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Electric and Magnetic Field

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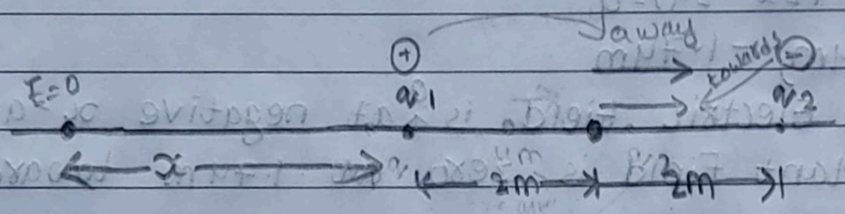
Unit-2

1. A charge $q_1 = 3\times10^{-6}$ C is located at the origin of the x-axis. A second charge $q_2 = -5\times10^{-6}$ C is also on the x axis 4m from the origin in the positive x-direction.

a) Calculate the electric field at the mid-point on the line joining the two charges. 18×10^{3} N/m

b) At what point on the line is the resultant field zero?
1.9m toward left from origin



a. soln:-

$$|E_1| = \left|k\frac{q_1}{r_1^2}\right| = \left|9\times10^9 \times \frac{3\times10^{-6}}{2^2}\right| = 6750 \text{ N/C}$$

$$|E_2| = \left|k\frac{q_2}{r_2^2}\right| = \left|9\times10^9 \times \frac{-5\times10^{-6}}{2^2}\right| = 11250 \text{ N/C}$$

From Figure,

Net Electric Field $= E_1 + E_2$ [as both are along positive x-axis]

$= 6750 + 11250$

$= 18000$ N/C

$= 18\times10^3$ N/C

b) soln:-

From above we know that, E = 0 cannot be at any point betn q_1 and q_2. So, E = 0 must lie towards left from origin,

$E_{net} = E_1 - E_2$, $E = E_1 + E_2 = 0$

$0 = E_1 - E_2$ $\quad E_1 = -E_2$

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or, $E_1 = E_2$ and, $|E_1| = |E_2|$

or, $\cancel{K}\dfrac{3\times 10^{-6}}{x^2} = \cancel{K}\dfrac{5\times 10^{-6}}{(x+4)^2}$

or, $3(x^2+8x+16) = 5x^2$

or, $3x^2+24x+48 = 5x^2$

or, $2x^2-24x-48=0$

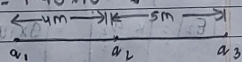
or, $2(x^2-12x-24)=0$

or, $x = 13.74m, -1.74m$

Since, ~~th.~~ Electric field is 0 at negative x-axis, thus, The resultant Field is zero at 1.74 m toward left from origin.

2. Three charges $q_1 = 3\times10^{-6}$ C, $q_2 = -5\times10^{-6}$ C and $q_3 = -8\times10^{-6}$ C are positioned on a st. line. Find the potential energy of the charges. Ans = 1.43×10^{-2} J

Soln:



Electric potential Energy (U) =

$$K\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= 9\times10^9\left(\frac{(3\times10^{-6})\times(-5\times10^{-6})}{4} + \frac{(3\times10^{-6})(-8\times10^{-6})}{5} + \frac{(-5\times10^{-6})(-8\times10^{-6})}{9}\right)$$

$= 1.43\times10^{-2}$ J

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3. Assume that the electron in a hydrogen atom is essentially in a circular orbit of radius 0.5×10^{-10} m and rotates about the nucleus at the rate of ~~1014~~ 10^{14} times per second. What is the magnetic moment of hydrogen atom due to the orbital motion of the electron? 1.26×10^{-25} Am^2

Soln:

Magnetic dipole moment, μ = area × current

or, $\mu = iA$

or, $\mu = i\times\pi r^2$

Where, ~~is the single~~ i is the current due to single electron, Because current is defined as the amount of charge passing per unit time,

$$i = ev$$

Then,

$\mu = i\times\pi r^2$

$= e\times v\times\pi\times r^2$

$= (1.6\times10^{-19})\times10^{14}\times3.14\times(0.5\times10^{-10})^2$

$= 1.256\times10^{-25}$ Am^2

4. A current of 50 A is established in a slab of copper 0.5 cm thick and 2 cm wide. The slab is placed in a magnetic field B of 1.5 T. The magnetic field is perpendicular to the plane of the slab and to the current. The free electron concentr-ation in copper is 8.4×10^{28} electrons/m³. What will be the magnitude of the Halls voltage across the width of the slab? 1.12×10^{-6} V

