ISTANBUL TECHNICAL UNIVERSITY

DEPT. OF MECHANICAL ENGINEERING

MKM506E | Modelling & Control of Mechanical Systems

Final Exam

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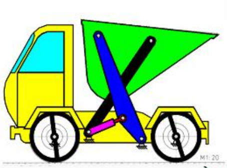
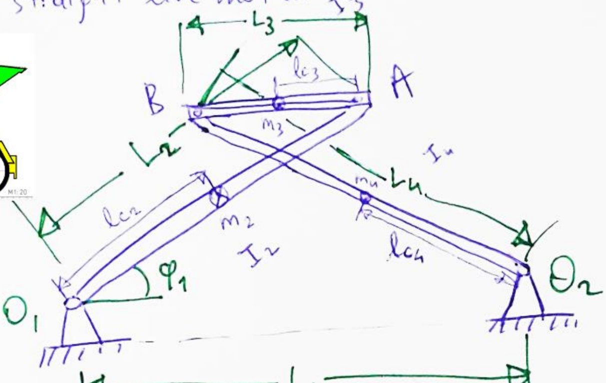
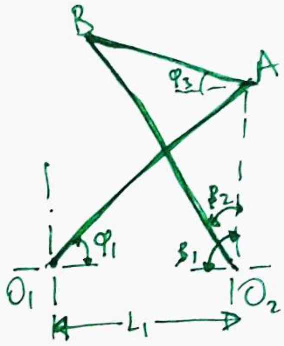
28 June 2020 | ISTANBUL

Chebyshev's linkage, which is fourbar linkage that generates approximate straight line motion





(Figure.1.)



(Figure 2.)

- $X_A = L_2\cos(\varphi_1)$ ...... (1)
- $Y_A = L_2\sin(\varphi_1)$ ...... (2)
- $\dot{X}_A = -L_2\sin(\varphi_1)\dot{\varphi}_1$ ...... (5)
- $\dot{Y}_A = L_2\cos(\varphi_1)\dot{\varphi}_1$ ...... (6)

*To complete the description of the entire mechanism we only need define the trajectory of point B, and we be able to describe both of remaining links. I write the motion of point B to be as,

- $X_B = L_4\cos(\varphi_2)$ ...... (3)
- $Y_B = L_4\sin(\varphi_2)$ ...... (4)
- $\dot{X}_B = -L_4\sin(\varphi_2)\dot{\varphi}_2$ ...... (7)
- $\dot{Y}_B = L_4\cos(\varphi_2)\dot{\varphi}_2$ ...... (8)

*Now, it is only a matter of computing the angle φ_2 and since it is agreed on φ_1 being the input angle, it is in this form: $\varphi_2 = f(\varphi_1)$. Basicly, I want to write the output angle as a function of the input angle. Figure 2. shows that, if I draw a line between $\overline{AO_2}$ and I compute it as follows,

- $\overline{AO_2}^2 = L_1^2 + L_2^2 - 2L_1L_2\cos(\varphi_1)$ ...... (9)

and then, it is possible to write the angles β_1 and β_2 as,

- $\beta_1 = \arcsin\left[\dfrac{L_2\sin(\varphi_1)}{\overline{AO_2}}\right]$ ...... (10)
- $\beta_2 = \arccos\left[\dfrac{L_4^2 + \overline{AO_2}^2 - L_3^2}{2L_4\overline{AO_2}}\right]$ ...... (11)

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• φ_2 can be written as a subtraction of the two,

• $\varphi_2 = \beta_1 - \beta_2$ ——— (12)

// There are some limitations about joint angle limit. Now, I neglected all limitation.

1) Lagrange Formulation

* The lagrange solution of the entire system is given by,

• $L = T - V$ ——— (13)

T: Total Kinetic Energy
V: Total Potential Energy

• After constructing the expression of Lagrangian, we can determine the equation of motion of the entire system by:

$$\left(\frac{\partial L}{\partial q_i}\right) - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] = 0 \quad \text{——— (14)}$$

• I first determine the total kinetic energy of the system

$$*\ T = \frac{1}{2} I_2 \dot{\varphi}_1^2 + \frac{1}{2} I_3 \dot{\varphi}_3^2 + \frac{1}{2} I_4 \dot{\varphi}_2^2 + \frac{1}{2} m_3 V_{c_3}^2 \quad \text{——— (15)}$$

$$\cdot\ V_{c_3}^2 = X_{c_3}^2 + Y_{c_3}^2 = L_2^2 \dot{\varphi}_1^2 + l_{c_3}^2 \dot{\varphi}_3^2 + 2 L_2 l_{c_3} \dot{\varphi}_1 \dot{\varphi}_3 \cos(\varphi_1 - \varphi_3)$$

$$\boxed{T = \frac{1}{2} I_2 \dot{\varphi}_1^2 + \frac{1}{2} I_3 \dot{\varphi}_3^2 + \frac{1}{2} I_4 \dot{\varphi}_2^2 + \frac{1}{2} m_3 \left[L_2^2 \dot{\varphi}_1^2 + l_{c_3}^2 \dot{\varphi}_3^2 + 2 L_2 l_{c_3} \dot{\varphi}_1 \dot{\varphi}_3 \cos(\varphi_1 - \varphi_3)\right]} \quad (16)$$

• Then I determine the potential energy of the system as:

$$*\ V = m_2 g y_{c_2} + m_3 g y_{c_3} + m_4 g y_{c_4} \quad \text{——— (17)}$$

• $y_{c_2} = l_{c_2} \sin(\varphi_1)$

• $y_{c_3} = L_1 \sin(\varphi_1) + l_{c_3} \sin(\varphi_3)$

• $y_{c_4} = l_{c_4} \sin(\varphi_2)$

$$\boxed{V = m_2 g\, l_{c_2} \sin(\varphi_1) + m_3 g \left[L_1 \sin(\varphi_1) + l_{c_3} \sin(\varphi_3)\right] + m_4 g\, l_{c_4} \sin(\varphi_2)} \quad (18)$$

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If I substitute constant values into the equations (16) and (18)

$$\boxed{\begin{array}{l} * L_1 = 4m,\ L_2 = 5m,\ L_3 = 2m,\ L_4 = 5m,\ l_{c_2} = 2.5,\ l_{c_3} = 1m\ l_{c_4} = 2.5m \\ m_2 = 2kg,\ m_3 = 1kg,\ m_4 = 2kg,\ \boxed{I = \frac{1}{2} m l_4^2};\ I_2 = 25kgm^2\ I_3 = 2kgm^2 \\ I_4 = 25kgm^2 \end{array}}$$

$$* T = \frac{1}{2} 25 \dot{\varphi}_1^2 + \frac{1}{2} . 2 \dot{\varphi}_3^2 + \frac{1}{2} . 25 . \dot{\varphi}_2^2 + \frac{1}{2} . 1 [5^2 \dot{\varphi}_1^2 + 1^2 \dot{\varphi}_3^2 + 2.5 . 1^2 \dot{\varphi}_1 \dot{\varphi}_3 \cos(\varphi_1 - \varphi_3)$$

$$. T = \underline{12.5 \dot{\varphi}_1^2} + \dot{\varphi}_3^2 + 12.5 \dot{\varphi}_2^2 + \underline{12,5 \dot{\varphi}_1^2} + 0.5 \dot{\varphi}_3^2 + 5 \dot{\varphi}_1 \dot{\varphi}_3 \cos(\varphi_1 - \varphi_3)$$

$$\boxed{. T = 25 \dot{\varphi}_1^2 + 12.5 \dot{\varphi}_2^2 + 1.5 \dot{\varphi}_3^2 + 5 \dot{\varphi}_1 \dot{\varphi}_3 \cos(\varphi_1 - \varphi_3)\ \#} \quad \text{- - ——} (19)$$

$$* V = 2.9,81.2,5. \sin(\varphi_1) + 1.9,81(4 \sin(\varphi_1) + 1 \sin(\varphi_3)) + 2.9,81 \overset{2.5}{} \sin(\varphi_2)$$

$$. V = 49.05 \sin(\varphi_1) + 39,24 \sin(\varphi_1) + 9.81 \sin(\varphi_3) + 49.05 \sin(\varphi_2)$$

$$\boxed{. V = 88.29 \sin(\varphi_1) + 49.05 \sin(\varphi_2) + 9.81 \sin(\varphi_3)} \quad \text{- - - -} (20)$$

$$\boxed{\begin{array}{l} * L = T - V = 25 \dot{\varphi}_1^2 + 12.5 \dot{\varphi}_2^2 + 1.5 \dot{\varphi}_3^2 + 5 \dot{\varphi}_1 \dot{\varphi}_3 \cos(\varphi_1 - \varphi_3) - 88.29 \sin(\varphi_1) \\ \quad - 49.05 \sin(\varphi_2) - 9.81 \sin(\varphi_3) \end{array}} \quad \text{- -} (21)$$

$(\cos\varphi_1 \cos\varphi_3 + \sin\varphi_1 \sin\varphi_3)$

$$* \left(\frac{\partial L}{\partial \varphi_i}\right) - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\varphi}_i}\right] = 0$$

\# Before the lagrange solution, the Equation must be linearized around zero point by using small-angle approximation, for $\varphi_1, \varphi_2, \varphi_3$. Then, I'll execute lagrange equation

* According to small-angle approximation;

