Problem 7-1

What funds will be available 10 years from now if $10,000 is deposited at a nominal interest rate of 6% compounded semiannually?

Solution:

$$F = 10000 \cdot \left(1 + \frac{0.06}{2}\right)^{2 \cdot 10} = \underline{\underline{\$18061}}_{\text{ANS}}$$

Problem 7-2

The original cost for a distillation tower is $50,000, and the useful life of the tower is estimated to be 10 years. How much must be placed annually in an annuity at an interest rate of 6% to obtain sufficient funds to replace the tower at the end of 10 years? If the scrap value of the distillation tower is $5,000, determine the asset value (i.e., the total book value of the tower) at the end of 5 years based on straight line depreciation.

Solution:

If the scrap value is $5000 after 10 years, then straight-line depreciation means that the column is declining in value at a rate of $4500 per year, so the value after 5 years is:

$$50000 - 4500 \cdot 5 = \underline{\underline{\$27500}}_{\text{ANS}}$$

If $5,000 is available from the scrap value, then $5,000 of the $50,000 replacement cost will come from salvage, so the future value of the uniform series is $45,000. Therefore:

$$A = F \cdot (A/F, i, n) = F \cdot (A/F, 6\%, 10) = 45,000 \cdot 0.0759 = \$3,416$$

$$A = F \cdot (A/F, i, n) = F \cdot \frac{i}{(1+i)^N - 1} = \$45,000 \cdot \frac{.06}{(1+.06)^{10} - 1} = \$3,414$$

Problem 7-4

Derive an expression for capitalized cost based on annual discrete interest compounding. *Capitalized cost* is defined as the sum of the original cost C_V of the equipment or asset plus the amount P that must be invested when the original equipment or asset is purchased so that when the original equipment or asset is replaced in N years at a cost C_R, the value of the investment equals P plus C_R.

Solution:

Compound interest: $C_C = C_V + P$ (from problem statement)

$F = P \cdot (1+i)^N$ (compound interest equation)

$F = C_R + P$ (also, from problem statement)

This equation says that the future value is sufficient to cover replacement of the equipment plus enough principal for a new investment. Equating the two future values gives us

$$C_R + P = P \cdot (1+i)^N \;\Rightarrow\; P = \frac{C_R}{(1+i)^N - 1}$$

$$C_C = C_V + \frac{C_R}{(1+i)^N - 1}$$

ANS

Problem 7-5

A heat exchanger is to be used in a heating process. A standard type of heat exchanger with a negligible scrap value costs $20,000 and will have a useful life of 6 years. Another type of heat exchanger with equivalent design capacity is priced at $34,000 but has a useful life of 10 years and a scrap value of $4000. Assume an effective compound interest rate of 6% per year, and that the replacement cost of each heat exchanger is the same as that of the original exchanger. Determine which heat exchanger is cheaper by comparing the capitalized cost of each. See Problem 7-4 for a definition of capitalized cost.

Solution Using Derived Equation from Problem 7-4:

Exchanger A: $$C_{CA} = C_{VA} + \frac{C_{RA}}{(1+i)^N - 1} = 20000 + \frac{20000}{(1+.06)^6 - 1} = \$67,788$$

Exchanger B: $$C_C = C_V + \frac{C_{RB}}{(1+i)^N - 1} = 34000 + \frac{34000 - 4000}{(1+.06)^{10} - 1} = \$71,934$$

Exchanger A is cheaper by $4,146 in terms of capitalized cost.
ANS

Alternate Solution Using Equation on Page 231 of the FE Reference Manual:

$$C_C = C_V + P \text{, and } P = \frac{A}{i}$$

Heat Exchanger A: $$P = \frac{A}{i} = \frac{F \cdot (A/F, i, N)}{i} = \frac{20000 \cdot 0.1434}{0.06} = \$47,800$$

$$C_C = C_V + P = \$20,000 + \$47,800 = \$67,800$$

Heat Exchanger B: $$P = \frac{A}{i} = \frac{F \cdot (A/F, i, N)}{i} = \frac{30000 \cdot 0.0759}{0.06} = \$37,950$$

$$C_C = C_V + P = \$34,000 + \$37,950 = \$71,950$$

Exchanger A is cheaper by $4,150 in terms of capitalized cost.
ANS

Problem 7-9

The fixed capital investment for an existing chemical plant is \$20 million. Annual property taxes amount to 1% of the fixed-capital investment, and state income taxes are 5% of the gross earnings. The net income after all taxes is \$2 million, and the federal income taxes amount to 35% of gross earnings. If the same plant had been constructed for the same fixed capital investment but at a location where property taxes were 4% of the fixed capital investment and the state income taxes were 2% of the gross earnings, what would be the net income per year after taxes, assuming all other cost factors were unchanged?

