Exercise 1

1)

$$\dot{X}_1 = \begin{bmatrix} \dot{e}_1 \\ \ddot{e}_1 \\ \dot{e}_2 \\ \ddot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -4C\alpha/m\dot{x} & \frac{4C\alpha}{m} & \frac{-2C\alpha(l_f - l_r)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C\alpha(l_f - l_r)}{I_z\dot{x}} & \frac{2C\alpha(l_f - l_r)}{I_z} & \frac{-2C\alpha(l_f^2 + l_r^2)}{I_z\dot{x}} \end{bmatrix} X_1 + \begin{bmatrix} 0 & 0 \\ \frac{2C\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C\alpha l_f}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

Put the values

$$\dot{X}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -42.35/\dot{x} & 42.35 & -3.6/\dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.247}{\dot{x}} & 0.247 & \frac{-6.66}{\dot{x}} \end{bmatrix} X_1$$

$$X_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 21.17 & 0 \\ 0 & 0 \\ 2.39 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} \quad -(1)$$

$$\dot{X}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X_2 + \begin{bmatrix} 0 & 0 \\ 0 & 1/1888.1 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} \quad -(2)$$

" For observability; we take $C_1 = [1,0,0,0]$ for e_1
$C_2 = [0,1,0,0]$ for $\dot{e}_1$; $C_3 = [0,0,1,0]$ for e_2
$C_4 = [0,0,0,1]$ for $\dot{e}_2$; ∴ we find
for all the speed when we measure e_1
the rank $(Q) = 4$; " ∴ it is observable only
if we measure e_1

$\dot{x}_1^0$ = 2m/s;

For x_1

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -21.175 & 42.35 & -1.8 \\ 0 & 0 & 0 & 1 \\ 0 & -0.134 & +0.247 & 3.33 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 & 0 \\ 21.17 & 0 \\ 0 & 0 \\ 2.34 & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}.$$

$$P_1 = \begin{bmatrix} 0 & 0 & 0.0002 & 0 & -0.0095 & 0 & 0.096 & 0 \\ 0 & 0 & -0.0095 & 0 & 0.0967 & 0 & -2.097 & 0 \\ 0.0002 & 0 & 0 & 0 & 0.0001 & 0 & 0.0007 & 0 \\ 0 & 0 & 0.0001 & 0 & 0.0007 & 0 & -0.0094 & 0 \end{bmatrix} \times 10^5$$

$$\text{rank}(P) = 4 \quad \therefore \text{ controllable}$$

$\dot{x}^0$ = 5m/s

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -8.47 & 42.35 & -0.72 \\ 0 & 0 & 0 & 1 \\ 0 & -0.05 & +0.24 & 1.5 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & 0 & 0.0021 & 0 & -0.018 & 0 & 0.163 & 0 \\ 0 & 0 & -0.018 & 0 & 0.16 & 0 & -1.37 & 0 \\ 0.0021 & 0 & 0.0002 & 0 & 0.002 & 0 & 0.0012 & 0 \\ 0 & 0 & 0.0002 & 0 & 0.0012 & 0 & -0.0093 & 0 \\ 0.0002 & 0 & & & & & & \end{bmatrix}$$

$$\text{rank}(P_2) = 4 \quad \therefore \text{controllable}$$

III^rd for $\dot{x} = 8$ m/s

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -5.29 & 12.35 & -0.45 \\ 0 & 0 & 0 & 1 \\ 0 & -0.030 & 0.29 & 0.85 \end{bmatrix}$$

rank $(P_3) = 4$ ∴ controllable

In the program I have attached the P & Q & rank for each speed & for diff C

2) We notice that the overall stability (poles $L<0$) is achieved when speed is less than 30; (∵ all poles $L<0$) and the car is controllable at any speed since it is of full rank for any speed between (1-40) thus making it fully controllable

