Artificial Intelligence (III/II)

Course Code: CT-653
(Module#2)

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Chapter#2

Problem Solving

✓ Class Outline

1. Defining problems as a state space search
2. Goal Formulation and Problem formulation
3. Well-defined problems, Constraint satisfaction problem
4. Production systems

Defining problems as a state space search

Problem

- Problem is what (a task which is) to be done.

Problem Characteristics

1. Is the problem decomposable into small sub-problems which are easy to solve?
2. Can solution steps be ignored or undone?
3. Is the universe of the problem is predictable?
4. Is a good solution to the problem is absolute or relative?
5. Is the solution to the problem a state or a path?
6. What is the role of knowledge in solving a problem using artificial intelligence?
7. Does the task of solving a problem require human interaction?

Defining problems as a state space search...

Problem Classification

1. Ignorable: Intermediate actions can be ignored; water-jug problem.
2. Recoverable: the actions can be implemented to go the initial state; 8-puzzle game.
3. Irrecoverable: The actions cannot help to reach the previous state; Tic-Tac-Toe
4. Decomposable: The problem can be broken into similar ones; word puzzle game.

Defining problems as a state space search...

Problem-solving in AI

- A simplest agent, *reflex-agent* which bases its actions on direct mapping states into actions, fails to operate in an environment when mapping is too large store and learn.
- One type of *goal-based agent* known as a problem-solving agent uses atomic representation with no internal states visible to the problem-solving algorithms.
- The problem-solving agent performs precisely by defining problems and its several solutions, so it is goal-driven agent which focuses on future actions and satisfying the goal.

Steps performed by Problem-solving agent

1. Goal Formulation
2. Problem Formulation
3. Searching
4. Execution

Goal Formulation and Problem formulation

1 Goal Formulation

- This is the first and simplest step in problem-solving.
- It organizes the steps/sequence required to formulate one goal out of multiple goals as well as actions to achieve that goal.
- Goal formulation is based on the current state and the agent's performance measure.

2 Problem Formulation

- This step is the most important – which decides what actions sequences should be taken to reach the goal formulated.
- Collecting detail information about the problem, It defines the problems precisely with initial states, final states and acceptable solutions.

✓ Five components involved in problem formulation:

1. Initial State,
2. Actions,
3. Transition Models,
4. Goal Test,
5. Path Costs.

Goal Formulation and Problem formulation...

② Goal Formulation...

1. Initial State: It is the starting state of the agent towards its goal formulated.
2. Actions: It is the description of the possible actions available to the agent.
3. Transition Model: It describes what each action does while moving next state.
4. Goal Test: It determines if the given state is a goal state.
5. Path Cost: It assigns a numeric cost to each path that follows the goal. The problem-solving agent selects a cost function, which reflects its performance measure. An optimal solution has the lowest path cost among all the solutions.

Goal Formulation and Problem formulation...

③ Searching

- Find the most appropriate technique of sequence among all possible techniques – optimal solution.

④ Execution

- once the search (algorithm) returns a solution (optimal) to the problem, the solution is then executed by the agent.

State Space Representation

- A state space essentially consists of a set of nodes representing states of the problem solving techniques.
- Arcs/transitions between nodes represent the legal moves from one state to another (from initial state towards goal states).

Well-defined problems, Constraint satisfaction problem

Well-Defined Problems

A problem is said as well-defined problem if the problems is composed of (can be described by) following components:

1. initial state: From where the solution is started.
2. operator or successor functions: Rules to move from one state to another states.
3. state space: Area or scope in which problem is to be solved.
4. path: the possible solution directing the movement.
5. path cost: the total movements in a particular direction.
6. goal: final state(desired state which is to be achieved)

Well-defined problems, Constraint satisfaction problem...

Constraint satisfaction problems (CSPs)

- Constraint satisfaction problems are the problems which must be solved under given constraints.
- The focus must be on not to violate the constraint while solving such problems

It consists of the following:

- A finite set of variables which stores the solution (V = V1, V2, V3,....., Vn).
- A set of discrete values known as domain from which the solution is picked (D = D1, D2, D3,.....,Dn)
- A finite set of constraints (C = C1, C2, C3,......, Cn)

Constraint Satisfaction Problem

Constraint propagation terminates for one of two reasons:

- *Contradiction detected* i.e. no consistent solution for given problem within under the given constraints.
- Propagation has *run off stream* and there are no further changes that can be made on the basis of current knowledge.

