Problem Set 3 - Solutions

Problem 2.24

A stream of warm water is produced in a steady-flow mixing process by combining 1.0 $\frac{kg}{s}$ of cool water at 25 °C with 0.8 $\frac{kg}{s}$ of hot water at 75 °C. During mixing, heat is lost to the surroundings at a rate of 30 $\frac{kJ}{s}$. Assume the specific heat of water is constant at 4.18 $\frac{kJ}{kg \cdot K}$.

What is the temperature of the warm water stream?

SOLUTION

In[ ]:=
```
2*2.3
```
Out[ ]=
```
4.6
```

The enthalpy balance (mechanical energy balance) on the mixer:

In[3]:= HoldForm[...]

Out[3]=

$$\Delta H = Q$$

In[11]:= HoldForm[...]

Out[11]=

$$H_3^t - H_1^t - H_2^t == Q$$

In[12]:= HoldForm[...]

Out[12]=

$$\dot{m}_3 C_{P3} \Delta T_3 - \dot{m}_1 C_{P1} \Delta T_1 - \dot{m}_2 C_{P2} \Delta T_2 == -30$$

In[13]:= HoldForm[...]

Out[13]=

$$1.8 \times 4.18 (T - T_{ref}) - 1. \times 4.18 (25 - T_{ref}) - 0.8 \times 4.18 (75 - T_{ref}) == -30$$

Assign a value to T_{ref}. It can be anything because all terms containing T_{ref} cancel.

In[ ]:=
```
Tref = 0;
```

Write the Mathematica equation and solve it:

In[ ]:=
```
1.8*4.18*(T - Tref) - 1.0*4.18*(25 - Tref) - 0.8*4.18*(75 - Tref) == -30 // Solve
```
Out[ ]=
```
{{T → 43.23498}}
```

The temperature of the warm water stream is 43.235 °C. //ANS

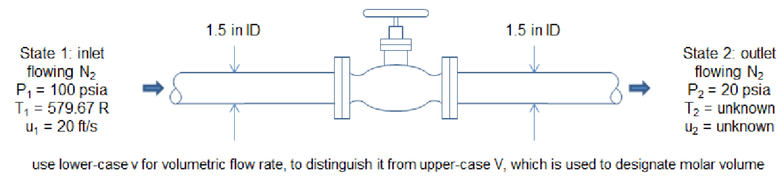
Problem 2.28

Nitrogen flows at steady state through a horizontal, insulated pipe with inside diameter of 1.5 inches. A pressure drop results from flow through a partially opened valve. Just upstream from the valve the pressure is 100 psia, the temperature is 120 °F, and the average velocity is 20 $\frac{ft}{s}$. Assume for nitrogen that $\frac{PV}{T}$ is constant, $C_V = \frac{5}{2} R$, and $C_P = \frac{7}{2} R$. Values of the gas constant R are given in App. A.

If the pressure just downstream from the valve is 20 psia, what is the temperature?

SOLUTION

Process Sketch:



Solution Outline:

The inlet volumetric flow rate (v_1) can be calculated from the given inlet velocity (u_1) and the inlet cross-sectional area.

Once v_1 is known, the outlet volumetric flow rate v_2 can be calculated since $\frac{PV}{T}$ is constant.

The outlet velocity (u_2) is then calculated from the volumetric flow rate (v_2) and the area.

However, state 2 has two unknowns, T_2 and u_2. We must recognize that we need two independent equations to solve for the two unknowns.

The two equations are the ideal gas law and the mechanical energy balance.

Use the ideal gas law to reduce the mechanical energy balance to a single equation with only one unknown (T_2), and then solve for the unknown.

Mechanical Energy Balance:

Out[▫]//TraditionalForm=

$$\Delta H + \frac{\Delta u^2}{2\, g_c} + \frac{g\, \Delta z}{g_c} = Q + W_s \Rightarrow \Delta H + \frac{\Delta u^2}{2\, g_c} = 0 \Rightarrow C_P\, \Delta T + \frac{\Delta u^2}{2\, g_c} = 0$$

Since C_P and T_1 are known, the heat capacity term ($C_P\, \Delta T$) is a function only of T_2. As will be shown in"OUTLET CONDITIONS", the velocity term $(\Delta u^2 / 2\, g_c)$ is also a function of only T_2. This allows us to solve for T_2, as shown below.

