CH402 Chemical Engineering Process Design

Class Notes L17

Interest

A Chemical Engineering Approach to Continuous Interest

Important Features of Interest Computations

(Important Uses as an Engineering Tool)

Simple Interest Model

Simplest Method to Account for the Time-Value of Money

Allows the use of an amount of money P for a specified period of time.

P is the principal in a loan or the present value in a cash flow analysis.

for one "period" $I = P \cdot i$

for N periods $I = P \cdot i \cdot N$

I is the interest payment

i is the interest rate

F is called the future value

payback $F = P + I$

$F = P \cdot (1 + i \cdot N)$ "Simple Interest"

$(F / P, i, n) = (1 + i \cdot N)$

"Discrete single-payment future-worth factor"

Read this as "to F given P"

New Term!
Very Important!
(memorize)

Example 1 – $1000 is borrowed for 5 years at 10% annual interest

$F = P \cdot (1 + i \cdot N) = 1000 \cdot (1 + .1 \cdot 5) = \$1500$

$(F / P, i, n) = 1.5$

FEE

Compound Interest Model – Interest Earns Interest

Funds (P) deposited into an account that accrues interest at specified times in the future.
Interest payments are due at the end of each specified time period.

Formulas for discrete single-payment future-worth factors are tabulated in the FE Reference Manual, pp. 230 and Table 7-3 in PTW p. 299

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\dfrac{i}{(1+i)^n - 1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\dfrac{i(1+i)^n}{(1+i)^n - 1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\dfrac{(1+i)^n - 1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\dfrac{(1+i)^n - 1}{i(1+i)^n}$
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\dfrac{(1+i)^n - 1}{i^2(1+i)^n} - \dfrac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\dfrac{(1+i)^n - 1}{i^2} - \dfrac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\dfrac{1}{i} - \dfrac{n}{(1+i)^n - 1}$

"Sinking Fund"

A sinking fund is used by companies that have floated debt in the form of bonds to gradually save money and avoid a large lump-sum payment at maturity. Some bonds are issued with the attachment of a sinking fund feature.
A sinking fund is essentially a savings account.

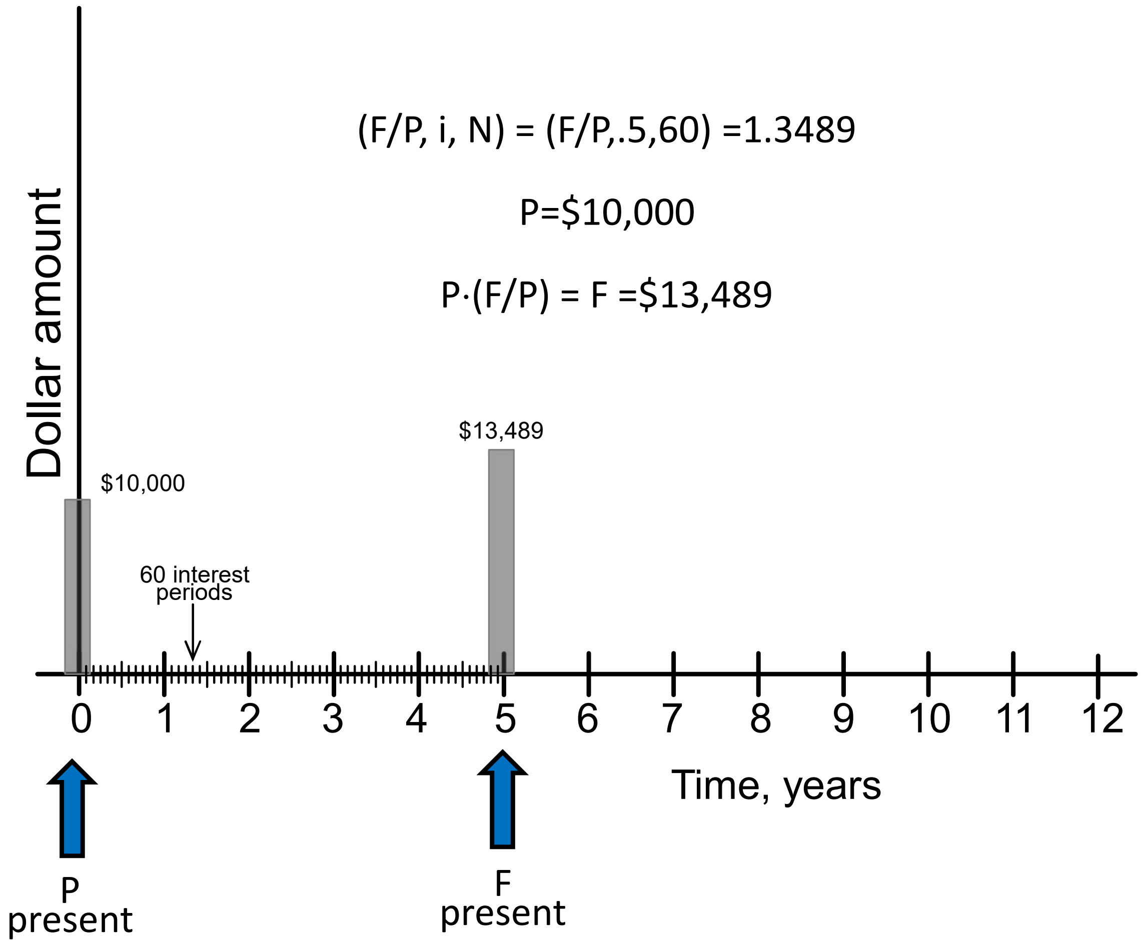
https://www.investopedia.com/terms/s/sinkingfund.asp#toc-understanding-a-sinking-fund

Example 2 – \$10,000 earns 6% annual interest compounded monthly for 5 years. What is the amount in the account at the end of that time?

