Lecture 5

The classical linear regression model

Review of key concepts from Lecture 3 and 4

- Simple linear regression model
  - ARM = B0 + B1 (age – 6) + e, $e \sim N(0,\sigma^2)$, independent

Review of key concepts from Lecture 3 and 4

- Sex adjusted relationship between ARM and age
  - ARM = B0 + B1 (age – 6) + B2 Female + e, $e \sim N(0,\sigma^2)$, independent

Review of key concepts from Lecture 3 and 4

- Height adjusted relationship between ARM and age
  - ARM = B0 + B1 (age – 6) + B2 (HT – 62) + e, $e \sim N(0,\sigma^2)$, independent

Review of key concepts from Lecture 3 and 4

- Effect modification: Is the ARM vs. age relationship the same or different by sex
  - ARM = B0 + B1 (age – 6) + B2 Female + B3 (age – 6) Female + e, $e \sim N(0,\sigma^2)$, independent

Multiple Linear Regression Model

- Y is a random variable representing the outcome of interest in the population
- The explanatory variables, X_1, X_2, …, X_p are fixed/known (not random or measured with error)
- Sample of size n is observed, data are:

$$Y_i = \mu_i(\beta, X_i) + \varepsilon_i$$

- X is the design matrix
- X_i is the row of the design matrix corresponding to subject i

Multiple Linear Regression Model

$$Y_i = \mu_i(\beta, X_i) + \varepsilon_i$$

- Systematic component:
  - $\mu_i(\beta, X_i)$
- ε_i is the random components:
- The least squares solution finds the values of β that minimize:

Least squares solution: simple linear regression

Maximum likelihood inference in MLR

- Start with the MLR:

- Other notation:

Likelihood function definition

- Model:

- Probability density function:

- Likelihood function:

Maximum likelihood estimation under gaussian residuals

- Likelihood function

Maximum likelihood estimation under gaussian residuals

- Log Likelihood Function

Maximum likelihood estimation under gaussian residuals

- Solution for β_j

Maximum likelihood estimation under gaussian residuals

- Solution for β_j

Maximum likelihood estimation under gaussian residuals

- Solution for σ^2

MLEs for simple linear regression

MLEs for simple linear regression

MLEs for simple linear regression

Take away messages

Take away messages

Next time….

- Vector / Matrix representation of MLR
- Geometry of least squares
- Distribution of MLEs for regression parameters