$\forall$ = for all

Function of one variable

① Cartesian Product

Let A and B be any two non-empty sets, then their cartesian product is denoted by $A \times B$ read as 'A cross B' and defined all the possible ordered pairs.

$$A \times B = \{(a, b); a \in A, b \in B\}$$

② Relation

It is subset of the cartesian product, i.e $R \subseteq A \times B$.

③ Function

Let A and B be any two non-empty sets and f be the function from set A to B. i.e $f: A \rightarrow B$, in which each element of set A has the correspond unique images in the second set B.

- Injective (one to one) function

Each element of set A has the corresponding single image in set B. ie $\forall x \in A$ and $y \in B$,

such; $x \neq y \Rightarrow F(x) \neq F(y)$

Equivalently, $F(x) = F(y) \Rightarrow x = y$

Eg:- $F(x) = 2x + 1$, is one to one defined from R to R.

since,

let $x_1, x_2 \in R$ then, $F(x_1) = 2x_1 + 1$ and $F(x_2) = 2x_2 + 1$

By defn; $F(x_1) = F(x_2) \Rightarrow 2x_1 + 1 = 2x_2 + 1$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

Here, $F(x_1) = F(x_2) \Rightarrow x_1 = x_2$, So, it is injective function.

- Surjective (onto) function

Let 'f' be the function defined from set A to set B i.e. $f: A \to B$ is called surjective, if each element of set B has corresponding pre-image in set A (i.e range = co-domain).

Eg:-

$f: R \to R$ defined by $F(x) = 6x - 8$

Since, let $y \in R$ then, $F(x) = y = 6x - 8$

or, $\frac{y+8}{6} = x \in R$

Note:

a) Even Function

Let $f: A \to B$ be a function defined from set A to set B, if it satisfies the condition:

$F(-x) = F(x)$ is called the even function as well as symmetric about y-axis.

Eg:- $F(x) = 3x^4 + 2x^2$, $g(x) = \cos x + 2\sec(x)$

b) Odd Function

Let $F: A \to B$ be a function defined from set A to another set B, if it satisfies the condition $F(-x) = -F(x)$ is called odd as well as symmetric about origin.

Eg:- $F(x) = 8x^3 - 6x$, $g(x) = 3\tan x - 6\sin x$ etc.

c) Equal function

Let f and g are any two functions, if they are equal

* Domain of (f) = Dom. of g
* Range of (f) = Range of g

Imp d) Piecewise defined function

A piecewise defined function is one which is defined using two or more formulae in different parts of domain.

Eg:- $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Imp # Eg:- A function f is defined by

$F(x) = \begin{cases} 1-x & \text{for } x \leq -1 \\ x^2 & \text{for } x > -1 \end{cases}$, evaluate $F(-2)$, $F(-1)$ and $F(0)$ and sketch the graph.

Soln:-

Here, the give function is piecewise defined function.

Now,

$F(-2) = 1 - (-2) = 1 + 2 = 3$

$F(-1) = 1 - (-1) = 1+1 = 2$

and, $F(0) = 0^2 = 0$

Since, the ~~first~~ function:

$F(x) = 1 - x$, $x \leq -1$ represent the linear function.

