
Problem Set 8 - Solutions

Problem 5.2

A Carnot engine receives 250 kJ s⁻¹ of heat from a heat source reservoir at 525 °C and rejects heat to a heat-sink reservoir at 50 °C. What are the power developed and the heat ejected?

Solution

Use equation 5.7 to compute efficiency, and then use 5.6 to get the work, based on the heat received.

```
In[*]:=  $\eta = 1 - \frac{50 + 273.15}{525 + 273.15}$ 
Out[*]= 0.5951262
```

```
In[*]:=  $Q_H = 250 \cdot \frac{\text{kJ}}{\text{sec}}; (*\text{heat in is positive}*)$ 
 $W = -\eta * Q_H (*\text{work out is negative}*)$ 
Out[*]=  $-\frac{148.7816 \text{ kJ}}{\text{sec}}$ 
```

The power developed is -148.782 kJ/sec = -148.782 kW. //ANS

To compute the heat ejected, use eq 2.3 (first law) with $\Delta U=0$, that is $Q_C+Q_H+W=0$:

```
In[*]:=  $Q_C = -Q_H - W$ 
Out[*]=  $-\frac{101.2184 \text{ kJ}}{\text{sec}}$ 
```

```
In[*]:=  $\text{Solve}[Q_C + 250 + (-148.782) == 0, Q_C]$ 
Out[*]= { {QC → -101.218} }
```

Heat rejected is -101.218 kJ/sec = -101.218 kW. //ANS

Problem 5.6

Which is the more effective way to increase the thermal efficiency of a Carnot engine: to increase T_H with T_C constant, or to decrease T_C with T_H constant? For a real engine, which would be the more practical way?

Solution

```
In[*]:= Clear["Global`*"]
```

```
In[*]:= (*Extra typesetting function used to add parentheses*)
parens[e_] := DisplayForm[RowBox[{"(", MakeBoxes[e], ")"}]]
(*This is not part of the solution*)
```

Since η is a function of two variables, T_C and T_H , that is, $\eta = \eta(T_C, T_H)$, a general solution is obtained from the partial derivatives to answer this question:

```
Out[*]//TraditionalForm=
```

$$\eta = 1 - \frac{T_C}{T_H}$$

The relationship between the partial derivatives and the total differential is shown below.

```
Out[*]//TraditionalForm=
```

$$d\eta = \left(\frac{\partial \eta}{\partial T_C} \right)_{T_H} dT_C + \left(\frac{\partial \eta}{\partial T_H} \right)_{T_C} dT_H$$

The partial derivatives represent the slopes of the function η in the T_C and T_H directions.

```
Out[*]//TraditionalForm=
```

$$\left(\frac{\partial \eta}{\partial T_C} \right)_{T_H} = -\frac{1}{T_H}$$

```
Out[*]//TraditionalForm=
```

$$\left(\frac{\partial \eta}{\partial T_H} \right)_{T_C} = -T_C \left(-\frac{1}{T_H^2} \right) = \frac{1}{T_H} \frac{T_C}{T_H}$$

Examine the partial derivative of η with respect to T_C . First note that T_H is always positive. Therefore, this partial derivative is always negative. This means that the η function has a negative slope in the T_C direction. Therefore, η decreases as T_C increases. Or, in the other direction, η increases as T_C decreases. The problem statement specifically asks about decreasing T_C , which clearly increases η .

Now look at the other partial derivative. Since T_H and T_C are always positive, the derivative of η with respect to T_H is always positive, the slope is positive in the T_H direction, and η increases as T_H increases. The problem asks about increasing T_H , which increases η .

Now compare the “size” of the two partial derivatives to see which one is larger and therefore causes a larger change. Use absolute values to compare the sizes of the derivatives. By premise, the cold reservoir temperature is less than the warm reservoir temperature. Since $T_C < T_H$,

```
Out[*]//TraditionalForm=
```

$$\frac{1}{T_H} \frac{T_C}{T_H} < \left| -\frac{1}{T_H} \right|$$

Therefore,

Out[*]//TraditionalForm=

$$\left| \left(\frac{\partial \eta}{\partial T_H} \right)_{T_C} \right| < \left| \left(\frac{\partial \eta}{\partial T_C} \right)_{T_H} \right|$$

Therefore, since the derivative with respect to T_C is larger, decreasing T_C with T_H held constant is more effective at increasing thermal efficiency η than increasing T_H with T_C held constant. This is a general solution and applies for all $T_C > 0$, all $T_H > 0$, and for $T_H > T_C$.

