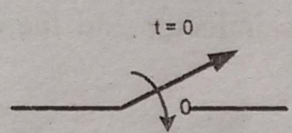
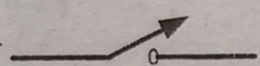
Initial Conditions

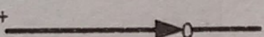
 # Initial Conditions: The value of dependent variables and its derivatives just after any change is made is known as Initial condition. Example, $i(0^+)$ = ' i ' is the dependent value at $t = 0^+$ condition.

 # Changing Element [Switch]:



(i) Initially it was opened at $t = 0^-$ and going for closed at $t = 0^+$

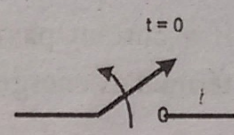
At $t = 0^-$ 

At $t = 0^+$ 

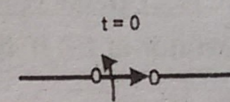
(ii) $t = 0^+$ the time just after the switch is closed.

(iii) $t = 0^-$ the time just before the switch is closed.

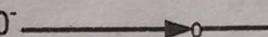
(iv) $i(0^+)$ the value of the current just after the switch is closed.

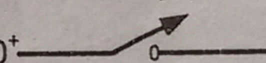


OR



(i) Initially it was closed at $t = 0^-$ and going open for at $t = 0^+$

At $t = 0^-$ 

At $t = 0^+$ 

(ii) $t = 0^+$ the time just after the switch is opened.

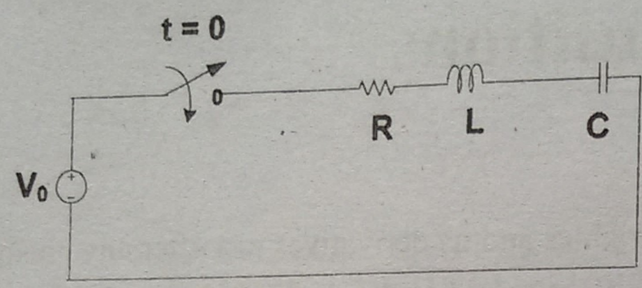
(iii) $t = 0^-$ the time just before the switch is opened.

(iv) $i(0^+)$ the value of the current just after the switch is opened.

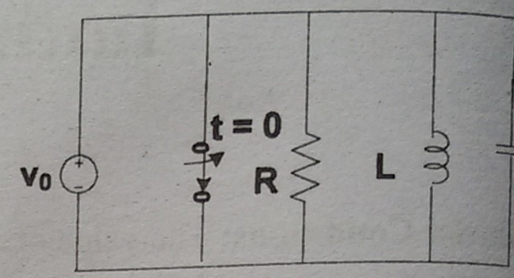
<u># Initial condition for different circuit elements:</u>

While evaluating initial condition for different circuit elements, we should know about two conditions for better understanding (a) De-energized condition (b) Energized condition.

(a).<u>De- energized condition:</u> If a current doesn't pass through the inductor and there is no voltage across capacitor at $t = 0^-$ then it is termed as De-energized element.

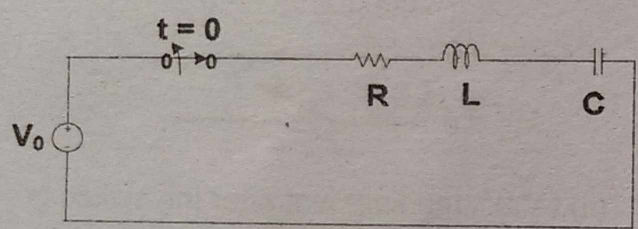


'OR'

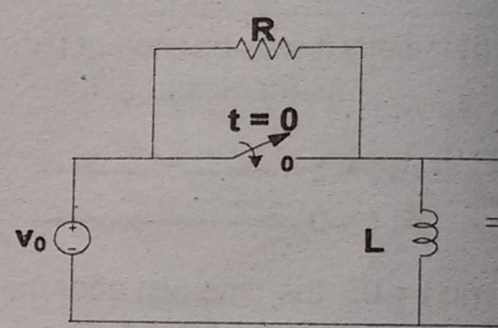


At, $t = 0^-$ there is zero value of current passing through inductor and capacitor for both circuit shown in above figures.

