11th i-CoMSE Workshop: Mesoscale Particle-Based Modeling

Mississippi State University
July 21–25, 2025

## Session 3: Bottom-up coarse graining

Bottom-up coarse graining

- Objective: use information from a high-resolution (e.g., atomistic) model to parameterize a low-resolution (e.g., mesoscale) model

- The low-resolution model contains fewer "degrees of freedom" (e.g., particles) and is thus less computationally intensive, enabling simulation of larger length and time scales
  - Fewer particle-particle interactions to calculate
  - Removal of fast motions (e.g., bond vibrations) allowing for larger integration time step

- Two common strategies (often used in combination) for solutions
  1. Coarse grain the solute – combine multiple atoms into mesoscale beads
  2. Coarse grain the solvent – remove completely (implicit) or simplify (explicit)

Solute coarse graining

- Coarse grain (CG) groups of atoms into single mesoscale beads

- Determine effective interactions between mesoscale beads from simulations of atomistic model
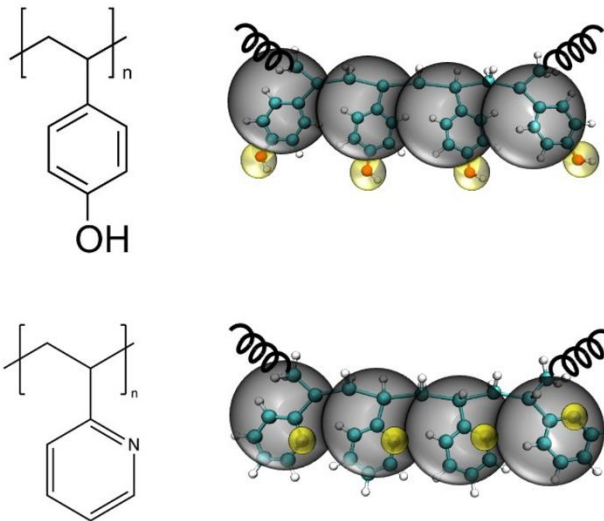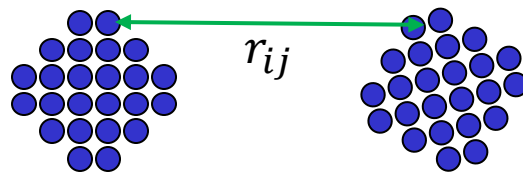

Kapoor et al., Polymers 12, 2764, 2020
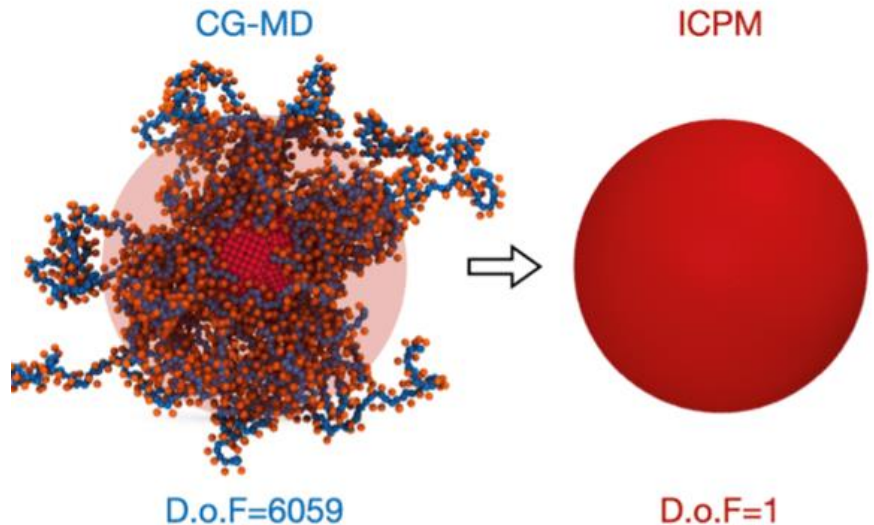
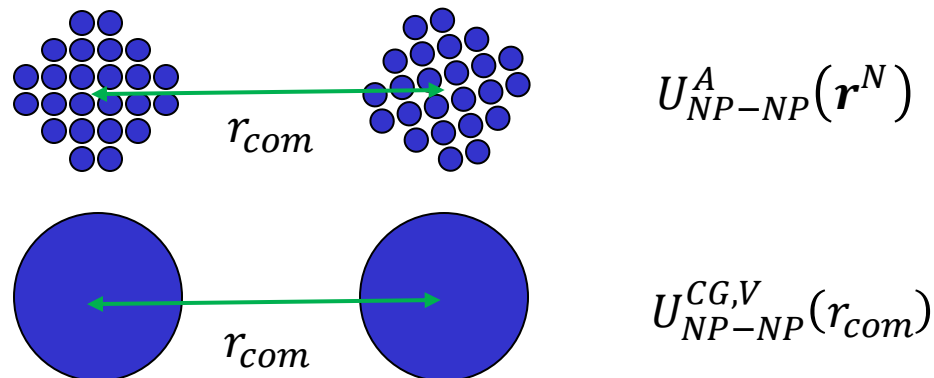**Zhenghao et al., Macromolecules 9, 3259, 2023**

