

Summary of Lectures 5+6 on Centering in Multi-level Models (MLMs)

$$Y_{ij} = \underbrace{\beta_0 + b_{0i}}_{\beta_{0i}} + \beta_1 X_{ij} + \varepsilon_{ij}$$

$\varepsilon_{ij} \stackrel{\text{iid}}{\sim} G(0, \sigma^2)$
 $b_{0i} \stackrel{\text{iid}}{\sim} G(0, \tau^2)$
 $\varepsilon_{ij} \perp b_{0i}$

Decompose $X_{ij} = X_{ij} - \bar{X}_{..} + \bar{X}_{..}$

GRAND MEAN

$$\begin{aligned} Y_{ij} &= \beta_0 + b_{0i} - \beta_1 \bar{X}_{..} + \beta_1 (X_{ij} - \bar{X}_{..}) \\ &= (\beta_0 - \beta_1 \bar{X}_{..}) + b_{0i} + \beta_1 (X_{ij} - \bar{X}_{..}) \end{aligned}$$

$$= \beta_0^* + b_{\bar{x}_j} + \beta_1(x_j - \bar{x}_{..})$$

Centering with grand mean changes
the intercept only; same plot
except for center of x-axis

Centering with $\bar{x}_{i\cdot}$

$$x_{ij} = x_{ij} - \bar{x}_{i\cdot} + \bar{x}_{i\cdot} \quad \text{cluster mean}$$

$$\begin{aligned} Y_{ij} &= \beta_0 + b_{0i} + \beta_1(x_{ij} - \bar{x}_{i\cdot} + \bar{x}_{i\cdot}) + \varepsilon_{ij} \\ &= \underbrace{\beta_0 + b_{0i}}_{\text{within cluster effect of } x} + \underbrace{\beta_1 W(x_{ij} - \bar{x}_{i\cdot})}_{\text{between cluster effect of } x} + \underbrace{\beta_1 B \bar{x}_{i\cdot}}_{\text{between cluster effect of } x} + \varepsilon_{ij} \end{aligned}$$

Contextual effect : effect of context
($\bar{x}_{i\cdot}$) for individuals with the same value
of $\underline{\boxed{x_{ij}}}$!!

A

$$Y_{ij} = \beta_0 + b_{0i} + \beta_{1W}(x_{ij} - \bar{x}_{i\cdot}) + \beta_{1B}\bar{x}_{i\cdot} + \epsilon_{ij}$$

same value
of $x_{ij} - \bar{x}_{i\cdot}$,
not x_{ij}

Instead use:

$$Y_{ij} = \beta_0 + b_{0i} + \beta_1 x_{ij} + \beta_2 \bar{x}_{i\cdot} + \epsilon_{ij}$$

contextual effect

$$\textcircled{1}: Y_{ij} = \beta_0 + b_{0i} + \beta_{1W} \bar{X}_{ij} - \beta_{1B} \bar{X}_{i0} + \beta_{2B} \bar{X}_{i0} + \epsilon_{ij}$$
$$= \beta_0 + b_{0i} + \beta_{1W} \bar{X}_{ij} + (\beta_{1B} - \beta_{1W}) \bar{X}_{i0} + \epsilon_{ij}$$

$$\beta_2 = (\beta_{1B} - \beta_{1W}) \bar{X}_{i0}$$

Contextual effect

Compositional Effect

"when inter-group (inter-context) differences in the outcome (Y_{ij}) are attributable to differences in group (i) composition (ie. in the characteristics) of the individuals (Z_{ij}) that comprise the group (i)

"inter-context differences \Rightarrow estimate of contextual effect

are attributable to \Rightarrow contextual effects are reduced when Z_{ij} included in model

Objectives in Homework

II. does the composition of the school
(ethnicity, gender, SES) explain the
observed variation in outcome across
schools ?

I. to understand the variation in
average math achievement across
schools

III. do school-level variables account for
variation in Y_i across schools ?

$$Y_{ij} = \underbrace{\beta_0 + b_{0i}}_{\beta_{0i}} + \epsilon_{ij}$$

$$\epsilon_{ij} \sim G(0, \sigma^2)$$

$$b_{0i} \sim G(0, \tau^2)$$

I. What measures variation across schools in average \bar{Y}_{ij} ? _____
 Estimate _____ from data to answer I.

II. If size of variation across schools explained by ethnicity, gender, SES differences in children across schools ?

$$Y_{ij} = \beta_0 + b_{0i} + \beta_E (E_{ij} - \bar{E}_{..}) + \beta_G (G_{ij} - \bar{G}_{..}) \\ + \beta_{SES} (SES_{ij} - SES_{..}) + \epsilon_{ij}$$

Compare $\hat{\tau}_I^2$ with $\hat{\tau}_{II}^2$

Center by Grand or School mean?

III: Do school-level variables explain size of
 $\hat{\tau}_{II}^2$?

Model II + $\beta_{PAT} (PAT_i - PAT_o) +$
 $\beta_{\text{SMM}} (\% \text{Min}_i - \overline{\% \text{Min}_o}) +$

$\Rightarrow \hat{P}_{\underline{II}}^2$ - compare to $\hat{P}_{\underline{II}}^1$ and $\hat{P}_{\underline{I}}^2$

- BUT -

- "Contextual effect - vs
compositional effects
- brief google scholar view

$$Y_{ij} = \beta_0 + b_{0i} + \beta_{1W}(X_{ij} - \bar{X}_{..}) + \beta_{1C}(\bar{X}_{i..} - \bar{X}_{..}) \\ + \beta_Z(Z_{ij} - \bar{Z}_{..}) + \varepsilon_{ij}$$

Compare $\hat{\beta}_{1C}$ to $\hat{\beta}_{1C}^* | \beta_Z = 0$

Lecture 7

Linear Mixed Effects Models

-Review or
Short
Overview

- Extension of traditional linear model
- Some subset of the regression parameters vary randomly across subjects (schools, clusters, “independent units”)
- Mean response is modeled as
 - Fixed effects: shared characteristics of the entire population
 - Subject (“Cluster”)-specific effects: unique to individuals
- The variation among clusters in regression parameters is how we induce correlation structure into the model
 - All observations from the same subject share the same cluster-specific regression parameter(s) creating a link/correlation

Linear Mixed Effects Models

cluster

- In addition, these randomly varying or subject-specific parameters
 - Allow us to distinguish between different sources of variation in the data
 - Variation in responses at baseline across individuals
 - Variation in rate of change of responses across individuals
 - Random variation in the measurement process within an individual over time

cluster

clusters

cluster

Linear Mixed Effects Models

- Benefits of these models:
 - Partitioning variance into between vs. within subject variation or more generally assigning variation to different levels or combinations of levels
 - Flexible in terms of handling imbalance: in number of observations per person and variation in measurement times
 - Modeling non-constant variance
 - Are valid given common missing data models
 - Prediction:
 - Can predict/describe population mean trajectories
 - Can predict individual trajectories

cluster

Linear Mixed Effects Models

$$Y_i = X_i \beta + Z_i b_i + \varepsilon_i$$

X_i is the design matrix for the fixed population - level effects

β is the vector of population - level association parameters

Z_i is the design matrix for the subject - specific or random effects

b_i is the vector of subject - specific parameters

$b_i \sim N(0, G)$, G is some covariance matrix

$\varepsilon_i \sim N(0, \sigma_\varepsilon^2 R)$, R is some correlation matrix

Assume b_i and ε_i are independent.