$(F/P, i, N) = (F/P, .5, 60) = 1.3489$

$P = \$10{,}000$

$P \cdot (F/P) = F = \$13{,}489$



There are 60 months in 5 years, so interest is 0.005 (or .5%) per month

Numerical discrete single-payment future-worth factors are tabulated in
FEE Reference Manual, pp. 233-237

FEE, p. 233

Interest Rate Tables
Factor Table - i = 0.50%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9950	0.9950	0.0000	1.0050	1.0000	1.0050	1.0000	0.0000
2	0.9901	1.9851	0.9901	1.0100	2.0050	0.5038	0.4988	0.4988
3	0.9851	2.9702	2.9604	1.0151	3.0150	0.3367	0.3317	0.9967
4	0.9802	3.9505	5.9011	1.0202	4.0301	0.2531	0.2481	1.4938
5	**0.9754**	**4.9259**	**9.8026**	**1.0253**	**5.0503**	**0.2030**	**0.1980**	**1.9900**
6	0.9705	5.8964	14.6552	1.0304	6.0755	0.1696	0.1646	2.4855
7	0.9657	6.8621	20.4493	1.0355	7.1059	0.1457	0.1407	2.9801
8	0.9609	7.8230	27.1755	1.0407	8.1414	0.1278	0.1228	3.4738
9	0.9561	8.7791	34.8244	1.0459	9.1821	0.1139	0.1089	3.9668
10	**0.9513**	**9.7304**	**43.3865**	**1.0511**	**10.2280**	**0.1028**	**0.0978**	**4.4589**
11	0.9466	10.6770	52.8526	1.0564	11.2792	0.0937	0.0887	4.9501
12	0.9419	11.6189	63.2136	1.0617	12.3356	0.0861	0.0811	5.4406
13	0.9372	12.5562	74.4602	1.0670	13.3972	0.0796	0.0746	5.9302
14	0.9326	13.4887	86.5835	1.0723	14.4642	0.0741	0.0691	6.4190
15	**0.9279**	**14.4166**	**99.5743**	**1.0777**	**15.5365**	**0.0694**	**0.0644**	**6.9069**
16	0.9233	15.3399	113.4238	1.0831	16.6142	0.0652	0.0602	7.3940
17	0.9187	16.2586	128.1231	1.0885	17.6973	0.0615	0.0565	7.8803
18	0.9141	17.1728	143.6634	1.0939	18.7858	0.0582	0.0532	8.3658
19	0.9096	18.0824	160.0360	1.0994	19.8797	0.0553	0.0503	8.8504
20	**0.9051**	**18.9874**	**177.2322**	**1.1049**	**20.9791**	**0.0527**	**0.0477**	**9.3342**
21	0.9006	19.8880	195.2434	1.1104	22.0840	0.0503	0.0453	9.8172
22	0.8961	20.7841	214.0611	1.1160	23.1944	0.0481	0.0431	10.2993
23	0.8916	21.6757	233.6768	1.1216	24.3104	0.0461	0.0411	10.7806
24	0.8872	22.5629	254.0820	1.1272	25.4320	0.0443	0.0393	11.2611
25	**0.8828**	**23.4456**	**275.2686**	**1.1328**	**26.5591**	**0.0427**	**0.0377**	**11.7407**
30	0.8610	27.7941	392.6324	1.1614	32.2800	0.0360	0.0310	14.1265
40	0.8191	36.1722	681.3347	1.2208	44.1588	0.0276	0.0226	18.8359
50	0.7793	44.1428	1,035.6966	1.2832	56.6452	0.0227	0.0177	23.4624
60	0.7414	51.7256	1,448.6458	$\boxed{1.3489}$	69.7700	0.0193	0.0143	28.0064
100	**0.6073**	**78.5426**	**3,562.7934**	**1.6467**	**129.3337**	**0.0127**	**0.0077**	**45.3613**