(b). Energized condition: If a current pass through the inductor and there is voltage across capacitor at $t = 0^-$ then it is termed as energized element.



'OR'



At, $t = 0^-$ there is some value of current passing through inductor and capacitor for both circuit shown in above figures.

1. **For Inductor:**

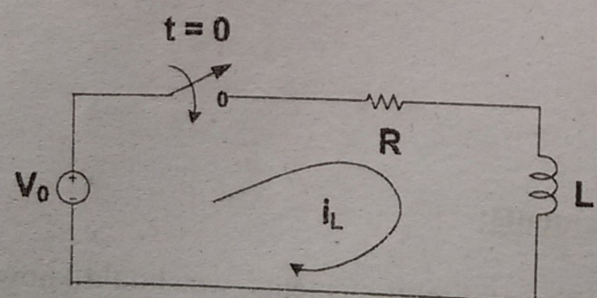


Fig .(i)

From the circuit, (De- energized)

$i_L(0^-) = 0$

Now let us consider the current through conductor can change instantaneously (i.e. $i_L(0^+)$ have some current value)

$i_L(0^-) = 0$

$i_L(0^+) = \text{Constant Value}$

We know,

$$V_L = L\frac{di_L}{dt}$$

$di_L = i_L(0^+) - i_L(0^-) = \text{Constant Value}$

$dt = 0^+ - 0^- \approx 0$

Thus,

$$V_L = L * \frac{\text{Constant}}{0}$$

$$V_L = \infty$$

This is impossible because there is no any such practical voltage source. Thus our assumption of inductor current change instantaneously is wrong. Hence we can conclude that current through inductor cannot change instantaneously.

i.e. $i_L(0^-) = i_L(0^+)$ ---------------------- (i)

Equation (i) is known as continuity relation for Inductor.

> **Note:** Current through inductor cannot change instantaneously but voltage does.
>
> I.e. $\mathbf{i_L(0^-) = i_L(0^+)}$
>
> And $\mathbf{V_L(0^-) \neq V_L(0^+)}$

Now,

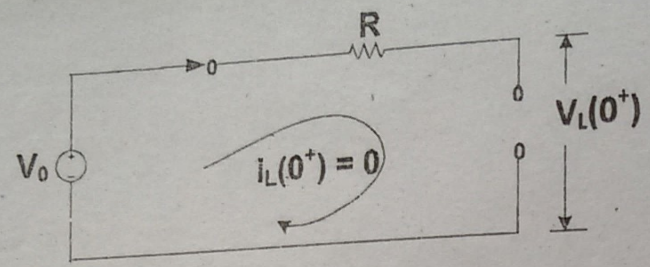
From continuity relation of inductor in Fig.(i)

$i_L(0^-) = i_L(0^+) = 0$

[This shows that at $t = 0^+$ there should be a zero current in inductor, and to be zero current there must be an open circuit in place of Inductor.]

Also, $V_L(0^-) = 0$

So, circuit of Fig.(i) at $t = 0^+$ is



Thus $V_L(0^+) = V_0$

$i_L(0^+) = 0$

Thus we conclude that the <u>De-energized inductor</u> can be <u>replaced</u> by <u>open circuit</u> at <u>$t = 0^+$</u>

And

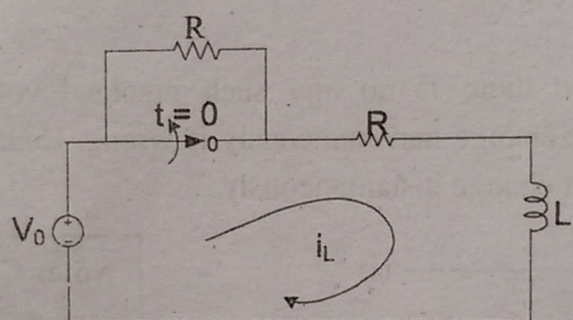


Fig (ii)

From the circuit, (Energized)