$\sin\theta = \theta$

$\cos\theta = 1$

$$\boxed{* 25 \dot{\varphi}_1^2 + 12.5 \dot{\varphi}_2^2 + 1.5 \dot{\varphi}_3^2 + 5 \dot{\varphi}_1 \dot{\varphi}_3 + 5 \dot{\varphi}_1 \dot{\varphi}_3 \varphi_1 \varphi_3 - 88.29 \varphi_1 - 49.05 \varphi_2 - 9.81 \varphi_3} \quad \text{-} (22)$$

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for φ_1 :

$$\left(\frac{\partial L}{\partial \varphi_1}\right) - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot\varphi_1}\right] = 0 \cdot \left(\frac{\partial L}{\partial \varphi_1}\right) = 5\dot\varphi_1\dot\varphi_3\varphi_3 - 88.29$$

$$\cdot \frac{d}{dt}\left[\frac{\partial L}{\partial \dot\varphi_1}\right] = 50\ddot\varphi_1 + 5\ddot\varphi_3 + 5\ddot\varphi_3\dot\varphi_1\dot\varphi_3$$

$$\Rightarrow \boxed{5\dot\varphi_1\dot\varphi_3\varphi_3 - 88.29 - 50\ddot\varphi_1 - 5\ddot\varphi_3 - 5\ddot\varphi_3\dot\varphi_1\dot\varphi_3 = 0} \ \# \text{---} (23)$$

for φ_2 : $\left(\frac{\partial L}{\partial \varphi_2}\right) - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot\varphi_2}\right] = 0$;

$$\left(\frac{\partial L}{\partial \varphi_2}\right) = -49.05 \quad ; \frac{d}{dt}\left[\frac{\partial L}{\partial \dot\varphi_2}\right] = 25\ddot\varphi_2 \quad \Rightarrow \boxed{-49.05 - 25\ddot\varphi_2 = 0} \ \# \text{---} (24)$$

for φ_3 : $\left(\frac{\partial L}{\partial \varphi_3}\right) - \frac{d}{dt}\left[\frac{\partial L}{\partial \dot\varphi_3}\right] = 0$

$$\left(\frac{\partial L}{\partial \varphi_3}\right) = 5\dot\varphi_1\dot\varphi_3\varphi_1 - 9.81 \quad , \frac{d}{dt}\left[\frac{\partial L}{\partial \dot\varphi_3}\right] = 3\ddot\varphi_3 + 5\ddot\varphi_1 + 5\ddot\varphi_1\dot\varphi_1\dot\varphi_3$$

$$\Rightarrow \boxed{5\dot\varphi_1\dot\varphi_3\varphi_1 - 9.81 - 3\ddot\varphi_3 - 5\ddot\varphi_1 - 5\ddot\varphi_1\dot\varphi_1\dot\varphi_3 = 0} \ \# \text{---} (25)$$

2) State Space Form

$$-50\ddot\varphi_1 + (-5 - \dot\varphi_1\dot\varphi_3)\ddot\varphi_3 + 5\dot\varphi_1\dot\varphi_3\varphi_3 - 88.29 = 0$$

$$-25\ddot\varphi_2 - 49.05 = 0$$

$$(-5 - 5\dot\varphi_1\dot\varphi_3)\ddot\varphi_1 - 3\ddot\varphi_3 + 5\dot\varphi_1\dot\varphi_3\varphi_1 - 9.81 = 0$$

$$\begin{bmatrix} -50 & 0 & (-5-\dot\varphi_1\dot\varphi_3) \\ 0 & -25 & 0 \\ (-5-\dot\varphi_1\dot\varphi_3) & 0 & -3 \end{bmatrix} \begin{Bmatrix} \ddot\varphi_1 \\ \ddot\varphi_2 \\ \ddot\varphi_3 \end{Bmatrix} + \begin{bmatrix} 5\dot\varphi_3\varphi_3 & 0 & 0 \\ 0 & -25 & 0 \\ (-5-5\dot\varphi_1\dot\varphi_3) & 0 & 5\dot\varphi_1\varphi_1 \end{bmatrix} \begin{Bmatrix} \dot\varphi_1 \\ \dot\varphi_2 \\ \dot\varphi_3 \end{Bmatrix} + \begin{bmatrix} -88.29 \\ -9.81 \\ -9.81 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3) After lagrange solution, the equations must be linearized around zero point by using small-angle approximation. According to small-angle approximation; $\sin\theta = \theta$, $\cos\theta = 1$, multiplication of two angular velocity term is zero. Equation (23), (24), and (25) will be linearized at this step.

$$* \; -50\ddot{\varphi}_1 - 5\ddot{\varphi}_3 - 88.29 = 0 \quad (26)$$

$$* \; -25\ddot{\varphi}_2 - 49.05 = 0 \quad (27)$$

$$* \; -3\ddot{\varphi}_3 - 5\ddot{\varphi}_1 - 9.81 = 0 \quad (28)$$

Transfer Functions

$$* \; -25\varphi_2(s)s^2 = 49.05 \;; \quad \boxed{\varphi_2(s) = \frac{-49.05}{25s^2}} \quad (29)$$

$$* \; -50\ddot{\varphi}_1 - 5\ddot{\varphi}_3 - 88.29 = 0, \quad -3\ddot{\varphi}_3 - 5\ddot{\varphi}_1 - 9.81 = 0$$

$$\boxed{\ddot{\varphi}_1 = \frac{88.29 + 5\ddot{\varphi}_3}{-50}} \rightarrow \boxed{-3\ddot{\varphi}_3 + \left[\frac{88.29 + 5\ddot{\varphi}_3}{+10}\right] - 9.81 = 0}$$

$$\rightarrow -2.5\ddot{\varphi}_3 - 0.981 = 0 \;; \quad \ddot{\varphi}_3 = \frac{0.981}{-2.5} \quad \ddot{\varphi}_3 = -0.3924$$

$$\boxed{\varphi_3(s) = \frac{-0.3924}{s^2}} \quad \text{if we put } \ddot{\varphi}_3 \text{ into equation (26)} \quad (30)$$

$$\boxed{\varphi_1(s) = \frac{-1.72656}{s^2}} \quad (31)$$

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4) Is the system controllable and observable?

$$\boxed{\begin{aligned}\dot{X} &= AX + Bu\\ y &= CX + Du\end{aligned}}$$

* It is controllable if...

$C = [B \; AB \; A^2B \; ... \; A^{n-1}B]$

$rank(C) = n$

* It is observable if...

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}, \; rank(O) = n$$

* this part is realized in MATLAB. "obsv" command is used for observability, "ctrb" command is used for controbility, and for rank "rank" command is used. Codes are shown in APPENDIX.

*** Answer:** My sistem is observable and not controllable.

5) For my system. DC motor input is V output is Torque. So, transfer function of DC motor will be

* $\frac{\text{Voltage}}{\text{Torque}}$. $R = 20\,\Omega$, $K_t = 1\,Nm/A$, $K_b = 3Vs/rad$, $J = 2\,Nms^2/rad$, $f = 0.1\,Nms/rad$

* $R_a i_a(t) + L_a \frac{di_a}{dt} + V_b(t) = V_a(t)$ ——— (32)

* $V_b(t) = K_b \frac{d\theta(t)}{dt}$ ——— (33)

Laplace Transform;

* $I_a(s)[R_a + L_a s] + K_b\Theta(s)s = V_a(s)$ ——— (34)

* $I_a(s) = \frac{V_a(s) - K_b\Theta(s)s}{(R_a + L_a s)}$ ——— (35)

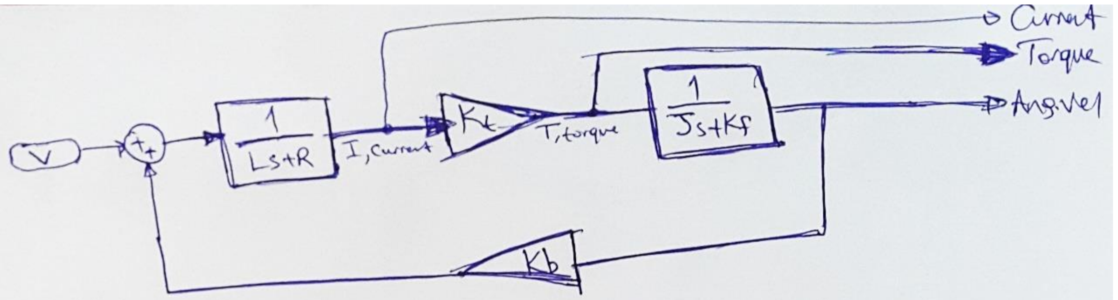

Figure 1. General DC motor Diagram

* Until this point, I obtain the first block diagram shown in Figure 1

$$* T_m(s) = K_t I_a(s) = K_t\left[\frac{V_a(s) - K_b\theta(s)s}{(R_a + L_a s)}\right] \quad (36)$$

also $T_m(s) = \omega(s)(Js + B)$ (37)

$$* \ \omega(s) = \frac{T_m(s)}{Js + B} \quad (38)$$

$$* \ \omega(s) = \theta(s)s \quad (39)$$

If we put equation (39) into equation (36), we obtain,

$$* T_m(s) = K_t\left[\frac{V_a(s) - \frac{T_m(s)}{(Js+B)}}{(R_a + L_a s)}\right] = \frac{K_t V_a(s) - K_b\theta(s)s}{(R_a + L_a s)} - \frac{K_t K_b T_m(s)}{(R_a + L_a s)(Js + B)} \quad (40)$$

$$* T_m(s)\left[1 + \frac{K_t K_b}{(R_a + L_a s)(Js + B)}\right] = \frac{K_t V_a(s)}{(R_a + L_a s)} \quad (41)$$

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$$* \; T_m(s)\left[\frac{(R_a+L_as)(Js+B)+K_tK_b}{(R_a+L_as)(Js+B)}\right] = \frac{K_t V_a(s)}{(R_a+L_as)} \quad \text{---} \quad (42)$$

If L_a value is neglected, transfer function will be as follows:

$$* \; \frac{T_m(s)}{V_a(s)} = \frac{JK_ts+BK_t}{(JR_as+(BR_a+K_tK_b))} \quad \text{---} \quad (43)$$

- If we put given value into equation (43), we obtain the transfer function of DC motor as follows:

$$\boxed{\frac{T_m(s)}{V_a(s)} = \frac{2s+0.1}{40s+5}} \quad \text{---} \quad (44)$$

If we add DC motor into our system, the Torque of DC motor will be input for our system. Equations 26-28 will be arranged.