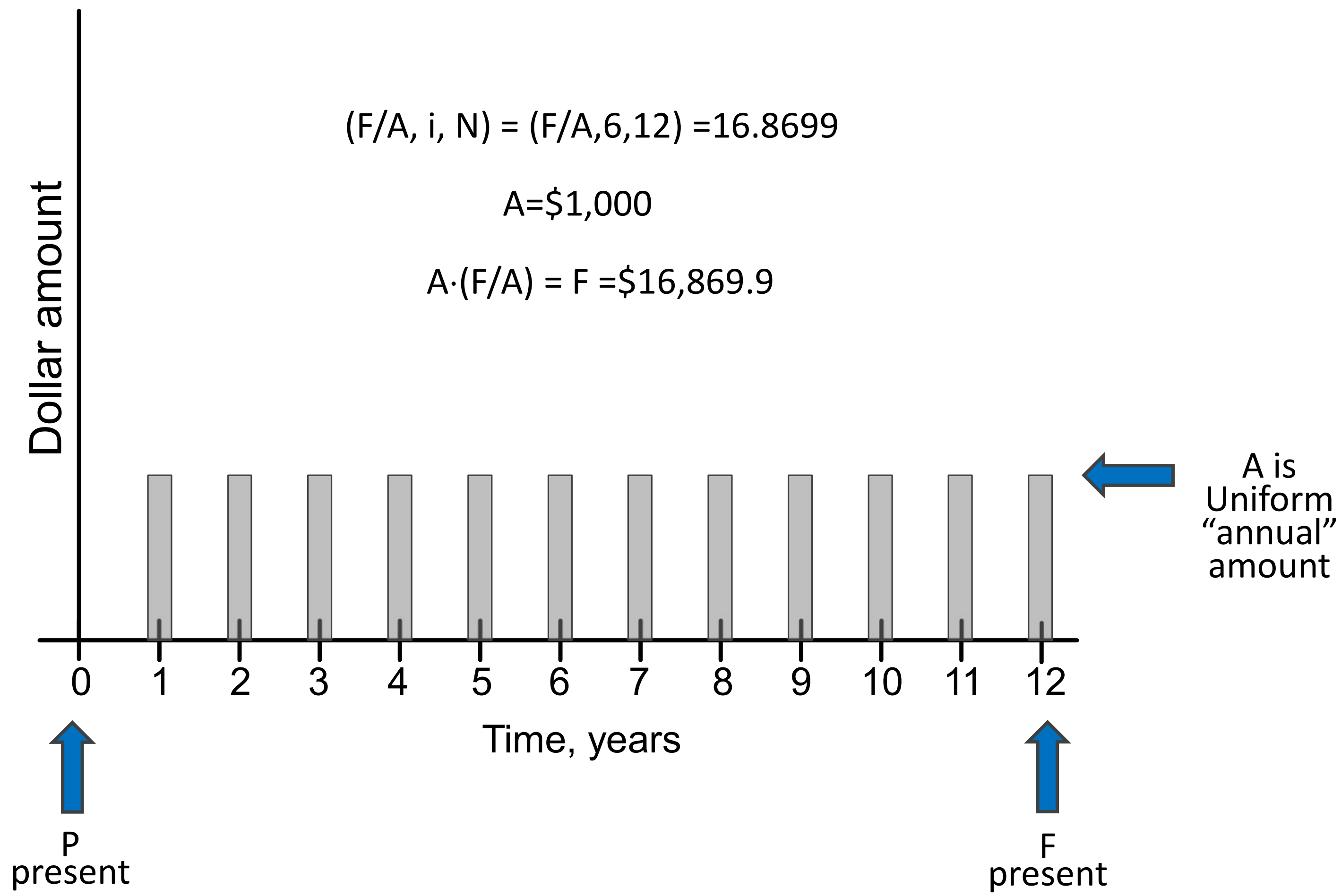
Other values at i=.5, 1, 1.5, 2, 4, 6, 8, 10, 12, and 18%

FEE

Problem 7-1

What funds will be available 10 years from now if $10,000 is deposited at a nominal interest rate of 6% compounded semiannually?

Example 3 – $1000 added annually (at end of year) for 12 years to an account that earns 6% annual interest compounded annually. What amount of money is in the account at the end of year 12?



The sum of the payments is $12,000

Problem 7-2

The original cost for a distillation tower is $50,000, and the useful life of the tower is estimated to be 10 years (and the scrap value at that time is $5000). How much must be placed annually in an annuity at an interest rate of 6% to obtain sufficient funds to replace the tower at the end of 10 years? If the scrap value of the distillation tower is $5,000, determine the asset value (i.e., the total book value of the tower) at the end of 5 years based on straight line depreciation.

What happens if we allow the **interest** to **earn interest** for **more periods**?

$$F = P \cdot (1+i)^N$$

Interest is compounded once per period.

\$10,000 earning interest at .5% per period for 60 periods is $10{,}000(1+.005)^{60} = 13{,}488.50$

Banks and other financial institutions often use the term “effective annual interest” or “nominal interest” to represent the interest over one year.

$$F = P \cdot \left(1+\frac{r}{n}\right)^{n \cdot t}$$

t is time in years
n is the number of interest periods per year.

0.5% per month for 60 months is equivalent to nominal interest of 6% per year

\$10,000 at an effective annual (nominal) interest of 6% compounded monthly for 5 years is $10{,}000(1+.06/12)^{12 \cdot 5} = 13{,}488.50$

Problem 7-4 – Capitalized Cost

Derive an expression for capitalized cost based on annual discrete interest compounding. *Capitalized cost* is defined as the sum of the original cost C_v of the equipment or asset plus the amount P that must be invested when the original equipment or asset is purchased so that when the original equipment or asset is replaced in N years at a cost C_R, the value of the investment equals P plus C_R.

Problem 7-5 – Capitalized Cost

A heat exchanger is to be used in a heating process. A standard type of heat exchanger with a negligible scrap value costs $20,000 and will have a useful life of 6 years. Another type of heat exchanger with equivalent design capacity is priced at $34,000 but has a useful life of 10 years and a scrap value of $4000. Assume an effective compound interest rate of 6% per year, and that the replacement cost of each heat exchanger is the same as that of the original exchanger. Determine which heat exchanger is cheaper by comparing the capitalized cost of each. See Problem 7-4 for a definition of capitalized cost.

Questions?

Supplemental Slides

Problem 7-9

The fixed capital investment for an existing chemical plant is $20 million. Annual property taxes amount to 1% of the fixed-capital investment, and state income taxes are 5% of the gross earnings. The net income after all taxes is $2 million, and the federal income taxes amount to 35% of gross earnings. If the same plant had been constructed for the same fixed capital investment but at a location where property taxes were 4% of the fixed capital investment and the state income taxes were 2% of the gross earnings, what would be the net income per year after taxes, assuming all other cost factors were unchanged?