P2-Ex1

November 6, 2024

[13]:
```
import numpy as np
import matplotlib.pyplot as plt
#v int array from 1 to 40
vs = np.arange(1, 41)
#create a empty array ln
ln = np.zeros_like(vs, dtype=float)

lr = 1.39
lf = 1.55
Ca = 20000
Iz = 25854
m = 1888.6


#for xdot=2,5 and 8 , P and Q matrix
xdots=[2,5,8]
for xdot in xdots:
    A = np.array([[0, 1, 0, 0], [0, -4*Ca / (m * xdot), 4*Ca / m, -(2*Ca*(lf - lr))/(m*xdot)], [0, 0, 0, 1], [0, -(2*Ca*(lf - lr)) / (Iz * xdot), (2*Ca*(lf - lr)) / Iz, (-2*Ca*(np.power(lf, 2) + np.power(lr, 2))) / (Iz * xdot)]])
    B = np.array([[0], [2*Ca / m], [0], [(2 * Ca* lf) / Iz]])
    C1 = np.array([[1, 0, 0, 0], [0, 0, 0, 0],[0, 0, 0, 0],[0, 0, 0, 0]])
    C2 = np.array([[0, 0, 0, 0], [0, 1, 0, 0],[0, 0, 0, 0],[0, 0, 0, 0]])
    C3 = np.array([[0, 0, 0, 0], [0, 0, 0, 0],[0, 0, 1, 0],[0, 0, 0, 0]])
    C4 = np.array([[0, 0, 0, 0], [0, 0, 0, 0],[0, 0, 0, 0],[0, 0, 0, 1]])
    #make an array of C which is a combination of C1,C2,C3 and C4


    p = np.hstack([B, A @ B, A @ A @ B, A @ A @ A @ B])

    #and the rank of the matrix
    print("For xdot=",xdot)
    print("Rank of P matrix ", np.linalg.matrix_rank(p))

    print("P matrix")
    print(p)
```

```
    print("If we are measuring E1")
    q1=np.vstack([C1, C1 @ A, C1 @ A @ A, C1 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q1))
    print("Q matrix")
    print(q1)
    print(end="\n\n")

    print("If we are measuring E1dot")
    q2=np.vstack([C2, C2 @ A, C2 @ A @ A, C2 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q2))
    print("Q matrix")
    print(q2)
    print(end="\n\n")

    print("If we are measuring E2")
    q3=np.vstack([C3, C3 @ A, C3 @ A @ A, C3 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q3))
    print("Q matrix")
    print(q3)
    print(end="\n\n")

    print("If we are measuring E2dot")
    q4=np.vstack([C4, C4 @ A, C4 @ A @ A, C4 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q4))
    print("Q matrix")
    print(q4)
    print(end="\n\n")
```

```
For xdot= 2
Rank of P matrix  4
P matrix
[[ 0.00000000e+00  2.11797098e+01 -4.52643363e+02  9.70650280e+03]
 [ 2.11797098e+01 -4.52643363e+02  9.70650280e+03 -2.06189084e+05]
 [ 0.00000000e+00  2.39808153e+00 -1.06625495e+01  9.23711699e+01]
 [ 2.39808153e+00 -1.06625495e+01  9.23711699e+01 -1.51376540e+03]]
If we are measuring E1
Rank of Q matrix 4
Q matrix
[[   1.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            1.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.          -21.17970984   42.35941968   -1.69437679]
```

```
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.          448.78982514 -897.57965028   83.92730562]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]]


If we are measuring E1dot
Rank of Q matrix 3
Q matrix
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -2.11797098e+01  4.23594197e+01 -1.69437679e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  4.48789825e+02 -8.97579650e+02  8.39273056e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -9.51562612e+03  1.90312522e+04 -1.93941845e+03]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]


If we are measuring E2
Rank of Q matrix 3
Q matrix
[[ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          1.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          1.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.         -0.12377195  0.2475439  -3.35313685]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          3.03647828 -6.07295656 11.70078692]
```

```
 [ 0.          0.          0.          0.        ]]


If we are measuring E2dot
Rank of Q matrix 2
Q matrix
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.23771950e-01  2.47543900e-01 -3.35313685e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  3.03647828e+00 -6.07295656e+00  1.17007869e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -6.57599581e+01  1.31519916e+02 -5.04522346e+01]]


For xdot= 5
Rank of P matrix  4
P matrix
[[ 0.00000000e+00  2.11797098e+01 -1.81057345e+02  1.63836877e+03]
 [ 2.11797098e+01 -1.81057345e+02  1.63836877e+03 -1.40710886e+04]
 [ 0.00000000e+00  2.39808153e+00 -4.26501982e+00  1.52780368e+01]
 [ 2.39808153e+00 -4.26501982e+00  1.52780368e+01 -1.02661158e+02]]
If we are measuring E1
Rank of Q matrix 4
Q matrix
[[   1.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           1.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.          -8.47188394   42.35941968   -0.67775071]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.          71.80637202 -359.03186011   49.01028143]
 [   0.           0.           0.           0.        ]
```

```
 [   0.            0.            0.            0.        ]
 [   0.            0.            0.            0.        ]]


If we are measuring E1dot
Rank of Q matrix 3
Q matrix
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -8.47188394e+00  4.23594197e+01 -6.77750715e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  7.18063720e+01 -3.59031860e+02  4.90102814e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -6.10761689e+02  3.05380844e+03 -4.73433952e+02]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]


If we are measuring E2
Rank of Q matrix 3
Q matrix
[[ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          1.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          1.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.         -0.04950878  0.2475439  -1.34125474]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.48583652 -2.42918262  2.08006278]
 [ 0.          0.          0.          0.        ]]


If we are measuring E2dot
Rank of Q matrix 2
```

```
Q matrix
[[ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          1.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.         -0.04950878  0.2475439  -1.34125474]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.48583652 -2.42918262  2.08006278]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.         -4.21893202 21.09466009 -5.54835274]]


For xdot= 8
Rank of P matrix  4
P matrix
[[ 0.00000000e+00  2.11797098e+01 -1.13160841e+02  7.01888933e+02]
 [ 2.11797098e+01 -1.13160841e+02  7.01888933e+02 -3.83204707e+03]
 [ 0.00000000e+00  2.39808153e+00 -2.66563739e+00  6.32972667e+00]
 [ 2.39808153e+00 -2.66563739e+00  6.32972667e+00 -2.76845127e+01]]
If we are measuring E1
Rank of Q matrix 4
Q matrix
[[   1.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           1.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.          -5.29492746   42.35941968   -0.4235942 ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.          28.04936407 -224.39491257   44.95741255]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]]


If we are measuring E1dot
```