T_M: Torque of motor

$$* \; -50\ddot{\varphi}_1 - 5\ddot{\varphi}_3 - 88.29 = T_M$$

$$* \; -25\ddot{\varphi}_2 - 49.05 = 0$$

$$* \; -3\ddot{\varphi}_3 - 5\ddot{\varphi}_1 - 9.81 = 0$$

*DC motor effects just joint.1.

$$\boxed{\ddot{\varphi}_1 = \frac{T_m + 5\ddot{\varphi}_3 + 88.29}{-50}} \quad \text{---} \quad (45)$$

if we put $\ddot{\varphi}_1$ into equation (28)

$$\boxed{\frac{\varphi_3(s)}{V(s)} = \frac{0.04T_m(s) - 0.3924}{s^2}} \quad (48)$$

$$\boxed{\frac{\varphi_1(s)}{V(s)} = \frac{-0.02T_M - 1.7658}{s^2}} \quad (46)$$

$$\boxed{\varphi_2(s) = \frac{-49.05}{25s^2}} \quad \text{--} \quad (47)$$

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* Transfer Function of DC Motor $= \frac{\text{Torque}}{\text{Voltage}} = \frac{2s+0.1}{40s+5}$

* I will add DC motor transfer function into the system's transfer function according to equations (46), (47), (48). It is obtained by MATLAB.

$$\boxed{\text{for } \frac{\varphi_1(s)}{V(s)} = \frac{0.07063\,s + 0.003532}{40s^3 + 5s^2}} \quad \text{(49)}$$

$$\boxed{\text{for } \frac{\varphi_3(s)}{V(s)} = \frac{0.0157\,s + 0.0007848}{40s^3 + 5s^2}} \quad \text{(50)}$$

6) Find the poles and zeros of the system with and without the DC motor.

It is found by MATLAB.

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5) Simulink Diagram of the System

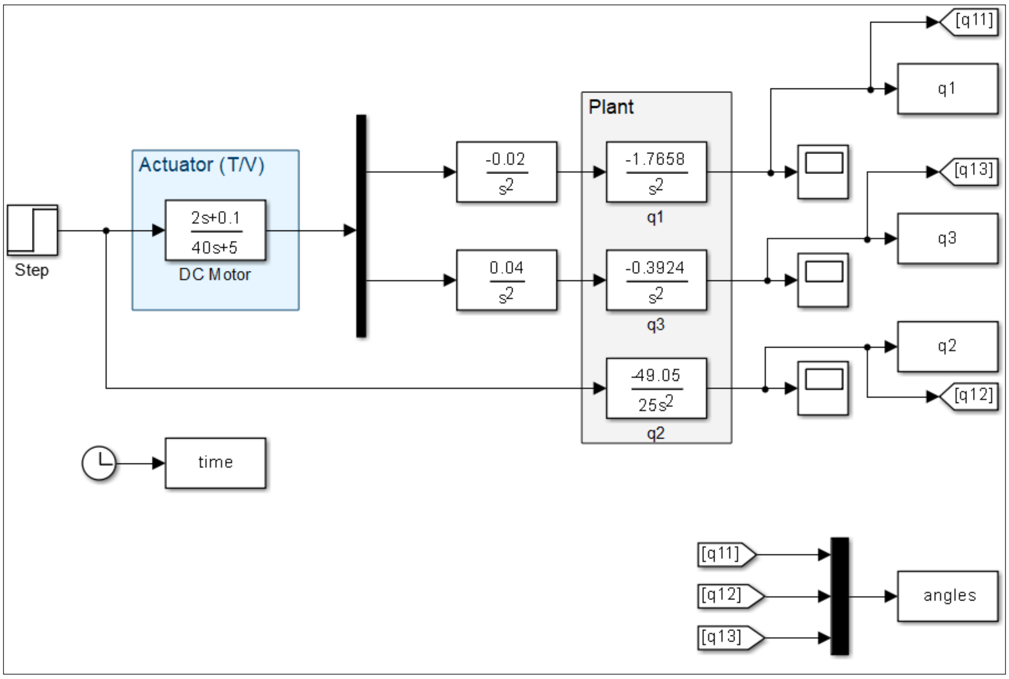

Figure 1 Plant Model with DC Motor Model Simulink Diagram

6) The Poles and Zeros Of The System

Without DC Motor

For φ_1

```
numq1=[-1.7658];
denumq1=[1 0 0];
q1tf=tf(numq1,denumq1);
pzmap(q1tf)
```

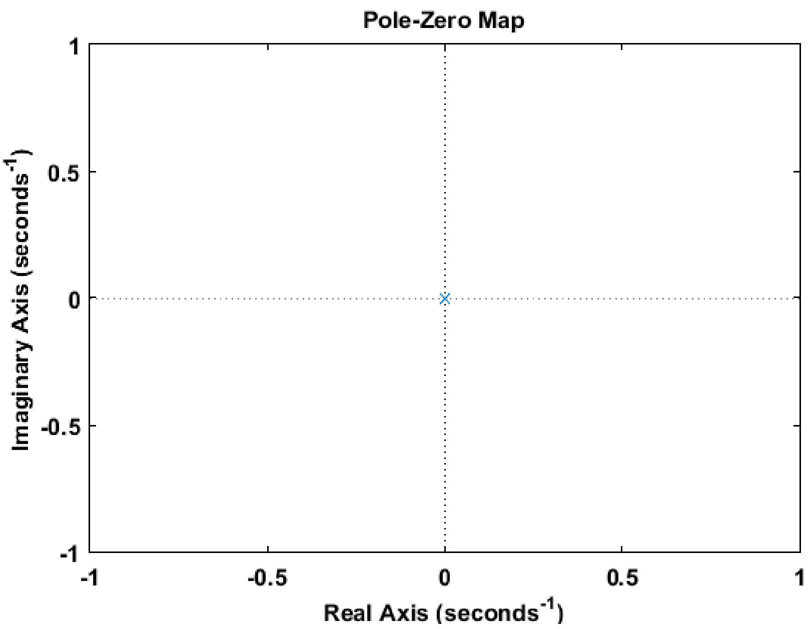

Figure 2 Pole-Zero Map of φ_1

For φ_2

```
numq2=[-49.05];
denumq2=[25 0 0];
q2tf=tf(numq2,denumq2);
pzmap(q2tf)
```

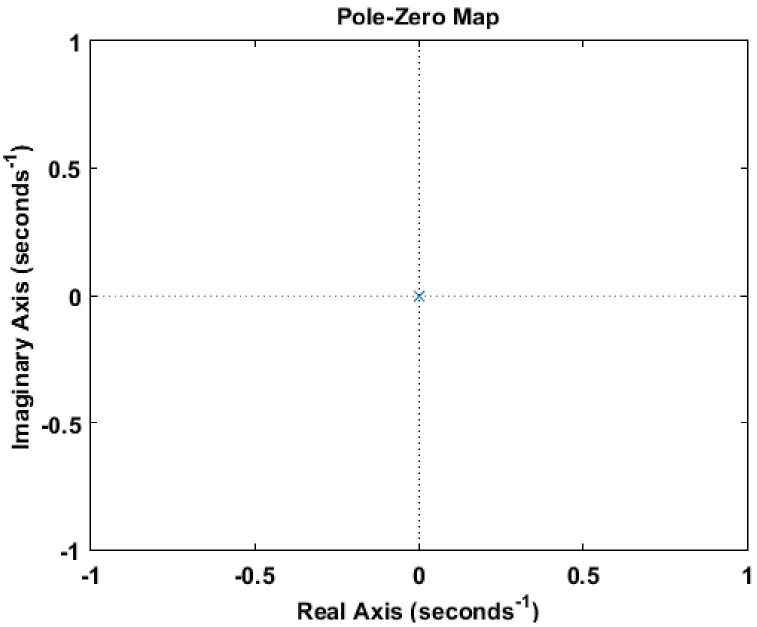

Figure 3 Pole-Zero Map of φ_2

For φ_3

```
numq3=[-0.3924];
denumq3=[1 0 0];
q3tf=tf(numq3,denumq3);
pzmap(q3tf)
```

