Mathematica Quiz, AY26-1

Weight: This quiz is worth 60 points.

Scope: The quiz covers the “Getting Started” Document.

Instructions: There is no time limit for this assignment. You may not get assistance from any other cadets or instructors. You may refer to the “Getting Started” document as much as necessary. You may also search the documentation center or google as much as necessary. Submit your completed Mathematica notebook file in Canvas with a cover sheet.

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Problem 1

(3 points each.) In Part (a), define the function $f(x) = \sin\left(x^2\right)$ in Mathematica, and then use *f(x)* to evaluate the limits in Parts (b) through (d) below.

(a)

Out[ ]//TraditionalForm=

$$f(x) = \sin\left(x^2\right)$$

(b)

Out[ ]//TraditionalForm=

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

(c)

Out[ ]//TraditionalForm=

$$\lim_{x \to \frac{\pi}{2}} f(x)$$

(d)

Out[ ]//TraditionalForm=

$$\lim_{x \to \sqrt{\pi}} f'(x)$$

Problem 2

(3 points each.) Use Mathematic to find an exact solution for x for each of the given functions in Parts (a) through (d).

(a)

Out[ ]//TraditionalForm=

$$7 x^2 + 3 x - 5 = a$$

(b)

Out[ ]//TraditionalForm=

$$x^2 + 6x + 9 = 0$$

(c)

Out[ ]//TraditionalForm=

$$x^2 - 2x + 5 = 0$$

(d)

Out[ ]//TraditionalForm=

$$x^3 - 2x^2 + 7x + 26 = a$$

Problem 3

(3 points each.) Enter the following expressions in Mathematica and then use the ReplaceAll function to make the substitution a=1 in each of the three expressions.

(a)

Out[ ]//TraditionalForm=

$$x^3 - 2x^2 + 7x + 26 = a$$

(b)

Out[ ]//TraditionalForm=

$$\sin(a x^2) + \cos(a x^2) = 0$$

(c)

Out[ ]//TraditionalForm=

$$e^{-a t}$$

Problem 4

(3 points each.)

(a) For the following function, find all exact solutions for *x* (the answer is a list of three rules):

Out[ ]//TraditionalForm=

$$3x^3 - 3x^2 + 17x + 6 = a$$

(b) Use the list of rules (solutions) from part (a) and the “Part” function to isolate the real solution.

(c) Use your answer for part (b) and the “N” function to create a new function that reports the approximate numerical value of *x* as a function of *a*.

(d) Use your function from part (c) to create a plot of *x* versus *a*, with *a* varying from negative infinity to positive infinity and with a *PlotRange* of -5 to 5.

(e) Use your function from part (c) and “Solve” to find the value of *a* that makes *x=0*.

Problem 5

(2 points each.) Use the natural language function to find the following:

(a) Heat capacity of water.

(b) Critical pressure of argon.

(c) Critical temperature of oxygen.

(d) Van der Waals constants for nitrogen.

(e) SRK equation of state.

(f) Convert 1.01 grams per mL to lbs per cubic foot.

Mathematica Quiz, AY26-1, Solutions

Problem 1

Part (a)

In[ ]:= `f[x_] = Sin[x^2]`

Out[ ]=

$\text{Sin}[x^2]$

Part (b)

In[ ]:= $\text{Limit}\left[\frac{f[x+\Delta x]-f[x]}{\Delta x}, \Delta x \to 0\right]$

Out[ ]=

$2\,x\,\text{Cos}[x^2]$

Part (c)

In[ ]:= $\text{Limit}\left[f[x], x \to \frac{\pi}{2}\right]$ // N

Out[ ]=

0.624266

In[ ]:= $\text{Limit}\left[f[x], x \to \frac{\pi}{2}\right]$

Out[ ]=

$\text{Sin}\left[\frac{\pi^2}{4}\right]$

Part (d)

In[ ]:= $\text{Limit}\left[f'[x], x \to \sqrt{\pi}\right]$ // N

Out[ ]=

-3.544908

In[ ]:= $\text{Limit}\left[f'[x], x \to \sqrt{\pi}\right]$

Out[ ]=

$-2\sqrt{\pi}$

Problem 2

Part (a)

In[ ]:= $\text{Solve}\left[7\,x^2 + 3\,x - 5 == a, x\right]$

Out[ ]=

$\left\{\left\{x \to \frac{1}{14}\left(-3-\sqrt{149+28\,a}\right)\right\}, \left\{x \to \frac{1}{14}\left(-3+\sqrt{149+28\,a}\right)\right\}\right\}$

