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Report

ON

Numerical Integration(Trapezoidal Rule)

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ABSTRACT

The trapezoidal rule is a numerical integration method to be used to approximate the integral or the area under a curve. The integration of [a, b] from a functional form is divided into n equal pieces, called a trapezoid.

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INTRODUCTION

Numerical integration is used to obtain approximate answers for definite integrals that cannot be solved analytically. Numerical integration is a process of finding the numerical value of a definite integral.

$$I = \int_a^b f(x)\,dx,$$

When a function y=f(x) is not known explicitly. But we give only a set of values of the function y=f(x) corresponding to the same values of x.

Trapezoidal rule is based on the Newton-Cotes formula that if one approximates the integrand of the integral by an nth order polynomial, then the integral of the function is approximated by the integral of that nth order polynomial. Integration of polynomials is simple and is based on the calculus. Trapezoidal rule is the area under the curve for a first order polynomial (straight line).

What is Trapezoidal Method

In numerical analysis, the trapezoidal rule or method is a technique for approximating the definite

$$I = \int_a^b f(x)\,dx,$$

It also known as Trapezium rule.

The trapezoidal rule works by approximating the region under the graph of the function as a trapezoid and calculating its area.
The trapezoidal rule may be viewed as the result obtained by averaging the left and right Riemann sums, and is sometimes defined this way. The integral can be even better approximated by partitioning the integration interval, applying the trapezoidal rule to each subinterval, and summing the results. In practice, this

"chained" (or "composite") trapezoidal rule is usually what is meant by "integrating with the trapezoidal rule".

The approximation becomes more accurate as the resolution of the partition increases (that is, for larger decreases). When the partition has a regular spacing, as is often the case, the formula can be simplified for calculation efficiency.

As discussed below, it is also possible to place error bounds on the accuracy of the value of a definite integral estimated using a trapezoidal rule.

General Formula of Integration

Substituting $n = 1$ in the relation (3) and neglecting all differences greater than the first we get

$$I_1 = \int_{x_0}^{x_0+h} f(x)\,dx = h\left[y_0 + \frac{1}{2}\Delta y_0\right]$$

$$= \frac{h}{2}(2y_0 + y_1 - y_0) = \frac{h}{2}(y_0 + y_1),$$

for the first subinterval $[x_0, x_0 + h]$,

similarly, we get

$$I_2 = \int_{x_0+h}^{x_0+2h} f(x)\,dx = \frac{h}{2}(y_1 + y_2),$$

$$I_3 = \int_{x_0+2h}^{x_0+3h} f(x)\,dx = \frac{h}{2}(y_2 + y_3),$$

$$\vdots$$

$$I_n = \int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left(y_{n-1} + y_n\right),$$

for the other integrals.

Adding $I_1, I_2, ..., I_n$

we get $I_1 + I_2 + ... + I_n$

$$= \int_{x_0}^{x_0+h} f(x)dx + \int_{x_0+h}^{x_0+2h} f(x)dx + \int_{x_0+2h}^{x_0+3h} f(x)dx + ... + \int_{x_0+(n-1)h}^{x_0+nh} f(x)dx$$

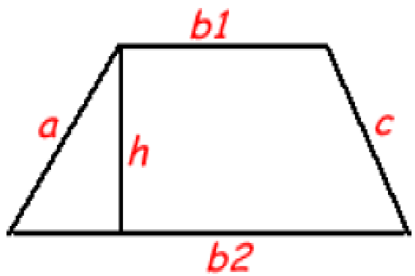
$$= \frac{h}{2}\left[y_0 + y_1\right] + \frac{h}{2}\left[y_1 + y_2\right] + \frac{h}{2}\left[y_2 + y_3\right] + ... + \frac{h}{2}\left[y_{n-1} + y_n\right]$$

$$\Rightarrow \int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + ... + y_{n-1}\right)\right],$$

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + ... + y_{n-1}\right)\right]. \qquad (\text{}$$

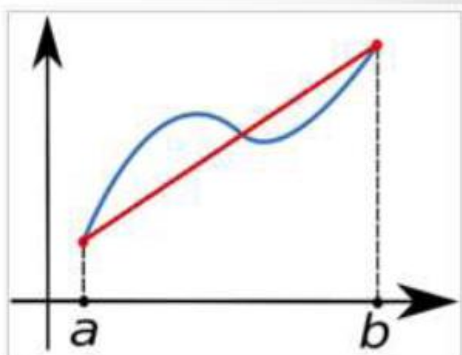
How it works

Trapezoid is an one kind of rectangle which has 4 sides and minimum two sides are parallel



$$\text{Area } A=\left(\frac{b1+b2}{2}\right)h$$

The trapezoidal rule works by approximating the region under the graph of the function as a trapezoid and calculating its area in limit.