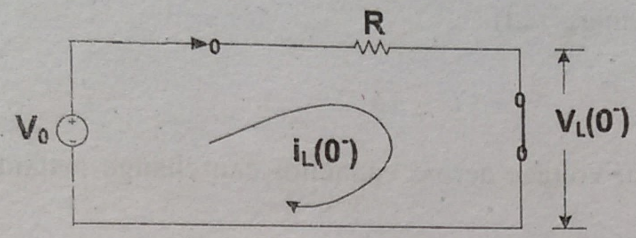
At this circuit for time at $t = 0^-$ all the voltage and current attains their maximum value or final value as switch was in this position (ON) for very long time. Thus $i_L(0^-) = I_0 = \frac{V_0}{R}$(constant)

$$V_L(0-) = L\frac{dI_L}{dt} = L\frac{dI_0}{dt} = L * 0$$

$$V_L(0-) = 0$$

[This shows that at $t = 0^-$ there should be a zero voltage across inductor, and to be zero voltage across there must be a short circuit in place of inductor.]

So, Circuit at $t = 0^-$ is



From the circuit $I_L(0^-) = \frac{V_0}{R} = I_0$

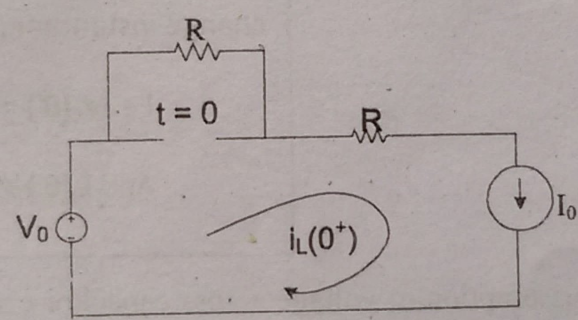
And $V_L(0-) = 0$

Thus we conclude that the inductance of energized condition is replaced by short circuit at $t = 0^-$ or $t = \infty$.

Note: $t = 0^-$ and $t = \infty$ are same

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = I_0$

Now, circuit at $t = 0^+$ is



From the circuit $i_L(0^+) = I_0$

And $V_L(0^+) = V_0 - I_0 * 2R$

Thus we conclude that the energized inductor is replaced by current source at $t = 0^+$

2. **For Capacitor :**

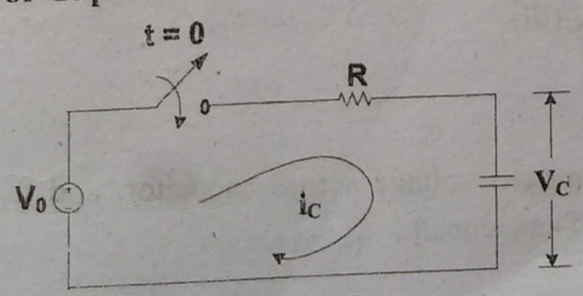


Fig (iii)

From the circuit, (De- energized)

$$V_C(0^-) = 0$$

Now let us assume that voltage across capacitor can change instantaneously (i.e. $V_C(0^+)$ have some current value)

$$V_C(0^-) = 0$$

$$V_C(0^+) = \text{Constant Value}$$

We know,

$$i_C = C\frac{dV_C}{dt}$$

$$dV_C = V_C(0^+) - V_C(0^-) = \text{Constant Value}$$

$$dt = 0^{+} - 0^{-} \approx 0$$

Thus,

$$i_C = C * \frac{\text{Constant}}{0}$$

$$i_C = \infty$$

Note: Voltage across capacitor cannot change instantaneously but current does.

I.e. $V_C(0^-) = V_C(0^+)$

And $i_C(0^-) \neq i_C(0^+)$

This is impossible thus our assumption of voltage across capacitor can change instantaneously is wrong. Hence we can conclude that voltage across capacitor cannot change instantaneously.

i.e. $V_C(0^-) = V_C(0^+)$ ------------------ (i)

Equation (i) is known as continuity relation for capacitor.