Random Intercept Model

$$Y_{ij} = \beta_0 + \beta_1 t_{ij} + b_i + \varepsilon_{ij}$$

$$= \beta_0 + b_{0i} + \beta_1 t_{ij} + \varepsilon_{ij}$$

$$= \beta_0 + \beta_1 t_{ij} + b_i + \varepsilon_{ij}$$

$$\varepsilon_{ij} \sim G(0, \sigma^2)$$

$$b_{0i} \sim G(\mu_0, \tau^2)$$

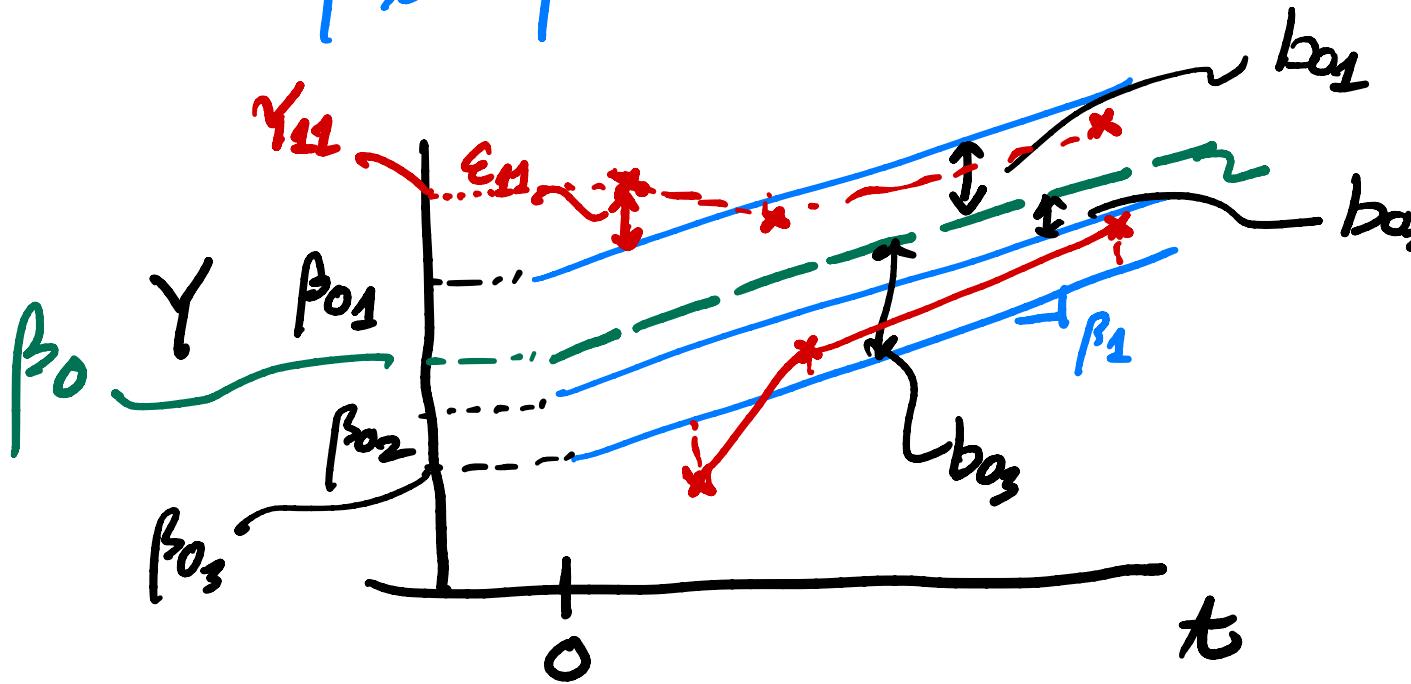
NOTE: remainder b_i are independent of ε_{ij} , simplest case is to assume ε_{ij} independent

Random Intercept Model

- Two-stage random effects formulation

$$Y_{ij} = \beta_{0i} + \varepsilon_{ij}$$

$$\beta_{0i} = \beta_0 + b_{0i}, \quad b_{0i} \sim G(0, \tau^2)$$



Mean Responses

cluster

- Subject-specific mean:

$$E(Y_{ij}|b_i) = \beta_0 + \beta_1 t_{ij} + b_i$$

- Population-level mean:

$$E(Y_{ij}) = E[E(Y_{ij}|b_i)] = E[\beta_0 + \beta_1 t_{ij} + b_i] = \beta_0 + \beta_1 t_{ij}$$

Variance Estimates

- Variance $= E\{(\beta_0 + \beta_1 t_{ij} + b_i + \varepsilon_{ij})^2\}$

$$\begin{aligned} Var(Y_{ij}) &= Var(\beta_0 + \beta_1 t_{ij} + b_i + \varepsilon_{ij}) \\ &= Var(b_i + \varepsilon_{ij}) = \sigma^2_b + \sigma^2 \end{aligned}$$
- Covariance $= E(b_i^2) + E(\varepsilon_{ij}^2) + 2\text{cov}(b_i, \varepsilon_{ij}) =$

$$\begin{aligned} \text{Cov}(Y_{ij}, Y_{ik}) &= \text{Cov}(b_i + \varepsilon_{ij}, b_i + \varepsilon_{ik}) = E\{(b_i + \varepsilon_{ij})(b_i + \varepsilon_{ik})\} \\ &= \text{Cov}(b_i, b_i) = \sigma^2_b = E(b_i^2) + E(b_i \varepsilon_{ij}) + E(b_i \varepsilon_{ik}) \end{aligned}$$
- Correlation $\text{Corr}(Y_{ij}, Y_{ik}) = \frac{\sigma^2_b}{\sigma^2_b + \sigma^2}$

Variance Estimates

- Linear random intercept model
 - Assumes constant variance, independent within subject residuals
 - Exchangeable correlation structure

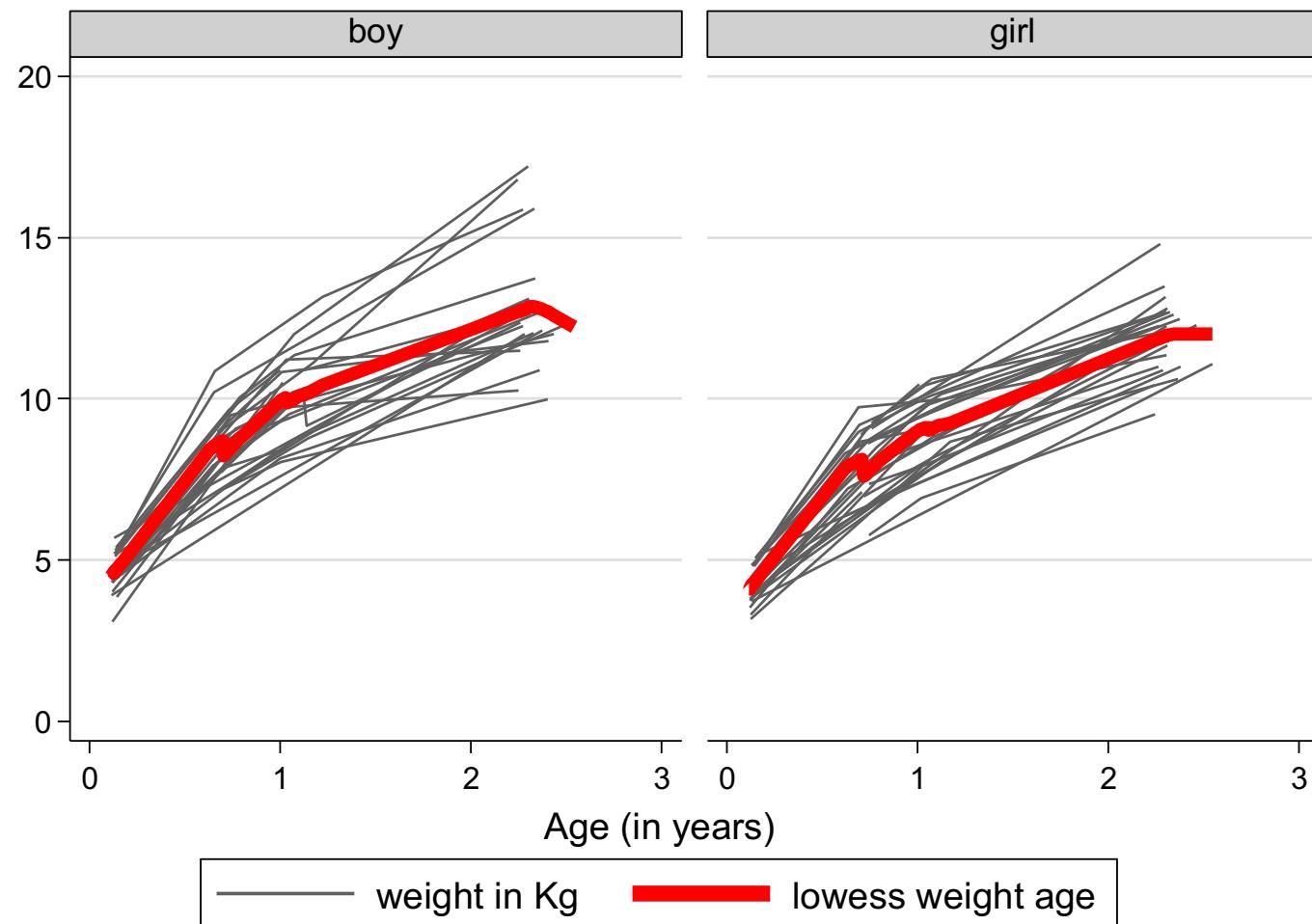
$$\text{Corr}(Y_{ij}, Y_{ik}) = \frac{\sigma^2_b}{\sigma^2_b + \sigma^2}$$

- Within subject correlation can also be interpreted as:
 - The percentage of the total variation in Y that is attributable to differences across subjects relative to natural variation within subject.

