P2-Ex1

November 6, 2024

```python
[13]: import numpy as np
      import matplotlib.pyplot as plt
      #v int array from 1 to 40
      vs = np.arange(1, 41)
      #create a empty array ln
      ln = np.zeros_like(vs, dtype=float)


      lr = 1.39
      lf = 1.55
      Ca = 20000
      Iz = 25854
      m = 1888.6



      #for xdot=2,5 and 8 , P and Q matrix
      xdots=[2,5,8]
      for xdot in xdots:
          A = np.array([[0, 1, 0, 0], [0, -4*Ca / (m * xdot), 4*Ca / m, -(2*Ca*(lf -␣
       ↪lr))/(m*xdot)], [0, 0, 0, 1], [0, -(2*Ca*(lf - lr)) / (Iz * xdot), (2*Ca*(lf␣
       ↪- lr)) / Iz, (-2*Ca*(np.power(lf, 2) + np.power(lr, 2))) / (Iz * xdot)]])
          B = np.array([[0], [2*Ca / m], [0], [(2 * Ca* lf) / Iz]])
          C1 = np.array([[1, 0, 0, 0], [0, 0, 0, 0],[0, 0, 0, 0],[0, 0, 0, 0]])
          C2 = np.array([[0, 0, 0, 0], [0, 1, 0, 0],[0, 0, 0, 0],[0, 0, 0, 0]])
          C3 = np.array([[0, 0, 0, 0], [0, 0, 0, 0],[0, 0, 1, 0],[0, 0, 0, 0]])
          C4 = np.array([[0, 0, 0, 0], [0, 0, 0, 0],[0, 0, 0, 0],[0, 0, 0, 1]])
          #make an array of C which is a combination of C1,C2,C3 and C4



          p = np.hstack([B, A @ B, A @ A @ B, A @ A @ A @ B])

          #and the rank of the matrix
          print("For xdot=",xdot)
          print("Rank of P matrix ", np.linalg.matrix_rank(p))

          print("P matrix")
          print(p)
```

```
    print("If we are measuring E1")
    q1=np.vstack([C1, C1 @ A, C1 @ A @ A, C1 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q1))
    print("Q matrix")
    print(q1)
    print(end="\n\n")


    print("If we are measuring E1dot")
    q2=np.vstack([C2, C2 @ A, C2 @ A @ A, C2 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q2))
    print("Q matrix")
    print(q2)
    print(end="\n\n")


    print("If we are measuring E2")
    q3=np.vstack([C3, C3 @ A, C3 @ A @ A, C3 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q3))
    print("Q matrix")
    print(q3)
    print(end="\n\n")


    print("If we are measuring E2dot")
    q4=np.vstack([C4, C4 @ A, C4 @ A @ A, C4 @ A @ A @ A])
    print("Rank of Q matrix",np.linalg.matrix_rank(q4))
    print("Q matrix")
    print(q4)
    print(end="\n\n")
```

```
For xdot= 2
Rank of P matrix  4
P matrix
[[ 0.00000000e+00  2.11797098e+01 -4.52643363e+02  9.70650280e+03]
 [ 2.11797098e+01 -4.52643363e+02  9.70650280e+03 -2.06189084e+05]
 [ 0.00000000e+00  2.39808153e+00 -1.06625495e+01  9.23711699e+01]
 [ 2.39808153e+00 -1.06625495e+01  9.23711699e+01 -1.51376540e+03]]
If we are measuring E1
Rank of Q matrix 4
Q matrix
[[   1.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           1.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.           0.           0.           0.        ]
 [   0.         -21.17970984   42.35941968   -1.69437679]
```

```
[  0.             0.             0.             0.         ]
[  0.             0.             0.             0.         ]
[  0.             0.             0.             0.         ]
[  0.           448.78982514 -897.57965028   83.92730562]
[  0.             0.             0.             0.         ]
[  0.             0.             0.             0.         ]
[  0.             0.             0.             0.         ]]
```

If we are measuring E1dot
Rank of Q matrix 3
Q matrix
```
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -2.11797098e+01  4.23594197e+01 -1.69437679e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  4.48789825e+02 -8.97579650e+02  8.39273056e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -9.51562612e+03  1.90312522e+04 -1.93941845e+03]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]
```

If we are measuring E2
Rank of Q matrix 3
Q matrix
```
[[ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           1.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           1.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.          -0.12377195   0.2475439   -3.35313685]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           0.           0.           0.        ]
 [ 0.           3.03647828  -6.07295656  11.70078692]
```

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```
 [ 0.          0.          0.          0.        ]]


If we are measuring E2dot
Rank of Q matrix 2
Q matrix
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.23771950e-01  2.47543900e-01 -3.35313685e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  3.03647828e+00 -6.07295656e+00  1.17007869e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -6.57599581e+01  1.31519916e+02 -5.04522346e+01]]


For xdot= 5
Rank of P matrix  4
P matrix
[[ 0.00000000e+00  2.11797098e+01 -1.81057345e+02  1.63836877e+03]
 [ 2.11797098e+01 -1.81057345e+02  1.63836877e+03 -1.40710886e+04]
 [ 0.00000000e+00  2.39808153e+00 -4.26501982e+00  1.52780368e+01]
 [ 2.39808153e+00 -4.26501982e+00  1.52780368e+01 -1.02661158e+02]]
If we are measuring E1
Rank of Q matrix 4
Q matrix
[[   1.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.          1.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.         -8.47188394   42.35941968  -0.67775071]
 [   0.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.          0.          0.          0.        ]
 [   0.         71.80637202 -359.03186011  49.01028143]
 [   0.          0.          0.          0.        ]
```

```
[   0.            0.            0.            0.          ]
[   0.            0.            0.            0.          ]]


If we are measuring E1dot
Rank of Q matrix 3
Q matrix
[[ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   1.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00  -8.47188394e+00   4.23594197e+01  -6.77750715e-01]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   7.18063720e+01  -3.59031860e+02   4.90102814e+01]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00  -6.10761689e+02   3.05380844e+03  -4.73433952e+02]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]
 [ 0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00]]


If we are measuring E2
Rank of Q matrix 3
Q matrix
[[ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            1.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            1.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.           -0.04950878   0.2475439   -1.34125474]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.            0.            0.          ]
 [ 0.            0.48583652  -2.42918262   2.08006278]
 [ 0.            0.            0.            0.          ]]


If we are measuring E2dot
Rank of Q matrix 2
```

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```
Q matrix
[[ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          1.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.         -0.04950878  0.2475439  -1.34125474]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.48583652 -2.42918262  2.08006278]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.         -4.21893202 21.09466009 -5.54835274]]


For xdot= 8
Rank of P matrix  4
P matrix
[[ 0.00000000e+00  2.11797098e+01 -1.13160841e+02  7.01888933e+02]
 [ 2.11797098e+01 -1.13160841e+02  7.01888933e+02 -3.83204707e+03]
 [ 0.00000000e+00  2.39808153e+00 -2.66563739e+00  6.32972667e+00]
 [ 2.39808153e+00 -2.66563739e+00  6.32972667e+00 -2.76845127e+01]]
If we are measuring E1
Rank of Q matrix 4
Q matrix
[[   1.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          1.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.         -5.29492746    42.35941968   -0.4235942 ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.         28.04936407 -224.39491257   44.95741255]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]
 [   0.          0.          0.          0.         ]]


If we are measuring E1dot
```

```
Rank of Q matrix 3
Q matrix
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -5.29492746e+00  4.23594197e+01 -4.23594197e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  2.80493641e+01 -2.24394913e+02  4.49574125e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00 -1.49910465e+02  1.19928372e+03 -2.73963550e+02]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]


If we are measuring E2
Rank of Q matrix 3
Q matrix
[[ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          1.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          1.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.         -0.03094299  0.2475439  -0.83828421]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.18977989 -1.51823914  0.96337159]
 [ 0.          0.          0.          0.         ]]


If we are measuring E2dot
Rank of Q matrix 2
Q matrix
[[ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          0.         ]
 [ 0.          0.          0.          1.         ]
```

```
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.         -0.03094299  0.2475439  -0.83828421]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.18977989 -1.51823914  0.96337159]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.          0.          0.          0.        ]
[ 0.         -1.03468036  8.27744287 -2.40620799]]
```

We find that if we measure E1 we find the system to be observable

```
[14]: ##The variation of the log of the ratio of the largest singular value to the␣
      ↪smallest singular value of the matrix P with respect to the velocity v
      for i in range(len(vs)):
          #create matrix a and b
          xdot = vs[i]
          A = np.array([[0, 1, 0, 0], [0, -4*Ca / (m * xdot), 4*Ca / m, -(2*Ca*(lf -␣
      ↪lr))/(m*xdot)], [0, 0, 0, 1], [0, -(2*Ca*(lf - lr)) / (Iz * xdot), (2*Ca*(lf␣
      ↪- lr)) / Iz, (-2*Ca*(np.power(lf, 2) + np.power(lr, 2))) / (Iz * xdot)]])
          B = np.array([[0], [2*Ca / m], [0], [(2 * Ca* lf) / Iz]])
          C = np.identity(4)
          D=np.zeros((4,1))
          #create matrix p
          p = np.hstack([B, A @ B, A @ A @ B, A @ A @ A @ B])
          #find the smallest and largest singular value
          u,s,v=np.linalg.svd(p)
          #append log to base 10 of the ratio of the largest singular value to the␣
      ↪smallest singular value to ln
          ln[i] = np.log10(s[0]/s[-1])

      plt.plot(vs, ln)
      plt.xlabel('Velocity')
      plt.ylabel('Log of singular value ratio')
```

```
[14]: Text(0, 0.5, 'Log of singular value ratio')
```
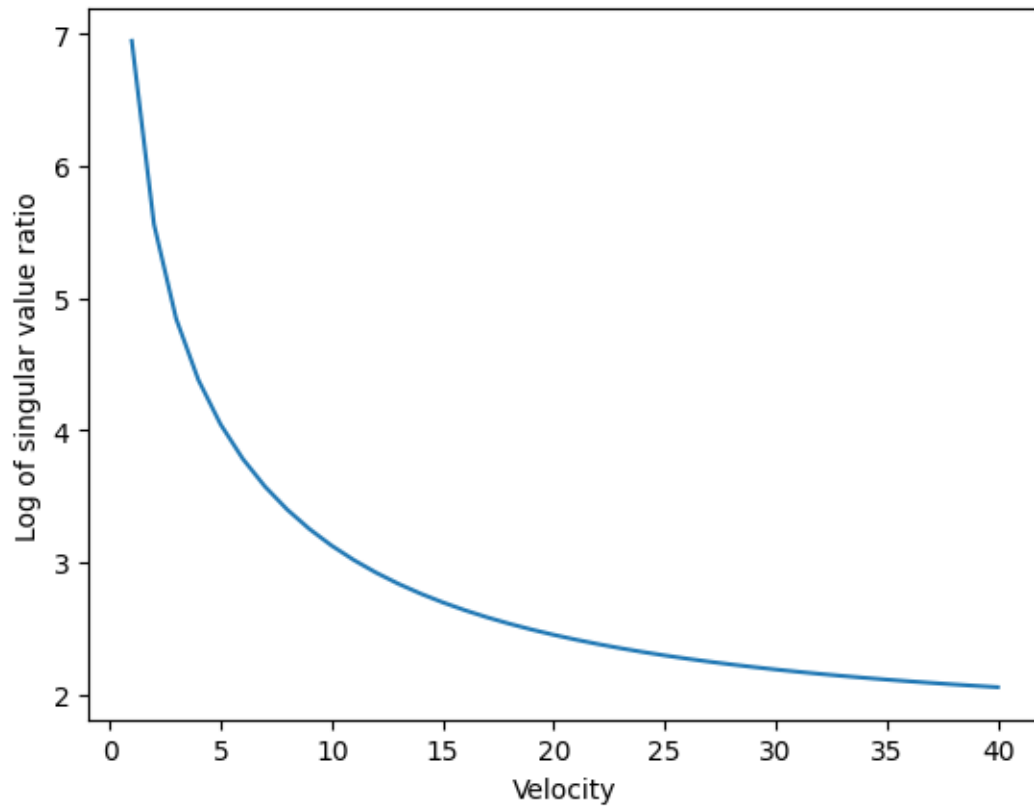
[15]:
```python
##the
from sympy import symbols

import control

p1 = []
p2 = []
p3 = []
p4 = []
lr = 1.39
lf = 1.55
Ca = 20000
Iz = 25854
m = 1888.6
for i in range(1,41):
    # create matrix a and b
    xdot = i
    A = np.array([[0, 1, 0, 0], [0, -4*Ca / (m * xdot), 4*Ca / m, -(2*Ca*(lf -␣
    ↪lr))/(m*xdot)], [0, 0, 0, 1], [0, -(2*Ca*(lf - lr)) / (Iz * xdot), (2*Ca*(lf␣
    ↪- lr)) / Iz, (-2*Ca*(np.power(lf, 2) + np.power(lr, 2))) / (Iz * xdot)]])
    B = np.array([[0], [2*Ca / m], [0], [(2 * Ca* lf) / Iz]])
```

```python
    C = np.identity(4)
    D=np.zeros((4,1))
    sys = control.StateSpace(A, B, C, D)

    poles = control.poles(sys)
    # append the real part of the poles to p1, p2, p3, p4
    p1.append(np.real(poles[0]))
    p2.append(np.real(poles[1]))
    p3.append(np.real(poles[2]))
    p4.append(np.real(poles[3]))



# plot the plots as 4 subplots vs velocity
plt.figure()
plt.subplot(2, 2, 1)
plt.plot(vs, p1)
plt.title('pole 1')
plt.subplot(2, 2, 2)
plt.plot(vs, p2)
plt.title('pole 2')
plt.subplot(2, 2, 3)
plt.plot(vs, p3)
plt.title('pole 3')
plt.subplot(2, 2, 4)
plt.plot(vs, p4)
plt.title('pole 4')
plt.tight_layout()
plt.show()
```