Solute coarse graining

- Consider two nanoparticles (NPs) containing N atoms total with positions $\boldsymbol{r}^N = \{\boldsymbol{r}_1 \ldots \boldsymbol{r}_N\}$ and total interaction energy $U_{NP-NP}^A(\boldsymbol{r}^N)$

$$U_{NP-NP}^A(\boldsymbol{r}^N) = \frac{1}{2} \sum_i \sum_j u(r_{ij})$$

Solute coarse graining

- Consider two nanoparticles (NPs) containing N atoms total with positions $\boldsymbol{r}^N = \{\boldsymbol{r}_1 \dots \boldsymbol{r}_N\}$ and total interaction energy $U_{NP-NP}^A(\boldsymbol{r}^N)$
- Mapping the NPs onto CG mesoscale beads

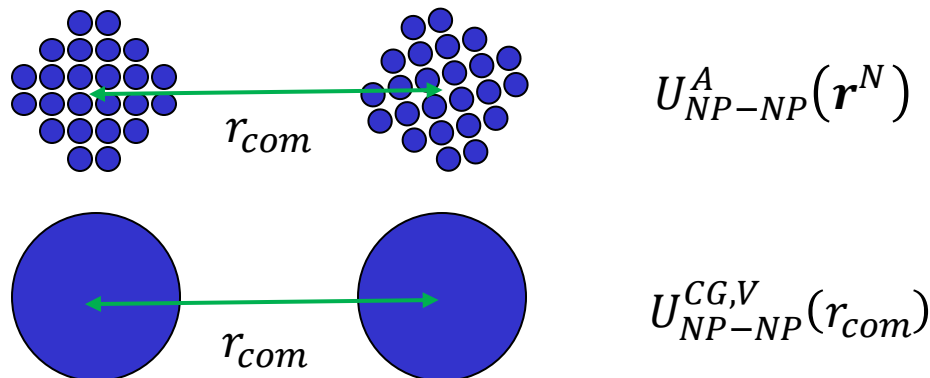$U_{NP-NP}^A(\boldsymbol{r}^N)$

$U_{NP-NP}^{CG,V}(r_{com})$

Solute coarse graining

- Consider two nanoparticles (NPs) containing N atoms total with positions $\boldsymbol{r}^N = \{\boldsymbol{r}_1 \dots \boldsymbol{r}_N\}$ and total interaction energy $U_{NP-NP}^A(\boldsymbol{r}^N)$