Growth-curve modeling

- “asian children weights.dta”
- Measurements of weight were recorded for children up to 4 occasions at roughly 6 weeks, and then at 8,12, and 27 months
- Goal: We want to investigate the growth trajectories of children’s weights as they get older
- Both the shape of the trajectories and the degree of variability are of interest

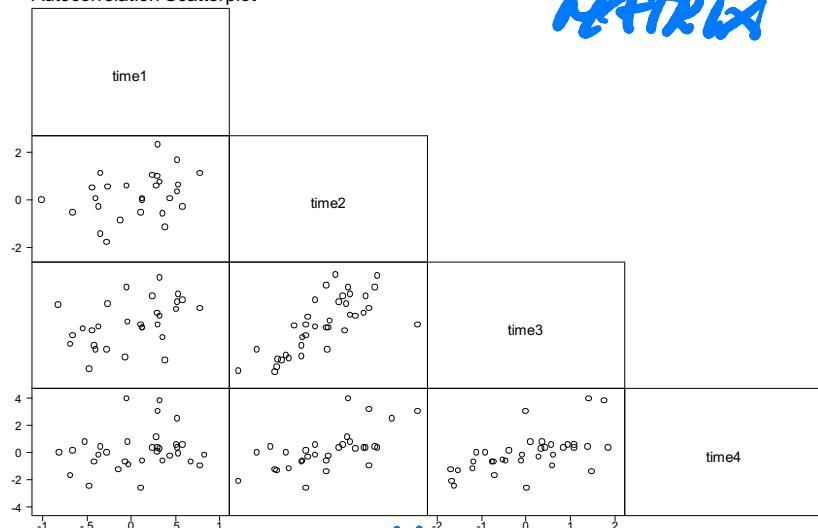
Observed Data



Graphs by gender

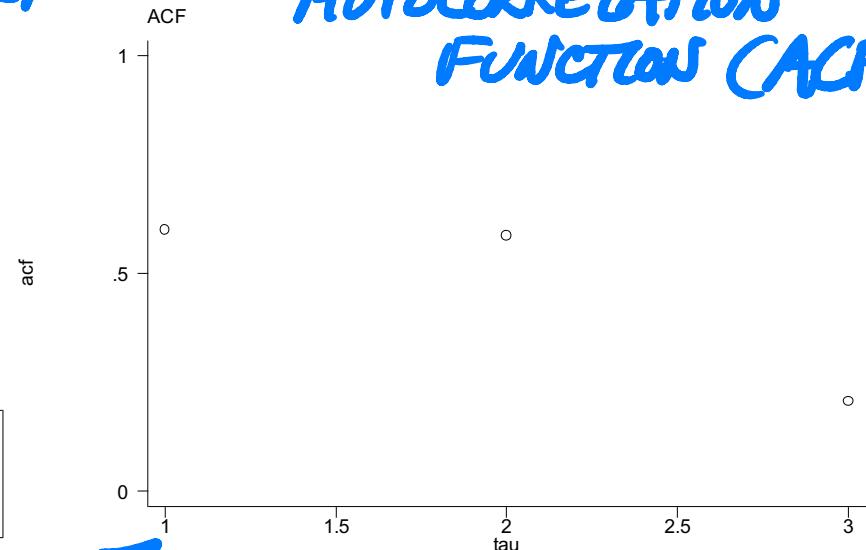
AUTOCORRELATION

Autocorrelation Scatterplot



SCATTERPLOT MATRIX

AUTOCORRELATION FUNCTIONS (ACF)



AUTOCORRELATION MATRIX

autocor wtres visit id

	time1	time2	time3	time4
time1	1.0000			
time2	0.3344	1.0000		
time3	0.4348	0.7388	1.0000	
time4	0.2060	0.6277	0.5938	1.0000

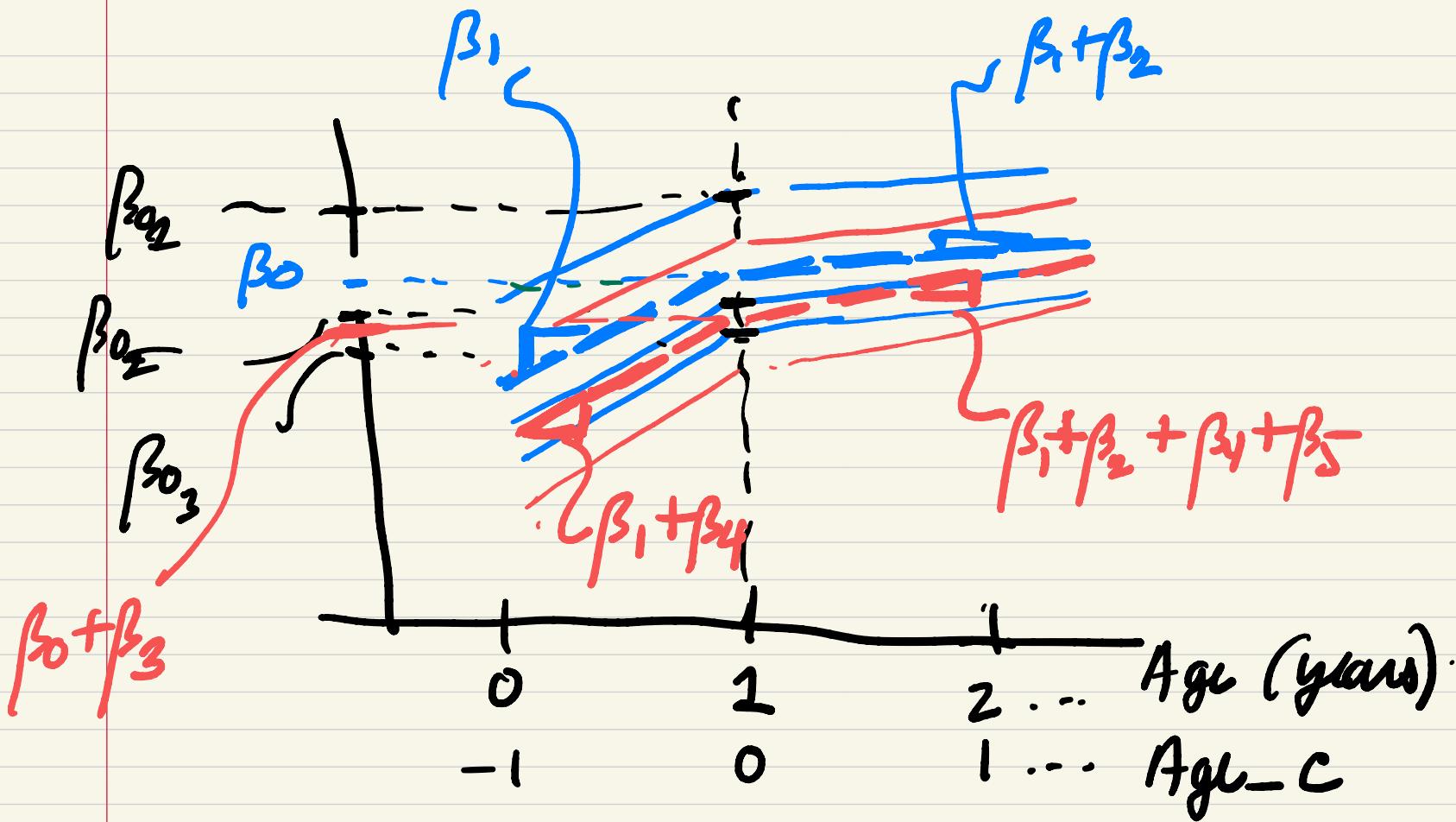
ACF	
1.	.600677
2.	.5878199
3.	.2059693

Random Intercept Model

- A reasonable model would be age, age-squared, gender and the interactions of the age terms and gender.
- Just to keep our model a bit more simple to interpret, we will use a linear spline model with knot at 1 year

```
gen age_c = age - 1  
gen age_sp = (age_c>0)*age_c  
gen age_c_girl = age_c*girl  
gen age_sp_girl = age_sp*girl  
mixed weight age_c age_sp girl age_c_girl age_sp_girl || id: , var
```

$$N_{ij} = \beta_0 + \beta_1(Age_{c_{ij}}) + \beta_2(Age_{c_{ij}})^+ + \beta_3 \underline{\varphi}_i + \\ \beta_4 Age_{c_{ij}} * \underline{\varphi}_i + \beta_5 (Age_{c_{ij}})^+ * \underline{\varphi}_i + \epsilon_{ij}$$



