atmospheric pressure.

The manometer pressure is given by ρgh , where h is the difference in height between the two mercury levels. English units require the conversion factor g_c , so the pressure difference in English units is $\frac{1}{g_c} \rho gh$. Also, density must be in English units:

$$\text{In[}]:= \left(\frac{13.543 \text{ gm}}{\text{cm}^3} \right) * \left(\frac{2.20462 \text{ lb}_m}{1000 \text{ gm}} \right) * \left(\frac{100 \text{ cm}}{39.3701 \text{ in}} \right)^3$$

$$\text{Out[}]:= \frac{0.489\ 270\ 5 \text{ lb}_m}{\text{in}^3}$$

To get absolute pressure, add the gauge pressure to the atmospheric pressure (25.62in+29.86in), then apply the manometer equation:

$$\text{In[}]:= \left(\frac{0.489\ 270\ 5 \text{ lb}_m}{\text{in}^3} \right) * \left(\frac{32.243 \text{ ft}}{\text{sec}^2} \right) * (25.62 \text{ in} + 29.86 \text{ in}) / \left(\frac{32.174 \text{ ft} * \text{lb}_m / \text{sec}^2}{1\text{bf}} \right)$$

$$\text{Out[}]:= \frac{27.20294 \text{ lbf}}{\text{in}^2}$$

The absolute pressure in the vessel is 27.20294 psia //ANS.

Many cadets got an answer of 27.23 psia. Since the density was given as 13.543 g/cm^3 , this is a significant difference. This problem happens when cadets conversion factors from the internet without accounting for density in the conversion factors. If cadets got this answer, they should try to reconcile their answers with the approved solution in their resubmission. This is demonstrated below.

END OF APPROVED SOLUTION

An Easier Way? (AI/Natural Language):

It may seem like AI is the easy way to solve this problem and we may be tempted to do this. However, the exact solution will depend on the language used to pose the question. For example:

In[48]:=

how many psia in (25.620+29.860) inches of mercury? »
UnitConvert[(25.62 + 29.86) inHg, "PoundsForce" / "Inches" ^ 2]

Out[48]=

$$27.24923 \text{ lbf/in}^2$$

However, Mathematica made some assumptions using its AI and its language interpreter. For example, we do not know the exact local acceleration due to gravity or the density of mercury used by Mathematica in this calculation.

At 0 °C the density of mercury is 13.595 g/cm³ and at sea level, $g = 32.174 \text{ ft/s}^2$. These values will reconcile the differences to some degree.

$$\text{In[49]:= } \left(\frac{13.595 \text{ gm}}{\text{cm}^3} \right) * \left(\frac{2.20462 \text{ lb}_m}{1000 \text{ gm}} \right) * \left(\frac{10^6 \text{ cm}^3}{35.3147 \text{ ft}^3} \right)$$

Out[49]=

$$\frac{848.7063 \text{ lb}_m}{\text{ft}^3}$$

$$\text{In[50]:= } \frac{\left(\left(\frac{848.7063 \text{ lb}_m}{\text{ft}^3} \right) * \left(\frac{32.174 \text{ ft}}{\text{s}^2} \right) * (25.62 \text{ in} + 29.86 \text{ in}) * \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) \right)}{\left(\frac{32.1740 \text{ ft} * \text{lb}_m}{\text{s}^2} \right)} * \left(\frac{1 \text{ ft}}{12 \text{ in}} \right)^2$$

Out[50]=

$$\frac{27.24897 \text{ lb}_f}{\text{in}^2}$$

The same situation arises when using internet browser AI or AI apps.

Problem 1.11

Liquids that boil at relatively low temperatures are often stored as liquids under their vapor pressures, which at ambient temperature can be quite large. Thus, n-butane stored as a liquid/vapor system is at a pressure of 2.581 bar for a temperature of 300 K. Large-scale storage ($> 50 \text{ m}^3$) of this kind is sometimes done in spherical tanks. Suggest two reasons why.

