Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech.
MA101

# 1st SEMESTER REGULAR EXAMINATION – 2023

SUBJECT NAME – Mathematics-I

Branch: ALL

Time – 3 Hours

Full Marks – 100

Answer all Questions from Part – A and Part – B
The figures in the right hand margin indicate marks.

## PART – A

1. Answer the following questions: [15 x 2]

(a) Find the asymptote of $r\theta = a$.

(b) Separate the intervals in which $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.

(c) State Cauchy Mean Value theorem.

(d) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, $xy \neq 0$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$.

(e) Evaluate $\Gamma\left(\frac{13}{2}\right)$.

(f) State Euler's theorem for homogeneous function.

(g) Write the parametric representation of the curve $4x^2 - 3y^2 = 12, z = 1$.

(h) Find the unit normal vector to the surface $z = \sqrt{x^2 + y^2}$ at the point $(6, 8, 10)$.

(i) Determine the length of the curve $\vec{r}(t) = a \cos t\ \hat{\imath} + a \sin t\ \hat{\jmath} + ct\ \hat{k}$ from $(a, 0, 0)$ to $(a, 0, 2\pi c)$.

(j) State Stoke's theorem.

(k) Find the area of the region under the cycloid $r = 1 + 2\cos\theta,\ 0 \leq \pi/2$.

(l) State Gauss divergence theorem.

(m) Find the smallest positive period of the function $\cos 2\pi x$.

(n) Determine whether the function $f(x) = |\sin 4x|, -\pi < x < \pi$ is odd or even .

(o) Find the Fourier transform of $f(x) = \begin{cases} e^{x}, & if\ x < 0, \\ e^{-x}, & if\ x > 0. \end{cases}$

## PART – B

2. Answer any two: [7 x 2]

(a) Verify Lagrange's Mean Value theorem for $f(x) = \log x$ in $[1, e]$.

(b) If the equation of the cycloid $x = a(t + sin\ t), y = a(1 - cos\ t)$, then prove that $\rho = 4a \cos\left(\frac{1}{2}t\right)$.

(c) Find all the asymptote of the curve

$$x^3 + 4x^2y + 4xy^2 + 5x^2 + 15xy + 10y^2 - 2y + 1 = 0.$$

3. Answer any two: [7 x 2]

(a) Show that the function $f(x,y) = \begin{cases} (x+y)\sin\left(\frac{1}{x+y}\right), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

is continuous at (0,0), but its partial derivative $f_x$ and $f_y$ do not exist at (0,0).

(b) For a positive number $m$, then prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

(c) Expand the function $f(x,y) = x^2 + xy - y^2$ by Taylor's theorem in powers of $(x - 1)$ and $(y + 2)$.

4. Answer any two: [7 x 2]

(a) Show that the differential form under the integral sign is exact and also evaluate

$$\int_{(0,0,0)}^{(4,1,2)} (2xy^2dx + 2x^2ydy + dz).$$

(b) Prove that $div(curl\ \vec{v}) = 0$.

(c) Evaluate $\iint_R e^{-(x^2+y^2)}dxdy$, where $R$ is the region $0 < x < \infty$ and $0 < y < \infty$.

5. Answer the followings: [7 x 2]

(a) Using Green's theorem, evaluate $\oint_c \vec{F}(\vec{r}).d\vec{r}$, where $\vec{F} = \sin y\ \hat{\imath} + \cos x\ \hat{\jmath}$, and $c$ is the boundary of the triangle with vertices $(0,0), (\pi, 0), (\pi, 1)$.

(b) Using divergence theorem evaluate the surface integral $\iint_S \vec{F}.\hat{n}\, dA$ where $\vec{F} = [2x^2, \frac{1}{2}y^2, -\cos\pi z]$ and $S$ is the surface of the tetrahedron with vertices $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$.

6. Answer the followings: [7 x 2]

(a) Find the Fourier series expansion for $f(x)$, where $f(x) = x^2, -\pi < x < \pi$ . and hence prove that $\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16}$ ..........