Random Intercept Model

Mixed-effects ML regression	Number of obs	=	189		
Group variable: id	Number of groups	=	68		
	Obs per group: min	=	1		
	avg	=	2.8		
	max	=	4		
	Wald chi2(5)	=	2471.22		
Log likelihood = -265.42862	Prob > chi2	=	0.0000		

weight	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]

age_c	6.466099	.2687824	24.06	0.000	5.939295 6.992903
age_sp	-4.5202	.3805891	-11.88	0.000	-5.266141 -3.774259
girl	-.9660133	.2890176	-3.34	0.001	-1.532477 -.3995493
age_c_girl	-.7398512	.3889635	-1.90	0.057	-1.502206 .0225032
age_sp_girl	.8015376	.5516641	1.45	0.146	-.2797043 1.882779
_cons	10.27181	.2032267	50.54	0.000	9.873495 10.67013

Random Intercept Model

weight	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age_c	6.466099	.2687824	24.06	0.000	5.939295	6.992903
age_sp	-4.5202	.3805891	-11.88	0.000	-5.266141	-3.774259
girl	-.9660133	.2890176	-3.34	0.001	-1.532477	-.3995493
age_c_girl	-.7398512	.3889635	-1.90	0.057	-1.502206	.0225032
age_sp_girl	.8015376	.5516641	1.45	0.146	-.2797043	1.882779
_cons	10.27181	.2032267	50.54	0.000	9.873495	10.67013

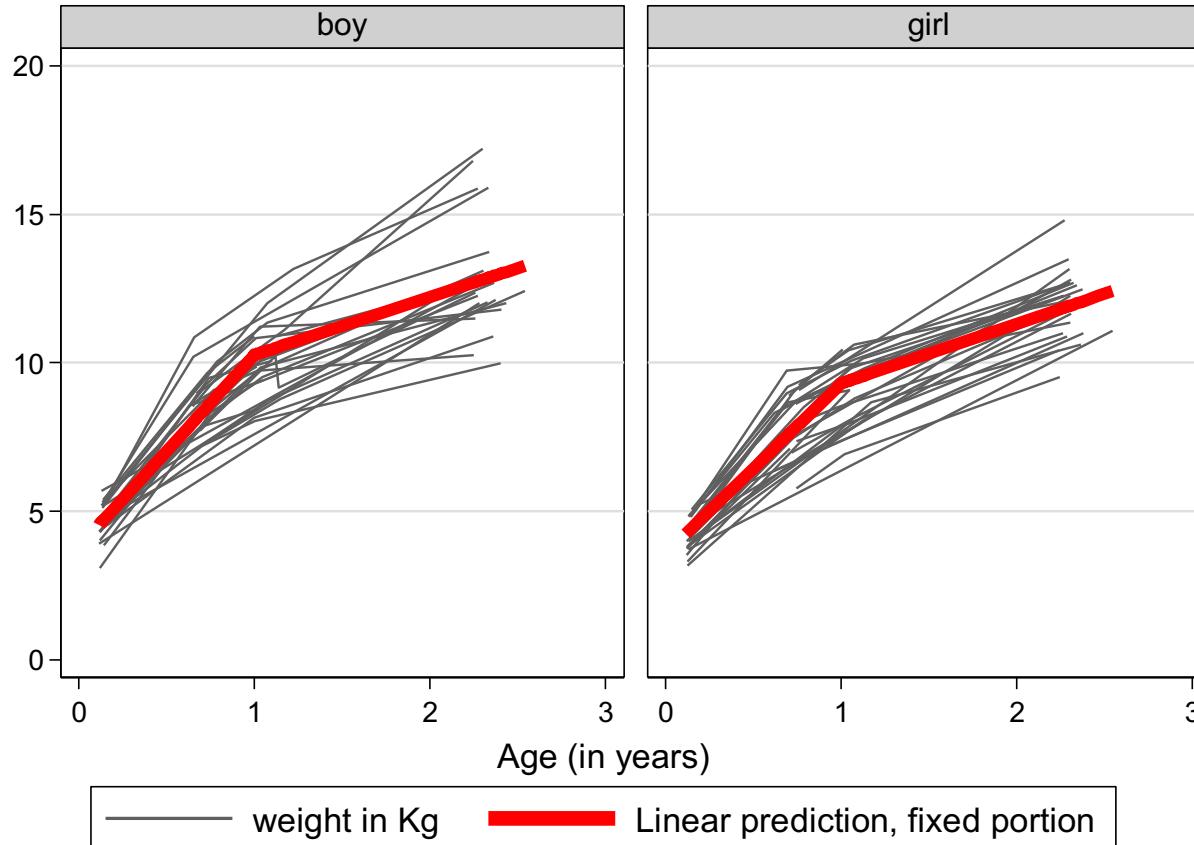
Interpretation:

Random Intercept Model

Random-effects Parameters		Estimate	Std. Err.	[95% Conf. Interval]	
<hr/>					
id: Identity					
<hr/>					
var(_cons)		.7161207	.1608949	.4610435	1.112322
<hr/>					
var(Residual)		.5727228	.0730425	.4460526	.7353648
<hr/>					
LR test vs. linear regression: chibar2(01) = 56.33 Prob >= chibar2 = 0.0000					

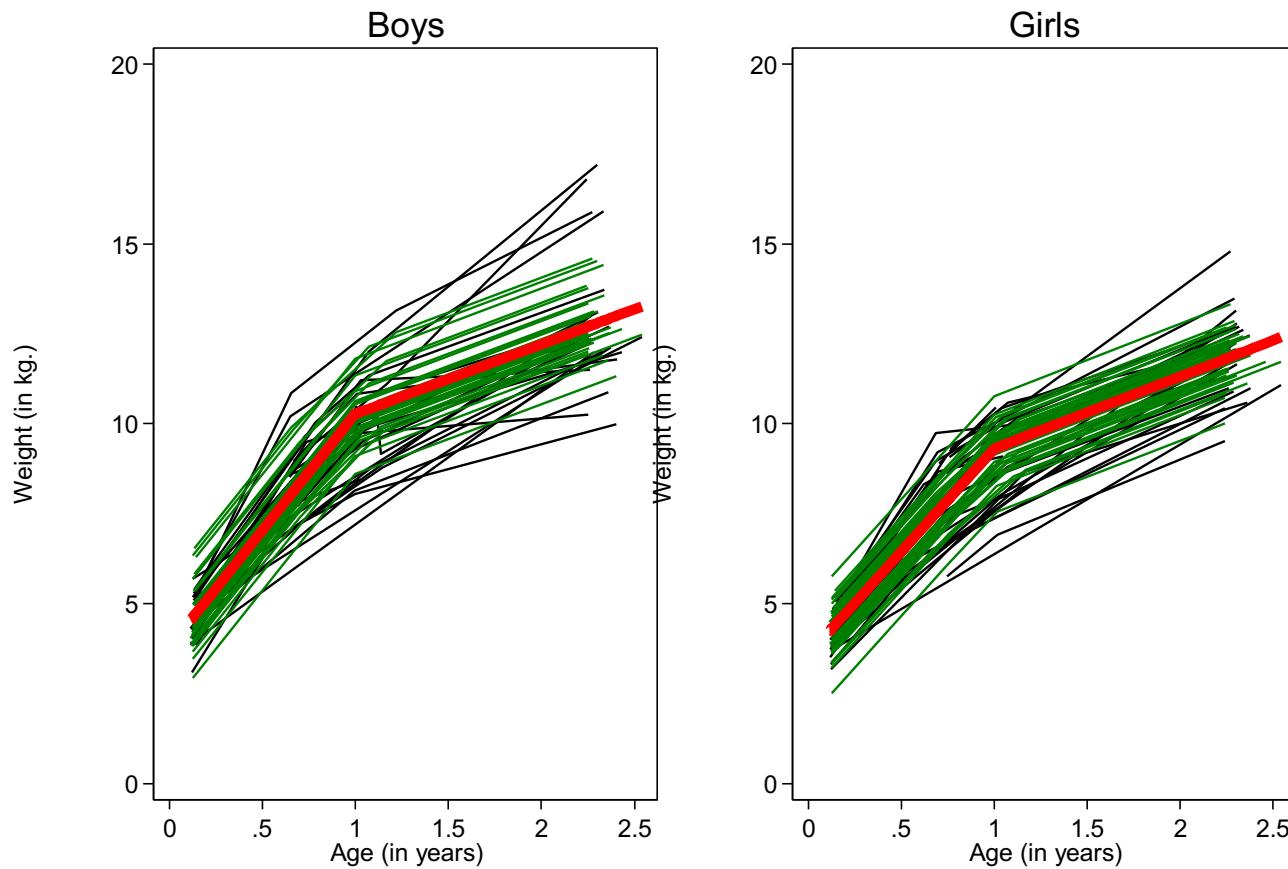
$$\text{Corr}(Y_{ij}, Y_{ik}) = \frac{0.72}{0.72 + 0.57} = 0.56$$

