project2

November 12, 2019

[41]:

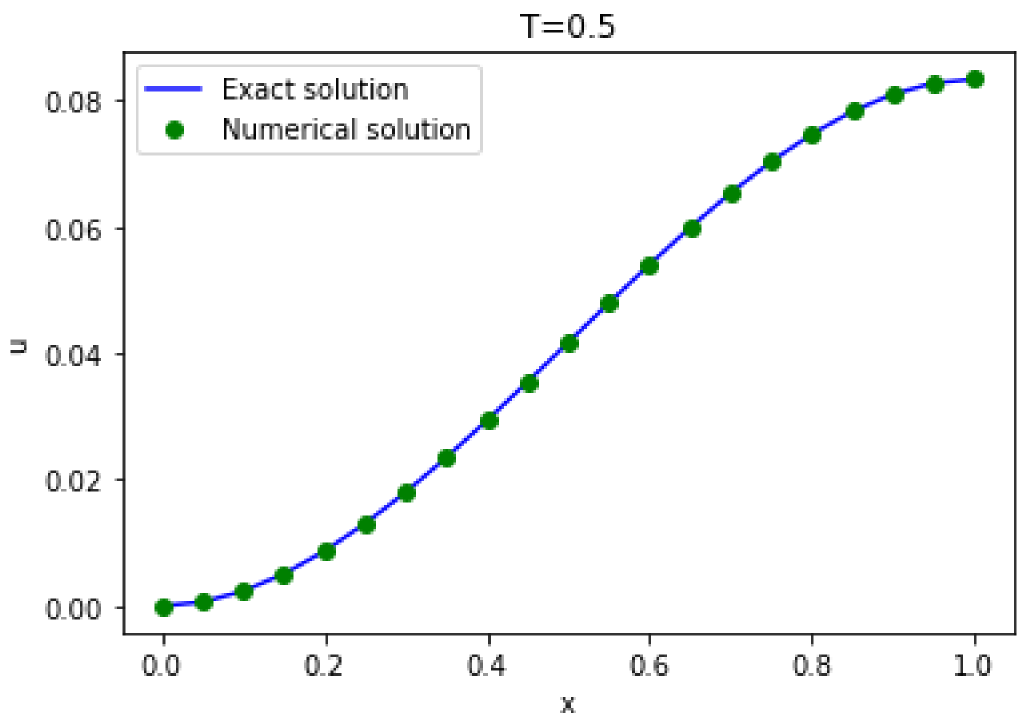
```
import matplotlib.pyplot as plt
from fenics import *
import numpy as np

def solver(N, I, f, alpha, rho, dt, T, P):
    """
    N: A list containg number of points in each direction, i.e [8,8,8] for 3D.
    I: An array of the initial condition
    f: The source term
    alpha: Might be a constant or a known function depending on u
    P: Degree of finite elements
    """
    def create_mesh(N):
        # Create mesh and define function space
        dim = len(N)
        if dim == int(1):
            #print('1D')
            mesh = UnitIntervalMesh(N[0])
        elif dim == int(2):
            #print('2D')
            mesh = UnitSquareMesh(N[0], N[1])
        else:
            #print('3D')
            mesh = UnitCubeMesh(N[0], N[1], N[2])
        return mesh


    # Create mesh and define function space
    mesh = create_mesh(N)
    V = FunctionSpace(mesh, 'P', P)

    # Define initial value
    u_1 = interpolate(I,V)

    # Define variational problem
    u = TrialFunction(V)
    v = TestFunction(V)
```