Now,

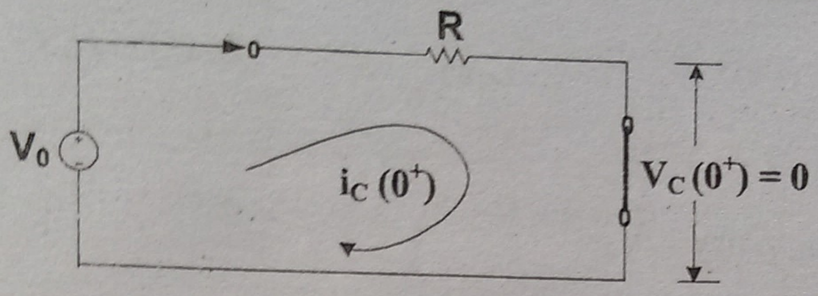
From continuity relation of capacitor in Fig.(iii)

$$V_C(0^-) = V_C(0^+) = 0$$

[This shows that at $t = 0^+$ there should be a zero voltage across capacitor, and to be zero voltage there must be a short circuit in place of capacitor.]

Also, $i_C(0^-) = 0$

So, circuit of Fig.(iii) at $t = 0^+$ is



Thus $V_C(0^+) = 0$

$$i_C(0^+) = \frac{V_0}{R} = I_0$$

Thus we conclude that the De-energized capacitor can be replaced by short circuit at $t = 0^+$

And

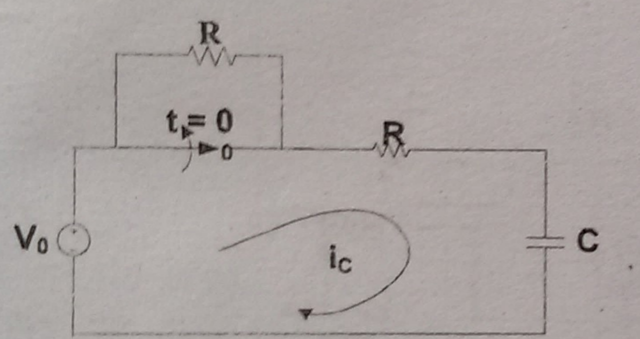


Fig (iv)

From the circuit, (Energized)

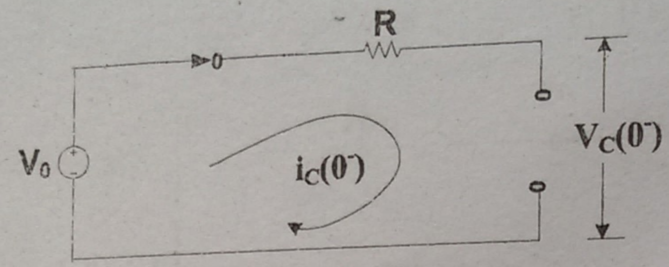
At this circuit for time at $t = 0^-$ all the voltage and current attains their maximum value or final value as switch was in this position (ON) for very long time. Thus $V_C(0^-) = V_0$ (constant)

$$i_C(0-) = C\frac{dV_C}{dt} = C\frac{dV_0}{dt} = C * 0$$

$$i_C(0-) = 0$$

[This shows that at $t = 0^-$ there should be a zero current through capacitor, and to be zero current there must be an open circuit in place of capacitor.]

So, Circuit at $t = 0^-$ is



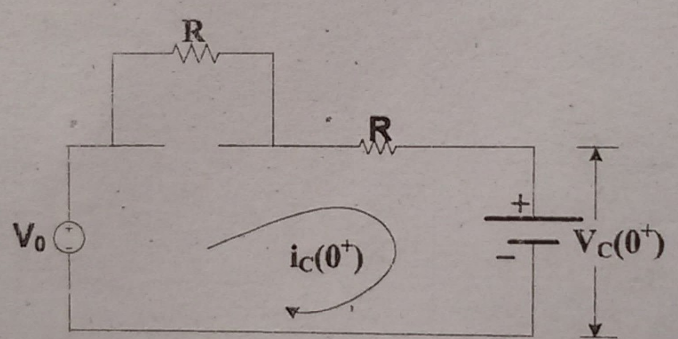
From the circuit $V_C(0^-) = V_0$

And $i_C(0-) = 0$

Thus we conclude that the capacitor of energized condition is replaced by open circuit at $t = 0^-$ or $t = \infty$.