Predicted Mean Growth



Graphs by gender

Predicted Mean vs. Individual Growth



Summary of Lecture 7

- Considered an example of how to fit a marginal model where measurements are unequally spaced
 - This is a common data feature
- Random Effects models
 - In these models, the within subject correlation is generated by subject specific effects (intercept, slopes)
 - This translates to linear regression models where the intercept and slopes are defined separately for each subject

Summary of Lecture 7, Part I

- Random intercept model
 - Assume a single predictor: time, where time = 0 is baseline
 - The random intercept or subject specific intercept is the expected (mean) value of the response for each subject at baseline
 - The model assumes that the mean response will change over time in the same linear fashion for each subject
 - If we assume the within subject residuals are independent, then this model is equivalent to a marginal exchangeable correlation model
 - Allows us to estimate the proportion of variation in the response that is attributable to differences across subjects relative to variation over time within a subject.

Inner-London School Data

- At age 16, students take Graduate Certificate of Secondary Education (GCSE) exams
 - Scores derived from the GCSE are used for schools comparisons
 - However, schools should be compared based upon their “value added”; the difference in GCSE score between schools after controlling for achievements before entering the school
- One measure of prior achievement is the London Reading Test (LRT)
 - taken by these students at age 11
- Goal: to investigate the relationship between GCSE and LRT and how this relationship varies across schools. Also identify which schools are most effective, taking into account intake achievement

(G) (L)

Inner-London School data:

- Outcome: score exam at age 16 (gcse)
- Covariate: reading test score at age 11 prior to enrollment in the school (lrt)
- Data are clustered within schools:
 - 4059 students clustered within 65 schools
 - Level 1: Student
 - Level 2: School

Inner-London School Data

- Analysis Goals:
 - Estimate the school-specific relationship between the exam score at age 16 and the score at age 11
 - Investigate how this association varies across schools
 - Rank the schools in terms of “performance”
 - Is there evidence that gender modifies the association between score exam at age 16 and score at age 11?
 - Modification at level 1: two level 1 predictors interacting
 - Does the type of school (mixed/boys/girls) explain part of the observed variation across schools?
 - A level 2 predictor interacting with a level 1 predictor

Exploratory Data Analysis

* Create some new variables

```
sort school student
```

* Generate the number of students within each school

```
by school: egen totalstudents = count(student)
```

* Generate a counter for the number of students within each school

```
by school: gen withinschoolcount = _n
```

