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A new key performance indicator model for demand forecasting in inventory management considering supply chain reliability and seasonality

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ABSTRACT

Forecasting demand and determining safety stocks are key aspects of supply chain planning. Demand forecasting involves predicting future demand for a product or service using historical data and other external and internal drivers. Stockouts and excess production can be reduced by accurately forecasting demand. This allows companies to plan production, inventory, and logistics more effectively. Companies maintain safety stocks in their inventory to protect against unexpected changes in demand or supply. A company must find the appropriate safety stock level to meet customer demands while avoiding excess inventory and carrying costs. Forecasting demand and determining safety stocks work together to help companies reduce costs, improve customer service, and optimize inventory levels. Key Performance Indicators (KPIs) are commonly used to measure model performance. Classical forecasting models mostly concern themselves with minimizing forecast errors. However, the impact on inventory costs is not directly considered. In this paper, we introduce a Key Performance Indicator to be used in the demand forecasting process that produces more efficient results in terms of inventory costs. We also propose a novel approach to determining the best level for safety stock. This approach considers logistic network supply reliability and seasonality indices identified within historical demand patterns. We use real-life data and show that the proposed method can improve efficiency in forecasting and safety stock levels by reducing the risk of stockouts and excess inventory.

1. Introduction

Supply chain systems have always struggled to forecast customers' future demand. Having an accurate estimation of demand in the future enables supply chain specialists to make better decisions in planning and taking efficient actions in operational processes. According to Fig. 1, as a general form of multi-echelon supply chain networks, demand forecasting aids supply chain planners in answering “how much product should be sent to each retail store”, “what is the optimal inventory level for each product to keep at each distribution center?”, “from each manufacturing plant, how much product should be produced and shipped?”, “what is the estimated amount of raw materials to be purchased from suppliers in the coming months?”, as well as many other questions. To answer these questions, practitioners and researchers have done extensive research in the last decades. In order to answer these questions, one of the most crucial parameters is customers’ future demand. As a result, supply chain systems will be properly prepared for the future by using efficient forecasting models. These models provide more reliable information for the supply chain planning processes.

The first step in demand forecasting problems is determining the granularity of forecasting by determining what level of aggregation we want to obtain the forecasts for. After defining the objectives and problem requirements, the forecasting time horizon and time bucket need to be determined. Following that, the most appropriate forecasting method(s) should be selected based on available historical demand data, objectives, limitations, etc. Moreover, in any forecasting model, a Key Performance Indicator (KPI) is required to measure the forecast's performance.(Vandeput Nicolas, 2020) Fig. 2 illustrates a general framework for building a supply chain demand forecasting model.

After gathering historical demand data, the data should be divided into training and test sets. To be able to find the proper forecasting method (s) and find the best values for the related parameters, we should keep the test data set aside and apply the forecasting model to the training data set. To obtain forecasts over the next periods, each candidate method must be applied to the training data set with different values for its parameter(s). Following that, the outputs must be compared to the actual values of the test set and the selected KPI(s) must be calculated. We can determine the best forecasting model and the optimum parameter values

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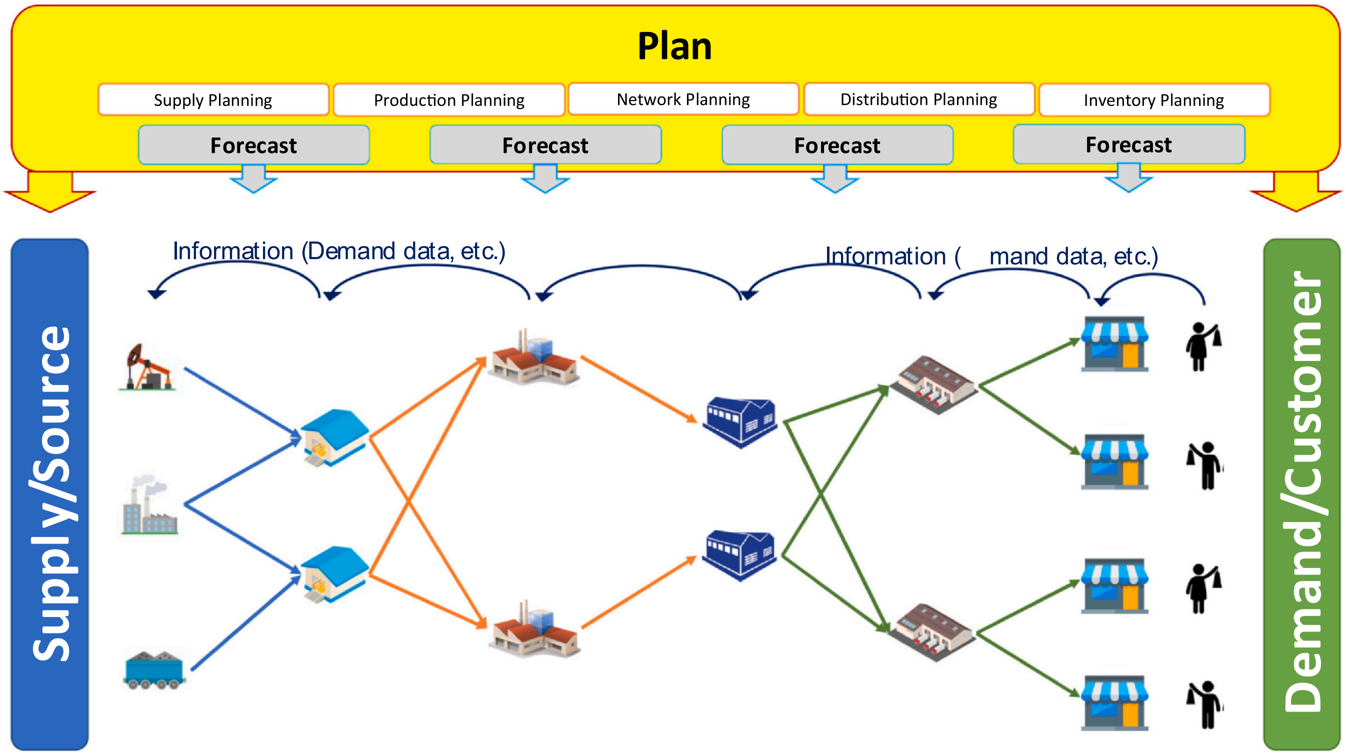

Fig. 1. A general representation of a multi-echelon supply chain network.

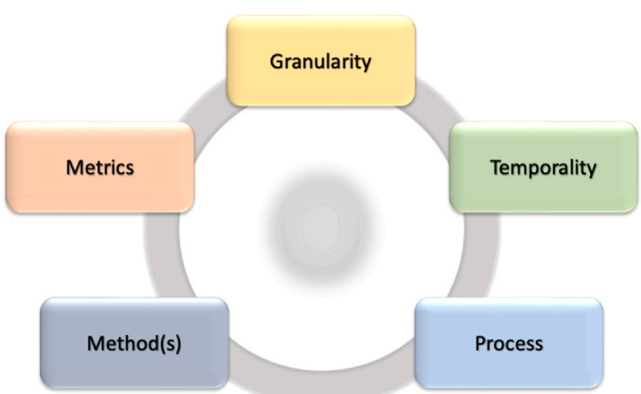

Fig. 2. A General Framework for Building a Supply Chain Demand Forecasting Model.

for our problem by comparing the KPI(s) for different forecasting methods and parameters. We have proposed a new KPI that leads to more efficient inventory cost and fill rate forecasts in this study. We will go another step further with this KPI and look for not only the most accurate forecasts but also forecasts that can help us decrease inventory costs while also improving fill rates.