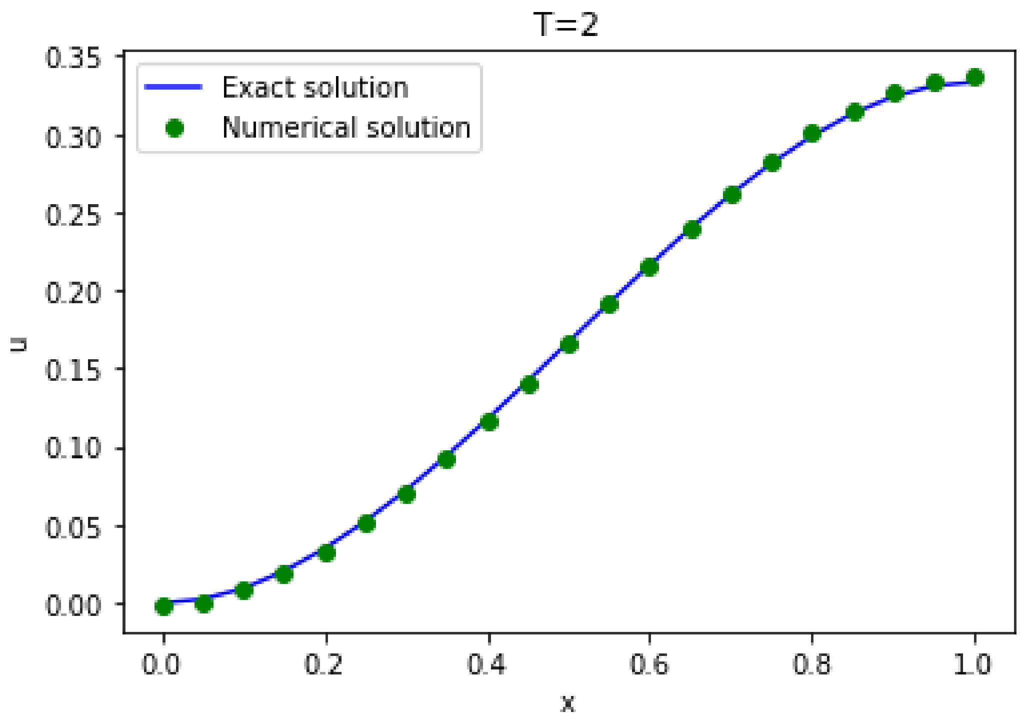
```
    a = (u*v + dt/rho*inner(alpha(u_1)*nabla_grad(u), nabla_grad(v)))*dx
    L = (u_1*v + (dt/rho)*f*v)*dx

    # Time-stepping
    u = Function(V)
    t=0
    num_steps = int(T/dt)
    for n in range(num_steps):
        # Update current time
        t += dt
        I.t = t
        f.t = t

        # Compute solution
        solve(a == L, u)

        # Update previous solution
        u_1.assign(u)

    return u_1, V


def task_d():
    """
    Testing a constant solution
    """
    I = Expression("7", degree=1)
    alpha = lambda u: 2
    f = Constant('0')
    T = 1; dt = 0.1
    rho = 1; P = 1
    interval, square, box = [8], [8,8], [8,8,8]

    for N in interval, square, box:
        u, V = solver(N, I, f, alpha, rho, dt, T, P)
        u_e = interpolate(I, V)
        error = np.abs(u_e.vector().get_local() - u.vector().get_local()).max()
        print('Using P%d elements in %dD gives the absolute error: %e \n' %(P,␣
 ↪len(N), error))



def task_e():
    """
    Testing analytical solution
    """
```