* EDA

* What is the distribution of number of students in each school

```
summ totalstudents if withinschoolcount==1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
totalstude~s	65	62.44615	29.74844	2	198

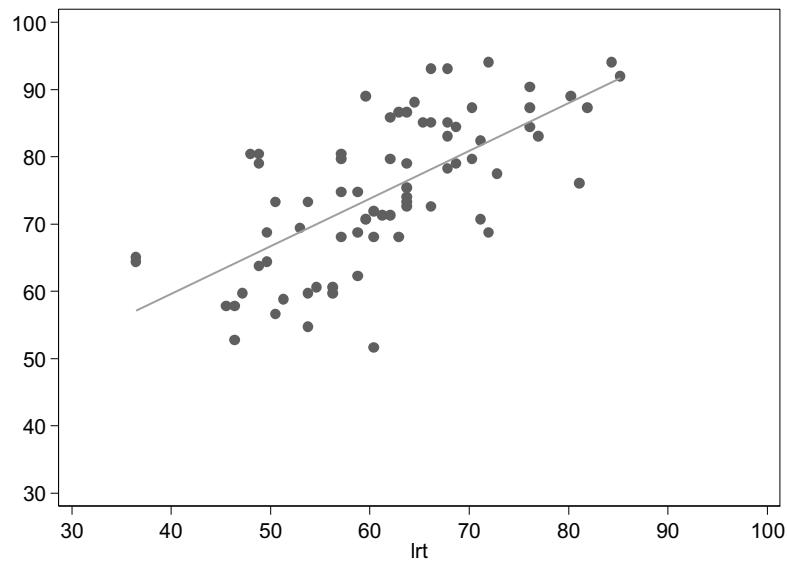
65 schools in the dataset:

Number of students ranges from 2 to 198, average 62

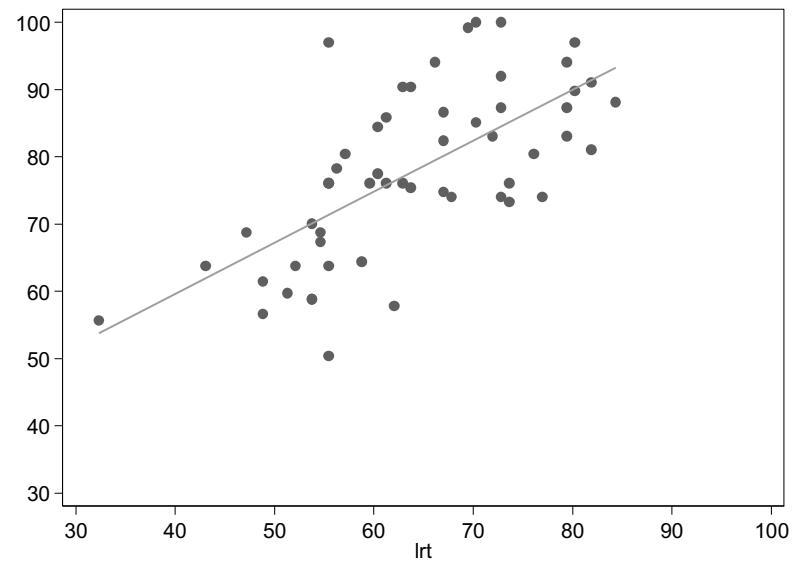
Exploratory Data Analysis

- Relationship between gcse and lrt among two schools (1 and 2)

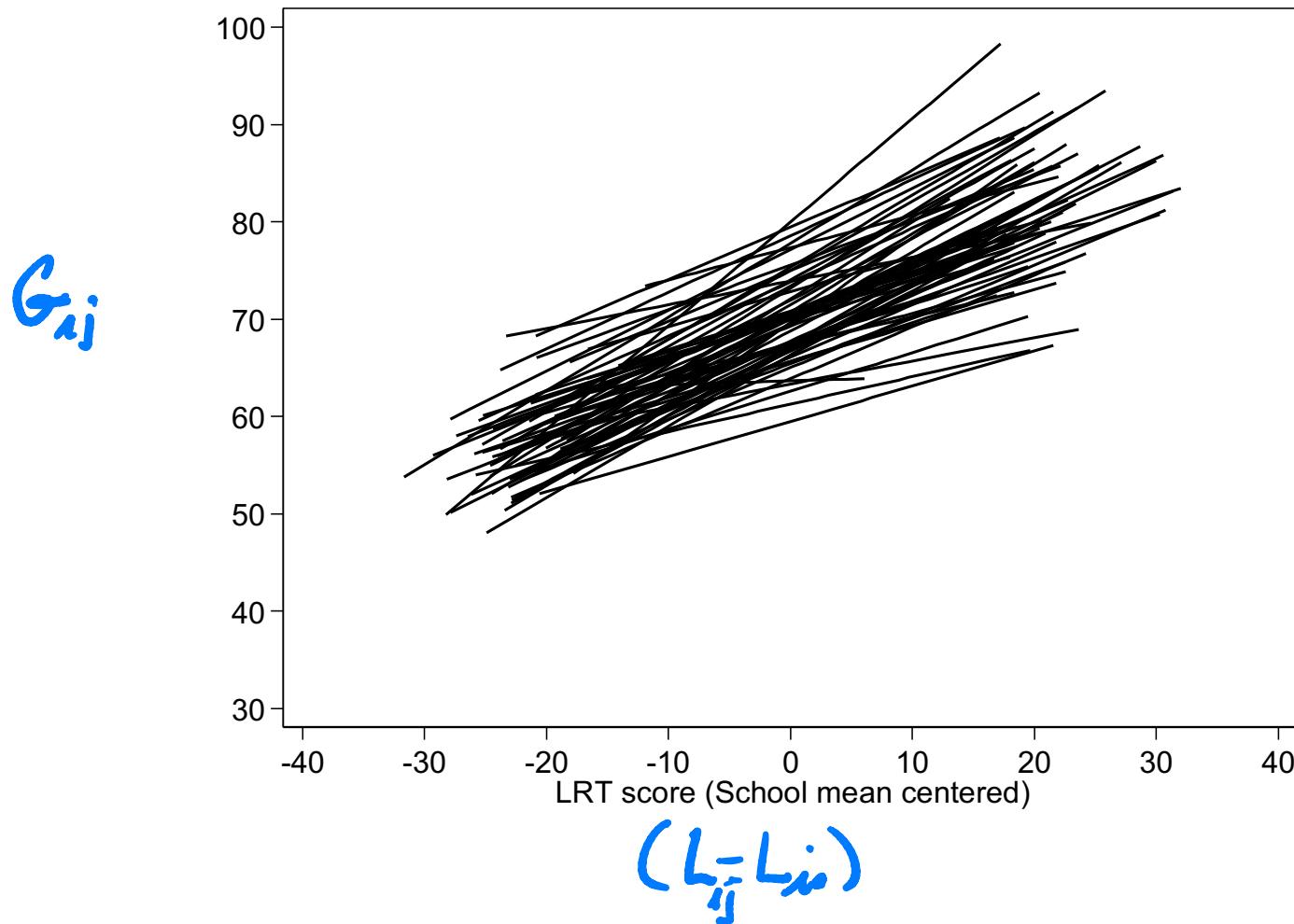
School: 1



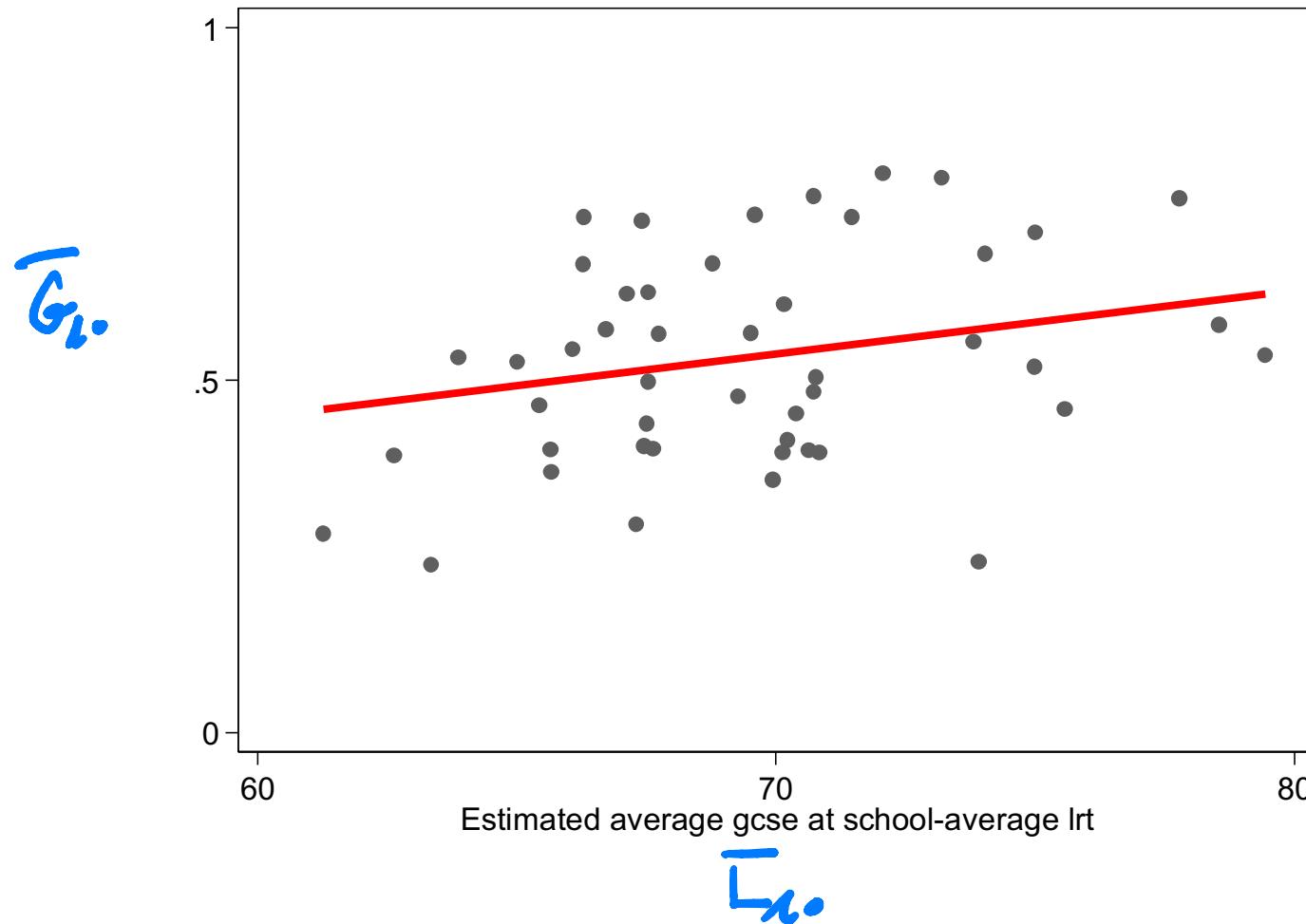
School: 2



School-specific relationships among schools with at least 5 students



Association between school-specific slope and intercept



Linear regression model with random intercept and random slope

Y_{ij} ^(G) gcse for student j in school i

x_{ij} ^(L) lrt for student j in school i

$\bar{x}_{i.}$ average lrt for school i

$$Y_{ij} = \beta_{0i} + \beta_{1i}(x_{ij} - \bar{x}_{i.}) + \varepsilon_{ij}$$

$$= [1, (x_{ij} - \bar{x}_{i.})] \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} +$$

$$[1, (x_{ij} - \bar{x}_{i.})] \begin{pmatrix} \beta_{0i} \\ \beta_{1i} \end{pmatrix} + \varepsilon_{ij}$$

$$= \tilde{x}_{ij} \beta + \tilde{z}_{ij} \beta_i + \varepsilon_{ij}$$

$$\beta_{0i} \sim N(\beta_0, \tau_1^2)$$

$$\beta_{1i} \sim N(\beta_1, \tau_2^2)$$

$$\text{cov}(\beta_{0i}, \beta_{1i}) = \tau_{12}$$

$$\varepsilon_{ij} \sim N(0, \sigma^2)$$

here $z_{ij} := x_{ij}$
 usually $z_{ij} \leq x_{ij}$
 "is contained in"

Alternative representations of the same model

$$Y_{ij} = b_{0i} + \beta_0 + (b_{1i} + \beta_1)(x_{ij} - \bar{x}_{i\cdot}) + \varepsilon_{ij}$$

