Quiz review

Find a_5

$$\sum_{i=1}^{5} (4a_i + 1) = 101 \longrightarrow 4\sum_{i=1}^{5} a_i + \sum_{i=1}^{5} 1 = 101$$

$$4\sum_{i=1}^{5} a_i + 5 = 101$$

$$\sum_{i=1}^{5} a_i = 24$$

$$* \quad \boxed{a_1 + a_2 + a_3 + a_4 + a_5 = 24}$$

$$\sum_{i=1}^{4} (a_i + i) = 120$$

$$\sum_{i=1}^{4} a_i + \sum_{i=1}^{4} i = 120$$

$$\frac{4(5)}{2} = 10$$

$$\sum_{i=1}^{4} a_i = 110 \qquad : \square \quad \boxed{a_1 + a_2 + a_3 + a_4 = 110}$$

$$a_5 = 24 - 110 = \boxed{-86}$$

$$\text{time} = 100 + \sum_{i=10}^{60} \Big(\underbrace{3(i+2) - 15}_{} \Big) \qquad 3i - 9$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$= 100 + 3\sum_{i=10}^{60} i - \sum_{i=10}^{60} 9$$

$$\Big(\sum^{60} \quad \sum^{9} \quad \sum^{60} 9$$

$$= 100 + 3\left(\sum_{i=1}^{60} i - \sum_{i=1}^{9} i\right) - \sum_{i=10}^{60} 9$$

$$100 + 3\left(\frac{(60)(61)}{2} - \frac{9(10)}{2}\right) - (60-10+1)(9)$$

$$= 100 + 3(1785) - 459$$

$$= \boxed{4996}$$

constant

$$\sum_{k=5}^{30} (2i) = (30-5+1)(2i)$$

$$= 26(2i) = 52i$$

Lab 9-2017

1. Review Quiz 3 ✓

2. Use mathematical induction to prove: $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}, n \geq 1$

Base case : $n = 1$

$L.S. = \sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1(2)} = \frac{1}{2}$ ✓

$R.S. = \frac{1}{1+1} = \frac{1}{2}$

Induction Hypothesis (Given): $n = k$ $\quad L.S = R.S$

$$\boxed{\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}}$$

We prove $n = k+1$

target : $\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k+1}{(k+1)+1}$

$L.S = \sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+1+1)}$

according to Induction Hypothesis $= \frac{k}{k+1} + \frac{1}{(k+1)(k+1+1)} =$

$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2}$

$R.S = \frac{k+1}{(k+1)+1} = \frac{k+1}{k+2}$ $\quad$ R.S = L.S. Target acheived :)

$$R.S = \frac{k+1}{(k+1)+1} = \frac{k+1}{k+2}$$ R.S = L.S. Target acheived ☺

3. What does the following function return? Can you prove that by mathematical induction?

```
MysteryFunction (positive integer n)

     i = 1
     j = 5
     while i ≠ n do
                 j = 2j - 3
                 i = i + 1
     end while
return j
```

Guess: $J_n = 2^n + 3$

n	1	2	3	4	5	6	...	n=10
returned j	$j_1 = 5$	$j_2 = 7$	$J_3 = 11$	$J_4 = 19$	$j_5 = 35$	$J_6 = 67$		$J_{10} = ?$

prove that the j returned from the code = the j returned from the formula

1) Base case : $n = 1$

j from the code = 5 ✓

j from the formula = $2^1 + 3 = 5$

2) Induction Hypothesis : for $n = k$

j returned from code = j returned from formula

j returned from code = $2^k + 3$ (Given)

Target : for $n = k+1$ j returned from code = j returned from formula

L.S: from the Code: R.S. = $2^{k+1}+3$

$$j_{k+1} = 2(j_k) - 3$$

$$= 2^1(2^k+3)-3$$ according to induction hypothesis

$$= 2^{k+1}+6-3$$

$$= 2^{k+1}+3 = R.S$$ target acheived.

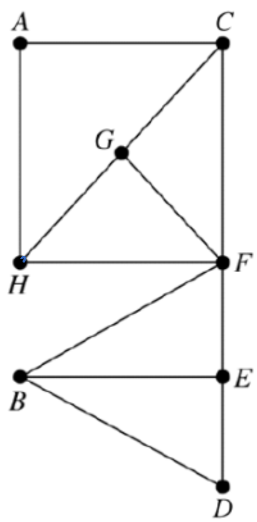
<u>Graphs and Trees</u>

1. Let G = (V, E) be graph in the figure below. How many paths are there from A to D? How many of these paths have length 6?

Path: no repeated vertices

trail: no repeated edges

walk: no restrictions.