(b) Find the Fourier cosine and sine transform of the function

$$f(x) = \begin{cases} k, & if\ 0 < x < a \\ 0, & if\ x > a. \end{cases}$$

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Course Outcome Assessment Scheme

| Cos | Questions | Marks | Total Mark |
|---|---|---|---|
| CO1: Mean Value Theorem, Increasing and decreasing function, Asymptote, Radius of curvature. | Q1. (a), (b), (c)<br>Q2. (a), (b), (c) | 27 | 22.31% |
| CO2: Continuity, Partial derivatives, Euler's Theorem, Taylor's Theorem, Gamma function. | Q1. (d), (e), (f)<br>Q3. (a), (b), (c) | 27 | 22.31% |
| CO3: Parametric representation of curve, length of curve, unit normal vector, Exact form and line integral, curl, divergence, double integral. | Q1. (g), (h), (i)<br>Q4. (a), (b), (c) | 27 | 22.31% |
| CO4: Stoke's theorem, Gauss divergence theorem, Green's theorem | Q1. (j), (k), (l)<br>Q5. (a), (b) | 20 | 16.52% |
| CO5: Periodic function, odd and even function, Fourier series, Fourier transform, Fourier Sine and Cosine transform. | Q1. (m), (n), (o)<br>Q6. (a), (b) | 20 | 16.52% |

Registration No. : | | | | | | | | | | |

*Total number of printed pages – 03*

B.Tech

MTH21101/MTH1001

## SUMMER QUARTER EXAMINATION – 2023

**Mathematics-1**

**BRANCH: ALL**

**Time – 3 Hours**

**Full Marks – 100**

Answer all Questions from Part – A and Part – B

The figures in the right hand margin indicate marks.

## PART – A

1. Answer the following questions: [2 x 15]

(a) State Rolle's mean value theorem.

(b) Separate the intervals in which the polynomial (4-$x^2$) is increasing or decreasing.

(c) Prove that 1-x<$e^{-x} < 1 - x + \frac{x^2}{2}$.

(d) State Taylor's theorem for a function of two variables.

(e) Using the $\epsilon - \delta$ approach, show that

$$\lim_{(x,y)\to(2,3)} 3x + 2y = 12.$$

(f) Write the first order partial order derivatives of the function f(x,y)=$ye^{-x}$.

(g) Under what condition vector field becomes incompressible.

(h) Evaluate the following double integral: $\int_0^2\int_0^4 (x^2 + y^2)dxdy$

(i) Examine whether the form (y dx - zxdy +z dz) is exact.

(j) State stokes theorem.

(k) Determine the parametric representation of the surface $x^2 + y^2 = 1$

(l) Using Green's theorem, evaluate the line integral $\oint_c F(r)\,.\,dr$ Where $\vec{F} = [y, x]$ and C is the boundary of the ellipse $\frac{x^2}{4} + \frac{y^2}{16}$.

(m) Define the concept of even and odd function.

(n) Determine the smallest positive period p of the function f(x)= cos 2x.

(o) Find the Fourier transform of following function

(a) $f(x) = \begin{cases} e^x & if\ -a < x < a \\ 0 & otherwise \end{cases}$

## PART - B

2. Answer any two: [7 x 2]

(a) If in the Cauchy mean value theorem f(x)=$e^x$ and F(x)=$e^{-x}$, show that 'c' is the arithmetic mean between a and b.

(b) Find the asymptotes of the curve $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 = 0$.

(c) Show that the curvature of the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the folium $x^3 + y^3 = 3axy$ is $\frac{-8\sqrt{2}}{3a}$.

3. Answer any two: [7 x 2]

(a) Show that the following function is discontinuous at (0,0).

$$f(x,y) = \begin{cases} \frac{x^2 - x\sqrt{y}}{x^2 + y}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$$

(b) If u(x,y)=$\frac{x^3+y^3}{x+y}$, $(x,y) \neq (0,0)$, then evaluate $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x}$.

(c) For positive real numbers m and n, prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

4. Answer any two: [7 x 2]

(a) given a curve C:r(t)= $t\hat{\imath}+t^3\hat{\jmath}$, find a tangent vector $\acute{r}(t)$ and the corresponding unit tangent vector $u(r)$, $\acute{r}$ and $u$ at the point P(1,1,0) and the tangent at P.

(b) find $f$ if $\vec{v} = grad\, f = \left[\frac{y}{z}, \frac{x}{z}, -\frac{xy}{z^2}\right]$

(c) prove that $curl\ (\vec{u} + \vec{v}) = curl\,\vec{u} + curl\ \vec{v}$.