- Mapping the NPs onto CG mesoscale beads
  - Group the atoms in each NP into a single mesoscale bead positioned at the center of mass
  - Perform simulations of the atomistic system (in vacuum)
  - Measure the average force $\left\langle f_{r_{com}}^V(\boldsymbol{r}^N) \right\rangle$ between the NPs as a function of the NP-NP center of mass separation distance $r_{com} = |\boldsymbol{r}_{2,com} - \boldsymbol{r}_{1,com}|$
  - Obtain effective CG potential between mesoscale beads via
    $U_{NP-NP}^{CG,V}(r_{com}; T) = -\int \left\langle f_{r_{com}}^V(\boldsymbol{r}^N) \right\rangle dr_{com}$

r_{com}

$U_{NP-NP}^A(\boldsymbol{r}^N)$

r_{com}

$U_{NP-NP}^{CG,V}(r_{com})$

Solute coarse graining

- Consider two nanoparticles (NPs) containing N atoms total with positions $\boldsymbol{r}^N = \{\boldsymbol{r}_1 \dots \boldsymbol{r}_N\}$ and total interaction energy $U_{NP-NP}^A(\boldsymbol{r}^N)$

- Effective CG potential $U_{NP-NP}^{CG,V}(r_{com}; T)$

  – "Potential of mean force" (PMF) along r_{com}: $\left\langle f_{r_{com}}^V(\boldsymbol{r}^N) \right\rangle = -\dfrac{dU_{NP-NP}^{CG,V}(r_{com};T)}{dr_{com}}$

  – CG potential obtained by integrating over all degrees of freedom (DoF) except r_{com}
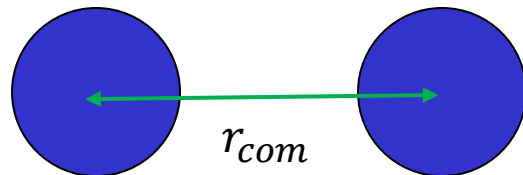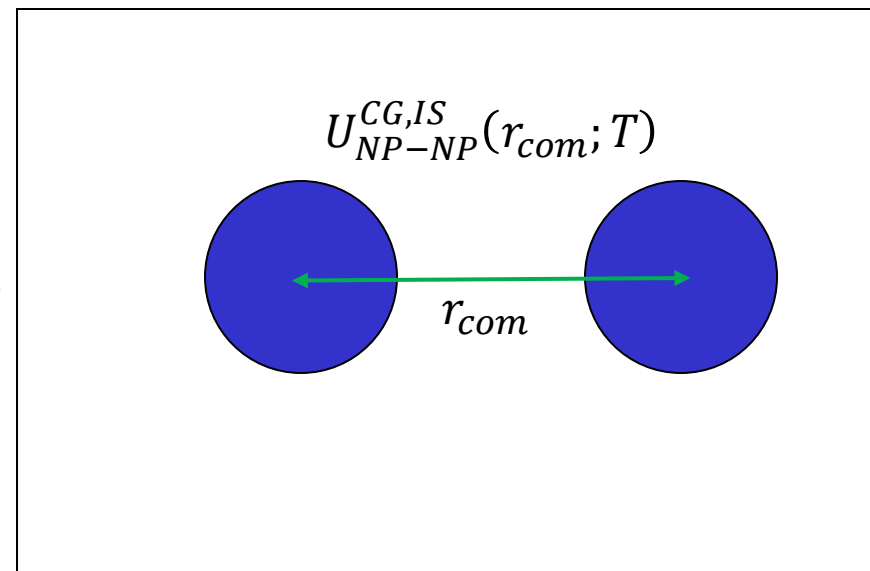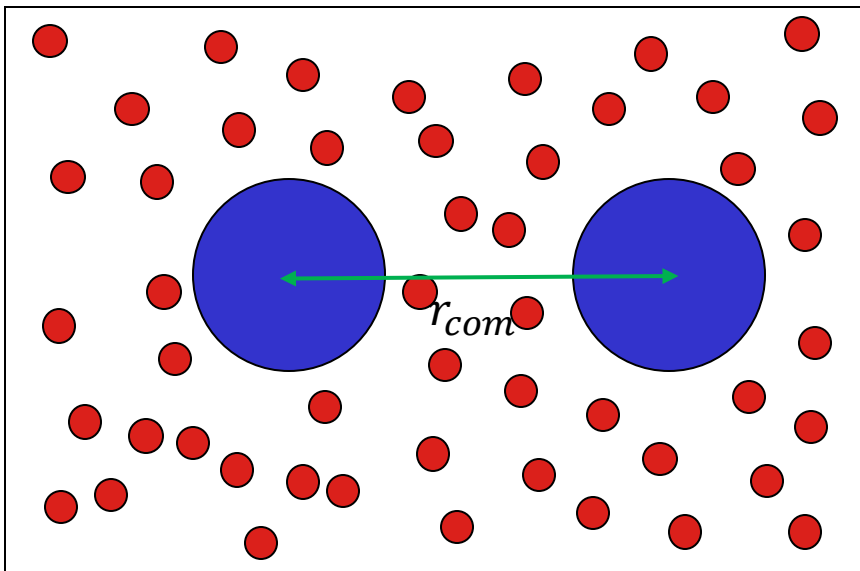

