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▣ Multiplicative inverse of -7 modulo 20 :

⇒ We need to find x such that $-7x \equiv 1 \pmod{20}$.

→ if $ax \equiv 1 \pmod{m}$, then x is the multiplicative inverse of a modulo m .

$$\rightarrow -7 \equiv 13 \pmod{20}$$

→ We are looking for an integer x such that $13x = 20k + 1$ for some integer k . We can test values of x :

$$\text{if } x=1, 13(1) = 13 \not\equiv 1 \pmod{20}$$

$$\text{if } x=2, 13(2) = 26 \not\equiv 1 \pmod{20}$$

$$\text{if } x=3, 13(3) = 39 \not\equiv 1 \pmod{20}$$

$$\text{" } x=4, 13(4) = 52 \not\equiv 1 \pmod{20}$$

$$\text{" } x=5, 13(5) = 65 \not\equiv 1 \pmod{20}$$

$$\text{if } x=6, 13(6) = 78 \not\equiv 1 \pmod{20}$$

$$\text{if } x=7, 13(7) = 91 \not\equiv 1 \pmod{20}$$

$$\text{if } x=8, 13(8) = 104 \not\equiv 1 \pmod{20}$$

if $x=9$,	$13(9) = 117 \equiv 17$	mod 20
if $x=10$,	$13(10) = 130 \equiv 10$	mod 20
if $x=11$,	$13(11) = 143 \equiv 3$	mod 20
if $x=12$,	$13(12) = 156 \equiv 16$	mod 20
if $x=13$,	$13(13) = 169 \equiv 9$	mod 20
if $x=14$,	$13(14) = 182 \equiv 2$	mod 20
if $x=15$,	$13(15) = 195 \equiv 15$	mod 20
if $x=16$,	$13(16) = 208 \equiv 8$	mod 20
if $x=17$,	$13(17) = 221 \equiv 1$	mod 20

Thus, $x=17$ is multiplicative inverse of 13 modulo 20.

\therefore The multiplicative inverse of -7 modulo 20 is 17.

* $-17 \text{ mod } 23$:

$$\begin{array}{r} 23 \overline{) -17} \quad (-1) \\ \underline{-23} \\ 6 \end{array}$$

$$\Rightarrow -17 = (-1 \times 23) + 6$$

$$\therefore -17 \text{ mod } 23 = 6 \text{ (Ans.)}$$

* Multiplication Inverse of $-13 \bmod 23$;

\Rightarrow The multiplicative inverse of a number $a \bmod m$ is a number x such that: $ax \equiv 1 \bmod m$.

In our case, we are looking for a number x such that:

$$-13x \equiv 1 \bmod 23$$

To simplify, we first convert -13 into a positive equivalent modulo 23.

$$-13 \bmod 23 = -13 + 23 = 10$$

So, the equation becomes:

$$10x \equiv 1 \bmod 23$$

Now, we find the integer x such that

$$10x \equiv 1 \bmod 23$$

$$\text{if } x=1, 10 \times 1 = 10 \not\equiv 1 \bmod 23$$

$$\text{if } x=2, 10 \times 2 = 20 \not\equiv 1 \bmod 23$$

$$\text{if } x=3, 10 \times 3 = 30 \equiv 7 \bmod 23$$

if $x = 4$, $10 \times 4 = 40 \equiv 17 \pmod{23}$

if $x = 5$, $10 \times 5 = 50 \equiv 4 \pmod{23}$

if $x = 6$, $10 \times 6 = 60 \equiv 14 \pmod{23}$

if $x = 7$, $10 \times 7 = 70 \equiv 1 \pmod{23}$

We found it: $10 \cdot 7 = 70 \equiv 1 \pmod{23}$

Since $-13 \equiv 10 \pmod{23}$ and $10^{-1} \pmod{23} = 7$

We conclude -

The multiplicative inverse of $-13 \pmod{23}$ is 7.

Ans. 7