5. Answer the followings: [7 x 2]

(a) using Green's theorem evaluate the line integral $\oint F(r).dr$ where F=$[\sin y, \cos x]$ C is the boundary of the triangle with vertices (0,0), $(\pi, 0)$, $(\pi, 1)$.

(b) evaluate $\oiint F.n\, dA$ where F=$[x^2, y^2, z^2]$ S: r=[u cos v, u sin v, 3v], $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$.

6. Answer the followings: [7 x 2]

(a) find the Fourier series of the periodic function of period $2\pi$,

$$f(x) = \begin{cases} -k, & if -\pi < x \leq 0 \\ k, & if\ 0 \leq x < \pi \end{cases}$$

(b) find the Fourier transform of the following function $f(x) = \begin{cases} e^x\ if\ x < 0 \\ e^{-x}\ if\ x > 0 \end{cases}$

10

B.TECH

MA10101

# $1^{ST}$ SEMESTER BACK EXAMINATION – 2022

## SUBJECT: MATHEMATICS-I

BRANCH (S): ALL

Time: 3 Hours

Max marks: 100

**Answer all Questions from Part – A and Part – B**

**The figures in the right hand margin indicate marks.**

## PART – A

**1. Answer the following questions:** [2x15]

(a) Find the parallel asymptotes of $x^2y + 2x^2 + 2y + 2 = 0$ to the coordinate axis.

(b) Very Rolle's theorem for $f(x) = 2+(x-1)^{2/3}$ in [0, 2].

(c) Write the relation between curvature and radius of curvature.

(d) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + 4z^2$ at the point P (1, 0, 1) in the direction of the vector $\vec{a} = i - j + k$.

(e) Prove that curl (grad f) = 0.

(f) Find the unit tangent vector of a cone $x^2 + y^2 = z^2$.

(g) Write the relation between beta and gamma function.

(h) Find by integration method, the area of triangle whose equations of sides are y = x, y = 0 and x = 2

(i) Find the length of the arc of the curve $y = \log secx$ from $x = 0$ to $x = \pi$.

(j) Find the mass of given density function $= x^2 + y^2 + 4z^2$ , in the box T : $|x| \le 1, |y| \le 3, |z| \le 2$

(k) Find the period of $\sin 2\pi x$ and $\cos\frac{\pi}{2}x$.

(l) Define periodic function.

(m) Test whether the function $f(x) = x$ , $(0 \le x \le 2\pi)$ is even or odd.

(n) Fid the Fourier sine transformation $f(x) = e^{-x}$.

(o)Write down the expanded form of Taylor series functions of two variables.

## PART-B

**Q2. (Answer any two)** [7x

(a) Find all the asymptotes of the curves $x^3 - x^2y - xy^2 + y^2 + 2x^2 - 4y^2 + 2xy + x + y + 1 =$

(b) Show that the curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the Folium$x^3 + y^3 = 3axy$ is $\frac{-8\sqrt{2}}{3a}$.

(c) Show that $\frac{x}{1+x} < \log(1 + x) < x$ for all$x > 0$.

**Q.3. (Answer any two)** [7

(a) Show that function $f(x,y)=\begin{cases}(x+y)\sin\left(\dfrac{1}{x+y}\right) & x+y\neq 0\\ 0 & x+y=0\end{cases}$

Is continuous at (0, 0) but its partial derivatives $f_x$ and $f_y$ does not exists at (0, 0).

(b) Find the relative maximum and minimum values of the functions

$f(x, y) = 2(x^2 - y^2) - x^4 + y^4.$

(c) If $u(x, y) = Cos^{-1}\left(\frac{x+y}{\sqrt{x}-\sqrt{y}}\right), x \neq y$ then prove that

$$i)\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}Cot\, u = 0$$

**Q.4. (Answer any two)**

(a) Evaluate the double integral $\int_1^5 \int_0^{x^2} (1 + 2x)\, e^{x+y} dxdy.$

(b) Show that the differential form under the integral sign of

$$\int_c [2xyz^2\, dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz]$$

Is exact, so that we have independence of path in any domain, and find the value of the i
from (0, 0, 1) to$(1, \frac{\pi}{4}, 2)$.

