

stacks (LIFO: Last-in First-out)

2 Main operations: Push(x): Add a new element to the top of stack.

Pop(): removes and returns the top of stack.

Additional ops: peek(), isEmpty(), size(), clear(), ...

↓
top of stack (do not remove)

Implementation: Arrays - Elements occupy arr[0..size-1]
Top of stack index = size - 1

Java: (old way): Stack<Integer> s = new Stack<>(),
(new way): Deque<Integer> s = new ArrayDeque<>(),

O(1) for push/pop . (O(1) amortized for push).

Applications of stacks:

1. Parsing of arithmetic expressions, Evaluation of exps.
2. Conversion of infix exp to postfix, Evaluation of postfix exp
3. Parsing of programming languages in compilers.
4. Memory management at run-time - functions - call.
5. Depth-first search (DFS)
6. Maze creation

⋮

Arithmetic expressions :

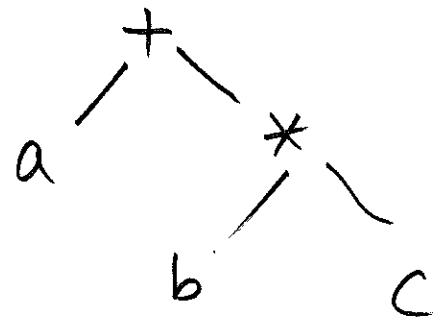
Infix

Postfix

Expression Tree

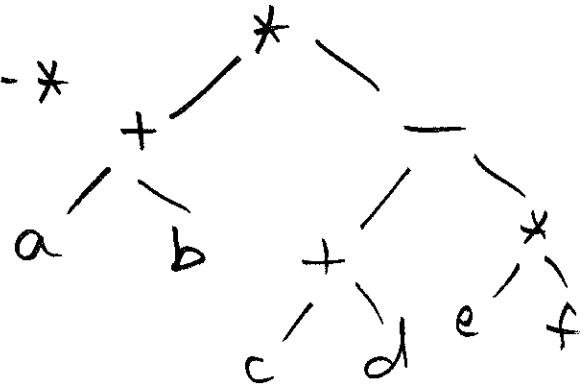
$a + b * c$

$a \ b \ c \ * \ +$



$(a + b) * (c + d - e * f)$

$a \ b \ + \ c \ d \ + \ e \ f \ * \ - \ *$



Defs:

Infix expressions: Operator comes between operands.

- Precedence of ops, Parentheses required, Associativity of operators.

Precedence: hierarchy:

- exponentiation (\wedge , xx)
- Multiplication ($*$, $/$, $\%$)
- Addition ($+$, $-$)

Associativity: $a \ op \ b \ op \ c = (a \ op \ b) \ op \ c$
Left associative

Right associative $\rightarrow = a \ op \ (b \ op \ c)$

Postfix exp: Operator comes after operands.

Unambiguous - without parentheses, precedence, associativity, etc.

Expression trees: binary trees.

internal nodes are operators
leaf nodes are operands. } - Parse tree of expression.

Parsing infix expressions:

(a) infix to postfix: Shunting Yard algorithm.

Stack for operators -

$$\begin{array}{c} O_n \leftarrow \text{top of stack.} \\ \vdots \\ O_2 \\ O_1 \\ \hline \text{bottom.} \end{array}$$

→ { For left associative ops,
 $\text{Prec}(O_{i+1}) > \text{Prec}(O_i)$

For right associative ops, $\text{Prec}(O_{i+1}) \geq \text{Prec}(O_i)$

Queue for Output.

Rules: Process input tokens one at a time.

(a) if token is "(" : Push "(" into stack

(b) if token = ")" : while top of stack is not "("
Pop from stack and add to Q

Pop '(' - discard

(c) if token = operator : if $\text{Prec}(\text{operator}) > \text{Prec}(\text{top of stack})$
Push operator on stack
else while $\text{Prec}(\text{op}) \leq \text{Prec}(\text{top of stack})$: Pop from stack → add to Q

(d) if token = operand: add it to Q.

End: Pop from stack \rightarrow add to Q until stack is empty.

Ex: Input: ~~a~~ * ~~b~~ * ~~c~~

Queue: a b c * +

Stack:

+

*
*

Ex 2: ~~(~~ ~~a~~ * ~~b~~ ~~)~~ * ~~(~~ ~~c~~ * ~~d~~ + ~~e~~ * ~~f~~ ~~)~~

Queue: a b + c d + e f * - *

Stack:

(

*
(

*
(

-
(

*
f
+
*

*