Solution:

Let x be the gross earnings. Net profits are the gross earnings after subtraction of federal, local, and state taxes:

$$x - .35 \cdot x - .01 \cdot 20 - .05 \cdot x = 2$$

Solving for x gives x=\$3.67 million. Now assume that the gross earnings do not change in the new location, but the state and local taxes do:

$$\text{New Net} = 3.67 - 3.67 \cdot .35 - .04 \cdot 20 - .02 \cdot 3.67 = 1.51$$

That is, the new net profits after taxes are \$1.51 million.
ANS

Problem 7-17

A laboratory piece of equipment was purchased for $35,000 and is estimated to be used for 5 years with a salvage value of $5,000. (a) Tabulate the annual depreciation allowances and year-end book values for the 5 years by using the (1) straight-line depreciation method, (2) the MACRS 5-yr recovery period depreciation method, and (3) the sum of digits depreciation method. (b) Compare the net present worth of each of the three depreciation methods assuming an interest rate of 6%.

Solution:

	purchase value		35000		
	salvage value		5000		
	replacement value		30000		
	recovery period		5		
	interest		0.06		
	straight line				
year	factors	d_j	book value	PWF	PW
	(=1/5)		35000		
1	0.2000	6000	29000	0.9434	5660
2	0.2000	6000	23000	0.8900	5340
3	0.2000	6000	17000	0.8396	5038
4	0.2000	6000	11000	0.7921	4753
5	0.2000	6000	5000	0.7473	4484
					NPW
					25274
	MACRS				
	factors	d_j	book value	PWF	PW
			35000		
1	0.2	6000	29000	0.9434	5660
2	0.32	9600	19400	0.8900	8544
3	0.192	5760	13640	0.8396	4836
4	0.1152	3456	10184	0.7921	2737
5	0.1152	3456	6728	0.7473	2583
6	0.0576	1728	5000	0.7050	1218
					NPW
					25579
	SOD				
	factors	d_j	book value	PWF	PW
			35000		
1	0.3333	10000	25000	0.9434	9434
2	0.2667	8000	17000	0.8900	7120
3	0.2000	6000	11000	0.8396	5038
4	0.1333	4000	7000	0.7921	3168
5	0.0667	2000	5000	0.7473	1495
					NPW
					26255

PWFs at 6% can be found in the table on page 235 of the FE Reference Book, v10.1; 1st five entries in the upper left corner of the table.

MACRS factors are found on page 232 of the FE Reference Book, v10.1.

SOD values are found in slide 16 from lesson 18.

Problem 7-18

A piece of equipment with an original cost of $10,000 and no salvage value has a depreciation allowance of $2381 during its second year of service when depreciated by the sum-of-the-digits method. What recovery period has been used?

Solution:

Use the S.O.D. equation from the lesson slides. S.O.D. used to be in the FE Reference Book and the equation appears on page 115 of the Eighth (printed) Edition. However, the equation does not appear in the online pdf version (4th Edition).

Using the equation presented in class with j=2 gives:

$$2381 = 10000 \cdot \frac{2\cdot(n+1-2)}{n\cdot(n+1)} \Rightarrow n = 1.4 \text{ or } 6 \text{ (2 positive real roots)}$$

Since we are already in the second year of depreciation, a recovery period of 1.4 years is too short and we can reject this root. The recovery period is n=6 years.

ANS

Additional Background on Sum-of-Digits Depreciation Method:

The sum-of-the-digits (sometimes called sum-of-the-years-digits) method gives results similar to those obtained with the declining balance (fixed percentage) method. Larger costs for depreciation are allotted during the early-life years than during the later years. The annual depreciation is based on the number of service-life years remaining and the sum of the arithmetic series of numbers from 1 to n, where n represents the total service life (recovery period). The yearly depreciation factor is the number of useful service-life years remaining divided by the sum of the arithmetic series. This factor times the total depreciable value at the start of the service life gives the annual depreciation allowance.

$$d_j = \text{depreciation for year } j = \frac{(n-j+1)}{\sum_{i=1}^{n} n}\left(C - C_{scrap}\right) = \frac{(n-j+1)}{n\cdot(n+1)}\left(C - C_{scrap}\right)$$

In this equation, C is the original cost of the equipment and C_{scrap} is the scrap or salvage value of the equipment at the end of the recovery period. A handy formula used here is for taking the sum of numbers from 1 to n:

$$\sum_{i=1}^{n} n = \frac{1}{2}\cdot n\cdot(n+1)$$

Problem 8-1

What total amount of funds before taxes will be available 10 years from now if $10,000 is placed in a savings account earning an interest rate of 6 percent compounded monthly? How many years will be required for this amount to double at the same interest rate compounded semiannually? What is the shortest time in years for the doubling to occur if continuous compounding is available?

