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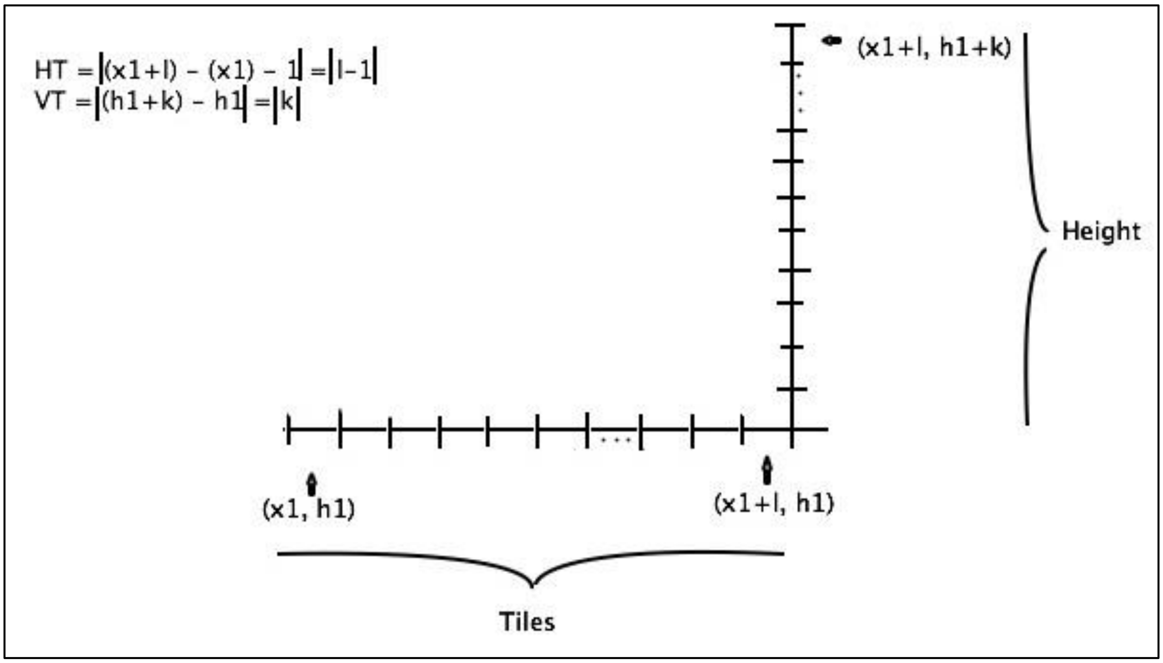
ECS 170 – Part 1, 1st Programming Assignment

## 1st Heuristics – Exponential

To calculate the heuristic value to go from a tile **T1** to another tile **T2** we need to keep track of 2 variables: minimal number of Horizontal Transitions (**HT**) and minimal number of Vertical Transitions (**VT**); these variables are deterministic, as shown below:




To find the better path between T1 and T2, we need to avoid large steps; in other words, we have to keep a constant height progression. This minimizes the value of our exponential cost function. The most reasonable choice for the change in height over the interval is $\frac{VT}{HT}$. There are different cases to consider here, however. For some points, the change in height will be higher than the change in distance, and for other points, the opposite is true. This creates a fraction in $\frac{VT}{HT}$. With testing, we find that simply using the decimal equivalent rather than separating our intervals into integer increments provides a higher upper bound.

Our total cost is simply the sum of HT steps, each with a constant change in height.

$$h = HT * e^{VT/_{HT}}$$

**2nd Heuristics – Division**

The second cost function provides new characteristics to take advantage of. Traveling downhill is extremely cheap, and traveling from a high node to a much lower node approaches zero cost. There is also a very cheap way to travel horizontally: by going to a height of one first. At a height of one, travelling to an adjacent node of equal height is $\frac{h1}{h2+1} = \frac{1}{2}$. The next advantage is that travelling up steep slopes is related to a percent increase in height instead of a scalar increase. For example, travelling from height 1 to height 10 is $\frac{10}{2}$, while the cost for the same height change, but starting at height of 100, is $\frac{110}{101}$. The first transition costs 5, while the second cost is very close to 1.

First I define several variables.

$$p2: \ current\ height$$
$$FD: \ flat\ distance\ to\ travel$$
$$CD: \ climbing\ distance\ to\ travel$$

I first calculate the climbing distance, or the horizontal distance over which I've decided to do my climbing. $CD\ = ceiling(\lg VT)$. I decide that once I reach an appropriate distance, I will double the height that I climb over every horizontal unit, until the next transition will put me at least at the target height. The first step will cost one exactly. All further climbing steps will cost between one and two if they all double in height. As height increases, this value is closer to a cost of two. Thus I upper bound the cost of climbing with

$$cost < 2(CD - 1) + 1$$

I then subtract from HT the climbing distance plus one more to account for the first step taken. $FD = HT - CD - 1$. Step one is to move to a height of one. In the second step, I move horizontally until I near the end state. Third step, I double my height over every horizontal transition until the goal state is reached. This necessarily underestimates the actual cost, as its efficiency comes from the idea that it is able to find a level path for most of the journey. It also relies on the idea that when our hero finally decides to climb the mountain, that he does so in parabolic fashion, constantly increasing his climbing height per horizontal unit travelled.

$$h = \frac{1}{p1 + 1} + \frac{FD}{2} + 2(CD - 1) + 1$$