2018-2019 140.651.01 – Methods in Biostatistics I
Sep 11, 2018
HW 1 Problem 1: hints

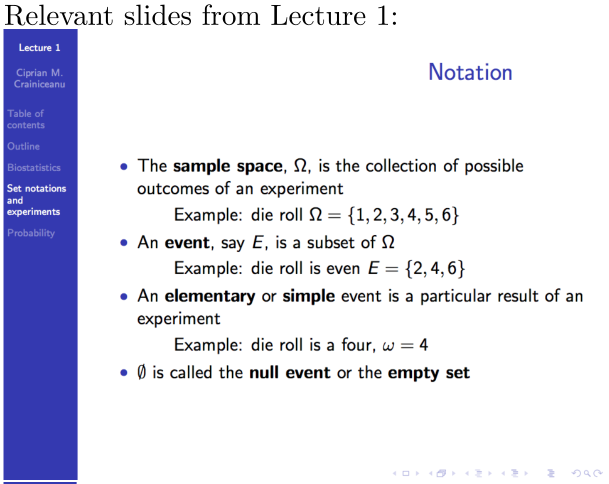
1 Problem 1

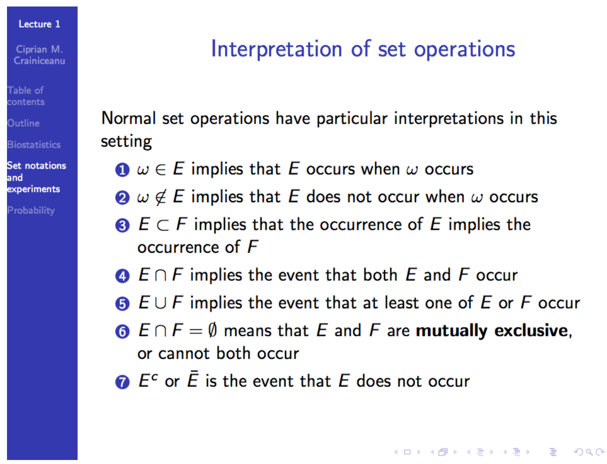
For HW1 Problem 1, using Venn diagram can be useful for seeing how you should proceed with the problem and visualizing it, but **is not enough** to constitute a proof of a problem.

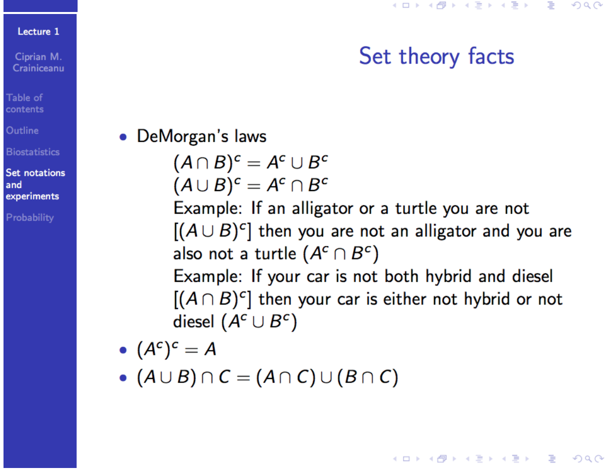
To provide a proof, you may use:

- the set definitions and properties given in Lecture 1,
- the properties of probability measure P given in Lecture 2.

Relevant slides from Lecture 1:







Relevant slides from Lecture 2:

Probability measures

A **probability measure**, P, is a real valued function from the collection of possible events so that the following hold

1. For an event $E \subset \Omega$, $0 \le P(E) \le 1$
2. $P(\Omega) = 1$
3. If E_i, $i = 1, \ldots, \infty$ are mutually exclusive events ($E_i \cap E_j = \emptyset$ for every $i \neq j$)

$$P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$$

This is called **countable additivity**

Lecture 2
Ciprian Crainiceanu
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Probability
Random variables
PMFs and PDFs
CDFs, survival functions and quantiles

Additivity

Part 3 of the definition implies **finite additivity**

$$P(\cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$$

where $\{E_i\}$, $i = 1, \ldots, n$ are mutually exclusive

For $n = 2$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

where $E_1 \cap E_2 = \emptyset$

1.1 Example: 1a

To show $P(\emptyset) = 0$, we proceed:

$$\begin{aligned}
\Omega &= \emptyset \cup \Omega && (1)\\
P(\Omega) &= P(\emptyset \cup \Omega) && (2)\\
P(\Omega) &= P(\emptyset) + P(\Omega) && (3)\\
1 &= P(\emptyset) + 1 && (4)\\
0 &= P(\emptyset) && (5)
\end{aligned}$$

Above is a sufficient proof. Below, we include additional explanation based on set properties and properties of probability measure P.

- Line (3): uses finite additivity of P for mutually exclusive sets $\emptyset$ and Ω
- Line (4): uses property 2. of probability measure P: $P(\Omega) = 1$

NOTE: above we used seemingly "surplus" representation of Ω: $\Omega = \emptyset \cup \Omega$. Using alternative representation / partition of a set into disjoint subsets and then applying finite additivity will be useful for other problems too, see Example below.

1.2 Example: hint for 1c

Note if $A \subset B$, then $B = A \cup (A^c \cap B)$ where A and $(A^c \cap B)$ are **disjoint**.