Soln:

N = number of charge carrying per unit volume

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We have,

$I = 50\ A$ $\qquad N = 8.4 \times 10^{28}$ electrons/m^3

thick~~ness~~, $l = 0.5\ cm = 5 \times 10^{-3} m$

width, thickness, $d^{or\ w} = 2\ cm = 0.02 m$

$B = 1.5\ T$

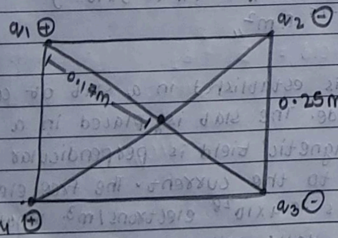
Then,

$$V_{Hall} = \frac{i\ B\ d}{q\ N\ A}$$

$$= \frac{50 \times 1.5 \times 0.02}{1.6 \times 10^{-19} \times 8.4 \times 10^{28} \times 1 \times 10^{-4}} \qquad \left(A = l \times w = 5 \times 10^{-3} \times 0.02 = 1 \times 10^{-4} m^2 \right)$$

$$= 1.11 \times 10^{-6}\ V$$

5. Four charges $q_1 = 3 \times 10^{-6} C$, $q_2 = -3 \times 10^{-6} C$, $q_3 = -3 \times 10^{-6} C$ and $q_4 = 3 \times 10^{-6} C$ are placed in a corner of a square 25 cm as shown in the Figure. What is the electric field at the centre of the square
zero



Here,

length = 25 cm = 0.25 m

diagonal = $l\sqrt{2}$ = 0.35 m

Then, $r = \frac{0.35}{2} = 0.17 m$

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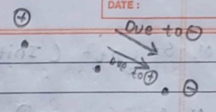


Electric field at centre due to,

q_1 and q_3, $E_A = |E_1| - |E_3|$ —— (i)

Again,

q_2 and q_4, $E_B = |E_4| - |E_2|$ —— (ii)

Now, $\qquad E = E_A + E_B$

Net Electric field, $\vec{E} = |E_1| - |E_3| + |E_4| - |E_2|$

or, $K\left(\frac{3 \times 10^{-6}}{r^2} - \frac{3 \times 10^{-6}}{r^2} + \frac{3 \times 10^{-6}}{r^2} - \frac{3 \times 10^{-6}}{r^2}\right)$

$= K \times 0$

$= 0$

∴ The Electric field at the centre of the square is zero.

6. Two large parallel plates are seperated by a distance of 5cm. The plates have equal but opposite charges that creates an electric field in the region between the plates. An α-particle ($q = 3.2 \times 10^{-19} C$, $m = 6.68 \times 10^{-27}$ kg) is released from the positively charged plate and it strikes the negatively charged plate 2×10^{-6} sec later. Assuming that the electric field bet^n the plates is uniform and perpendicular to the plates, what is the strength of the electric field?

Soln: $\qquad$ 522 N/C

charge of particle (q) = $3.2 \times 10^{-19} C$

mass of particle (m) = 6.68×10^{-27} kg

distance bet^n plates (s) = 5cm = 0.05 m

Time taken (t) = $2 \times 10^{-6} s$

Electric field Bet^n the plates (E) = ?

We have,

$$S = ut + \frac{1}{2}at^2$$

or, $S = \frac{1}{2} a t^2$ (Initially at rest, $u=0$)

or, $a = \frac{2S}{t^2}$ —— ①

Again, from 2nd law of motion,

$F = ma$ —— ②

and electrostatic force,

$F = qE$ —— ③

From ①, ② & ③,

$ma = qE$

or, $m \frac{2S}{t^2} = qE$

or, $E = \frac{2mS}{qt^2}$

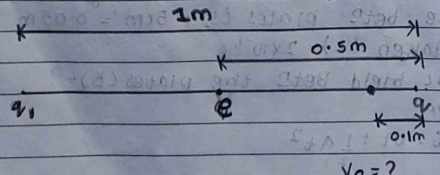
$= \frac{2 \times 6.68 \times 10^{-27} \times 0.05}{3.2 \times 10^{-19} \times (2 \times 10^{-6})^2}$