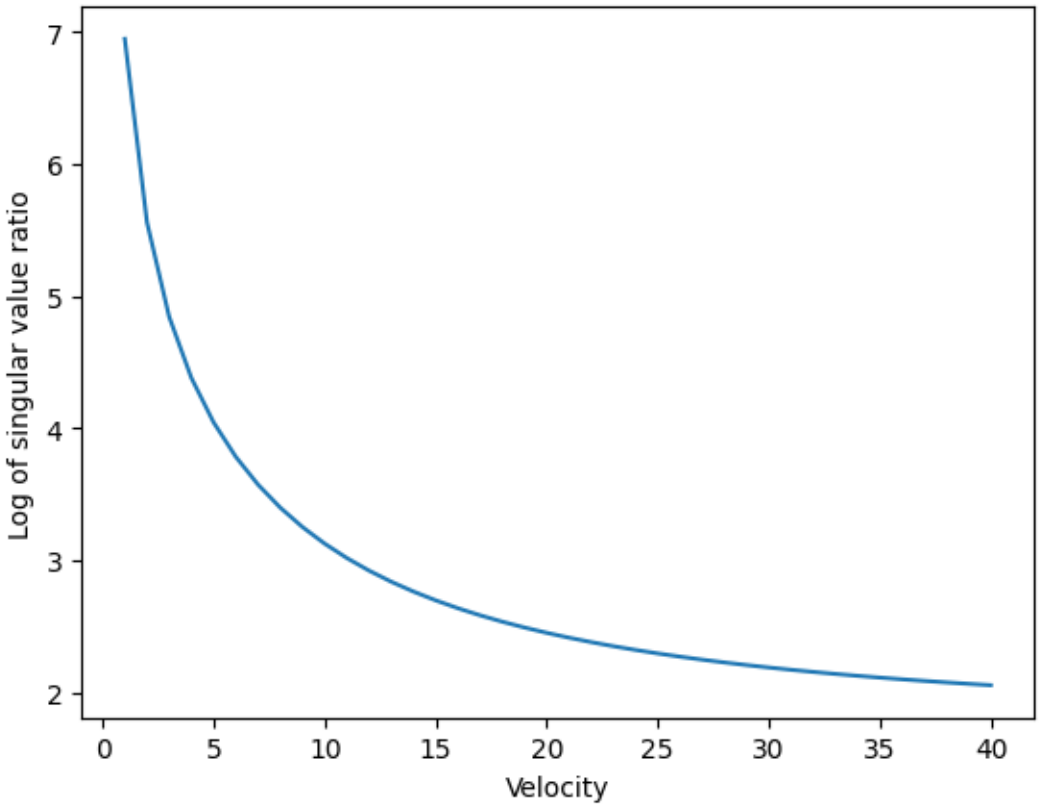
```
Rank of Q matrix 3
Q matrix
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -5.29492746e+00  4.23594197e+01 -4.23594197e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  2.80493641e+01 -2.24394913e+02  4.49574125e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.49910465e+02  1.19928372e+03 -2.73963550e+02]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]


If we are measuring E2
Rank of Q matrix 3
Q matrix
[[ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          1.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          1.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.         -0.03094299  0.2475439  -0.83828421]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.18977989 -1.51823914  0.96337159]
 [ 0.          0.          0.          0.        ]]


If we are measuring E2dot
Rank of Q matrix 2
Q matrix
[[ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          0.        ]
 [ 0.          0.          0.          1.        ]
```

```
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.         -0.03094299  0.2475439  -0.83828421]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.18977989 -1.51823914  0.96337159]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.         -1.03468036  8.27744287 -2.40620799]]
```

