

$$\begin{aligned}
P(X = x) &= \binom{n_1}{x} p^x (1-p)^{n_1-x} \\
P(Y = z-x) &= \binom{n_2}{z-x} p^{z-x} (1-p)^{n_2-z+x} \\
P(X+Y = z) &= \binom{n_1+n_2}{z} p^z (1-p)^{n_1+n_2-z} \\
P(X = x | X+Y = z) &= \frac{P(X = x, X+Y = z)}{P(X+Y = z)} \\
&= \frac{P(X = x, Y = z-x)}{P(X+Y = z)} \\
&= \frac{P(X = x)P(Y = z-x)}{P(X+Y = z)} \\
&= \frac{\binom{n_1}{x} p^x (1-p)^{n_1-x} \binom{n_2}{z-x} p^{z-x} (1-p)^{n_2-z+x}}{\binom{n_1+n_2}{z} p^z (1-p)^{n_1+n_2-z}} \\
&= \frac{\binom{n_1}{x} \frac{n_2!}{(z-x)!(n_2-z+x)!} p^z (1-p)^{n_1+n_2-z}}{\frac{(n_1+n_2)!}{z!(z-n_1-n_2)!} p^z (1-p)^{n_1+n_2-z}} \\
&= \frac{\binom{n_1}{x} \binom{n_2}{z-x}}{\binom{n_1+n_2}{z}} \text{ hypergeometric}
\end{aligned}$$