In[◦]:=
```
eq4 = 7/2 * R * (T2 - T1) == -1/(2 gc) (u2^2 - u1^2) * MW;
```

Inlet Conditions:

```
T1 = (120 + 459.67); (*given with converting from °F to Rankine*)
P1 = 100;(*given psia*)
u1 = 20; (*given ft/s*)
area = π/4 * (1.5/12)^2; (* =0.0122718 ft^2*)
v1 = u1 * area (* v1=u1*area=20 ft/s *0.0122718ft^2=0.245437 ft^3/s  *)
```

Out[◦]=

```
0.2454369
```

Outlet Conditions:

In[◦]:=
```
P2 = 20; (*given - psia*)
v2 = v1 * P1/P2 T2/T1 ;(*same units as v1*)
u2 = u1 * v2/v1 (*u2*A2*ρ2=u1*A1*ρ1; u2/v2 = u1/v1; same units as u1*)
```

Out[◦]=

```
0.1725119 T2
```

Outlet velocity u_2 is a function of T_2 only. When u_2 is added back to the enthalpy balance, you have one equation with one unknown (T_2).

Other Information:

In[◦]:=
```
R = 1545.; (*gas constant with units of ft*lbf/(lbmol*degR)*)
gc = 32.1740; (*ft*lbm/s^2 / lbf, page xiv*)
MW = 28.; (*lbm/lbmol*)
```

Solution of the Mechanical Energy Balance:

Take another look at eq4 now to see how it has changed after adding all of the specifications:

In[◦]:=
```
eq4
```

Out[◦]=

```
5407.5 (-579.67 + T2) == -0.435134 (-400 + 0.02976037 T2^2)
```

Now solve eq4 for T2:

In[∘]:=
```
Solve[eq4 && T2 > 0, T2]
```

Out[∘]=
```
{{T2 → 578.8996}}
```

In[∘]:=
```
578.9 - 459.67 (*Outlet temperature in °F //ANS*)
```

Out[∘]=
```
119.23
```

The outlet temperature is 119.23 °F. //ANS

Problem 2.38

CO_2 gas enters a water-cooled compressor at conditions P_1 = 15 psia and T_1 = 50 °F, and is discharged at conditions P_2 = 520 psia and T_2 = 200 °F. The entering CO_2 flows through a 4-inch-diameter pipe with a velocity of 20 $\frac{ft}{s}$, and is discharged through a 1-inch-diameter pipe. The shaft work supplied to the compressor is 5,360 $\frac{Btu}{lbmol}$.

Additional Information:

$H_1 = 307\ \frac{Btu}{lb_m}$ and $V_1 = 9.25\ \frac{ft^3}{lb_m}$

$H_2 = 330\ \frac{Btu}{lb_m}$ and $V_2 = 0.28\ \frac{ft^3}{lb_m}$

What is the heat-transfer rate from the compressor in $\frac{Btu}{hr}$?

SOLUTION

Solution Outline:

Use the open system energy balance and assume potential energy changes due to changes in elevation are negligible. Under these conditions, the balance reduces to:

Out[∘]//TraditionalForm=

$$\Delta H + \frac{\Delta u^2}{2\, g_c} + \frac{g\, \Delta z}{g_c} = Q + W_s \Rightarrow \Delta H + \frac{\Delta u^2}{2\, g_c} = Q + W_s$$

Recognize that we are given everything except the outlet velocity and the heat duty (u_2 and Q). Also recognize that mass is conserved so that the mass flow rate in is equal to the mass flow rate out ($m_1 = m_2$).

The inlet mass flow rate can be calculated from the given inlet velocity and specific volume, and pipe cross-sectional area. This is then equal to the outlet mass flow rate.

The outlet velocity u_2 can then be calculated from the mass flow rate, the given specific volume and pipeline cross-sectional area.

This leaves only a calculation of Q. So the strategy is to calculate u_2 from the flow rates and areas, and then use the energy balance to calculate Q.