$= 521.875$

∴ $E = 522$ N/C

7\. An electron is placed midway between two fixed charges $q_1 = 2.5 \times 10^{-10}$ C and $q_2 = 5 \times 10^{-10}$ C. If the charges are 1m apart, what is the velocity of electron when it reaches a point 10 cm from q_2? 1.125×10^6 m/s

soln:-



$V_e = ?$

Here,

$q_1 = 2.5 \times 10^{-10}$ C

$q_2 = 5 \times 10^{-10}$ C

Distance between q_1 and q_2 $(r) = 1$m

Now,

Distance travelled by e^- towards q_2 from initial position

$S = 0.5$ m $- 0.1$ m

$= 0.4$ m

from equation of motion,

$v^2 = u^2 + 2aS$

$v = \sqrt{2aS}$ —— ①

Again,

Electrostatic force betn:-

a) q_1 and e

$F_1 = K \frac{q_1 e}{r_1^2}$ —— ②

b) q_2 and e

$F_2 = K \frac{q_2 e}{r_2^2}$ —— ③

From ② and ③,

Net force $(F) = F_2 - F_1$

$= Ke \left(\frac{q_2}{r_2^2} - \frac{q_1}{r_1^2} \right)$

$= 1.6 \times 10^{-19} \times 9 \times 10^9 \left(\frac{5 \times 10^{-10} - 2.5 \times 10^{-10}}{0.5^2} \right)$

$= 1.44 \times 10^{-18}$ N

Then,

From Newton's 2^{nd} law of motion,

$F = ma = 1.44 \times 10^{-18}$

or, $a = \frac{1.44 \times 10^{-18}}{9.1 \times 10^{-31}}$

$\therefore a = 1.58 \times 10^{12}\ m/s^2$

Now,

Putting value of a in eqn ①

$v = \sqrt{2 \times 1.58 \times 10^{12} \times 0.4}$

$\therefore v = 1.125 \times 10^{6}\ m/s$

8. What force is experienced by the wire of length $l = 0.08$ at an angle of 20°. to the magnetic field direction carrying a current of 2A in a magnetic field of 1.4 T?

7.66×10^{-2} N

Soln:-

length (l) = 0.08 m

angle (θ) = 20°

current (i) = 2A

Magnetic field (B) = 1.4 T

Then,

Force (F) = $B i l \sin\theta$

$= 1.4 \times 2 \times 0.08 \times \sin(20°)$

$= 0.0766$ N

$= 7.6 \times 10^{-2}$ N

9. The earth's magnetic field is 4×10^{-5} T and is parallel to the surface of earth in the south-north direction. (Note that the earth's geographic north pole is the magnetic south pole). A wire of 2m long of mass. m = 9 gm is suspended by a string. The wire is also parallel to the earth's surface and carries a current of 150 A in the east-west direction. a) What is the tension on the string? b) What would be th tension if the current was in the west-east direction?

10.02×10^{-2} N, 7.62×10^{-2} N

Soln:-

$B = 4 \times 10^{-5}$ T

$l = 2$ m

$m = 9$ gm $= 9 \times 10^{-3}$ kg

$I = 150$ A

Then,

a) Tension in the string,

$T = mg + BIl$

$= 9 \times 10^{-3} \times 9.8 + 4 \times 10^{-5} \times 2 \times 150$

$= 0.1002$ N

$= 10.02 \times 10^{-2}$ N

b) Tension when current was in West-East direction,

$T = mg - BIl$

$= 0.0882 - 0.0762$

$= 0.0762$ N

$= 7.62 \times 10^{-2}$ N

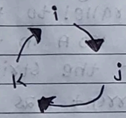
10. A proton is moving with the velocity $v = (3\times10^5 i + 7\times10^5 k)$ m/s in a region where there is a magnetic field $B = 0.4 j$ T. What is the force experienced by the proton?