In practical terms, the situation is the opposite. In a real engine, it is more practical to increase T_H with T_C held constant, because the cold reservoir for a working engine is normally the surrounding, and T_C is normally the ambient temperature, which is difficult to change by design. T_H can be changed by redesigning the engine to use a different fuel, or changing the amount of heat removed from the engine, or changing the pressure of the engine, etc.

Problem 5.8

With respect to 1 kg of liquid water:

- (a) Initially at 0 °C, it is heated to 100 °C by contact with a heat reservoir at 100 °C. What is the entropy change of the water? What is the entropy change of the heat reservoir? What is ΔS_{total} ?
- (b) Initially at 0 °C, it is first heated to 50 °C by contact with a heat reservoir at 50 °C, and then heated to 100 °C by contact with a heat reservoir at 100 °C. What is ΔS_{total} ?
- (c) Explain how the water might be heated from 0 to 100 °C so that $\Delta S_{\text{total}} = 0$.

Solution to Part (a)

```
In[*]:= m = 1; (*kg*)
Cp = 4.184; (*kJ/kg*K*)
T2 = 100 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)
```

Reservoir:

```
In[*]:= Qhot = -m * Cp * (T2 - T1); (*kJ*)
DeltaShot = Qhot / T2 (*kJ/K*)
Out[*]:= -1.121265
```

Water:

```
In[*]:= DeltaSwater = Integrate[m * Cp / T, {T, T1, T2}] (*kJ/K*)
Out[*]:= 1.305238
```

Total:

```
In[*]:= DeltaStotal = DeltaShot + DeltaSwater (*kJ/K*)
Out[*]:= 0.1839732
```

The ΔS of the hot reservoir is -1.12123 kJ/K. //ANS

The ΔS of the cold reservoir is 1.30524 kJ/K. //ANS

The total entropy change is 0.18397 kJ/K. //ANS

Solution to Part (b)

```
In[*]:= m = 1; (*kg*)
Cp = 4.184; (*kJ/kg*degC*)
T3 = 100 + 273.15; (*K*)
T2 = 50 + 273.15; (*K*)
T1 = 0 + 273.15; (*K*)
```

Reservoir 1 (50°C):

```
In[*]:= Qhot1 = -m * Cp * (T2 - T1); (*kJ*)

In[*]:= ΔShot1 =  $\frac{Qhot1}{T2}$  (*kJ/K*)

Out[*]:=
-0.6473774
```

Water, step 1 (0 to 50 °C):

```
In[*]:= ΔSw1 =  $\int_{T1}^{T2} \frac{m * Cp}{T} dT$  (*kJ/K*)

Out[*]:=
0.7033116
```

Reservoir 2 (100°C):

```
In[*]:= Qhot2 = -m * Cp * (T3 - T2); (*kJ*)

In[*]:= ΔShot2 =  $\frac{Qhot2}{T3}$  (*kJ/K*)

Out[*]:=
-0.5606325
```

Water, step 2 (50 to 100 °C):

```
In[*]:= ΔSw2 =  $\int_{T2}^{T3} \frac{m * Cp}{T} dT$  (*kJ/K*)

Out[*]:=
0.6019264
```

Total:

```
In[*]:= ΔStotal = ΔShot1 + ΔSw1 + ΔShot2 + ΔSw2 (*kJ/K*)

Out[*]:=
0.09722823
```

ΔS of the hot reservoir in step 1 is -0.64738 kJ/K and in step 2 is -0.56063 kJ/K. //ANS

ΔS of the cold reservoir in step 1 is 0.70331 kJ/K and in step 2 is 0.60192 kJ/K. //ANS

The total entropy change is is 0.097228 kJ/K. //ANS

Solution to Part (c)

The reversible heating of the water requires an infinite number of heat reservoirs covering the range of temperatures from 273.15 to 373.15 K, each one exchanging an infinitesimal quantity of heat with the water and raising its temperature by a differential increment.

