Problem Set 9 - Solutions

Problem 5.43

Heat in the amount of 150 kJ is transferred directly from a hot reservoir at $T_H = 550\,K$ to two cooler reservoirs at $T_1 = 350\,K$ and $T_2 = 250\,K$. The surroundings temperature is $T_\sigma = 300\,K$. If the heat transferred to the reservoir at T_1 is half that transferred to the reservoir at T_2, calculate:

(a) The entropy generation in kJ/K.

(b) The lost work.

(c) How could the process be made reversible?

Solution - Part (a)

In[ ]:=
```
QH = -150.; (*kJ*)
TH = 550; (*K*)
SH = QH / TH; (*kJ/kg*)

Q1 = 50;
T1 = 350;
S1 = Q1 / T1;

Q2 = 100;
T2 = 250;
S2 = Q2 / T2;
```

In[ ]:=
```
SG = SH + S1 + S2 (*kJ/kg*)
```

Out[ ]=
```
0.2701299
```

The total entropy generation is 0.27013 $\frac{kJ}{K}$. //ANS

Solution - Part (b)

Use equation 5.29:

In[ ]:=
```
Tσ = 300;
Wlost = Tσ * SG (*kJ*)
```

Out[ ]=
```
81.03896
```

The lost work is 81.039 kJ. //ANS

Solution - Part (c)

By the second law, the process can be made to be reversible if the total entropy generation is somehow made to be zero. This means that an entropy term must be added that compensates for the total entropy generated. One possibility is to couple some sort of additional reservoir that could remove entropy by heat transfer. Another possibility would be to add some kind of chemical reaction to consume entropy. //ANS

eq1 = 0 == $\frac{Q_H}{T_H} + \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$ + s

Out[ ]=

```
0 == 0.2701299 + s
```

Solve[eq1, s]

Out[ ]=

```
{{s → -0.2701299}}
```

One can add a device or chemical reaction with an entropy change of -0.27013 $\frac{kJ}{K}$ of entropy. //ANS

Problem 5.44

A nuclear power plant generates 750 MW. The reactor temperature is 315 °C and a river with water temperature of 20 °C is available.

(a) What is the maximum possible thermal efficiency of the plant, and what is the minimum rate at which heat must be discarded to the river?

(b) If the actual thermal efficiency of the plant is 60% of the maximum, at what rate must heat be discarded to the river, and what is the temperature rise of the river if it has a flow rate of 165 cubic meters per second?

Solution - Part (a)

In[∘]:=
```
Tc = 273.15 + 20; (*river temperature in K*)
Th = 273.15 + 315; (*reactor temperature in K*)
```

In[∘]:= $\eta_{max} = 1 - \frac{Tc}{Th}$ (*eq. 5.7*)

Out[∘]=
```
0.5015727
```

The maximum possible thermal efficiency η_{max} is 0.5016. //ANS

In[∘]:= $Q_H = 750 / \eta_{max}$ (*eq. 5.6*)

Out[∘]=
```
1495.297
```

In[∘]:= $Q_C = Q_H - 750$

Out[∘]=
```
745.2966
```

At least 745.3 MW of heat are discarded to the river. //ANS

Solution - Part (b)

In[∘]:= $\eta = .60 * \eta_{max};$
$Q_H = 750 / \eta;$

In[∘]:= $Q_C = Q_H - 750$

Out[∘]=
```
1742.076
```

At least 1742.2 MW of heat are discarded to the river at $\eta = 0.6\ \eta_{max}$. //ANS

To find the temperature increase of the river, use Qc = $\dot{m}$ Cp ΔT.

In[∘]:= $eq1 = \frac{165\ m^3}{s} * \frac{1000\ L}{m^3} * \frac{1\ kg}{L} * \frac{4.184\ kJ}{kg * degC} * \Delta T == 1742.16\ MW * \frac{1000\ kW}{MW} * \frac{1\ kJ / s}{kW}$; (*Qc=$\dot{m}Cp\Delta$T*)

In[ ]:=
```
Solve[eq1, ΔT]
```

Out[ ]=
```
{{ΔT → 2.523553 degC}}
```

The temperature increase of the river is 2.52 °C. //ANS

Problem 5.50

Problem 5.17

A Carnot engine operates between temperature levels of 600 K and 300 K. It drives a Carnot refrigerator, which provides cooling at 250 K and discards heat at 300 K. Determine a numerical value for the ratio of heat extracted by the refrigerator ("cooling load") to the heat delivered to the engine ("heating load").

Solution

Calculate the efficiency and work of the Carnot engine:

In[ ]:=
```
THE = 600.; (*Temperature of the engine heat reservoir*)
TCE = 300.; (*Temperature of the engine cold reservoir*)
```

In[ ]:= $\eta = 1 - \frac{TCE}{THE}$

Out[ ]=
```
0.5
```

In[ ]:=
```
WE = -QHE * η
```

Out[ ]=
```
-0.5 QHE
```

Work is negative because it is leaving the engine.

Calculate the COP and work of the Carnot refrigerator. Equations are shown in the Lesson 26 slide deck:

In[ ]:=
```
THR = 300.;
TCR = 250.;
```

In[ ]:= $COP = \frac{TCR}{THR - TCR}$

Out[ ]=
```
5.
```

In[ ]:=
```
WR = +QCR / COP
```

Out[ ]=
```
0.2 QCR
```

The work entering the refrigerator is equal to but opposite in sign to the work leaving the engine:

In[ ]:=
```
Simplify[WR == -WE]
```

Out[ ]=
```
1. QCR == 2.5 QHE
```

The ratio of the cooling load QCR to the heating load QHE is 2.5. In other words, $\frac{QCR}{QHE} = 2.5$. //ANS