

COLLATZ SEQUENCES

POETRY OF PROGRAMMING – CLOJURE ASSIGNMENTS

According to the Collatz conjecture, every natural number will produce a sequence sooner or later containing 1, if we iterate the following function.

$$\text{collatz}(x) = \begin{cases} 3x + 1 & \text{if } x \text{ is odd} \\ \frac{x}{2} & \text{if } x \text{ is even} \end{cases}$$

What number between 1 and 1000 produces the longest sequence?

Plan: We need a CLOJURE function `collatz` that calculates the above mathematical function.

```
(collatz 10)
5
(collatz 5)
16
```

With `iterate` we can automate the process of putting the function's output back into its input, but `iterate` produces a lazy list. From this lazy list we need take numbers only while we don't reach 1. So we need to write a predicate function `not-one?`.

```
(not-one? 5)
true
(not-one? 1)
false
```

Then we can use `take-while` to get the sequence of numbers before the iteration reaches 1. Let's define the `collatz` length as the length of this sequence. We can write a function `c-length` that calculates this.

```
(c-length 10)
6
```

We can `map` `c-length` on the numbers between 1 and 1000. Using `(apply max coll)` gives the maximal element of a collection `coll`, so we can easily find the longest sequence. Then `filter` with a suitable predicate function will tell which number(s) produced sequence(s) of maximal length.