SOLUTION

Reason 1:

For a given volume, the surface area to volume ratio is minimized in a sphere. What does this mean? For a unit volume, the surface area for a sphere is lower than for other shapes. Since heat transfer is proportional to surface area, the same volume of fluid experiences less heat transfer when contained inside a sphere as opposed to a cube or rectangle. Less heat flux means less evaporation, which means

the losses are lower. Engineers have to weigh the costs and benefits of using a more expensive spherical vessel to lower evaporative fluid losses versus less expensive cylindrical vessels with higher fluid losses. //ANS

Reason 2:

A sphere is inherently a very strong structure. The reason for this is the symmetrical and even distribution of stresses on the sphere's surfaces. Generally, this means that there are fewer weak points than in cylindrical vessels. Of course, this reasoning is true only for a "theoretical" sphere. In an actual spherical vessel, there will be imperfections and stress points at the welds between the plates used to fabricate the vessel, but all things being equal, the spherical vessel should be somewhat stronger. //ANS

Problem 1.12

The first accurate measurements of the properties of high-pressure gases were made by E.H. Amagat in France between 1869 and 1893. Before developing the dead-weight gauge, he worked in a mine shaft, and used a mercury manometer for measurements of pressure to more than 400 bar. Estimate the height of the manometer required.

SOLUTION

Since this is an estimate, we do not need highly precise constants. The acceleration of gravity is about $9.8 \frac{m}{s^2}$, and we can assume a density of mercury of about $13.5 \frac{gm}{cm^3}$, which corresponds to a temperature near $25^\circ C$.

Convert the pressure of 400 bar to base units:

$$\text{In[1]:= } 400 \text{ bar} * \frac{100000 \text{ Pa}}{\text{bar}} * \frac{1 \text{ N/m}^2}{\text{Pa}}$$

$$\text{Out[1]:= } \frac{40000000 \text{ N}}{\text{m}^2}$$

The pressure is equal to ρgh :

$$\text{In[2]:= } 13.5 \frac{g}{cm^3} * \frac{1 \text{ kg}}{1000 \text{ g}} * \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 * \frac{9.8 \text{ m}}{\text{sec}^2} * \frac{1 \text{ N}}{1 \text{ kg} * \text{m/sec}^2} * h$$

$$\text{Out[2]:= } \frac{132300. \text{ h N}}{\text{m}^3}$$

$$\text{In[3]:= } \text{eq1} = \frac{4. * 10^7 \text{ N}}{\text{m}^2} == \frac{132300 \text{ N}}{\text{m}^3} * (h)$$

$$\text{Out[3]:= } \frac{4. \times 10^7 \text{ N}}{\text{m}^2} == \frac{132300 \text{ h N}}{\text{m}^3}$$

Solve for h:

```
In[1]:= Solve[eq1, h]
```

```
Out[1]=
```

$$\{ \{ h \rightarrow 302.3432 \text{ m} \} \}$$

Therefore the height of the manometer was more than 302 m or 992 ft. That means Amagat's manometer was more than 3 football fields in length! //ANS

Problem 1.18

A gas is confined in a 1.25-ft-diameter cylinder by a piston, on which rests a weight. The mass of the piston and weight together is 250 lb_m. The local acceleration of gravity is 32.169 $\frac{\text{ft}}{\text{s}^2}$, and the atmospheric pressure is 30.12 inches of Hg (in_Hg).

(a) What is the force in lb_f exerted on the gas by the atmosphere, the piston, and the weight, assuming no friction between the piston and the cylinder?

(b) What is the pressure of the gas in psia?

(c) If the gas in the cylinder is heated, it expands, pushing the piston and weight upward. If the piston and weight are raised 1.7 ft, what is the work done by the gas in ft * lb_f? What is the change in potential energy of the piston and the weight?