$$b_{0i} \sim N(0, \tau_1^2)$$

$$b_{1i} \sim N(0, \tau_2^2)$$

$$\text{cov}(b_{0i}, b_{1i}) = \tau_{12}, \varepsilon_{ij} \sim N(0, \sigma^2)$$

$$Y_{ij} = \beta_{0i} + \beta_{1i}(x_{ij} - \bar{x}_{i\cdot}) + \varepsilon_{ij}$$

$$\beta_{0i} = \beta_0 + b_{0i}, b_{0i} \sim N(0, \tau_1^2)$$

$$\beta_{1i} = \beta_1 + b_{1i}, b_{1i} \sim N(0, \tau_2^2)$$

$$\text{cov}(\beta_{0i}, \beta_{1i}) = \tau_{12}, \varepsilon_{ij} \sim N(0, \sigma^2)$$

"MIXED"

Random coefficient models induce heteroskedastic error structure!

○ - random
█ - fixed

$$Y_{ij} = (b_{0i} + \beta_0) + (b_{1i} + \beta_1)(x_{ij} - \bar{x}_{i\cdot}) + \varepsilon_{ij}$$

$$Y_{ij} = (\beta_0 + \beta_1 x_{ij}) + (b_{0i} + b_{1i}(x_{ij} - \bar{x}_{i\cdot})) + \varepsilon_{ij}$$

$$\xi_{ij} = (b_{0i} + b_{1i}(x_{ij} - \bar{x}_{i\cdot})) + \varepsilon_{ij}$$

2 ξ_{ij}

$$\text{var}(\xi_{ij}) = \tau_1^2 + 2\tau_{12}(x_{ij} - \bar{x}_{i\cdot}) + \tau_2^2(x_{ij} - \bar{x}_{i\cdot})^2 + \sigma^2$$

$$\underline{\text{Var } b_{0i}} + 2C_2 \text{Cov}(b_{0i}, b_{1i}) + C_2^2 \text{Var } b_{1i} + \underline{\text{Var } \varepsilon_{ij}}$$

The total residual variance is said to be heteroskedastic because it depends on x

$$\tau_2^2 = \tau_{12} = 0 \quad \text{Model with random intercept only}$$

$$\text{var}(\xi_{ij}) = \tau_1^2 + \sigma^2$$

$$\text{Var}(c_1 Y_1 + c_2 Y_2) \quad \text{when } EY_1 = EY_2 = 0, c_1, c_2 \text{ constants}$$

$$= E\left\{\left[c_1 Y_1 + c_2 Y_2 - E(c_1 Y_1 + c_2 Y_2)\right]^2\right\}$$

$$= E\left\{c_1^2 Y_1^2 + c_2^2 Y_2^2 + 2c_1 c_2 Y_1 Y_2\right\}$$

$$= c_1^2 \text{Var} Y_1 + c_2^2 \text{Var} Y_2 + 2c_1 c_2 \text{Cov}(Y_1, Y_2)$$

