Modelling Dengue Fever Transmission Dynamics with Biological and Chemical Controls

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Introduction

- Dengue is the most widespread vector-borne disease causing about 50 million human infections worldwide every year.
- The disease is transmitted by the bite of an *Aedes* mosquito infected with any of the four serotypes, denoted as DENV-1, DENV-2, DENV-3, and DENV-4.
- There is no specific treatment for dengue. Prevention and control of the disease depend on vector control or interruption of vector-host contacts.
  - Vector control interventions can be grouped into three: ecological or mechanical, chemical, and biological controls.
- Mathematical model can be helpful in facilitating the understanding of mechanisms involved in the transmission dynamics and control of dengue.

Aim

- The interest of this study is to develop and analyse a mathematical model for dengue disease transmission and control using chemical open space spraying of insecticide) and biological (*Wolbachia*) controls.

The Model

The proposed dengue model subgroups human and mosquito populations into different epidemiological classes according to the definition given in Table 1.

Table 1: Definition of the state variables of model (1)

Human population (N_h):	S_h–Susceptible humans	E_h–Exposed humans
	A_h–Asymptomatic infectious humans	I_h–Infectious humans
	R_h–Recovered humans	
Mosquito population (N_v):	S_v–Susceptible mosquito	E_v–Exposed mosquitoes
	I_v–Infectious mosquitoes	

The model that governs the system of time-dependent ordinary differential equations under the flow described in Figure 1 is given as

$$\frac{dS_h}{dt} = \Lambda_h - \frac{b\beta_h I_v}{N_h} S_h - \mu_h S_h, \tag{1a}$$
$$\frac{dE_h}{dt} = \frac{b\beta_h I_v}{N_h} S_h - \sigma_h E_h - \mu_h E_h, \tag{1b}$$
$$\frac{dA_h}{dt} = \nu\sigma_h E_h - \gamma_{Ah} A_h - \mu_h A_h, \tag{1c}$$
$$\frac{dI_h}{dt} = (1-\nu)\sigma_h E_h - \gamma_{Ih} I_h - \mu_h I_h, \tag{1d}$$
$$\frac{dR_h}{dt} = \gamma_{Ah} A_h + \gamma_{Ih} I_h - \mu_h R_h, \tag{1e}$$
$$\frac{dS_v}{dt} = (1-\eta)\Lambda_v - \frac{b\beta_v(1-\psi)(\vartheta A_h + I_h)}{N_h} S_v - \mu_v S_v - \eta S_v, \tag{1f}$$
$$\frac{dE_v}{dt} = \frac{b\beta_v(1-\psi)(\vartheta A_h + I_h)}{N_h} S_v - \sigma_v E_v - \mu_v E_v - \eta E_v, \tag{1g}$$
$$\frac{dI_v}{dt} = \sigma_v E_v - \mu_v I_v - \eta I_v, \tag{1h}$$

with initial conditions given at time $t = 0$.