Some Problems that can be solved using CSP:

- CryptArithmetic (Coding alphabets to numbers.)
- n-Queen (In an n-queen problem, n queens should be placed in an nXn matrix such that no queen shares the same row, column or diagonal.)
- Map Coloring (coloring different regions of map, ensuring no adjacent regions have the same color)
- Crossword (everyday puzzles appearing in newspapers)
- Sudoku (a number grid)

Constraint Satisfaction Problem

Crypt-Arithmetic Rules

- First letter must be non-zero.
- Each variable must be unique.
- Carry over must be taken into account if Nothing is mentioned on question.
- Need to be careful about symbol of the operation.

Constraint Satisfaction Problem

Example#1

Solve the given CSP under given constraints.

$$
\begin{array}{cccc}
 & O & N & E \\
+ & O & N & E \\
\hline
 & T & W & O
\end{array}
$$

$$
\begin{array}{ccccc}
\underline{(C3|\quad)} & \underline{(C2|\quad)} & \underline{(C1|\quad)} & & \\
 & & (O|\quad) & (N|\quad) & (E|\quad) \\
+ & & (O|\quad) & (N|\quad) & (E|\quad) \\
\hline
 & & (T|\quad) & (W|\quad) & (O|\quad)
\end{array}
$$

✓Set of Variables

$X = \{ O, N, E, T, W \}$

✓Set of Domain

$D = \{0,1,2,3,4,5,6,7,8,9\}$

✓Set of Constraints C are:

1. Starting Letters O, T must not be Zero.
2. Each variable must be assigned uniquely.

Constraint Satisfaction Problem

O must be even as it is result of E+E i.e. 0,2,4,6,8 but 0 can't be Zero because it violates constraints no. 1 and O can't be 5,6,8 because it will give carry over in 3rd column.
So O can be either 2 or 4. let's say O = 2 with C2 as 0 which will result T = 4 and E = 1.

$$
\begin{array}{ccccc}
 & \underline{(C3|0)\ (C2|0)} & \underline{(C1|0)} & & \\
 & (O|2) & (N|\ \ ) & (E|1) & \\
+ & (O|2) & (N|\ \ ) & (E|1) & \\
\hline
 & (T|4) & (W|\ \ ) & (O|2) &
\end{array}
$$

As 1st column doesn't produce carry over, N+N = W, if we take N = 3 then W = 6

$$
\begin{array}{ccccc}
 & \underline{(C3|0)\ (C2|0)} & \underline{(C1|0)} & & \\
 & (O|2) & (N|3) & (E|1) & \\
+ & (O|2) & (N|3) & (E|1) & \\
\hline
 & (T|4) & (W|6) & (O|2) &
\end{array}
$$

Constraint Satisfaction Problem

Example#2

Solve the given CSP under given constraints.

	S	E	N	D
+	M	O	R	E
M	O	N	E	Y

✓Set of variables are

$X = \{S, E, N, D, M, O, R, Y\}$

✓Set of Domains are

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

✓Set of Constraints C are:

① Starting Letters M, S must not be Zero.

② Each variable must be assigned uniquely.

(C4|)	(C3|)	(C2|)	(C1|)	
	(S|)	(E|)	(N|)	(D|)
+	(M|)	(O|)	(R|)	(E|)
(M|)	(O|)	(N|)	(E|)	(Y|)

Constraint Satisfaction Problem

✓To begin, start in the 5th column, we must have M = 1.
✓To become M =1, carry from 5th column C4 = 1

(C4|1)	(C3|)	(C2|)	(C1|)	
	(S|)	(E|)	(N|)	(D|)
+	(M|1)	(O|)	(R|)	(E|)
(M|1)	(O|)	(N|)	(E|)	(Y|)

✓To go 4th column, either have
$1 + S + 1 = O + 10$,
or $S + 1 = O + 10$
✓meaning $S = O + 8$
or $S = O + 9$, and $O = 0$ or 1

(C4|1)	(C3|)	(C2|)	(C1|)	
	(S|)	(E|)	(N|)	(D|)
+	(M|1)	(O|0)	(R|)	(E|)
(M|1)	(O|0)	(N|)	(E|)	(Y|)

Constraint Satisfaction Problem

✓To go 3rd column, since E cannot equal N, we must have $1 + E + 0 = N$.
✓Since $C3 = 0$, E must be less than 9, and 2nd column must have carryover.

