# To Weight or not To Weight? That is the Question

- Question, Question, Question
- Bias - Variance Trade-off
- Beware highly variable weights
- In regression problems, regress on the weights too rather than weighting by them

# Lecture 11

Incorporating complex survey design information into MLMs

# Lecture 11 Outline

- Stratified Survey
- Cluster Survey
- Define sampling weights
- Analysis of survey data
  - Descriptive inference
    - Design based approach
  - Analytic Inference
    - Design based approach
    - Model based approach
    - What do we do in MLMs?
    - Example of MLM: PISA 2000 US Data
    - My practical recommendations

# Stratified Survey Design: An Example

- Program for International Student Assessment (PISA)
  - Conducted by the National Center for Educational Statistics
  - Coordinated by the Organization for Economic Development
  - System of international assessments that focus on 15-year-olds' capabilities in reading literacy, mathematics literacy, and science literacy.
  - 43 countries participate

# PISA: US

- Data from 2000 US survey
  - We will look at this later as an example
- Each administration focuses on one of the three subject areas in depth but assesses each of the three subject areas
- http://nces.ed.gov/surveys/pisa/index.asp

# PISA: Requirements for representative sample

- Represent the full population of 15-year-old students in each participating country and jurisdiction.
- Population is defined internationally as 15-year-olds attending both public and private schools in grades 7-12.
- Requires a minimum of 4,500 students from a minimum of 150 schools in each participating country and jurisdiction.
- Within schools, a sample of 35 students must be selected in an equal probability sample unless fewer than 35 students age 15 are available (in which case all students are selected).
- The school response rate target is 85 percent for all countries and jurisdictions.
  - A minimum participation rate of 65 percent of schools from the original sample of schools
  - A minimum participation rate of 80 percent of sampled students from schools within each country and jurisdiction

# US Study Design

- *U.S. sampling frame*
  - Common Core of Data (CCD) listing of public schools supplemented with the Private School Universe Survey (PSS) listing of private schools. Close to 100 percent complete listing of schools
- *U.S. sampling design in 2009*
  - Stratified systematic sample
  - First define 8 strata defined by school type (public or private) and region (Northeast, Central, West, Southeast).
    - "Students within strata are homogeneous
    - Schools are sampled from each strata, proportional to size
  - Second, students sampled at random from each school

# Stratified Sample

- Partition the population into *more* homogeneous groups
  - States, ethnicities, education level, SES
- Sample units within each strata
- This design reduces sampling error of strata-specific rates since all groups of interest are included in the sample
- This design will decrease statistical variance, i.e. increases precision *for estimates of strata-specific rates*
- This design is more efficient than a simple random sample *to estimate strata-specific rates*
  - Need smaller total sample size when you stratify relative to using a simple random sample

* over-sample some strata ⇒ less efficient estimate of pop means

# Cluster Sample

- Divide the population of interest into geographic areas or clusters
  - Often times use Census tract as the cluster
- Take a sample of the clusters
- Measurements are taken on a sample within each selected cluster
- WHO 30 x 7 design
  - Randomly select 30 clusters from the list of potential clusters
  - Randomly select 7 interview locations within each cluster
- Persons within clusters are heterogeneous; ideal if cluster means are similar
- Not an "efficient" design
  - Need larger sample size relative to performing a simple random sample of the entire population
  - But this design is convenient!

# Sampling weights

- Reflect the probability of the sampling unit (person, school, etc) of being selected for inclusion in the sample
- Typically presented as $w = 1 / Pr(selection)$
- In PISA 2009 design,
  - Assigned to each school in each strata
  - Roughly proportional to the size of the school
  - Students are selected at random from each school, so sampling weight is same for each student in selected school