For that,

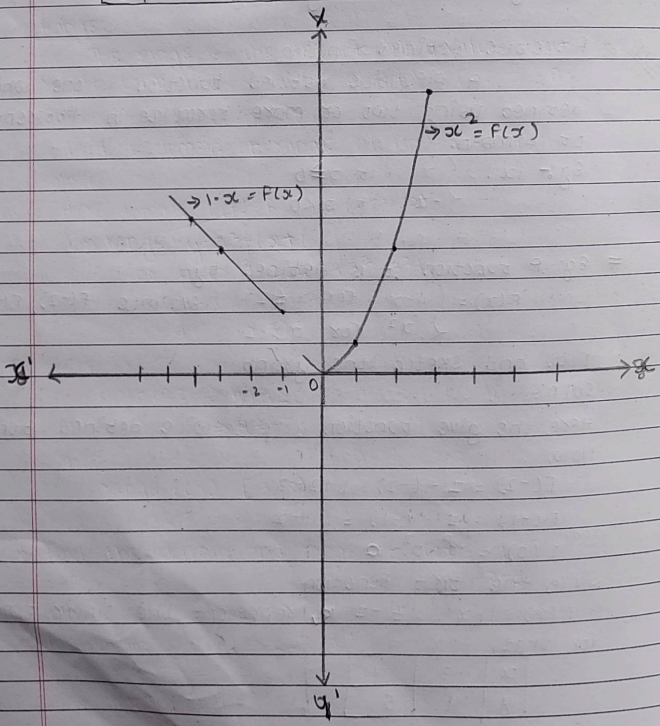
x	-3	-4
y	4	5

And, the second function,

$f(x) = x^2$, $x > -1$. It represent the parabolic curve.

For that,

x	1	2	3
y	1	4	9



(W) A function f is defined by $f(x) = |x|$. Evaluate $f(-3)$, $f(3)$ and $f(4)$ and sketch the graph.

Defⁿ → Soln:-

Here, the given function is piecewise defined function.

Now,

$$f|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Then,

$f(-3) = -(-3) = 3$

$f(3) = 3$

$f(4) = 4$

Since, the first function,

$f(x) = x$; $x \geq 0$ is represented by linear function.

For that,

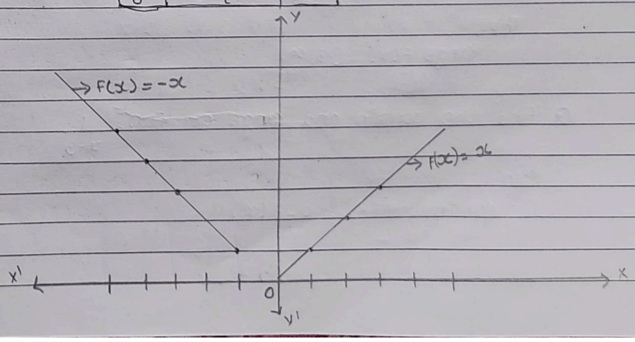
x	1	2	3
y	1	2	3

Again, the second function,

$f(x) = -x$; $x < 0$ is also represented by linear function.

For that,

x	-3	-4	-5
y	3	4	5



Domain:

Let $F: A \to B$ be a function, then the domain is the set of all x-values to which a function assigns a corresponding y-values. So, $D(f) = A$

Range:

The range is the set of all y-values that the result from the x-values. So, $R(F) = \{F(x) : x \in A\}$

Eg:-

Find a domain and range of $F(x) = x^2 - 6x + 6$

Soln:-

Here, domain $D(f) = R$ i.e $(-\infty, \infty)$

For range, $y = x^2 - 6x + 6$

or, $y = (x)^2 - 2 \cdot x \cdot 3 + 3^2 - 3^2 + 6$

or, $y = (x-3)^2 - 3$

or, $y + 3 = (x-3)^2$

since,

$(x-3)^2 \geq 0 \Rightarrow y + 3 \geq 0$

$\Rightarrow y \geq -3$

Thus,

range $F(x) = [-3, \infty)$

Q. Find the domain and range of $f(x) = \frac{1}{x-3}$

Soln:-

For, domain : $D(R) = R - F(x)$

For range : $y = \frac{1}{x-3}$

or, $x - 3 = \frac{1}{y}$

or, $x = \frac{1}{y} + 3$

$\therefore$ Range $R(f) = R - \{0\}$

Imp

Q. $y = \sqrt{6 - x - x^2}$

Here, for domain: $6 - x - x^2 \geq 0$

i.e $-(x^2 + x - 6) \geq 0$

or, $x^2 + x - 6 \leq 0$

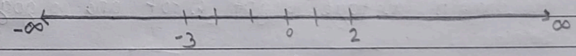
The corresponding eqn is, $x^2 + x - 6 = 0$

or, $x^2 + 3x + 2x - 6 = 0$

or, $(x+3)(x-2) = 0$

$\therefore x = -3$ and 2

Here,



The real number divided into three intervals.