We find that if we measure E1 we find the system to be observable

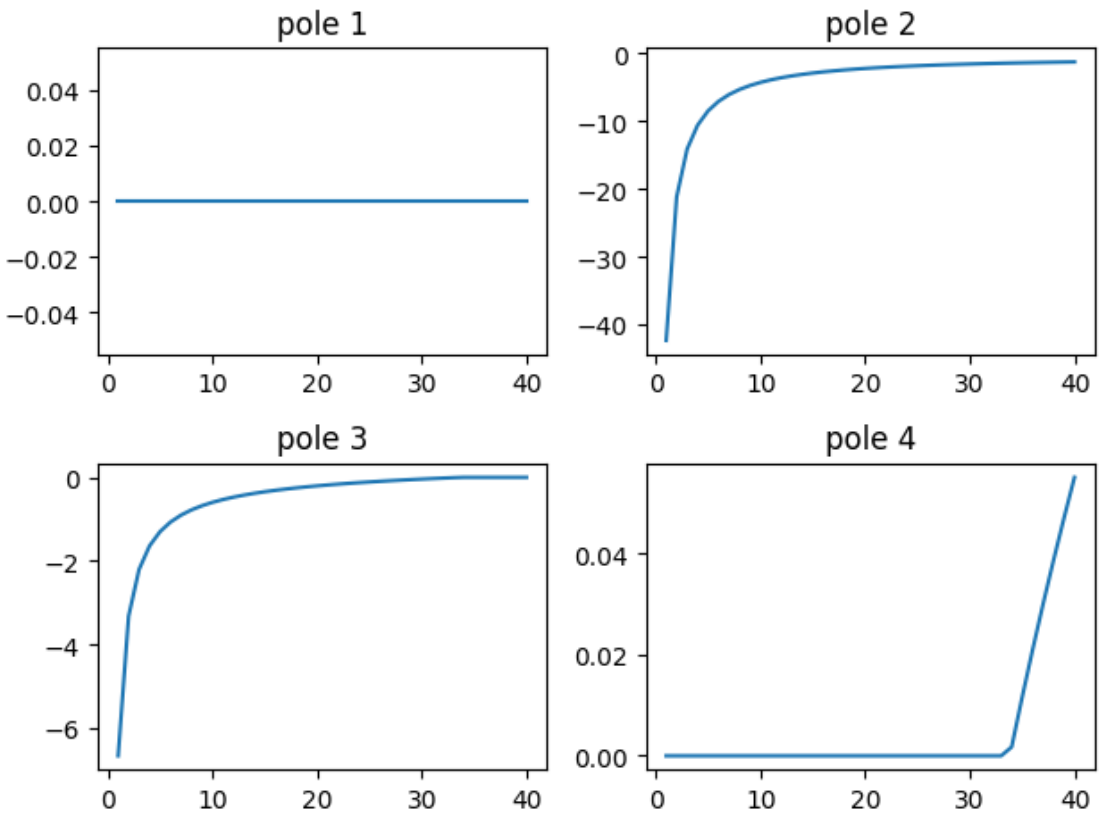
[14]:
```
##The variation of the log of the ratio of the largest singular value to the␣
 ↪smallest singular value of the matrix P with respect to the velocity v
for i in range(len(vs)):
    #create matrix a and b
    xdot = vs[i]
    A = np.array([[0, 1, 0, 0], [0, -4*Ca / (m * xdot), 4*Ca / m, -(2*Ca*(lf -␣
 ↪lr))/(m*xdot)], [0, 0, 0, 1], [0, -(2*Ca*(lf - lr)) / (Iz * xdot), (2*Ca*(lf␣
 ↪- lr)) / Iz, (-2*Ca*(np.power(lf, 2) + np.power(lr, 2))) / (Iz * xdot)]])
    B = np.array([[0], [2*Ca / m], [0], [(2 * Ca* lf) / Iz]])
    C = np.identity(4)
    D=np.zeros((4,1))
    #create matrix p
    p = np.hstack([B, A @ B, A @ A @ B, A @ A @ A @ B])
    #find the smallest and largest singular value
    u,s,v=np.linalg.svd(p)
    #append log to base 10 of the ratio of the largest singular value to the␣
 ↪smallest singular value to ln
    ln[i] = np.log10(s[0]/s[-1])

plt.plot(vs, ln)
plt.xlabel('Velocity')
plt.ylabel('Log of singular value ratio')
```

