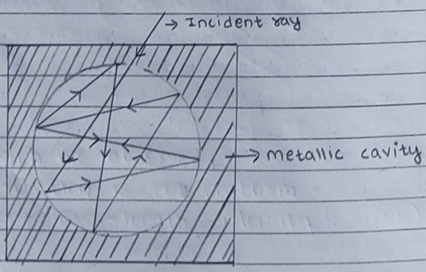
Fundamentals of Atomic Theory

Black body

A black body is an object that absorb all incident electromagnetic radiation that falls on it. No radiation passes through it and none is reflected. Despite the name, black bodies are not actually black as they radiate energy as well.

Black body Radiation

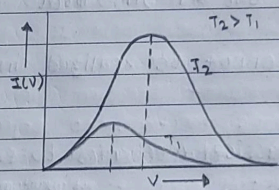
It is a thermal electromagnetic radiation within or surrounding a body in thermodynamics equilibrium with its environment, or emitted by a black body. It has a specific spectrum and intensity that depends on on the body's temperature but independent of the material of the body.



This property is valid for radiation corresponding to all wavelengths and to all angles of incidence.

Characteristics of spectrum of a black body

The observed variation of the spectral intensity $I(\nu)$ (power per unit area per unit frequency) of black body radiation as a function of frequency (ν) is shown in the figure.



1. The intensity reaches a maximum at some frequency ν_{max} ($\nu_{max} \propto T$)
2. The frequency ν_{max} as well as the height of the peak increases with temperature.
3. The spectrum is continous with a broad maximum.
4. The spectrum shifts towards higher frequency as the temperature increases.
5. The integral of $I(\nu)$ overall is called I_T, represents the energy emitted per unit time per area, regardless of the frequency and it is found to increase with fourth power of the temperature. This is known as Stefan Boltzman law ie.

spectral radiance →

$$I_T = \int_0^\infty I(\nu)\, d\nu = \sigma T^4 \quad \text{where}$$

or, $\boxed{E = \sigma T^4}$ σ = Stefan's constant = $5.67 \times 10^{-8}\ W/m^2K^4$

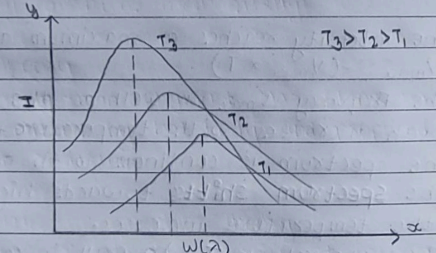
$\mathcal{E} = \sigma T^4$ (Perfectly black body)
$\mathcal{E} = e\sigma T^4$ (Not perfectly black)

For eg:-
To absorber,
$E = e\sigma (T_0^4 - T^4)$
$P = e\sigma (T_0^4 - T^4) A] \rightarrow$ Area

6. Black Body cannot be realized in practise.

* Failure of Classical Mechanics to explain Black Body Radiation

but quantum mechanics can explain.



From the graph we observed,

1. Energy distribution is not uniform. (for different wave-length, intensity is diff)

2. $I \propto \frac{1}{\omega}$ [Intensity increases as wavelength decreases]

3. Area under each curve will get is total energy emitted per unit time.

4. ~~As wavelength~~ temp increases, wavelength shift toward lower intensity side.

Failures

- The electrons in a hot object can vibrate with a range of frequencies, ranging from very few vibrat-ions per second to a huge number of vibrations per second.
  In Fact, there is no limit to how great the frequency can be.

- Classical Physics said that each frequency of vibrat-ion should have the same energy. Since there is no limit to how great the frequency can be, there is no limit to the energy of the vibrating electrons at high frequencies.
  This means that, according to classical physics, there should be no limit to the energy of the light produce by the electrons vibrating at high frequencies. ~~Experi~~
  Experimentally, the blackbody spectrum always becomes small at the left had side (short, wavelength, high frequency)
  i.e. $I \propto \frac{1}{\omega}$

- Black body radiation - Explain by quantum Theory

Different attempts by physist to explain the black body spectrum using law of classical, electromag-netism & thermodynamics ~~but~~ proved was unsuccessful. The quantum ideas introduced by Plank's to explain the spectrum of black body i.e to find the imperical mathematical expression for I(V) that would fit the experimental data called plank's theory, was further expanded by Albert Einstien ie light incident on a metal will cause electron to eject from the metal surface is called photoelectric effect, give theoritical and experimental support to describe black body radiation by quantum mechanically.