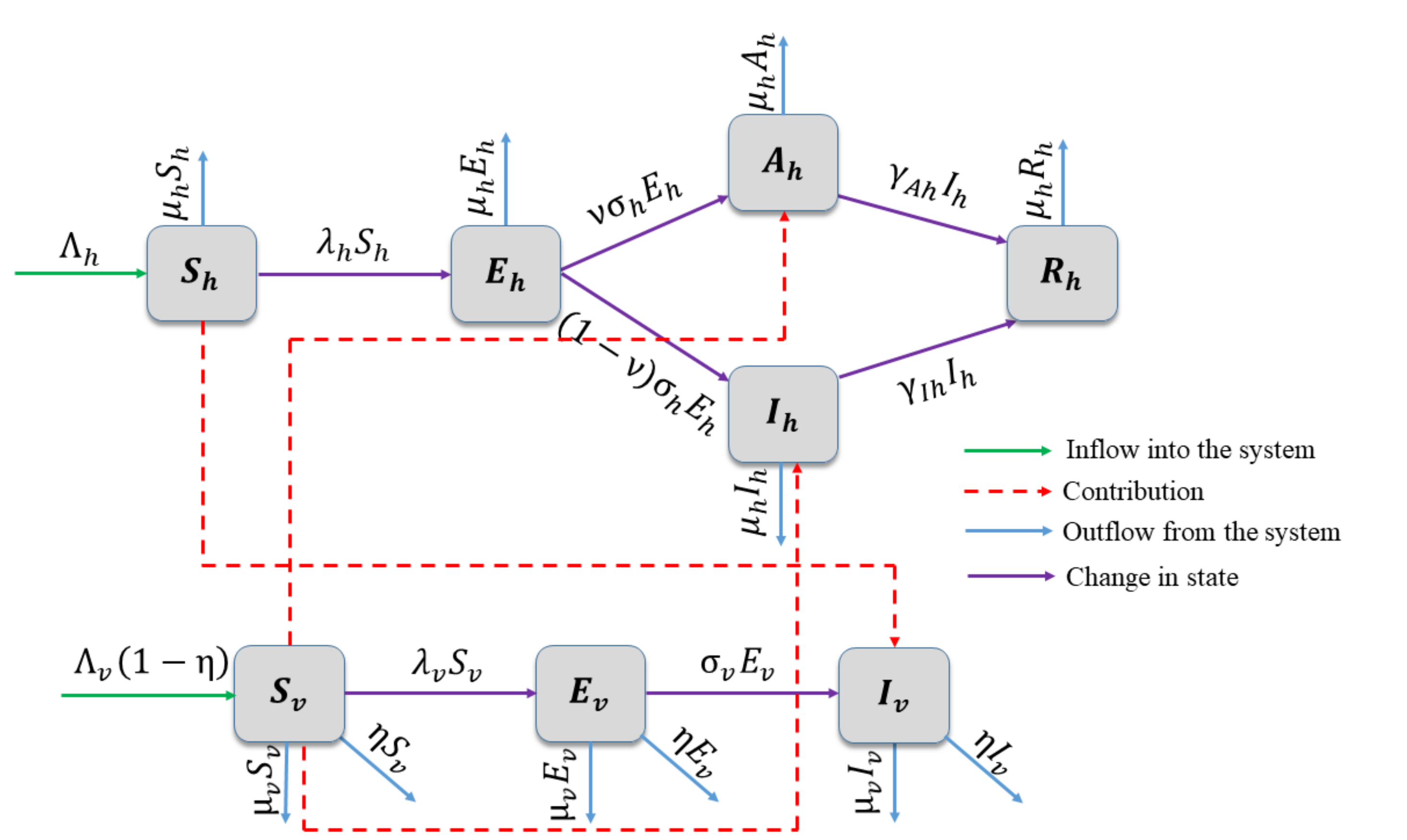


Figure 1: Flow diagram of model (1) where $\lambda_h = \frac{b\beta_h I_v}{N_h}$ and $\lambda_v = \frac{b\beta_v(1-\psi)(\vartheta A_h + I_h)}{N_h}$

Qualitative analysis of the basic properties of model (1)

- Disease-free equilibrium, $\mathcal{E}_0$:

$$\mathcal{E}_0 = (S_h^0, 0, 0, 0, 0, S_v^0, 0, 0) = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, 0, 0, \frac{\Lambda_v(1-\eta)}{(\eta+\mu_v)}\right).$$

- Basic reproduction number, $\mathcal{R}_0$:

$$\mathcal{R}_0 = \sqrt{\frac{b^2\beta_h\sigma_h(g_3\nu\vartheta + (1-\nu)g_2)\beta_v(1-\psi)\sigma_v S_v^0}{g_1 g_2 g_3 g_4 g_5(\mu_v+\eta)S_h^0}},$$

where $g_1 = \sigma_h + \mu_h$, $g_2 = \gamma_{Ah} + \mu_h$, $g_3 = \gamma_{Ih} + \mu_h$, $g_4 = \sigma_v + \mu_v + \eta$, and $g_5 = \eta + \mu_v$.

- Stability analysis:

Lemma 1

If $\mathcal{R}_0 < 1$, then the disease-free equilibrium (DFE) of the dengue model (1) *is locally asymptotically stable (LAS), and unstable if $\mathcal{R}_0 > 1$.*