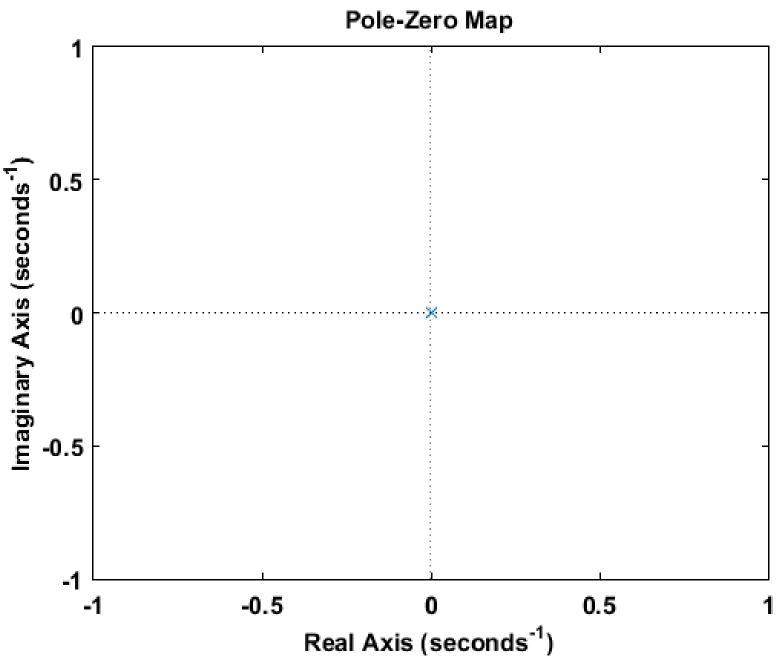

Figure 4 Pole-Zero Map of φ_3

With DC Motor

For φ_1

```
% With DC Motor
DCnum=[2 0.1];
DCdenum=[40 5];
DCtf=tf(DCnum,DCdenum);

sysnumq1=[-1.7658];
sysdenumq1=[1 0 0];
systf=tf(sysnumq1,sysdenumq1);

Gs=series(-0.02*DCtf,systf)

pzmap(Gs)
```

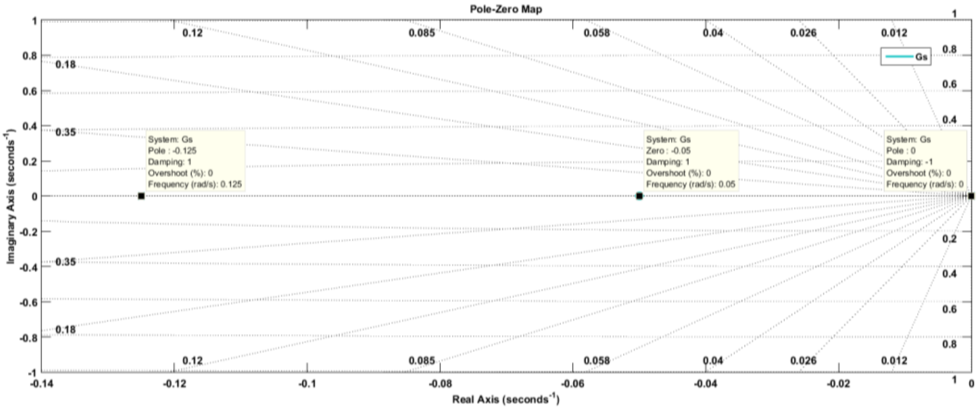

Figure 5 Poles - Zeros of the φ_1 with DC Motor

Poles=[0 , 0, -0.125];

Zeros=[-0.05];

For φ_2

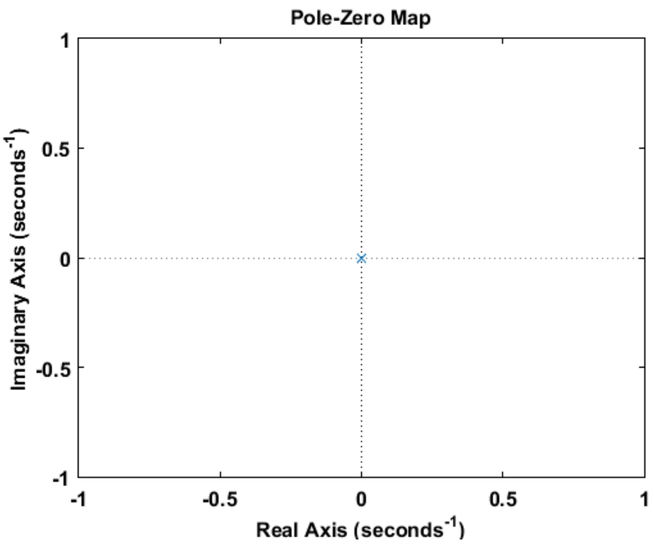

Figure 6 Pole-Zero Map of φ_2

For φ_3

```
DCnum=[2 0.1];
DCdenum=[40 5];
DCtf=tf(DCnum,DCdenum);

sysnumq2=[-0.3924];
sysdenumq2=[1 0 0];
systf=tf(sysnumq2,sysdenumq2);

Gs2=series(-0.02*DCtf,systf)
pzmap(Gs2)
```

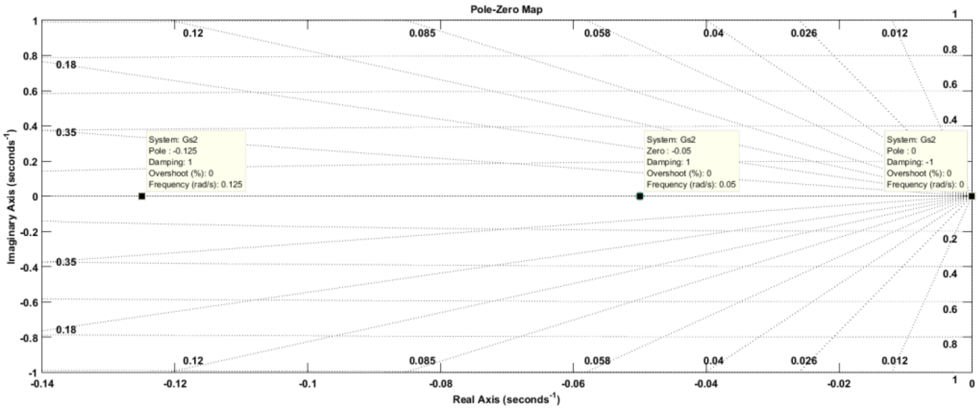

Figure 6 Poles - Zeros of the φ_3 with DC Motor

Poles=[0 , 0, -0.125];

Zeros=[-0.05];

7) Plot the step response of the mechanism powered by the DC motor where the input is the voltage input to the DC motor.

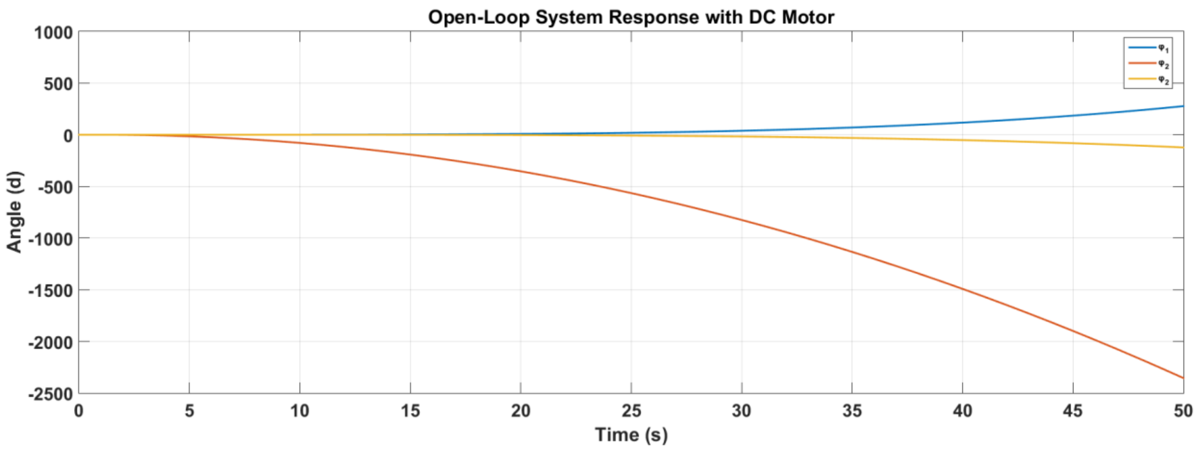

Figure 7 System Responce of $\varphi_1, \varphi_2, \varphi_3$ for 50 sec.

8) What are the rise time, settling time, percent overshoot, peak time and peak magnitude ? (namely, time domain performance measures)

Since system goes to infinitive, I could not find time domain performance criteria.