* Plank's Theory

In 1900, German physicist Max Planck had studied radiation emitted by hot body and proposed quantum theory of radiation

Main Postulates of quantum theory

- Atom emit or absorb energy at interval in form of energy packet or wave packet called quanta or photon.
- These energy are in discontinous or discrete manner (Photon is the piece of energy that help electron to run from one energy level to another).

Theory:-

The plank's introduced quantum ideas to explain the distribution of energy in the spectrum of a black body. From his hypothesis, "A system undergoing Harmoic motion with frequency (V) can only have and therefore can only emit energy given by ($E = nh\nu$)

where, $n = 1, 2, 3 \ldots$

h = Plank's constant $= 6.626 \times 10^{-34}$ Js"

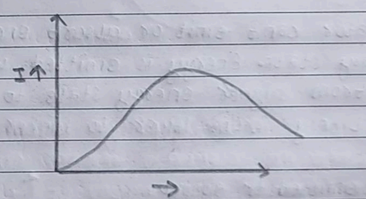


Fig:- plank's law / Theory

$E = h\nu$

- Energy is quantized (just be in discrete not continous)

He assumed that he atom in wall of Black body behaves as simple harmonic oscillator.

Mathematical Expression for I(V), by Planck's

$$I(\nu) = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp(h\nu/K_BT) - 1}$$

$$= \frac{2\pi h\nu^3}{c^2(e^{h\nu/K_BT} - 1)}$$

c = velocity of light

K_B = Boltzmann constant

T = absolute temperature

ν = Frequency of electromagnetic wave

Two assumption about harmonic oscillator

1 A simple harmonic oscillator have any orbitary value of energy but only those total energy is given as.

$$E = nh\nu \text{ ——— } (1)$$

where,

$n = 1, 2, 3 ---$ called quantum numbers

$h = 6.62 \times 10^{-3}$ JS as planck's constant

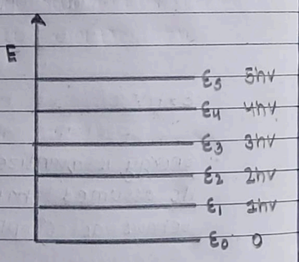
ν = Frequency of oscillation.

2 A oscillator can't emit or absorb energy when it is in stationary state. Energy is emitted when the oscillator jumps from higher energy state to lower energy state. Likely from lower to higher as well.

Energy emitted or absorbs between E_1 and E_2

$$\Delta E = E_2 - E_1$$
$$= (n_2 - n_1) h\nu$$

$$E_2 - E_1 = h\nu$$



Bohr's Atomic Model

A model of Hydrogen atom was proposed by Bohr in 1913. He assumed basically Rutherford nuclear model of the atom & tried to overcome the defects of Rutherford model. He proposed the following ~~two~~ postulates:-

1. An electron revolves round the nucleus in a stable circular orbit.

The centripetal force which keep electron moving in the circular orbit is provided by electrostatic force betn proton and electron. i.e

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \text{ ——— } (1)$$

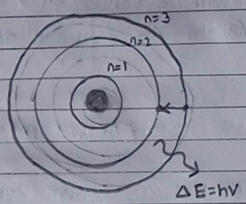
2. An electron cannot revolve round the nucleus in all possible orbits.

Electron can revolve round the nucleus only in those orbits for which angular momentum of an electron is equal to integral multiple of $\frac{h}{2\pi}$ i.e

$$mvr = \frac{nh}{2\pi} \text{ ——— } (2)$$

3. When an electron transits from outer stable orbit to inner stable orbit, it emit phon of energy equal to $h\nu$ i.e

$$E_{n_2} - E_{n_1} = h\nu \quad \text{------} \ ③$$



- Energy Spectrum of Hydrogen atom

When an electron revolves round the nucleus, the centripetal force equals to electrostatic force i.e

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

or, $$mv^2 r = \frac{e^2}{4\pi\epsilon_0} \quad \text{------} \ ①$$

Since $mvr = \frac{nh}{2\pi}$

or, $$v = \frac{nh}{2\pi m r} \quad \text{------} \ ②$$

Now,

Putting v in Eq - ①

$$m\frac{n^2h^2}{4\pi^2 m^2 r^2} r = \frac{e^2}{4\pi\epsilon_0}$$

or, $$r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$$

Therefore the radius of n^{th} permissible orit for Hydrogen atom.