From continuity relation for inductor, $V_C(0^-) = V_C(0^+) = V_0$

Now, circuit at $t = 0^+$ is



From the circuit $V_C(0^+) = V_0$

And $i_C(0^+) = \frac{V_0 - V_C}{2R} = 0$

Thus we conclude that the energized capacitor is replaced by Voltage source at $t = 0^+$

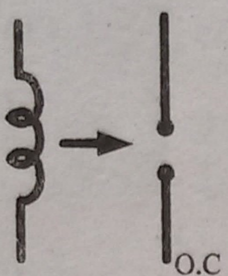
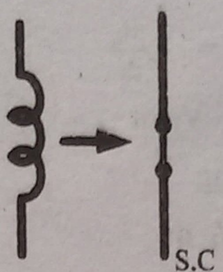
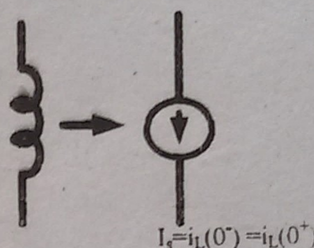
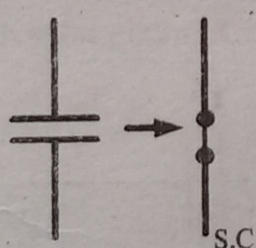
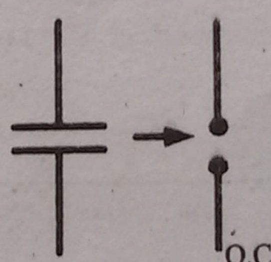
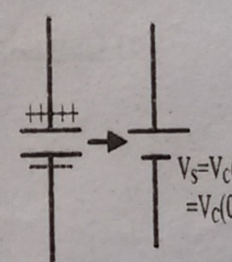
3. **For Resistor:**

And for resistor there will be:

. i.e. $i_R(0^-) \neq i_R(0^+)$

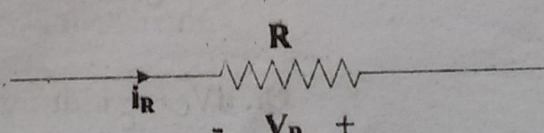
And $V_R(0^-) \neq V_R(0^+)$

Summary:

Circuit Elements	De-energized		Energized	
	At t=0⁻ or t=∞	At t= 0⁺	At t=0- or t=∞	At t= 0+
Inductor (L)	No need to change circuit elements			
Capacitor (C)	No need to change circuit elements			

 # Voltage – current (V-I) relation of different circuit elements:

(a) Resistance or Resistor (R)

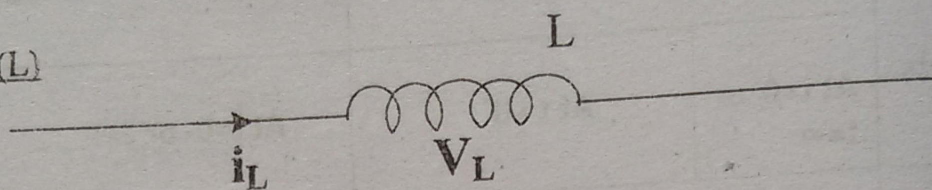


Here, $V_R = i_R * R$

$i_R = \frac{V_R}{R}$

The voltage across a resistance change instantaneously the current through it al changes instantaneously and vice versa.

(b) Inductor (L)



$$\text{Here, } V_L = L\frac{di_L}{dt}$$

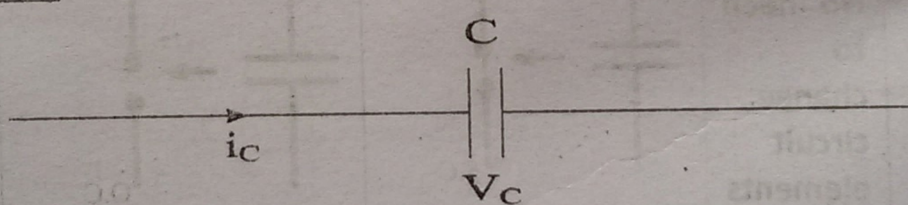
$$\text{Or, } di_L = \frac{1}{L} V_L\, dt$$

Integrating

$$i_L = \frac{1}{L}\int V_L\, dt$$

The current through an inductor cannot change instantaneously. Thus inductor acts open circuit for the newly applied energy at the instant of switching, but if I_0 current already flowing in the inductor before switching action, the inductor may considered as a current source I_0.