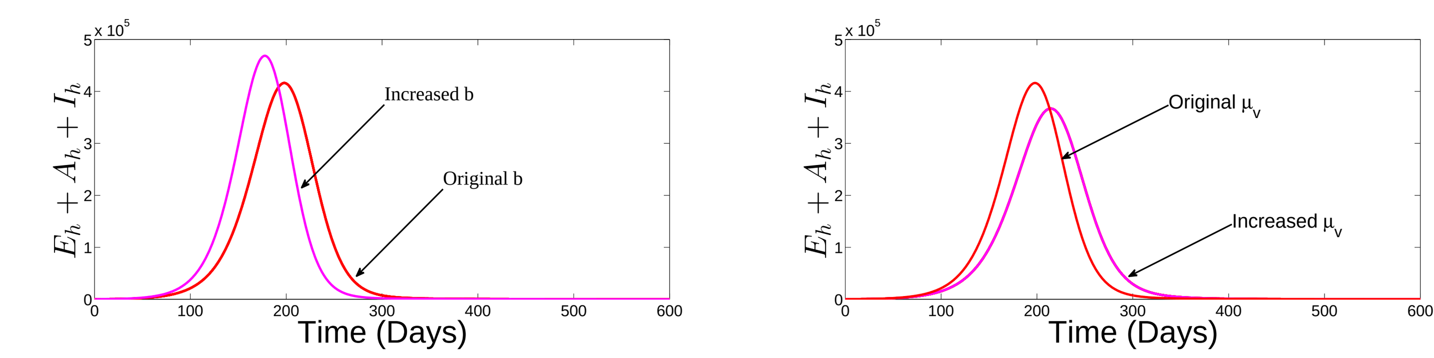
Theorem 2

If $\mathcal{R}_0 \leqslant 1$, then the DFE $\mathcal{E}_0$ of the dengue model (1) *is globally asymptotically stable (GAS), and it is unstable otherwise.*

The epidemiological implication of Theorem 2 is that dengue elimination is possible regardless of the initial sizes of the sub-populations of model 1 whenever $\mathcal{R}_0 \leqslant 1$. This result is graphically illustrated in Figure 7.

Numerical Simulations and Results

Sensitivity indices of $\mathcal{R}_0$ to the parameters of model (1)

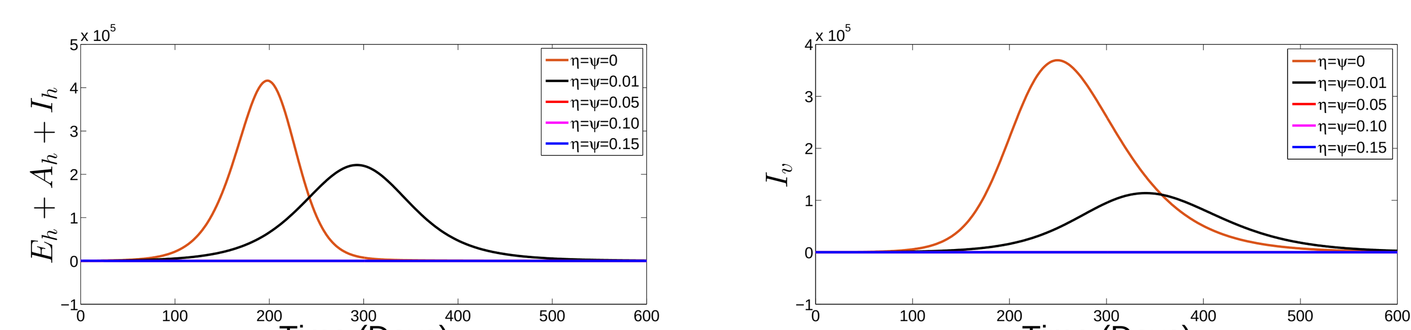


(a) Behaviour of the disease prevalence with time (b) Behaviour of the disease prevalence with time

Figure 2: Dynamics of the disease prevalence in human population with increased b and μ_v

- Figure 2 shows the sizes of the disease prevalence in human population with 10% increase in mosquito biting rate b and death rate μ_v.

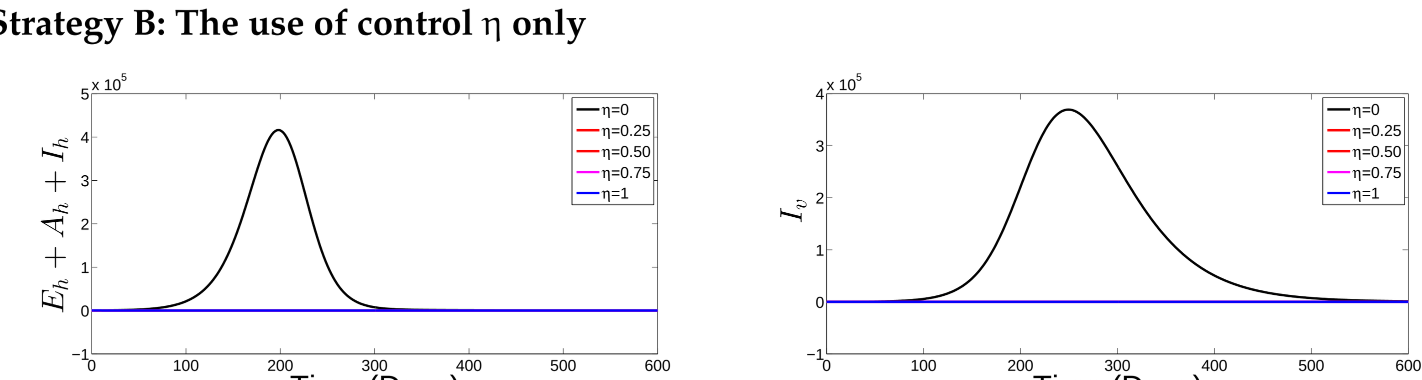
Strategy A: The use of controls η and ψ simultaneously



