

Let $n = 1, \dots, N$, $k = 1, \dots, K$, and constants $a_n \in (0, 1)$ with $\sum_n a_n = 1$ and $b_{nk} > 0$. Consider the system of equations

$$x_k = \frac{\sum_n a_n \left(\frac{b_{n,k} x_k^\beta}{\sum_k b_{n,k} x_k^\beta + 1} \right)}{1 - \sum_n a_n \left(\frac{\sum_k b_{n,k} x_k^\beta}{\sum_k b_{n,k} x_k^\beta + 1} \right)}, k = 1, \dots, K$$

for unknowns x_1, \dots, x_K . Write code that quickly and robustly finds the solution to this system of equations for any data $\{a_n, b_{nk}\}$.