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## Problem Set 9 - Solutions

### Problem 5.43

Heat in the amount of 150 kJ is transferred directly from a hot reservoir at  $T_H = 550\text{ K}$  to two cooler reservoirs at  $T_1 = 350\text{ K}$  and  $T_2 = 250\text{ K}$ . The surroundings temperature is  $T_\sigma = 300\text{ K}$ . If the heat transferred to the reservoir at  $T_1$  is half that transferred to the reservoir at  $T_2$ , calculate:

- (a) The entropy generation in kJ/K.
- (b) The lost work.
- (c) How could the process be made reversible?

### Solution - Part (a)

$$\begin{aligned} \text{In[*]} := & \quad Q_H = -150.; \text{ (*kJ*)} \\ & T_H = 550.; \text{ (*K*)} \end{aligned}$$

$$\begin{aligned} Q_1 &= 50.; \\ T_1 &= 350.; \end{aligned}$$

$$\begin{aligned} Q_2 &= 100.; \\ T_2 &= 250.; \end{aligned}$$

$$\text{In[*]} := S_G = \frac{Q_H}{T_H} + \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \quad \text{(*}\frac{\text{kJ}}{\text{K}}\text{*)}$$

$$\begin{aligned} \text{Out[*]} := & \\ & 0.27012987013 \end{aligned}$$

The total entropy generation is  $0.27013 \frac{\text{kJ}}{\text{K}}$ . //ANS

### Solution - Part (b)

Use equation 5.29:

$$\begin{aligned} T_\sigma &= 300.; \\ W_{\text{lost}} &= T_\sigma * S_G \text{ (*kJ*)} \\ & 81.038961039 \end{aligned}$$

The lost work is 81.039 kJ. //ANS

### Solution - Part (c)

By the second law, the process can be made to be reversible if the entropy generation is somehow made to be zero. One possibility is to couple some sort of additional reservoir that could remove

entropy by heat transfer. Another possibility would be to add flowing streams to adjust the entropy balance to zero. A third possibility would be to add some kind of chemical reaction to consume entropy. //ANS

$$\text{eq1} = 0 == \frac{Q_H}{T_H} + \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + s$$

Out[\*]=

$$0 == 0.27012987013 + s$$

**Solve[eq1, s]**

Out[\*]=

$$\{\{s \rightarrow -0.27012987013\}\}$$

One can add a device or chemical reaction that removes  $0.27013 \frac{\text{kJ}}{\text{K}}$  of entropy. //ANS

## Problem 5.44

A nuclear power plant generates 750 MW. The reactor temperature is 315 °C and a river with water temperature of 20 °C is available.

(a) What is the maximum possible thermal efficiency of the plant, and what is the minimum rate at which heat must be discarded to the river?

(b) If the actual thermal efficiency of the plant is 60% of the maximum, at what rate must heat be discarded to the river, and what is the temperature rise of the river if it has a flow rate of 165 cubic meters per second?

### Solution - Part (a)

```
In[*]:= Tc = 273.13 + 20; (*river temperature in K*)
Th = 273.13 + 315; (*reactor temperature in K*)
```

```
In[*]:= ηmax = 1 - Tc / Th (*eq. 5.7*)
```

```
Out[*]=
0.501589784571
```

The maximum possible thermal efficiency  $\eta_{\max}$  is 0.5016. //ANS

```
In[*]:= QH = 750 / ηmax; (*eq. 5.6*)
```

```
In[*]:= QC = QH - 750
```

```
Out[*]=
745.245762712
```

At least 745.3 MW of heat are discarded to the river. //ANS

### Solution - Part (b)

```
In[*]:= η = .60 * ηmax;
```

```
QH = 750 / η;
```

```
In[*]:= QC = QH - 750
```

```
Out[*]=
1742.07627119
```

At least 1742.2 MW of heat are discarded to the river at  $\eta = 0.6 \eta_{\max}$ . //ANS

```
In[*]:= eq1 = (165 m^3 / s) * (1000 L / m^3) * (1 kg / L) * (4.184 kJ / kg * degC) * ΔT == 1742.16 MW * (1000 kW / MW) * (1 kJ / s / kW); (*QC=ṁCpΔT*)
```

```
In[*]:= Solve[eq1, ΔT]
```

```
Out[*]=
{ {ΔT → 2.52355292891 degC} }
```

The temperature increase of the river is 2.52 °C. //ANS

## Problem 5.50

Ethylene vapor is cooled at atmospheric pressure from 830 to 35 °C by direct heat transfer to the surroundings at 25 °C. With respect to this surroundings temperature, what is the lost work of the process in kJ/mol?

Show that the same result is obtained as the work which can be derived from reversible heat engines operating with the ethylene vapor as the heat source and the surroundings as the sink. The heat capacity of ethylene is given in Table C.1 of App. C.

### Solution - Part (a)

This problem involves integrating the heat capacity polynomial. Recall that temperature must be in K and heat capacity in Appendix C is dimensionless.