9) Draw the Bode plot of the system. What are the phase margin, gain margin, resonant frequencies and bandwidth of the system ? (namely, frequency domain performance measures)

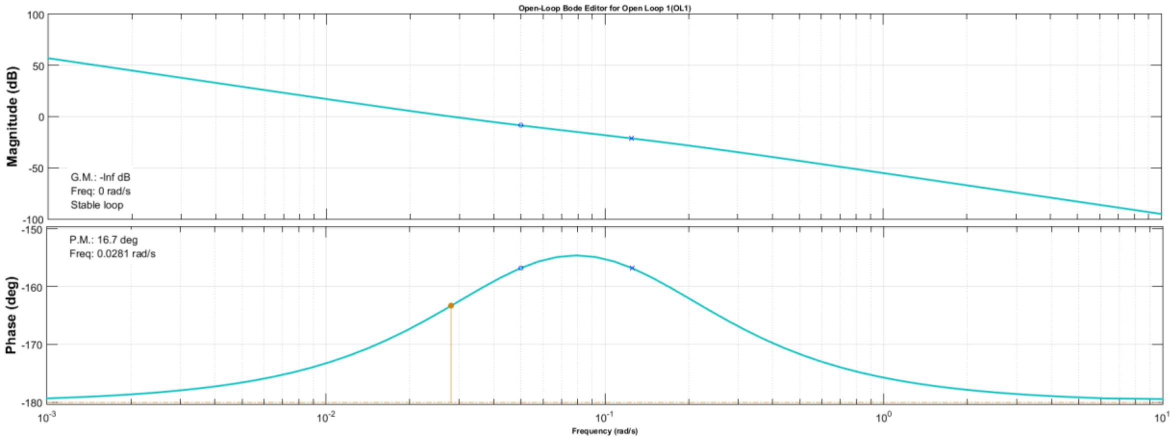

Figure 8 Bode plot of the φ_1

```
[Gm,Pm,Wcg,Wcp] = margin(Gs)
```

Gain Margin = 0

Phase Margin = 16.6741

```
Wcg =    0           % Gain Crossover frequency
Wcp =    0.0281      % Phase Crossover frequency
```

The "bandwidth" command returns NaN for models with infinite DC gain.

10) Design a PID for the mechanism using the Ziegler-Nichols rules and obtain the step response of the mechanism.

Table 1 Zeigler Nichols Method

Control Type	K_p	K_i	K_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2\ K_p/\ T_u$	-
PID	$0.60K_u$	$2\ K_p/\ T_u$	$K_p\ T_u/8$

There are some basic rules of Zeigler Nichols method. The first step is choosing correct K_p which oscillate the system. By this way, we fint the systems period Tu. Then this method is calculated according to Table 1 value.

It is arranged in MATLAB Simulink PID block diagram by tuning. PID parameters K_p , K_i, K_d found as follows:

K_p=350,7

K_i=32.6

K_d=893.2

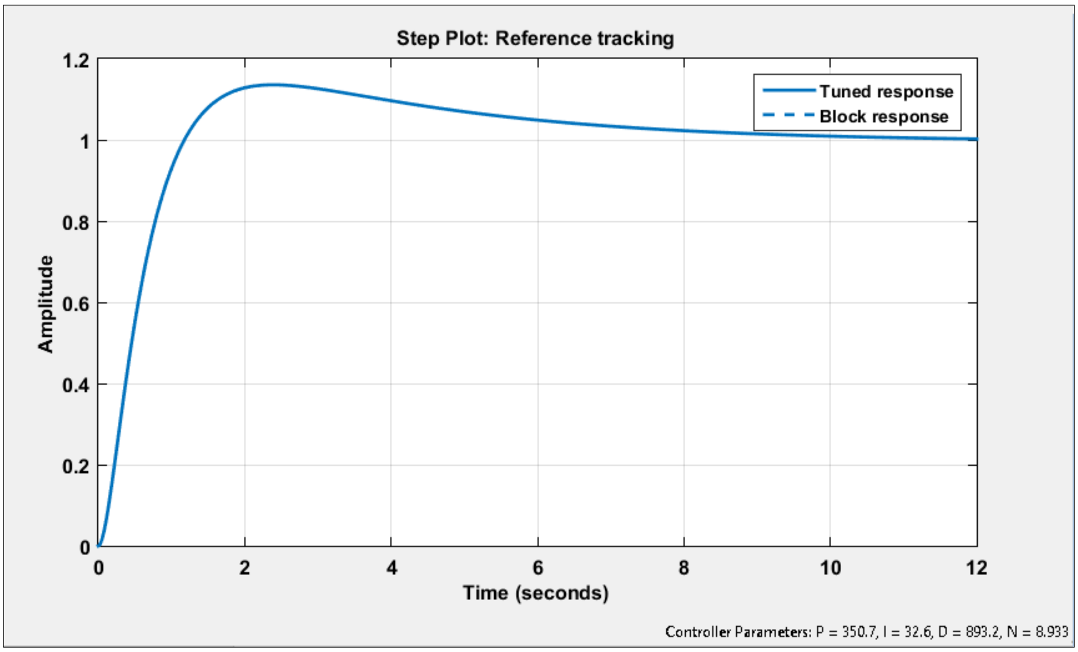

Figure 9 Step Response of the Mechanism

11) Draw the block diagram of the system. Find the corresponding transfer function of the mechanism controlled by the PID controller.

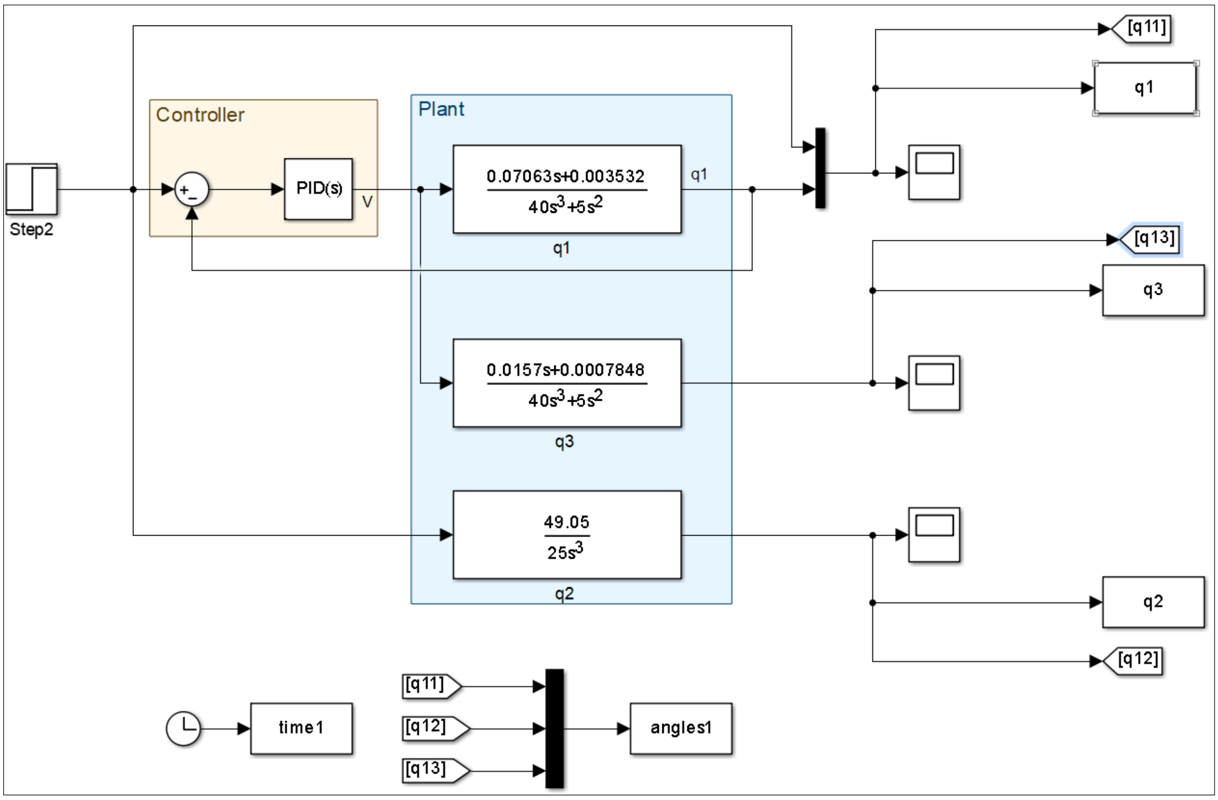

Figure 10 Block Diagram of the Mechanism Controlled by the PID Controller

12) What are the poles and zeros of the closed-loop system ? Draw the step response of the system.

