Optimizing Power Allocation using Convex optimization for wireless IoT network

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1 Introduction

Optimizing power allocation in a wireless IoT network is a common problem in resource management. This can be formulated as a convex optimization problem, which ensures a global optimum solution. Below, a mathematical formulation is provided for convex optimization of power allocation in a wireless IoT network.

2 Problem Description

Consider a wireless IoT network with N devices that need to transmit data to a base station or access point. The goal is to optimize the power allocation to these devices to minimize the total power consumption while satisfying the quality of service (QoS) constraints, such as data rate or signal-to-noise ratio (SNR) requirements.

3 Variables

- p_i : Transmit power of IoT device i
- r_i : Data rate or SNR requirement for IoT device i
- h_i : Channel gain between the IoT device i and the base station
- P_total : Total available power budget

4 Objective Function

The objective is to minimize the total power consumption while satisfying QoS constraints:

$$minimize \sum_{i=1}^{N} p_i \tag{1}$$

5 Constraints

- **Total Power Constraints**

  The sum of transmit powers should not exceed the total available power budget:

$$\sum_{i=1}^{N} p_i \leq P_{total} \tag{2}$$

- **QoS Constraints**

  Ensure that the data rate (or SNR) at each device is at least equal to the required value:

  For data rate constraints:

$$\frac{1}{2}\log_2(1 + \frac{p_i|h_i|^2}{\sigma^2}) \geq r_i \tag{3}$$

  For SNR constraints:

$$\frac{p_i|h_i|^2}{\sigma^2} \geq 2^{2r_i} - 1, \tag{4}$$

  Where:

  - $i = 1, 2, 3, ..., N$
  - $|h_i|^2$ is the squared magnitude of the channel gain between device i and the base station
  - σ^2 is the noise power

- **Non-negative Power Constraints** Ensure that the power allocation for each device is non-negative:

$$p_i \geq 0 \tag{5}$$

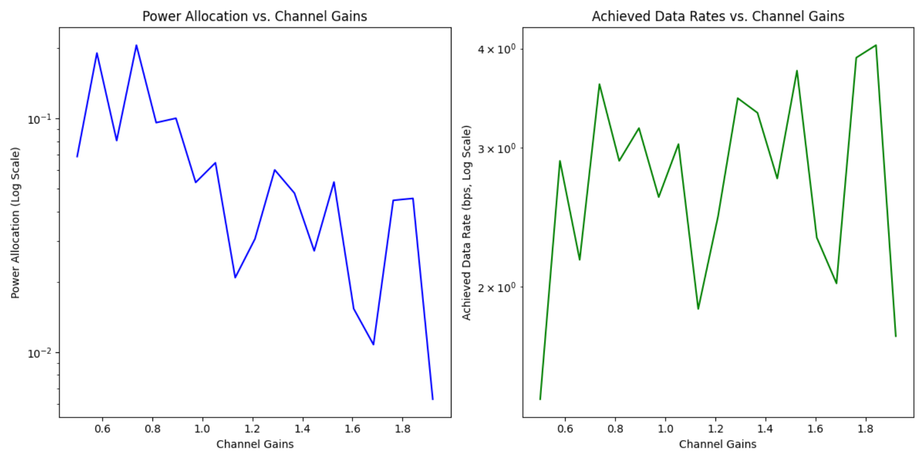
6 Solution and Code

To solve the power allocation problem using CVXPY with sample data in Python:

```
import cvxpy as cp
import numpy as np
import matplotlib.pyplot as plt


# Sample data
N = 20
P_total = 10.0
r = np.array([1.0, 2.0, 1.5, 2.5, 2.0, 2.2, 1.8, 2.1,
1.3, 1.7, 2.4, 2.3, 1.9, 2.6, 1.6, 1.4, 2.7, 2.8, 1.2, 2.9])
```

```
h = np.linspace(0.5, 2.0, N)  # Generate linearly spaced channel gains
sigma_sq = 0.01


# Variables
p = cp.Variable(N, nonneg=True)

# Objective function
objective = cp.Maximize(cp.sum(cp.log(1 + (p * h ** 2 / sigma_sq))))


# Constraints
constraints = [cp.sum(p) <= P_total]


for i in range(N):
    constraints.append(cp.log(1 + (p[i] * h[i] ** 2 / sigma_sq)) >= r[i])


# Create and solve the problem
problem = cp.Problem(objective, constraints)
problem.solve()


# Visualize both power allocation and data rate satisfaction with log scale
plt.figure(figsize=(12, 6))


# Power allocation vs. Channel Gains
plt.subplot(1, 2, 1)
plt.semilogy(h[:-1], power_allocation[:-1], 'b')  # Use semilogy to set a
logarithmic y-axis and remove the last point
plt.xlabel('Channel Gains')
plt.ylabel('Power Allocation (Log Scale)')
plt.title('Power Allocation vs. Channel Gains')


# Data rate satisfaction vs. Channel Gains with log scale
plt.subplot(1, 2, 2)
plt.semilogy(h[:-1], data_rates[:-1], 'g')  # Use semilogy to set a
logarithmic y-axis and remove the last point
plt.xlabel('Channel Gains')
plt.ylabel('Achieved Data Rate (bps, Log Scale)')
plt.title('Achieved Data Rates vs. Channel Gains')
```

```
plt.tight_layout()
plt.show()


print(f"Total Power Consumed: {problem.value}")
```



Total Power Consumed: 8.200904700939587.

7 Figure Description

- Power Allocation vs. Channel Gains (Left Figure):

  The left figure illustrates the power allocation across IoT devices concerning their respective channel gains. It portrays the inverse relationship between transmit power and channel quality. As channel gains improve, the allocated power decreases due to the logarithmic nature of the objective function, highlighting the trade-off in power allocation based on channel conditions.

- Achieved Data Rates vs. Channel Gains (Right Figure):

  The right figure shows how the achieved data rates vary as a function of channel gains for each IoT device. Devices with better channel conditions achieve higher data rates, reflecting the inherent behavior of wireless communication where strong channels lead to more efficient data transmission. The figure visually demonstrates the trade-off between channel quality and data rate satisfaction.