Soln:- $[1.92k - 4.48i]\times10^{-14}$ N

velocity of proton (v) = $(3\times10^5 i + 7\times10^5 k)$ m/s

Magnetic field (B) = $0.4 j$ T

Then,



$i \times j = k$

$j \times k = i$

$i \times k = -j$

$k \times j = -i$

$$F = q(\vec{B} \times \vec{v})$$
$$= q[(3\times10^5 \vec{i} + 7\times10^5 \vec{k})\, 0.4j]$$
$$= 1.6\times10^{-19}(1.2\vec{k} - 2.8\vec{i})\times10^5$$
$$= (1.92k - 4.48i)\times10^{-14} \text{ N}$$

11. A proton is accelerated through a potential difference of 200 V. It then enters a region where there is a magnetic field $B = 0.5$ T. The magnetic field is perpendicular to the direction of motion of proton. What is the force experienced by the proton? 1.568×10^{-14} N

Soln:

$B = 0.5$ T

$V = 200$ V

$m_p = 1.67\times10^{-27}$ kg

$q_p = 1.6\times10^{-19}$ C

Then,

we know,

PE = KE

$$qV = \frac{1}{2}mv^2$$

or, $1.6\times10^{-19}\times200 = \frac{1}{2}\times1.67\times10^{-27}\times v^2$

or, $v = \sqrt{\frac{3.2\times10^{-17}\times2}{1.67\times10^{-2}}}$

$\therefore$ $v = 195763.51$ m/s

Again,

$$F = Bqv\sin\theta$$
$$= 0.5\times1.6\times10^{-19}\times195763.51\times\sin90^\circ$$
$$= 1.568\times10^{-14} \text{ N}$$

12. A potential difference of 100 V is established betn two plates A and B, the plate B being high potential. A proton of charge $q = 1.6\times10^{-19}$C is released from plate B. What will be the velocity of the proton when it reaches plate A? 1.38×10^5 m/s

Soln:-

PE = KE

or, $qV = \frac{1}{2}mv^2$

or, $1.6\times10^{-19}\times100 = \frac{1}{2}\times1.67\times10^{-27}\times v^2$

or, $\sqrt{\frac{3.2\times10^{-17}}{1.67\times10^{-27}}} = v$

$\therefore$ $v = 1.38\times10^5$ m/s

$= 138425.70$ m/s

13. How far apart should two electrons be placed if the force exerted on each is equal to the weight of the electron in free space? 5.13 m

Soln:-

mass of electron, $m = 9.1\times10^{-31}$ kg

weight of electron, $W = mg$

$= 9.1\times10^{-31}\times 9.8$

$= 8.9\times10^{-30}$ N

we have,

$$mg = K\frac{q^2}{r^2}$$

or, $8.9\times10^{-30} = \dfrac{9\times10^9 \times (1.6\times10^{-19})^2}{r^2}$

or, $r^2 = 25.88$

or, $r = \sqrt{25.88}$

$\therefore$ $r = 5.08$ m

14. How many excess electrons must be placed on each of two small spheres spaced 3 cm apart if the force of repulsion between the spheres is to be 10^{-19} N? 625

Soln:-

$$F = K\frac{q^2}{r^2}$$

or, $10^{-19} = 9\times10^9 \times \dfrac{q^2}{(0.03)^2}$

or, $\sqrt{\dfrac{9\times10^{-23}}{9\times10^{9}}} = q$

$\therefore$ $q = 1\times10^{-32}$ C

Now,

For excess electrons,

$q = ne$

or, $1\times10^{-32} = n \times 1.6\times10^{-19}$

or, $n = \dfrac{1\times10^{-32}}{1.6\times10^{-19}}$

$\therefore$ $n = 625$

15. Find the amount of work an external agent must do in assembling four charges $+2\mu C$, $+3\mu C$, $+4\mu C$ and $+5\mu C$ at the vertices of a square of side 1cm, starting each charge from infinity. 57.8 J

Soln:-

$q_1 = +2\mu C = +2\times10^{-6}$ C

$q_2 = +3\mu C = +3\times10^{-6}$ C

$q_3 = +4\mu C = +4\times10^{-6}$ C

$q_4 = +5\mu C = +5\times10^{-6}$ C

Then,

$W_1 = \dfrac{q\,q_1}{4\pi\epsilon_0 r} = \dfrac{0\times q_1}{4\pi\epsilon_0 r} = 0$ J

$W_2 = K\dfrac{q_1 q_2}{r_{12}} = 9\times10^9 \times \dfrac{2\times10^{-6}\times 3\times10^{-6}}{0.01} = 5.4$ J

$W_3 = K\dfrac{q_1 q_3}{r_{13}} + K\dfrac{q_2 q_3}{r_{23}}$

$= K\left(\dfrac{q_1 q_3}{0.01\sqrt{2}} + \dfrac{q_2 q_3}{0.01}\right)$

$= 9\times10^9\left(\dfrac{2\times10^{-6}\times4\times10^{-6}}{0.01\sqrt{2}} + \dfrac{3\times10^{-6}\times4\times10^{-6}}{0.01}\right)$