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \quad \text{------} \ ③$$

Thus,

radius, $r_n \propto n^2$

The radius of inner most orbit called Bohr's radius,

$$r_0 = \frac{\epsilon_0 h^2}{\pi m e^2} \approx 0.529 \text{ Å} \quad [n=1]$$

From eqn ② we have,

$$v = \frac{nh}{2\pi m r}$$

or, $$v = \frac{nh}{2\pi m \frac{\epsilon_0 n^2 h^2}{\pi m e^2}}$$

$$= \frac{e^2}{2\epsilon_0 n h} \quad \text{------} \ ④$$

KE
PE $\rightarrow$
TE

The kinetic energy of electron,

$$E_K = \frac{1}{2} m v^2$$

$$\text{or, } E_K = \frac{1}{2} m \left(\frac{e^2}{2\epsilon_0 n h}\right)^2$$

$$\text{or, } \boxed{E_K = \frac{me^4}{8\epsilon_0^2 n^2 h^2}} \quad \text{—— (5)} \quad \text{Kinetic Energy}$$

The potential energy,

$$E_p = -\frac{e^2}{4\pi\epsilon_0 r_n}$$

$$\text{or, } E_p = -\frac{e^2}{4\pi\epsilon_0 \frac{\epsilon_0 n^2 h^2}{\pi m e^2}}$$

$$\text{or, } \boxed{E_p = -\frac{me^4}{4\epsilon_0^2 n^2 h^2}} \quad \text{—— (6)} \quad \text{Potential Energy}$$

Now,

Total energy of electron in n^{th} orbit,

$$E_n = E_K + E_p$$

$$\text{or, } E_n = \frac{me^4}{8\epsilon_0^2 n^2 h^2} - \frac{me^4}{4\epsilon_0^2 n^2 h^2}$$

$$\therefore \boxed{E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2}} \quad \text{—— (7)} \quad \text{Total Energy}$$

(Negative sign shows electron is bounded to nucleus)

From eq^n (7), total energy of electron in n^{th} orbit is inversly proportional to n^2. i.e $E_n \propto \frac{1}{n^2}$

Again, total energy of an electron in n^{th} orbit can be written as.

$$E_n = -\frac{9.1\times10^{-31} \times (1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2 (6.62\times10^{-34})^2 n^2}$$

$$= -\frac{13.6}{n^2} \text{ eV}$$

For $n = 1, 2, 3 ----$

$E_1 = -13.6$ eV

$E_2 = -3.4$ eV,

$E_3 = -1.5$ eV, ---

- Spectral lines Predicted by the Bohr Model

Total energy of electron in n^{th} orbit.

$$E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2} \quad \text{—— (1)}$$

When the electron makes transition from n_2^{th} orbit to n_1^{th} orbit, the Frequency ν of radiation emitted is given by,

Photon energy $\rightarrow$ $h\nu = E_{n_2} - E_{n_1}$

$$\text{or, } h\nu = -\frac{me^4}{8\epsilon_0^2 n_2^2 h^2} + \frac{me^4}{8\epsilon_0^2 n_1^2 h^2}$$

$$\text{or, } \frac{hc}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or, $\frac{1}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

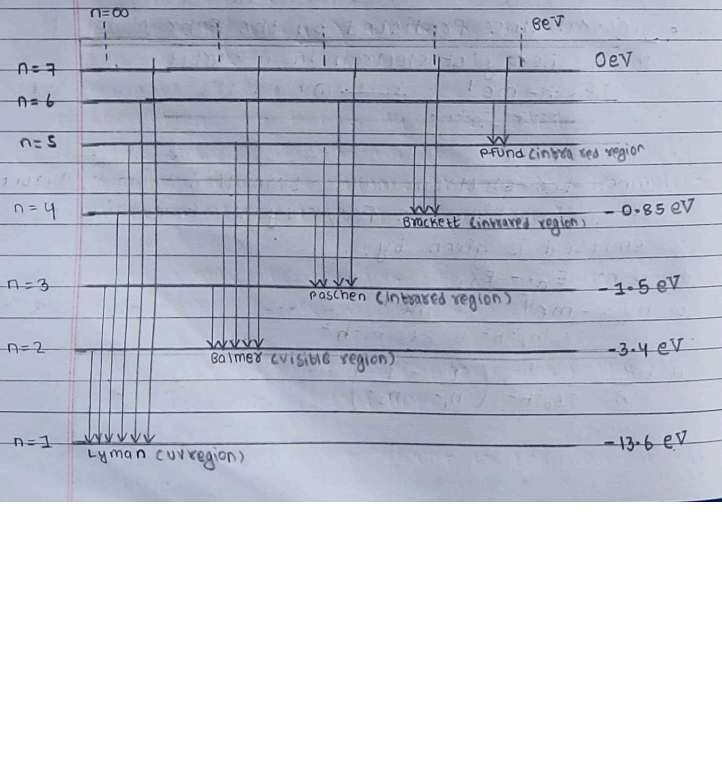
or, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