For φ_1

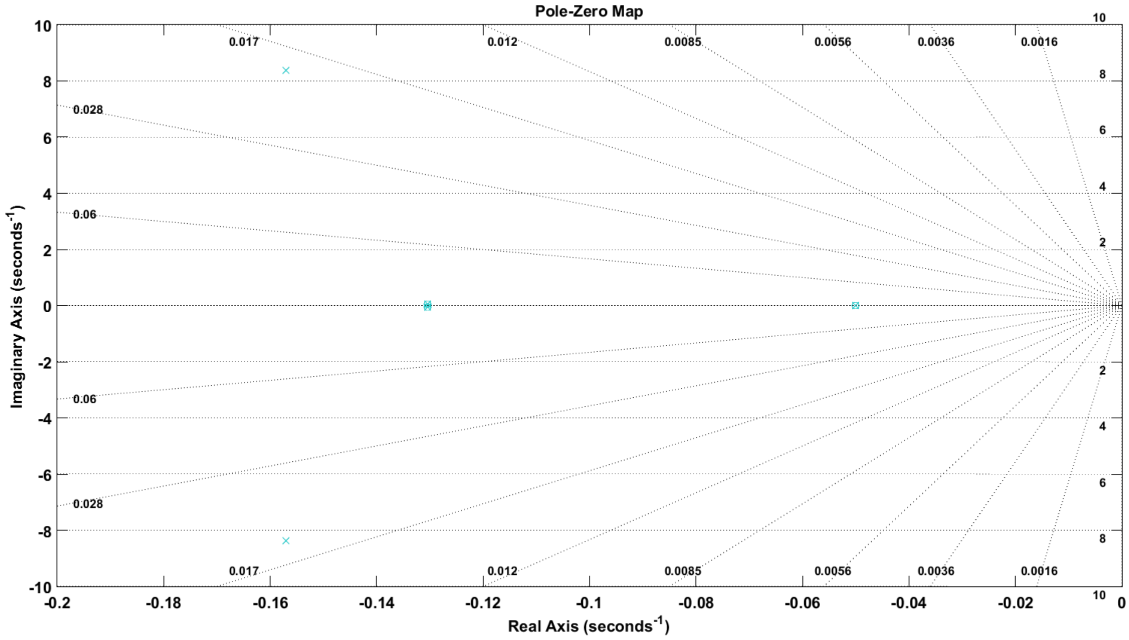

Figure 11 Poles-Zeros Map of Closed-Loop System

Poles= [-0.1570 + 8.3840i, -0.1570 - 8.3840i, -0.1305 + 0.0586i, -0.1305 - 0.0586i, -0.0500 + 0.0000i]

Zeros= [-0.1304 + 0.0586i, -0.1304 - 0.0586i, -0.0500 + 0.0000i]

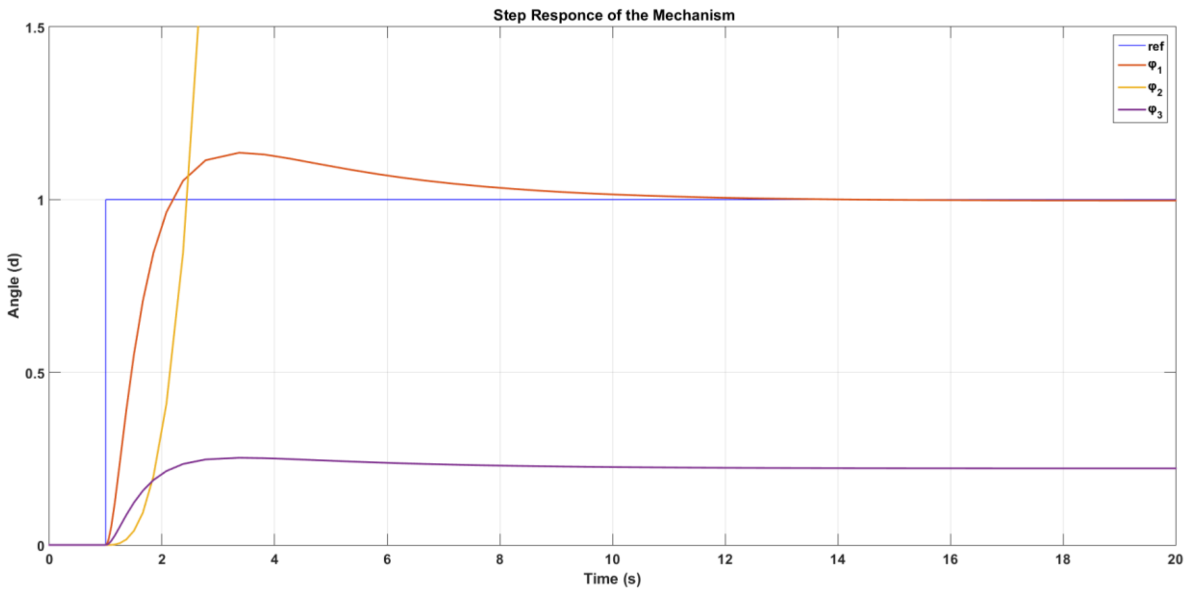

Figure 12 Step Response of the Mechanism

13) What is the system type (0, 1, or 2) ? What are the frequency and time domain performance measures (as listed in parts 8 and 9) ?

Performance and Robustness

	Tuned	Block
Rise time	0.8 seconds	0.8 seconds
Settling time	8.31 seconds	8.31 seconds
Overshoot	13.6 %	13.6 %
Peak	1.14	1.14
Gain margin	-30.5 dB @ 0.124 rad/s	-30.5 dB @ 0.124 rad/s
Phase margin	69 deg @ 1.64 rad/s	69 deg @ 1.64 rad/s
Closed-loop stability	Stable	Stable

Figure 13 Time Domain and Frequency Domain Response of the System

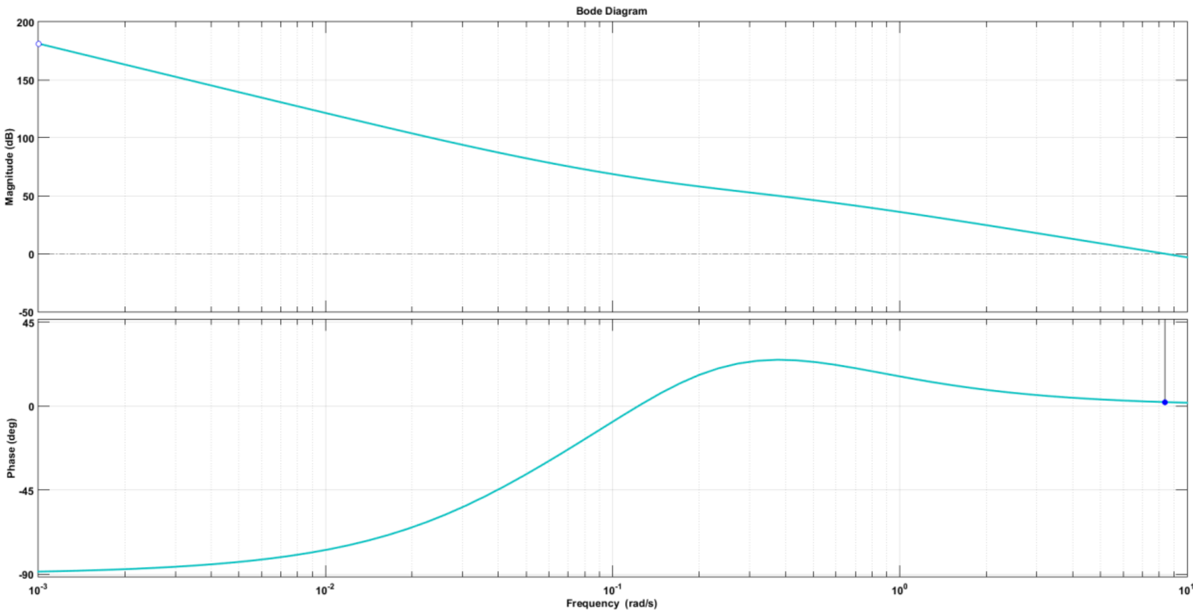

Figure 14 Bode Diagram of the System

15) Obtain the step response of the discrete-time implementation of the PID controller using a Zero-Order-Hold circuit and the sampling time of T=1, 0.1, and 0.01 second.

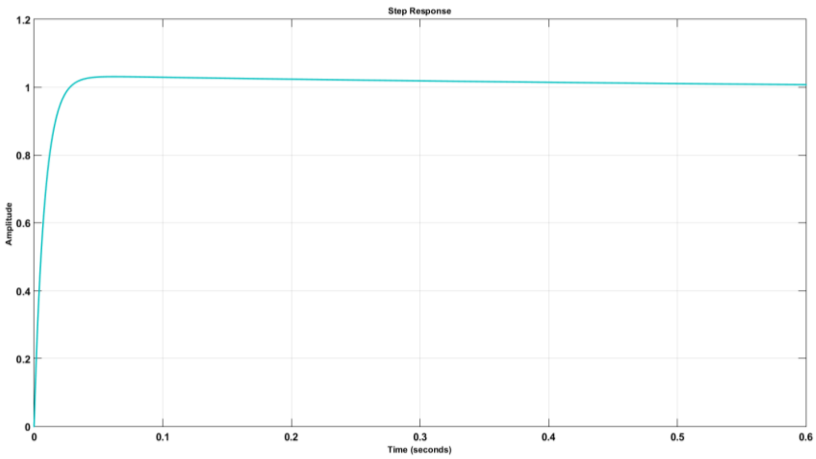

Figure 15 Step Response of the Discrete-Time Implementation of the PID Controller for T=1

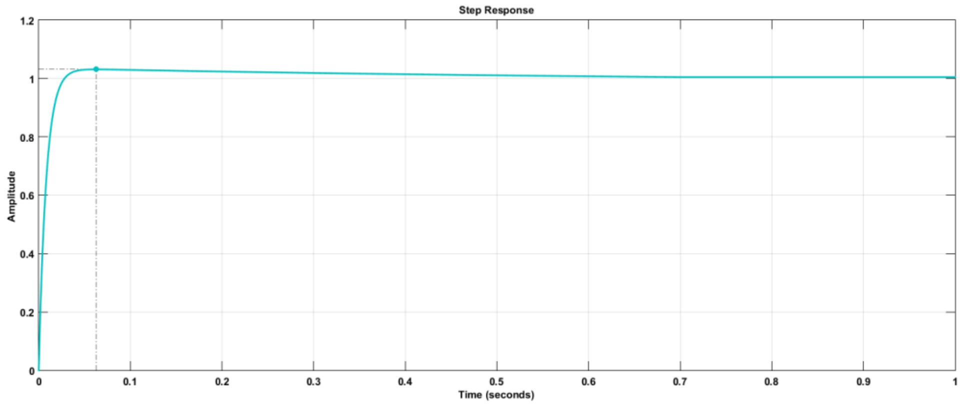

Figure 16 Step Response of the Discrete-Time Implementation of the PID Controller for T=0.1