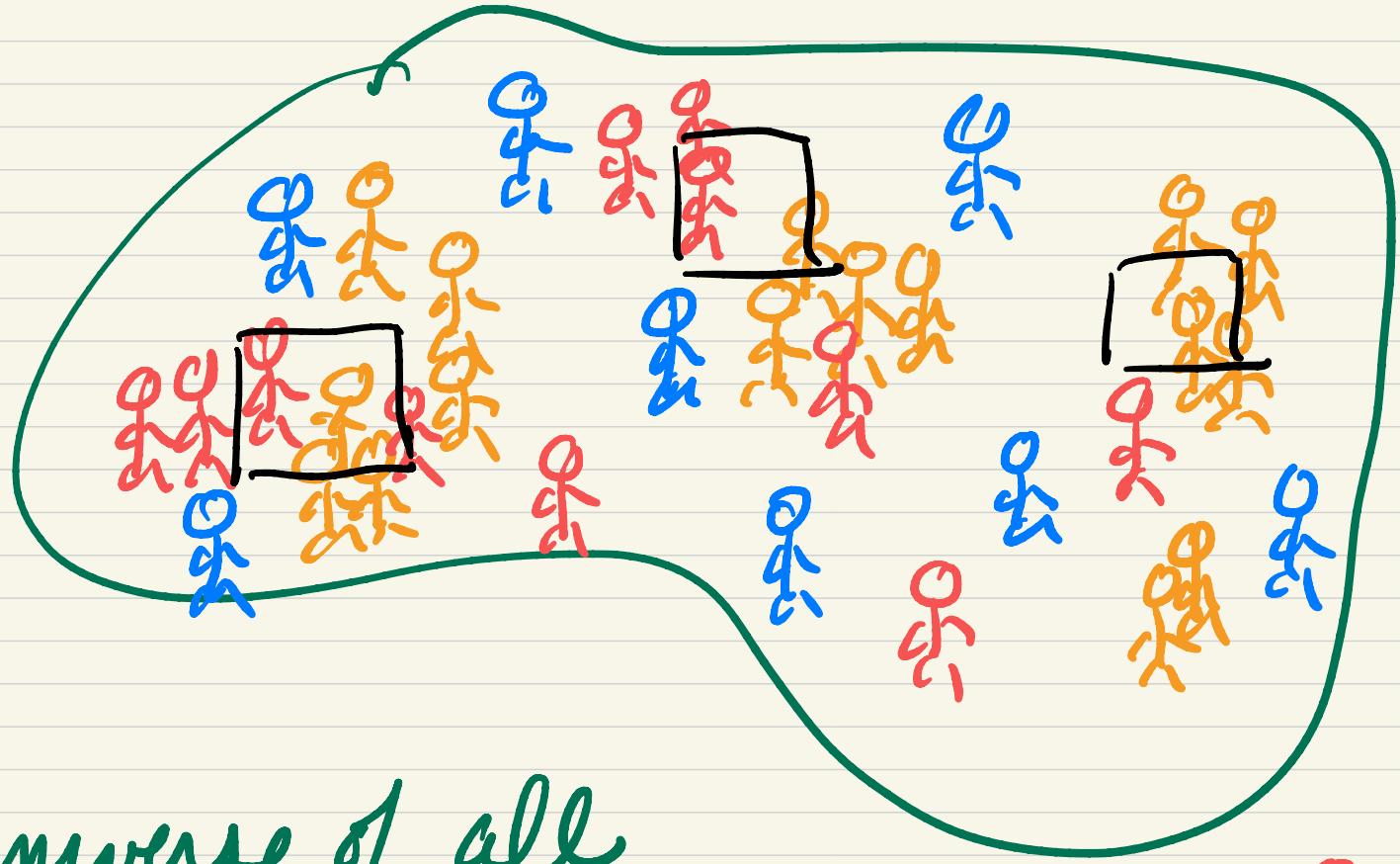
# Sampling Weights: Be Aware!

- Very common that the sampling weights contain additional adjustments!
  - Non-response: school-level, student-level
    - Imputation of item non-response within student assessments
  - Calibrate the sample to external control population
  - Weight trimming, to minimize impact of weights on estimation

wt = sampling wt × adjustment wt for non-response...

# Survey design produces correlated responses!

- Stratified random survey: PISA
  - Within strata (contextual variables for given types of people)
  - Within school correlation (contextual variables at the school level)
- Cluster survey
  - Within cluster (contextual variables based on geographic location)
  - Within person (genetic and temporal associations in individual responses over time)



universe of all people with 3 types :

# Analysis of Survey Data

- Descriptive inference:
  - Most folks agree that the survey design is important to incorporate into descriptive statistics via survey weights
  - Known as “Design-based inference”
  - Variance estimates for descriptive statistics are then also adjusted for the design
- Analytic inference:
  - Hypothesis testing within regression models
  - Quite a bit of debate over the appropriate methods for incorporating the survey design into regression based analysis
  - Design-based inference vs. Model-based inference

# Design-based descriptive inference

- Goal design-based inference:
  - estimate parameters associated with the target population incorporating features of the design
  - "impute" response for the non-sampled subjects
  - results generalize to target population
  - e.g. estimate the proportion of students that achieve reading proficiency in PISA among U.S 15 year olds

$Y_1 + Y_2 + \dots Y_N$ $N$ $n$ $Y_1 + Y_2 \dots Y_n$

in contrast to: are girls more likely than boys to be proficient?

