COVID-19 – Epi Model Documentation

1 Model

The model uses the following structure, with susceptible individuals (S) progressing to an exposed state (E), and then moving to either an undetected symptomatic infected state (UI) or an undetected asymptomatic state (UA). From there, symptomatic infected individuals can either die or recover, or move to a detected symptomatic state (DI), and asymptomatic individuals can only recover or move to a detected symptomatic state (DA).

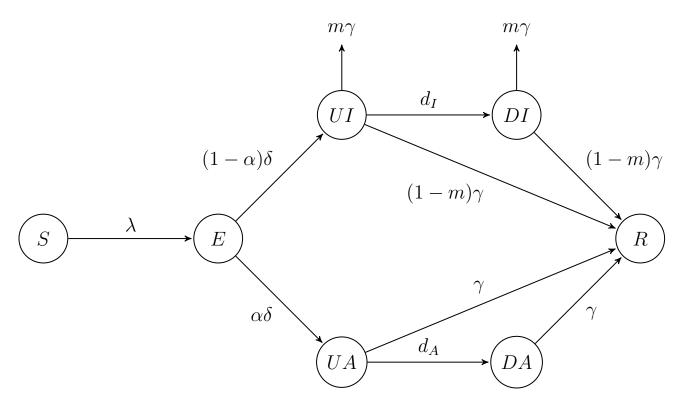


Figure 1: Model Structure

We assume that there are 3 age groups in the population, $i = \{< 20, 20 - 64, 65 +\}$ (3 strata total).

Let $\beta_{ij} = v_{ij}p$ and $\beta_{ij}^A = v_{ij}^Ap\kappa$. The model is defined by the following system of differential equations:

$$\frac{\delta S_i}{\delta t} = -\sum_{j=1}^6 \beta_{ji} S_i (UI_j + cDI_j) / N_j - \sum_{j=1}^6 \beta_{ji}^A S_i (UA_j + cDA_j) / N_j$$

$$\frac{\delta E_i}{\delta t} = \sum_{j=1}^6 \beta_{ji} S_i (UI_j + cDI_j) / N_j + \sum_{j=1}^6 \beta_{ji}^A S_i (UA_j + cDA_j) / N_j - \delta E_i$$

$$\frac{\delta UA_i}{\delta t} = \alpha_i \delta E_i - (\gamma + d_A) UA_i$$

$$\frac{\delta DA_i}{\delta t} = d_A UA_i - \gamma DA_i$$

$$\frac{\delta UI_i}{\delta t} = (1 - \alpha_i) \delta E_i - (m_i \omega + (1 - m_i) \gamma + d_I) UI_i$$

$$\frac{\delta DI_i}{\delta t} = d_I UI_i - (m_i \omega + (1 - m_i) \gamma) DI_i$$

$$\frac{\delta R_i}{\delta t} = (1 - m_i) \gamma (UI_i + DI_i) + \gamma (UA_i + DA_i)$$

2 Base Case Parameters

Table 1: Model Parameters

Parameter	Description	Value
v_{ij}	Average number of contacts with individuals in group j per	(See Below)
	person in group i	
p	Probability of transmission given contact with an infectious	0.05
	person	
κ	Reduction in transmission probability when asymptomatic	0.375
α_i	Proportion of cases in group i that do not go on to experience	(0.75, 0.3, 0.3)
	symptoms	
$\frac{1}{\delta}$	Average length of the incubation period	5 days
m_i	Mortality risk for group i	(0, 0.005, 0.1)
$\frac{1}{\gamma}$	Average duration of infection to recovery or death	5 days
d_i	Detection rate among symptomatic group	0.1
d_a	Detection rate among asymptomatic group	0
c	Proportion reduction in contact rate among detected group	0.5

2.1 Age distribution

We assume an age distribution representative of the United States with 25% of the population under 20, 59% between 20 and 64, and 16% at least 65 (United States Census Bureau, 2019). While individual municipalities may not reflect these averages, we find that these values are similar across major metropolitan areas (Figure 2), and we do not formally model uncertainty around these parameters.

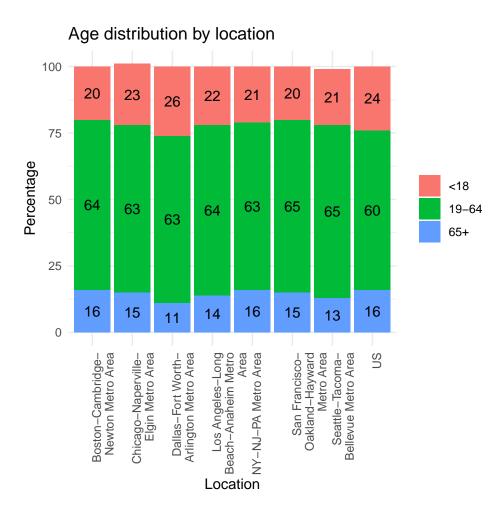


Figure 2: Age distribution by location. Due to rounding errors, total may not sum to 100%. (Source: American Community Survey 2018 via *Population Distribution by Age* (2019) and *Census profiles* (2020)).