(c) Evaluate the surface integrals $\iint_S F\, dA$ when $F = [x^2, 0, 3y^2]$ and S is the portion of the pla
$y + z = 1$ in the first octant.

**Q.5. (Answer the followings)** **[7**

**(a) Evaluate the line integral** $\int_c (F_1 dx + F_2 dy)$ using Greens theorem, if $F_1 = y^2 - 7y, F_2 = 2x$
and C the circle$x^2 + y^2 = 1$.

**(b)** Using Stokes theorem solve $\iint_S F\, dA$ for $F = yi + zj + xk$ and $S$ the paraboloid $z = f(x, y)$
$(x^2 + y^2)\ z \geq 0.$

**Q.6. (Answer the followings)** **[7**

**(a)** Find the Fourier series of $f(x) = \begin{cases} k, & -\pi/2 < x < \pi/2 \\ 0, & \pi/2 < x < 3\pi/2 \end{cases}$ and hence

Show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots\ldots\ldots\ldots\ldots = \frac{\pi}{4}.$

**(b)** Find the Fourier integrals of the function $f(x) = \begin{cases} 0 & if\ \ x < 0 \\ \frac{1}{2} & if\ \ x = 0 \\ e^{-x} & if\ \ x > 0 \end{cases}$

Registration No/ College Roll No:

Total Number of Pages: 2

B.TECH
MTH21101

# FIRST SEMESTER REGULAR EXAMINATION – 2022

## MATHEMATICS-I

BRANCH (S): ALL

Time: 3 Hours

Max marks: 100

**Answer all Questions from Part – A and Part – B**

**The figures in the right hand margin indicate marks.**

## PART – A

**1. Answer the following questions:** [2x15]

(a) Find the asymptotes of $y = \frac{x-2}{x+5}$ parallel to coordinate axes.

(b) Determine curvature of the curve $s = 5\tan\Psi$.

(c) Verify the Rolle's theorem for the function $f(x) = x^2$ on $[-1,1]$.

(d) Examine whether the function $f(x,y) = \frac{x^2+y}{x+y^2} + \sin(x^2+y^2)$ is homogeneous or not.

(e) If $u = e^{xy^2} + \sin(xy)$, then find the value of $\frac{\partial u}{\partial x}$ at the point (1,2).

(f) Evaluate $\beta(1, 2)$, where $\beta$ is stands for Beta function.

(g) Find the directional derivative of $f(x,y,z) = 2x + 2y + 2z$ at the point P (1, 2, 0) in direction of the vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

(h) Write the parametric representation of the line joining the points from$(3,1,-1)$ to $(3,2,-6)$.

(i) Evaluate the gradient of a scalar field $f(x,y,z) = xy + yz + zx$.

(j) Calculate the area bounded by the lines $x = 0, x = 1$ and $y = 2x^2 + 1$.

(k) Compute the Jacobian $J$ for the transformation $x = r\cos\theta, y = r\sin\theta$.

(l) Find the unit normal vector $\vec{N}$ of the surface $x^2 + y^2 + z^2 = 9$.

(m) Test whether the function $g(x) = x\sin x$ is even or odd.

(n) Find the period of $\cos 2x$ .

(o) If $f(x) = x^2 + x$ on $(-2,2)$, then calculate the value of Fourier coefficient $a_0$.

## PART-B

**Q2. (Answer any two)** [7x

(a) Find all types of the asymptotes for the curve $y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x$
$2y - 2x - 1 = 0$.

(b) Calculate the radius of curvature at any point on the cardioids $r = a(1-\cos\theta)$.

(c) State Lagrange's mean value theorem and then verify the theorem for the function $f(x) =$
$\sqrt{x-2}$ on the interval $[2,3]$.

**Q.3. (Answer any two)** [7

(a) Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$ for the following function

$$f(x,y) = \begin{cases} \frac{x^3+2y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(b) Find the linear and quadratic Taylor's series polynomial approximations to the function $f(x,y) = 2x^3 + 3y^3 - 4x^2y$ about the point (1,2).

(c) If $u = sec^{-1}(\frac{x^n+y^n}{x-y})$ for some $n > 0$ and $x \neq y$ then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (n-1)cot\, u.$$

**Q.4. (Answer any two)** **[7x2]**

(a) Calculate $curl(\vec{f})$ for $\vec{f} = x^2y\hat{\imath} - (z^3 - 3x)\hat{\jmath} + 4y^2\hat{k}$ and then show that $div\left(curl(\vec{f})\right) = 0$.