$(-\infty, -3)$, $(-3, 2)$ and $(2, \infty)$

Sign table

	Intervals	Sign of		Results
		$(x+3)$	$(x-2)$	$(x+3)(x-2)$
-4	$(-\infty, -3)$	−	−	+
0	$(-3, 2)$	+	−	−
3	$(2, \infty)$	+	+	+

From the above sign table, the domain $D(F) = [-3, 2]$

For range: $y = \sqrt{-(x^2 + x - 6)}$

$= \sqrt{-\{(x)^2 + 2 \cdot x \cdot \frac{1}{2} + (\frac{1}{2})^2 - (1/2)^2 - 6\}}$

$= \sqrt{-\{(x + 1/2)^2 - 1/4 - 6\}}$

$= \sqrt{-((x+1/2)^2 - (\frac{1+24}{4}))}$

$= \sqrt{-((x+1/2)^2 - (25/4)^2)}$

$= \sqrt{(5/2)^2 - (x+1/2)^2}$

when, $x = -\frac{1}{2}$ then, $y = \frac{5}{2}$

when, $x = 2$ then, $y = 0$

so range $R(F) = \left[0, \frac{5}{2}\right]$,,

Q. If $F(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find each function and its domain (a) fog b) gof c) fof and d) gog

IMP

Soln:-

Here, $F(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$

a) $Fog = F\{g(x)\} = F(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$
$= (2-x)^{1/4}$

Since,

$2 - x \geq 0$

$\Rightarrow -x \geq -2$

$\Rightarrow x \leq 2$

The domain $D(F) = (-\infty, 2]$

b) $gof = g\{f(x)\} = g(\sqrt{x}) = \sqrt{2 - \sqrt{x}}$

Since,

~~$2 - \sqrt{x} \geq 0$~~

~~$\Rightarrow -\sqrt{x} \geq -2$~~

$\sqrt{x} \geq 0$

$\Rightarrow x \geq 0$

$\therefore x = [0, \infty)$

and,

$2 - \sqrt{x} \geq 0$

or, $-\sqrt{x} \geq -2$

or, $\sqrt{x} \leq 2$

or, $x \leq 4$, $x \in (-\infty, 4]$

Thus, we have, $0 \leq x \leq 4$.

Domain $gof = [0, 4]$

c) $FoF = F\{F(x)\} = F(\sqrt{x}) = \sqrt{\sqrt{x}} = x^{1/4}$

Since,

$x \geq 0$

Thus, domain $D(F) = [0, \infty)$.

d) $gog = g\{g(x)\} = g(\sqrt{2-x}) = \sqrt{2 - \sqrt{2-x}}$

Since,

$2 - x \geq 0$

$\Rightarrow -x \geq -2$

$\Rightarrow x \leq 2$

$\therefore x = [2, \infty)$

Thus, we have, $-2 \leq x \leq 2$

Domain $gog = [-2, 2]$,,

and,

$2 - \sqrt{2-x} \geq 0$

$\Rightarrow -\sqrt{2-x} \geq -2$

$\sqrt{2-x} \leq 2$

$2 - x \leq 4$

$-x \leq 2$

$x \geq -2$

$\therefore x = [-2, \infty)$

Pg-no. 2~~8~~7

1~~4~~ ~~a)~~

a) $y=x^2$

b) ~~Soln:-~~ $y=x^3$

HW- c) $y=1/x$

d) $y=\log x$

HW- e) $y=3^x$

f) $y=\log_{\cancel{2}} \cancel{x}$

f) $y=\log_2 a$

(i) when, $y=0$ then,

$0=\log_2 x$

$\log_2 1=\log_2 x$

$\Rightarrow x=1$

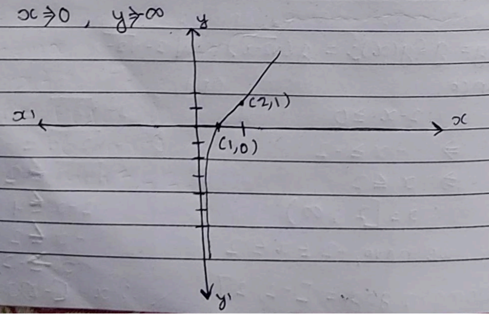
So, the curve passing through $(1,0)$

(ii) when. $x=2$

then, $y=\log_2 2=1$ $[\log_2 2=1]$

$\therefore$ It passes through $(2,1)$

(iii) when $x\Rightarrow 0$, $y\Rightarrow\infty$



10.