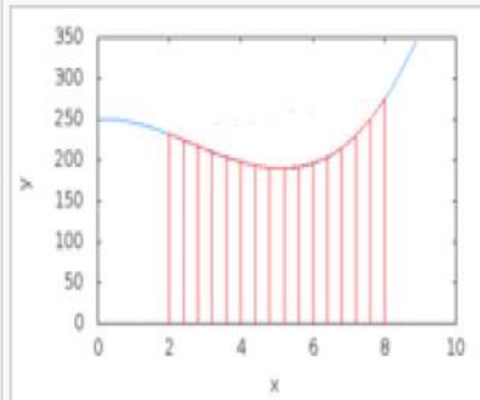


The function *f*(*x*) (in blue) is approximated by a linear function (in red).

It follows that,

$$\int_a^b f(x)\,dx \approx \frac{(b-a)}{2}[f(a)+f(b)]$$

The trapezoidal rule
approximation improves
With More strips , from
This figure we can clearly
See it



An animation showing how the trapezoidal rule approximation improves with more strips.

<u>History of trapezoidal method</u>

Trapezoidal Rule," by Nick Trefethen and André Weideman. It deals with a fundamental and classical issue in numerical analysis—approximating an integral.

Trefethen

By focusing on up-to-date covergence of recent results

Advantages

There are many alternatives to the trapezoidal rule, but this method deserves attention because of

- Its ease of use
- Powerful convergence properties
- Straightforward analysis

Application of Trapezoidal Rule

- The trapezoidal rule is one of the family members of numerical-integration formula.
- The trapezoidal rule has faster convergence.
- Moreover, the trapezoidal rule tends to become extremely accurate than periodic functions.

Problem & Solution

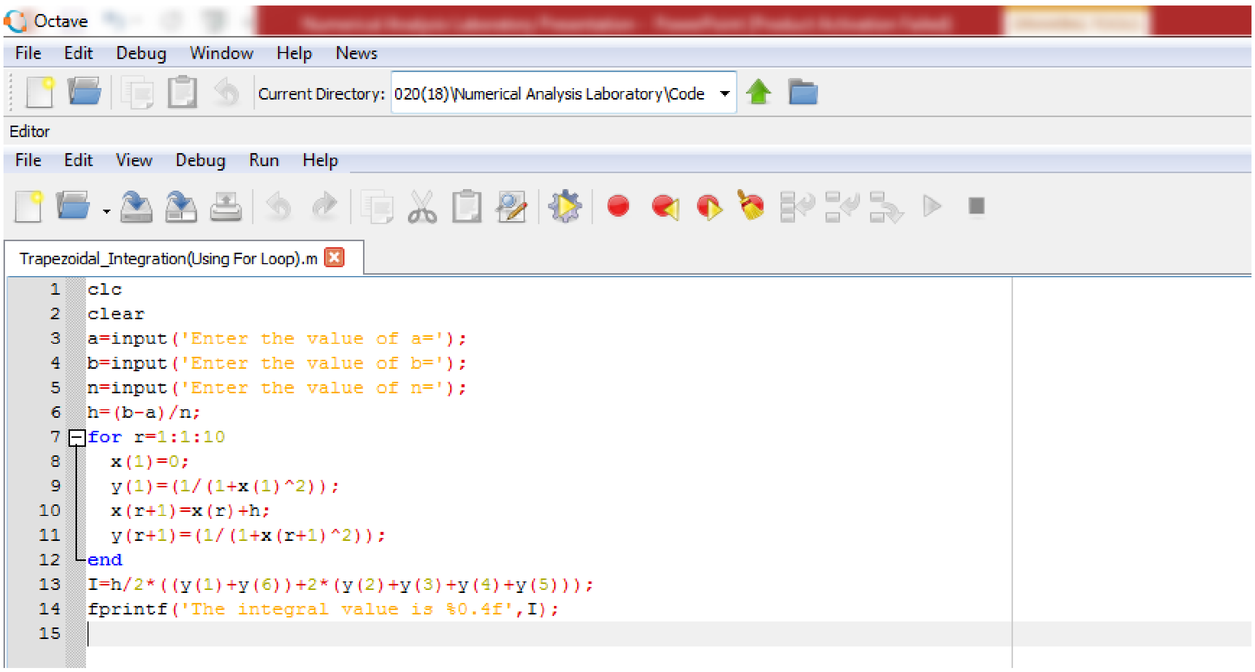
Problem: Find the value of $\int_0^1 \frac{dx}{1+x^2}$, *taking 5 subinterval by Trapezoidal rule, correct to five significant figures. Also compare it with its exact value.*