(c) Capacitor (C)



$$\text{Here, } i_C = \frac{dq}{dt} \qquad [q = VC]$$

$$\text{Or, } i_C = \frac{dCV_c}{dt}$$

$$\text{Or, } i_C = C\frac{dV_c}{dt}$$

$$\text{Or, } dV_C = \frac{1}{C} i_C\, dt$$

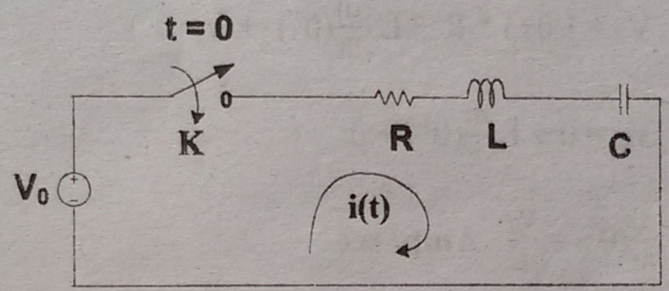
Integrating

$$V_C = \frac{1}{C}\int i_C\,dt$$

For a system of fixed capacitance, the voltage cannot change instantaneously. Thus for a suddenly applied energy source a capacitor is equivalent to a short circuit. Hence on connecting an uncharged capacitor to an energy source, a current flows instantaneously.

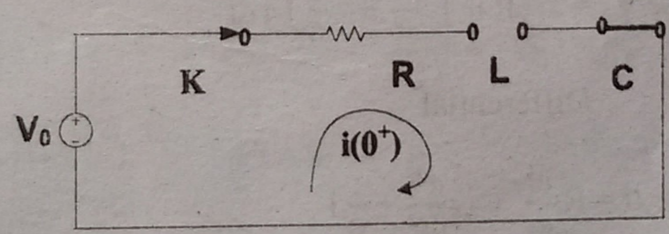
Initial conditions in the case of a series RLC network:

In the circuit shown below switch K is open for a long time at $t = 0$ switch is going to close then find $i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$.



Solution:

It's a de-energized condition, so at $t = 0^+$ inductor is replace with open circuit and capacitor is replace with short circuit. Circuit at $t = 0^+$ is shown below

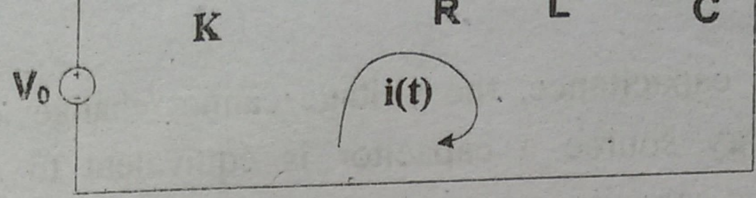


So we know by inspection

$i_L(0^-) = 0 = i_L(0^+)$, $V_C(0^-) = 0 = V_C(0^+)$

$i(0^+) = i_L(0^+) = 0$ **Amp**

Now, to find $\frac{di}{dt}(0^+)$, apply KVL at $t > 0$ in the circuit,



$$V_0 = V_R + V_L + V_C$$

$$\text{Or, } V_0 = i * R + L\frac{di}{dt} + V_C \quad \text{------------------- (i)}$$

