# ABilal19

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1 a) $\left\| x_i - \sum_{j=1}^{k} z_{ij} v_j \right\|_2^2 = \left( x_i - \sum_{j=1}^{k} z_{ij} v_j \right)^T \left( x_i - \sum_{j=1}^{k} z_{ij} v_j \right)$ ✯ saw soln

$= x_i^T x_i - x_i \sum_{j=1}^{k} z_{ij} v_j^T - x_i^T \sum_{j=1}^{k} z_{ij} v_j + \left( \sum_{j=1}^{k} z_{ij} v_j \right)^T \left( \sum_{j=1}^{k} z_{ij} v_j \right)$

$= x_i^T x_i - 2 \sum_{j=1}^{k} z_{ij} v_j^T x_i + \left( \sum_{j=1}^{k} z_{ij} v_j \right)^T \left( \sum_{j=1}^{k} z_{ij} v_j \right)$

$= x_i^T x_i - 2 \sum_{j=1}^{k} z_{ij} v_i^T x_i + \sum_{j=1}^{k} z_{ij} v_i v_j^T z_{ij}^T$ $(z_{ij} = x_i^T v_j)$

$+ \sum_{j=1}^{k} v_j^T x_i x_i^T v_j$ $(v_i^T v_j = 1 \text{ iff } i=j)$

$= x_i^T x_i - \frac{1}{2} \sum_{j=1}^{k} v_j^T x_i x_i^T + \sum_{j=1}^{k} v_j^T x_i x_i^T v_j$

$= x_i^T x_i - \sum_{j=1}^{k} v_i^T x_i x_i^T v_j$

b) $J_k = \frac{1}{n} \sum_{i=1}^{n} \left( x_i^T x_i - \sum_{j=1}^{k} v_j^T x_i x_i^T v_j \right)$

$= \frac{1}{n} \sum_{i=1}^{n} x_i^T x_i - \sum_{i=1}^{k} v_j^T \frac{1}{n} \left( \sum_{i=1}^{n} x_i x_i^T \right) v_i$ ✯ saw soln

$= \frac{1}{n} \sum_{i=1}^{n} x_i^T x_i - \sum_{j=1}^{k} v_i^T \sum v_j = \frac{1}{n} \sum_{i=1}^{n} x_i^T x_i - \sum_{i=1}^{k} \lambda$

✯ saw soln

c) if $k=d$, $J_k=0$.

$\sum_{j=1}^{d} \lambda_j = \frac{1}{n}\sum_{i=1}^{n} x_i^T x_i$ ✡ saw soln

$J_k = \underbrace{\frac{1}{n}\sum_{i=1}^{n} x_i^T x_i - \sum_{j=1}^{k} \lambda_j + \sum_{i=k+1}^{d} \lambda_i}_{\text{by partitioning the sum}} = \sum_{j=k+1}^{d} \lambda_j$

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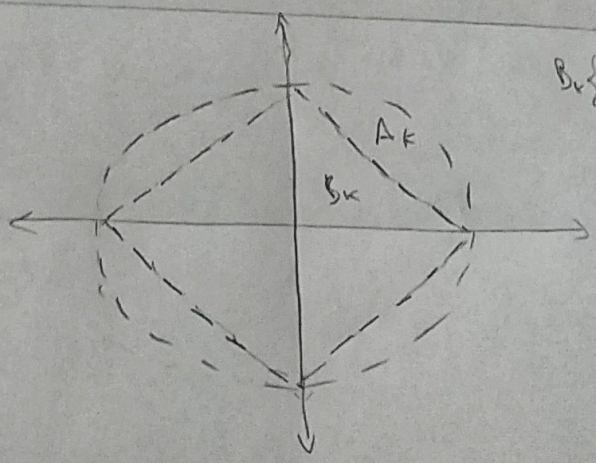
2)




$B_k\{x : \|x\|_1 \le k\}$ for $k=1$
(metric space ball)

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Optimization

$\min : f(x)$
subj to: $\|x\|_p \le k$

Lagrangian:

$$\mathcal{L}(w,\beta) = f(w) + \sum_{i=1}^{l} \beta_i h_i(w)$$

$$\inf_x \sup_{\lambda \ge 0} \mathcal{L}(x,\lambda) = \inf_x \sup_{\lambda \ge 0} f(x) + \lambda(|x|_p - k)$$ ✡ saw soln

flip inf and sup.

$$\sup_{\lambda \ge 0} \inf f(x) + \lambda(|x|_p - k) = \sup_{\lambda \ge 0} g(\lambda)$$ ✡ saw soln