Uniform Cash Flows Per Period

A = uniform amount per period (FEE Ref.)
L = loan payment per period (PTW Book)

FEE, p. 230

---

factors:

(F/P,i,N)

$$(1+i)^N$$

(A/F,i,N)

$$\frac{i}{(1+i)^N - 1}$$

(A/P,i,N)

$$\frac{i \cdot (1+i)^N}{(1+i)^N - 1}$$

P·(A/P,i,N)

$$A = L = \frac{P \cdot i \cdot (1+i)^N}{(1+i)^N - 1}$$

Calculating Uniform Cash Flows in Excel

Example 7-3

1. Determine loan amount, nominal interest, and periodic interest
2. Determine length of loan
3. Calculate periodic interest factors
4. Calculate Summation term in eq 7-29
5. Determine L using eq 7-29
6. Periodic interest for period j using eq 7-20
7. Periodic principal for period j using eq 7-19
8. Remaining principal using eq 7-21
9. Repeat from step 5

$$A = L = \frac{P_0 \cdot \left[1 + i \cdot \sum_{j=1}^{N} (1+i)^{j-1}\right]}{\sum_{j=1}^{N} (1+i)^{j-1}}$$

$I_j = i \cdot P_j$

$p_j = L - I_j$

$P_j = P_{j-1} - p_{j-1}$

Calculating Uniform Cash Flows Algebraically

Another method for calculating future cash flows (loan payments)

specify:
- principle
- interest
- interest periods per year
- number of years

determine future value of principle using (F/P,I,N):

$$F = P \cdot \left(1 + \frac{r}{m}\right)^{mN}$$

solve for L in annuity equation using (F/A,I,N):

$$F = L \cdot \frac{\left(1 + \frac{r}{m}\right)^{mN} - 1}{\frac{r}{m}}.$$

Set F’s equal and solve for L.

Future value of annuity payments must equal future value of principle

How do we know the interest rate? (Good question)

Sources of Capital

Source of capital	Indicated interest or dividend, %/y	Interest or dividend before taxes, %/y	Interest or dividend after taxes, %/y
Bonds	5	5	3.25
Bank loans	8	8	5.2
Preferred stock	8	12.3	8
Common stock	n/a	13.8	9

Tax rate = 0.35

Actual interest or dividend rates depend on <u>market</u> values.

Example 7-3

A loan of $100,000 at a nominal interest rate of 10 percent per year is made for a repayment period of 10 years. Determine the constant payment per period, the interest and principal paid each period, and the remaining unpaid principal at the end of each period using end-of-month payments. Assume 12 equal-length months per year.

Work in class with cadets using Excel.

Example 1 – Future value of $10,000 at a periodic interest rate of 0.005 for 60 interest periods

$10,000 earning interest at .005 per period (i=.5%) for 60 periods is

$$10{,}000\cdot(1+.005)^{60} = 13{,}488.50$$

"Future Value" $F = 10{,}000\cdot\left(1+.005\right)^{60} = 13{,}488.50$

$$F = P\cdot\left(1+i\right)^{N} \quad \Rightarrow \quad \frac{F}{P} = \left(1+i\right)^{N}$$

Discreet Single Payment Future Worth Factor:

(F/P, i, N)

(F/P, .5, 60)

