

Lecture 7

In-class exercise

Using the HS&B data,

1. Specify a linear mixed model to estimate the school-specific relationship between math achievement and SES after adjusting for student gender and minority status
2. Fit the model and interpret the random slope for SES variance component

For now, let's ignore all the ranking questions that I posed in the original document; we will get back to these when we talk about hospital rankings.

Y_{ij} = math achievement for student j from school i

$\left. \begin{array}{l} \text{ses}_{ij} \\ \overline{\text{ses}_i} \end{array} \right\}$ will group mean center SES

$\left. \begin{array}{l} \text{female}_{ij} \\ \text{minority}_{ij} \end{array} \right\}$ assume no contextual effect of gender or minority status

Student level model:

$$Y_{ij} = \beta_{0i} + \beta_{1i} (\text{ses}_{ij} - \overline{\text{ses}_i}) + \beta_2 \text{female}_{ij} + \beta_3 \text{minority}_{ij} + \epsilon_{ij},$$

$\epsilon_{ij} \sim N(0, \sigma^2)$

School-level model:

$$\beta_{0i} = \beta_0 + b_{0i}, \quad b_{0i} \sim N(0, \tau_0^2)$$

$$\beta_{1i} = \beta_1 + b_{1i}, \quad b_{1i} \sim N(0, \tau_1^2)$$

$$\text{Cov}(b_{0i}, b_{1i}) = \tau_{12}$$

Mixed-effects ML regression
Group variable: newid

Number of obs = 7,042
Number of groups = 156

Obs per group:

min = 14
avg = 45.1
max = 67

Log likelihood = -22766.002

Wald chi2(3) = 588.48
Prob > chi2 = 0.0000

mathach	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
centeredSES	1.929271	.1238028	15.58	0.000	1.686623	2.17192
female	-1.212396	.1665529	-7.28	0.000	-1.538834	-.8859589
minority	-3.063004	.2117642	-14.46	0.000	-3.478054	-2.647953
_cons	14.185	.235785	60.16	0.000	13.72287	14.64713

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
newid: Unstructured				
var(center~S)	.4779385	.254601	.1682408	1.357728
var(_cons)	6.172218	.8055378	4.779151	7.971349
cov(center~S,_cons)	-.4232893	.335151	-1.080173	.2335946
var(Residual)	35.71282	.6152384	34.52711	36.93926

LR test vs. linear model: chi2(3) = 731.88 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

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For the average school, the expected math achievement score is 1.93 points greater per standard deviation increase in student SES after adjusting for student gender and minority status.

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Our model allows for the school level gender and minority status adjusted relationship between math achievement and SES to vary from school to school (i.e. the school specific slopes). We estimate that roughly 95% of the schools will have slopes that range from $1.93 \pm 1.96 \sqrt{0.48} = 0.57$ to 3.29