In addition to demand forecasting itself, the linkage between demand forecasting and safety stock level determination is vital for optimizing inventory management and ensuring customer satisfaction in any supply chain system. However, to further enhance this process, it is crucial to consider two key factors separately: seasonality indices and supply reliability factors. Firstly, seasonality indices play a pivotal role in determining safety stock levels. Many products experience fluctuations in demand based on seasonal variations, holidays, or specific events. By analyzing historical data and identifying these patterns, companies can develop a more accurate forecast for future demand. This enables them to adjust their safety stock levels accordingly, ensuring adequate inventory availability during peak seasons and avoiding excess inventory during slower periods. By aligning safety stock levels with seasonal demand patterns, companies can optimize their inventory investment and enhance operational efficiency. Neglecting to account for seasonality indices can lead to stockouts during high-demand periods or excessive inventory during low-demand periods, resulting in lost sales and increased holding costs. Secondly, supply reliability factors are crucial in determining safety stock levels. Supply chains often face disruptions, such as delays in transportation, production bottlenecks, or unforeseen events impacting suppliers. It is essential to assess the reliability of the supply chain partners and consider the potential risks associated with their performance. By incorporating supply reliability factors into safety stock calculations, companies can determine the buffer stock required to mitigate supply chain risks effectively. For products with higher risk levels due to supply disruptions, a larger safety stock may be necessary to ensure uninterrupted availability and customer satisfaction. Failing to consider supply reliability factors can leave companies vulnerable to supply chain disruptions, resulting in delayed order fulfillment, dissatisfied customers, and a damaged reputation. To optimize safety stock level determination using seasonality indices and supply reliability factors, companies can leverage advanced analytics and machine learning algorithms. These technologies can process vast amounts of data, including historical sales records, weather patterns, promotional activities, supplier performance, and external market indicators. By utilizing these data sources, companies can develop more accurate demand forecasts, identify seasonal demand patterns, and assess the reliability of their supply chain partners. This information can then be used to calculate optimal safety stock levels that align with both demand variations and potential supply disruptions. By considering seasonality indices and supply reliability factors separately in safety stock level determination, companies can achieve several benefits. Firstly, they can proactively manage inventory levels based on expected demand fluctuations, reducing the risk of stockouts or excess inventory. Secondly, by considering supply reliability, companies can mitigate the impact of supply chain disruptions, ensuring a reliable flow of products to meet customer demands. Additionally, optimizing safety stock levels helps reduce inventory holding costs as inventory is maintained at appropriate levels. Ultimately, these improvements lead to enhanced customer service levels by ensuring product availability during peak seasons and minimizing order fulfillment delays. In conclusion, the importance of considering seasonality indices and supply reliability factors in safety stock level determination cannot be overstated. By incorporating these factors into the decision-making process, companies can optimize inventory management, minimize stockouts, reduce holding costs, and improve customer satisfaction. Leveraging advanced analytics and machine learning algorithms can facilitate accurate demand forecasts and supply chain risk assessments, enabling companies to determine optimal safety stock levels. Ultimately, these practices contribute to a well-functioning supply chain system and a competitive advantage in the marketplace.

In addition to enhancing the efficiency of the demand forecasting model and linking it with safety stock determination, this paper aims to address some key challenges that arise in supply chain management. One significant challenge is the consideration of seasonality, which refers to the cyclical patterns and fluctuations in demand that occur during specific periods or events. By incorporating seasonality into the safety stock determination process, companies can better align their inventory levels with expected demand variations, ensuring optimal stock availability during peak seasons and avoiding overstocking during off-peak periods. Moreover, another critical challenge in supply chain management is supply reliability. Unforeseen disruptions in the supply chain, such as delays in transportation, production bottlenecks, or supplier-related issues, can have a significant impact on a company's ability to fulfill customer demand. By taking into account supply reliability factors, such as historical supplier performance, delivery lead times, and potential risks associated with suppliers, companies can establish appropriate safety stock levels to mitigate the impact of these disruptions. This proactive approach helps ensure a consistent flow of products to meet customer demands, even in the face of unforeseen supply chain disturbances.

To achieve the objectives of this paper, we propose a novel forecasting KPI that incorporates inventory costs, which provides a more comprehensive perspective on the accuracy and effectiveness of the forecasting model. By considering the implications of inventory costs such as shortages and excess inventories, companies can make more informed decisions about their safety stock levels and overall inventory management strategies.

Additionally, our proposed method for determining safety stock levels takes into account the impact of seasonality and supply reliability on the logistics network. By considering these parameters, we can set the most appropriate safety factor for each Stock Keeping Unit (SKU) at each node of the logistics network. This approach ensures that sufficient safety stock is maintained at every period and location within the distribution network, minimizing the risk of stockouts and optimizing customer service levels.

In summary, this paper not only focuses on enhancing the demand forecasting model and linking it with safety stock determination but also addresses the challenges of seasonality and supply reliability. By considering these factors, companies can optimize their inventory levels, improve customer service, mitigate supply chain disruptions, and ultimately enhance their overall supply chain performance.

The following will provide a very quick overview of the topic's background. Afterward, the proposed method will be explained in detail, and then a real data set will be used to evaluate its efficiency. In the end, the results will be discussed, and conclusions will be drawn.

2. Topic Background

Supply chain planning is the process of coordinating and aligning various activities involved in the production, delivery, and distribution of goods and services. It involves forecasting demand, determining inventory levels, scheduling production, allocating resources, and managing logistics [15,52].

Integrated planning is a form of cooperation between an organization and its supply chain partners as well as collaboration among different functions in its supply chain system. The objective is to integrate and effectively coordinate multiple financial, informational, and physical flows within and outside the organization. This results in the effective and efficient management of intra-organizational and inter-organizational processes [20,41]. As a result of supply chain integration, it is often possible to align processes and results between an organization and its supply chain members [34].

In every supply chain system, numerous crucial processes are carried out on a regular basis. Among these processes, forecasting future demand and determining safety stock levels are particularly vital in supply chain planning. The objective of this paper is to combine these two consecutive and essential processes into a single model, which requires a meticulous approach. The integration of demand forecasting and safety stock level determination is crucial for achieving optimal inventory management and meeting customer demands efficiently. By accurately forecasting future demand, companies can make informed decisions about the quantity and timing of their inventory replenishment. Simultaneously, determining appropriate safety stock levels helps buffer against uncertainties in demand and supply chain disruptions. To integrate these processes effectively, careful consideration must be given to various factors. Firstly, historical data, market trends, and relevant information should be leveraged to develop an accurate demand forecast. This forecast serves as the foundation for determining the required safety stock levels. Additionally, factors such as lead times, supplier reliability, and seasonality need to be carefully analyzed to ensure that safety stock levels are adjusted accordingly. The integration of demand forecasting and safety stock level determination into a unified model offers several benefits. It facilitates a more streamlined and efficient decision-making process by providing a comprehensive view of inventory requirements based on anticipated demand fluctuations and potential disruptions. By integrating these processes, companies can optimize their inventory levels, minimize costs associated with excess inventory or stockouts, and enhance customer satisfaction. However, it is crucial to approach this integration with caution. The accuracy of the demand forecast and the appropriateness of safety stock levels heavily influence supply chain performance. Inaccurate demand forecasts can lead to suboptimal inventory levels, resulting in lost sales or increased holding costs. Similarly, incorrect determination of safety stock levels can disrupt the balance between stock availability and cost-effectiveness. Totally we can say that integrating the processes of demand forecasting and safety stock level determination is a critical aspect of supply chain planning. This paper aims to merge these two processes into a unified model, emphasizing the need for careful consideration and attention to various factors. By achieving a successful integration, companies can enhance their inventory management, mitigate supply chain risks, and ultimately improve overall supply chain performance.

Demand forecasting is always one of the basic challenges in supply chains. This is followed by a detailed examination of this approach and the literature in this field. Demand forecasting is an essential tool for launching upcoming products, planning production, determining necessary inventory levels, and creating optimal distribution methods. Mistakes between low and high estimates can be very costly. Forecasting demands more than the actual may lead to an excessive increase in the investments in production and inventory. This may result in financial waste and reduce profitability [33]. The other side of the coin is that underestimating demand can have negative outcomes. When the company forecasts less than the actual level, it cannot increase mobility, invest in new technology, and plan for the future. Thus, if the actual demand exceeds the predicted amount, we lose opportunities and potential customers. However, we also make some customers unhappy because we cannot meet their demands, which negatively impacts our business [12]. There are different methods to forecast future demand. These methods will be explained in the following sections as statistical methods and machine learning algorithms.