Mechanical Energy Balance:

In[∘]:=

$$eq5 = (H2 - H1) + \frac{1}{2 * gc} * \left(u2^2 - u1^2\right) * \frac{.000947831}{.737562} == Q + \frac{Ws}{MW};$$

$$(* \frac{.000947831 Btu}{.737562 ft * lbf} \text{ conversion factors from Appendix A} *)$$

In[∘]:= **eq5**

Out[∘]=

$$-H1 + H2 + \frac{0.0006425433\left(-u1^2 + u2^2\right)}{gc} == Q + \frac{Ws}{MW}$$

CHECK UNITS FOR USE OF CONVERSION FACTOR:

$$\frac{\left(\frac{ft}{s}\right)^2}{\frac{ft*lbm/s^2}{lbf}} \text{ (*without conversion factor from App. A*)}$$

Out[∘]=

$$\frac{ft\ lbf}{lbm}$$

$$\frac{\left(\frac{ft}{s}\right)^2}{\frac{ft*lbm/s^2}{lbf}} * \frac{Btu}{ft*lbf} \text{ (*with conversion factor from App. A*)}$$

Out[∘]=

$$\frac{Btu}{lbm}$$

Inlet Conditions:

In[∘]:= **H1 = 307; (*$\frac{Btu}{lb_m}$, given*)**

u1 = 20; (*velocity in ft/s, given*)

V1 = 9.25; (*specific volume in $\frac{ft^3}{lbm}$, given*)

area1 = $\frac{\pi}{4} * \left(\frac{4.}{12}\right)^2$; (*cross-sectional area in ft^2*)

v1 = u1 * area1; (*volumetric flow rate in $\frac{ft^3}{s}$ *)

m1 = $\frac{v1}{V1}$ * 3600 (*mass flow rate in lbm/hr*)

Out[∘]=

679.2633

In[∘]:= $\frac{\pi}{4} * \left(\frac{4.}{12}\right)^2$

Out[∘]=

0.08726646

In[∘]:= **679.2633 / 3600**

Out[∘]=

0.1886843

Outlet Conditions:

In[∘]:=
```
m2 = m1; (*in lbm/hr from conservation of mass*)

H2 = 330; (*Btu/lb_m, given*)
V2 = 0.28;(*specific volume in ft^3/lbm, given*)

area2 = π/4 * (1./12)^2 ;(* =0.00545415 ft^2*)
u2 = ((m2 / 3600) * V2)/area2 (*velocity in ft/s, 2.26, L7 Slide 4*)
```

Out[∘]=

9.686486

In[∘]:= eq5

Out[∘]=

$$23 - \frac{0.1967287}{gc} == Q + \frac{Ws}{MW}$$

Other Information:

In[∘]:=
```
gc = 32.1740; (*ft*lbm/s^2 / lbf, lookup on page xiv*)
MW = 44.01; (*lb_m/lbmol, lookup on page 665*)
Ws = 5360;(*Btu/lbmol, given*)
```

In[∘]:= eq5

Out[∘]=

22.99389 == 121.7905 + Q

In[∘]:= ans = Solve[eq5, Q]

Out[∘]=

{{Q → -98.79662}}

In[∘]:= Q = Q /. ans[[1]]

Out[∘]=

-98.79662

Dimensional Analysis of Q:

Dimensions of Q are $\frac{\text{Btu}}{\text{lb}_m}$ from either the dimensions of H or the kinetic energy term.

$$\frac{\text{lbf}}{\text{ft} * \text{lbm} / \text{s}^2} * \left(\frac{\text{ft}}{\text{s}}\right)^2$$

Out[ ]=

$$\frac{\text{ft lbf}}{\text{lbm}}$$

In[ ]:=

$$\frac{\text{lbf}}{\text{ft} * \text{lbm} / \text{s}^2} * \left(\frac{\text{ft}}{\text{s}}\right)^2 * \frac{\text{Btu}}{\text{ft} * \text{lbf}}$$

Out[ ]=

$$\frac{\text{Btu}}{\text{lbm}}$$

Convert Q from $\frac{\text{Btu}}{\text{lb}_m}$ to $\frac{\text{Btu}}{\text{hr}}$ using m_1 (in $\frac{\text{lb}_m}{\text{hr}}$):

In[ ]:=

$$\text{Q} \frac{\text{Btu}}{\text{lb}_\text{m}} * \text{m1} \frac{\text{lb}_\text{m}}{\text{hr}}$$

Out[ ]=

$$-\frac{67\,108.91\ \text{Btu}}{\text{hr}}$$

The heat transfer rate from the compressor is -67,108.9 $\frac{\text{Btu}}{\text{hr}}$. //ANS

Problem 2.40

One kilogram of air is heated reversibly at constant pressure from an initial state of 300 K and 1 bar until its volume triples. Assume for air that $\frac{PV}{T} = 83.14 \frac{\text{bar}\cdot\text{cm}^3}{\text{mol}\cdot K}$ and $C_P = 29 \frac{J}{\text{mol}\cdot K}$.