- Observed data and the sampling weights are the foundation of the design-based descriptive inference

# Simple example of a weighted survey

- Assume 4 students from Northeast public schools
- Two schools
- Design a study to estimate total number with proficiency in reading

Q3

One possible probability sample: choose any pair of persons with equal probability

n = 2

| Label | School | Reading Proficiency |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 0 |
| 3 | 1 | 1 |
| 4 | 2 | 1 |
| Known | Known | Unknown |

Truth 3/4 proficient

2

Since we are choosing pairs out of 4 subjects, each selected subject will have probability of 2/4 to be selected, so weight is 2 for each subject.

weight = 1/Probability of being sampled

# Simple Random Sample: n=2 out of 4 total

All possible samples

Known from design

| Persons selected | Measured values | Estimated total | Pr(sample) |
|---|---|---|---|
| 1,2 | 1,0 | 2*1+2*0=2 | 1/6 |
| 1,3 | 1,1 | 2*1+2*1=4 | 1/6 |
| 1,4 | 1,1 | 2*1+2*1=4 | 1/6 |
| 2,3 | 0,1 | 2*0+2*1=2 | 1/6 |
| 2,4 | 0,1 | 2*0+2*1=2 | 1/6 |
| 3,4 | 1,1 | 2*1+2*1=4 | 1/6 |

Total = 1/6 (2+4+4+2+2+4) = 3

Randomization Distribution

# Stratified Sample: 1 from each school

| Persons selected | Measured values | Estimated total | Pr(sample) |
|---|---|---|---|
| 1,4 | 1,1 | 3*1+1*1=4 | 1/3 |
| 2,4 | 0,1 | 3*0+1*1=1 | 1/3 |
| 3,4 | 1,1 | 3*1+1*1=4 | 1/3 |

Total = 1/3 (4+1+4) = 3

Randomization Distribution

# Design-based estimate of total

- Horvitz-Thompson (unbiased and consistent) estimator for the population total
  - Foundation for all design-based analyses

$$\hat{T} = \sum_{i=1}^{n} \frac{1}{\pi_i} Y_i = \sum_{i=1}^{n} w_i Y_i$$

$$Y_i = \begin{cases} 0 \text{ not proficient} \\ 1 \text{ proficient} \end{cases}$$

$$\pi_i = \text{probability that person i is selected in the sample}$$

$$w_i = \frac{1}{\pi_i} = \text{sampling weight for person i}$$

$$n = \text{number of subject sampled}$$

# Design-based estimator of population mean

$$N = \sum_{i=1}^{n} \frac{1}{\pi_i} \text{ is known}$$

$$\hat{T} = \sum_{i=1}^{n} \frac{1}{\pi_i} Y_i$$

$$\overline{Y} = \hat{T} / N$$

Straight-forward to estimate variance of the mean for the population!

# Design-based inference

- Most descriptive statistics of interest can be calculated based on totals/means
- Variance estimates:
  - Can get tricky!
  - Taylor series linearization
    - Well defined theory for all survey designs
    - Requires calculation of partial derivatives which may be hard
  - Replication methods
    - Jackknife methods: leave one out
    - Bootstrap or Balanced Repeated Replication: special case of jackknife when two clusters per strata
    - Easy to implement

# Available Software

- SAS:
  - Proc's SURVEYSELECT, SURVEYMEANS, SURVEYFREQ
- R:
  - "survey" package
  - Then svymean, svytotal, svyby
- Stata:
  - svyset
  - Then svy: mean, proportion, ratio, total

# Analysis of Survey Data

- Descriptive inference:
  - Most folks agree that the survey design is important to incorporate into descriptive statistics via survey weights
  - Known as "Design-based inference" 
  - Variance estimates for descriptive statistics are then also adjusted for the design
- Analytic inference:
  - Hypothesis testing within regression model
  - Quite a bit of debate over the appropriate methods for incorporating the survey design into regression based analysis
  - Design-based inference vs. Model-based inference