$U_{NP-NP}^{CG,V}(r_{com})$

Solvent coarse graining

- CG nanoparticles (NP) in a bath of N solvent (or depletant) particles

- Typical experimental system: each 10 nm diameter nanoparticle is surrounded by > 10^6 solvent molecules under dilute conditions (0.1 volume fraction)
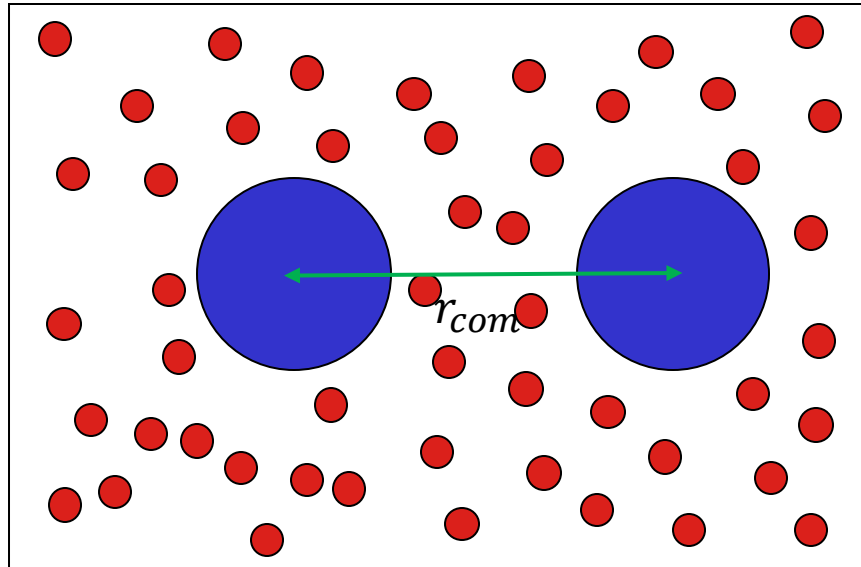
Solvent coarse graining

- CG nanoparticles (NP) in a bath of N solvent (or depletant) particles

- Determine effective NP-NP potential $U_{NP-NP}^{CG,IS}(r_{com}; T)$ by integrating out the solvent DoF

  - Perform simulations of the CG NPs in the solvent (S)
  - Measure the average force $\langle f_{r_{com}}^{S}(r_{com}; \boldsymbol{r}^N) \rangle$ along r_{com}
  - Obtain CG potential via $U_{NP-NP}^{CG,IS}(r_{com}; T) = -\int \langle f_{r_{com}}^{S}(r_{com}, \boldsymbol{r}^N) \rangle \, dr_{com}$

Solvent coarse graining

- This is an example of an implicit solvent (IS) model

$$U_{NP-NP}^{CG,IS}(r_{com}; T) = U_{NP-NP}^{CG,V}(r_{com}; T) + \Delta u^{IS}(r_{com}; T)$$

- CG potentials depend on T due to canonical average

$$\langle f_{r_{com}}(\boldsymbol{r}^N) \rangle = -\frac{\int \left(f_{r_{com}}(\boldsymbol{r}^N) e^{-U(\boldsymbol{r}^N)/k_B T} \right) \delta[r_{com} - \hat{r}_{com}(\boldsymbol{r}^N)] d\boldsymbol{r}^N}{\int f_{r_{com}}(\boldsymbol{r}^N) e^{-U(\boldsymbol{r}^N)/k_B T} \delta[r_{com} - \hat{r}_{com}(\boldsymbol{r}^N)] d\boldsymbol{r}^N}$$

- CG potentials also depend on species concentrations via $\boldsymbol{r}^N$!