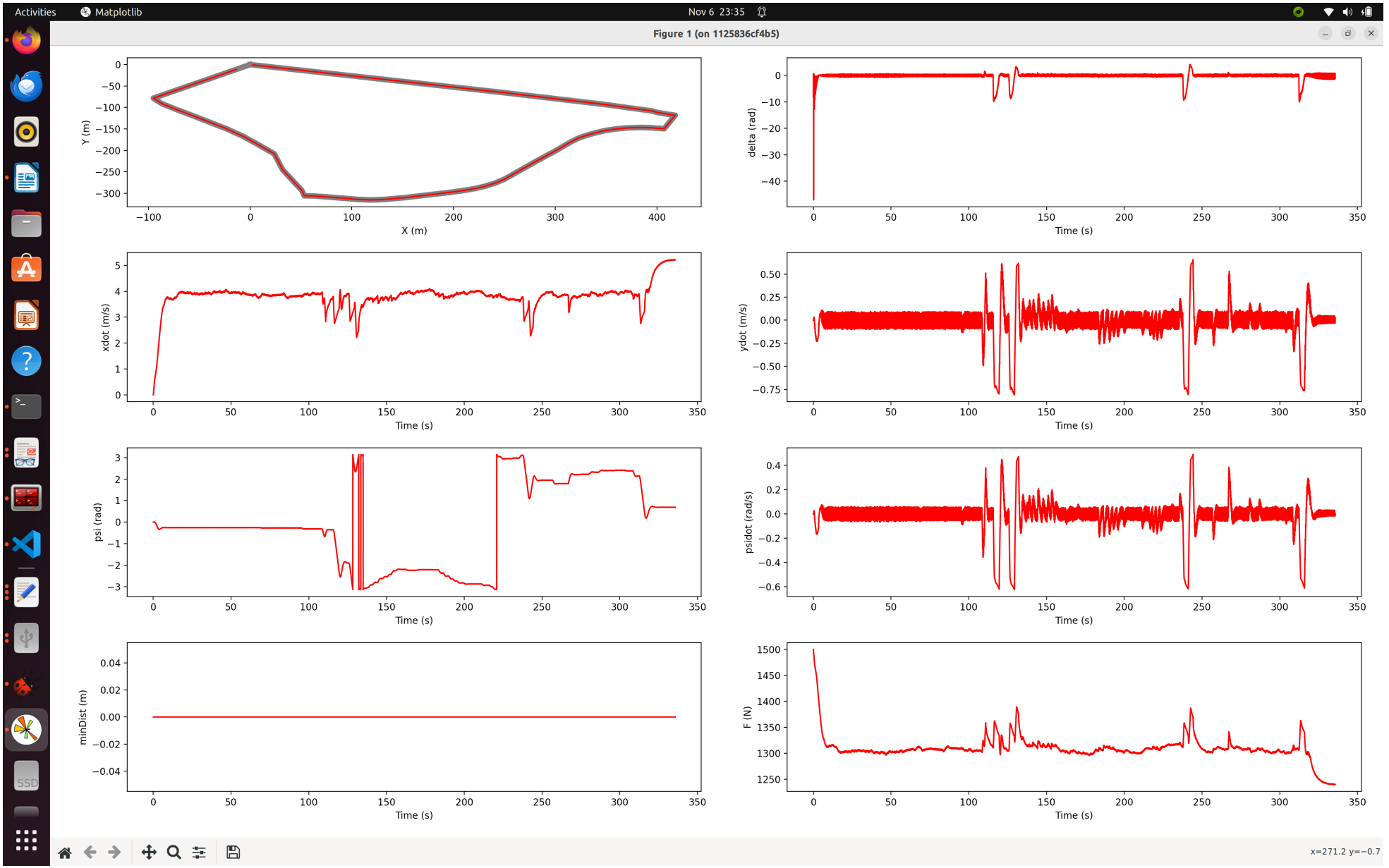
[14]:
```
Text(0, 0.5, 'Log of singular value ratio')
```