$$\Rightarrow \quad (F/P,.5,60) = \left(1+0.005\right)^{60} = 1.3489$$

FEE

FEE Manual v10.1 page 231

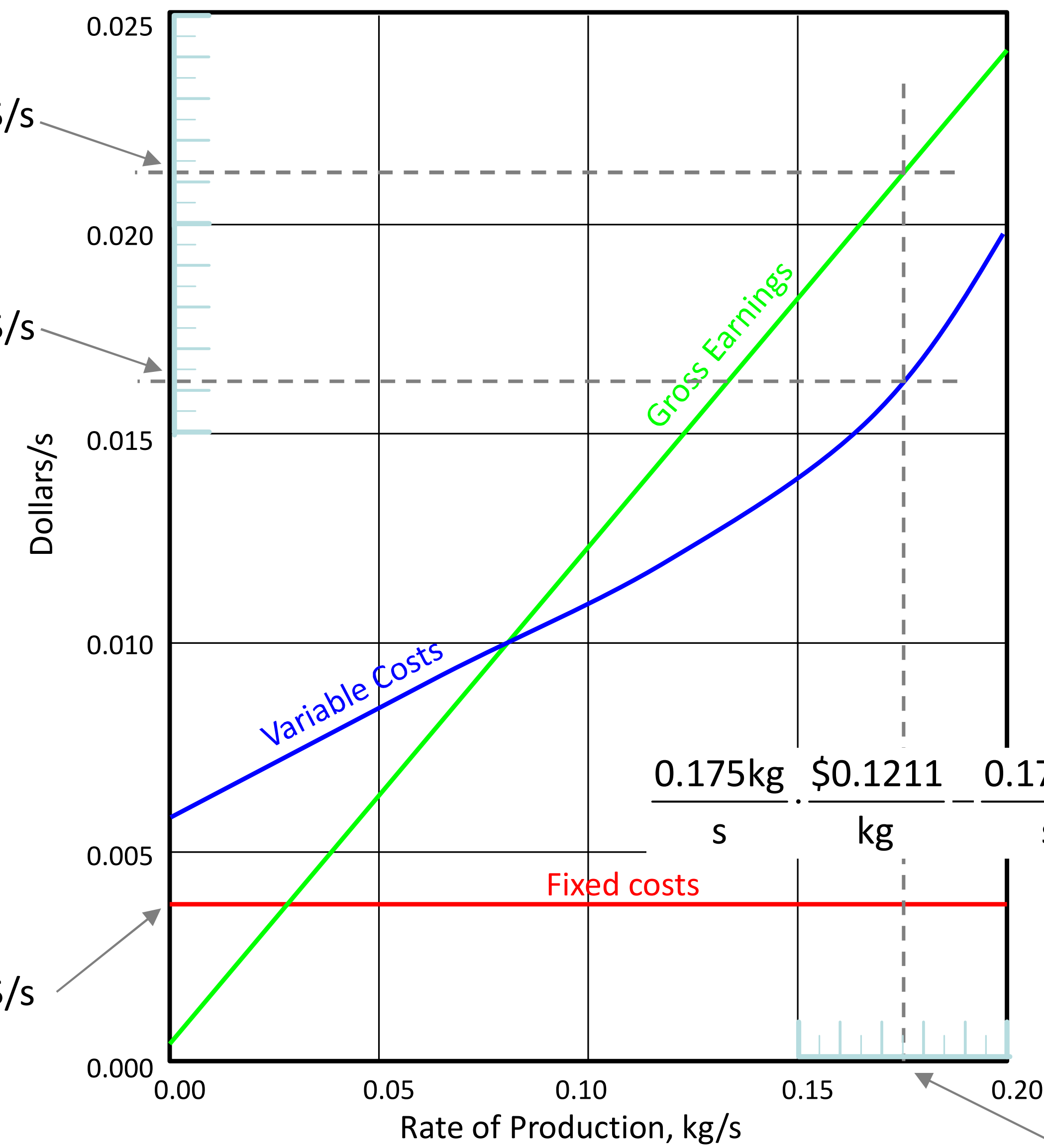
Break-Even Analysis – Ex1

Lesson 14 reading, pages 226-232 – breakeven chart – page 231 figure 6-3; FEE p. 231

