BST 140.651 Final Exam

Notes:

- You may not use a calculator for this exam.
- You may use your single formula sheet.
- Please be neat and write legibly. Use the back of the pages if necessary.
- Good luck!

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printed name

1. You simulate 10 variables from a normal distribution with mean 0 and variance 1 and 100 more from a normal distribution with mean 5 and variance 1. You repeat this process (simulating a total of 110 normals) $I = 10,000$ times. Let S_{1i}^2 and S_{2i}^2 and $\bar{X}_{1i}$ and $\bar{X}_{2i}$ be the sample means and variances for sample $i = 1, \ldots, I$, respectively. Answer the following (it is not necessary to solve for final decimal numbers):

   A. About what number will $\frac{1}{I}\sum_{i=1}^{I}(S_{1i}^2 + S_{2i}^2)$ be close to?

   B. Let $D_i = \bar{X}_{2i} - \bar{X}_{1i}$. About what number will $\bar{D} = \frac{1}{I}\sum_{i=1}^{I} D_i$ be close to?

   C. About what number will $\frac{1}{I-1}\sum_{i=1}^{I}(D_i - \bar{D})^2$ be close to?

2. You glue together a quarter, nickel, penny and dime (in that order) to obtain a funny shaped coin with a head on the small side and a tail on larger one. You claim that the coin is fair while a friend claims that it should have probability of a head of 25%. Your friend flips the coin 5 times to obtain 2 heads and 3 tails. Write out a number that would compare the relative evidence of the two hypotheses. (You do not need to calculate the final number, simply plug into the relevant equations.)

3. The Poisson mass function is for a random count of events for a process having been monitored for a fixed (non-random) time t is given by:

$$\frac{(\lambda t)^x \exp(-\lambda t)}{x!} \quad \text{for} \quad x = 0, 1, \ldots.$$

Suppose that $x_1, \ldots, x_N$ are independent counts of events with associated monitoring times $t_1, \ldots, t_N$. Argue that the maximum likelihood estimate of λ is

$$\hat{\lambda} = \frac{\sum_{i=1}^N x_i}{\sum_{i=1}^N t_i}.$$

4. Gray matter brain volume in middle aged men of a certain population is normally distributed with a mean of $1,000$cc with a standard deviation of 80cc. Answer the following (solve for the final numbers)

   A. For a randomly drawn subject from this population, what is the probability of him having a brain volume larger than $1,120$cc?

   B. For a sample of 64 men from this population, what is the probability that their sample average brain volume is below $1,011$cc?

5. Consider the previous problem. In a new population, a sample of 9 men yielded a sample average brain volume of $1,100$cc and a standard deviation of 30cc. Give and interpret a 95% interval for the mean brain volume in this new population? (Solve for a final interval simplifying calculations as needed.)

6. A friend is study hypertension and wants to estimate the prevalence (percentage of people) having hypertension in a specific population using a 95% Wald interval on a sample of n subjects
$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$
How large would n have to be to have the margin of error ($1/2$ the width of the confidence interval) no larger than $.01$ regardless of the value of $\hat{p}$.

7. Suppose that $\hat{\theta}$ is an estimator of population parameter θ. Moreover, assume that $\frac{\hat{\theta}-\theta}{SE_{\hat{\theta}}}$ is standard normally distributed for large n, where $SE_{\hat{\theta}}$ is the standard error of $\hat{\theta}$. Let $Z_{1-\alpha/2}$ be the $1-\alpha/2$ quantile from the standard normal distribution. Argue that

$$\hat{\theta} \pm Z_{1-\alpha/2} SE_{\hat{\theta}}$$

is a confidence interval for θ with coverage probability $1-\alpha$.