where $R = \frac{me^4}{8\epsilon_0^2 h^3 c}$ is called **Rydberg's constant.**

$$R = \frac{(9.1\times10^{-31})\times(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2\times(6.62\times10^{-34})^3\times(3\times10^8)}$$

$$R = 1.091\times10^7\ m^{-1}$$

Energy level Diagram of Hydrogen atom



1. Lymen series

When an electron jumps from second, third ---etc orbit to the first orbit, we get spectral lines which lie in ultraviolet region called Lymen series.

For Lymen series,

$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$ where, $n_2 = 2, 3, 4, ---$

2. Balmer series

When an electron jumps from third, forth ---etc orbit to the second orbit, we get spectral lines which lie in visible region called Balmer series.

For Balmer series,

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$ where, $n_2 = 3, 4, 5 ----$

Note:- The first line in the series (For $n_2 = 3$) is called H_α line, the second line (For $n_2 = 4$) is called H_β line and so on.

3. Paschen series

When an electron jumps from fourth, Fifth--- etc orbit to the third orbit, we get spectral lines which lie in infrared region called Paschen series.

For Paschen series,

$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$ where, $n_2 = 4, 5, 6 ----$

4. Brackett series

When an electron jumps from fifth, sixth---- etc orbit to fourth orbit, we get spectral lines which lie in far infrared region called Brackett series.

For Brackett series,

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right) \text{ where } n_2 = 5, 6, 7 \ldots$$

5. Pfund series

When an electron jumps from sixth, seventh--- etc orbit to the fifth orbit, we get spectral lines which lie far infrared region called Pfund series.

for Pfund series,

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right) \text{ where, } n_2 = 6, 7, 8 \ldots$$

Limitations of Bohr's Hydrogen Model

- Bohr's theory could explain the spectral lines of hydrogen atom but could not explain the spectra of multi-electron atom (other than hydrogen).

- Bohr's theory does not give explanation, why only circular orbits are possible around the nucleus.

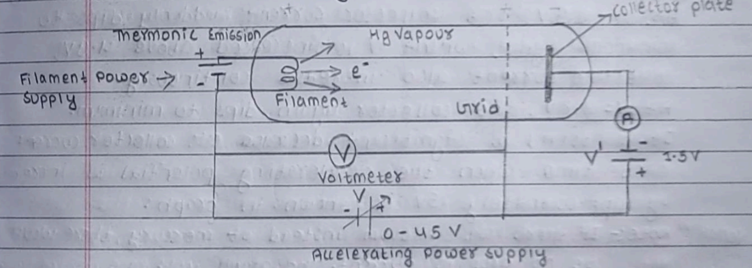
- Bohr's theory doesnot give explation, about the relative intensities of spectral line.

- Bohr's theory does not account for the wave nature of electron.

VVI Franck-Hertz Experiment

It demonstrate the existence of excited state state of mercury atom, helping to confirm the quantum theory which predicted that electron occupied only discrete, quantized energy state.

Experimental arrangement:



It consists of glass tube filled with mercury vapour at pressure 1mm of Hg. Electron are emitted by heated filament and accelerated towards grid. We have taken 3 electrode i.e cathode, anode and grid which are connected by two battery. Ammeter is used to measure current. We emit the electron by thermonic emission with the help of supply of battery. Grid is used to accelerate electron (because of +ve). If electron have enough energy it passes from grid to anode. If not it remain as it is.

Note:- we add vapour of gases, so that electrons collide with the atom of gases and it's energy is lost during collision and KE of electron change and there is inellastic collision.

Electron while passing strikes with mercury.

