

Problem 5.2

A Carnot engine receives 250 kJ s⁻¹ of heat from a heat source reservoir at 525 deg C and rejects heat to a heat-sink reservoir at 50 deg C. What are the power developed and the heat rejected?

Solution

Use equation 5.7 to compute efficiency, and then use 5.6 to get the work, based on the heat received.

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In[*]:= 
$$\eta = 1 - \frac{50 + 273.15}{525 + 273.15}$$

Out[*]= 0.595126229406

In[*]:= 
$$Q_H = 250 \frac{\text{kJ}}{\text{sec}}; (*\text{heat in is positive}*)$$


$$W = -\eta * Q_H (*\text{work out is negative}*)$$

Out[*]= 
$$- \frac{148.781557351 \text{ kJ}}{\text{sec}}$$


(*Power developed is -148.782 kJ/sec = -148.782 kW. //ANS*)

Use eq 2.3 (first law) with ΔU=0 to compute heat ejected, that is QC+QH+W=0:

In[*]:= 
$$Q_C = -Q_H - W$$

Out[*]= 
$$- \frac{101.218442649 \text{ kJ}}{\text{sec}}$$


In[*]:= 
$$\text{Solve}[Q_C + 250 + (-148.782) == 0, Q_C]$$

Out[*]= { { Q_C → -101.218 } }
```

(*Heat rejected is -101.218 kJ/sec = -101.218 kW. //ANS*)

Problem 5.6

Which is the more effective way to increase the thermal efficiency of a Carnot engine: to increase T_H with T_C constant, or to decrease T_C with T_H constant? For a real engine, which would be the more practical way?

Solution

We have to start by interpreting what is meant by “more effective.” Since a derivative is a rate of change, it is a natural choice in this context for assessing effectiveness. For example, since η is a function of both T_C and T_H , we can take the derivatives of η with respect to T_H and T_C , while holding the other constant, and compare them to look for a greater than or less than relationship. In other words, write the definition of the efficiency, then take $\left(\frac{\partial \eta}{\partial T_H}\right)_{T_C}$ and $\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H}$ and then compare the results.

$$\eta = 1 - \frac{T_C}{T_H};$$

$$\frac{\partial \eta}{\partial T_H}$$

$$\frac{T_C}{T_H^2}$$

$$\frac{\partial \eta}{\partial T_C}$$

$$-\frac{1}{T_H}$$

Note that the partial derivative of η with respect to T_C is equal to $-\frac{1}{T_H}$ and the partial derivative of η with respect to T_H is equal to $-\frac{T_C}{T_H^2}$. We see that the latter includes the former. That is,

$$\left(\frac{\partial \eta}{\partial T_H}\right)_{T_C} = \frac{T_C}{T_H^2} = \frac{1}{T_H} \left(\frac{T_C}{T_H}\right) = \left|\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H}\right| \left|\left(\frac{T_C}{T_H}\right)\right| = \left|\frac{-1}{T_H}\right| \left|\left(\frac{T_C}{T_H}\right)\right|.$$

Since by definition $T_C < T_H$, then $\frac{T_C}{T_H} < 1$. Therefore, in terms of absolute magnitudes,

$$\left|\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H}\right| > \left|\left(\frac{\partial \eta}{\partial T_H}\right)_{T_C}\right|.$$

This means that lowering T_C will *always* be more effective at increasing η than raising T_H .

Interpreting the negative sign: Notice that $\left(\frac{\partial \eta}{\partial T_C}\right)_{T_H}$ is negative. This means that lowering the T_C increases η , which is completely consistent with the problem statement (decrease T_C with T_H constant).

For real engines, it is more difficult to lower the temperature of the cold reservoir than it is to raise the temperature of the hot, because the cold reservoir is normally closely connected to the ambient conditions, which will not change significantly unless we change the geographic location to Antarctica or outer space, which will not help much for more local operations.

Problem 5.8

With respect to 1 kg of liquid water:

(a) Initially at 0 deg C, it is heated to 100 deg C by contact with a heat reservoir at 100 deg C. What is the entropy change of the water? What is the entropy change of the heat reservoir? What is ΔS_{total} ?

(b) Initially at 0 deg C, it is first heated to 50 deg C by contact with a heat reservoir at 50 deg C, and then heated to 100 deg C by contact with a heat reservoir at 100 deg C. What is ΔS_{total} ?

(c) Explain how the water might be heated from 0 deg C to 100 deg C so that $\Delta S_{\text{total}} = 0$.

Solution to Part (a)

```
In[11]:= m = 1; (*kg*)
Cp = 4.184; (*kJ/kg*degC*)
T2 = 100 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)
ΔT = T2 - T1; (*degC*)
Qhot = m * Cp * ΔT; (*kJ*)
ΔShot =  $\frac{-Q_{\text{hot}}}{T_2}$  (*kJ/K*)
ΔSwater = m *  $\int_{T_1}^{T_2} \frac{C_p}{T} dT$  (*kJ/K*)
ΔStotal = ΔShot + ΔSwater (*kJ/K*)

Out[17]=
-1.12126490687

Out[18]=
1.30523808247

Out[19]=
0.183973175596

(*The ΔS of the hot reservoir is -1.12123 kJ/K//ANS*)
(*The ΔS of the cold reservoir is 1.30524 kJ/K//ANS*)
(*The total entropy change is is 0.18397 kJ/K//ANS*)
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Solution to Part (b)

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In[20]:= m = 1; (*kg*)
Cp = 4.184; (*kJ/kg*degC*)
T2 = 100 + 273.15; (*K*)
Tb = 50 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)

