EEE 506 – Project 1

Consider the following ARMA(2,2) system:

$$y(t) = 1.3435y(t-1) - 0.9025y(t-2) + e(t) + 1.3435e(t-1) + 0.9025e(t-2)\ ,$$

where $e(t)$ is a real-valued, i.i.d. Gaussian process.

1. If we view $y(t)$ as the output of a pole-zero linear time-invariant system with input $e(t)$, what are the poles of this system, and what are the zeros? Is the system stable? Is it minimum-phase?

2. Plot $\phi(\omega)$, the true power spectral density of $y(t)$ versus the angular frequency variable ω. The horizontal axis should be $\omega \in [0, \pi]$, and the vertical axis should be in dB scale.

3. Generate samples of $y(t)$ by passing white Gaussian noise through a filter. Make sure to discard initial samples of $y(t)$ to avoid the transient edge effects. So, for example, if you want to generate $N = 2048$ samples, you can generate $N = 3000$, and discard the first 952 samples.

4. Plot three periodograms for this data for $N = 128, 512$ and 2048 data points where the vertical axis is in dB and the horizontal axis is ω.

5. For $N = 2048$, plot the Bartlett's periodogram for $M = 128, 256, 512, 1024$. Does the estimate get more noisy or less noisy (i.e., variance) with increasing M? What about the bias, does it get more or less biased with increasing M?

6. Do the same for Welch's periodogram with 50% overlap. Experiment with two different tapering windows and see if you can reduce the bias of the Bartlett periodogram above for the same amount of variance.

7. For the same sample for $N = 2048$ plot the Blackman-Tukey estimate with a rectangular window of size $M = 128, 256, 512, 1024$. Does it get more noisy, or less noisy as M increases? What about the bias? Is it better, worse, or about the same as the estimators above?

8. For $N = 2048$ plot the spectral density estimator you get from the modified Yule-Walker method for two different values of M and compare them.

9. Do the same for the two-stage least-square method. Use $K = 8$, and $K = 20$.

10. The two-stage least-square method requires you to approximate $y(t)$ as an AR process of order K as a first step, before it estimates the ARMA parameters and the spectral density. Use that preliminary AR approximation to estimate the power spectral density for the two values of K you considered ($K = 8$, and $K = 20$). In other words, you will generate the data $y(t)$ using the same ARMA model as always, but pretend that it really is an AR model of order K with values of K given above, and use that AR estimate to get a spectral density estimate. Of course this AR estimate will be worse than the two-stage approach above. Compare the two.

Write a report commenting on the results. Make sure you follow these guidelines:

- Type your report.

- The report should be self-contained.
- All notation should be defined.
- Someone knowledgable about spectral estimation (like your future self in 10 years) should be able to follow your report.
- The plots should be in the format described above.
- Keep the report short by incorporating multiple plots on the same figure (which also helps the comparisons), and also by putting multiple plots on the same page. Make sure it is clear which curve is which by using legends or some other way.
- Keep it short.