If we increase voltage, e^- moves faster towards Grid

i.e voltage $\propto$ K.E $\propto$ I

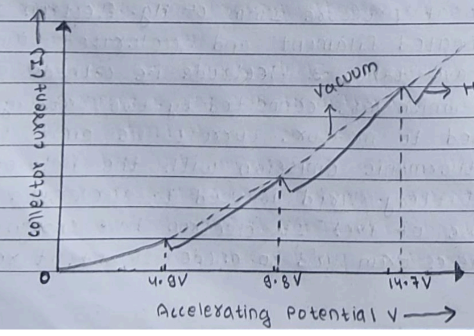
When V' is constant, V is gradually increased and there will be no collector current for V<V'. And V is increased collector current also increase continously. At, V = 4.9 V, the collector current suddenly dips to minimum. Again, when V is increased above 4.9V, collector current also increases continously.

At V = 9.8 V, the collector again dips to minimum.

It is found a significant decrease of collector current each time when the accelerating potential is increased by approximately 5V as shown in graph.

To remember Note:- If there was vacuum instead of mercury, there would not be no drop in current. The graph would have pointed upward.

Explanation of graph.



In the absence of any vapour, i.e vaccum, I-V is shown by dotted line.

There is no collector current for V<V'. Above this value the collector current increases continously. When the accelerating power supply reaches a value 4.9 volt, current becomes maximum and then suddenly dips to a minimum value. Again the collector current increases gradually till another maximum is reached. When the p.d is just 9.8 V, then suddenly current again dips steeply to minimum so that every 4.9 V significant decrease in the collector current occurs.

The fact that there is no drop in current V = 4.9 V indicates that the electron do not loose energy through collision until they have 4.9 eV of K.E. When the electron collide with a heavy atom such as Hg, there are two possible kind of collisions: Elastics and InElastics. **In case of elastic collision** the total energy of both particle before and after collision is same. The requirement that the total energy & momentum be conserved leads to the fact that the KE of the light particle (electron) is hardly changed, the velocity is slightly reversed; so they will eventually be able to overcome the small retarding voltage, V and contribute to the collector plate. No drop in current will be caused by this kind of collision. **In case of inelastic collision,** the external K.E of the colliding particles (e^- & Hg) becomes internal energy (mercury atoms absorb energy and gets excited). Some of the electrons may loose enough energy to prevented from reaching the plate by retarding potential V. i.e drop in the current should occur for any value of V', so that the 1^{st} excited state of 'Hg' (the smallest amount of energy that the mercury can absorb) is 4.9eV, above the ground state. 2x4.9eV = 9.8eV is the 2^{nd} excited state and so on.

Each time there is inelastic collision, the mercury atoms will be excited and return to ground state by emitting photons.

The wavelength of radiation from the tube was found to be 2536 Å corresponding to transition from 1^{st} excited state to ground state. The energy of the photon of this wavelength.

$$E = h\nu = h\frac{c}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{2536\times10^{-10}} = 4.9\ eV$$

This experiment shows the existence of discrete energy levels in mercury atoms.

Limitations of Franck-Hertz Experiment

- This experiment is not able to distinguish betn excitation and ionization potential.
- This method is not suitable for strongly electron gases like oxygen, flourine (because these gases attract electrons strongly).
- The actual value of critical potential is slightly less than the observe value (because the velocity of electron is not zero).

De-Broglie Hypothesis

Light (dual-nature)

Particle | Wave

- Photoelectric effect
- Compton effect

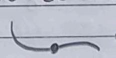
- Interference
- Diffraction
- Polarization

Matter (e^-)

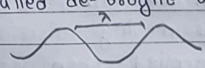
Particle | Wave

De-Broglie proposed that just as light has both (wave & particle) like properties, Matters (Specially electrons) also have wave like properties.

The waves associated with moving particle are called de-Broglie waves or matter waves.



The wave length associated to matter waves is called de-Broglie wavelength.



~~de~~broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv}$

where, $p = mv$ is momentum of particle

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- **Derivation of de-Broglie Wavelength**

According to Planck's theory of radiation,

$E = h\nu$ ——— ①

where,

E = energy of photon,

ν = frequency of photon.

According to Einstein mass energy relation.

$E = mc^2$ ——— ②

Then,

From eqⁿ ① and ②

$h\nu = mc^2$

or, $\frac{hc}{\lambda} = mc^2$

or, $\frac{h}{\lambda} = mc = p$

or, $\lambda = \frac{h}{p}$ ——— ③

For a particle moving with velocity v,

$$\boxed{\lambda = \frac{h}{mv}} \text{ ——— ④}$$

De-broglie wavelength of an electron.