Put $t = 0^+$

$$\text{Or, } V_0 = i(0+) * R + L\frac{di}{dt}(0^+) + V_C(0^+)$$

$$\text{Or, } V_0 = 0 + L\frac{di}{dt}(0^+) + 0$$

$$\frac{di}{dt}(0^+) = \frac{V_0}{L} \text{ Amp/ sec}$$

Now, to find $\frac{d^2i}{dt^2}(0^+)$, differential equation(i) w.r.t 't' , we get

$$V_0 = i * R + L\frac{di}{dt} + V_C$$

$$\text{Or, } V_0 = i * R + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Differential

$$\text{Or, } 0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

Put $t = 0^+$

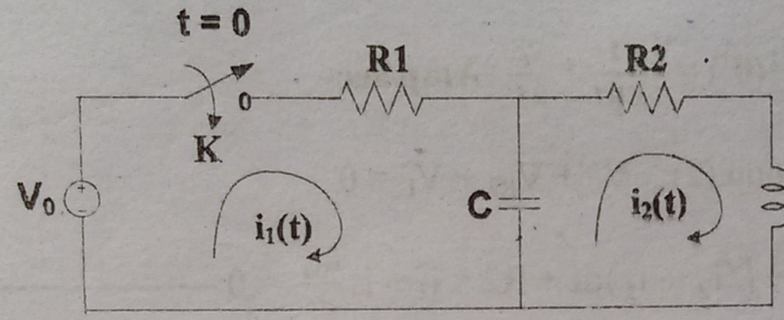
$$\text{Or, } 0 = R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{1}{C}i(0^+)$$

$$\text{Or, } 0 = R * \frac{V_0}{L} + L\frac{d^2i}{dt^2}(0^+) + 0$$

$$\frac{d^2i}{dt^2}(0^+) = -\frac{RV_0}{L} \text{ Amp/ sec}^2$$

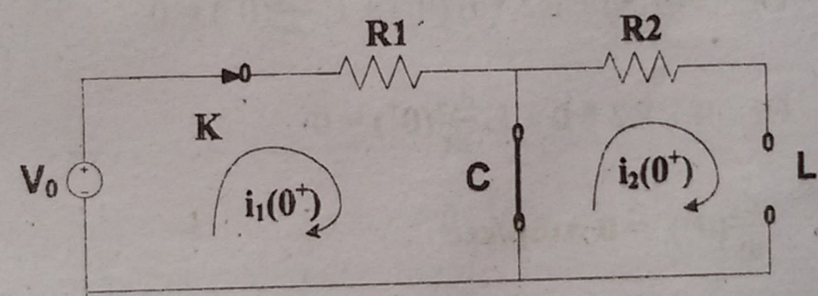
Initial conditions in the case of a two-loop RLC network:

In the circuit shown below switch K is open for a long time at $t = 0$ switch is closed then find $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$ $\frac{d^2i_1}{dt^2}(0^+)$ and $\frac{d^2i_1}{dt^2}(0^+)$.



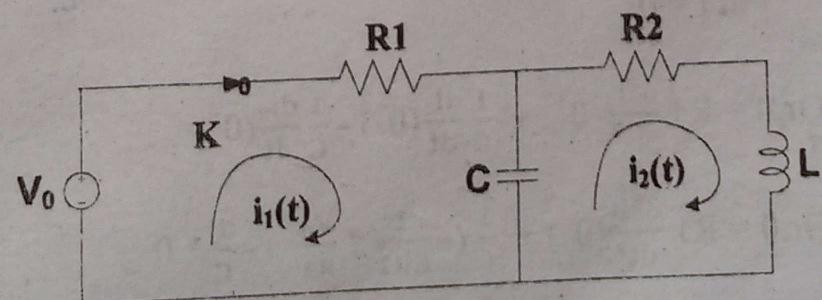
Solution:

It's a de-energized condition, so at $t = 0^+$ inductor is replace with open circuit and capacitor is replace with short circuit. Circuit at $t = 0^+$ is shown below



By inspection, $i_L(0^-) = i_2(0^-) = i_2(0^+) = 0$ and $i_1(0^+) = \frac{V_0}{R1}$ Also ,$V_C(0^-) = V_C(0^+) = 0$

Now, to find $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$ apply KVL at $t > 0$ in the circuit,



From loop (1): $V_0 = V_{R1} + V_C$

Or, $V_0 = R1 * i_1 + \frac{1}{C}\int(i_1 - i_2)dt$ Differential w.r.t't'

Or, $0 = R1\frac{di_1}{dt} + \frac{1}{C}(i_1 - i_2)$ ---------------- (i)