a) $F(x)=2-0.4x$ $\Rightarrow D(F)=R=(-\infty,\infty)$

b) $F(x)=\sqrt{x}-5$

$\sqrt{x}\geq 0$

For the domain,

$F(x)=y=\sqrt{x}-5$

or, $y-5=\sqrt{x}$

$x\geq 0$

or, $y-5\geq 0$

or, $y\geq -5$

$D(F)=[-5,\infty)$

11.

Domain of f/g

$f(x)=x^3+2x^2$

$g(x)=3x^2-1$

$\frac{f}{g}=\frac{x^3+2x^2}{3x^2-1}$

for domain,

$3x^2-1=0$

i.e $x=\pm\frac{1}{\sqrt{3}}$

$\therefore$ Domain $=R-\left\{\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right\}$

12. Find, Domain,

$F(x) = x + \frac{1}{x}$ and

$g(x) = \frac{x+1}{x+2}$

Fog

Here,

$F(g(x))$

$= F\left(\frac{x+1}{x+2}\right)$

$= \frac{x+1}{x+2} + \frac{x+2}{x+1}$

$= \frac{x^2+2x+1+x^2+4x+4}{x^2+x+2x+2}$

$= \frac{2x^2+6x+5}{x^2+3x+2} = \frac{2x^2+6x+5}{(x+2)(x+1)}$

Here,

the domain of $g(x) = R - \{-2\}$

the domain of $Fog(x) = R - \{-2,-1\}$

13. $F(x) = 3x-2$

$g(x) = \sin x$

$h(x) = x^2$

$Fogoh = F(g(x^2))$

$= F(\sin x^2)$

$= 3\sin x^2 - 2$

14 Find Fog

(1) $F(x) = (2x+x^2)^4$

Let, $F(x) = Fog$

Now, $g(x) = 2x+x^2$

and $F(x) = x^4$

Also,

$Fog = F(g(x))$

$= F(2x+x^2)$

$= (2x+x^2)^4$

$= F(x)$

b) $U(t) = \frac{\tan t}{1+\tan t}$

Let, $F(x) = Fog$

Now,

$g(x) = \tan t$

$F(x) = \frac{x}{1+x}$

Then,

$Fog = F(g(x))$

$= F(\tan t)$

$= \frac{\tan t}{1+\tan t}$

Date ______
Page ______

15. $F(x) = \sqrt{\sqrt{x} - 1}$