Consider a electron is moving with velocity v when the potential **V** is applied then,

$eV = \frac{1}{2}mv^2$

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or, $v = \sqrt{\frac{2eV}{m}}$

De-Broglie wavelength,

$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$

In terms of kinetic energy,

KE of the moving particle,

$E_K = \frac{1}{2}mv^2$

or, $v = \sqrt{\frac{2E_K}{m}}$

De-Broglie wavelength,

$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE_K}}$

De-Broglie wavelength in thermal equilibrium

Consider a moving particle in thermal equilibrium at temperature T, the kinetic energy of the particle is

$E_K = \frac{3}{2}K_B T$ $\quad$ $K_B = 1.38 \times 10^{-23} JK^{-1}$ is Boltzmann constant

Since,

$\lambda = \frac{h}{\sqrt{2mE_K}} = \frac{h}{\sqrt{2m\frac{3}{2}K_B T}}$

De-Broglie wavelength, $\lambda = \frac{h}{\sqrt{4mK_B T}}$

* Experimental verification of de-Broglie Hypothesis

Davisson and Germer Experiment

The Davisson-Germer in 1927 demonstrated the wave nature of the electron, conforming the de-Broglie hypothesis.

Experimental setup:

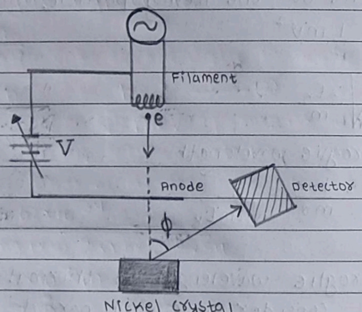

Fig ①

The experimental setup for Davisson and Germer experiment is shown as in fig ①. The electrons from a heated filament are accelerated by variable voltage V towards nickel crystal.

Here, The kinetic energy of electron is $E_K = eV$

The electrons which strike on the nickel get scattered and its intensity can be detected for different angle and various voltage by using detector. At certain angles there can be found a peak in the intensity of the scattered electron beam. This peak indicated wave behaviour for the electrons, and could be interpreted by the Bragg law to give values for the lattice spacing in the nickel crystal. While rotating the detector, the maximum intensity of scattered beam of electrons can be at values $V = (54V)$ and $\phi = 50^\circ$ as shown in fig ②

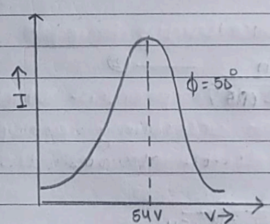

Fig-②

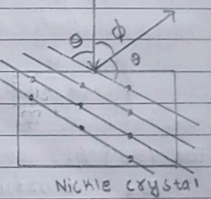

Fig-③

It concludes that if the electrons behave as particles only, then for any incoming angle will be same number of particles rebounding at a similar angle regardless of their velocity and potential as shown in fig ③. The intensity is uniform for different value of ϕ.

This proves that diffraction of electron as a wave and hence Davisson and Germer experiment is evidence of de-Broglie hypothesis.

→

If the electron of mass 'm' is accelerated through the potential V with velocity 'v' then

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} \quad \text{——— (1)}$$

Then, we know,
From de-Broglie hypothesis,

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} \quad \text{——— (2)}$$

For V=54 V,

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times54}}$$

$$= 1.67\times10^{-10}\,m$$

$$\approx 1.67\,\mathring{A}$$

Again when x-ray beam is incident on the nickel crystal, from Bragg's law of diffraction.

$$2d\sin\theta = n\lambda \quad \text{——— (3)}$$

For, Nickel crystal; $d = 0.91\,\mathring{A}$
From fig (3),

$$\theta + \phi + \theta = 180^\circ \quad \therefore \theta = 65^\circ \quad , \text{since } \phi = 50^\circ$$

Now, from eqn (3)

$$\lambda = \frac{2d\sin\theta}{n} \quad \text{taking } n=1 \text{ and } \theta = 65^\circ$$

or, $\lambda = \frac{2\times0.91\times\sin65^\circ}{1} \approx 1.65\,\mathring{A}$

This is nearly equal to wavelength of electron beam. This agreement between the wavelength of electron beam and x-ray shows that electron beam produces same type of diffraction patter as produced by x-rays.

Thus the experiment confirms the de-Broglie hypothesis that moving particles (electrons) behave as wave.