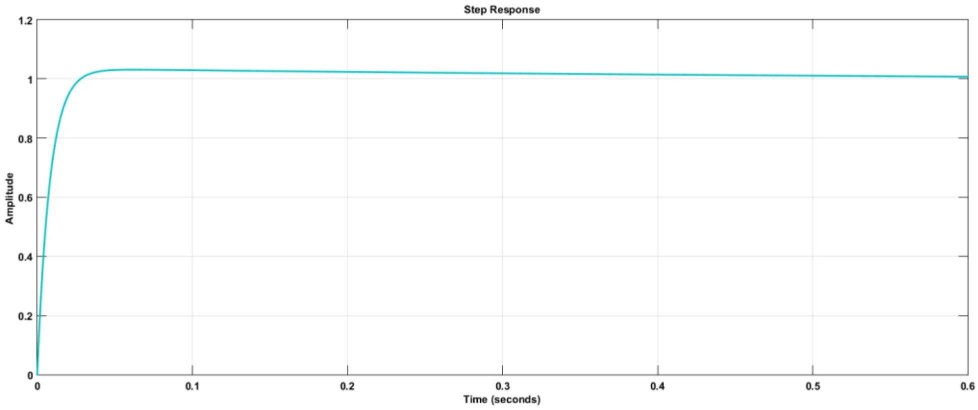

Figure 17 Step Response of the Discrete-Time Implementation of the PID Controller for T=0.01

```
%% Discrete Control Codes written in MATLAB
T = 1;  %Sampling Time
d_sys = c2d(sys_cl,T,'zoh'); % Converts continuous-time dynamic
system to discrete time
% impulse(d_sys);
% step(d_sys)
% Discrete-Time State Space Model which is obtained by "c2d" command
a =[  0.9999  0  -0;0.0005 1 0;0 0.0005 1];
b = [ 0.0005;0;0];
c = [0  -0.0883  -0.0044];
d=0;
Q=[1 0 0;0 1 0;0 0 1];
R=[1];
K=lqr(a,b,Q,R);    %   K = 11.5605   18.2677   10.0000
sys_discrete= ss(a-b*K, b, c, d);
step(sys_discrete) %Closed loop
TfLQRDiscrete=K*inv(s*eye(3)-a)*b; % Transfer Function is obtained
Kp=21.1;
Ki=26;
Kd=0;
H=1;
Gc=pid(Kp,Ki,Kd);
DscConPID=feedback(Gc*TfLQRDiscrete,H);
step(DscConPID);
```

16-17-18) Design an LQR controller for the continuous time mechanism model powered by a DC motor using Matlab. The goal should be better percent overshoot and rise time than those of the PID. Then, find the corresponding transfer function of the mechanism controlled by the LQR state-feedback.

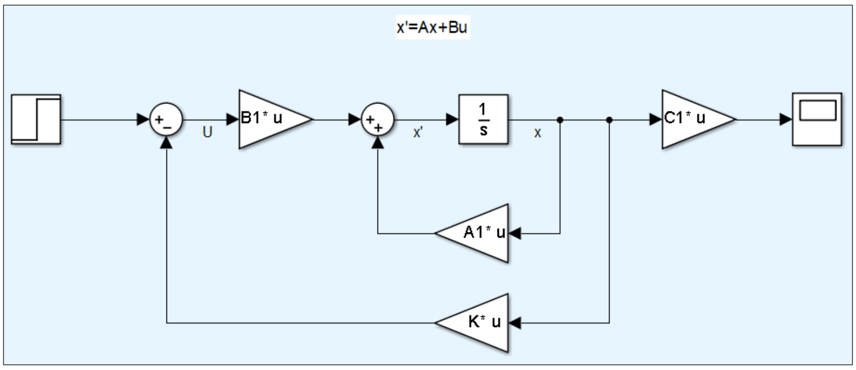

Figure 18 LQR Controller Design of the System

For Q=[1 0 0;0 1;0 0 1]; R=[1]; K =[2.2931 2.4158 1.0000]

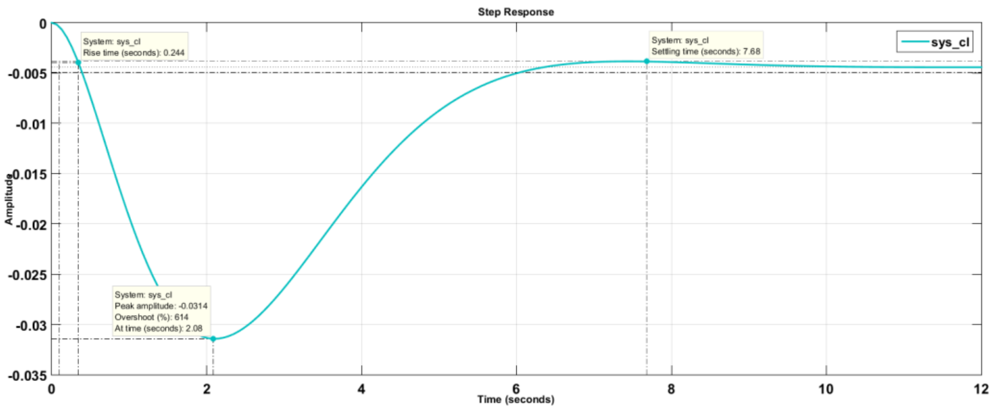

Figure 19 LQR Response of the System For Q=eye(3) R=1

For Q=[100 0 0;0 100 0;0 0 100]; R=[1]; K =[11.5605 18.2677 10.0000]

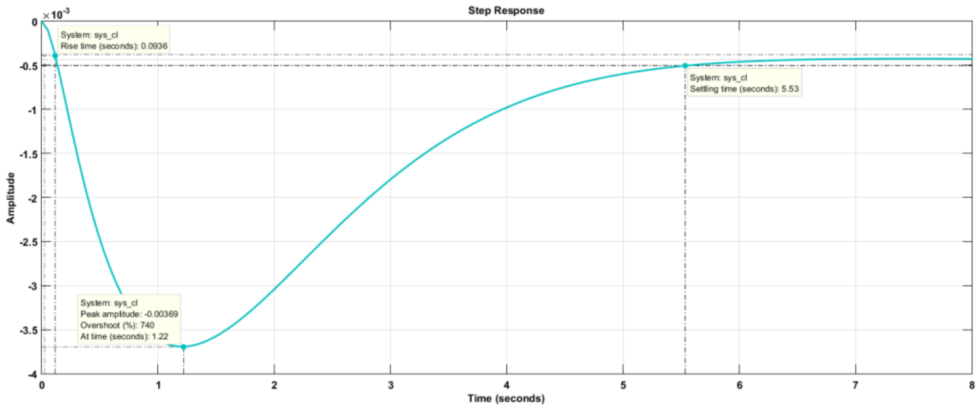

Figure 20 LQR Response of the System for Q=100*eye(3) R=1

For Q=[1000 0 0;0 1000 0;0 0 1000]; R=[1]; K =[33.2145 55.7546 31.6228]

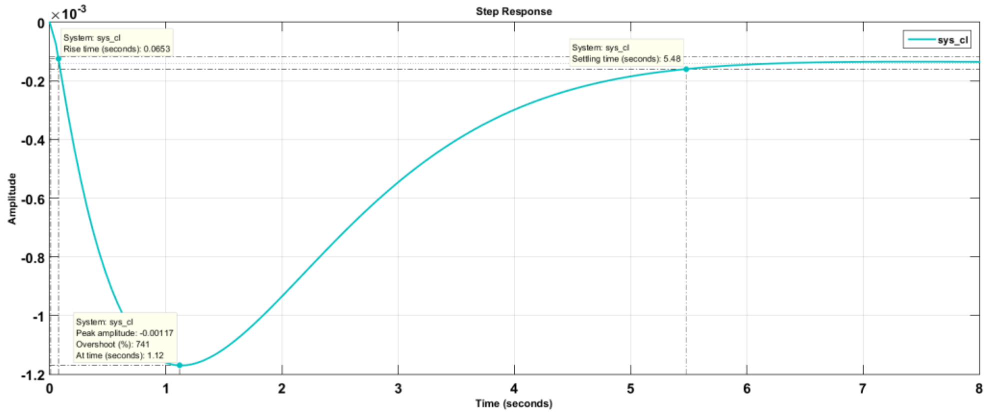

Figure 21 LQR Response of the System For Q=1000*eye(3) R=1

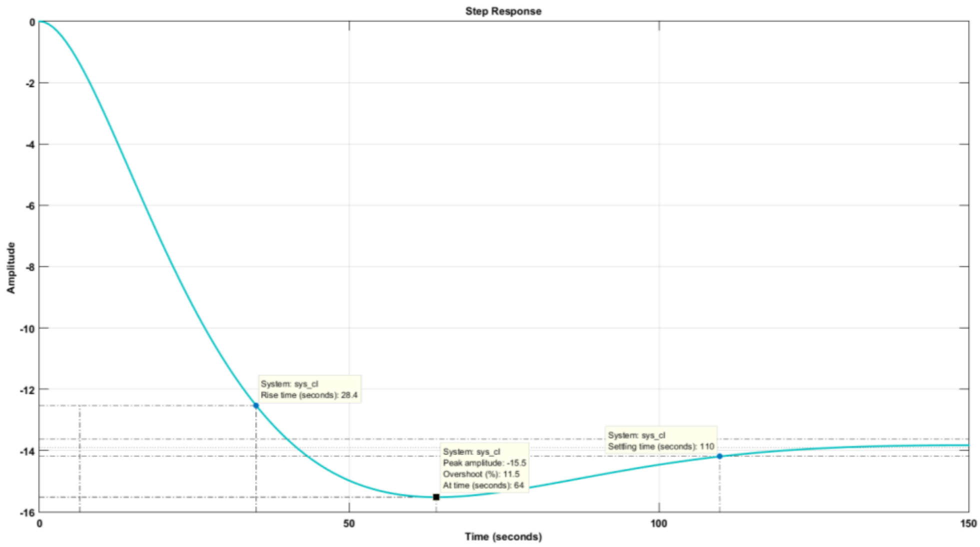

Figure 22 LQR Response of the System for Q= Q=10^-7*[100 0 0;0 0.001 0;0 0 1] R=1