Earnings = 0.0212 $/s

Variable Costs = 0.0162 $/s

Fixed Costs = 0.0038 $/s



Rate of production = 0.175 kg/s

product or market price

$$\frac{0.175kg}{s}\cdot x=\frac{\$0.0212}{s}$$

$$x=\frac{\$0.1211}{kg}$$

production cost

$$\frac{0.175kg}{s}\cdot y=\frac{\$0.0162}{s}$$

$$y=\frac{\$0.0926}{kg}$$

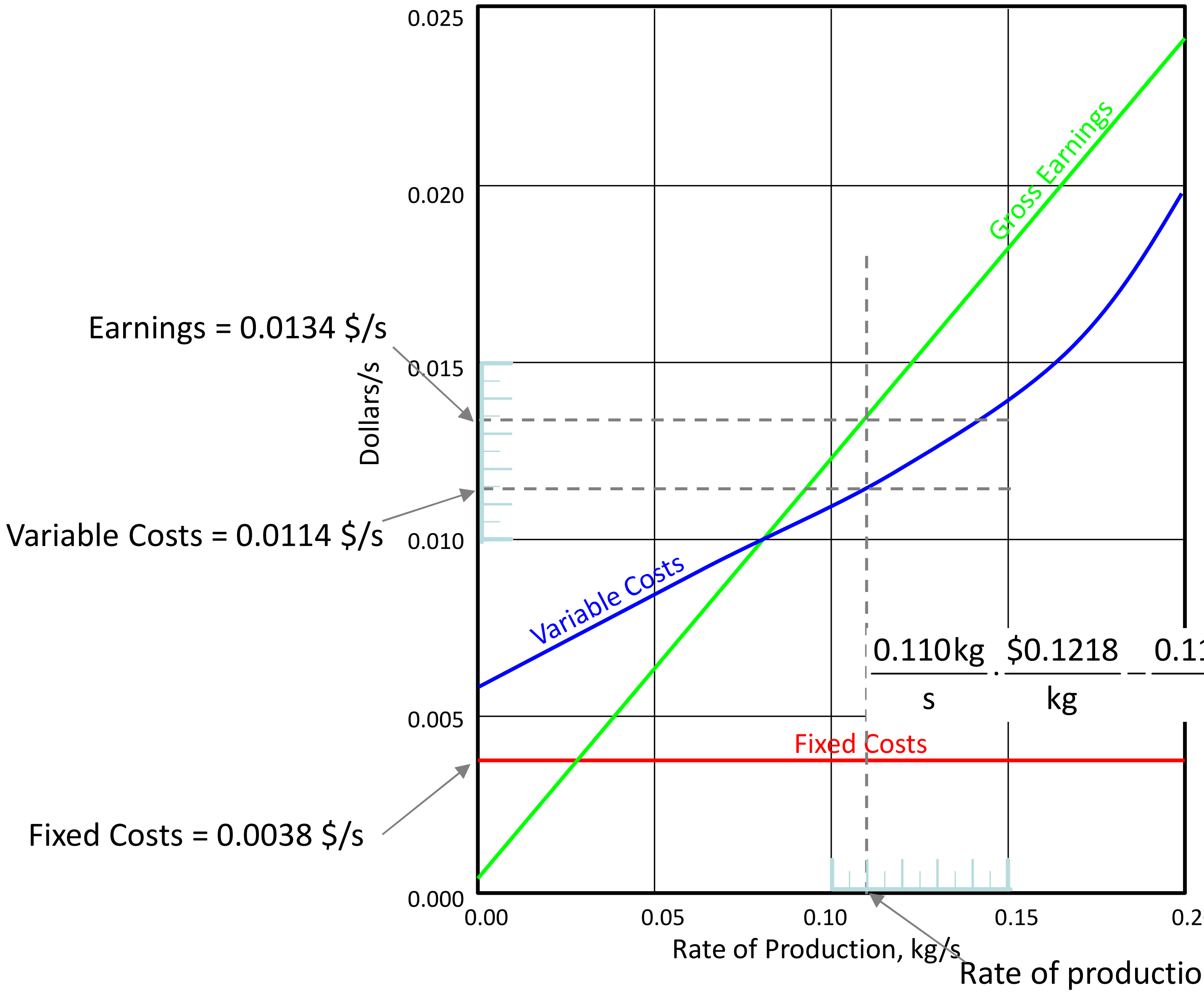
+

$$\frac{0.175kg}{s}\cdot\frac{\$0.1211}{kg}-\frac{0.175kg}{s}\cdot\frac{\$0.0926}{kg}-0.0038=\frac{\$0.0012}{s}$$

Operating at profit

Break-Even Analysis – Ex2

Figure 6-3; FEE p. 231; finding the “true” breakeven point iteratively



product or market price

$$\frac{0.110\text{kg}}{\text{s}}\cdot x = \frac{\$0.0134}{\text{s}}$$

$$x = \frac{\$0.1218}{\text{kg}}$$

production cost

$$\frac{0.110\text{kg}}{\text{s}}\cdot y = \frac{\$0.0114}{\text{s}}$$

$$y = \frac{\$0.1036}{\text{kg}}$$

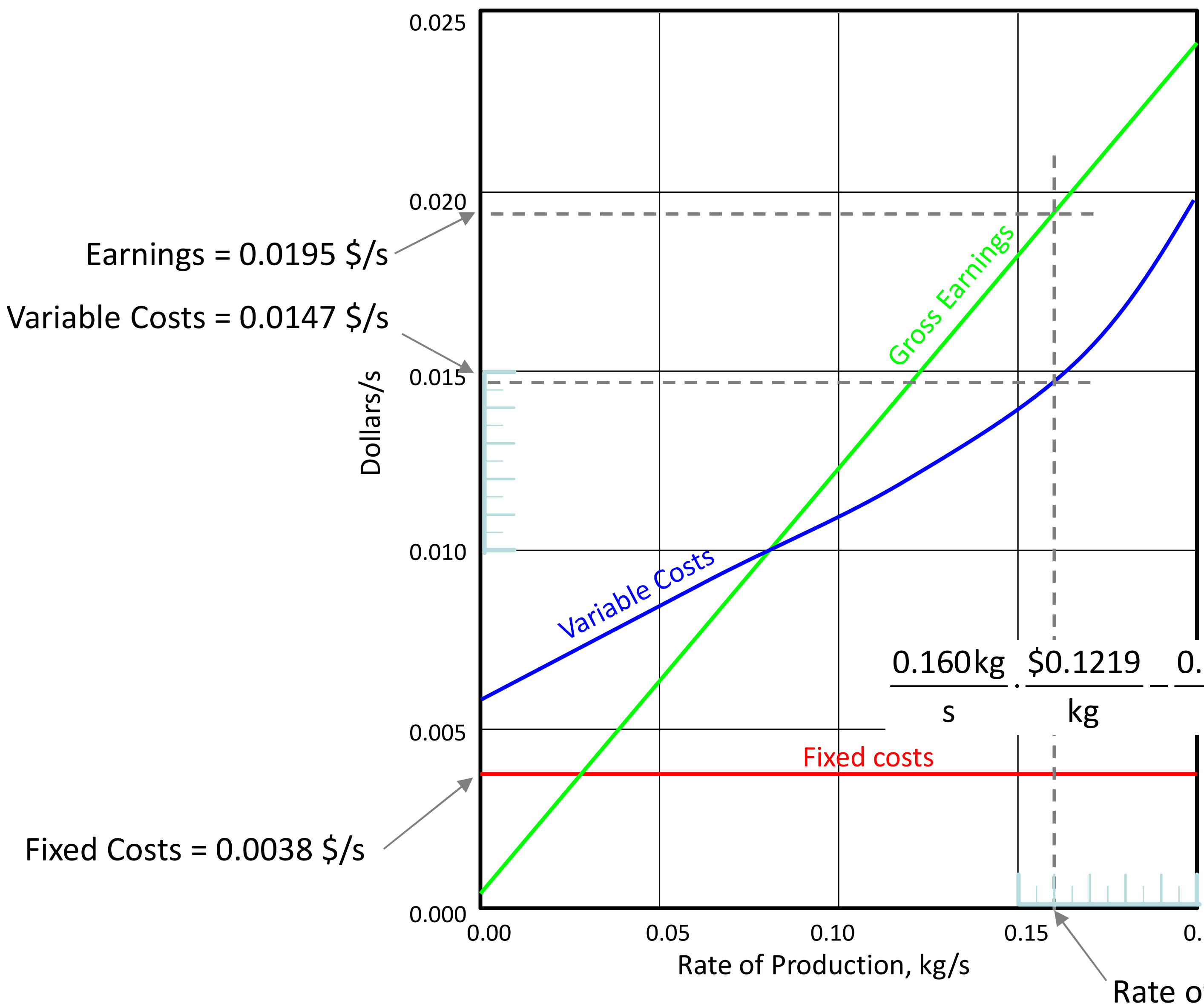
$$\frac{0.110\text{kg}}{\text{s}}\cdot\frac{\$0.1218}{\text{kg}} - \frac{0.110\text{kg}}{\text{s}}\cdot\frac{\$0.1036}{\text{kg}} - 0.0038 = -\frac{\$0.0049}{\text{s}}$$

