100

Survival Analysis HW1

Jiyang Wen, 2nd year Biostatistics PhD student

September 11, 2019

1 Problem 1

(a) Survival function $S(t) = \exp\{-\int_0^t \lambda(s)ds\} = \exp\{-ct\}$ if $t \geq 0$ and S(t)=1 if $t < 0$.
Density function $f(t) = \lambda(t)S(t) = c\exp\{-ct\}I(t \geq 0)$.
When c=2, set $S(t) = 0.5$, then median survival time $t = \frac{1}{2}ln2$.
When c=5, set $S(t) = 0.5$, then median survival time $t = \frac{1}{5}ln2$.
When c=11, set $S(t) = 0.5$, then median survival time $t = \frac{1}{11}ln2$.
(b) Density function $f(t) = \frac{d}{dt}\{1 - S(t)\} = \theta\beta t^{\beta-1}\exp\{-\theta t^{\beta}\}$
Hazard function $\lambda(t) = \frac{f(t)}{S(t)} = \theta\beta t^{\beta-1}$

2 Problem 2

(a) The assumption is to say that the instantaneous failure rate at t given survival up to t is increasing linearly with time t.
(b) Survival function $S(t) = \exp\{-\int_0^t \lambda(s)ds\} = \exp\{-\frac{1}{2}bt^2\}$ if $t \geq 0$ and S(t)=1 if $t < 0$.
(c) Density function $f(t) = \lambda(t)S(t) = bt\exp\{-\frac{1}{2}bt^2\}I(t \geq 0)$.
(d) See the figures below.

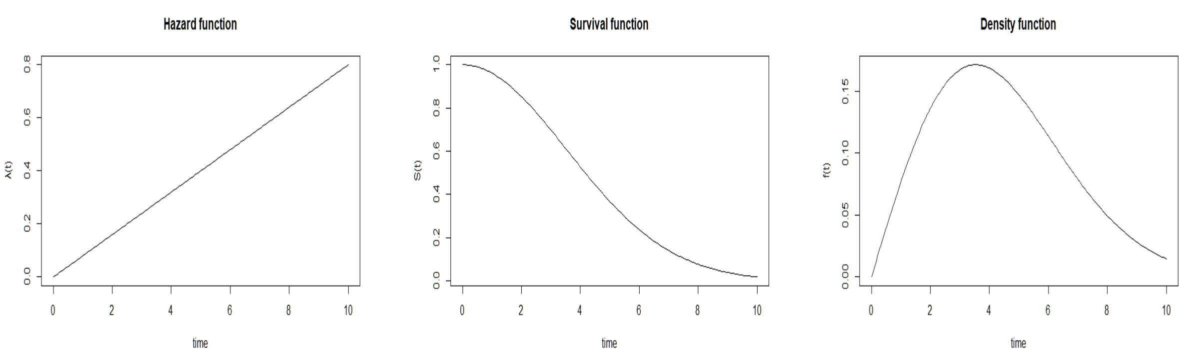

Figure 1: Hazard, density and survival function plot

3 Problem 3

(a) and (b) We can have that

$$\begin{aligned} S(t) &= P(Y \geq t) = P(T \geq t, C \geq t) = P(T \geq t)P(C \geq t) \\ &= \begin{cases} 1 & t \leq 0 \\ (1 - \frac{t}{\theta})I(0 < t < \min\{\theta, c_0\}) & t > 0 \end{cases} \end{aligned}$$

Therefore density function for Y is

$$f(t) = \frac{d}{dt}\{1 - S(t)\} = \begin{cases} \frac{1}{\theta}I(0 < t < \theta) & \theta < c_0 \\ \frac{1}{\theta}I(0 < t < c_0) + \frac{\theta - c_0}{\theta}I(t = c_0) & \theta \geq c_0 \end{cases}$$

.

(c) Note that Δ can only be 0 or 1. We can have that

$$P(\Delta = 1) = P(T \leq C) = P(T \leq c_0) = \begin{cases} 1 & \theta < c_0 \\ \frac{c_0}{\theta} & \theta \geq c_0 \end{cases}$$

$$P(\Delta = 0) = 1 - P(T \leq C) = 1 - P(T \leq c_0) = \begin{cases} 0 & \theta < c_0 \\ 1 - \frac{c_0}{\theta} & \theta \geq c_0 \end{cases}$$

4 Problem 4

(a) and (b) We can have that

$$S(t) = P(Y \geq t) = P(T \geq t, C \geq t) = P(T \geq t)P(C \geq t) = \begin{cases} 1 & t \leq 0 \\ \exp\{-5t\}\frac{10-t}{10}I(t < 10) & t > 0 \end{cases}$$

Therefore density function for Y is $f(t) = \frac{d}{dt}\{1 - S(t)\} = \exp\{-5t\}(\frac{51}{10} - \frac{t}{2})I(0 < t < 10)$.
(c) Note that Δ can only be 0 or 1. We can have that

$$P(\Delta = 1) = P(T \leq C) = \int_0^{10}\int_0^s 5\exp\{-5y\} \cdot \frac{1}{10}dyds = 1 - \frac{1}{50}(1 - e^{-50})$$

$$P(\Delta = 0) = 1 - P(T \leq C) = \frac{1}{50}(1 - e^{-50})$$

5 Problem 5

(a) The question is equivalent to $P(T \leq C) = 0.4$. Therefore we can have that

$$P(T \leq C) = \int_0^{10}\int_0^s \theta\exp\{-\theta y\} \cdot \frac{1}{5}dyds = 1 - \frac{1}{5\theta}(1 - \exp\{-5\theta\}) = 0.4$$

By solving this equation, we can derive that $\theta = 0.2252522$.
(b) First randomly generate T from $\exp(\theta)$ and C from $Unif(0, 5)$ independently. Then take $Y = \min\{T, C\}$ and set $\Delta = 1$ if $T \leq C$, $\Delta = 0$ otherwise. Repeat the above procedure for n times.