(C4|1)	(C3|0)	(C2|1)	(C1|)	
	(S|9)	(E|)	(N|)	(D|)
+	(M|1)	(O|0)	(R|)	(E|)
(M|1)	(O|0)	(N|)	(E|)	(Y|)

✓$1 + E = N$ since 2nd column has carryover $C2 = 1$.
✓To go to 2nd column,
$N + R = E + 10$, OR
$N + R + 1 = E + 10$.
✓substituting $1 + E = N$ above,
$(1 + E) + R = E + 10 \Rightarrow R = 9$
which is not possible.
$(1 + E) + R + 1\ (C1) = E + 10$
$\Rightarrow R = 8$

(C4|1)	(C3|0)	(C2|1)	(C1|1)	
	(S|9)	(E|)	(N|)	(D|)
+	(M|1)	(O|0)	(R|8)	(E|)
(M|1)	(O|0)	(N|)	(E|)	(Y|)

Constraint Satisfaction Problem

- ✓ To go 1st column, D + E = Y and must have carryover.
- ✓ Since Y cannot be 0 or 1, we need D + E $\geq$ 12
- ✓ Since 9 and 8 are taken for S and R, we can have 5 + 7 = 12 or 6 + 7 = 13.
- ✓ Therefore, either D = 7 or E = 7.
- ✓ If E = 7, then E + 1 = N so N = 8 which is not possible.
- ✓ Since R = 8, we must have D = 7, meaning E is either 5 or 6.
- ✓ If E = 6, then N = 7 which is not possible as D = 7.
- ✓ So, we must have E = 5 and N = 6.

This means D + E = 7 + 5 = 12, and thus Y = 2 & $\Rightarrow$ N = 6.

(C4|1)	(C3|0)	(C2|1)	(C1|1)	
	(S|9)	(E|5)	(N|6)	(D|7)
+	(M|1)	(O|0)	(R|8)	(E|5)
(M|1)	(O|0)	(N|6)	(E|5)	(Y|2)

Production systems

Production systems/Production Rule System

- A production system in AI is a kind of cognitive architecture that provides artificial intelligence based on a set of rules.
- That is $\rightarrow$ it consists of a set of rules, each consisting of a left hand side (pattern) that determines the applicability of rules (condition) and a right side that describes the operation to be performed if the rule is applied (action).
- **Rules** recognize the condition, and the **actions** part has the knowledge of how to deal with the condition.

Production systems

Production systems/Production Rule System

AI production system has three main elements:

- **Global Database:** The primary database which contains all the information necessary to successfully complete a task.
- **A set of Production Rules:** A set of rules that operates on the global database. Each rule consists of a precondition and postcondition that the global database either meets or not.
- **Control System:** A control system that acts as decision making component, decides which production rule should be applied. It stops computation or processing when a termination condition is met on the database.

Production systems

Features of production system:

1. **Simplicity**: IF-THEN structure makes each sentence/production rule unique in the production system – which makes the knowledge representation simple and the production rules more readable.
2. **Modularity**: The knowledge available is coded in modular/discrete pieces by the production system, which makes it easy to add, modify, or delete the information without any side effects.
3. **Modifiability**: This feature allows for the modification of the production rules. The rules are first defined in the skeletal form and then modified to suit an application.

Production systems

Features of production system:

5. **Knowledge-intensive**: The system only stores knowledge. All the rules are written in the English Language and the representation ease the semantics problem.

Advantages of Production System

- Offers modularity as all the rules can be added, deleted, or modified individually.
- Separate control system and knowledge base.
- An excellent and feasible model that imitates human problem-solving skills.
- Beneficial in real-time applications and environment.
- Offers language independence.

Production systems...

Water Jug Problem:

You are given two unlabeled empty water jugs x, y that can hold 4 ltrs and 3 ltrs of water respectively. Now fill the water jug x with exactly 2 ltrs keeping jug y empty from the water pool.

Note: Neither jug has any measuring marking on it.

State Space Representation:

- state of the problem will be represented with tuple (x,y).
- x represents the amount of water in 4-liter jug and y represent the amount of water in 3-liter jug.
- Note that $0 \leq x \leq 4$ and $0 \leq y \leq 3$
- **initial state:** (0,0); **goal state:** (2,y) where $0 \leq y \leq 3$

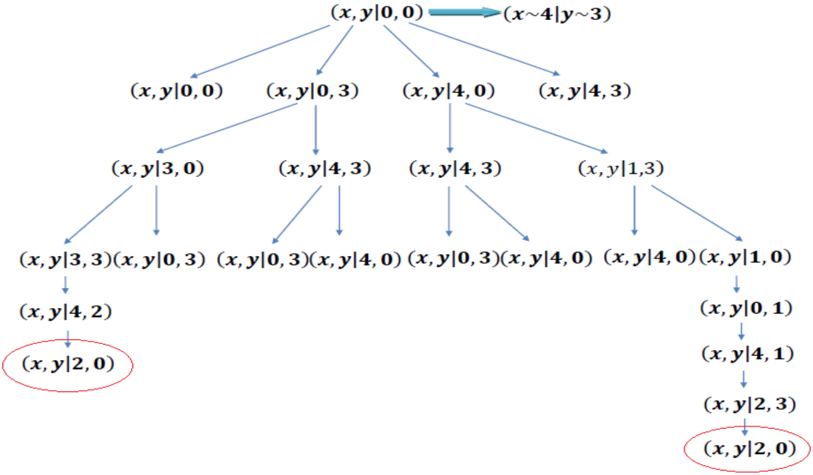
Production systems...