Operating at loss

FEE Manual v10.1 page 231

Break-Even Analysis – Ex3

Lesson 14 reading, pages 226-232 – breakeven chart – page 231 figure 6-3; FEE p. 130



product or market price

$$\frac{0.160\,kg}{s}\cdot x = \frac{\$0.0195}{s}$$

$$x = \frac{\$0.1219}{kg}$$

production cost

$$\frac{0.160\,kg}{s}\cdot y = \frac{\$0.0147}{s}$$

$$y = \frac{\$0.0919}{kg}$$

+

$$\frac{0.160\,kg}{s}\cdot\frac{\$0.1219}{kg} - \frac{0.160\,kg}{s}\cdot\frac{\$0.0919}{kg} - 0.0038 = \frac{\$0.0010}{s}$$

Operating at profit

FEE Manual v10.1 page 231

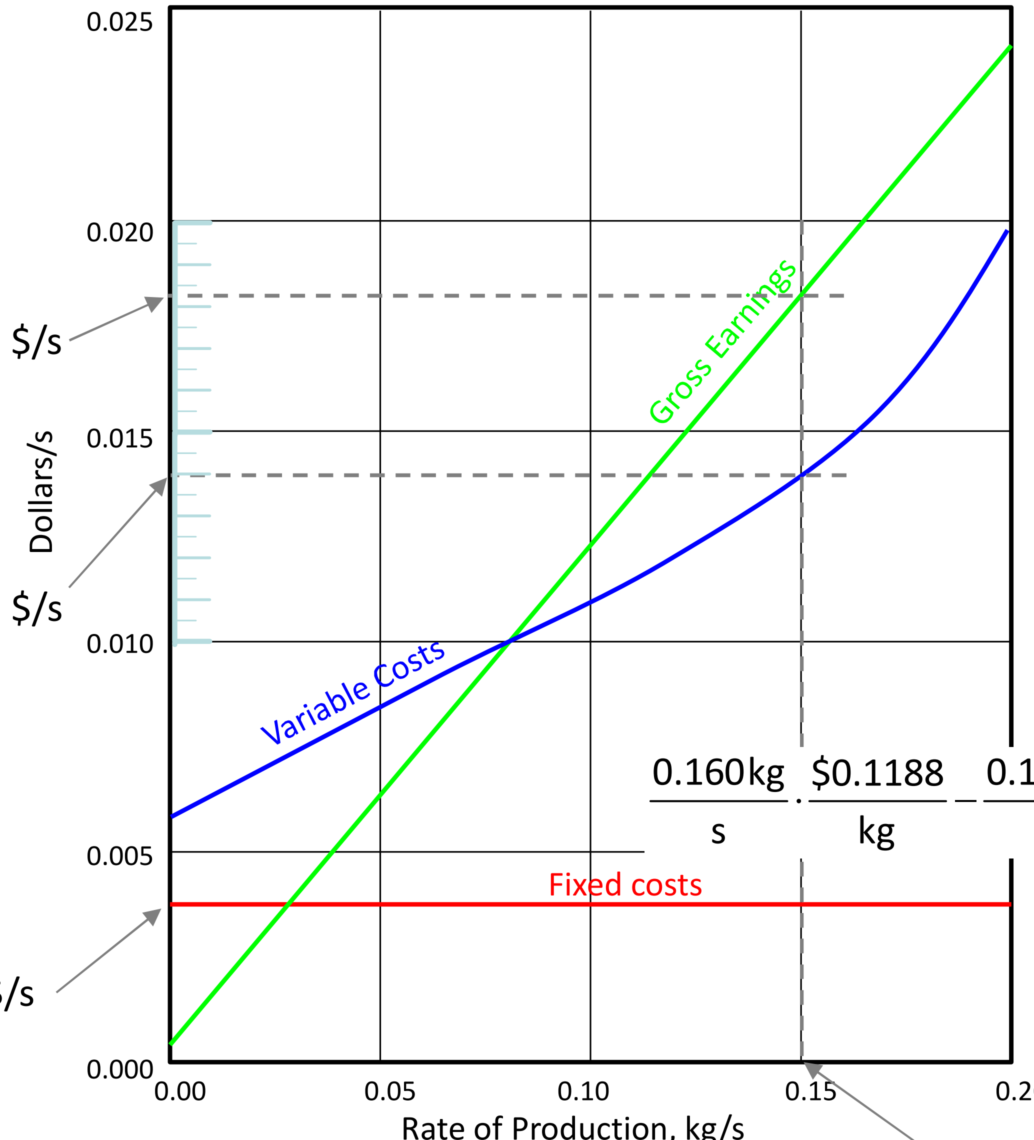
Break-Even Analysis – Ex4

Lesson 14 reading, pages 226-232 – breakeven chart – page 231 figure 6-3; FEE p. 130