Demand forecasting in the supply chain is a crucial process that helps companies predict future demand for their products or services. This allows them to make informed decisions about production, inventory management, and logistics [50]. There are several different approaches to demand

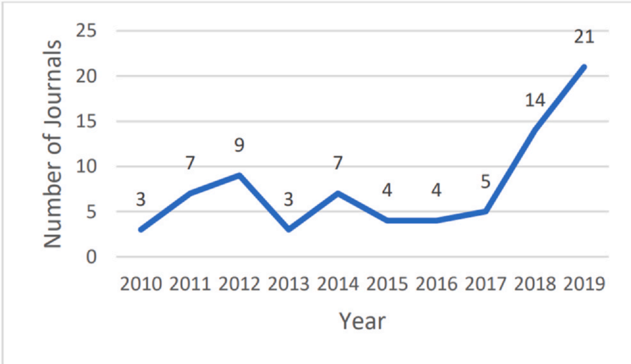

Fig. 3. The number of articles in the field of prediction with machine learning [1].

forecasting, including time series analysis, causal analysis, and qualitative analysis as well as machine learning algorithms. Time series analysis is one of the most common methods used in demand forecasting. It involves analyzing historical data to identify patterns and trends that can be used to predict future demand. Time series methods include Moving Averages (MA), Exponential Smoothing (ES), and Autoregressive Integrated Moving Average (ARIMA) models. Hyndman, Athanasopoulos [27,38], Causal analysis methods, such as regression analysis and econometrics, are used to identify the factors that drive demand and to estimate the impact of these factors on future demand. Archer [5] This approach can be useful for understanding the underlying causes of demand and for making predictions about how changes in these factors will affect demand. Merkuryeva et al., [39] Qualitative analysis methods, such as Delphi and market research, involve gathering input from experts and stakeholders to make predictions about future demand. These methods can be useful for understanding the subjective factors that influence demand, such as consumer preferences and market trends [58,35]. Machine learning methods have also been used in recent years for demand forecasting, such as Random Forests, Neural Networks, and Bayesian Networks. These methods have been found to be more accurate and efficient in comparison to traditional methods. Carbonneau et al., [13,47].

In the last two decades, human technical abilities to generate and collect data have increased rapidly. Factors such as the widespread use of barcodes for commercial products, the use of computers in business, the growth of science and technologies during the 4th industrial revolution, and progress in data collection tools have led to the collection of various data in organizations [1]. Using data mining and machine learning algorithms (ML algorithms) has also been developed in recent years. One of the areas where data and machine learning algorithms are used is demand forecasting. For example [1], a study has reviewed prediction articles with machine learning algorithms. In their study, 1870 articles were collected from Scopus and Web of Science databases related to the subject. Finally, 79 articles about ML-based demand forecasting were selected. Their findings showed that, as shown in Fig. 3, the trend of using ML algorithms for demand forecasting problems is increasing in recent years.

Also, their results showed that among the various algorithms for forecasting, neural networks, support vector regression, and time-series methods are among the best algorithms for demand forecasting. In general, comparing the three approaches of neural networks, regression, and time series, the pros and cons of each are shown in Table 1.

Demand forecasting performance is often evaluated using Key Performance Indicators (KPIs) such as accuracy, bias, and error measures. These KPIs provide a way to measure the effectiveness of a forecasting model and identify areas for improvement.

Accuracy measures, such as Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Symmetric Mean Absolute Percentage Error (SMAPE) are commonly used to evaluate the overall performance of a forecasting model. These measures compare the predicted demand with the actual demand, and the smaller the error, the better the accuracy of the model. Makridakis [36,37] Bias measures, such as Mean Forecast Error (MFE) and Mean Absolute Deviation (MAD) are used to evaluate if a forecasting model has a consistent bias. A positive bias would mean that the model is consistently over-forecasting and a negative bias means the model is consistently under-forecasting [13]. Error measures, such as forecast error and prediction intervals, are used to evaluate the uncertainty of the forecasting model. These measures give an indication of how much the forecast can vary from the actual demand and can be used to assess the risk of forecasting [46]. Overall, the literature on demand forecasting performance KPIs highlights the importance of using multiple measures to evaluate the effectiveness of a forecasting model and identify areas for improvement. The choice of measure will depend on the nature of the data, the forecasting method used, and the objective of the forecasting.

Many studies have been done on demand forecasting with ML algorithms. Vithitsoontorn, Chongstitvatana [57] used time series algorithms to predict dairy products for production planning. Their research carried out a direct multi-stage demand forecasting approach on 8 dairy products over 5 years. Their results showed that both statistical and deep learning methods are appropriate for demand forecasting. So, the Autoregressive Integrated Moving Average (ARIMA) method predicted the future in an average straight line and showed the best result in a few staggered series, while Long Short-Term Memory Networks (LSTM) predicted the future value following a season of time series. Also, Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Scaled Error (MASE) metrics had been used, and a final evaluation metric of their proposed model was calculated by using these three evaluation metrics. Falatouri et al., [18] studied demand forecasting in the retail supply chain using time series algorithms. The data used in this study consists of 37 months of sales of an Austrian retailer. Then, Seasonal Autoregressive Integrated Moving Average (SARIMA) and LSTM models were developed. Their results showed that LSTM had better results for constant-demand products, while SARIMA was better for products with seasonal behavior. In addition, the results were compared with Seasonal Auto-Regressive Integrated Moving Average with eXogenous factors (SARIMAX) considering the external factor of advertising, and it was concluded that SARIMAX was more appropriate for products with advertising. To evaluate the prediction model, Mean Absolute Percentage Error (MAPE) and RMSE metrics were used, and for the four products that were predicted, the values obtained from these metrics were aligned in 50% of cases. (D.-K [30].) had also focused on a study with the aim of predicting the daily tourism demand in South Korea. In their study, a Multi-Head Attention CNN model (MHAC) model was proposed. MHAC uses a 1D convolutional neural network for analyzing the temporal patterns and also includes an attentional mechanism for considering input variables

Table 1
Comparison of demand forecasting approaches [45].

Method Description	Neural Network	Regression	Time-series
Advantages	Able to specify non-linear relationships between dependent and independent variables, detect probable correlation between independents	Suitable for high-volume data, using linear and non-linear kernels to build the model	Better recognition and understanding of time patterns
Disadvantages	Tendency to overfit, getting stuck in local optimum	Results related to the determination of relevant parameters and absolute non-optimization	Failure to build non-linear models

correlation. The forecasting framework presented in their article was applied to predict the changes of incoming tourists in South Korea while exogenous variables such as politics, disease, season, and attractiveness of Korean culture are considered. Finally, MAPE, RMSE, and the Empirical Correlation Coefficient (CORR) metrics had been used to evaluate the presented model and provided better output in all time periods and data volumes. Goli et al., [24] focused on the use of meta-heuristics and artificial intelligence in the problems of forecasting the demand for different products. They presented a combined and integrated framework including statistical tests, time series neural networks, and improved Multi-Layer Perceptron (MLP), Adaptive Neuro-Fuzzy Inference Systems (ANFIS), and Support Vector Machine (SVR) considering meta-heuristic algorithms with the aim of predicting dairy products' demands in Iran. To do this, in the first step, they identified a number of both economic and social criteria which have a probable impact on dairy demand. In the next step, Pearson's correlation coefficient was calculated for two variables and based on that, ineffective indicators were removed. In the forecasting phase, MLP, ANFIS, and SVR models were created using some meta-heuristic algorithms such as Gray Wolf Optimization (GWO). They concluded that their proposed combined method has a better ability to forecast the demand for dairy products. Zubaidi et al., [59] focused on the problem of urban water demand forecasting and for this purpose, they used the combination of Artificial Neural Network (ANN) and Backward Search Algorithm (BSA-ANN) to adjust and optimize the parameters. The dataset they used was related to the amount of water consumption in South Africa between 2007 and 2016. Their proposed BSA-ANN model has better root mean square error (RMSE) than other prediction models. Abbasimehr et al., [2] also presented an optimal demand forecasting model using the LSTM network. Their proposed method had a hyper-parameter tunning for LSTM hyper-parameters via grid search method compared to methods including autoregressive integrated moving average (ARIMA), exponential smoothing (ES), artificial neural network (ANN), K nearest neighbors (KNN), recurrent neural network (RNN), support vector machines (SVM) and single layer LSTM. The evaluation metrics of their presented model were SMAPE and RMSE, and the results showed that according to the evaluation metrics, the algorithm presented in their article won the best rank. Qu et al., [44] tried to predict demand aiming to price optimization for the semi-luxury supermarket sector. They developed a regression tree and used the past 2.5 years of sales data from various stores in 45 different regions for model training. The evaluation metric of their model was MAPE, according to their findings, it was observed that the random tree model has the least error among the used algorithms. The summary of demand forecasting studies is depicted in Table 2.

Inventory management has been an active research topic in management science during the last decade. In today's world, which has been affected by the digital revolution and the technologies of the Industry 4.0 revolution, organizations are always facing new challenges due to the diversity of customers and a variety of products. Therefore, developing a framework that can automatically check the optimal or near-optimal strategy for different demands and create an inventory control system based on that is essential.