$= 15.9$ J

$$W_4 = K\left(\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$$

$$= K\left(\frac{2\times10^{-6}\times5\times10^{-6}}{0.01} + \frac{3\times10^{-6}\times5\times10^{-6}}{0.01\sqrt{2}} + \frac{4\times10^{-6}\times5\times10^{-6}}{0.01}\right)$$

$$= K\left(1\times10^{-9} + 1.06\times10^{-9} + 2\times10^{-9}\right)$$

$$= 9\times\cancel{10^{9}}\times4.06\times\cancel{10^{-9}}$$

$$= 36.54\ J$$

Now,

Amount of work done $(W) = W_1 + W_2 + W_3 + W_4$

$= (0 + 5.4 + 15.9 + 36.54)\ J$

$= 57.84\ J$

16. Calculate the value of two equal charges if they repel one another with a force of 0.1 N when situated 50 cm apart in vacuum. What would be the size of the charges if they were situated in an insulating liquid whose permittivity was ten times that of the vacuum? 1.7 μC, 5.4 μC

Soln:-

$q_1 = q_2 = q$ (Magnitude of both charges are equal)

Force $(F) = 0.1\ N$

$r = 50\ cm = 0.5\ m$

Then,

We know,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

or, $0.1 = 9\times10^{9}\ \frac{q^2}{(0.5)^2}$

or, $q = \sqrt{2.777\times10^{-12}}$

$\therefore q = 1.66\times10^{-6}\ C = 1.66\ \mu C$

Again,

If it is situated in an insulating liquid of $10\epsilon_0$.

$$F = \frac{1}{4\pi 10\epsilon_0}\frac{q_1 q_2}{r^2}$$

or, $0.1 = \frac{9\times10^9}{10}\ \frac{q^2}{(0.5)^2}$

or, $q^2 = 2.77\times10^{-11}$

$\therefore q = 5.27\times10^{-6}\ C$

or,

$q = 5.27\ \mu C$

17. Three point charges are arranged along x-axis. Charge $q_1 = +3\mu C$ is at the origin and charge $q_2 = -5\mu C$ is at $x = 0.2\,m$ and charge $q_3 = -1.5\ \mu C$. Where is q_3 located if the net force on q_3 is 7N in the negative x-direction? 0.144 m

Soln:-

$q_1 = +3\mu C = +3\times10^{-6}\,C$

$q_2 = -5\mu C = -5\times10^{-6}\,C$

$q_3 = -1.5\ \mu C = -1.5\times10^{-6}\,C$

Then,

Force of attraction between q_1 and q_2 along +ve x direction is given as,

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

or, $= 9\times10^{9}\times\frac{3\times10^{-6}\times5\times10^{-6}}{(0.2)^2}$

$= 3.375\,N$

Let us say the charge q_3 is at distance 'x' from charge q_1
We know that q_3 will attract q_1, towards left along -ve direction and this force is given as,

$F_{net} = 7N$

or, $F_2 - 3.375 = 7$

$\therefore F_2 = 10.375 N$

We have,

$F_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r^2}$

or, $10.375 = \frac{9\times10^9 \times 3\times10^{-6} \times 1.5\times10^{-6}}{r^2}$

or, $10.375 = \frac{0.0405}{r^2}$

or, $r = \sqrt{\frac{0.0405}{10.375}}$

$\therefore r = 0.0624 m$,

18. A slice of indium antimonide is 2.5 mm thick and carries a current of 150 mA. A magnetic field of flux density 0.5 T correctly applied produces a maximum hall voltage of 8.75 mV bet^n the edges of the slice. Calculate the no. of free charge carriers per unit volume. 14×10^{22} m^{-3}

Soln:-

$i = 150 mA = 0.15 A$

$l = 2.5 mm = 2.5\times10^{-3} m$

$B = 0.5 T$

$V_H = 8.75 mV = 8.75\times10^{-3} V$

no. of free charge carriers (n) = ?

Now,

We know,

$N = \frac{iB}{V_H e l}$

or, $N = \frac{0.15\times0.5}{8.75\times10^{-3}\times1.6\times10^{-19}\times2.5\times10^{-3}}$

$\therefore N = 2.14\times10^{22} m^{-3}$

Number of free charge carriers per unit volume is $2.14\times10^{-12} m^{-3}$.