(*First Step*)
ΔT1b = Tb - T1; (*degC*)
Qhot1b = m * Cp * ΔT1b; (*kJ*)
ΔShot1b =  $\frac{-Q_{hot1b}}{T_b}$ 
ΔSwater1b =  $m \int_{T_1}^{T_b} \frac{C_p}{T} dT$ 
(*Second Step*)
ΔTb2 = T2 - Tb; (*degC*)
Qhotb2 = m * Cp * ΔTb2; (*kJ*)
ΔShotb2 =  $\frac{-Q_{hotb2}}{T_2}$  (*kJ/K*)
ΔSwaterb2 =  $m \int_{T_b}^{T_2} \frac{C_p}{T} dT$  (*kJ/K*)
(*Total, kJ/K //ANS*)
ΔStotal = ΔShot1b + ΔSwater1b + ΔShotb2 + ΔSwaterb2

Out[27]=
-0.647377378926

Out[28]=
0.703311642572

Out[31]=
-0.560632453437

Out[32]=
0.601926439898

Out[33]=
0.0972282501067

(*ΔS of the hot reservoir in step 1 is -.64738 kJ/K//ANS*)
(*ΔS of the cold reservoir in step 1 is .70331 kJ/K//ANS*)
(*ΔS of the hot reservoir in step 2 is -.56063 kJ/K//ANS*)
(*ΔS of the cold reservoir in step 2 is .60192 kJ/K//ANS*)
(*The total entropy change is is 0.097228 kJ/K//ANS*)

```

Solution to Part (c)

The reversible heating of the water requires an infinite number of heat reservoirs covering the range of temperatures from 273.15 to 373.15 K, each one exchanging an infinitesimal quantity of heat with the water and raising its temperature by a differential increment.

Problem 5.10

An ideal gas, $C_p = (7/2)R$, is heated in a steady-flow heat exchanger from 70 deg C to 190 deg C by another stream of the same gas, which enters at 320 deg C. The flow rates of the two streams are the same, and heat losses from the exchanger are negligible.

(a) Calculate the molar entropy changes of the two gas streams for both parallel and countercurrent flow in the exchanger.

(b) What is ΔS_{total} in each case?

(c) Repeat parts (a) and (b) for countercurrent flow if the heating stream enters at 200 deg C.

Solution

This problem is answered using Equation 5.14, with constant C_p and assuming constant pressure. Note that the outlet temperature of the hot gas in both cases is equal to 200 deg C. This is because the gas and flow rates are identical in the hot and cold streams.

$$\begin{aligned} (* \text{ } mC_p\Delta T_c &= -mC_p\Delta T_h *) \\ (* \Delta T_c &= -\Delta T_h *) \\ (* 190 - 70 &= - (T_{\text{hout}} - 320) *) \end{aligned}$$

```
In[ ]:= Solve[ (190 - 70) == - (Thout - 320) ]
Out[ ]:= { {Thout -> 200} }
```

```
In[ ]:= (*Gas constant and heat capacity*)
R = 8.314; (*J/mol*K*)
Cp = (7 / 2) R;
```

Solution, Part (a)

$$\begin{aligned} (*\text{Cold-side temperatures, K}*) \\ 190 + 273.15 \\ 70 + 273.15 \end{aligned}$$

```
Out[ ]:= 463.15
```

```
Out[ ]:= 343.15
```

$$\Delta S_{\text{cold}} = C_p * \text{Log} \left[\frac{(463.15)}{(343.15)} \right] \quad (* // \text{ANS}, \frac{\text{J}}{\text{mol} * \text{K}} *)$$

```
Out[ ]:= 8.72630433134
```

$$\begin{aligned} (*\text{Hot-side temperatures, K}*) \\ (200 + 273.15) \\ (320 + 273.15) \end{aligned}$$

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Out[ ]:= 473.15
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Out[ ]:= 593.15
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$$\Delta \text{Shot} = \text{Cp} * \text{Log} \left[\frac{(473.15)}{(593.15)} \right] \quad (// \text{ANS}, \frac{\text{J}}{\text{mol} * \text{K}} *)$$

Out[]=

-6.57738825009

Solution, Part (b)

$$\Delta \text{Scold} + \Delta \text{Shot} \quad (// \text{ANS}, \frac{\text{J}}{\text{mol} * \text{K}} *)$$

Out[]=

2.14891608125

Solution, Part (c)

$$(* \text{ mCp} \Delta \text{Tc} = -\text{mCp} \Delta \text{Th} *)$$

$$(* \Delta \text{Tc} = -\Delta \text{Th} *)$$

$$(* 190 - 70 = -(\text{Thout} - 320) *)$$

In[]:= Solve[(190 - 70) == - (Thout - 200)]

Out[]=

{ {Thout -> 80} }

$$(* \text{Cold-side temperatures, K} *)$$

$$190 + 273.15$$

$$70 + 273.15$$

Out[]=

463.15

Out[]=

343.15

$$\Delta \text{Scold} = \text{Cp} * \text{Log} \left[\frac{(463.15)}{(343.15)} \right] \quad (// \text{ANS}, \frac{\text{J}}{\text{mol} * \text{K}} *)$$

Out[]=

8.72630433134

In[]:= (*Hot-side temperatures*)

$$(80 + 273.15)$$

$$(200 + 273.15)$$

Out[]=

353.15

Out[]=

473.15

$$\Delta \text{Shot} = \text{Cp} * \text{Log} \left[\frac{(353.15)}{(473.15)} \right] \quad (// \text{ANS}, \frac{\text{J}}{\text{mol} * \text{K}} *)$$

Out[]=

-8.51202688302

```
 $\Delta S_{\text{cold}} + \Delta S_{\text{hot}} \left( \frac{\text{J}}{\text{mol} \cdot \text{K}} \right)$ 
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```
Out[ ]=
```

```
0.214277448322
```