Earnings = 0.0182 \$/s

Variable Costs = 0.0140 \$/s

Fixed Costs = 0.0038 \$/s



Rate of production = 0.150 kg/s

product or market price

$$\frac{0.150kg}{s}\cdot x = \frac{\$0.0182}{s}$$

$$x = \frac{\$0.1213}{kg}$$

production cost

$$\frac{0.150kg}{s}\cdot y = \frac{\$0.0140}{s}$$

$$y = \frac{\$0.0933}{kg}$$

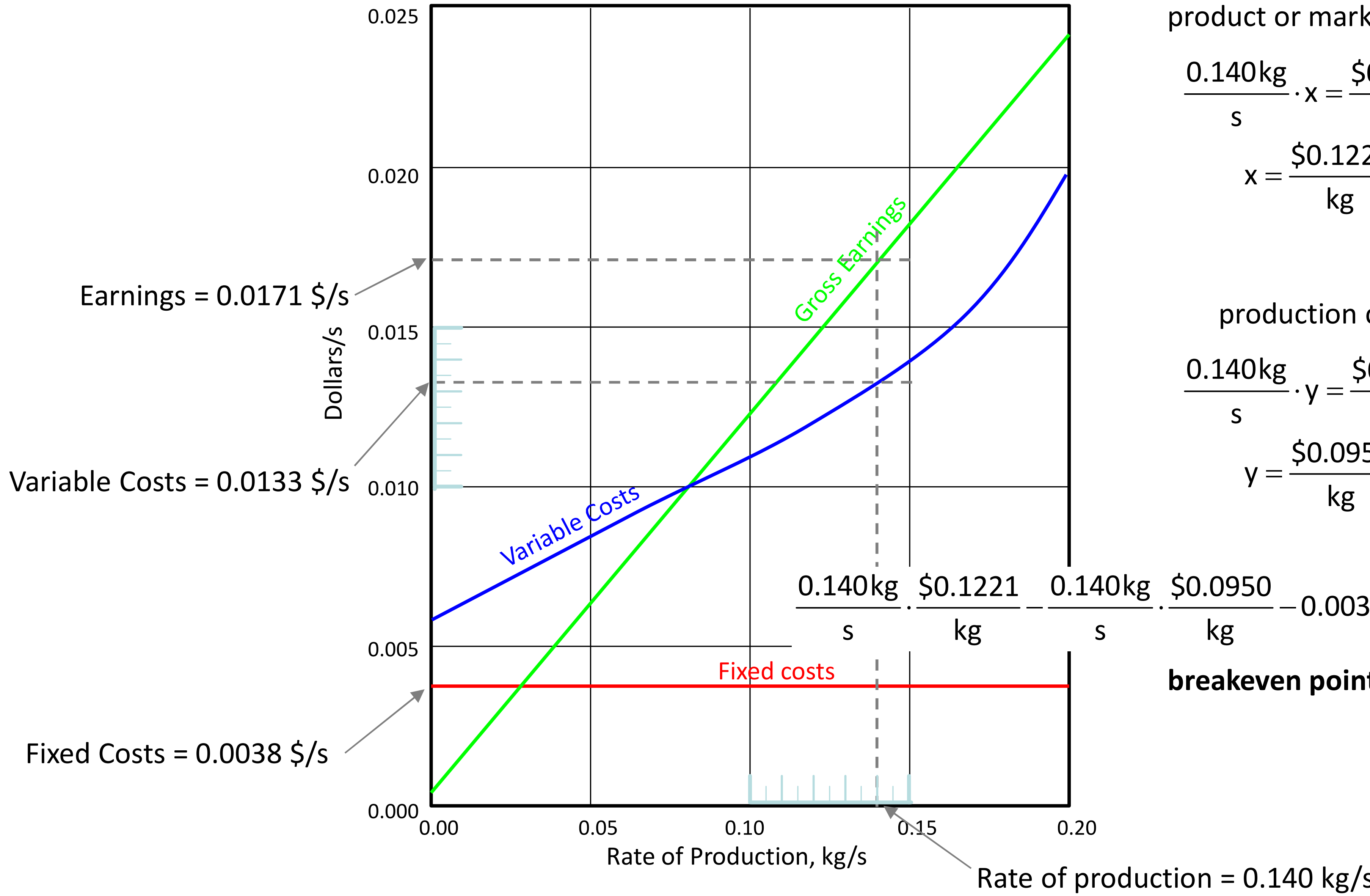
$$\frac{0.160kg}{s}\cdot\frac{\$0.1188}{kg} - \frac{0.160kg}{s}\cdot\frac{\$0.0925}{kg} - 0.0038 = \frac{\$0.0004}{s}$$

+

Operating at profit

Break-Even Analysis – Ex5

Lesson 14 reading, pages 226-232 – breakeven chart – page 231 figure 6-3; FEE p. 130



product or market price

$$\frac{0.140\,kg}{s}\cdot x = \frac{\$0.0171}{s}$$

$$x = \frac{\$0.1221}{kg}$$

production cost

$$\frac{0.140\,kg}{s}\cdot y = \frac{\$0.0133}{s}$$

$$y = \frac{\$0.0950}{kg}$$

$$\frac{0.140\,kg}{s}\cdot\frac{\$0.1221}{kg} - \frac{0.140\,kg}{s}\cdot\frac{\$0.0950}{kg} - 0.0038 = \frac{\$0.0000}{s}$$

breakeven point