Part (b)

In[ ]:= `Solve[x^2 + 6 x + 9 == 0, x]`

Out[ ]=

$\{\{x \to -3\}, \{x \to -3\}\}$

Part (c)

In[ ]:= `Solve[x^2 - 2 x + 5 == 0, x]`

Out[ ]=

$\{\{x \to 1 - 2\,i\}, \{x \to 1 + 2\,i\}\}$

Part (d)

In[ ]:= `Solve[x^3 - 2 x^2 + 7 x + 26 == a, x]`

Out[ ]=

$$\Bigg\{\Bigg\{x \to \frac{2}{3} - \frac{17 \times 2^{1/3}}{3\left(-812 + 27\,a + 3\sqrt{3}\sqrt{25148 - 1624\,a + 27\,a^2}\right)^{1/3}} + \frac{\left(-812 + 27\,a + 3\sqrt{3}\sqrt{25148 - 1624\,a + 27\,a^2}\right)^{1/3}}{3 \times 2^{1/3}}\Bigg\},$$

$$\Bigg\{x \to \frac{2}{3} + \frac{17\left(1 + i\sqrt{3}\right)}{3 \times 2^{2/3}\left(-812 + 27\,a + 3\sqrt{3}\sqrt{25148 - 1624\,a + 27\,a^2}\right)^{1/3}} - \frac{\left(1 - i\sqrt{3}\right)\left(-812 + 27\,a + 3\sqrt{3}\sqrt{25148 - 1624\,a + 27\,a^2}\right)^{1/3}}{6 \times 2^{1/3}}\Bigg\},$$

$$\Bigg\{x \to \frac{2}{3} + \frac{17\left(1 - i\sqrt{3}\right)}{3 \times 2^{2/3}\left(-812 + 27\,a + 3\sqrt{3}\sqrt{25148 - 1624\,a + 27\,a^2}\right)^{1/3}} - \frac{\left(1 + i\sqrt{3}\right)\left(-812 + 27\,a + 3\sqrt{3}\sqrt{25148 - 1624\,a + 27\,a^2}\right)^{1/3}}{6 \times 2^{1/3}}\Bigg\}\Bigg\}$$

Problem 3

Part (a)

In[ ]:= `x^3 - 2 x^2 + 7 x + 26 == a /. a → 1`

Out[ ]=

$26 + 7\,x - 2\,x^2 + x^3 == 1$

Part (b)

In[ ]:= `Sin[a x^2] + Cos[a x^2] == 0 /. a → 1`

Out[ ]=

$\mathrm{Cos}[x^2] + \mathrm{Sin}[x^2] == 0$

Part (c)

In[∘]:=
```
Exp[-a t] /. a → 1
```
Out[∘]=

$$e^{-t}$$

Problem 4

Part (a)

In[∘]:=
```
ans = Solve[3 x^3 - 3 x^2 + 17 x + 6 == a, x]
```
Out[∘]=

$$\left\{\left\{x \to \frac{1}{3} - \frac{16 \times 2^{1/3}}{3\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}} + \frac{\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}}{3 \times 2^{1/3}}\right\},\right.$$

$$\left\{x \to \frac{1}{3} + \frac{8 \times 2^{1/3}\left(1 + i\sqrt{3}\right)}{3\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}} - \frac{\left(1 - i\sqrt{3}\right)\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}}{6 \times 2^{1/3}}\right\},$$

$$\left.\left\{x \to \frac{1}{3} + \frac{8 \times 2^{1/3}\left(1 - i\sqrt{3}\right)}{3\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}} - \frac{\left(1 + i\sqrt{3}\right)\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}}{6 \times 2^{1/3}}\right\}\right\}$$

Part (b)

In[∘]:=
```
ans[[1]]
```
Out[∘]=

$$\left\{x \to \frac{1}{3} - \frac{16 \times 2^{1/3}}{3\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}} + \frac{\left(-103 + 9\,a + \sqrt{26\,993 - 1\,854\,a + 81\,a^2}\right)^{1/3}}{3 \times 2^{1/3}}\right\}$$

