#### Lecture 20

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 $\mathsf{Outline}$ 

The delt

Derivation o the delta method

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- 1 Review two sample binomial results
- 2 Delta method

Derivation o the delta method

## Two sample binomials results

Recall  $X \sim \text{Bin}(n_1, p_1)$  and  $Y \sim \text{Bin}(n_2, p_2)$ . Also this information is often arranged in a 2×2 table:

$$\begin{array}{c|cccc} n_{11} = x & n_{12} = n_1 - x & n_1 \\ n_{21} = y & n_{22} = n_2 - y & n_2 \\ \end{array}$$

$$\bullet \hat{RD} = \hat{p}_1 - \hat{p}_2$$

$$\hat{SE}_{\hat{RD}} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

• 
$$\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2}$$

$$\hat{SE}_{\log \hat{RR}} = \sqrt{\frac{(1-\hat{p}_1)}{\hat{p}_1 n_1} + \frac{(1-\hat{p}_2)}{\hat{p}_2 n_1}}$$

• 
$$\hat{OR} = \frac{\hat{p}_1/(1-\hat{p}_1)}{\hat{p}_2/(1-\hat{p}_2)} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

$$\hat{SE}_{\log \hat{OR}} = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$$

$$CI = Estimate \pm Z_{1-\alpha/2}SE_{Fst}$$

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The delta

Derivation of the delta method

- delta method can be used to obtain large sample standard errors
- Formally, the delta methods states that if

$$rac{\hat{ heta}- heta}{\hat{SE}_{\hat{ heta}}} 
ightarrow \mathrm{N}(0,1)$$

then

$$\frac{f(\hat{\theta}) - f(\theta)}{f'(\hat{\theta})\hat{SE}_{\hat{\theta}}} \to N(0,1)$$

- Asymptotic mean of  $f(\hat{\theta})$  is  $f(\theta)$
- Asymptotic standard error of  $f(\hat{\theta})$  can be estimated with  $f'(\hat{\theta})\hat{SE}_{\hat{\theta}}$

Derivation of the delta method

- $\theta = p_1$
- $\hat{ heta}=\hat{p}_1$
- $\hat{SE}_{\hat{\theta}} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}}$
- $f(x) = \log(x)$
- f'(x) = 1/x
- $\frac{\hat{ heta}- heta}{\hat{SE}_{\hat{ heta}}} 
  ightarrow \mathrm{N}(0,1)$  by the CLT
- Then  $\hat{SE}_{\log \hat{p}_1} = f'(\hat{\theta})\hat{SE}_{\hat{\theta}}$

$$=rac{1}{\hat{
ho}_1}\sqrt{rac{\hat{
ho}_1(1-\hat{
ho}_1)}{n_1}}=\sqrt{rac{(1-\hat{
ho}_1)}{\hat{
ho}_1n_1}}$$

And

$$\frac{\log \hat{p}_1 - \log p_1}{\sqrt{\frac{(1-\hat{p}_1)}{\hat{p}_1 p_1}}} \to \mathrm{N}(0,1)$$

Derivation of the delta method

# Putting it all together

Asymptotic standard error

$$\begin{aligned}
\operatorname{Var}(\log \hat{R}R) &= \operatorname{Var}\{\log(\hat{p}_{1}/\hat{p}_{2})\} \\
&= \operatorname{Var}(\log \hat{p}_{1}) + \operatorname{Var}(\log \hat{p}_{2}) \\
&\approx \frac{(1-\hat{p}_{1})}{\hat{p}_{1}n_{1}} + \frac{(1-\hat{p}_{2})}{\hat{p}_{2}n_{2}}
\end{aligned}$$

- The last line following from the delta method
- The approximation requires large sample sizes
- The delta method can be used similarly for the log odds ratio

Derivation of the delta method • If  $\hat{\theta}$  is close to  $\theta$  then

$$\frac{f(\hat{\theta}) - f(\theta)}{\hat{\theta} - \theta} \approx f'(\hat{\theta})$$

So

$$\frac{f(\hat{\theta}) - f(\theta)}{f'(\hat{\theta})} \approx \hat{\theta} - \theta$$

Therefore

$$\frac{f(\hat{\theta}) - f(\theta)}{f'(\hat{\theta})\hat{SE}_{\hat{\theta}}} \approx \frac{\hat{\theta} - \theta}{\hat{SE}_{\hat{\theta}}}$$