CS301 Assignment 5

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1 Decision Problem

Input: Given a set T of projects, where each $t \in T$ is associated with a positive integer e_t denoting the effort, a positive integer p_t denoting the profit. Other inputs are integers c (denoting the maximum effort), and k (denoting the largest profit).
Problem: Does there a subset of T exists such that total effort of the projects in subset is less than or equal to c and total profit of the projects in subset is greater than or equal to k?
Output: YES, if the subset of T exists with desired conditions.
NO, if the subset of T does not exists with desired conditions.

2 Showing the problem is in NP

A problem is in NP if it is possible to verify a guess solution in polynomial time. In this problem a guess solution G is a subset of T that can consists of n many projects(t's) at most. When a guess solution is considered, this solution can be verified by calculating total effort and profit then checking whether they satisfies the desired conditions in the optimization problem or not:

To check the conditions in the problem, trace all the projects and
Calculate total effort ($e'_t s$) $\Theta(n)$
Calculate total profit ($p'_t s$) $\Theta(n)$
After calculating total effort and profit:
Check whether total effort is less than or equal to c or not $\Theta(1)$
Check whether total profit is greater than or equal to k or not $\Theta(1)$
Deciding the output is YES/NO by combining above two statements. $\Theta(1)$

As it can be seen from the above algorithm:
Asymptotic time complexity of the verify is $2\Theta(n) + 3\Theta(1)$ which is $\Theta(n)$ in polynomial time.
Since a guess solution to this optimization problem can be verified in polynomial time, then this problem is in NP.