SOLUTION

Part (a)

Calculate the force due to the piston and the weight:

$$\text{In[1]} := 250 \text{ lb}_m * (32.169 \text{ ft} / \text{s}^2) * \frac{1 \text{ lb}_f}{\frac{32.174 \text{ ft} * \text{lb}_m}{\text{s}^2}}$$

```
Out[1]=
```

$$249.9611 \text{ lb}_f$$

Calculate the atmospheric pressure in psia using the conversion factors from Appendix A. Additional conversion factor: 1 Torr = 1 mmHg.

$$\text{In[2]} := 30.12 \text{ in}_Hg * \frac{100 \text{ cm}_Hg}{39.3701 \text{ in}_Hg} * \frac{10 \text{ mm}_Hg}{1 \text{ cm}_Hg} * \frac{1 \text{ torr}}{1 \text{ mm}_Hg} * \frac{14.5038 \text{ psia}}{750.061 \text{ torr}}$$

```
Out[2]=
```

$$14.79359 \text{ psia}$$

Calculate the piston area:

$$\text{In[3]} := \text{area} = \pi * \left(1.25 \text{ ft} * \frac{12 \text{ in}}{\text{ft}} \right)^2 / 4$$

```
Out[3]=
```

$$176.7146 \text{ in}^2$$

Now calculate the force due to atmospheric pressure:

$$\text{In}[\circ]:= \frac{14.7936 \text{ lb}_f}{1 \text{ in}^2} * \text{area}$$

$$\text{Out}[\circ]:= 2614.245 \text{ lb}_f$$

Now calculate the total force due to weight, piston, air atmosphere:

$$\text{In}[\circ]:= \text{force} = 249.961 \text{ lb}_f + 2614.24 \text{ lb}_f$$

$$\text{Out}[\circ]:= 2864.201 \text{ lb}_f$$

The total force is 2864.2 lb_f. //ANS

Part (b)

The pressure of the gas in psia is the force divided by the piston area:

$$\text{In}[\circ]:= \frac{2864.2 \text{ lb}_f}{176.715 \text{ in}^2}$$

$$\text{Out}[\circ]:= \frac{16.20802 \text{ lb}_f}{\text{in}^2}$$

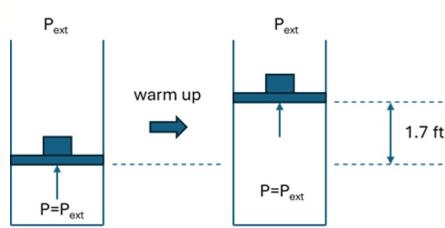
The pressure of the gas is 16.208 psia. //ANS

Part (c)

The gas in the piston is heated so the pressure goes up initially, but the system responds by the piston moving out to restore the equilibrium.

Because the piston is frictionless and the initial and final positions are in mechanical equilibrium, the initial and final pressures in the piston are equal to the pressure outside of the piston, which is constant and equal to 16.2801 psia. The force of the gas is pressure times area, where the area is 176.7 in² from part (a).

The work done by the gas is the force times the displacement. Since the displacement of the piston and the direction of the force from the gas inside the piston are in the same direction, the work done by the gas on the piston is positive. (We could also say that the work done on the gas is negative.)



(*work=force*displacement, or*)
(*work=pressure*area*displacement)

$$\text{In}[\circ]:= \frac{16.2081 \text{ lb}_f}{\text{in}^2} * 176.7 \text{ in}^2 * 1.7 \text{ ft}$$

$$\text{Out}[\circ]= \\ 4868.751 \text{ ft lb}_f$$

Therefore, the system did 4869.16 ft*lb_f of work on the surroundings. //ANS.

Change in potential energy is given by m*g*h:

$$\text{In}[\circ]:= 250 \text{ lb}_m * 32.169 \frac{\text{ft}}{\text{s}^2} * 1.7 \text{ ft} * \frac{1 \text{ lb}_f}{\frac{32.174 \text{ ft} * \text{lb}_m}{\text{s}^2}}$$

$$\text{Out}[\circ]= \\ 424.934 \text{ ft lb}_f$$

The change in potential energy is 424.934 ft*lb_f. //ANS

Problem 1.20

Verify that the SI unit of kinetic and potential energy is the joule.

SOLUTION

This problem deals with converting energy units. Students must know how to convert kinetic and potential energy from base units into Joules.

