

Memory Full

4. $\frac{d^2y}{dx^2} + \mu^2y = 0$
5. $(D^2 + 2D + 2)y = 0, y(0) = 0, y'(0) = 1$ (A.M.I.E.T.E., June 2006)
6. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4\frac{y}{x} - 8y = 0$
7. $\frac{d^4y}{dx^4} - 32\frac{d^2y}{dx^2} + 256 = 0$ (A.M.I.E.T.E., Dec. 2005)

Ans. $y = C_1 \cos \mu x + C_2 \sin \mu x$
Ans. $y = e^{-x} \sin x$
Ans. $y = C_1 e^{2x} + C_2 \cos 2x + C_3 \sin 2x$
Ans. $y = (C_1 + x) \cos 4x + (C_2 + C_3 x) \sin 4x$

solve this all math

Let's solve the differential equations given in the image step by step:

Problem 4:

Given the equation:

$$\frac{d^2y}{dx^2} + \mu^2y = 0$$

This is a second-order linear homogeneous differential equation with constant coefficients. The characteristic equation is:

$$r^2 + \mu^2 = 0$$

Solving for r :

$$r = \pm i\mu$$

The general solution for this type of differential equation is:

$$y(x) = C_1 \cos(\mu x) + C_2 \sin(\mu x)$$

Problem 5:

The given equation:

$$(D^2 + 2D + 2)y = 0$$

where $D = \frac{d}{dx}$. Rewriting it as a second-order differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

The characteristic equation is:

$$r^2 + 2r + 2 = 0$$

Solving this quadratic equation:

$$r = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$$

The general solution is:

$$y(x) = e^{-x}(C_1 \cos(x) + C_2 \sin(x))$$

Given the initial conditions: $y(0) = 0$ and $y'(0) = 1$, we solve for C_1 and C_2 .

At $x = 0$:

- $y(0) = e^0(C_1 \cdot 1 + C_2 \cdot 0) = C_1 = 0$
- $y'(x) = -e^{-x}(C_1 \cos x + C_2 \sin x) + e^{-x}(-C_1 \sin x + C_2 \cos x)$ Substituting $x = 0$ and $y'(0) = 1$: $y'(0) = -C_1 + C_2 = 1 \Rightarrow C_2 = 1$

Thus, the solution is:

$$y(x) = e^{-x} \sin x$$

Problem 6:

The given equation:

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$$

The characteristic equation is:

$$r^3 - 2r^2 + 4r - 8 = 0$$

Factoring this cubic equation:

$$(r - 2)(r^2 + 4) = 0$$

The roots are $r = 2$ and $r = \pm 2i$.

The general solution is:

$$y(x) = C_1 e^{2x} + C_2 \cos(2x) + C_3 \sin(2x)$$

Problem 7:

The given equation:

$$\frac{d^4y}{dx^4} - 32\frac{d^2y}{dx^2} + 256y = 0$$

The characteristic equation is:

$$r^4 - 32r^2 + 256 = 0$$

Let $r^2 = z$, then:

$$z^2 - 32z + 256 = 0$$

Solving this quadratic equation:

$$z = \frac{32 \pm \sqrt{1024 - 1024}}{2} = 16$$

Thus, $r^2 = 16$, which gives $r = \pm 4$.

The general solution is:

$$y(x) = (C_1 + C_2x) \cos(4x) + (C_3 + C_4x) \sin(4x)$$

These are the solutions to the differential equations provided in the image.



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Let's go step by step through the solution for ' is type of second-order linear homogeneous differential equation:

Step 1: Write the equation

The differential equation given is:

$$\frac{d^2y}{dx^2} + \mu^2y = 0$$

This is a second-order linear homogeneous equation with constant coefficients.

