Appendix 2: Prior distributions and multinomial-Dirichlet mixture

Appendix 2.1

The following vague prior distributions were used in the model. Parameter descriptions associated with parameter symbols can be found in Table 1 of the main text:

Trap size selectivity parameters:

$$h_M^{max} \sim Uniform(0, 0.1)$$

$$h_M^{A} \sim Uniform(35, 60)$$

$$h_M^{\sigma} \sim Uniform(3, 10)$$

$$h_F^{max} \sim Uniform(0, 0.1)$$

$$h_F^{k} \sim Uniform(0.1, 1.5)$$

$$h_F^{0} \sim Uniform(30, 100)$$

$$h_S^{max} \sim Uniform(0, 0.1)$$

$$h_S^{k} \sim Uniform(0.1, 1.5)$$

$$h_S^{0} \sim Uniform(30, 100)$$

Natural mortality parameters:

$$\beta_\alpha \sim Uniform(0, 50)$$

$$\beta_\theta \sim Uniform(0, 150)$$

$$\alpha \sim Uniform(0, 10000)$$

Overwinter mortality parameters:

$$\alpha^o_\alpha \sim Uniform(0, 50)$$

$$\alpha^o_\theta \sim Uniform(0, 150)$$

Seasonal growth parameters:

$$\sigma^G \sim Uniform(0.01, 4)$$

Overdispersion observation process (see Appendix 2.2):

$$\rho \sim Beta(1, 1)$$

Initial population density and annual recruitment:

$$\mu^A \sim Uniform(3.25, 4.5)$$

$$\sigma^A \sim Uniform(0.1, 1)$$

$$\mu^R \sim Uniform(1, 25)$$

$$\sigma^R \sim Uniform(0.01, 20)$$

$$\mu^\lambda \sim Uniform(1, 1000000)$$

$$\sigma^\lambda \sim Uniform(0, 10000)$$

$$\lambda^A \sim Uniform(1, 1000000)$$

Since the model was estimated hierarchically, with the seasonal growth model was estimated first with the size-at-age data (D2), and the marginal posterior distributions from this first model were used in the second model with the time series data (D1) and mark-recapture data (D3). Informative priors used in the second model for parameters A, k, x_∞, and d_s can be found in Table A1.2 in Appendix 1.2.4.

Appendix 2.2

Similarly to how a beta-binomial replaces a single probability with a Beta distribution of probabilities among binomial draws to account for overdispersion, the Dirichlet-multinomial mixture replaces a single vector of probabilities (that sum to one) with a Dirichlet distribution of such vectors to account for overdispersion in counts among traps.

The parameter α^D amount of overdispersion in the Dirichlet-multinomial mixture distribution and generates the conditional probability of capture, $p^C_{t,j,y}(x)$, across individual traps.

The parameter α^D is linked to the sampled parameter, ρ:

$$\alpha^D = p^C_{t,j,y}(x) \times p^D$$

$$p^D = \frac{1-\rho}{\rho}$$