Gather information:

$$T_1 = 273.15 + 830; \text{ (*initial ethylene T*)}$$

$$T_2 = 273.15 + 35; \text{ (*final ethylene T*)}$$

$$T_\sigma = 273.15 + 25; \text{ (*surroundings T*)}$$

(\*gas constant, needed for Cp\*)

$$R = 8.314; \text{ (* } \frac{\text{J}}{\text{mol}\cdot\text{K}} \text{ *)}$$

(\*Cp coefficients from App. C\*)

$$a = 1.424;$$

$$b = 14.394 \times 10^{-3};$$

$$c = -4.392 \times 10^{-6};$$

$$C_p = a + b \cdot T + c \cdot T^2;$$

Ethylene is changing T so S and H both change. Calculate the  $\Delta S$  by integrating  $\frac{C_p}{T}$  with respect to T. Calculate the  $\Delta H$  by integrating  $C_p$  with respect to T.

$$\text{In[*]:= } \Delta S_{fs} = R \cdot \int_{T_1}^{T_2} \frac{C_p}{T} dT \text{ (*in units of } \frac{\text{J}}{\text{mol}\cdot\text{K}} \text{ *)}$$

Out[\*]=

$$-89.7532549813$$

$$\text{In[*]:= } Q_{\text{ethylene}} = R \cdot \int_{T_1}^{T_2} C_p dT \text{ (* } Q=\Delta H \text{ at constant pressure, in units of J/mol*)}$$

Out[\*]=

$$-60563.0087621$$

$$\text{In[*]:= } W_{\text{lost}} = T_\sigma \cdot \Delta S_{fs} - Q_{\text{ethylene}} \text{ (*equation 5.31*)}$$

Out[\*]=

$$33803.0757894$$

The lost work is  $33.8031 \frac{\text{kJ}}{\text{mol}}$ . //ANS

## Solution - Part (b)

To analyze this process as a reversible engine with the Carnot equations, we need values for  $Q_H$  and  $Q_C$ . It should be clear that the ethylene vapor is the heat source. That is, the heat added to the engine (the system) comes from the ethylene. This is  $Q_H$  in the reversible (Carnot) cycle, and the value is the same as it was before, but must be opposite in sign because it enters the engine.

$Q_H = -Q_{\text{ethylene}}$  (\*Heat transferred ( $Q_H$ ) is positive with respect to the engine.\*)

Out[\*]=

60563.0087621

To obtain  $Q_C$ , we need to understand a couple of things. First, equation 5.31 (shown below) gives the lost work in terms of entropy change of the flowing streams and  $Q$  dissipated to the surroundings. Second, the lost work is zero for a reversible process. Since this  $Q$  is heat transferred to the surroundings, it is also  $Q_C$  in the reversible (Carnot) cycle. Because the temperature change of the ethylene is the same as it was in part (a), the entropy change of the ethylene is also the same. So we calculate the  $Q$  that zeros the lost work in equation 5.31 and make this equal to  $Q_C$  in the Carnot engine.

In Equation 5.31,  $W_{\text{lost}} = T_\sigma \Delta S - Q$ , and  $W_{\text{lost}} = 0$  for a reversible process.

$\text{sol} = \text{Solve}[0 == T_\sigma * \Delta S_{fs} - Q, Q]$  (\*J/mol\*)

Out[\*]=

{ {  $Q \rightarrow -26759.9329727$  } }

$Q_C = Q /. \text{sol}[[1]]$  (\*heat transferred is negative with respect to the system.\*)

Out[\*]=

-26759.9329727

$Q_C = -26759.9$ ; (\*heat transferred is negative with respect to the system.\*)

Finally, use the first law  $\Delta U = Q + W$  (equation 2.3), where  $\Delta U = 0$  for a cyclic process, and where  $Q = Q_H + Q_C$  as explained on page 181. This gives  $W = -Q_H - Q_C$ .

$W_{\text{heatengine}} = -Q_H - Q_C$  (\*J/mol\*)

Out[\*]=

-33803.1087621

The work derived from reversible heat engine is  $-33.8031 \frac{\text{kJ}}{\text{mol}}$ . The sign is negative because work is leaving the system.

The absolute value is the same value as obtained in part (a). //ANS

## Problem 5.17

A Carnot engine operates between temperature levels of 600 K and 300 K. It drives a Carnot refrigerator, which provides cooling at 250 K and discards heat at 300 K. Determine a numerical value for the ratio of heat extracted by the refrigerator (“cooling load”) to the heat delivered to the engine (“heating load”).

### Solution

Calculate the efficiency and work of the Carnot engine:

```
In[*]:= THE = 600.; (*Temperature of the engine heat reservoir*)
TCE = 300.; (*Temperature of the engine cold reservoir*)
```

```
In[*]:=  $\eta = 1 - \frac{TCE}{THE}$ 
Out[*]:=
0.5
```

```
In[*]:= WE = QHE *  $\eta$ 
Out[*]:=
0.5 QHE
```

Calculate the COP and work of the Carnot refrigerator. Equations are shown in the Lesson 26 slide deck:

```
In[*]:= THR = 300.;
TCR = 250.;

In[*]:=  $COP = \frac{TCR}{THR - TCR}$ 
Out[*]:=
5.
```

```
In[*]:= WR = QCR / COP
Out[*]:=
0.2 QCR
```

Equate the engine work and refrigerator work and simplify:

```
In[*]:= Simplify[WE == WR]
Out[*]:=
1. QCR == 2.5 QHE
```

The ratio of the cooling load to the heating load is  $2.5 \left( \frac{QCR}{QHE} = 2.5 \right)$ . //ANS