Problem 5.10

An ideal gas, $C_p = (7/2)R$, is heated in a steady-flow heat exchanger from 70 to 190 °C by another stream of the same gas, which enters at 320 °C. The flow rates of the two streams are the same, and heat losses from the exchanger are negligible.

(a) Calculate the molar entropy changes of the two gas streams for both parallel and countercurrent flow in the exchanger.

(b) What is ΔS_{total} in each case?

(c) Repeat (a) and (b) for countercurrent flow if the heating stream enters at 200 °C.

Solution to Part (a):

The outlet temperature of the hot gas in both co-current and counter-current flow is 200 °C:

(*Level 1 enthalpy balance gives $m \cdot C_p \cdot \Delta T_c = -m \cdot C_p \cdot \Delta T_h$ *)

(*Both m and C_p cancel, so $\Delta T_c = -\Delta T_h$ *)

(* $190 - 70 = - (T_{\text{out}} - 320)$ *)

In[*]:= Solve[(190 - 70) == - (Thout - 320)]

Out[*]=

{ { Thout → 200 } }

In[*]:= (*Gas constant and heat capacity*)

R = 8.314; (*J/mol*K*)

Cp = (7 / 2) R; (*J/mol*K*)

Cold stream:

70 + 273.15 (*Cold-side inlet T in K *)

190 + 273.15 (*Cold-side outlet T in K *)

Out[*]=

343.15

Out[*]=

463.15

Use the integrated form of Equation 5.10, with constant C_p and assuming constant pressure:

In[*]:= $\Delta S_{\text{cold}} = C_p * \text{Log}\left[\frac{463.15}{343.15}\right] - R * \text{Log}\left[\frac{P}{P}\right]$

Out[*]=

8.726 304

The molar entropy change of cold stream gas in co-current (parallel) flow is $8.726 \frac{\text{J}}{\text{mol}\cdot\text{K}}$ //ANS

Hot stream:

```
In[*]:= 320 + 273.15 (*Hot-side inlet temperature in K *)
200 + 273.15 (*Hot-side outlet temperature in K *)

Out[*]=
593.15

Out[*]=
473.15
```

```
In[*]:= ΔShot = Cp * Log[ $\frac{473.15}{593.15}$ ] - R * Log[ $\frac{P}{p}$ ]

Out[*]=
-6.577 388
```

The molar entropy change of hot stream gas in co-current flow is $-6.577 \frac{J}{mol \cdot K}$ //ANS

The molar entropy changes of the gas streams in counter-current and co-current flow are the same because all stream temperatures are the same. //ANS

Solution to Part (b) - total entropy change:

```
In[*]:= ΔScold + ΔShot

Out[*]=
2.148 916
```

The total molar entropy change of hot and cold gas streams in co-current (parallel) flow is $2.149 \frac{J}{mol \cdot K}$ //ANS

Solution to Part (c):

Countercurrent only.

The outlet temperature of the hot gas is 80 °C:

```
In[*]:= (*190-70==-(Thout-200) *)
Solve[(190 - 70) == -(Thout - 200)]

Out[*]=
{{Thout -> 80}}
```

Cold stream:

No addition calculations are needed. The cold stream entropy change is the same as part (a) because the cold-stream inlet and outlet temperatures are the same. As in part (a), the molar entropy change of cold stream gas in co-current flow is $8.726 \frac{J}{mol \cdot K}$ //ANS

Hot stream:

The hot stream temperatures are different, so the entropy change needs to be recalculated:


```
In[*]:= (*Hot-side temperatures*)
(80 + 273.15)
(200 + 273.15)
```

```
Out[*]=
353.15
```

```
Out[*]=
473.15
```

```
In[*]:= ΔShot = Cp * Log[ (353.15)
(473.15) ] - R * Log[ P
P ]
```

```
Out[*]=
-8.512 027
```

The molar entropy change of hot stream gas in co-current (parallel) flow is $-8.512 \frac{\text{J}}{\text{mol}\cdot\text{K}}$ //ANS

Total entropy change:

```
In[*]:= ΔScold + ΔShot
```

```
Out[*]=
0.214 277 4
```

The total molar entropy change of hot and cold gas streams in co-current (parallel) flow is $0.214 \frac{\text{J}}{\text{mol}\cdot\text{K}}$ //ANS