Table 2 LQR Responses List of the

Q and R values	Rise Time (t_r)	Overshoot(%)	Settling Time (t_s)
Q=eye(3) R=1	0.244s.	614	7.68 s.
Q=100*eye(3) R=1	0.0939s.	740	5.53 s
Q=1000*eye(3) R=1	0.0653	741	5.48
Q=10^-7*[100 0 0;0 0.001 0;0 0 1]; R=1	28.4	11.5	110

For the last experiment, I've obtained the best overshoot value, especially if I compare PID controller overshoot shown in Figure 13, among LQR values. However, Rise time and settling time have increased.

For my system, dump truck, the last experiment may be beneficial. Let me think this vehicle is dustcart, overshoot is very critical criteria for environment and people living around this environment.

For the best Q and R values, global search algorithm should be used.

Then,

In order to obtain corresponding transfer function of the mechanism controlled by the LQR state-feedback, I use this code in MATLAB:

```
TfLQR=K*inv(s*eye(3)-A)*B;
```

I obtain this transfer function:

$$\frac{0.06946\, s^5+0.02846\, s^4+0.004174 s^3+0.0002523\, s^2}{s^6+0.375\, s^5+0.04688 s^4+0.001953 s^3}$$

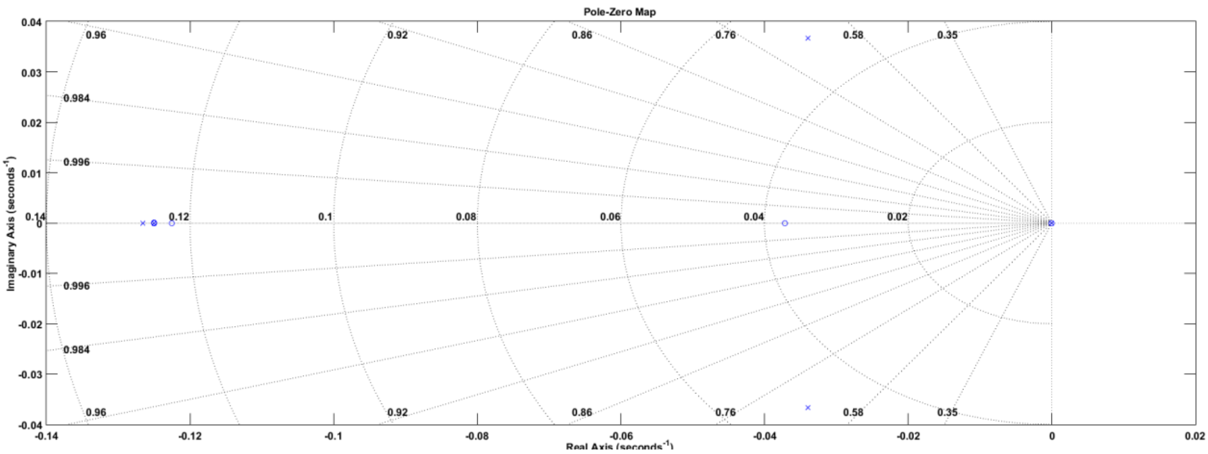

Figure 23 Poles- Zeros Map of LQR Controlled System

Poles= [-0.1250 + 0.0000i, -0.1250 - 0.0000i, -0.1250 + 0.0000i, 0.0000 + 0.0000i, -0.0000 + 0.0000i, -0.0000 + 0.0000i];

Zeros=[-0.1250 + 0.0000i, -0.1250 - 0.0000i, -0.1225 + 0.0000i, -0.0372 + 0.0000i, 0.0000 + 0.0000i];

APPENDIX

MATLAB Codes

```
%% MKM506E | Modelling and Control of Mechanical Systems
%% Lecturer: Prof.Dr.Ata Muğan
%% Abdurrahim Bilal Özcan | 514191026
%%
clc,clear all

num=-0.3924;
denum=[1 0 0];

[A,B,C,D]=tf2ss(num,denum)
%% State-Space Model
A=[0 0 1;0 0 0;0 0 0];
B=[0;0;1];
C=[0 0 -1.72656;0 0 -0.3924;0 0 -1.96];
D=[0;0;0];
%
% [NUM,DENUM]=ss2tf(A,B,C,D)
%% Transfer Function

s=tf('s');
TrafFnc=C*inv(eye(3)*s^2-A)*B+D;

%% Controllability and Observability

Qb=obsv(A,C);
if(rank(Qb)== rank(A))
    disp('Given System is Observable')
else
     disp('Given System is not Observable')
end

Cr=ctrb(A,B);
if(rank(Cr)== rank(A))
    disp('Given System is Controllable')
else
     disp('Given System is not Controllable')
end

%% Poles ans zeros of the system with without DC Motor
% With DC Motor
%For q_1
DCnum=[2 0.1];
DCdenum=[40 5];
DCtf=tf(DCnum,DCdenum);

sysnumq1=[-1.7658];
sysdenumq1=[1 0 0];
systf=tf(sysnumq1,sysdenumq1);

Gs=series(-0.02*DCtf,systf);
controlSystemDesigner('bode',Gs);  %Bode Diagram for q_1
margin(Gs);
```

```
[Gm,Pm,Wcg,Wcp] = margin(Gs); % Gain and phase margins and crossover
frequencies
fb = bandwidth(Gs)
controlSystemDesigner('bode',Gs);  %Bode Diagram for q_1

pzmap(Gs)

%For q_3
sysnumq2=[-0.3924];
sysdenumq2=[1 0 0];
systf=tf(sysnumq2,sysdenumq2);

Gs2=series(-0.02*DCtf,systf)
pzmap(Gs2)

%% Without DC Motor

numq1=[-1.7658];
denumq1=[1 0 0];
q1tf=tf(numq1,denumq1);
pzmap(q1tf)

numq3=[-0.3924];
denumq3=[1 0 0];
q3tf=tf(numq3,denumq3);
pzmap(q3tf)

numq2=[-49.05];
denumq2=[25 0 0];
q2tf=tf(numq2,denumq2);
pzmap(q2tf)

%% Ziegler - Nichols Method
KU=180;    %From Simulation
TU=3.9;     %From Simulation
[KP, KI, KD] = ZieglerNichols(KU,TU,'NoOvershoot')

%% Poles-Zeros Map of Closed-Loop System

% For q1
DCnum=[2 0.1];
DCdenum=[40 5];
DCtf=tf(DCnum,DCdenum);

sysnumq1=[-1.7658];
sysdenumq1=[1 0 0];
systf=tf(sysnumq1,sysdenumq1);

C=pid(350.7,32.6,893,2);

Control=series(DCtf,systf);


last=feedback(C*Control,-1); %feedback
```

```
controlSystemDesigner('bode',last);  %Bode Diagram for q_1

% For q3
DCnum=[2 0.1];
DCdenum=[40 5];
DCtf=tf(DCnum,DCdenum);

numq3=[-0.3924];
denumq3=[1 0 0];
q3tf=tf(numq3,denumq3);
C=pid(350.7,32.6,893,2);

Control=series(DCtf,systf);

last=feedback(C*Control,-1); %feedbask

controlSystemDesigner('bode',last);  %Bode Diagram for q_1


%% LQR Controller Design


num=[-3.532 -0.1766];
denum=[40 5 0 0];
[A,B,C,D]=tf2ss(num,denum);  % I pass state-space form from transfer
function

A1=[-0.125 0 0;1 0 0;0 1 0];
B1=[1;0;0];
C1=[0 -0.0883 -0.0044];
D1=0
Q=10^-7*[100 0 0;0 0.001 0;0 0 1];
R=[1];
K=lqr(A1,B1,Q,R)    %   K = 11.5605   18.2677   10.0000

sys_cl = ss(A1-B1*K, B1, C1, D1);
step(sys_cl) %Closed loop

%% Poles and Zeros of the LQR Controlled System

TfLQR=K*inv(s*eye(3)-A1)*B1
pole(TfLQR)
zero(TfLQR)
controlSystemDesigner('bode',TfLQR);
```