Problem 5.43

Heat in the amount of 150 kJ is transferred directly from a hot reservoir at T_H=550 K to two cooler reservoirs at T_1=350 K and T_2=250 K. The surroundings temperature is T_σ = 300 K. If the heat transferred to the reservoir at T_1 is half that transferred to the reservoir at T_2, calculate:

(a) The entropy generation in kJ/K.

(b) The lost work.

(c) How could the process be made reversible?

Solution to (a)

$Q_H = -150.;$ (*kJ*)
$T_H = 550;$ (*K*)

$Q_1 = 50;$
$T_1 = 350;$

$Q_2 = 100;$
$T_2 = 250;$

$$S_G = \frac{Q_H}{T_H} + \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \quad (*\frac{kJ}{K}*)$$

0.27012987013

(*The total entropy generation is 0.27013 $\frac{kJ}{K}$. //ANS*)

Solution to (b)

Use equation 5.29:

$T_\sigma = 300;$
$W_{lost} = T_\sigma * S_G$ (*kJ*)

81.038961039

(*The lost work is 81.03896 kJ. //ANS*)

Solution to (c)

By the second law, the process can be made to be reversible if the entropy generation is somehow made to be zero. One possibility is to couple some sort of additional reservoir that could remove entropy by heat transfer. Another possibility would be to add flowing streams to adjust the entropy balance to sero. A third possibility would be to add some kind of chemical reaction to consume entropy. //ANS

Problem 5.44

A nuclear power plant generates 750 MW; the reactor temperature is 315 deg C and a river with water temperature of 20 deg C is available.

(a) What is the maximum possible thermal efficiency of the plant, and what is the minimum rate at which heat must be discarded to the river?

(b) If the actual thermal efficiency of the plant is 60% of the maximum, at what rate must heat be discarded to the river, and what is the temperature rise of the river if it has a flow rate of 165 cubic meters per second?

Solution, Part (a)

In[1]:= Tc = 273.13 + 20;(*river temperature in K*)
Th = 273.13 + 315; (*reactor temperature in K*)

In[4]:= $\eta_{max} = 1 - \frac{Tc}{Th}$ (*eq. 5.7*)

Out[4]= 0.501589784571

(*The maximum possible thermal efficiency η_{max} is 0.5016. //ANS*)

In[7]:= $Q_H = 750 / \eta_{max}$; (*eq. 5.6*)

In[8]:= $Q_C = Q_H - 750$

Out[8]= 745.245762712

(*At least 745.3 MW of heat are discarded to the river. //ANS*)

Solution, Part (b)

In[9]:= $\eta = .60 * \eta_{max}$;
$Q_H = 750 / \eta$;

In[11]:= $Q_C = Q_H - 750$

Out[11]=

1742.07627119

(*At least 1742.2 MW of heat are discarded to the river at η=0.6η_{max}. //ANS*)

In[12]:= $eq1 = \frac{165\,m^3}{s} * \frac{1000\,L}{m^3} * \frac{1\,kg}{L} * \frac{4.184\,kJ}{kg * degC} * \Delta T == 1742.16\,MW * \frac{1000\,kW}{MW} * \frac{1\,kJ / s}{kW}$; (*Qc=$\dot{m}Cp\Delta$T*)

In[13]:= Solve[eq1, ΔT]

Out[13]=

$\{\{\Delta T \to 2.52355292891\ degC\}\}$

(*The temperature increase of the river is 2.52 °C. //ANS*)

Problem 5.50

Ethylene vapor is cooled at atmospheric pressure from 830 to 35 deg C by direct heat transfer to the surroundings at 25 deg C. With respect to this surroundings temperature, what is the lost work of the process in kJ/mol? Show that the same result is obtained as the work which can be derived from reversible heat engines operating with the ethylene vapor as the heat source and the surroundings as the sink. The heat capacity of ethylene is given in Table C.1 of App. C.

Solution

Part (a)

This problem involves integrating the heat capacity polynomial. Recall that temperature must be in K and heat capacity in Appendix C is dimensionless.

In[ ]:=
```
(*gather information*)
T1 = 273.15 + 830; (*initial ethylene T*)
T2 = 273.15 + 35; (*final ethylene T*)
Tσ = 273.15 + 25; (*surroundings T*)

(*gas constant, needed for Cp*)
R = 8.314; (* J/(mol*K) *)
(*Cp coefficients from App. C*)
a = 1.424;
b = 14.394 * 10^-3;
c = -4.392 * 10^-6;
Cp = a + b * T + c * T^2;
```

Ehtylene is changing T so S and H both change. Calculate the ΔS by integrating $\frac{C_P}{T}$ with respect to T. Calculate the ΔH by integrating C_P with respect to T.