The problem of multi-period inventory management has been focused on for several decades. Kaplan [29,17], developed optimal conditions for the base and (s, S) inventory policies under finite and infinite horizons based on dynamic programming, respectively. Although the optimality of basic inventory policies has been proven under different settings [22,28]. Calculation of the optimal parameter of such a policy is computationally intractable in general cases. Levi et al. (2007) developed approximate algorithms using dual equilibrium techniques. Another option is using heuristic algorithms. However, the stated policies and approaches do not have special knowledge and attention to the seller's demand and delivery time while such information is important for the correct decision-making with the least errors of the managers. It is better to predict the demand in inventory control models first, and based on that, make a decision for the safety stock and (s, S) policies.

Safety stock, also known as buffer stock, is the extra inventory that a company holds in order to mitigate the risk of stockouts or running out of items to fulfill customer demand. Safety stock is intended to provide a buffer against uncertainty in demand, lead times, or other factors that can affect the availability of items. Safety stock is an important component of inventory management in the supply chain. It helps to ensure that customer demand is met, even when there are unexpected fluctuations in demand or disruptions in the supply chain [7].

But another paradigm that is always discussed in inventory control issues is safety stock. There is a strong correlation between the keywords of inventory control and safety stock in the literature. In [25] the authors investigated this correlation by analyzing 95 articles from 2000 to 2020. Also, as shown in this research, 88% of the solution methods in the topic of inventory management and safety stock use optimization approaches, and only 12% of them are hybrid and simulation approaches.

Today, many methods are used for more accurate estimation of safety stock in inventory control problems. For example [11], presented a new approach based on simulation to define inventory policies and optimize inventory for supply chain planning. The inventory policies considered were *(r, Q)* and *(s, S)* policies. In the *(r, Q)* inventory policy, an order is placed for *Q* units every time the inventory level reaches the *r* level, while in the *(s, S)* policy, the inventory is checked at predefined intervals. If the inventory balance is less than level *s*, an order is given to restore level S. In addition, four safety stock formulas are proposed to address demand uncertainty: (1)

Table 2
Summary of some of the demand forecasting studies.

Author	Objective	Method	Evaluation metrics	Case study
Vithitsoontorn, Chongstitvatana [57]	Product forecasting in production planning	Time-series	RMSE MAE MASE Fill rate	dairy products
Falatouri et al.,[18]	Demand forecasting at the supply chain level	Time-series	MAPE RMSE	retail
(D.-K[30].)	Predicting the number of customers	MHAC	MAPE RMSE CORR	Tourism
Goli et al.,[24]	Forecasting product demand	Metaheuristic algorithm and neural network	MSE R^2 MAE	—
Zubaidi et al.,[59]	Forecasting consumption of urban people	Artificial neural network optimized with backward search algorithm (BSA-ANN)	RMSE coefficient of efficiency	Water consumption
Abbasimehr et al.,[2]	Demand forecasting at the supply chain level	Networked LSTM	SMAPE RMSE	—
Qu et al.,[44]	Demand forecasting and price optimization	Decision Tree Regressor	MAPE	Semi-luxury retail

capacity proportional, (2) capacity proportional considering risk pooling effect, (3) explicit risk pooling, and (4) time-guaranteed service. The proposed models enable the optimization of safety stock, and base inventory levels together considering material flow in supply chain planning. Also, (M [31].) used machine learning methods to predict demand with the aim of optimizing the safety level for mass customization in smart production. In their study, it has been stated that mass production in a certain period of time can both guarantee quality and benefit the organization from cost aspects. In this regard, in their study, in order to reach the optimal production point and the optimal inventory level, the demand was predicted using the ARIMA time series algorithm, and then, using mathematical modeling, the final model of the research aimed at optimizing the costs and the inventory level Reassurance was designed. Another study [32] stated that although the optimization of safety inventory has been studied for more than 60 years, most organizations used traditional methods to optimize it. One of the new approaches of Reinforcement Learning (RL) is a subset of artificial intelligence, which removes the concern of simplifying existing control models by simulating the black box. In their paper, three RL methods were provided to optimize the trust inventory. Their findings showed that RL can optimize both the safety stock level and order quantity as inventory policy decision variables. Therefore, it can be seen that if machine learning algorithms and artificial intelligence methods, in general, can be used in optimization problems, the possibility of reaching more accurate answers will increase.

There are different approaches to forecasting models. In the past, predictions were often made as points, which increased the error in estimating values [21]. Therefore, point estimation causes information loss, which has an influence on the next optimization step. On the other hand, if a complete prediction of the random variable distribution can be provided, all the required information can be considered in the optimization and inventory control model. Bertsimas, Kallus [9] used a non-parametric method in their recent study to approximate the distribution of a random variable with a weighted empirical distribution. However, due to computational difficulties, this criterion is not used for numerical testing using real-world datasets that have time series characteristics. In another study [3], proposed a non-parametric method with the objective of predicting conditional density using a kernel mixture network. Also, they were created by a deep neural network that the densities were determined as a linear combination. In general, studies that have extensively combined the topic of demand forecasting with inventory control models are rarely found in the literature. Among the few studies in this field, we can refer to the article [19] which stated that if organizations use past data and consider various variables, they can provide more accurate planning by using data mining algorithms. This planning is for all resources, including goods, employees, and customer needs. The solution method used in this study is a neural network that considers variables such as holidays, elections, regional conflicts, Valentine's Day, Christmas holidays, disease invasion, different seasons, increase and decrease of the dollar, and hot and cold weather to predict demand and manage inventory. Also [48], with the aim of facilitating the optimization of the inventory level and thus improving the order and inventory management system, created a neural network to predict demand in the software environment. To analyze their data, the area of major business with fasteners has been used. Their proposed network includes TRAINGDX as a training function and TANSIG as a transfer function and has a 6–8–1 architecture that had better results than other architectures. In another study [16], addressed the issue that supplying raw materials and having a backup order is a fundamental activity in inventory management. In their study, different ML algorithms were used to predict the demand values and other desired parameters in the existing models, and the algorithms were compared. Variables used in their study include current inventory level, recorded transportation time, transportation quantity, sales forecast for 3, 6, and 9 months, sales amount in 1, 3, 6, and 9 month periods, minimum safety stock level, the purchase risk and the number of orders. Also, in the study of [54], they also presented a data-driven framework to optimize inventory control. In their study, a new inventory cost minimization framework was proposed, which was based on decision tree models. Table 3 shows the summary of inventory management and forecasting studies.

Integrated forecasting and inventory planning in the supply chain is a process of combining demand forecasting with inventory management to optimize inventory levels and improve supply chain efficiency. By combining forecasts of future demand with current inventory levels, companies can make informed decisions about how much to produce or purchase, when to order, and how much safety stock to hold [49]. One approach to integrated forecasting and inventory planning is the use of inventory models, such as the Economic Order Quantity (EOQ) model and the Newsvendor model. These models take into account demand forecasts and inventory costs to determine optimal inventory levels [23]. Another approach is the use of simulation models, which allow companies to test different inventory policies and demand scenarios to see how they will impact inventory levels and costs. Güller et al., [26] A third approach is to use optimization methods, such as linear programming, to determine the optimal inventory levels and production schedules that will minimize costs and meet demand [42]. In recent years, companies have been using advanced analytics, such as machine learning and artificial intelligence, to improve the accuracy of demand forecasting and optimize inventory levels [40,43]. The projects and papers conducted on integrated forecasting and inventory planning in the supply chain highlight the importance of combining demand forecasting with inventory management to optimize inventory levels and improve supply chain efficiency. Different approaches and methods can be used to achieve this goal, such as inventory models, simulation models, optimization methods, and advanced analytics.

Although several studies have explored the integration of demand forecasting and safety stock determination in supply chain systems, there remains a research gap in terms of effectively incorporating inventory costs, seasonality, and supply reliability factors into the decision-making process. While existing literature has primarily focused on forecasting accuracy and traditional KPIs, such as forecast errors, there is a need to explore alternative

Table 3
Summary of some articles on inventory management and forecasting.

