



JOHNS HOPKINS  
BLOOMBERG SCHOOL  
of PUBLIC HEALTH

## Lecture 11

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### Finish Model Checking

### Implementation of WLS and robust variance estimation in R

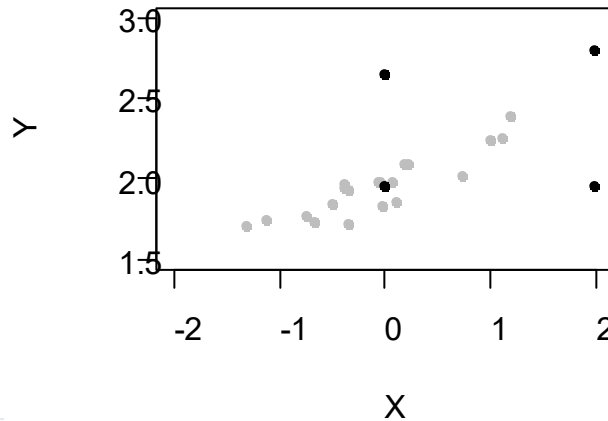
# Key Assumptions by Order of Importance

- ▶  $E(Y|X) = X\beta$ , i.e. the mean model is “correctly” specified
  - ▶ Misspecification of  $X\beta$  can lead to biased  $\beta$  / misinterpretations
  - ▶ Omitted variable Bias
  - ▶ Correct functional form for continuous  $X$
- ▶ Residuals are independent
  - ▶ This assumption is violated due to the design of the study
  - ▶ Longitudinal study
  - ▶ Clustered design
  - ▶ Show today: ignoring the correlation will impact  $Var(\hat{\beta})$  and derive weighted least squares
- ▶ Variance of residuals is constant
  - ▶ Often the variance is a function of some  $X$
  - ▶ Show today: same impact and solution as violation of independence
- ▶ Residuals are normally distributed
  - ▶ CLT, bootstrap procedure
- ▶ There are not a small number of highly influential observations
  - ▶ Sensitivity analyses



# Leverage and Influence

- ▶ Leverage: A measure of how far an individual's predictors ( $X_i$ ) are from the mean  $X_i$ 
  - ▶ Hat matrix:  $h_{ii} = \frac{(X_i - \bar{X})^2}{\sum (X_i - \bar{X})^2}$
- ▶ Influence: An observation ( $Y_i, X_i$ ) such that including this value would greatly change the fitted values:  $\hat{\beta}$  and  $\hat{Y}$ .



# Influence statistics

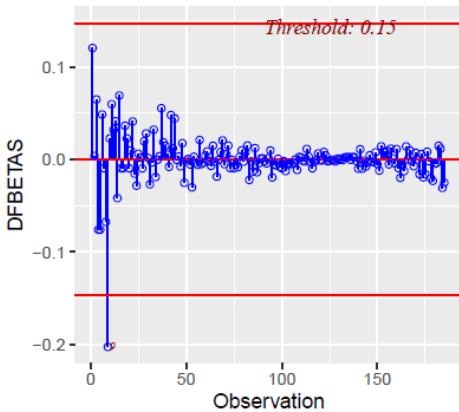
There are several influence statistics that are used in practice:

- $DBETA_{ij} = \hat{\beta}_j - \hat{\beta}_{j(-i)}$
- $DBETAS_{ij} = \frac{DBETA_{ij}}{\hat{se}(\hat{\beta}_{j(-i)})}$
- $DFIT_i = \hat{Y}_i - \hat{Y}_{i(-i)}$
- $DFITS_i = \frac{DFIT_i}{\hat{se}(\hat{Y}_{i(-i)})}$

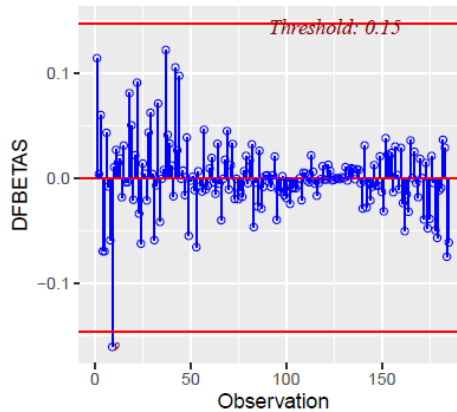


# Example: Nepali Anthropometry Data

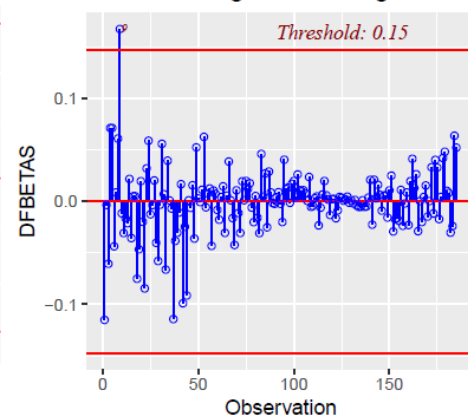
Influence Diagnostics for (Intercept)