* Heisenberg Uncertainty Principle

Statement:-

It is impossible to specify precisely and simultaneously the values of both member of positive pairs (cannonically conjugates) of physical variables that describes the behaviour of an atomic systems.

According to him, the position and momentum, energy and time, and angular position and angular momentum are cannonically conjugate. simultaneously to any desired degree of accuracy.

If a position (x) has an uncertainity(Δx) ~~to any desired~~

$\hbar \Rightarrow h$ bar
or,
h cut

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and the corresponding momentum component (p) has uncertainty (ΔP) then uncertainities are bound to be related in general by an inequality.

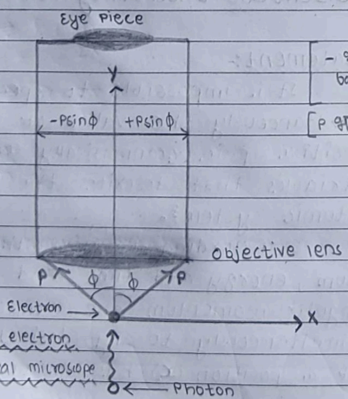
i.e $\Delta x . \Delta p \geq \hbar$ $\quad \boxed{\hbar = \frac{h}{2\pi}} = \frac{6.62 \times 10^{-34}}{2\pi} = 1.054 \times 10^{-34}$ Js

Similarly, $\Delta E . \Delta t \geq \hbar$ (Energy and time)

$\Delta L . \Delta \theta \geq \hbar$ (Angular momentum & angular position)

Physical Origin of Uncertainty Principle

Bohr's proposed an experiment which illustrates the physical origin of uncertainty principle, namely, the majoring process itself introduces the uncertainty.



[- भनेको origin bata left भएकोले]

[P भनेको momentum]

Fig:- Looking at a electron with hypothetical microscope

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Suppose that we want to determine the position of an electron, we can look at electron using hypothetical microscope. For this we have to locate the photons scattered when incident on electron as shown in the figure. Any photon scattered by electron with in an angle 2ϕ will be focused by microscope and detected by eye piece.

The collision of photon will the electron will change the momentum of electron. The photon may enter the objective lens within the angular range $+\phi$ to $-\phi$. The x-component of momentum could be any value between $-p\sin\phi$ and $+p\sin\phi$. The uncertainty of x-component of momentum of photon is,

$$\Delta P_x (\text{photon}) = 2 \Delta P_x (\text{photon}) \sin\phi \quad \text{——— (1)}$$

Again,
The uncertainty of x-component of momentum of electron also has the same magnitude i.e.

$$\Delta P_x (\text{electron}) = 2 \Delta P_x (\text{photon}) \sin\phi \quad \text{——— (2)}$$

Then,
The Momentum of photon is given by,

$$P = \frac{h}{\lambda} \quad \text{——— (3)}$$

$$\therefore \Delta P_x (\text{electron}) = 2\, h/\lambda \sin\phi \quad \text{——— (4)}$$

→

wave function = $\psi \Rightarrow$ psi (pronunciation → साई)

In this process of locating the electron, we have introduced the uncertainty in momentum.
From eqn ③, uncertainty in momentum can be reduced by
a) reducing ϕ,
b) using photons of longer wavelength.
These two factors lead to increase uncertainty in position of electron.

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Matter Wave & Uncertainty Principle

The quantum particle propagates in a space is given by the wave function,

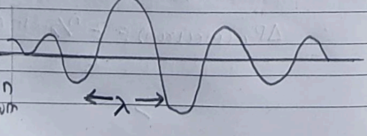
$$\psi(x,t) = A\sin(Kx - \omega t) \quad \text{——— ①}$$

$$\omega = 2\pi F \Rightarrow F = \frac{\omega}{2\pi}$$

$$K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K}$$

$$v = F\lambda = \frac{\omega}{K}$$

(In Quantum mechanics, particle is considered as wave)
So, position is not fixed
↓
uncertain

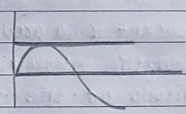


The particle can be found with equal probability at any point in space i.e the particle is unlocalized ($\Delta x = \infty$).
So, the uncertainty in momentum, $\Delta P_x = 0$.
This is the agreement with uncertainty principle.