Probability refers to chance of getting each possible random sample ⟹ caused by sampling design

Inference based upon set of all hypothetical samples

"Randomization" distribution

# Use of Sampling Weights in Mixed Models for Multilevel Data

Main ideas to this point

1. Sample surveys use clustered designs for practicality.

2. Responses $Y_{ijk}$... are correlated in nature and because of

the design

③ Unbiased estimators of population totals (means) are obtained by weighting sampled responses inversely proportional to probability of inclusion in sample

Horvitz-Thompson Estimator

$$\hat{Y}_{TOTAL} = \sum_{i=1}^{n} \frac{1}{\pi_i} Y_i$$

## ④ Design-based -vs- Model-based inference ("Analytic inference")

# Analytic Inference

- Goal is to specify a model for $f(Y|X,\pi)$ where $w = 1/\pi$ represent the sampling weights (provide information about design).
- When can we ignore the weights/design and use standard regression methods:
  - Okay if design is "non-informative"
    " " $\Longrightarrow$ $f(Y|X,\pi) = f(Y|X)$
  - Produce biased results if design is informative

$$f(Y|X,\pi) = f(Y|X,\pi)$$

if $\pi = \pi(x)$, then $f(Y|X,\pi(x)) = f(Y|X)$

# What to do if design is informative?

- If the design is informative,
  - Design-based approach
    - Specify a model: $f(y_i|x_i)$
    - Then weight observations according to $\pi_i$
    - Weighted regression: Natural extension of descriptive inference based on randomization distribution
  - Model-based approach
    - Incorporate design features into the regression model itself
    - Model $f(y_i|x_i, \pi_i)$ directly
      - i.e. some function of the weights are included as a predictor, as are strata information, etc.
    - Likelihood based approach

# Design-based approach

- Estimators are approximately unbiased and robust to model misspecification, but not design limitations
- Inefficient; larger standard errors than model-based methods
- Hypothesis testing: limited to Wald type tests
- Model diagnostics: must think critically
- Most popular statistical software packages provide appropriate options

# Model-based approach

- Likelihood based estimation
- Efficient; smallest standard errors if model is specified correctly
- Does not require specialized software
- Model diagnostics:  we already know these
- Inference:  likelihood ratio tests
- Results ~~will~~ may be sensitive to model misspecification!
  - Missing key design elements

# Multi-level models for survey data

Consider a simple MLM

- PISA data
- Ignore the strata for now
- Level 1 variable: student
- Level 2 variable: school

$$y_{ij} = \beta_0 + U_{0i} + \varepsilon_{ij}$$

$$U_{0i} \sim N(0, \tau^2)$$

$$\varepsilon_{ij} \sim N(0, \sigma^2)$$

and assume random effects are independent

i denotes the school, j denotes the student and $y_{ij}$ is the reading proficiency score (continuous) for student j within school i

# Recall estimation in mixed models

- Maximize the likelihood

$$L(\beta_0, \sigma^2, \tau^2 \mid Y)$$ What we observe!

$$= \prod_i \int \prod_j f(y_{ij} \mid U_{0i}, \beta_0, \sigma^2) f(U_{0i} \mid \tau^2) dU_{0i}$$

What we assume to be true

In the linear case, there is closed form solution

In non-linear case, there is no-closed form solution.

Requires special estimation procedures

# Design-based mixed models?

- You could include key design features within the model
  - i.e. fixed effects of strata, we are already clustering by school
  - Ignore sampling weights if you think they are non-informative
- **Design-adjusted mixed models**
  - Extension of the design-based methods to mixed models
  - Weighted likelihood approach
    - Pseudo-likelihood

# Sampling weights

- Sampling weights represent the probability of selection (plus additional corrections) and are measured at the various levels within the hierarchy

$$w_{ij} = 1/\pi_{ij}, \pi_{ij} = \pi_i \times \pi_{j|i}$$

$$w_i = 1/\pi_i$$

$$w_{j|i} = 1/\pi_{j|i}$$

- In some cases, we may not have available the selection probability information at each level
  - In many publicly available datasets, we just get the overall weight