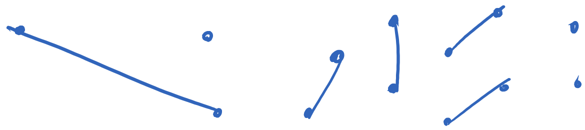
A → F	F → D
ACF	FED
AHF	FBD
ACGF	FBED
AHGF	FEBD
ACGHF	
AHGCF	

# of The paths of length 6 = 8

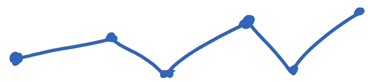
the number of paths from A to D: 6x4 = 24

2. Consider a simple[1] graph G = (V, E) with 30 edges. $E = 30$

a) What is the maximum number of vertices that G can have? Infinite

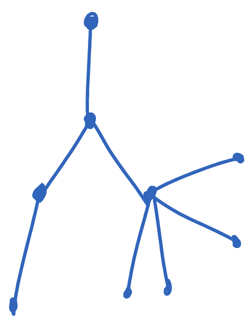


b) If G is connected, what is the maximum number of vertices that G can have? 31



3. Give an example of a connected graph G where removing any edge results in a disconnected graph. Can you generalize this result?

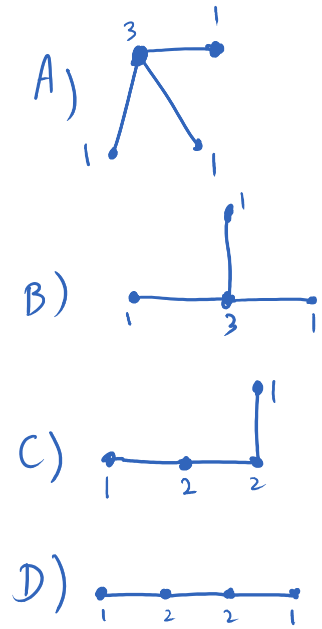
A tree is an example of such graph.



$V = 9$ $E = 8$

$\boxed{|V| = |E| + 1}$ ← tree equation

This is an example of graph isomorphism:

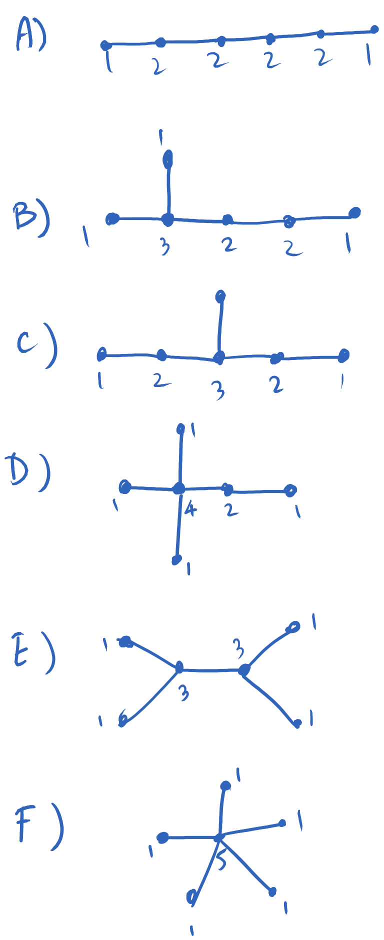


A and B are isomorphic

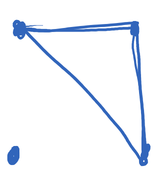
C) and D) are isomorphic

A and C or B and D are non-isomorphic

4. Draw all non-isomorphic trees on six vertices.



5. Give an example of an undirected graph G = (V, E) where |V| = |E| + 1 but graph is not a tree.



$|V| = 4$

$|E| = 3$

6. If a tree has three vertices of degree 2, four vertices of degree 3, four vertices of degree 4, and two vertices of degree 5, how many vertices with degree 1 does it have?

let be x

# of vertices	Degree
3	2
4	3
4	4
2	5
x	1

Tree equation: $|V| = |E| + 1$

$$\boxed{\sum \deg(v_i) = 2|E|}$$ General simple Graph equation

$$|V| = 3+4+4+2+x$$
$$= 13+x$$

$$\sum \deg v_i = 2+2+2+3+3+3+3+4+4+4+4+5+5+x(1)$$

$$\sum \deg v_i = 44+x = 2|E|$$

$$22+\frac{x}{2} = |E|$$

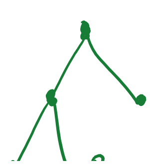
$$|V| = |E|+1 \longrightarrow 13+x = 22+\frac{x}{2}+1$$

$$\frac{x}{2} = 10, \quad \boxed{x=20}$$

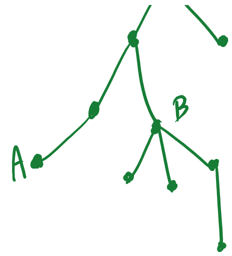
7. Let T = (V, E) be a tree with 10 vertices. How many distinct paths are there in T?

$|V| = 10$ $\quad |V| = |E|+1$

$|E| = 9$



$|E| = 9$



Note that for any two points A and B on a tree there is a unique path that connects them. Thus, the number of paths on a tree is equal to the number of ways we can choose two points A and B.

So the answer is: $C(10,2) = 45$