Exercise 1

1)

$$\dot{X}_1 = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -4C_0/m\dot{x} & \frac{4C_0}{m} & \frac{-2C_0(l_f-l_r)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_0(l_f-l_r)}{I_z\dot{x}} & \frac{2C_0(l_f-l_r)}{I_z} & \frac{-2C_0(l_f^2+l_r^2)}{I_z\dot{x}} \end{bmatrix} X_1 + \begin{bmatrix} 0 & 0 \\ \frac{2C_0}{m} & 0 \\ 0 & 0 \\ \frac{2C_0 l_f}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

Put the values

$$\dot{X}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -42.35/\dot{x} & 42.35 & -3.6/\dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.247}{\dot{x}} & 0.247 & \frac{-6.66}{\dot{x}} \end{bmatrix} X_1 + \begin{bmatrix} 0 & 0 \\ 21.17 & 0 \\ 0 & 0 \\ 2.39 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} \quad -(1)$$

$$X_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{X}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X_2 + \begin{bmatrix} 0 & 0 \\ 0 & 1/1888.1 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} \quad -(2)$$

"For observability; we take $C_1 = [1,0,0,0]$ for e_1, $C_3 = [0,0,1,0]$ for e_2
$C_2 = [0,1,0,0]$ for $\dot{e}_1$; $C_4 = [0,0,0,1]$ for $\dot{e}_2$; ∴ we find for all the speed when we measure e_1 the rank$(Q) = 4$; ∴ "it is observable only if we measure e1

$\dot{x}_1 = 2\,m/s$

For X_1

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -21.175 & 42.35 & -1.8 \\ 0 & 0 & 0 & 1 \\ 0 & -0.124 & +0.247 & 3.33 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & 0 \\ 21.17 & 0 \\ 0 & 0 \\ 2.39 & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} =$$

$$P_1 = \begin{bmatrix} 0 & 0 & 0.0002 & 0 & -0.0095 & 0 & 0.096 & 0 \\ 0.0002 & 0 & -0.0095 & 0 & 0.0967 & 0 & -2.097 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 & 0 & 0.0007 & 0 \\ 0 & 0 & 0.0001 & 0 & 0.0007 & 0 & -0.0096 & 0 \end{bmatrix} \times 10^5$$

$\text{rank}(P) = 4 \quad \therefore$ controllable

$\dot{x} = 5\,m/s$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -8.47 & 42.35 & -0.72 \\ 0 & 0 & 0 & 1 \\ 0 & -0.05 & +0.24 & 1.5 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & 0 & 0.0021 & 0 & -0.018 & 0 & 0.163 & 0 \\ 0.0021 & 0 & -0.018 & 0 & 0.16 & 0 & -1.37 & 0 \\ 0 & 0 & 0.0002 & 0 & 0.002 & 0 & 0.0012 & 0 \\ 0.0002 & 0 & 0.0002 & 0 & 0.0012 & 0 & -0.0023 & 0 \end{bmatrix}$$

$\text{rank}(P_2) = 4 \quad \therefore$ controllable

IIIrd for $\dot{x} = 8$ m/s

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -5.29 & 12.35 & -0.45 \\ 0 & 0 & 0 & 1 \\ 0 & -0.030 & 0.29 & 0.83 \end{bmatrix}$$

rank (P_3) = 4 $\therefore$ controllable

In the program I have attached the P & Q & rank for each speed & for diff C

2) We notice that the overall stability (poles $<=0$) is achieved when speed is less than 30; ($\because$ all poles $<=0$)

and the car is controllable at any speed since it is of full rank for any speed between (1-40) thus making it fully controllable