In[ ]:= $\Delta S_{fs} = R * \int_{T1}^{T2} \frac{Cp}{T} \, dT$ (*in units of $\frac{J}{mol*K}$ *)

Out[ ]= -89.7532549813

In[ ]:= $Q_{ethylene} = R * \int_{T1}^{T2} Cp \, dT$ (* Q=ΔH at constant pressure, in units of J/mol*)

Out[ ]= -60 563.0087621

In[ ]:= $W_{lost} = T\sigma * \Delta S_{fs} - Q_{ethylene}$ (*equation 5.31*)

Out[ ]= 33 803.0757894

(*The lost work is 33.80308 kJ/mol. //ANS*)

Part (b)

To analyze this process as a reversible engine with the Carnot equations, we need values for Q_H and Q_C. It should be clear that the ethylene vapor is the heat source. That is, the heat added to the engine (the system) comes from the ethylene. This is Q_H in the reversible (Carnot) cycle, and the value is the same as it was before, but must be opposite in sign because it enters the engine.

```
QH = -Qethylene (*Heat transferred (QH) is positive with respect to the engine.*)
```

Out[ ]=

```
60 563.0087621
```

To obtain Q_C, we need to understand a couple of things. First, equation 5.31 (shown below) gives the lost work in terms of entropy change of the flowing streams and Q dissipated to the surroundings. Second, the lost work is zero for a reversible process. Since this Q is heat transferred to the surroundings, it is also Q_C in the reversible (Carnot) cycle. Because the temperature change of the ethylene is the same as it was in part (a), the entropy change of the ethylene is also the same. So we calculate the Q that zeros the lost work in equation 5.31 and make this equal to Q_C in the Carnot engine.

```
(*Equation 5.31 Wlost=TσΔS-Q and Wlost=0 for a reversible process*)
sol = Solve[0 == Tσ * ΔSfs - Q , Q] (*J/mol*)
```

Out[ ]=

```
{{Q → -26 759.9329727}}
```

In[ ]:=

```
Qc = Q /. sol[[1]] (*heat transferred is negative with respect to the system.*)
```

Out[ ]=

```
-26 759.9329727
```

In[ ]:=

```
QC = -26 759.9; (*heat transferred is negative with respect to the system.*)
```

Finally, use the first law ΔU=Q+W (equation 2.3), where ΔU=0 for a cyclic process, and where Q=Q_H+Q_C as explained on page 181. This gives W=-Q_H-Q_C.

In[ ]:=

```
Wheatengine = -QH - QC (*J/mol*)
```

Out[ ]=

```
-33 803.1087621
```

```
(*This is the same value as obtained in part (a). //ANS*)
(*The sign is negative because work is leaving the system.*)
```

Problem 5.17

A Carnot engine operates between temperature levels of 600 K and 300 K. It drives a Carnot refrigerator, which provides cooling at 250 K and discards heat at 300 K. Determine a numerical value for the ratio of heat extracted by the refrigerator ("cooling load") to the heat delivered to the engine ("heating load").

Solution

Calculate the efficiency and work of the Carnot engine:

In[ ]:=
```
THE = 600.;(*Temperature of the engine heat reservoir*)
TCE = 300.;(*Temperature of the engine cold reservoir*)
```

In[ ]:=
$$\eta = 1 - \frac{TCE}{THE}$$

Out[ ]=
```
0.5
```

In[ ]:=
```
WE = QHE * η
```

Out[ ]=
```
0.5 QHE
```

Calculate the COP and work of the Carnot refrigerator:

```
(*Equations are shown in the Lesosn 26 slide deck.*)
```

In[ ]:=
```
THR = 300.;
TCR = 250.;
```

In[ ]:=
$$COP = \frac{TCR}{THR - TCR}$$

Out[ ]=
```
5.
```

In[ ]:=
```
WR = QCR / COP
```

Out[ ]=
```
0.2 QCR
```

Equate the engine work and refrigerator work and simplify:

In[ ]:=
```
Simplify[WE == WR]
```

Out[ ]=
```
1. QCR == 2.5 QHE
```

```
(* The ratio of heat extracted by the refrigerator
  QCR to the heat delivered to the engine QHE is 2.5. //ANS*)
```