Author	Objective	Method	Indicators
Sustrova[48]	Inventory management with neural network	Artificial neural network with TRAINGDX training function and TANSIG transfer function and 6–8–1 architecture	Demand history in the last three months, quarterly demand in the last three years, current inventory level, price, shipping cost
Farhat, Owayjan [19]	Inventory management with neural network	Improved Neural network with ERP	Holidays, elections, regional conflicts, Valentine's Day, Christmas holidays, invasion of diseases, different seasons, rise and fall of the dollar and hot and cold weather.
De Santis et al., [16]	Inventory management using machine learning	Forest GBOOST BLAG	Current inventory level, recorded shipping time, shipping quantity, sales forecast for 3, 6, and 9 months, sales amount for 1, 3, 6, and 9 month periods, minimum safety stock level, purchase risk level, and confidence back order amount Fallen
Theodorou et al., [54]	Optimizing inventory control using data-driven frameworks	Time Series Mathematical model	Current inventory levels, safety stock levels, raw material prices, inflation, customer demand, seasons, holidays, political events

forecasting KPIs that consider the financial implications of inventory costs. Furthermore, limited attention has been given to the simultaneous consideration of seasonality and supply reliability in determining safety stock levels. Existing approaches often overlook the impact of these factors on the logistics network, resulting in suboptimal safety stock allocations across nodes. Therefore, there is a research gap in developing a method that comprehensively accounts for seasonality and supply reliability parameters to set appropriate safety factors for each SKU at each node within the logistics network, ensuring sufficient safety stock availability throughout different periods. To bridge this gap, this paper aims to improve the efficiency of the demand forecasting model and establish a robust link with safety stock determination. By introducing a novel forecasting KPI that incorporates inventory costs rather than relying solely on forecast errors, this study seeks to enhance the accuracy and effectiveness of demand forecasting. Additionally, the proposed method will consider the impact of seasonality and supply reliability of the logistics network in determining safety stock levels, aiming to optimize inventory investment and mitigate supply chain risks. By addressing these research gaps, this study will contribute to the existing body of knowledge in supply chain management by providing a more comprehensive approach to integrating demand forecasting and safety stock determination. It will shed light on the significance of considering inventory costs, seasonality, and supply reliability factors when making inventory-related decisions, ultimately enhancing supply chain performance, cost-effectiveness, and customer satisfaction.

3. Problem statement

In a real business case, supply chain planners are mostly requested to figure out the forecasts of future demand at each distribution center in the logistics network, along with the most appropriate safety stock levels. The mentioned logistics network is a multiple-multiple network consisting of Manufacturing Plants (MP: where the products are produced and sent to the distribution centers), Central Distribution Centers (CDC: where the products are received in large quantities and sent to the other distribution centers), and Regional Distribution Centers (RDC: where the products are received from manufacturing plants or central distribution centers and from there customers are served). There are thousands of products with different demand scales, different values, and different historical demand patterns that must be forecasted. There is a different lead time between the distribution centers and manufacturing plants since they are located in different countries around Europe. There is a possibility of late arrivals, causing the lead time to be uncertain. In light of data related to late arrivals in recent years, we can estimate the expected lead times for the upcoming period. Orders or shipments sometimes aren't delivered in full by suppliers in real life. In other words, it is possible that the distribution centers will receive orders less than what they ordered. A large number of problems arise from this inaccuracy in inventory planning. Based on the information provided in the given problem, we can estimate the accuracy of order fulfillment in future periods by analyzing the available historical data over the last few years. The accuracy of receiving shipments from each supply node can be estimated using data sets in which we have the accuracy (in percentage) of the shipments received from each supplier. For a logistics system to determine the safety stock levels for a group of products, many factors must be taken into account. Among the most important drivers are customers' demands, network configurations, reliability KPIs, and product characteristics. The future demand of the customers must be estimated as accurately as possible since it is one of the most important parameters. As a result, we will have a better idea of what buffer inventory we will need for future periods. Logistics configuration gives us a better idea of lead times and allows us to find some correlations among different nodes in the network concerning late deliveries. It is also common in real-world cases to consider different safety factors for different products depending on their value margin importance Fig. 4.

Another important parameter that should be considered is the reliability of supply networks when determining safety stock. This has not been much discussed in the literature. Therefore, it is extremely important to consider how accurate the receiving shipments will be. For this purpose, analyzing the past performance of the suppliers' network is highly useful.

4. Model development

In this study, we've proposed a 2-phase method to tackle this problem. In the first phase, future demand for each Stock-Keeping Unit (SKU) will be forecasted by implementing a new inventory-related KPI and in the second phase, the most optimum safety stock level for each product at each distribution center will be determined considering a novel method to determine the safety factors. The novelty of the first phase is considering an inventory-related KPI instead of using classical forecasting metrics like Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), etc. and the novelty of the second phase is considering supply reliability factors and seasonality indices as two influencing parameter in safety stock level determination.

First of all.

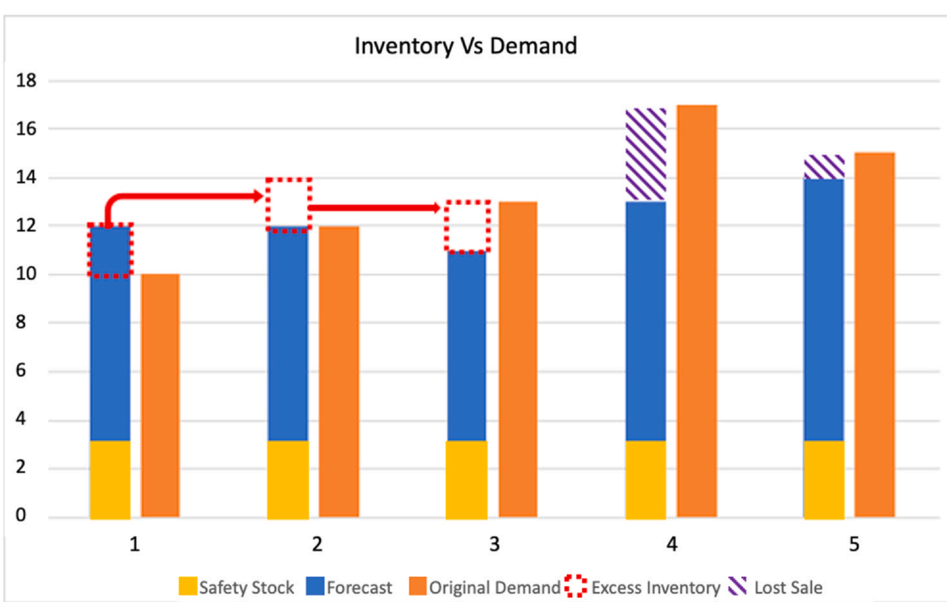

Fig. 4. An example to show how we can calculate the proposed KPI.

4.1. THETA-ATA combination method

Theta method is a statistical method introduced by Petropoulos and Nikolopoulos that in its standard form works with a decomposition of the original time series into two theta lines, then extrapolation of the theta lines to the future periods, and a combination of the extrapolated lines to obtain the forecasts [6]. The first and the second theta lines represent the long-term and short-term behavior of the original time series. ATA method is another statistical method introduced in 2017 that is an alternative to two major forecasting approaches: Exponential Smoothing (ES) and ARIMA. ATA method works by modifying the smoothing constants of various exponential smoothing models in a way that the smoothing constant becomes a function of t (t indicates the period of time). This modification helps to deal with initialization problems in the exponential smoothing methods and makes the optimization process easier. Also with ATA, it is possible to assign past information equal weights [53]. Tadayonrad and Ndiaye have proposed a new forecasting model by combining Theta and ATA methods by which we can benefit from the advantages of both methods [51]. In the standard form of the Theta method, the first theta line is extrapolated by simple linear regression but in the proposed model, ATA is used to represent the long-term component (trend) of the time series.

In the classical Theta method, a prior step is also needed if there is a seasonality in the original data set where seasonality removal is done using the multiplicative decomposition method. In the literature, additive and multiplicative decomposition methods are among the most popular methods to deseasonalize the time series. It has been always a matter of debate as to whether use the multiplicative or additive decomposition method. To do so, many researchers have proposed to visualize the time series and based on the pattern seen in the historical data, select one of those two methods. But in real cases (like the dataset that we are working on) in which we have the historical data of thousands of items, it is very time-consuming to use visualization to tackle this issue. In this paper, we are going to add three additional contributions to the mentioned work. First, we will introduce a new heuristic method to distinguish the seasonality patterns of the historical data and second, we will propose an inventory-related KPI to be used as the forecasting performance measurement KPI.

Here is the first contribution of this paper by which we propose a heuristic algorithm to detect seasonality and find the seasonality pattern (additive or multiplicative).