(b) Show that the integral $\int_C [3x^2dx + 2yzdy + y^2dz]$is independent of path in any domain in space and find its value if the curve $C$ has the initial point $A(0,1,2)$ and the terminal point in $B(1,-1,7)$.

(c) Evaluate the double integral $\int_1^5 \int_0^{x^2} (1 + 2x)e^{x+y}dxdy$.

**Q.5. (Answer the following)** **[7x2]**

(a) Verify Green's theorem for $F = [3x - y, x + 5y]$ on the curve $C: x^2 + y^2 = 1$.

(b) Use Gauss Divergence theorem to evaluate $\iint_S \vec{F} \cdot \vec{n}\, ds$, where $F(x,y,z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $S$ is the solid enclosed by $x + y + z = 1,\ x = 0,\ y = 0,\ z = 0$.

**Q.6. (Answer the following)** **[7x2]**

(a) Find the Fourier transform of the function $f(x) = \begin{cases} e^x & if \quad x < 0 \\ e^{-x} & if \quad x > 0 \end{cases}$.

(b) If $f(x) = x + x^2$ on $-\pi < x < \pi$, then find the Fourier series expansion of $f(x)$ and from it deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$.

Registration No/ College Roll No:

Total Number of Pages: 3

**B. TECH**
**MA10101**

**FIRST SEMESTER (REGULAR/BACK) EXAMINATION (DEC. /JAN.) – 2016-17**
**[2015 ADMISSION ONWARDS]**
**SUBJECT: MATHEMATICS-1**
**BRANCH (S):ALL**
**Time: 3 Hours**
**Max marks: 100**

**Answer all Questions from Part – A and Part – B**
**The figures in the right hand margin indicate marks.**

## PART – A

**Q1. Answer the following questions:** **[2x15]**

a) Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on the interval $[1, 3]$.

b) Find the radius of curvature of the curve $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2}$.

c) Find the asymptotes of the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

d) Show that the function $f(x, y) = \frac{x-y}{x+y}$ is not continuous at the point $(0, 0)$ .

e) Find the divergence of the vector function $\vec{V} = x\hat{\imath} + xy^2\hat{\jmath} + xy^2z^3\hat{k}$.

f) Show that the following flow field is irrigational where $\vec{V} = (x^2 + xy^2)i + (y^2 + x^2y)j$.

g) If $\vec{F} = 3xyi - y^2j$, evaluate $\int_C \vec{F}.\vec{dr}$ where $C$ is the arc of the parabola $y = 2x^2$ from $P(0,0)$ to $Q(1,2)$.

h) Evaluate in terms of gamma functions, the integral $\int_0^\infty e^{-x^4}dx$.

i) Evaluate $\int_0^5 \int_0^y e^{x+2y}\,dxdy$.

j) State Gauss Divergence theorem.

k) Find a unit normal vector of the surface represented by $x + 2y + 3z = 10$.

l) Using Green's Theorem evaluate the line integral $\int_c f(\vec{r})d\vec{r}$, where $\vec{F} = [x, y]$ and C is the boundary of the ellipse $\frac{x^2}{4} + \frac{y^2}{16}$.

m) Write the formula of Fourier sine and cosine integrals.

n) Examine whether the function $f(x) = e^x \cos\frac{n\pi x}{10}$ is even or odd.

o) Find the period of $f(x) = \cos\frac{n\pi x}{10}$.

## PART-B

**Q2. (Answer any two)** **[7x2]**

a Verify Lagrange's Mean Value Theorem for the function $f(x) = (x-3)(x-6)(x-9)$ on the interval $[3, 5]$.

b. Find the asymptotes of the following curve $y^3 - x^2y + 2xy^2 - y + 1 = 0$.

c. Find the radius of curvature of $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the folium $x^3 + y^3 - 3axy = 0$.

**Q.3. (Answer any two)** **[7x2]**

a. Show that the following function is continuous at $(a, b)$ where $f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is not continuous at $(0,0)$, but its partial derivatives $f_x$ and $f_y$ exist at $(0,0)$.

b. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

c. Show that $f_{xy}$ and $f_{yx}$ are not continuous at $(0,0)$ where $f(x,y) = \begin{cases} \frac{xy^3}{x+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$.