Production Rules or Set of Operators:

state	Current State (left) and condition	Next State (Right)	Definition
1	(x, y) & $x < 4$	(4, y)	Fill jug x
2	(x, y) & $y < 3$	(x, 3)	Fill jug y
3	(x, y) & $0 < x \leq 4$	(0, y)	Empty jug x
4	(x, y) & $0 < y \leq 3$	(x, 0)	Empty jug y
5	(x, y) & $x + y \geq 4$	(4, y-(4-x))	Fill jug x from jug y
6	(x, y) & $x + y \geq 3$ & $x > 0$	(x-(3-y), 3)	Fill jug y from jug x
7	(x, y) & $x + y \leq 4$	(x+y, 0)	Add water from jug y to jug x
8	(x, y) & $x + y \leq 3$	(0, x+y)	Add water from jug x to jug y

Production systems...

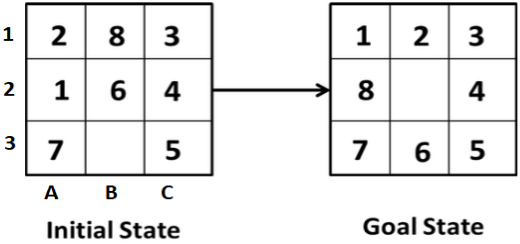
Water Jug Problem



Production systems...

Eight Puzzle Problem

Solve the given 8-puzzle game as shown in the figure:



Production systems...

Production Rules or Set of Operators

State	Current State (left)	Next State (Right)	Definition
1	Blank Space(1,A)	(1,A) $\rightarrow$ (1,B) (1,A) $\rightarrow$ (2,A)	Move Right Move Down
2	Blank Space(1,B)	(1,B) $\rightarrow$ (1,A) (1,B) $\rightarrow$ (2,B) (1,B) $\rightarrow$ (1,C)	Move left Move Down Move Right
3	Blank Space(1,C)	(1,C) $\rightarrow$ (1,B) (1,C) $\rightarrow$ (2,C)	Move Left Move Down
4	Blank Space(2,A)	(2,A) $\rightarrow$ (2,B) (2,A) $\rightarrow$ (1,A) (2,A) $\rightarrow$ (3,A)	Move Right Move Up Move Down
5	Blank Space(2,B)	(2,B) $\rightarrow$ (1,B) (2,B) $\rightarrow$ (3,B) (2,B) $\rightarrow$ (2,A) (2,B) $\rightarrow$ (2,C)	Move Up Move Down Move Left Move Right

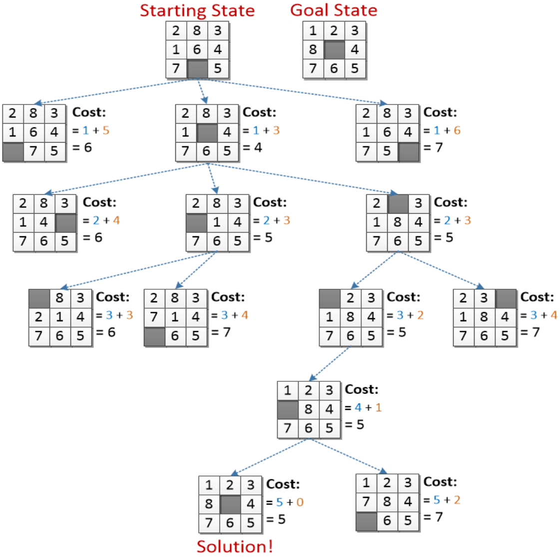
Production systems...

Production Rules or Set of Operators

State	Current State (left)	Next State (Right)	Definition
6	Blank Space(2,C)	(2,C) $\rightarrow$ (1,C) (2,C) $\rightarrow$ (3,C) (2,C) $\rightarrow$ (2,B)	Move Up Move Down Move Left
7	Blank Space(3,A)	(3,A) $\rightarrow$ (2,A) (3,A) $\rightarrow$ (3,B)	Move Up Move Right
8	Blank Space(3,B)	(3,B) $\rightarrow$ (3,A) (3,B) $\rightarrow$ (3,C) (3,B) $\rightarrow$ (2,B)	Move Left Move Right Move Up
9	Blank Space(3,C)	(3,C) $\rightarrow$ (3,B) (3,C) $\rightarrow$ (2,C)	Move Left Move Up

Production systems...

Eight Puzzle Problem



Module Assignment – As You Go

Module#2 Assignment is available at MS-Team.

Submission Deadline: 27th January 2023 (*Before 3:00 PM*)