2.2 Cases and proportion of cases detected in the United States

We paramaterize the model using the number of cases reported daily from the Centers for Disease Control and Prevention (CDC), but there is major uncertainty around these values because not all cases are detected. To model uncertainty around this key parameter, we use an expert survey conducted by McAndrew and Reich (2020) and assume that 13% of all cases have been detected. This assumption is further supported by recent research from Li et al. (2020b), which estimates that 86% of all cases are undetected. We do not formally model any uncertainty around this detection probability, aside from evaluating a scenario where all cases are detected.

2.3 Contact rates between strata

Very limited primary data exists on age-stratified contacts in the US. Therefore, we rely on methods that project contact rates derived from surveys in other contexts and US demographics. We use the age-structured contact matrix for the US estimated by Prem et al. (2017), which was derived by projecting the POLYMOD Mossong et al. (2008, 2017) survey data to US demographics Prem et al. (2017). Since we are interested in 3 broad age-classes, we bin the projected contacts according to US Census Bureau age-distribution estimates for 2018 (United States Census Bureau (2019)). This provided the following baseline contact matrix, where v_{ij} is the average number of daily contacts with individuals in group j per person in group i (i.e., v_{12} represents the number of daily contacts where an infectious individual in group 2 could infect a susceptible individual in group 1):

$$\begin{bmatrix} 9.7619 & 2.2706 & 0.2923 \\ 6.6876 & 10.3334 & 0.8853 \\ 0.9157 & 1.0141 & 1.2495 \end{bmatrix}$$

2.4 Probability of Transmission

The probability of transmission is determined by calibrating the model to an empirical estimate of the basic reproduction number R_0 . Both Li et al. (2020a) and Riou and Althaus (2020) estimate R_0 at 2.2; following Peak et al. (2020) we assume a 95% confidence interval of (1.46, 3.31) and calibrate the transmission probability to the R_0 estimate of 2.2. The calibrated value for the transmission probability per contact is 0.05.

Additionally, we assume that the rate of transmission from asymptomatic individuals is systematically lower than the transmission probability from symptomatic individuals. There is no empirical estimate for this reduction in transmission, however, so we assume that the transmissibility of asymptomatic infection is 37.5% the rate of symptomatic infection. This assumption reflects a middle ground between the 50% value used by Zhao et al. (2020) (which is similar to the estimate for influenza) and the 25% value used by Prem et al. (2020) (which they derived from Liu et al. (2020)).

2.5 Proportion of asymptomatic infections

We assume that, for individuals 20 years and older, 30% of infections are asymptomatic. This assumption is supported by findings from Nishiura et al. (2020), which found that 30.8% (95% CI: 7.7, 53.8) of infected individuals among Japanese citizens evacuated from Wuhan, China were asymptomatic. Similarly, Mizumoto et al. (2020) estimate that, among individuals on the Diamond Princess Cruise ship, 17.9% (95% CI: 15.5, 20.2) were asymptomatic.

There is less available data on the proportion of asymptomatic infections among individuals younger than 20 years, but we assume that 75% of such infections are asymptomatic. This assumption is also supported by the data from the Diamond Princess Cruise ship – among the 6 cases detected in individuals under 20 years old as of February 20, 2020, 4 of the cases were asymptomatic (Russell et al., 2020).

2.6 Age-specific mortality rates

We assume that the mortality risks among symptomatic individuals are 0%, 0.5%, and 10% for the 0-19, 20-64, and 65+ age groups, respectively. These assumptions are consistent with estimates from Riou et al. (2020), which adjusts crude fatality rates from Hubei province, China to account for delayed mortality and unidentified cases. They estimate that the case fatality risk is less than 0.05% among individuals 19 years of age and younger, between 0.19% and 2.7% for individuals between 20-59 years old, and is at least 9.5% for individuals older than 60. (For comparison, the Epidemiology Working Group for NCIP Epidemic Response (2020) reports the crude case fatality risk from all reported Chinese cases through February 11, 2020, as 0.10% for individuals aged 0-19 (1 deaths out of 965 cases), 0.98% for ages 20-59 (193 deaths out of 19,790 cases), and 5.96% for ages 60 and older (829 deaths out of 13,909) – the high proportion of unreported mild cases leads to a lower adjusted CFR for the younger age groups, and the delayed mortality leads to a higher adjusted CFR for the older age group.)