**Q.4. (Answer any two)** **[7x2]**

a. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

b. Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, n)$.

c. Evaluate the line integral $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ where $C$ is the square formed by the lines $x =$ and $y = \pm 1$.

**Q.5. (Answer the followings)** **[7x2**

a. Using Green's theorem evaluate the line integral $\int_C [(xy + y^2)dx + x^2dy]$ where $C$ is bounded by $y = x$ and $y = x^2$.

b. Evaluate the Line Integral $\int_C F.r'(s)ds$ by Stokes's theorem where $F = [y^2, z^2, x^2]$ and C: the boundary of the portion of the paraboloid $x^2 + y^2 = z, y \geq 0, z \leq$

**Q.6. (Answer the followings)** **[7**

a. Find the Fourier series of $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$ with $f(x)$ is periodic with period 2.

b. Find Fourier cosine transform of $f(x) = \frac{e^{-ax}}{x}$.

Registration No/ College Roll No:

Total Number of Pages: 2

B. TECH
BS1101

# FIRST SEMESTER (BACK) EXAMINATION (DEC.) – 2017
## [2014 ADMISSION BATCH]
## Subject: Mathematics-I

BRANCH (S): All

Time : 3 Hours

Max marks : 70

**Answer Question No.1 which is compulsory and any Five from the rest.**
**The figures in the right hand margin indicate marks.**

**Q1. Answer the following questions:** [2x10]

a) Find the order and degree of the following differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{2/3}=\frac{d^3y}{dx^3}$$

b) Form a differential equation for a circuit with resistance $R=10\Omega$, $L=-0.2H$ and emf $E=25V$.

c) Find the integrating factor of $y'+ycosx=xe^{sinx}$.

d) Solve $x^2y''+5y'=0$.

e) Find the asymptotes parallel to the coordinate axes of the curve $2xy^2-3x^2y+x^2-y^2+2xy-y-1=0$

f) Find the radius of curvature at any point on the curve $y^2=4ax$ .

g) What does convergence of power series mean? How would you test it?

h) What is the rank of the matrix $\begin{bmatrix}3 & 2 & -1\\ 0 & -2 & 4\\ 0 & 2 & 5\end{bmatrix}$.

i) Define Hermitian matrix and orthogonal matrix.

j) Are the vectors $(1,-1,),(2,1),(-3,4)$ linearly independent?

**Q2.** Solve the following differential equations

a) $y'+y\,tan\,x=\sin 2x$

b) $(2x+e^y)dx+xe^y dy=0$

**Q3.** Find the general solution of the following differential equations:

a) $y''+2y'-35y=e^{2x}+\sin 3x$

b) $xy''-y'=(3+x)x^2e^x$

Q4. a) Find all asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$

b) Find the radius of curvature of the curve $r = a(1 + \cos\theta)$ at the point where the tangent is parallel to the initial line.

Q5. a) Find the power series solution of $y'' + 8xy' - 4y = 0$

b) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2 - 1)^n]$

Q6. a) Solve the following systems of equations by Gauss elimination method

$-x + y + 2z = 2, 3x - y + z = 6, -x + 3y + 4z = 4.$

b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -9 \\ -2 & -4 & 19 \\ 2 & -1 & 2 \end{bmatrix}$.

Q7. a) Find the eigen values and the corresponding eigen vectors of the following matri

$$\begin{bmatrix} 15 & 6 & -12 \\ 4 & 10 & -2 \\ -4 & 8 & -7 \end{bmatrix}$$

b) Find out what type of conic section is represented by the quadratic form $9{x_1}^2 - 6x_1x_2 + {x_2}^2$ and transform it to principal axes .