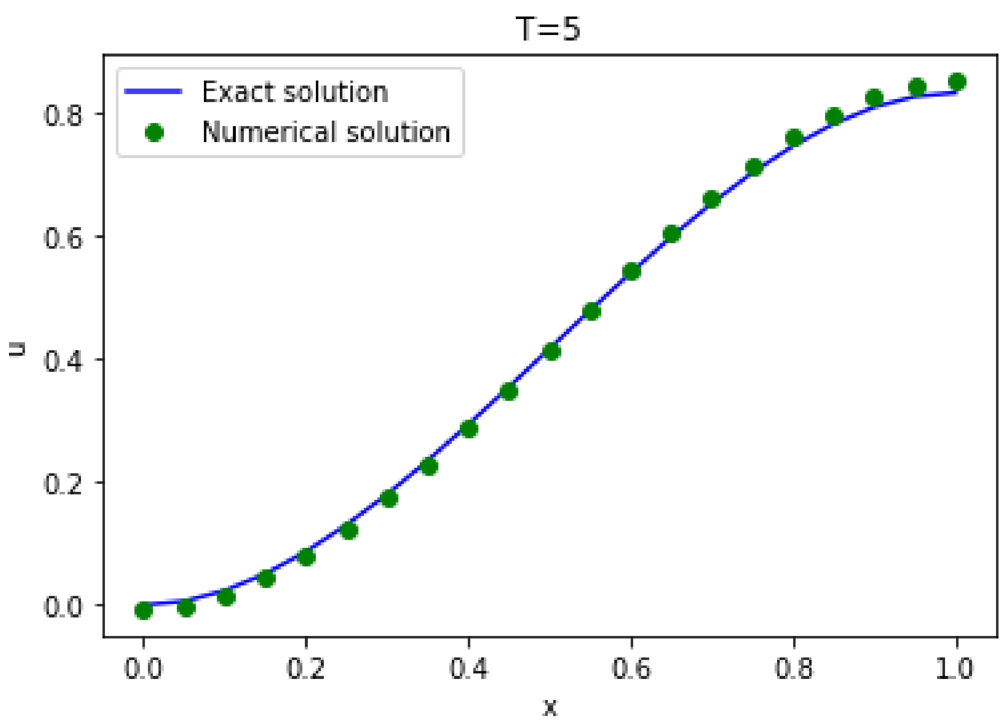
```
    I = Expression("cos(pi*x[0])", degree=1)
    alpha = lambda u: 1
    f = Constant('0')
    T = 0.5; dt = 0.1
    rho = 1; P = 1
    u_N = Expression('exp(-pi*pi*t)*cos(pi*x[0])', t=T, degree=1)

    for round in range(4):
        h = dt
        N = [int(1./np.sqrt(dt)), int(1./np.sqrt(dt))]
        u, V = solver(N, I, f, alpha, rho, dt, T, P)
        u_e = interpolate(u_N, V)
        e = u_e.vector().get_local() - u.vector().get_local()
        E = np.sqrt((np.sum(e**2))/u.vector().get_local().size)
        print('h=%.4f, E/h=%.4f, N=%d \n' %(h, float(E)/h, N[0]))
        dt /= 2


def task_f():
    N = [20]; dt = 0.5
    rho = 1; P = 1

    I = Constant('0')
    u_N = Expression('t*pow(x[0],2)*(0.5 - x[0]/3.)', t=0, degree=3)
    alpha = lambda u: 1 + u**2
    f = Expression('-rho*pow(x[0],3)/3 + rho*pow(x[0],2)/2 +␣
↪8*pow(t,3)*pow(x[0],7)/9 - \
                    28*pow(t,3)*pow(x[0],6)/9 + 7*pow(t,3)*pow(x[0],5)/2 - \
                    5*pow(t,3)*pow(x[0],4)/4 + 2*t*x[0] - t', rho=rho, t=0,␣
↪degree=1)


    for T in [0.5, 2, 5]:
        u, V = solver(N, I, f, alpha, rho, dt, T, P)
        u_N.t = T
        u_e = interpolate(u_N, V)

        x = np.linspace(0, 1, 20+1)
        plt.plot(x, u_e.vector().get_local()[::-1], 'b')
        plt.plot(x, u.vector().get_local()[::-1], 'go')
        plt.legend(['Exact solution','Numerical solution'])
        plt.xlabel('x')
        plt.ylabel('u')
        plt.title('T=%s' %T)
        plt.show()
```

```
task_d()
task_e()
task_f()
```

```
Using P1 elements in 1D gives the absolute error: 1.305622e-13

Using P1 elements in 2D gives the absolute error: 2.140510e-13

Using P1 elements in 3D gives the absolute error: 3.641532e-14

h=0.1000, E/h=0.1488, N=3

h=0.0500, E/h=0.1267, N=4

h=0.0250, E/h=0.1187, N=6

h=0.0125, E/h=0.1086, N=8
```




T=2
Exact solution
Numerical solution
u
x
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.0
0.2
0.4
0.6
0.8
1.0


T=5
Exact solution
Numerical solution
u
x
0.0
0.2
0.4
0.6
0.8
0.0
0.2
0.4
0.6
0.8
1.0

```
[ ]:
```