(a) Disease prevalence vs time (b) Infectious mosquito subpopulation vs time

Figure 3: Simulated results of model (1) with effect of the combination of insecticide (η) and *Wolbachia* (ψ) controls

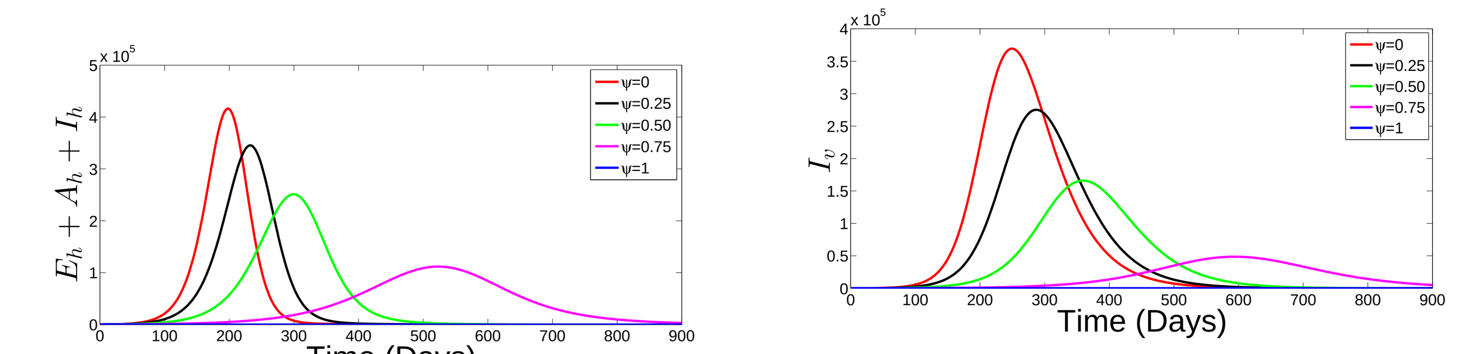
Strategy B: The use of control η only



(a) Disease prevalence vs time (b) Infectious mosquito subpopulation vs time

Figure 4: Simulated results of model (1) with effect of insecticide control η only

Strategy C: The use of control ψ only



(a) Behaviour of the disease prevalence with time (b) Behaviour of the infectious mosquito subpopulation with time

Figure 5: Simulated results of model (1) with effect of *Wolbachia* control ψ only