For kinetic energy, start with $\frac{1}{2} mv^2$:

$$\text{In}[\circ]:= \text{kg} * \left(\frac{\text{m}}{\text{s}}\right)^2 * \frac{1 \text{ N}}{\frac{1 \text{ kg} * \text{m}}{\text{s}^2}} * \frac{1 \text{ J}}{1 \text{ N} * \text{m}}$$

$$\text{Out}[\circ]= \\ \text{J}$$

For potential energy, start with m*g*h:

$$\text{In}[\circ]:= \text{kg} * \frac{\text{m}}{\text{s}^2} * \text{m} * \frac{1 \text{ N}}{\frac{1 \text{ kg} * \text{m}}{\text{s}^2}} * \frac{1 \text{ J}}{1 \text{ N} * \text{m}}$$

$$\text{Out}[\circ]= \\ \text{J}$$

In both cases the units reduce to J. //ANS

Problem 1.22

The turbines in a hydroelectric plant are fed by water falling from a 50-m height. Assuming 91% efficiency for conversion of potential to electrical energy, and 8% loss of the resulting power in transmission, what is the mass flow rate of water required to power a 200-watt light bulb?

SOLUTION

Students must know how to convert potential energy into *energy flow rate*. The following conversion factors into base energy units are important:

$$\frac{\text{kg} \cdot \text{m}}{\text{s}^2} (*\text{N}*)$$

$$\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} (*\text{Nm or joule}*)$$

$$\frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} (*\text{Nm/s or Watt}*)$$

$$\frac{\text{kg}}{\text{m} \cdot \text{s}^2} (*\text{N/m}^2 \text{ or pascal}*)$$

Use the conversion factor for Watts to convert bulb power to base energy units:

$$\text{In[1]:= } \text{BulbPower} = 200 \text{ W} * \frac{\frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}}{\text{W}} \quad (*\text{Light bulb power}*)$$

$$\text{Out[1]= } \frac{200 \text{ kg m}^2}{\text{s}^3}$$

Use ρgh to convert the height of the reservoir to base energy units:

$$\text{In[2]:= } \text{HydrostaticPressure} = \frac{1000 \text{ kg}}{\text{m}^3} * \frac{9.8 \text{ m}}{\text{s}^2} * 50 \text{ m} \quad (*\rho * g * h*)$$

$$\text{Out[2]= } \frac{490000. \text{ kg}}{\text{m s}^2}$$

By comparison to the units for Watts, these two terms differ by multiplication of ρgh by volumetric flow in $\frac{\text{m}^3}{\text{s}}$. To illustrate, divide them:

$$\text{In[3]:= } \frac{\text{BulbPower}}{\text{HydrostaticPressure}}$$

$$\text{Out[3]= } \frac{0.0004081633 \text{ m}^3}{\text{s}}$$

Now add the efficiencies, noting that any losses must increase the flow rate to account for the lost energy. We do this by dividing by the efficiencies.

$$\text{In[4]:= } \frac{\text{BulbPower}}{0.91 * (1 - .08) * \text{HydrostaticPressure}}$$

$$\text{Out[4]= } \frac{0.0004875338 \text{ m}^3}{\text{s}}$$

$$\text{In[5]:= } \frac{0.000487534 \text{ m}^3}{\text{s}} * \frac{1000 \text{ kg}}{\text{m}^3}$$

$$\text{Out[5]= } \frac{0.487534 \text{ kg}}{\text{s}}$$

The mass flow rate required is $0.4875 \frac{\text{kg}}{\text{s}}$ or $1,755 \frac{\text{kg}}{\text{h}}$. //ANS

Problem 1.27

Energy costs vary greatly with energy source. For example, coal is about \$35.00/ton, gasoline has a pump price of \$2.75/gallon, and electricity as about \$0.10/kWhr. Conventional practice is to put these on a common basis by expressing them in \$/GJ. For this purpose, assume gross heating values of 29 MJ/kg for coal and $37 \text{ GJ}/\text{m}^3$ for gasoline.

- (a) Rank order the three energy sources with respect to energy cost in \$/GJ.
- (b) Explain the large disparity in the numerical results of Part (a).