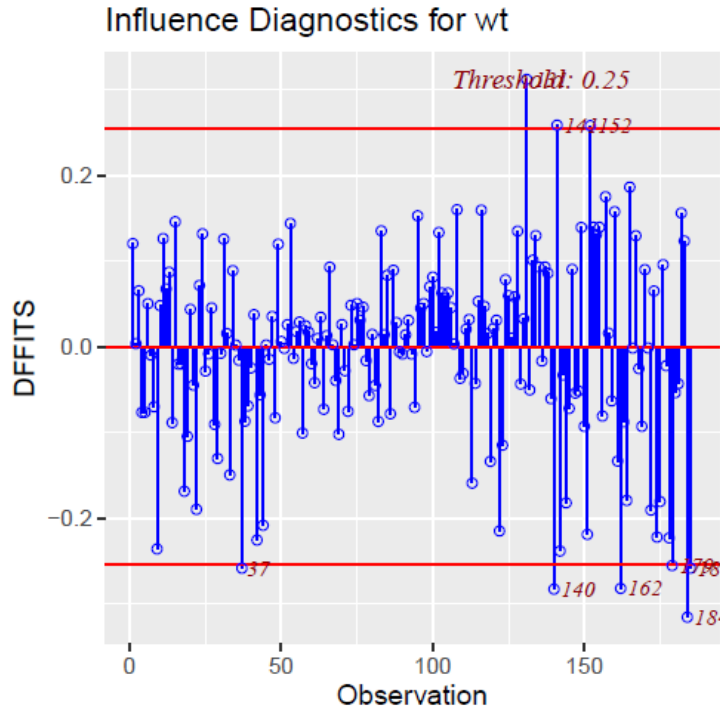
Influence Diagnostics for agesp6



Influence Diagnostics for age



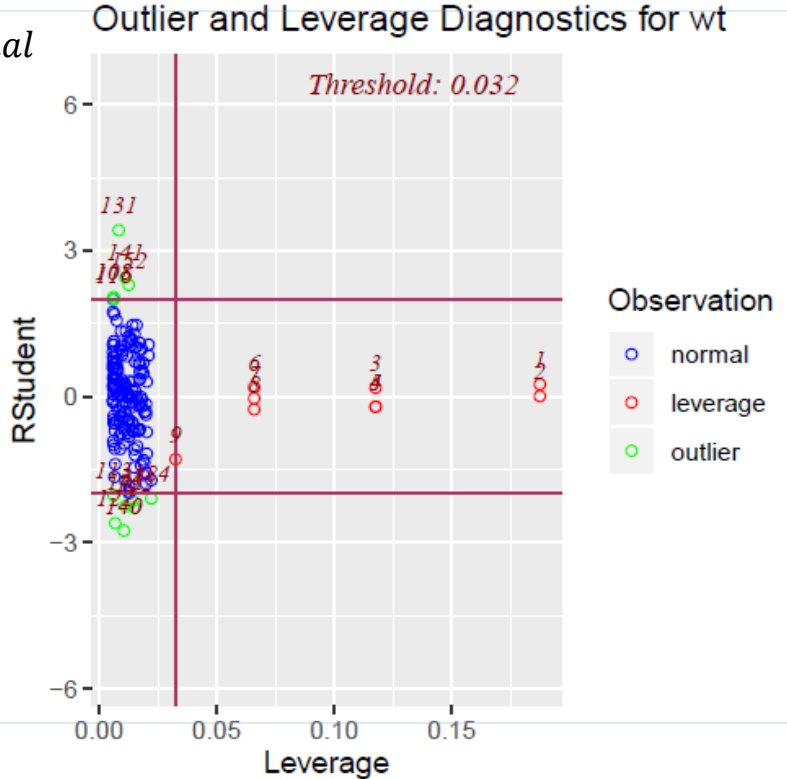
# Example: Nepali Anthropometry Data



## Example: Nepali Anthropometry Data

- $RStudent$ : studentized residual

$$\text{Studentized Residual} = \frac{y_i - \hat{y}_{(i)}}{se(y_i - \hat{y}_{(i)})} = \frac{y_i - \hat{y}_i}{\sqrt{MSE_{(i)}(1 - h_{ii})}}$$



# Implementation of WLS in R

- ▶ For the remainder of the lecture, we will work through some analyses to demonstrate how to fit WLS in R
- ▶ In addition, I will show one approach for obtaining robust variance estimates for different working correlation models. Here we will use the *gee* package in R.
- ▶ See Handout 11.