- Generally, must re-compute CG potentials at different T and species concentrations

- Accurately computing potentials of mean force often requires enhanced (biased) sampling techniques

Mapping to the CG model and back

- No unique approach for defining a CG model → how to group atoms into CG beads?

- Increasing complexity does not always improve accuracy!

single-bead   double-bead   triple-bead
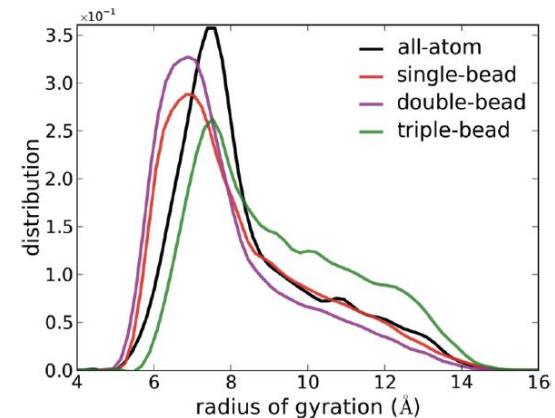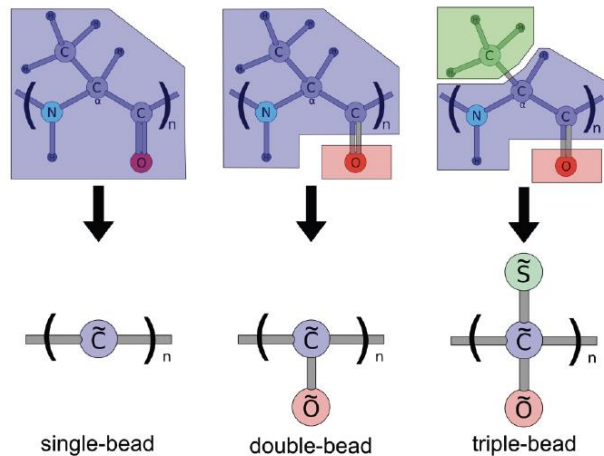
Carmichael and Shell., JPCB 116, 8383, 2012

- The inverse problem of creating an atomistic model from a CG configuration is known as "backmapping" → no exact solution since information is irreversibly lost in CG procedure

Other CG approaches

- Determining effect CG potentials from the potential of mean force is a common strategy

- The PMF can be computed by routes that do not involve directly integrating the mean force (e.g., histograms of r_{com})

- Other related but distinct bottom-up approaches include

    - Force matching [Ercolessi and Adams, EPL 26, 583, 1994]: determine CG potential that minimize residual error with instantaneous energies, forces, and stresses from atomistic simulations

    - Relative entropy method [Shell, J. Chem. Phys. 129, 144108, 2008.]: determine CG potential that maximize overlap with statistical distributions from atomistic simulations

Exercise

- Measure the mean force between two WCA colloids immersed in a bath of smaller WCA depletants

$$u_{WCA}(r_{ij}) = 4\varepsilon \left[\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^{6} \right] + \varepsilon \text{ for } r < 2^{1/6}\sigma_{ij}; \text{ 0 otherwise}$$

$$R_c = 2.5\sigma, R_d = 0.5\sigma, \phi_d = 0.04, kT = 1.0$$

- Compute the mean force as function of r_{ij}
  - $\langle f(r_{ij}) \rangle = \langle f_{WCA}(r_{ij}) \rangle + \langle f_{Dep.}(r_{ij}) \rangle$

- Determine the depletion contribution
  - $\langle f_{Dep.}(r_{ij}) \rangle = \langle f(r_{ij}) \rangle - f_{WCA}(r_{ij})$

- Integrate $\langle f_{Dep.}(r_{ij}) \rangle$ to determine $u_{Dep.}(r_{ij})$

- Compare $u_{Dep.}(r_{ij})$ with AO theory