2.7 Incubation and Latent period

We assume an incubation period of 5 days, consistent with the existing literature. Li et al. (2020a) reported a mean incubation period of 5.2 days in the first 425 confirmed patients from Wuhan, China; Linton et al. (2020) estimate a mean period of 5.6 days for patients in China (both inside and outside Wuhan); Lauer et al. (2020) estimates the median incubation period at 5.1 days for patients outside Hubei province, China; while Xu et al. (2020) reports a slightly lower median period of 4 days for patients in Zhejiang province, China, and Backer

et al. (2020) reports a slightly higher mean period of 6.4 days for travelers from Wuhan, China.

Additionally, we assume that the incubation and latent periods coincide and that there is no pre-symptomatic infectious period. This is likely not an accurate assumption, given reports of pre-symptomatic transmission, but unfortunately no data is available on the length of the latent period itself. This assumption will be revisited and updated in the model as more data becomes available.

2.8 Duration of Infectious Period

There is limited evidence available on the duration of the infectious period. We assume the duration of the infectious period is 5 days, and that it coincides with the entire duration of disease. This assumption aligns with Prem et al. (2020), which modeled a length of both 3 and 7 days.

3 Interventions

3.1 School Closing

To calculate the substitution of contacts resulting from the closure of schools, we consider weekend contacts as a proxy for contacts given school closures. Using the *socialmixr*(Funk (2018)) R package, we create separate contact matrices for the weekends and weekdays from the POLYMOD survey data in 8 European countries, and use the *ratio* between elements in these two contact matrices for each country to inform the relative change in contacts that would result from school closures. This process generated the following contact matrix applicable during school closures:

$$\begin{bmatrix} 6.5340 & 2.4083 & 0.5171 \\ 7.5894 & 11.5437 & 1.1518 \\ 1.5687 & 1.4475 & 1.4256 \end{bmatrix}$$

3.2 Other Social Distancing

We also consider other social distancing measures that limit other forms of contact. To do this, we construct separate compartments of "socially distant" individuals in each age group. These individuals have a constant, scalar reduction in their contact matrix, denoted by e, giving the following system of differential equations (where the subscript Q represents a socially distant compartment) and :

$$\begin{split} \frac{\delta S_i}{\delta t} &= -\sum_{j=1}^6 \beta_{ji} S_i (UI_j + cDI_j)/N_j - \sum_{j=1}^6 \beta_{ji}^A S_i (UA_j + cDA_j)/N_j \\ \frac{\delta E_i}{\delta t} &= \sum_{j=1}^6 \beta_{ji} S_i (UI_j + cDI_j)/N_j + \sum_{j=1}^6 \beta_{ji}^A S_i (UA_j + cDA_j)/N_j - \delta E_i \\ \frac{\delta UA_i}{\delta t} &= \alpha_i \delta E_i - (\gamma + d_A) UA_i \\ \frac{\delta DA_i}{\delta t} &= d_A UA_i - \gamma DA_i \\ \frac{\delta UI_i}{\delta t} &= (1 - \alpha_i) \delta E_i - (m_i \omega + (1 - m_i) \gamma + d_I) UI_i \\ \frac{\delta DI_i}{\delta t} &= d_I UI_i - (m_i \omega + (1 - m_i) \gamma) DI_i \\ \frac{\delta R_i}{\delta t} &= (1 - m_i) \gamma (UI_i + DI_i) + \gamma (UA_i + DA_i) \\ \frac{\delta S_{Qi}}{\delta t} &= -\sum_{j=1}^6 \beta_{ji} S_i (UI_j + cDI_j)/N_j - \sum_{j=1}^6 \beta_{ji}^A S_i (UA_j + cDA_j)/N_j - \delta E_i \\ \frac{\delta UA_i}{\delta t} &= \alpha_i \delta E_i - (\gamma + d_A) UA_i \\ \frac{\delta DA_i}{\delta t} &= d_A UA_i - \gamma DA_i \\ \frac{\delta UI_i}{\delta t} &= d_I UI_i - (m_i \omega + (1 - m_i) \gamma) DI_i \\ \frac{\delta BI_i}{\delta t} &= d_I UI_i - (m_i \omega + (1 - m_i) \gamma) DI_i \\ \frac{\delta R_i}{\delta t} &= (1 - m_i) \gamma (UI_i + DI_i) + \gamma (UA_i + DA_i) \end{split}$$

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