Effect of controls on $\mathcal{R}_0$

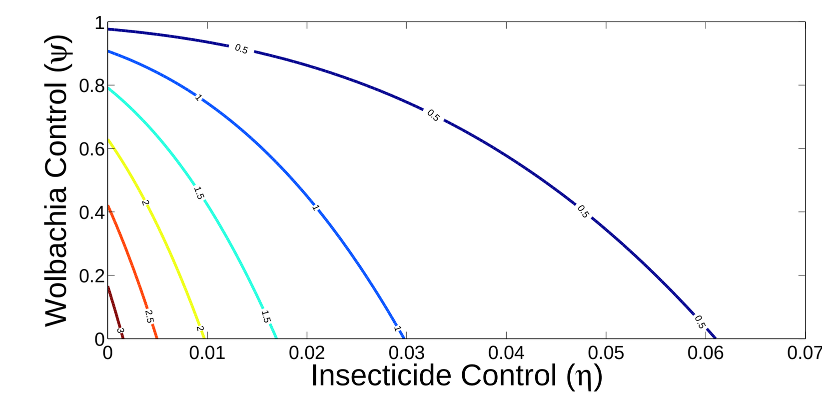
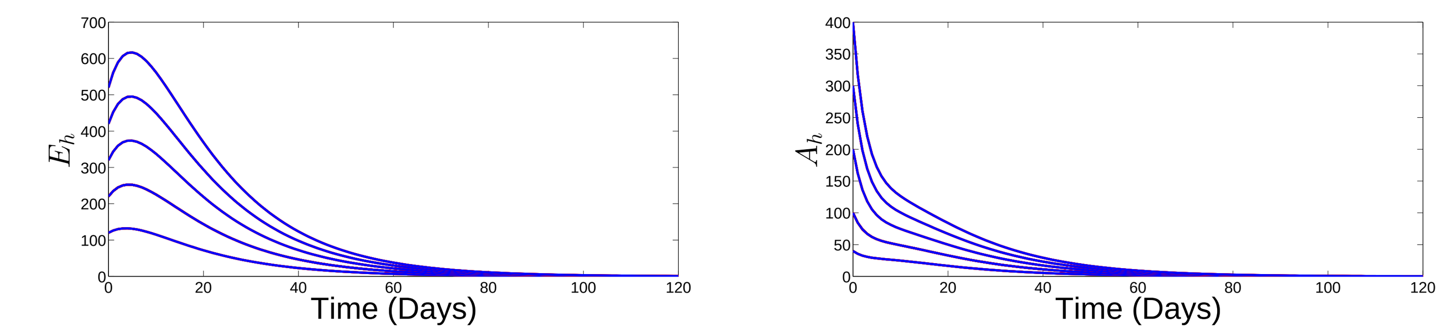
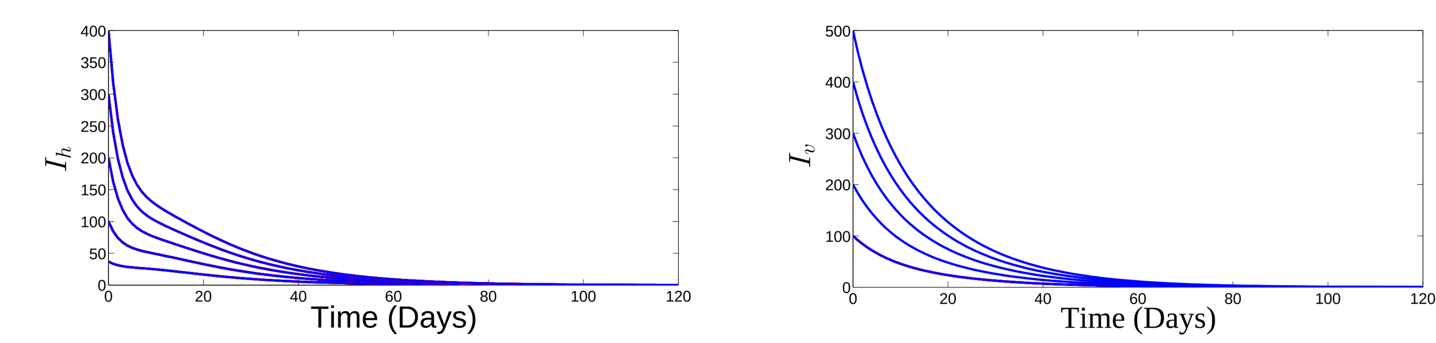


Figure 6: Effect of controls η and ψ on $\mathcal{R}_0$



(a) Subpopulation of exposed human (b) Subpopulation of asymptomatic infectious human



(c) Subpopulation of infectious human (d) Subpopulation of infectious mosquito

Figure 7: Dynamics of the global asymptotic disease-free equilibrium $\mathcal{E}_0$ of model (1)

Summary and Conclusion

- This study has presented and analysed a compartmental model to gain insight into the transmission dynamics and control of dengue using insecticide and *Wolbachia* controls.
- Theoretical analysis of the model shows that the model has a DFE which is both LAS and GAS whenever the control induced reproduction number, $\mathcal{R}_0$, satisfies $\mathcal{R}_0 < 1$, and unstable otherwise.
- The results obtained from sensitivity analysis reveal that mosquito biting rate among other model parameters contributes most significantly to the transmission and spread of dengue.
- The results of simulation suggest that dengue prevalence can be reduced significantly using insecticide and *Wolbachia* with lesser efforts of insecticiding.
- The pieces of information provided by this study may be helpful to the concerned authorities to bring dengue disease under control when an outbreak occurs.

Future Work

Further study needs to be conducted by addressing some other aspects such as:

- Integrating human personal protection control measures with insecticide and *Wolbachia* controls.
- Employing optimal control theory to derive optimal insecticide, *Wolbachia*, and personal protection controls required to effectively control the transmission and spread of dengue in a community.

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