SOLUTION

Part (a)

Coal:

$$\text{In[}]:= \frac{35 \text{ dollars}}{\text{ton}} * \frac{1 \text{ ton}}{2000 \text{ lbs}} * \frac{2.20462 \text{ lbs}}{\text{kg}} * \frac{1 \text{ kg}}{29 \text{ MJ}} * \frac{1000 \text{ MJ}}{\text{GJ}}$$

Out[]=

$$\frac{1.330374 \text{ dollars}}{\text{GJ}}$$

Gasoline:

$$\text{In[}]:= \frac{2.75 \text{ dollars}}{\text{gal}} * \frac{264.172 \text{ gal}}{1 \text{ m}^3} * \frac{1 \text{ m}^3}{37 \text{ GJ}}$$

Out[]=

$$\frac{19.63441 \text{ dollars}}{\text{GJ}}$$

Electricity:

$$\text{In[}]:= \frac{0.1000 \text{ dollars}}{\text{kWhr}} * \frac{2.77778 * 10^{-7} \text{ kWhr}}{1 \text{ J}} * \frac{10^9 \text{ J}}{\text{GJ}}$$

Out[]=

$$\frac{27.7778 \text{ dollars}}{\text{GJ}}$$

Coal < Gasoline < Electricity. //ANS

Part (b)

Gasoline and electricity are more expensive because they are more highly refined forms of energy requiring more infrastructure. Converting coal to electricity requires a power plant, a train line to move the coal, plus electrical power grid. Burning coal directly to produce heat energy requires only a fire box. Gasoline is similar. Gasoline is a highly refined form of crude oil, which is essentially liquid coal. The crude has to be transported into a refinery for conversion to gasoline. //ANS

Problem 1.29

A laboratory reports the following vapor-pressure (P^{sat}) data for a particular organic chemical:

t/ °C	$P^{\text{sat}} / \text{kPa}$
-18.5	3.18
-9.5	5.48
0.2	9.45
11.8	16.9
23.1	28.2
32.7	41.9
44.4	66.6
52.1	89.5
63.3	129
75.5	187

Correlate the data by fitting them to the Antoine equation:

$$\ln P^{\text{sat}} / \text{kPa} = A - \frac{B}{T/K + C}$$

That is, find numerical values of parameters A, B, and C by an appropriate regression procedure. Discuss the comparison of correlated and experimental values. What is the predicted normal boiling point of this chemical (i.e., the temperature at which the vapor pressure is 1 atm).

SOLUTION

Fitting the Data to the Antoine Equation:

The data is first entered into Excel because I find that easiest. The data is in a file on Canvas (in Modules→Lesson3). There you will find a link for “Student Excel Data for Problem 1.29.” Download this data to your computer. Or, you can just type it into Excel and save the file:

Insert the complete path and use *SemanticImport*:

```
In[1]:= filename =
"C:\\\\Users\\\\andrew.biaglow\\\\OneDrive - West Point\\\\Desktop\\\\Problem_1-29_Data.xlsx";
ds = SemanticImport[filename];
```

Convert the data to “Normal” form:

```
In[2]:= data = Normal[ds]
Out[2]= {{-18.5, 3.18}, {-9.5, 5.48}, {0.2, 9.45}, {11.8, 16.9}, {23.1, 28.2},
{32.7, 41.9}, {44.4, 66.6}, {52.1, 89.5}, {63.3, 129.}, {75.5, 187.}}
```

Define the model:

```
In[3]:= model = Exp[a - b/(t + c)]
```

```
Out[3]= E^(a - b/(c + t))
```

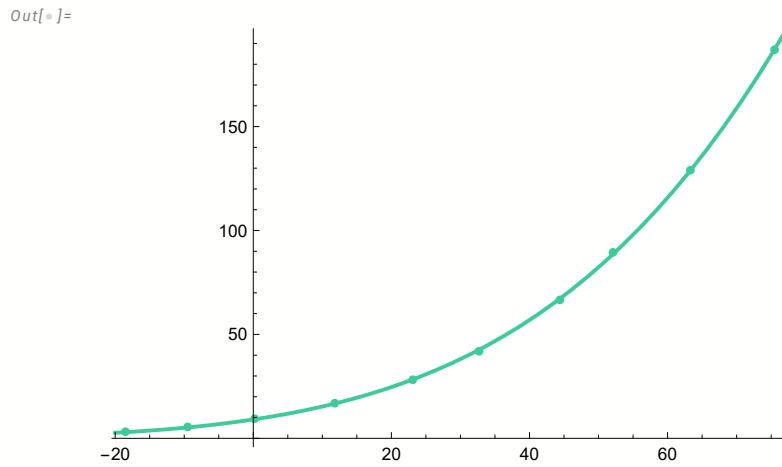
Find the best fit using the *FindFit* function:

```
In[6]:= fit = FindFit[data, model, {{a, 14.}, {b, 2800}, {c, 200}}, t]
Out[6]= {a → 13.42104, b → 2289.586, c → 204.0965}
```