Solution:

```
(*Problem 7-1 for reference*)
```

In[1]:= $10\,000 * \left(1 + \frac{.06}{2}\right)^{20}$

Out[1]= 18 061.1

```
(*Problem 8-1*)

(*Case 1: 6% APR compounded monthly for ten years*)
```

$10\,000 * \left(1 + \frac{.06}{12}\right)^{120}$

Out[2]= 18 194.

```
(*Case 2: Doubling time at 6% APR compounded semiannually*)
```

$20\,000 == 10\,000 \left(1 + \frac{.06}{2}\right)^{2*n}$

Out[3]= $20\,000 == 10\,000\ 1.03^{2\,n}$

In[4]:= FindRoot[%, {n, 11}]

Out[4]= {n → 11.7249}

```
(*Case 3: Doubling time at 6% APR compounded continuously*)
20 000 == 10 000 * Exp[.06 * n]
```

Out[5]= $20\,000 == 10\,000\ e^{0.06\,n}$

In[6]:= FindRoot[%, {n, 11}]

Out[6]= {n → 11.5525}

Problem 8-2

A proposed chemical plant will require a fixed-capital investment of $10 million. It is estimated that the working capital will be 25 percent of the total investment. Annual depreciation costs are estimated to be 10 percent of the fixed-capital investment. If the annual profit will be $3 million, determine the percent return on the total investment and the payout period.

Solution:

$$\mathrm{FCI} = \$10{,}000{,}000$$

$$\mathrm{WC} = 0.25 \cdot \mathrm{TCI}$$

$$\mathrm{TCI} = \mathrm{FCI} + \mathrm{WC} \Rightarrow \mathrm{FCI} + 0.25 \cdot \mathrm{TCI} \Rightarrow \therefore\ \mathrm{TCI} = \$13{,}333{,}333$$

$$\text{annual depreciation} = \mathrm{D_j} = 0.10 \cdot \mathrm{FCI} = \$1{,}000{,}000$$

$$\text{annual profit} = \mathrm{A_j} = \$3{,}000{,}000 = \mathrm{N_{p,j}} + \mathrm{D_j} \Rightarrow \therefore\ \mathrm{N_{p,j}} = \$2{,}000{,}000$$

$$\mathrm{ROI} = \frac{\mathrm{N_{p,j}}}{\mathrm{TCI}} = \frac{\$2{,}000{,}000/\mathrm{yr}}{\$13{,}333{,}333} = 0.15/\mathrm{yr}$$

$$\text{payout period} = \mathrm{PBP} = \frac{\mathrm{FCI}}{\mathrm{A_j}} = \frac{\$10{,}000{,}000}{\$3{,}000{,}000/\mathrm{yr}} = 3.33\ \mathrm{yr}$$

Problem 8-4

Two pumps are being considered for pumping water from a reservoir. Installed cost and salvage value for the two pumps are given below:

	Pump A	**Pump B**
Installed cost	\$20,000	\$25,000
Salvage value	\$2,000	\$4,000

Pump A has a service life of 4 years. Determine the service life of pump B at which the two pumps are competitive if the annual effective interest rate is 15%. Competitiveness refers to the requirement that the installed cost of the two pumps plus the amount that must be invested at the time of installation so that sufficient interest will be earned over the service life (when added to the salvage value) to replace the pumps at the original cost [be the same] (Capitalized cost must be the same).

Solution:

The definition of competitiveness given above is the same as capitalized cost (see Problems 7-4 and 7-5).

$$CC = Cv + \frac{C_R}{(1+i)^N - 1}$$

where

$$CC = \text{capitalized cost}$$

$$C_v = \text{original equipment cost}$$

$$C_R = \text{replacement cost}$$

Equate the capitalized costs for the individual units and solve for the unknown service life N:

$$20000 + \frac{18000}{(1+.15)^4 - 1} = 25000 + \frac{21000}{(1+.15)^N - 1}$$

$$N = 5.32 \text{ years}$$

Problem 8-6

A design engineer is evaluating two pumps for handling a corrosive solution. The information on the pumps is the following:

	Pump A	Pump B
Installed cost	\$15,000	\$22,000
Service life, years	2	5

Determine the annual interest rate at which the two pumps are considered competitive. Neglect salvage value. See Problem 8-4 for the definition of competitiveness. Which pump would you recommend?

Solution:

As in Problem 8-4, competitiveness is the same as capitalized cost (see Problems 7-4 and 7-5).

$$CC = Cv + \frac{C_R}{(1+i)^N - 1}$$

where

$$CC = \text{capitalized cost}$$

$$C_v = \text{original equipment cost}$$

$$C_R = \text{replacement cost}$$

Equate the capitalized costs for the individual units and solve for the unknown interest i:

$$15000 + \frac{15000}{(1+i)^2 - 1} = 22000 + \frac{22000}{(1+i)^5 - 1}$$

$$i = 0.626\,.$$

To judge the two pumps, we need to see what happens at a reasonable interest rate, such as 15%:

$$CC_{A,.15} = 15000 + \frac{15000}{(1+.15)^2 - 1} = \$61,512$$

$$CC_{B,.15} = 22000 + \frac{22000}{(1+.15)^5 - 1} = \$43,753$$

Pump B is significantly cheaper and is recommended.