#### Lecture 19

Ciprian M Crainiceanu

contents

Outline

Relative measures

The relative

The odds ratio

## Lecture 19

## Ciprian M Crainiceanu

Department of Biostatistics Johns Hopkins Bloomberg School of Public Health Johns Hopkins University

September 8, 2020

# Table of contents

Outlin

measures

risk

⊺he odds rati

- 1 Table of contents
- 2 Outline
- 3 Relative measures
- 4 The relative risk
- **5** The odds ratio

#### Lecture 19

Ciprian M Crainiceanu

Table of contents

Outline

Relative measures

The relative risk

- 1 Define relative risk
- Odds ratio
- 3 Confidence intervals

risk

The odds rati

# Motivation

- Consider a randomized trial where 40 subjects were randomized (20 each) to two drugs with the same active ingredient but different expedients
- Consider counting the number of subjects with side effects for each drug

	Side		
	Effects	None	total
Drug A	11	9	20
Drug B	5	15	20
Total	16	14	40

# Comparing two binomials

- Let  $X \sim \operatorname{Binomial}(n_1, p_1)$  and  $\hat{p}_1 = X/n_1$
- Let  $Y \sim \operatorname{Binomial}(n_2, p_2)$  and  $\hat{p}_2 = Y/n_2$
- We also use the following notation:

$n_{11} = X$	$n_{12}=n_1-X$	$n_1 = n_{1+}$
$n_{21}=Y$	$n_{22}=n_2-Y$	$n_2 = n_{2+}$
<i>n</i> <sub>2+</sub>	$n_{+2}$	

risk

The odds ratio

- Last time, we considered the absolute change in the proportions, what about relative changes?
- Relative changes are often of more interest than absolute, eg when both proportions are small
- The **relative risk** is defined as  $p_1/p_2$
- The natural estimator for the relative risk is

$$\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{X/n_1}{Y/n_2}$$

• The standard error for  $\log \hat{RR}$  is

$$\hat{SE}_{\log \hat{R}R} = \left(\frac{(1-p_1)}{p_1 n_1} + \frac{(1-p_2)}{p_2 n_2}\right)^{1/2}$$

 Exponentiate the resulting interval to get an interval for the RR The relative risk

The odds ratio

The odds ratio is defined as

$$\frac{\mathsf{Odds}\;\mathsf{of}\;\mathsf{SE}\;\mathsf{Drug}\;\mathsf{A}}{\mathsf{Odds}\;\mathsf{of}\;\mathsf{SE}\;\mathsf{Drug}\;\mathsf{B}} = \frac{p_1/(1-p_1)}{p_2/(1-p_2)} = \frac{p_1(1-p_2)}{p_2(1-p_1)}$$

• The sample odds ratio simply plugs in the estimates for  $p_1$  and  $p_2$ , this works out to have a convenient form

$$\hat{OR} = \frac{\hat{\rho}_1/(1-\hat{\rho}_1)}{\hat{\rho}_2/(1-\hat{\rho}_2)} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

(cross product ratio)

• The standard error for  $\log \hat{OR}$  is

$$\hat{SE}_{\log \hat{O}R} = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$$

 Exponentiate the resulting interval to obtain an interval for the OR The odds ratio

### Some comments

- Notice that the sample and true odds ratios do not change if we transpose the rows and the columns
- For both the OR and the RR, taking the logs helps with adherence to the error rate
- Of course the interval for the log RR or log OR is obtained by taking

Estimate 
$$\pm Z_{1-\alpha/2}SE_{Estimate}$$

- Exponentiating yields an interval for the OR or RR
- Though logging helps, these intervals still don't perform altogether that well

Table of contents

Outlin

measures

risk

- For the relative risk,  $\hat{p}_A = 11/20 = .55$ ,  $\hat{p}_B = 5/20 = .25$
- $\hat{RR}_{A/B} = .55/.25 = 2.2$
- $\hat{SE}_{\log \hat{R}R_{A/B}} = \sqrt{\frac{1-.55}{.55 \times 20} + \frac{1-.25}{.25 \times 20}} = .44$
- Interval for the log RR:  $\log(2.2) \pm 1.96 \times .44 = [-.07, 1.65]$
- Interval for the RR: [.93, 5.21]

Table of contents

Relative

measures
The relati

• 
$$\hat{OR}_{A/B} = \frac{11 \times 15}{9 \times 5} = 3.67$$

• 
$$\hat{SE}_{\log \hat{O}R_{A/B}} = \sqrt{\frac{1}{11} + \frac{1}{9} + \frac{1}{5} + \frac{1}{15}} = .68$$

• Interval for log OR: 
$$\log(3.67) \pm 1.96 \times .68 = [-.04, 2.64]$$

The odds ratio

• For the risk difference

$$\hat{RD}_{A-B} = \hat{p}_A - \hat{p}_B = .55 - .25 = .30$$

• 
$$\hat{SE}_{\hat{R}D_{A-B}} = \sqrt{\frac{.55 \times .45}{20} + \frac{.25 \times .75}{20}} = .15$$

• Interval:  $.30 \pm 1.96 \times .15 = [.15, .45]$ 

Table of

Outlin

measure

The relative

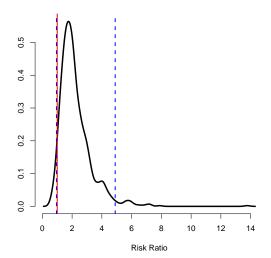


Table of

Outlin

measures

The relative