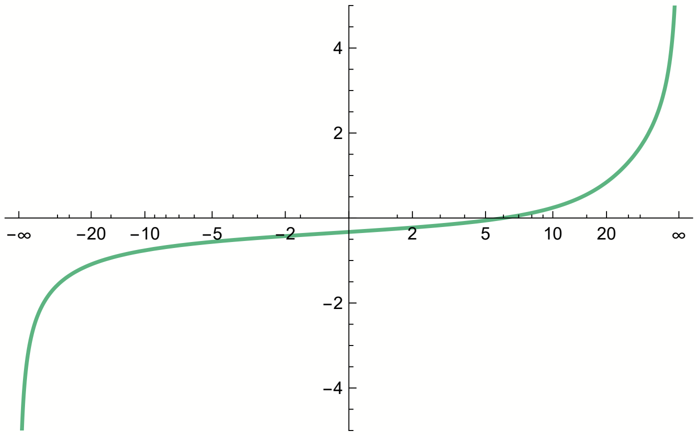
Part (c)

In[∘]:=
```
f[a_] = x /. ans[[1]] // N
```
Out[∘]=

$$0.333\,333\,3 - \frac{6.719\,579}{\left(-103. + 9.\,a + \sqrt{26\,993. - 1\,854.\,a + 81.\,a^2}\right)^{1/3}} + 0.264\,566\,8\left(-103. + 9.\,a + \sqrt{26\,993. - 1\,854.\,a + 81.\,a^2}\right)^{1/3}$$

Part (d)

In[ ]:=
```
Plot[f[a], {a, -∞, +∞}, PlotRange → {-5, 5}]
```
Out[ ]=



Part (e)

In[ ]:=
```
Solve[f[a] == 0, a]
```
Out[ ]=
```
{{a → 6.}}
```

In[ ]:=
```
f[6]
```
Out[ ]=
```
0.
```

Problem 5

Part (a)

In[ ]:=

heat capacity of water »

Result

Out[ ]=

gas	1.865 J/(g K) (joules per gram kelvin difference)
liquid	4.18 J/(g K) (joules per gram kelvin difference)

In[ ]:=

specific heat of water »

↳ Result

Out[ ]=

gas	1.865 J/(g K) (joules per gram kelvin difference)
liquid	4.18 J/(g K) (joules per gram kelvin difference)

Part (b)

In[ ]:=

critical pressure of argon

argon ELEMENT [critical pressure p_c]

Out[ ]=

4.898 MPa

Part (c)

In[ ]:=

critical temperature of oxygen »

↳ Result

154.59 K

Out[ ]=

154.59 K

Part (d)

In[ ]:=

van der waals constants for nitrogen

↳ Result

```
TableForm[{{1.37 L^2bar/mol^2}, {0.0387 L/mol}}, TableHeadings → {{"a", "b"}, None}]
```

Out[ ]//TableForm=

a	1.37 L^2bar/mol^2
b	0.0387 L/mol

Part (e)

In[ ]:=

Soave-Redlich-Kwong equation of state

Using closest Wolfram|Alpha interpretation: **Redlich–Kwong equation**

More interpretations: Soave

Computational Inputs:

Calculate pressure

- Redlich-Kwong constant a: 0.336 pascal meter to the sixth square root kelvins per mole squared
- Redlich-Kwong constant b: 0.0011 L/mol
- molar volume: 0.0245 L/mol
- temperature: 298.15 K

Compute

Input interpretation

Redlich–Kwong equation of state

Equation

$$P = \frac{R\,T}{V_m - b} - \frac{a}{\sqrt{T}\;V_m\,(V_m + b)}$$

P	pressure
a	Redlich-Kwong constant a
b	Redlich-Kwong constant b
V_m	molar volume
T	temperature
R	molar gas constant ($\approx$ 8.314 J/(mol K))

Input values

Redlich-Kwong constant a	0.336 Pa m^6 $K^{1/2}$/mol^2 (pascal meter to the sixth square root kelvins per mole squared)
Redlich-Kwong constant b	0.0011 L/mol (liters per mole)
molar volume	0.0245 L/mol (liters per mole)
temperature	298.2 K (kelvins)

Result — More units — ☑ Step-by-step solution

pressure	74.91 MPa (megapascals) = 10 865 psi (pounds-force per square inch) = 1.565×10^6 lbf/ft^2 (pounds-force per square foot)

WolframAlpha

Part (f)

In[ ]:=

convert 1.01 grams per mL to lbs per cubic foot

```
UnitConvert[1.01 g/mL, "Pounds" / "Feet"^3]
```

Out[ ]=

63.05224 lb/ft^3