Detecting the Seasonality Pattern using a Heuristic Algorithm:

I. Find the autocorrelation coefficients for each time series.
The autocorrelation coefficient for lag k denoted as r_k can be obtained by Eq. 1:

$$r_k = \frac{\sum_{i=1}^{n-k} (d_i - d_{avg})(d_{i+k} - d_{avg})}{\sum_{i=1}^{n} (d_i - d_{avg})^2} \tag{1}$$

where r_k is the autocorrelation coefficient for lag k
d_i is the actual value of the original demand time series at period i, n is the number of the available demand values, and d_{avg} is the average of the original demand over the given time series. Based on the values of r_k calculated in for different values for k, we should compare them with the threshold rule based on percentiles in a Normal distribution. On this rule, a time series is seasonal if:

$$|r_m| \geq q_{\left(1-\frac{\alpha}{2}\right)} \sqrt{\frac{1}{n} \quad \left(1 + 2\sum_{k=1}^{m-1} r_k^2\right)} \tag{2}$$

Where m is the periodicity of the data (4 for quarterly data, 12 for monthly data, etc.) and the value of 1.645 refers to the 90% confidence level in a Normal distribution.
If a seasonality lag is found in this step, go to the next step. If we find a seasonal pattern with a lag of l, considering that there are n observations in the original time series we can say that there are ($L = \left\lfloor \frac{n}{l} \right\rfloor$) seasons in the entire time series.
II. Find the approximated trend line using linear regression ($\bar{d}$)
III. Calculate the absolute value of the difference between each original observation and $\bar{d}$ as follows: $e_i = |d_i - \bar{d}|$
IV. Calculate the average of the absolute values for each season separately. For $j = 1, 2, ..., L$, $\bar{e}_j = \frac{(\sum_{j=(j-1)*l+1}^{j*l} e_j)}{l}$.
V. Calculate $\bar{e}'_j = \left|1 - \frac{\bar{e}_j}{(\sum_{k=1}^{j} \bar{e}_k / j)}\right|$; $j = 1, 2, ..., L$
VI. Calculate $\bar{\bar{e}} = STD(\bar{e}'_j)$; STD refers to standard deviation.

If $1 - Tr \leq \bar{\bar{e}} \leq 1 + Tr$ then the seasonality pattern could be considered an Additive pattern.

Else the seasonality pattern could be considered a Multiplicative pattern.

Note that Tr is a threshold that could be set between 0,2 and 0,3 based on the simulations we made.

4.2. Forecasting with THETA-ATA and considering an inventory-related KPI

In a supply chain system, demand forecasting is done to enable the system to be prepared for the future. It means that a balance between supply and demand is needed to make sure that all organizational objectives will be respected such as customers' service level as well as inventory costs.

This paper proposes a new metric to evaluate the forecasting model's performance and optimize it as efficiently as possible. Using the obtained forecasts in the previous step and the safety stock levels determined for each SKU, we evaluate the efficiency of the forecasting model by calculating a weighted sum of shortages and excess inventories that will occur in the test set.

As can be seen in Table 4, this metric is calculated in the following way. There are a variety of parameters that can be used to determine W^1 and W^2. For example, lost sales and holding costs of each SKU could be considered as W^1 and W^2 respectively. The safety stock level for each SKU (i) at each distribution center (j) which is supplied from each supplier (k) is calculated by the following equation:[8].

$$SS_{ijk} = SF(sl)^* \sigma_{ijk}^{F} {}^* \sqrt{LT_{jk}} \tag{3}$$

Where:

i	Represents the SKU (product)	$i = 1, 2, ..., p$
j	Represents the destination (distribution center)	$j = 1, 2, ..., m$
k	Represents the source	$k = 1, 2, ..., n$
σ_{ijk}^{F}	The standard deviation of forecast error for SKU i in destination (distribution center) j which is supplied from source k	
LT_{jk}	The lead time between source k and destination j	
sl	The service level	

In this paper, we use the proposed inventory KPI instead of using the usual KPIs like MAE, RMSE, etc. to measure the performance of forecasts. Undoubtedly, one of the main use of the outputs of the demand forecasting phase is to increase customer service levels and decrease inventory costs.

Where: ($EI_0 = 0$).

The total value of the KPI for each iteration is $\sum_{i=1}^{n} \sum_{j=1}^{n} (W_j^{1*} EI_{ij} + W_j^2 \quad {}^* LS_{ij})$.

Here is a particular example of one SKU over 5 periods.

In Table 5 and Fig. 5, we show a very simple example of how the proposed KPI works.

Table 4
Inventory-related KPI calculation.

Period/ Product	Forecast	Safety Stock	Safety Stock + Forecast	Original Demand	Condition	Excess Inventory	Shortage (Lost Sale)	Inventory Level at the end of the period	Shortage Cost (W1 =2)	Holding Cost (W2 =0,5)	Total expected holding and shortage costs
Period #i Product #j	F_{ij}	SS_{ij}	$F_{ij} + SS_{ij}$	D_{ij}	$if\, F_{ij} + SS_{ij} + EI_{i-1,j} > D_{ij}$	$EI_{ij} = F_{ij} + SS_{ij} + EI_{i-1,j} - D_{ij}$	0	EI_{ij}	0	$W_j^1 * EI_{ij}$	$W_j^1 * EI_{ij} + W_j^2 \ * LS_{ij}$
					$if D_{ij} > (F_{ij} + SS_{ij} + EI_{i-1,j})$	0	$LS_{ij} = D_{ij} - (F_{ij} + SS_{ij} + EI_{i-1,j})$	0	$W_j^2 \ * LS_{ij}$	0	

4.3. Safety stock with supply reliability factors

Service level is an important parameter in safety stock determination. In classical methods, the service level is often considered a constant value that is mostly determined based on the Cycle Service Level (CLS) and Fill Rate (FR) [14,55]. In this paper, we are going to go one step further and consider some important drivers that affect these service level indicators. In general, there are many variables influencing fill rate and cycle service level such as inaccurate shipments, damages and defects, and so on. Here we will focus on some of the main indicators that represent the reliability of suppliers and the logistics network. Based on SCOR 12.0 the rate of Delivery Quantity Accuracy (receiving orders from the upstream suppliers) and the Freight Carrier's Delivery Performance are among the indicators that could be monitored to assess the accuracy of receiving shipments and shipments' early/late arrivals in the logistics system [4]. Tracing the first indicator, we can estimate how much accurate will probably be the coming shipments in the next periods. And analyzing the second metric could help us to estimate the lead time as accurately as possible. This will lead us to decrease the possibility of shortage (in case of late arrivals or receiving the products less than the initial order) and excess stock (in case of early deliveries or receiving the products more than the initial order).

4.3.1. Forecasting supply reliability factors

In this problem, we have the historical data of Delivery Quantity Accuracy that indicates to what extent have been accurately matched the receiving shipments from the sources (Suppliers: manufacturing plants and central distribution centers) with the initial orders that we have put for each demand node (distribution centers). A framework has been proposed by Bono *et. al.* in which the future forecast of this KPI for the entire logistics network for future periods is obtained [10]. Using this framework, we can forecast what the future status of the logistics network will be in terms of suppliers' reliability. In that work, an LSTM RNN has been implemented to obtain the forecasts of the targeted KPI. The procedure for obtaining forecasts in this framework is as follows:

1. Data Pre-processing: Building a Network Model
2. Forecasting Procedure
   a. From Networks to Data
      i. Select a node to forecast
      ii. Select features (correlated routes/linked nodes)
      iii. Create the data frame with variables
   b. Forecasting Model: From Data to LSTM RNN
      i. Define the window to use
      ii. Modify dataset for multivariate time series
      iii. Define model parameters
      iv. Train model
      v. Evaluate
3. Post-processing: From Forecasts to Network

In the first step, a graph of nodes and their connections is drawn based on the historical data for each period in the past.[1] In this step, for each period in the historical data, one specific graph could be drawn. In each graph, the number of shipments sent/received from/at each node determines the size of that node (or each route), and the value of the KPI (Delivery Quantity Accuracy) for that node (or each route) compared to the thresholds determines the color of each node (or each route) that indicates the status of that node (or that route) in that period. (Fig. 6).

In the second step, a node is selected and then the neighbor nodes that are connected to the selected node are determined as the input variables (because of correlations investigated between the value of KPI at each node and the values for the neighbor nodes). Having the original dataset including the time series of the selected nodes, the subdataset is ready to be used in the next steps Table 6.

[1] The network is created by Networkx Python library and drawn using Gephi

Table 5
A numerical example to show how we can calculate the proposed KPI.