Q8. a) Show that $J_{\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \sin x$

b) Find the inverse of the following matrix by Gauss-Jordan Method

$$\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}.$$

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Total No. Pages: 3



B. TECH
MA10101

**FIRST SEMESTER (REGULAR/BACK) EXAMINATION–2018-19**
**[2018/2017/2016/2015 Admission Batch]**
**Subject Name: MATHEMATICS-I**
**BRANCH (S): ALL**
**Time: 3 Hours**
**Max marks: 100**
**Answer all Questions from Part - A and Part - B**
**The figures in the right hand margin indicate marks.**

## PART-A

**Q.1 Answer the following questions:** **[2×15]**

a) Verify Rolle's theorem for $f(x) = x^2$, $x \in [-1, 1]$.

b) Determine the intervals in which $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.

c) Define curvature. Write the relation between curvature and radius of curvature.

d) Show that $f(x,y) = \begin{cases} \frac{x^2 - x\sqrt{y}}{x^2 + y} & ;(x,y) \neq (0,0) \\ 0 & ;(x,y) \neq (0,0) \end{cases}$ ) is discontinuous at the point (0,0).

e) If $u(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right), x > 0, y > 0$ then evaluate $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy}$.

f) Represent the curve $4x^2 - 3y^2 = 12, z = 1$ parametrically.

g) Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

h) Prove that $B(m, n) = B(n,m)$.

i) Write the relation between Beta and Gamma function and compute $\Gamma(4.5)$.

j) Evaluate $\int_0^2 \int_0^4 (x^2 + y^2)\,dx\,dy$.

k) Using Green's theorem evaluate $\oint F(r)\,.\,dr$ counter clockwise of $F = [x^2 e^y, y^2 e^x]$, C: the rectangle with vertices (0, 0), (2, 0), (2, 3), (0, 3).

l) Find the directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at the point P:(2,1,3) in the direction of $\vec{a} = \hat{i} - 2\hat{k}$.

m) Determine the smallest positive period p of the $\cos\left(\frac{2\pi n x}{k}\right)$.

n) Test whether the function f(x) = sinx + cosx is even or odd or neither odd nor even.

o) Find the Fourier cosine transform of $f(x) = e^{-2x}$.

## PART-B

**Q.2) (Answer Any Two)** [7x2]

a) Find all the asymptotes of the curve
$$2x^3 - x^2 y + 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$$

b) Prove that for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ , $\rho=\frac{a^2b^2}{p^3}$ where $p$ is the perpendicular from the centre on the tangent at the point $(x,y)$.

c) If in the Cauchy's Mean value theorem, $f(x)=e^x$ , $F(x) = e^{-x}$.

Show that $c$ is arithmetic mean between $a$ and $b$.

**Q.3) (Answer Any Two)** [7x2]

a) Show that $f(x,y)=\begin{cases}\frac{x^2-y^2}{|x|+|y|} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)\neq(0,0)\end{cases}$ is continuous at the point $(0,0)$ but its partial derivatives $f_x$ and $f_y$ do not exist at $(0,0)$.

b) Find the linear Taylor series polynomial approximation to the function $f(x,y) = 2x^3 + 3y^3 - 4x^2y$ about the point $(1,2)$. Also obtain the maximum absolute error for linear approximation in the region $|x - 1| < 0.01$ an $|y - 2| < 0.1$.

c) If $u(x,y) = \cos^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}}), x > 0, y < 1$ then prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{2}\cot u.$$

**Q.4) (Answer Any Two)** [7x2]

a) Prove that $\Gamma(n)\Gamma(n+1)=\frac{\pi}{\sin n\pi}$

b) Find the area of the region lying above $x - axis$, and included between the circle $x^2 + y^2 = 2ax$ and the parabola $y^2 = ax$.

c) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

**Q.5) (Answer the followings)** [7x2]

a) Show that the form under the integral sign is exact and evaluate the integral $\int_{(0,\pi)}^{\left(2,\frac{\pi}{2}\right)} e^x(\cos y dx - \sin y dy)$.

b) Evaluate $\iint_S F.n dA$ by the Divergence theorem for $F = [x^2, 0, z^2]$, S : the box , $|x| \leq 1, |y| \leq 3, |z| \leq 2$.