Calculate W, Q, ΔU, and ΔH for the process. Report your answers in kJ.

SOLUTION

The key to this problem is reversibility. “Heated reversibly at constant pressure” means internal & external pressures are the same and W=-PΔV.

Make a sketch of the process:

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \text{State 1} \\ P_1 = 1\text{ bar} \\ T_1 = 300\,K \\ V_1 = 24\,942\,\frac{\text{cm}^3}{\text{mol}} \end{pmatrix} \Longrightarrow \begin{pmatrix} \text{State 2} \\ P_2 = 1\text{ bar} \\ T_2 = \text{unknown} \\ V_2 = \text{unknown} \end{pmatrix}$$

Find the new molar volume V_2, new temperature T_2, and moles n:

```
(*use PV/T=constant*)
```

In[∘]:= $\mathtt{eq1} = \frac{1\,\text{bar} * V}{300\,K} == \frac{83.14\,\text{bar} * \text{cm}^3}{\text{mol} * K};$

In[∘]:=
```
Solve[eq1, V]
```

Out[∘]=

$$\left\{\left\{V \to \frac{24\,942.\ \text{cm}^3}{\text{mol}}\right\}\right\}$$

In[∘]:=
```
V1 = V /. %[[1]][[1]]
```

Out[∘]=

$$\frac{24\,942.\ \text{cm}^3}{\text{mol}}$$

In[∘]:=
```
V2 = 3 * V1
```

Out[∘]=

$$\frac{74\,826.\ \text{cm}^3}{\text{mol}}$$

In[∘]:=
```
T1 = 300 K;
```

In[∘]:= $T2 = \frac{T1}{V1}\,(V2)$

Out[∘]=

900. K

In[ ]:= **V2 - V1**

Out[ ]=

$$\frac{49\,884.\ \text{cm}^3}{\text{mol}}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \text{State 1} \\ P_1 = 1\ \text{bar} \\ T_1 = 300\ K \\ V_1 = 24\,942\ \frac{\text{cm}^3}{\text{mol}} \end{pmatrix} \Longrightarrow \begin{pmatrix} \text{State 2} \\ P_2 = 1\ \text{bar} \\ T_2 = 900.\ K \\ V_2 = 74\,826\ \frac{\text{cm}^3}{\text{mol}} \end{pmatrix}$$

In[ ]:= **1000 / 28.97**

Out[ ]=

34.51847

6

In[ ]:= $\mathbf{MW = \frac{28.97\ g}{mol}}$**; (*molar mass of air*)**

In[ ]:= $\mathbf{n = \frac{1000\ g}{MW}}$

Out[ ]=

34.51847 mol

Calculate the enthalpy change ΔH:

In[ ]:= $\mathbf{Cp = \frac{29\ J}{mol * K}}$**;**

In[ ]:= **ΔH = n * Cp * (T2 - T1)**

Out[ ]=

600 621.3 J

ΔH = 600,621J = 600.6 kJ. //ANS

Calculate heat, Q:

Q = ΔH (*constant pressure*)

Out[ ]=

600 621.3 J

Q = ΔH = 600,621J = 600.6 kJ. //ANS

Calculate the work, W:

(*W=-PΔV, constant pressure*)

$$P = 1\,\text{bar} * \frac{\frac{10^5\,\text{kg}}{\text{m}*\text{s}^2}}{\text{bar}}$$ (*Conversion factor in Appendix A*)

Out[ ]=

$$\frac{100\,000\,\text{kg}}{\text{m}\,\text{s}^2}$$

$$W = -P * (V2 - V1) * n * \frac{1\,\text{m}^3}{10^6\,\text{cm}^3} * \frac{1\,\text{J}}{\frac{1\,\text{kg}*\text{m}^2}{\text{s}^2}}$$ (*factors in Appendix A*)

Out[ ]=

−172 191.9 J

W = -172,192 J = -172.192 kJ. //ANS

Calculate the internal energy change ΔU: