Project 1

Exercise 1

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix} \quad ; \quad S_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} \qquad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x}\dot{\psi} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \left(\frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\right) - \left(\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right)\right) \\ \dot{\psi} \\ \frac{2l_fC_\alpha}{I_z}\left(\delta - \left(\frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\right) - \frac{2l_rC_\alpha}{I_z}\left(-\left(\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right)\right) \end{bmatrix}$$

$$\frac{\partial S_1}{\partial y} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \frac{\partial S_1}{\partial \dot{y}} = \begin{bmatrix} 1 \\ 0 + \frac{2C_\alpha}{m}\left(\cos\delta\left(-\frac{1}{\dot{x}}\right) - \frac{1}{\dot{x}}\right) \\ 0 \\ \frac{2l_fC_\alpha}{I_z}\left[-\frac{1}{\dot{x}}\right] - \frac{2l_rC_\alpha}{I_z}\left[-\frac{1}{\dot{x}}\right] \end{bmatrix}$$

$$\frac{\partial S_1}{\partial \psi} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \frac{\partial S_1}{\partial \dot{\psi}} = \begin{bmatrix} 0 \\ -\dot{x} + \frac{2C_\alpha}{m}\left[\cos\delta\left(-\frac{l_f}{\dot{x}}\right) + \frac{l_r}{\dot{x}}\right] \\ 1 \\ \frac{2l_fC_\alpha}{I_z}\left[-\frac{l_f}{\dot{x}}\right] - \frac{2l_rC_\alpha}{I_z}\left[\frac{l_r}{\dot{x}}\right] \end{bmatrix}$$

$$\dot{S}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \end{bmatrix}$$

$$\frac{\partial \dot{S}_2}{\partial x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \frac{\partial \dot{S}_1}{\partial \delta} = \begin{bmatrix} 0 \\ \frac{2C_\alpha}{m}\left(-\sin\delta\left(\delta - \dot{y} + \frac{l_f\dot{\psi}}{\dot{x}}\right) + \cos\delta\right) \\ 0 \\ \frac{2l_f}{I_z}C_\alpha \end{bmatrix}$$

$$\frac{\partial \dot{S}_2}{\partial \dot{x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \frac{\partial \dot{S}_1}{\partial F} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial \dot{S}_2}{\partial \delta} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \frac{\partial \dot{S}_2}{\partial u} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

After linear about eq $\dot{y}=0;\ \ddot{y}=0;\ \psi=0;\ \dot{\psi}=0$

$\Rightarrow \underline{\underline{\delta=0}}$

$$\dot{\delta S_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{m\dot{x}} & 0 & -\dot{x}+\frac{2C_\alpha}{m\dot{x}}\left[l_R-l_f\right] \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}\left[l_R-l_f\right] & 0 & -\frac{2C_\alpha}{I_z\dot{x}}\left[l_R^2+l_f^2\right] \end{bmatrix} \delta S_1 + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2l_fC_\alpha}{I_z} & 0 \end{bmatrix} \delta u$$

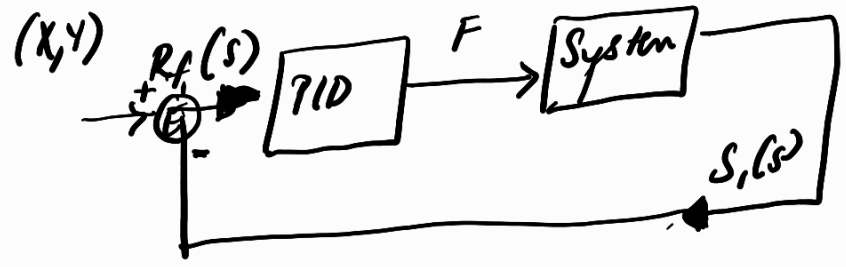
Putting the values

$$\dot{\delta S_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{42.36}{\dot{x}} & 0 & -\dot{x}-\frac{3.388}{\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{0.247}{\dot{x}} & 0 & -\frac{6.7}{\dot{x}} \end{bmatrix} \delta S_1 + \begin{bmatrix} 0 & 0 \\ 21.18 & 0 \\ 0 & 0 \\ 2.398 & 0 \end{bmatrix} \delta u$$

$$\delta \dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \delta S_2 + \begin{bmatrix} 0 & 0 \\ 0 & \gamma_m \end{bmatrix} \delta u + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} - fg \end{bmatrix}$$

$$\delta \dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \delta S_2 + \begin{bmatrix} 0 & 0 \\ 0 & \gamma / 1888.6 \end{bmatrix} \delta u + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} - fg \end{bmatrix}$$

Esercizio 2:



$\psi_{desired} = Y_{desired} - Y$

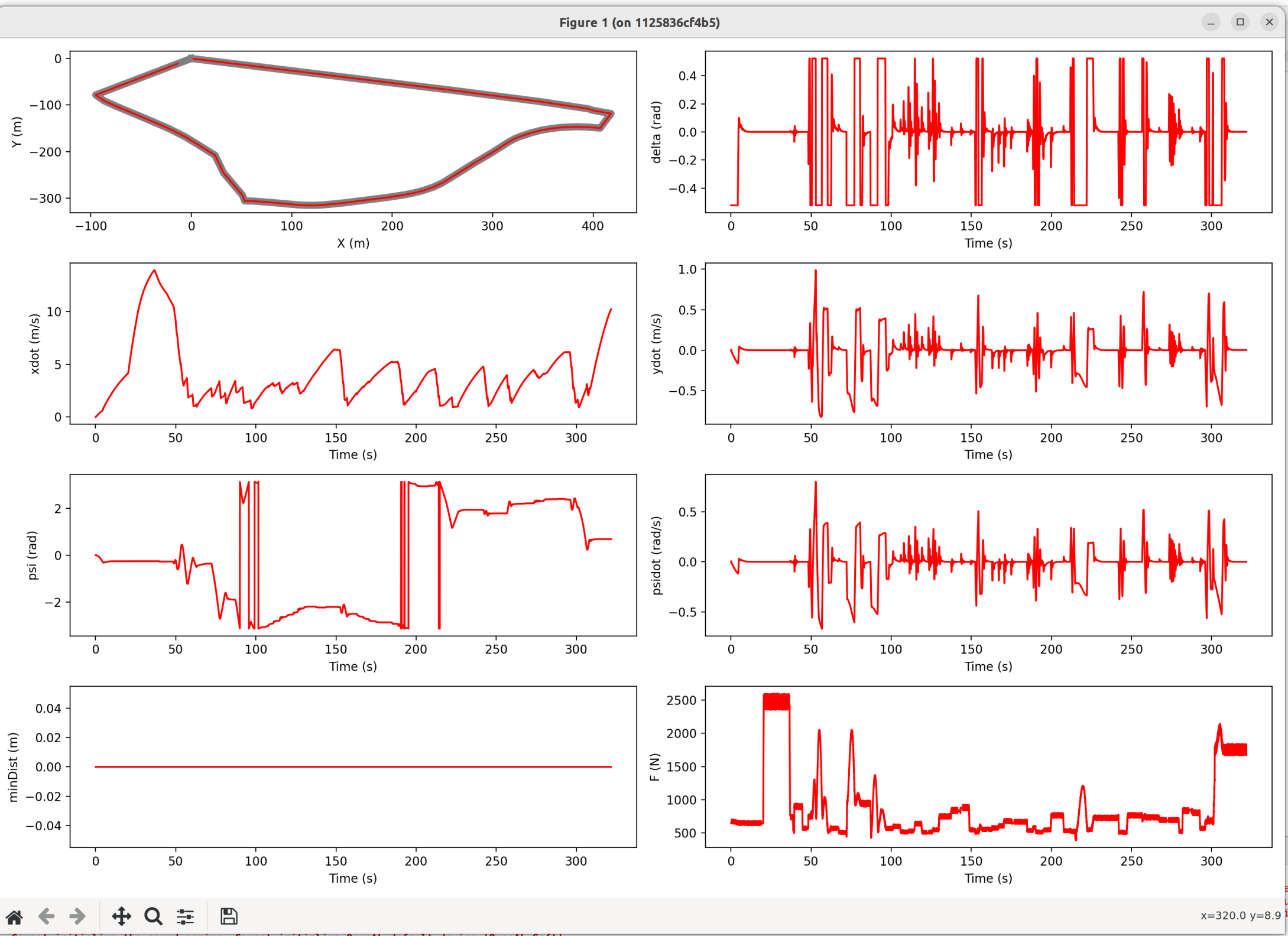


Figure 1 (on 1125836cf4b5)
Y (m)
X (m)
delta (rad)
Time (s)
xdot (m/s)
Time (s)
ydot (m/s)
Time (s)
psi (rad)
Time (s)
psidot (rad/s)
Time (s)
minDist (m)
Time (s)
F (N)
Time (s)
x=320.0 y=8.9