Period	Forecast	Safety Stock	Safety Stock + Forecast	Original Demand	Inventory Level at the end of the period	Shortage (Lost Sale)	Excess Inventory	Shortage Cost (W1 = 2)	Holding Cost (W2 = 0,5)
#1	9	3	12	10	2	0	2	0	1
#2	9	3	12	12	2	0	2	0	1
#3	8	3	11	13	0	0	0	0	0
#4	10	3	13	17	-4	-4	0	8	0
#5	11	3	14	15	-1	-1	0	2	0

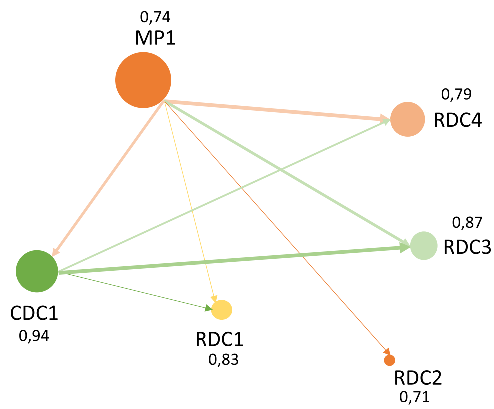

Fig. 5. A simplified example of the targeted KPI (Delivery Quantity Accuracy) network for a particular period (t).

In order to make the initial sub-dataset usable to apply LSTM RNN, it needs to be formatted properly. To do so, an algorithm described by Vandeput is an efficient approach to preparing the train and test sets [56]. Table 7–9.

As the historical data of the targeted KPI, in this case, is weekly data, the values of the KPI in 4 consecutive weeks (which is equal to one month) are considered as X_train and the value of the KPI in the very next week is considered as Y_train.

Like any other forecasting problem, the last bucket(s) could be kept aside to be used for validating the forecasting performance.

Once the sub-datasets are prepared, model parameters (Batch Size and Epochs) and loss function (forecast performance measurement function: in this case Mean Absolute Error) need to be determined to be able to apply the model to the sub-datasets.

Then the model should be trained on the training sub-datasets to find the optimum parameters for each node separately. Finally, by gathering all the forecasted values for all nodes through the entire logistics network, the future status of the network could be plotted. The output of this model is also a table including all the forecasted values of the targeted KPI for all nodes. Therefore, we can use this table to sort the nodes based on their future status in terms of this supply reliability KPI which leads us to a better understanding of potential future supply disruptions. Then we can give higher values to the service level parameter in the nodes that we expect a higher possibility of supply disruptions. In Table 10 we can see an example of how we convert the

Fig. 6. Total shortage and excess inventory costs by implementing two scenarios.

Table 6
The initial sub-dataset for a selected node including the weekly data.

Variables	M1				M2				M3			
	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
# 1	82	99	80	68	90	74	53	89	62	63	97	89
# 2	95	99	86	99	97	90	93	99	81	90	84	99
# 3	80	75	70	73	76	75	74	73	75	80	71	74
…	…	…	…	…	…	…	…	…	…	…	…	…
# m	97	84	81	94	94	89	94	91	80	97	91	88

Table 7
Converting the initial sub-dataset to input 4 periods as input and 1 period as output in the learning phase.

loop	Variables	M1				M2				M3			
		W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
1	# 1	82	99	80	68	90							
1	# 2	95	99	86	99	97							
1	# 3	80	75	70	73	76							
1	…	…	…	…	…	…							
1	# m	97	84	81	94	94							
2	# 1		99	80	68	90	74						
2	# 2		99	86	99	97	90						
2	# 3		75	70	73	76	75						
2	…		…	…	…	…	…						
2	# m		84	81	94	94	89						
3	# 1			80	68	90	74	53					
3	# 2			86	99	97	90	93					
3	# 3			70	73	76	75	74					
3	…			…	…	…	…	…					
3	# m			81	94	94	89	94					
…	…				…	…	…	…	…	…	…	…	…
L	# 1								89	62	63	97	89
L	# 2								99	81	90	84	99
L	# 3								73	75	80	71	74
L	…								…	…	…	…	…
L	# m								91	80	97	91	88

Table 8
The final converted sub-dataset.

loop	Variables	X_train				Y_train
1	# 1	82	99	80	68	90
1	# 2	95	99	86	99	97
1	# 3	80	75	70	73	76
…	…	…	…	…	…	…
1	# m	97	84	81	94	94
2	# 1	99	80	68	90	74
2	# 2	99	86	99	97	90
2	# 3	75	70	73	76	75
…	…	…	…	…	…	…
2	# m	84	81	94	94	89
3	# 1	80	68	90	74	53
3	# 2	86	99	97	90	93
3	# 3	70	73	76	75	74
…	…	…	…	…	…	…
3	# m	81	94	94	89	94
…	…	…	…	…	…	…

Table 9
The test set.

loop	Variables	X_test				Y_test
1	# 1	89	62	63	97	89
1	# 2	99	81	90	84	99
1	# 3	73	75	80	71	74
…	…	…	…	…	…	…
1	# m	91	80	97	91	88

forecasted values of the supply reliability KPI to the relevant service level for each node in the logistics network.

In the given table, $\hat{F}_{DQA}$ is the forecasted value of the targeted KPI in the future periods. Also, $Tr_j^D and Tr_k^S$ indicate the predefined thresholds that need to be determined by the decision-makers.

4.3.2. *Forecasting lead time*

Another important parameter that highly affects the safety stock levels is lead time. To estimate this parameter, we use the historical data of Freight Carrier's Delivery Performance to investigate the late and early deliveries in the last periods and forecast the future lead time for each route (each combination of a supply node and a demand node) in the logistics network. To do so we have implemented LSTM RNN on the mentioned dataset to obtain the expected values of the lead times.

4.3.3. *Determining safety stock levels considering supply reliability factors and uncertain lead times as well as seasonality impact*

Here we propose a new method to determine safety stock levels by taking supply reliability factors and seasonality patterns into account and also assuming the lead time as an uncertain parameter. Despite the classical methods where the service level is a constant parameter, in this paper the service level is considered a dynamic parameter that varies for each SKU and in different seasons and at each node in the logistics network. To consider the seasonality impact in determining the most appropriate service level, we have used the seasonality factors obtained in Section 4.1 where using the autocorrelation method we could investigate the seasonality lags and obtain the seasonality factors by multiplicative/additive decomposition method. To do this we need to interpret the seasonality indices properly to convert them to service level factors. Here is an example of how we convert the seasonality indices using a heuristic approach.

SKU	Seasonality pattern	Season 1				Season 2				Season 3			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
# 1	Additive or Multiplicative	s11	s12	s13	s14	s21	s22	s23	s24	s31	s32	s33	s34

For each SKU at each DC:

$$sl_{ijl}^{P} \quad = Tr_{ij}^{P} + (1 - Tr_{ij}^{P})*(s_l - s_l^{\min})/(s_l^{\max} - s_l^{\min}) \tag{4}$$

An example of seasonality indices for 2 particular SKUs:

SKU	Seasonality pattern	Season 1				Season 2				Season 3			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
# 1	Multiplicative	1,2	1	0,8	1,4	1,2	1	0,8	1,4	1,2	1	0,8	1,4
# 2	Additive	4	2	-2	-4	4	2	-2	-4	4	2	-2	-4

Seasonality indices converted to service level factors:

SKU	Seasonality pattern	Season 1				Season 2				Season 3			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
# 1	Multiplicative	0,89	0,79	0,69	0,99	0,89	0,79	0,69	0,99	0,89	0,79	0,69	0,99
# 2	Additive	0,99	0,79	0,69	0,99	0,89	0,79	0,69	0,99	0,89	0,79	0,69	0,99

Having obtained the service level factors based on supply reliability factors and seasonality indices, the final service level factor for each SKU at each distribution center could be calculated using the following equation:

$$SS_{ijkl} = SF\,(average\,(sl_{ijl}^{P},\, sl_{j}^{D},\, sl_{k}^{S}))*\sigma_{ijk}^{F}*\sqrt{\hat{LT}_{jk}} \tag{5}$$

i	Represents the SKU (product)	$i = 1, 2, \ldots, p \quad j = 1, 2, \ldots, m \quad k = 1, 2, \ldots, n$
j	Represents the destination (distribution center)	
k	Represents the source	
l	Represents the period (to determine the service level based on seasonality index)	
σ_{ijk}^{F}	The standard deviation of forecast error for SKU i in destination (distribution center) j which is supplied from source k	
$\hat{LT}_{jk}$	The estimated lead time between source k and destination j in the next periods	
sl_{ijl}^{P}	The service level parameter calculated according to the seasonality index of the SKU i at distribution center j at period m	
sl_{j}^{D}	The service level parameter calculated according to the reliability factor of the destination j	
sl_{k}^{S}	The service level parameter calculated according to the reliability factor of source k	

5. Test and validation

We implemented the proposed forecasting method including the heuristic seasonality pattern detection as well as the inventory-related KPI on Python and using some libraries like Pandas and NumPy we could run the model over a large real dataset including the historical demand data of 19,961 SKUs in different distribution centers in a logistics network. In this dataset, we had the monthly demand data over five years (60 periods). As explained earlier, machine learning algorithms were used to forecast supply reliability KPIs and lead times between supply nodes and destination nodes. In addition, the proposed heuristics approach was applied to detect seasonality patterns and obtain seasonality indices. Therefore, the safety factor for each SKU at each period and at each particular distribution center was determined considering the seasonality indices for that SKU and the supply reliability of that node. Following the definition of possible scenarios for the related parameters of the Theta-ATA method, a combination of values was examined, and the results were obtained at each iteration. Once the safety stock levels were calculated using the method outlined

Table 10
The scheme used to convert the forecasted values of the supply reliability KPI to the relevant service.