[15]:
```
##the
from sympy import symbols

import control

p1 = []
p2 = []
p3 = []
p4 = []
lr = 1.39
lf = 1.55
Ca = 20000
Iz = 25854
m = 1888.6
for i in range(1,41):
    # create matrix a and b
    xdot = i
    A = np.array([[0, 1, 0, 0], [0, -4*Ca / (m * xdot), 4*Ca / m, -(2*Ca*(lf - lr))/(m*xdot)], [0, 0, 0, 1], [0, -(2*Ca*(lf - lr)) / (Iz * xdot), (2*Ca*(lf - lr)) / Iz, (-2*Ca*(np.power(lf, 2) + np.power(lr, 2))) / (Iz * xdot)]])
    B = np.array([[0], [2*Ca / m], [0], [(2 * Ca* lf) / Iz]])
```

```
    C = np.identity(4)
    D=np.zeros((4,1))
    sys = control.StateSpace(A, B, C, D)

    poles = control.poles(sys)
    # append the real part of the poles to p1, p2, p3, p4
    p1.append(np.real(poles[0]))
    p2.append(np.real(poles[1]))
    p3.append(np.real(poles[2]))
    p4.append(np.real(poles[3]))



# plot the plots as 4 subplots vs velocity
plt.figure()
plt.subplot(2, 2, 1)
plt.plot(vs, p1)
plt.title('pole 1')
plt.subplot(2, 2, 2)
plt.plot(vs, p2)
plt.title('pole 2')
plt.subplot(2, 2, 3)
plt.plot(vs, p3)
plt.title('pole 3')
plt.subplot(2, 2, 4)
plt.plot(vs, p4)
plt.title('pole 4')
plt.tight_layout()
plt.show()
```


pole 1
0.04
0.02
0.00
−0.02
−0.04
0
10
20
30
40
pole 2
0
−10
−20
−30
−40
0
10
20
30
40
pole 3
0
−2
−4
−6
0
10
20
30
40
pole 4
0.04
0.02
0.00
0
10
20
30
40


Activities
Matplotlib
Nov 6 23:35
Figure 1 (on 1125836cf4b5)
Y (m)
X (m)
delta (rad)
Time (s)
xdot (m/s)
Time (s)
ydot (m/s)
Time (s)
psi (rad)
Time (s)
psidot (rad/s)
Time (s)
minDist (m)
Time (s)
F (N)
Time (s)
x=271.2 y=−0.7