From de-Broglie theorem,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{——— ①}$$

If $v = 0$
$\lambda = \infty$



Matter wave exist only in particle (moving)
If $v = \infty$
$\lambda = 0$

(Appears like straight line)

$$KE = \frac{3}{2}KT$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{3}{2}KT$$

$$\Rightarrow m^2v^2 = 3mKT$$

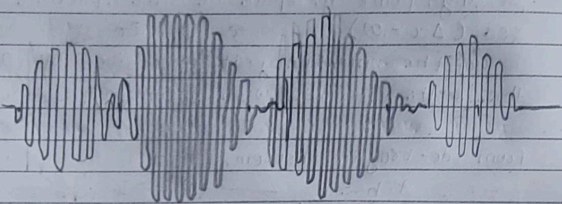
$$mv = \sqrt{3mKT}$$

Now, eqn ① be

$$\boxed{\lambda = \frac{h}{\sqrt{3mKT}}}$$

This is relation.

Group Velocity: Velocity of Wave Packet



A wave packet is a group of waves with slightly different wavelengths interferring with one another in a way that amplitude of the group (evelope) is non-zero.

- Wave packet is localized a good representation for a particle.
- The ~~wave packet as a wh~~ velocities of the individual waves which superpose to produce the wave packet repr-esenting the particle are different.
- The wave packet as a whole has a different velocity velocity from the waves that comprise it.

Phase velocity :- The rate at which the pase of the wave propagates in space.

Group velocity :- The rate at which the envelope of the wave packet propagates.

Consider two waves with slightly different frequency and wavelength.

$\psi_1 = A\sin(kx - \omega t)$ and

$\psi_2 = A\sin[(k+\Delta k)x - (\omega+\Delta\omega)t]$

when these waves superimposed, resulting wave:

$$\psi = \psi_1 + \psi_2 = A\sin(kx-\omega t) + A\sin[(k+\Delta k)x - (\omega+\Delta\omega)t]$$

or, $$\psi = 2A\sin\frac{kx-\omega t+(k+\Delta k)x-(\omega+\Delta\omega)t}{2}\cos\frac{kx-\omega t-(k+\Delta k)x+(\omega+\Delta\omega)t}{2}$$

or, $$\psi = 2A\cos\left(\frac{\Delta\omega t-\Delta kx}{2}\right)\sin\left(\frac{2kx+\Delta kx-2\omega t-\Delta\omega t}{2}\right)$$

Since $\Delta k << k$ and $\Delta\omega << \omega$

$$\psi = 2A\cos\left(\frac{\Delta\omega t-\Delta kx}{2}\right)\sin(kx-\omega t) \quad \text{—— (1)}$$

The second term of Eq^n (1) represents a wave having same frequency and same wave length as the original wave. whereas 1st term represent the amplitude (periodically varying) of wave packet.

The velocity with which each component wave propagates is called wave velocity or phase velocity given by,

$$v = \frac{\omega}{k}$$

$$v_{group} = \frac{(\Delta\omega/2)}{(\Delta k/2)} = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} \quad \text{—— (2)}$$

$E = hf = \frac{h}{2\pi}2\pi f = \hbar\omega$

Eq^n (2) can be written as,

$$v_{group} = \frac{d(\hbar\omega)}{d(\hbar k)} = \frac{dE}{dp} \quad \text{—— (3)}$$

$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$

We have KE of particle, $E = \frac{p^2}{2m}$

So,

$$v_{group} = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m} = \frac{mv}{m} = v \quad \text{---} (4)$$

Thus group velocity equals to particle velocity.

Relation Between Phase Velocity & Group Velocity

We have,

Phase velocity : $v_{phase} = \frac{\omega}{K}$ and Group velocity: $v_{group} = \frac{d\omega}{dK}$

So, $v_{group} = \frac{d\omega}{dK} = \frac{d}{dK}(v_{phase} K)$

or, $v_{group} = v_{phase} + K \frac{dv_{phase}}{dK}$

or, $v_{group} = v_{phase} + K \frac{dv_{phase}}{d\lambda} \frac{d\lambda}{dK}$

- Since, $\frac{d\lambda}{dK} = \frac{d}{dK}\left(\frac{2\pi}{K}\right) = -\frac{2\pi}{K^2}$

Now, $v_{group} = v_{phase} - K \frac{dv_{phase}}{d\lambda} \frac{2\pi}{K^2} = v_{phase} - \frac{2\pi}{K} \frac{dv_{phase}}{d\lambda}$

or, $v_{group} = v_{phase} - \lambda \frac{d}{d\lambda}(v_{phase})$

This is the relation between phase velocity and group velocity.