let, $F(x) = Fogoh$

Now,

$h(x) = \sqrt{x}$

$g(x) = x - 1$

$F(x) = \sqrt{x}$

(ii) $H(h) = \sec^4 \sqrt{x}$

Let, $H(h) = Fogoh$

Now,

$h(x) = \sqrt{x}$

$g(x) = \sec x$

$F(x) = x^4$

HW

Pg - ~~25~~ 27

1~~8~~. c. $y = 1/x$

or,

x	-2	-1	0	1	2
y	$-1/2$	-1	unde	1	$1/2$

(i) when, $y = 1$, ~~$x=1$~~,

then, $1 = \frac{1}{x}$

$\therefore x = 1$

so, the curve passes through $(1, 1)$

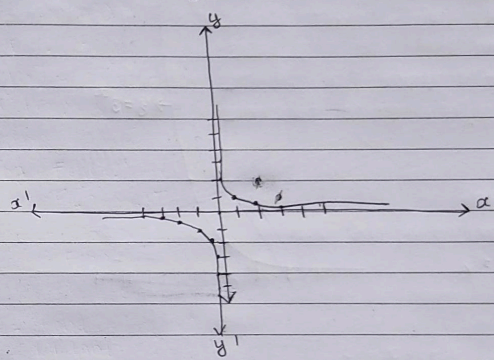
(ii) when, $x = 2$,

then, $y = \frac{1}{2}$

so, the curve passes through $(2, 1/2)$

Date ______
Page ______

(iii) when, $x = 0$ then $y \Rightarrow \infty$



e) $y = 3^x$

Soln:-

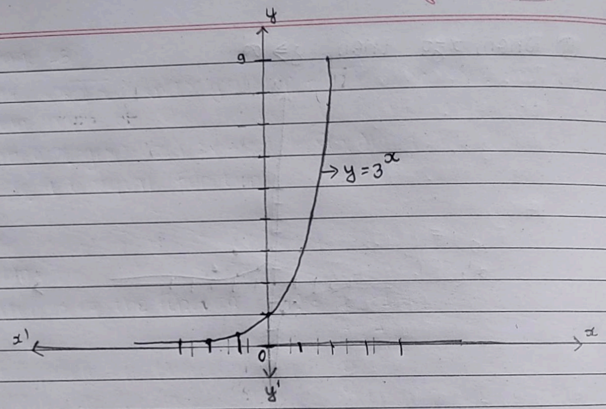
Let $F(x) = 3^x$

Then,

x	-2	-1	0	1	2
y	$1/9$	$1/3$	1	3	9

Now,

we sketch the graph with the help of the above point.



a. Soln:-

Let, $F(x) = x^2$

Then,

x	0	1	2	3	-1	-2	-3
y	0	1	4	9	1	4	9

Now, we sketch the graph with the help of the above point,

b. Soln:-

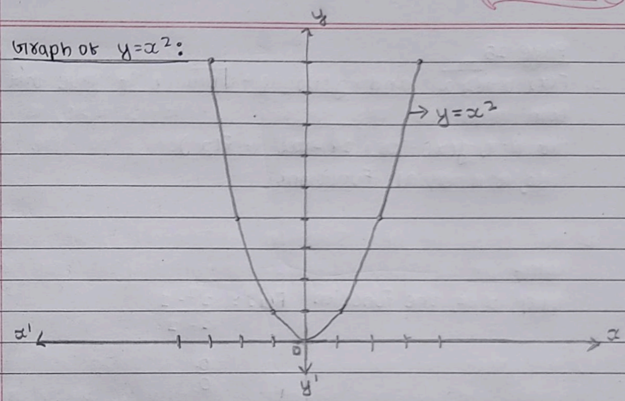
Let, $F(x) = x^3$

Then,

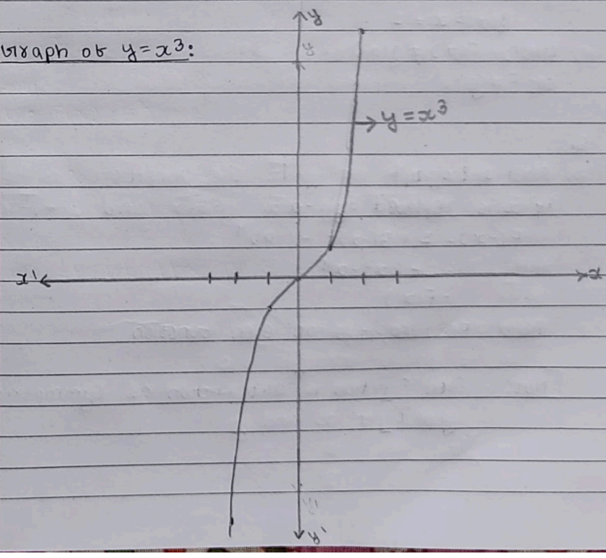
x	0	1	2	-1	-2
y	0	1	8	-1	-8

Now, we sketch the graph with the help of the above point,

Graph of $y = x^2$:



Graph of $y = x^3$:



Page-25

1. $F(x) = x + \sqrt{2-x}$ and $g(u) = u + \sqrt{2-u}$, is it true that $f = g$?

soln:-

if $u = x$, $g(x) = x + \sqrt{2-x} = F(x)$

so, it is equal functions.