```
mixed gcse lrt_groupc || school: lrt_groupc, cov(uns) mle stddev
```

Mixed-effects REML regression
Group variable: school

Number of obs = 4059
Number of groups = 65

Obs per group: min = 2
avg = 62.4
max = 198

Log restricted-likelihood = -14020.718
Wald chi2(1) = 768.50
Prob > chi2 = 0.0000

gcse	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
<hr/>					
lrt_groupc	.5529766	.0199474	27.72	0.000	.5138805 .5920727
_cons	69.83566	.5415451	128.96	0.000	68.77426 70.89707

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]
<hr/>			
school: Unstructured			
sd(lrt_gr~c)	.1202955	.0190615	.0881804 .1641069
sd(_cons)	4.220941	.3997349	3.505889 5.081833
corr(lrt_gr~c,_cons)	.5481782	.13837	.2241939 .7630512
<hr/>			
sd(Residual)	7.432694	.0838333	7.270187 7.598834
<hr/>			
LR test vs. linear regression:	chi2(3) = 835.32	Prob > chi2	= 0.0000

Interpretation of fixed effects

gcse	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]

lrt_groupc	.5529766	.0199474	27.72	0.000	.5138805 .5920727
_cons	69.83566	.5415451	128.96	0.000	68.77426 70.89707

Interpretation of random effects

Random-effects Parameters		Estimate	Std. Err.	[95% Conf. Interval]
school: Unstructured				
sd(lrt_gr~c)		.1202955	.0190615	.0881804 .1641069
sd(_cons)		4.220941	.3997349	3.505889 5.081833
corr(lrt_gr~c, _cons)		.5481782	.13837	.2241939 .7630512
sd(Residual)		7.432694	.0838333	7.270187 7.598834

Interpretation of random effects

Random-effects Parameters		Estimate	Std. Err.	[95% Conf. Interval]
school: Unstructured				
sd(lrt_gr~c)		.1202955	.0190615	.0881804 .1641069
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sd(Residual)		7.432694	.0838333	7.270187 7.598834

$$Y_{ij} = \underbrace{x_{ij}\beta}_{\alpha} + \underbrace{z_{ij}b_i}_{\gamma} + \epsilon_{ij}$$

$$\tilde{Y}_{ii} = \underbrace{x_{ii}\beta}_{n_x x_1} + \underbrace{z_{ii}b_i}_{n_z x_p \text{ per } n_z x_g \text{ g}_x} + \epsilon_{ii}$$

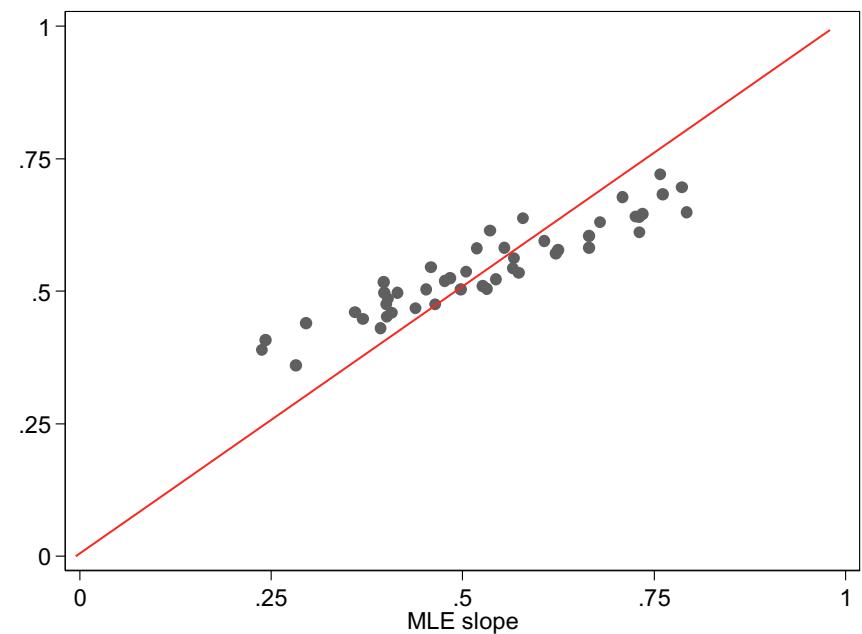
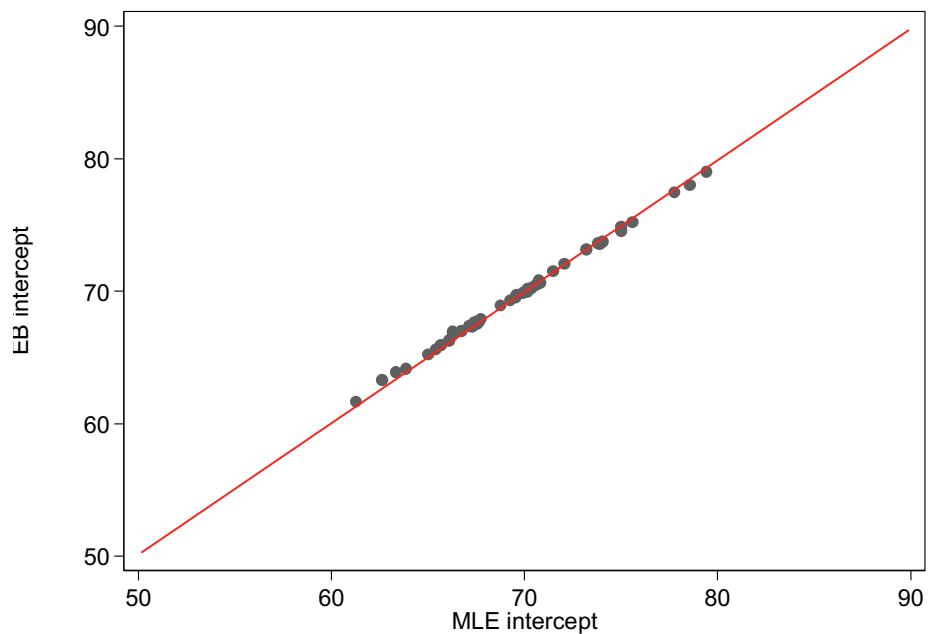
$$Y_{ii} \sim G(x_{ii}\beta, z_{ii}Dz_i'(\sigma^2 I_{n_z})).$$

$$E(b_{ii}|Y_{ii}) = E(b_{ii}) + \text{Cov}(b_{ii}, Y_{ii}) \text{Var}(Y_{ii})^{-1} (Y_{ii} - EY_{ii}).$$

$$\begin{matrix} 2 \times 1 \\ n_z x_1 \end{matrix} = 0 + Dz_i' (z_i Dz_i' + \sigma^2 I)^{-1} (Y_{ii} - x_{ii}\beta)$$

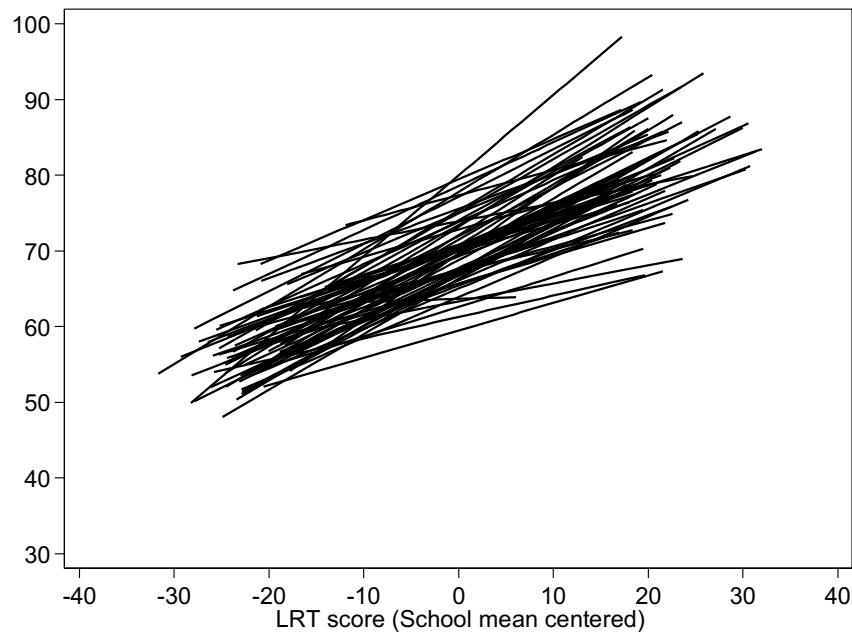
EB-replace D, σ^2, β by their estimates

Impact of shrinkage on intercepts and slopes

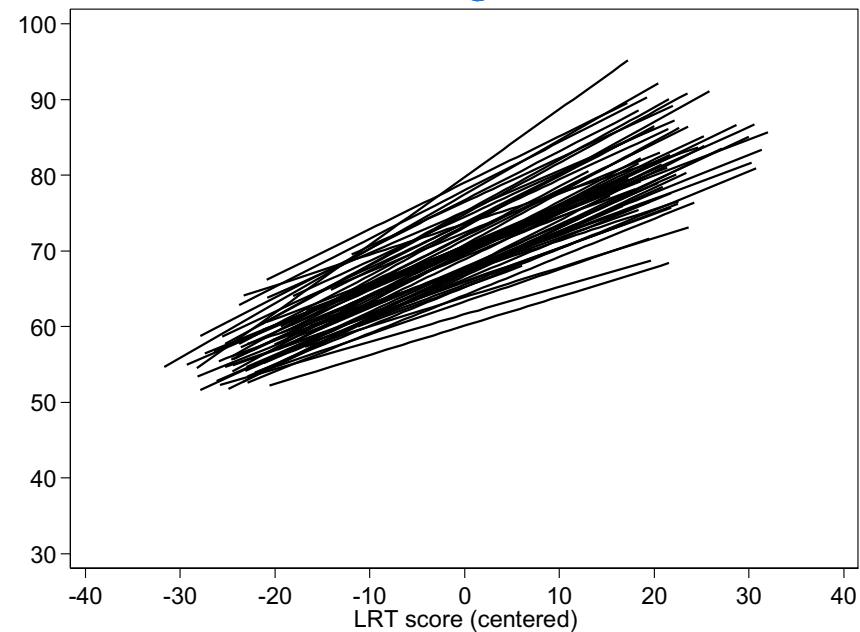


Impact of shrinkage on predicted association between gcse and lrt

individual curves



EB curves



Goal: Rank the schools in terms of performance

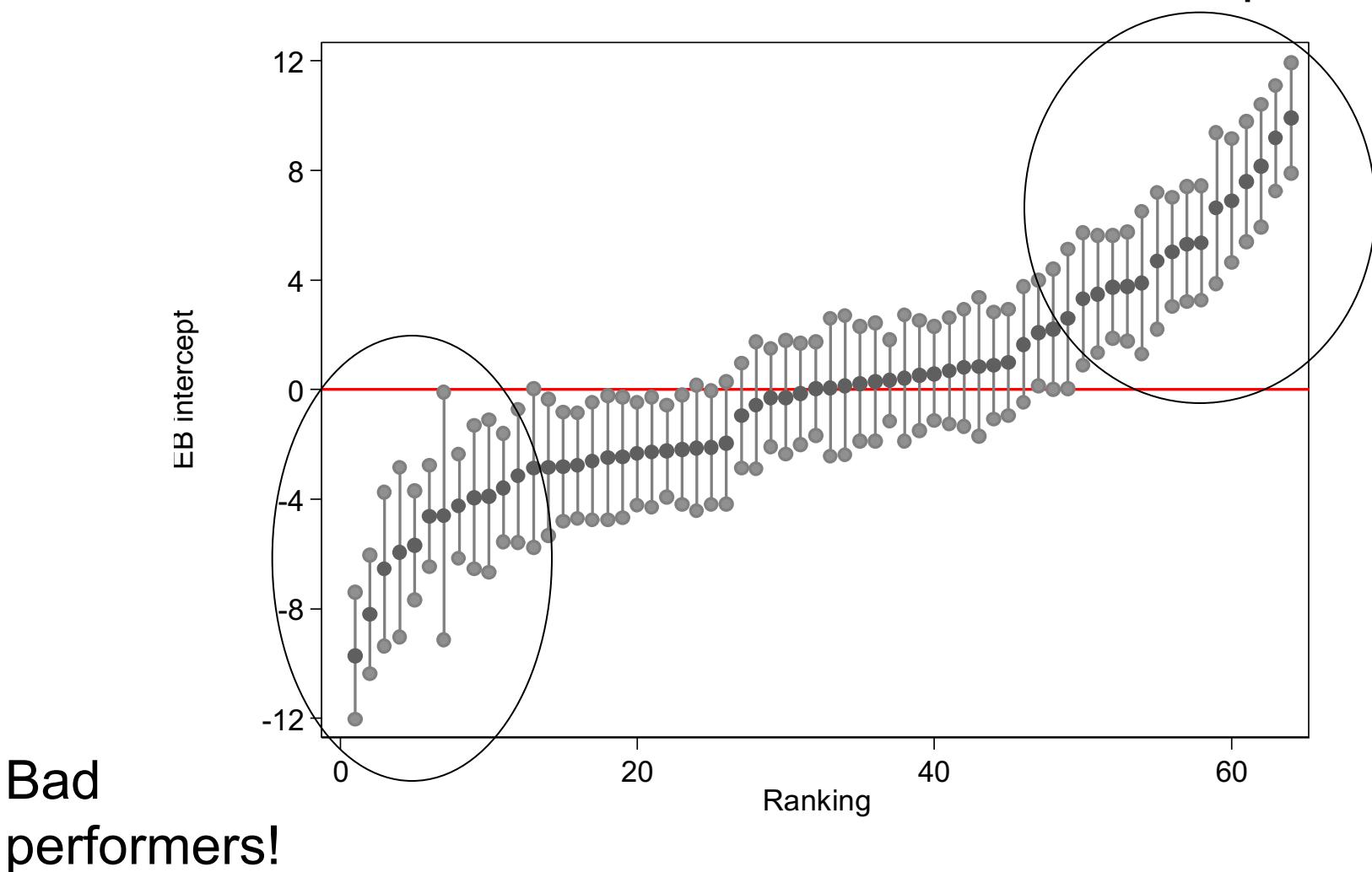
What is the appropriate measure to use from the analysis?

- random intercept?
- random slope?

What is the interpretation of the EB estimates of the intercepts?

School Rankings

Good performers!



Lecture 7 Summary

- Reviewed linear random coefficient models for multilevel data
 - within cluster factor not time
- Since we are working in linear case, coefficients have both a marginal and conditional interpretation
- Random slope models allow for us to understand heterogeneity across clusters in within cluster associations
- Cluster-level summaries allow for ranking of clusters