**Q.6) (Answer the followings)** [7x2]

a) Find the Fourier series of $f(x)=\frac{x^2}{2}, -\pi < x < \pi$ and hence prove that $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\frac{1}{25}-\cdots=\frac{\pi^2}{2}$

b) Prove that

$$(i) = \int_0^{\infty}\frac{\cos x\omega+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases}0 & if\ x<0\\ \frac{\pi}{2} & if\ \ x=0\\ \pi e^{-x} & if\ x<0\end{cases}$$

$$(ii) = \int_0^{\infty}\frac{1-\cos\pi\omega}{\omega}\sin x\omega\ d\omega=\begin{cases}\frac{\pi}{2} & if\ 0<x<\pi\\ 0\ if\ x>\pi\end{cases}$$

21/12/15
181

Registration No/ College Roll No:

Total Number of Pages: 3

**B. TECH**
MA10101

# FIRST SEMESTER (REGULAR/BACK-15/16) EXAMINATION – 2019-20
## [2015 Admission Batch onwards]

BRANCH (S) - ALL

SUBJECT NAME: Mathematics-1

Time: 3 Hours

Max Marks: 100

Answer all Questions from Part – A and Part – B

The figures in the right hand margin indicate marks.

## PART – A

**Q1. Answer the following questions:** [2x15]

a) Find the radius of curvature at origin of the curve

$$2x^4 - 3y^4 + 4x^2y + xy - y^2 + 2x = 0.$$

b) Find the interval in which the function $f(x) = x^3 + 8x^2 + 5x - 2$ is increasing & decreasing.

c) Calculate the unit normal to the surface $y = 2x^2 - 1$ at $(1, -1)$.

d) Find the directional derivative of $f(x,y,z) = x^4 + yz^3$ at the point $(2, 1, -1)$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

e) Calculate the area bounded by the lines $y = 0$, $y = 1$ and $y = x^2$.

f) Find the parametric form of a straight line passes through the point $(0,2)$ to $(1,4)$.

g) Find the gradient of the function $f(x,y) = x^2 + 2y$ at the point (1 , 2).

h) Write the relation between volume integral and surface integral.

i) Find the normal vector $N = r_u \times r_v$ of the surface $r(u, v) = [u\cos v, u\sin v, 3u]$.

j) Determine the fundamental period of $\sin 2x$.

k) Examine whether the function $e^{2x}$ is either odd or even.

l) Find the asymptote of the curve $r\theta = a$.

m) State Euler's theorem for homogeneous function of two variables.

n) Consider the Fourier series of the function $f(x) = x^2$, $-\pi \le x \le \pi$, find the value of $a_0$.

o) State Lagrange's mean value theorem.

**PART-B** [7x2]

Q2. (Answer any two)

a. Find the asymptotes of the curve $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$.

b. Using Lagrange Mean value theorem, show that $\frac{x}{1+x} < \log(1+x) < x, \forall x > 0$ and

hence , show that $0 < [\log(1+x)]^{-1} - x^{-1} < 1 \ \forall x > 0$,

c. Show that the curvature at the point $(\frac{3a}{2}, \frac{3a}{2})$ on the Folium $x^3 + y^3 = 3axy$ is $\frac{-8\sqrt{2}}{3a}$.

[7x2]

Q.3. (Answer any two)

a. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ ; $x \neq y$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

b. Find the minimum value of $x^2 + y^2 + z^2$ subjected to the condition $xyz = a^3$ .

c. Show that the function $f(x,y) = \begin{cases} \frac{x^2 - y^2}{x - y} & \text{if } (x,y) \neq (1,1) \\ 2 & \text{if } (x,y) \neq (1,1) \end{cases}$ is continuous &

differentiable at (1,1).

[7x2]

Q.4. (Answer any two)

a. Evaluate $\int_0^\infty x^{\frac{1}{2}} e^{-x^3} dx$

b. Show that the form under the integral sign is exact in the plane or in space and evaluate the

value of integral $\int_{(0,0,0)}^{4,1,2} (3y dx + 3x dy + 2z dz)$.

c. Calculate the line integral $_C\int f(\vec{r}) d\vec{r}$ for the following data $f = [y^2, -x^2]$, $C$ be the

straight line segment from (0,0) to (1,4).

[7

Q.5. (Answer the followings)

a. Evaluate the line integral $\oint_c f(\vec{r}) d\vec{r}$ by using Green's theorem where $f = [\tan 0.2x, x^5 y]$,

$R: x^2 + y^2 \leq 25, y \geq 0$

b. Evaluate the surface integral $\iint_s f.n.dA$, where

$f = [3x^2, y^2, 0]$ , $S: r = [u, v, 2u + 3v]$, $0 \leq u \leq 2$, $-1 \leq v \leq 1$.