2.

soln:-

When, $x = 1$, the function $F(x) = \frac{1^2 - 1}{1 - 1}$

$= \frac{0}{0}$

$g(x) = x = 1$

since, $F(x) \neq g(x)$

so, $F \neq g$.

3.

a) $F(x) = 1 + 3x^2 - x^4$

if x is replaced by : $-x$

$F(-x) = 1 + 3(-x)^2 - (-x)^4$

$= 1 + 3x^2 - x^4$

$= F(x)$

$F(-x) = F(x)$. So, it is an even function.

[Note:- Even function is also known as symmetric about y-axis]

9. The Number N (in millions) of US cellular phone subscribers is shown in the table. (Midyear estimates are given).

t	1996	1998	2000	2002	2004	2006
N	44	69	109	141	182	233

The number of cell-phone subscribers at mid year in $2001 = \frac{109+141}{2} = 125$ million

The number of cell-phone subscribers at midyear in $2005 = \frac{182+233}{2} = 207.5$ million.

10.

a. Slope (m) = 2

$y = mx + c$

The required eq^n of family of linear functions with slope 2 is $y = 2x + c$

b. Here,

$F(2) = 1$

then,

$1 = 2 \times 2 + c$

$\therefore c = -3$

$\therefore$ The required linear eq^n is $y = 2x - 3$

Note:

Vertically and horizontally shifting graphs

i) The graph of the function $y = F(x+a)$; $a > 0$, is the graph of $y = F(x)$ shifted horizontally to the left by 'a' units.

ii) The graph of the function $y = F(x-a)$; $a > 0$, is the graph of $y = F(x)$ shifted horizontally to the right by 'a' units

iii) The graph of function $y = F(x) + a$; $a > 0$ is the graph of $y = F(x)$ shifted vertically by 'a' units

iv) The graph of function $y = F(x) - a$; $a > 0$ is the graph of $y = F(x)$ shifted vertically downward by 'a' units.

5 marks 21. Draw the graph of $y = \sqrt{x}$. Use transformations to graph $y = \sqrt{x} - 3$, $y = \sqrt{x-3}$, $y = -\sqrt{x}$, $y = 3\sqrt{x}$ and $y = \sqrt{-x}$

Soln.

Here,

$y = \sqrt{x}$

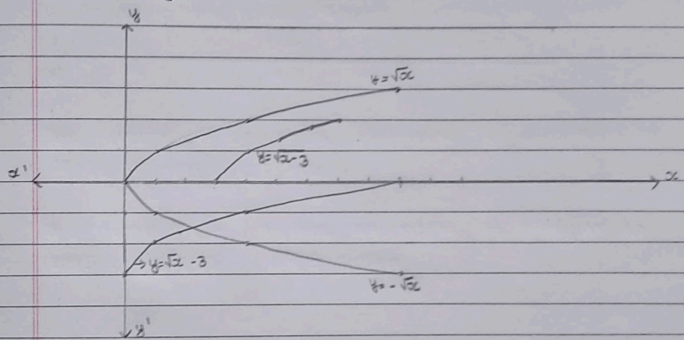
i) When $x = 0$, then $y = 0$

so, it passes through $(0,0)$

ii) Various values of x and y

x	1	4	9
y	1	2	3

iii) Domain = Range = $[0, \infty)$



19.

a) $F(x) = 3x - 2$

$g(x) = \sin x$

$h(x) = x^2$

Then,

$$\begin{aligned} Fogoh &= F(g(h(x))) \\ &= F(g(x^2)) \\ &= F(\sin x^2) \\ &= 3\sin(x)^2 - 2 \end{aligned}$$

b) $F(x) = \sqrt{x-3}$

$g(x) = x^2$

$h(x) = x^3 + 2$

Then,

$$\begin{aligned} Fogoh &= F(g(h(x))) \\ &= F(g(x^3+2)) \\ &= F((x^3+2)^2) \\ &= \sqrt{(x^3+2)^2 - 3} \\ &= \sqrt{x^6 + 4x^3 + 4 - 3} \\ &= \sqrt{x^6 + 4x^3 + 1} \end{aligned}$$