# Pseudo-likelihood Approach

$$L(\beta, \sigma^2, \tau^2 \mid Y)$$

$$= \prod_i w_i \int \prod_j w_{j|i} f(y_{ij} \mid x_{ij}, U_{0i}, \beta, \sigma^2) f(U_{0i} \mid \tau^2) dU_{0i}$$

School-level weight

Weight for student within school i

See Pfefferman, 1993, Feder, Nathan, Pfefferman 2000 and Rabe-Hesketh, Skrondal 2006

$$\log L(\beta, \sigma_1^2, \tau^2 / y) =$$

$$\log \prod_i \left[ \int \prod_j \left\{ f(y_{ij} / u_{oi}, \beta, \sigma^2) \right\}^{w_{j|i}} f(u_{oi} / \tau^2) \, du_{oi} \right]^{w_i}$$

$$\frac{\partial \log L}{\partial \begin{matrix} \beta \\ \sigma^2 \\ \tau^2 \end{matrix}} = \sum_i w_i \left( w_{j|u} \, E_{u_i / y} \left( \frac{\partial \log f}{\partial \begin{matrix} \beta \\ \sigma^2 \\ \tau^2 \end{matrix}} \right) \right) = 0$$

# How do we handle the weights?

$$\text{Level 2 weight}: w_i = \frac{1}{\pi_i}$$

$$\text{Level 1 weight}: w_{j|i} = \frac{1}{\pi_{\text{j|i}}}$$

$$\text{So that}: \; w_{ij} = w_i w_{j|i}$$

# How do we handle the weights?

NOTE: estimation within the mixed models framework MAY be sensitive to rescaling of the weights

This was NOT an issue in marginal models

Rescaled level-1 weights are recommended by Pfeffermann et al (1998) and Skinner and Holmes (2003)

Several proposed scaled weights for instance :

$$w^{r}_{j|i} = \frac{w_{j|i}}{\overline{w}_{j|i}}$$

So that the average rescaled weights within school $i$ is 1.

Check sensitivity of results to weighting scale!

# Available Statistical Software

- Stata: gllamm, mixed, **melogit, meqrlogit**
- HLM:
  - http://www.ssicentral.com/hlm/index.html
  - $425
- M-plus:
  - http://www.statmodel.com/
  - $600-$1000

# Special Note at this point

- Say that sampling of clusters was a simple random sample
  - i.e. each cluster had same probability of selection
  - Then we can ignore these weights in the MLM analysis
  - But if the units within the clusters are not a simple random sample, then the level 1 weight is important
- Say that the selection of units within a cluster are a simple random sample
  - i.e. each unit is selected with the same probability
  - Then we can ignore the level-1 weight
  - But if clusters are sampled proportional to size or any other non-equal probability method, then level-2 weights need to be considered.

# Example: PISA 2000 US sample REVISED

- The goals of the analysis will be to:
  - quantify the variation in reading proficiency across U.S. schools
  - determine if the composition of the students within the schools explains the observed variation across schools (excluding SES)
  - quantify the effect of SES (via the contextual effect)
- Account for the survey design within the analysis and determine if the results are sensitive to inclusion of the design elements.

# Data includes the following:

| | |
|---|---|
| id: | school id (numeric) |
| pass_read: | dummy variable for being proficient in reading (1: proficient, 0: not) |
| female: | dummy variable for student being female |
| isei: | international socioeconomic index |
| high_school: | dummy variable for highest education level by either parent being high school |
| college: | dummy variable for highest education level by either parent being college |
| test_lang: | dummy variable for English being spoken at home (NOTE: the reading proficiency test is administered in English) |
| one_for: | dummy variable for one parent being foreign born |
| both_for: | dummy variable for both parents being foreign born |
| w_fstuwt: | student-level survey weight |
| wnrschbq: | school-level survey weight |

# Outcome: indicator for proficiency in reading

1. Random intercept for school and no covariates
   - Goal here is to estimate the variance of the random intercept for school
2. Compositional effects (composition of X values within cluster affects cluster-average outcome )
   - Add the school level summaries of gender, highest level of education among parents (high_school, college), foreign born parents (one_for, both_for) and English is primary language at home (test_lang)
   - Estimate the variance of the random intercept for school
3. Contextual effects of SES (cluster average X affects the individual outcomes )
   - Model 2 plus the within and between school SES variables
   - Estimate the contextual effect by taking the difference in the between effect and within effect.