The forecast of "Delivery Quantity Accuracy" for next period ($\hat{F}_{DQA}$)	The value assigned to the Service Level (for each source or destination node)
$\hat{F}_{DQA} < Tr_j^D\, or Tr_k^S (e.\ g.\ 75\%)$	sl_k^S or $\quad sl_j^D = sl_\alpha^S\, or sl_\alpha^D (e.\ g.\ 95\%)$
$Tr_j^D\, or Tr_k^S \le \hat{F}_{DQA} < Tr_j^D\, or Tr_k^S + 10\%$	sl_k^S or $\quad sl_j^D = sl_\alpha^S\, or sl_\alpha^D - 5\%$
$Tr_j^D\, or Tr_k^S + 10\% \le \hat{F}_{DQA} < Tr_j^D\, or Tr_k^S + 20\%$	sl_k^S or $\quad sl_j^D = sl_\alpha^S\, or sl_\alpha^D - 10\%$
$\hat{F}_{DQA} \ge Tr_j^D\, or Tr_k^S + 20\%$	sl_k^S or $\quad sl_j^D = sl_\alpha^S\, or sl_\alpha^D - 15\%$

Table 11
Total shortage and excess inventory costs considering the forecasts obtained using MAE as the forecast KPI and safety stock levels calculated using the classical method.

Number of periods in the test set	Theta2	SES	ATA	Theta2-ATA
13	5272819,00	5286452,00	5421625,00	5255590,00
14	5226858,00	5221772,00	5392910,00	5209587,00
16	5112724,00	5080583,00	5227334,00	5070093,00
Average	5204133,67	5196269,00	5347289,67	5178423,33

Table 12
Total shortage and excess inventory costs considering the forecasts obtained using the proposed KPI and safety stock levels calculated using the proposed method with new service level factors.

Number of periods in the test set	Theta2	SES	ATA	Theta2-ATA
13	5183114,00	5172773,00	5215342,00	4954681,00
14	4894918,00	5125832,00	5185091,00	4901684,00
16	4665779,00	4940024,00	4997411,00	4674807,00
Average	4914603,00	5079543,00	5132614,00	4843724,00
Compared to the classic approach	5,6%	2,2%	4,0%	**6,5%**

previously, the proposed KPI was calculated. In this KPI we have compared the inventory level (sum of safety stock level and the forecasted value) to the actual original demand value on the test set to see to what extent we have allocated the proper inventory level to each DC. Here the goal is minimizing the risk of stock out and excess inventory simultaneously. Afterward, the best set of parameters was chosen by comparing the KPI values for each iteration. (See Appendix, Fig. 7).

6. Results

We first obtained the forecasts by implementing the proposed forecasting method (Theta-ATA method using Mean Absolute Error as the forecasting KPI) and calculated the shortage and excess inventory costs in this scenario considering different lengths of the training set and test set. The original dataset consists of 60 periods (on a monthly basis), and we have kept the test set with different lengths (13, 14, and 16 periods) aside at each iteration. Moreover, we have optimized all the parameters within the models to find the best values that give the least value of MAE (Table 11).

In the second scenario, the classical forecasting KPI has been replaced by the proposed inventory-related KPI and obtained the safety stock levels using the proposed method as well as the forecasts on different lengths of training and test sets (Table 12). In this case, the goal of our forecasting model is to minimize inventory costs (shortages and excess inventories) on the test set.

Here below is the comparison of the results from both scenarios shown. (Fig. 6).

Where:

Theta2:Refers to the classical Theta method with 2 theta lines.

SES:Refers to the Simple Exponential Smoothing method.

ATA:Refers to ATA method.

Theta2-ATA:Refers to the combination of Theta and ATA in which ATA is used to extrapolate the first theta line and other methods (e.g., moving average, simple/double exponential smoothing) are used for the second theta line.

7. Conclusions and future research directions

In conclusion, demand forecasting and safety stock determination are crucial components of a successful supply chain system. The ability to forecast demand accurately helps companies plan production, manage inventory levels, and prevent stockouts in the event of unexpected demand or supply disruptions. Optimizing inventory levels, reducing costs, and improving customer service can be achieved by using effective forecasting and inventory management strategies. In turn, this means a more profitable and efficient supply chain. Nevertheless, it is important to keep in mind that the forecasts and inventory decisions are always uncertain and are dependent on the specific context and industry of the company, so it is important to validate and adapt the approach accordingly. During the demand forecasting phase, we proposed a new KPI that considers inventory costs while we look for the best forecasting method and most appropriate parameter values. Additionally, a heuristic algorithm was introduced in order to identify seasonal patterns more accurately. Furthermore, machine learning algorithms were used to forecast supply reliability factors and lead times. As a result of the forecasting phase, safety stock levels were calculated based on the outputs of the forecasting phase (forecasting demand, forecasting supply reliability factors, and forecasting lead time). In this step, a new approach was proposed to estimate the most appropriate service level for each SKU at each distribution center using supply reliability factors and seasonality indices. Finally, the model was tested on a real data set to prove its efficiency. Comparing the proposed method to the classical method, determining safety stock levels by the proposed method, and optimizing the forecasting method with inventory-related KPIs lead to fewer inventory costs. Future works can also use machine learning algorithms to predict demand and to optimize threshold values for calculating service levels can be determined optimally. Also, using a more enriched dataset, including additional information on the groups of products, will allow us to carry out a more detailed analysis of the proposed method's impact on different groups of products.

Declaration of Competing Interest

None.

Appendix

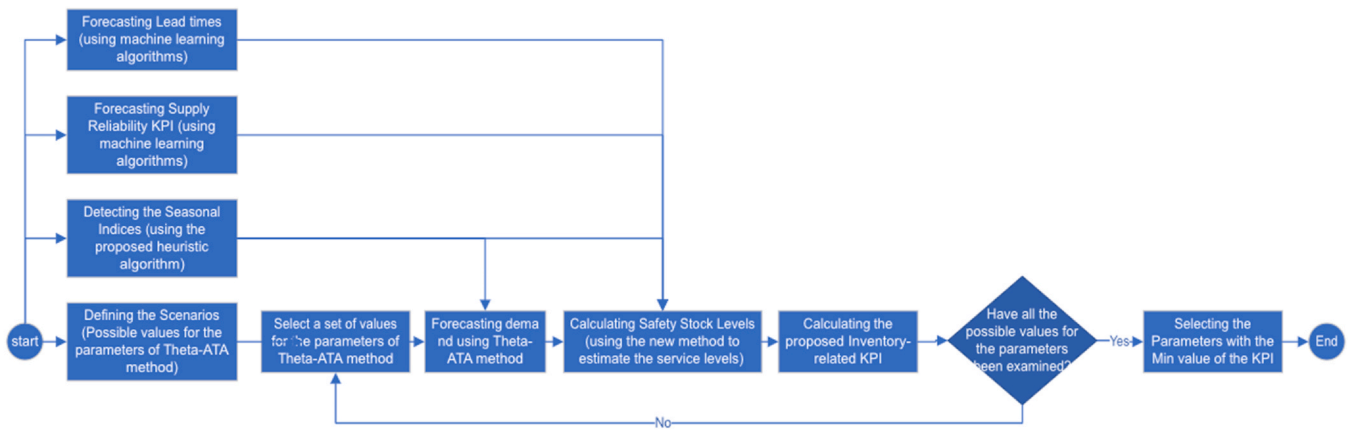

Fig. 7. The framework of testing the proposed model on the real dataset.

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