Solution:

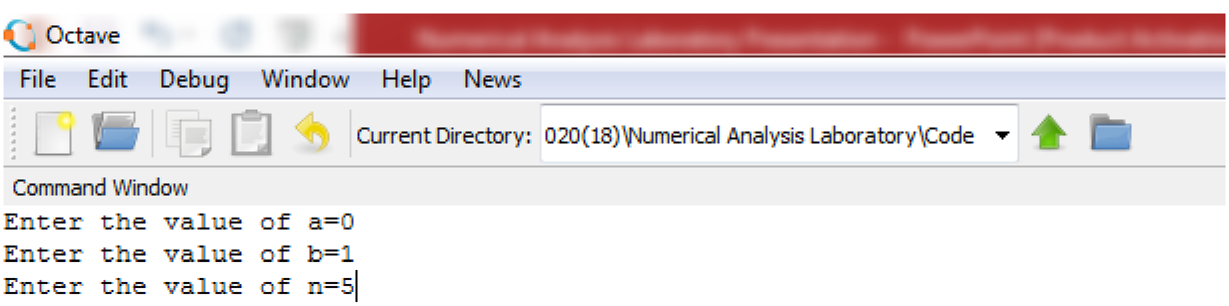
```
clc
clear
a=input('Enter the value of a=');
b=input('Enter the value of b=');
n=input('Enter the value of n=');
h=(b-a)/n;
for r=1:1:10
  x(1)=0;
  y(1)=(1/(1+x(1)^2));
  x(r+1)=x(r)+h;
```

```
  y(r+1)=(1/(1+x(r+1)^2));
end
I=h/2*((y(1)+y(6))+2*(y(2)+y(3)+y(4)+y(5)));
fprintf('The integral value is %0.4f',I);
```

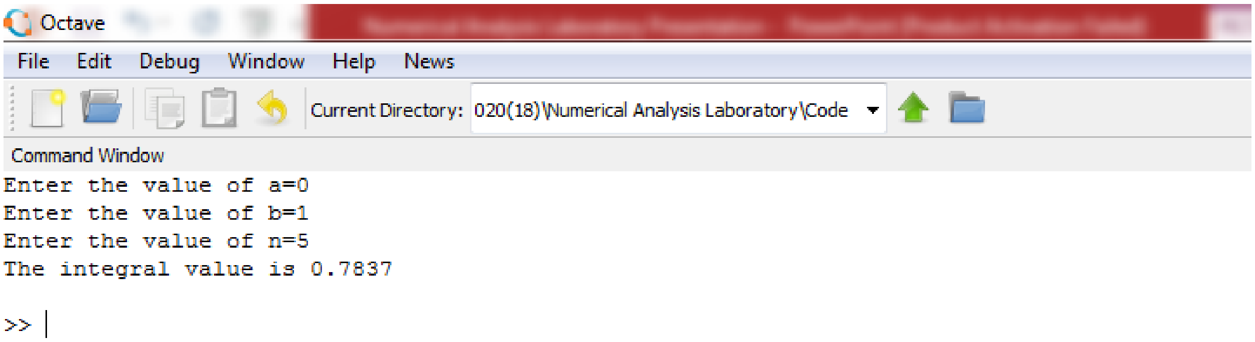
Code using octave



Input the value



Input & Output value



Conclusion

Trapezoidal Method can be applied accurately for non periodic function, also in terms of periodic integrals. when periodic functions are integrated over their periods, trapezoidal looks for extremely accurate.

References

- http://en.wikipedia.org/wiki/Trapezoidal_rule
- http://blogs.siam.org/the-mathematics-andhistory-of-the-trapezoidal-rule/
- And various relevant websites