| Covariates* | Random Intercept Only | Random Intercept plus school characteristics | Random Intercept plus school characteristic plus SES |
|---|---|---|---|
| Proportion Female | | 0.98 (0.87, 1.09) | 1.04 (0.94, 1.15) |
| Proportion of parents with High School education | | 1.14 (0.95, 1.36) | 1.03 (0.87, 1.21) |
| Proportion of parents with College education | | 1.37 (1.16, 1.62) | 1.04 (0.88, 1.22) |
| Proportion with one foreign born parent | | 1.02 (0.83, 1.26) | 0.90 (0.75, 1.09) |
| Proportion with both foreign born parents | | 1.16 (0.98, 1.36) | 1.07 (0.93, 1.25) |
| Proportion speaking English at home | | 1.36 (1.09, 1.69) | 1.24 (1.01, 1.51) |
| Student SES (centered) | | | 1.02 (1.01, 1.02) |
| Mean SES | | | 1.08 (1.06, 1.11) |
| Contextual effect: SES | | | 1.06 (1.04, 1.09) |
| Random intercept variance | 0.82 (0.17) | 0.44 (0.11) | 0.23 (0.08) |

- The covariates represent the school-level proportion of or mean characteristic unless otherwise noted. The odds ratios for the school-level proportion of the characteristic represent the relative odds per 10% difference in the characteristic.

# Incorporate the Survey Weights

```
bys id: egen mean_schwt = mean(w_fstuwt)
gen wt1 = w_fstuwt / mean_schwt
rename wnrschbw wt2

* Random intercept - weighted

gllamm pass_read, i(id) family(binomial) adapt pweight(wt) from(a)

* Compositional model (ignoring SES) - weighted

gllamm pass_read mean_female mean_high_school mean_college ///
mean_one_for mean_both_for mean_test_lang, i(id) family(binomial) ///
adapt from(b) pweight(wt) eform

* Contextual effect of SES - weighted

gllamm pass_read isei_c mean_isei mean_female mean_high_school
mean_college ///
mean_one_for mean_both_for mean_test_lang, i(id) family(binomial) ///
adapt from(c) pweight(wt) eform
lincom mean_isei - isei_c, eform
```

# Comparison of unweighted and weighted results

| Covariates* | Unweighted Contextual Model | Weighted Contextual Model |
|---|---|---|
| Proportion Female | 1.04 (0.94, 1.15) | 1.23 (1.03, 1.46) |
| Proportion of parents with High School education | 1.03 (0.87, 1.21) | 1.03 (0.87, 1.21) |
| Proportion of parents with College education | 1.04 (0.88, 1.22) | 0.96 (0.76, 1.20) |
| Proportion with one foreign born parent | 0.90 (0.75, 1.09) | 1.01 (0.80, 1.28) |
| Proportion with both foreign born parents | 1.07 (0.93, 1.25) | 1.07 (0.84, 1.35) |
| Proportion speaking English at home | 1.24 (1.01, 1.51) | 1.17 (0.86, 1.59) |
| Student SES (centered) | 1.02 (1.01, 1.02) | 1.02 (1.01, 1.03) |
| Mean SES | 1.08 (1.06, 1.11) | 1.10 (1.05, 1.15) |
| Contextual effect: SES | 1.06 (1.04, 1.09) | 1.08 (1.03, 1.13) |
| Random intercept variance | 0.23 (0.08) | 0.27 (0.11) |

# How do you know the design is informative?

- Use your knowledge of the design and your research question and plan accordingly
- You could assess impact of accounting for the weights by:
  - Refit the model ignoring the sampling weights/clusters
  - Compare the model results; looking for substantial changes in estimated regression coefficients
  - Simple statistic to calculate is $\left|\hat{\beta}_w - \hat{\beta}_{nw}\right| / se(\hat{\beta}_{nw})$
    - If this is "large" then you know the weights are informative
  - We know the standard error estimates will generally increase when we add sampling weights

# Summary of Approach

① Need to estimate "designed-for" totals : weight, but beware large $\hat{Y}_i$'s. Use $\sqrt{wt}$ and sensitivity check BIAS-VARIANCE TRADE-OFF

② More interested in causal relationships or associations ($X \rightarrow Y$?)

Don't weight:

use multi-level models including weights as "confounders" to reduce bias

or

use pseudo-likelihood with sampling weights, beware large weights, sensitivity analysis