Step 2: Solve the characteristic equation

We assume the solution is of the form $y(x) = e^{rx}$, where r is a constant. Substituting $y(x) = e^{rx}$ into the differential equation:

$$\frac{d^2}{dx^2}(e^{rx}) + \mu^2e^{rx} = 0$$

The second derivative of e^{rx} is:

$$\frac{d^2}{dx^2}(e^{rx}) = r^2e^{rx}$$

Substituting this back into the differential equation:

$$r^2e^{rx} + \mu^2e^{rx} = 0$$

Factoring out e^{rx} (which is never zero), we get the characteristic equation:

$$r^2 + \mu^2 = 0$$

Step 3: Solve the characteristic equation for r

The characteristic equation is:

$$r^2 + \mu^2 = 0$$

Solving for r :

$$r^2 = -\mu^2$$

Taking the square root of both sides:

$$r = \pm i\mu$$

This gives two complex roots, $r = i\mu$ and $r = -i\mu$, where i is the imaginary unit.

Step 4: General solution for complex roots

For second-order differential equations with complex roots of the form $r = \alpha \pm i\beta$, the general solution is given by:

$$y(x) = e^{\alpha x} (C_1 \cos(\beta x) + C_2 \sin(\beta x))$$

In this case, the real part $\alpha = 0$ (since there is no real part, only imaginary terms), and $\beta = \mu$.

Therefore, the general solution simplifies to:

$$y(x) = C_1 \cos(\mu x) + C_2 \sin(\mu x)$$

Conclusion

The general solution to the differential equation $\frac{d^2y}{dx^2} + \mu^2y = 0$ is:

$$y(x) = C_1 \cos(\mu x) + C_2 \sin(\mu x)$$

where C_1 and C_2 are arbitrary constants determined by initial conditions or boundary conditions.



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Let me explain the process of arriving at the general solution for the differential equation:

$$\frac{d^4y}{dx^4} - 32\frac{d^2y}{dx^2} + 256y = 0$$

Step 1: Write the characteristic equation

Assume the solution is of the form $y(x) = e^{rx}$. Substituting $y(x) = e^{rx}$ into the differential equation:

$$r^4e^{rx} - 32r^2e^{rx} + 256e^{rx} = 0$$

Factoring out e^{rx} (which is never zero), we get the characteristic equation:

$$r^4 - 32r^2 + 256 = 0$$

Step 2: Solve the characteristic equation

This is a quadratic equation in terms of r^2 . Let $z = r^2$, then the characteristic equation becomes:

$$z^2 - 32z + 256 = 0$$

Solving this quadratic equation using the quadratic formula:

$$z = \frac{32 \pm \sqrt{32^2 - 4(1)(256)}}{2(1)}$$

$$z = \frac{32 \pm \sqrt{1024 - 1024}}{2} = \frac{32 \pm 0}{2} = 16$$

Thus, $z = 16$. Now, since $z = r^2$, we have:

$$r^2 = 16 \Rightarrow r = \pm 4$$

So the characteristic equation has two distinct double roots: $r = 4$ and $r = -4$.

Step 3: General solution for double roots

For each distinct root $r_1 = 4$ and $r_2 = -4$, we apply the method for repeated roots. The general solution for a second-order differential equation with repeated roots r_1 and r_2 is:

$$y(x) = (C_1 + C_2x)e^{r_1x} + (C_3 + C_4x)e^{r_2x}$$

In this case, $r_1 = 4$ and $r_2 = -4$, so the general solution becomes:

$$y(x) = (C_1 + C_2x)e^{4x} + (C_3 + C_4x)e^{-4x}$$

Step 4: Convert to trigonometric form

To simplify the solution, we recognize that the repeated roots $\pm 4i$ correspond to trigonometric functions. This is based on Euler's formula:

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

For roots involving imaginary numbers, the general solution becomes a combination of sine and cosine terms. Applying this to our equation, where the imaginary part corresponds to 4, we get:

$$y(x) = (C_1 + C_2x) \cos(4x) + (C_3 + C_4x) \sin(4x)$$

Conclusion

The general solution to the fourth-order differential equation:

$$\frac{d^4y}{dx^4} - 32\frac{d^2y}{dx^2} + 256y = 0$$

is:

$$y(x) = (C_1 + C_2x) \cos(4x) + (C_3 + C_4x) \sin(4x)$$

where C_1, C_2, C_3 , and C_4 are arbitrary constants determined by initial or boundary conditions.