The numerical values of A, B, and C from the regression are A = 12.421, B = 2289.586, and C = 204.097. //ANS

Comparison of Experimental and Correlated Values with Plots:

```
In[7]:= plot1 = ListPlot[data];
plot2 = Plot[model /. fit, {t, -20, 100}];
Show[plot1, plot2]
```



Discussion: Based on the plot, the fit is very good. //ANS

Additional discussion: We could also be more quantitative using sum of squares. This is discussed in the supplement below.

Predicted Normal Boiling Point

Wikipedia: The normal boiling point is the temperature at which the vapor pressure is equal to an atmospheric pressure of 1 atm.

```
In[8]:= kilopascals in 1.0 atmosphere
UnitConvert[1. atm, "Kilopascals"]
```

Out[8]= 101.325 kPa

```
In[9]:= NSolveValues[(model /. fit) == 101.325, t, Reals]
Out[9]= {56.00372}
```

The predicted boiling point is 56.0037 deg C. //ANS

END OF APPROVED SOLUTION

Problem 1.29 Supplement - Comparison Using Sum of Squares - Not Required

Create a list of temperatures:

```
In[1]:= t = ds[[All, 1]] // Normal
Out[1]= {-18.5, -9.5, 0.2, 11.8, 23.1, 32.7, 44.4, 52.1, 63.3, 75.5}
```

Create a list of pressures from the given data:

```
In[2]:= p1 = ds[[All, 2]] // Normal
Out[2]= {3.18, 5.48, 9.45, 16.9, 28.2, 41.9, 66.6, 89.5, 129., 187.}
```

Create a list of calculated pressures:

```
In[3]:= p2 = model /. fit(*calculated pressure*)
Out[3]= {2.958483, 5.234272, 9.15105, 16.71027,
28.31755, 42.6078, 67.17388, 88.60643, 128.8343, 187.1941}
```

Absolute value of difference between data and calculated data:

```
In[4]:= residuals = Abs[p2 - p1]
Out[4]= {0.2215167, 0.2457277, 0.2989502, 0.1897279,
0.1175522, 0.7078043, 0.5738818, 0.8935747, 0.1657225, 0.1941205}
```

Average residual error:

```
In[5]:= Plus @@ residuals / Length[t]
Out[5]= 0.3608579
```

This means the average residual error is about 0.4 kPa. This is about 2% average error. You can see this by calculating the individual percent errors and then averaging them:

```
In[6]:= 100 * residuals / p1
Out[6]= {6.965934, 4.484082, 3.163494, 1.122651, 0.4168519,
1.68927, 0.8616844, 0.9984075, 0.128467, 0.1038078}
```

```
In[7]:= Plus @@ (100 * residuals / p1) / Length[t]
Out[7]= 1.993465
```

We can also look as sum of squares error:

```
In[8]:= SQ = (p2 - p1)^2
Out[8]= {0.04906965, 0.06038209, 0.08937122, 0.03599669,
0.01381853, 0.5009869, 0.3293404, 0.7984757, 0.02746394, 0.03768277}
```

```
In[6]:= SSQ = Plus @@ SQ
Out[6]= 1.942588
```

(*This is a tiny little bit better than Excel which gave SSQ=1.950.*)

```
In[7]:= averageError = Sqrt[SSQ / Length[t]]
Out[7]= 0.440748
```

This percentage 0.044% is about the same as the answer obtained above, 0.36%.

More Discussion: The average error in the calculated pressures is about 0.44 kPa, which is consistent with the precision given in the original data points. // ANS