Put $t = 0^+$

Or, $0 = R1\,\frac{di_1}{dt}(0^+) + \frac{1}{C}\,i_1(0^+) - \frac{1}{C}\,i_2(0^+)$

Or, $0 = R1\,\frac{di_1}{dt}(0^+) + \frac{1}{C} * \frac{V_0}{R1} - \frac{1}{C} * 0$

$\frac{di_1}{dt}(0^+) = -\frac{1}{CR1} * \frac{V_0}{R1}$ **Amp/sec**

From loop (2): $V_C + V_{R2} + V_L = 0$

Or, $\frac{1}{C}\int(i_2 - i_1)dt + R2 * i_2 + L\frac{di_2}{dt} = 0$ ------------------- (ii)

Or, $V_C + R2 * i_2 + L\frac{di_2}{dt} = 0$

Put $t = 0^+$

Or, $V_C(0^+) + R2 * i_2(0^+) + L\frac{di_2}{dt}(0^+) = 0$

Or, $0 + R2 * 0 + L\frac{di_2}{dt}(0^+) = 0$

$\frac{di_2}{dt}(0^+) = 0$ **Amp/sec**

Now, to find $\frac{d^2i_1}{dt^2}(0^+)$ and $\frac{d^2i_1}{dt^2}(0^+)$, differential equation (i) and (ii),

From equation (i) after differential, $0 = R1\,\frac{d^2i_1}{dt^2} + \frac{1}{C}\frac{di_1}{dt} - \frac{1}{C}\frac{di_2}{dt}$

Put $t = 0^+$

Or, $0 = R1\,\frac{d^2i_1}{dt^2}(0^+) + \frac{1}{C}\frac{di_1}{dt}(0^+) - \frac{1}{C}\frac{di_2}{dt}(0^+)$

Or, $0 = R1\,\frac{d^2i_1}{dt^2}(0^+) + \frac{1}{C}\left(-\frac{1}{CR1} * \frac{V_0}{R1}\right) - \frac{1}{C} * 0$

$\frac{d^2i_1}{dt^2}(0^+) = \frac{V_0}{C^2R1^3}$ **Amp/sec^2**

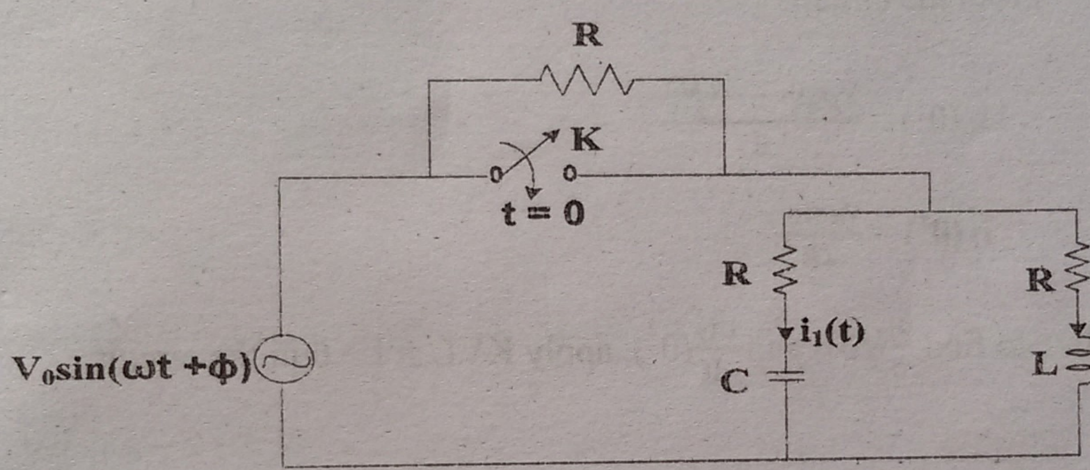
From equation (ii) after differential, $\frac{1}{C}\,i_2 - \frac{1}{C}\,i_1 + R2\,\frac{di_2}{dt} + L\,\frac{d^2i_2}{dt^2} = 0$

$$\text{Or,}\quad \frac{1}{C} i_2(0^+) - \frac{1}{C} i_1(0^+) + R2 \frac{di_2}{dt}(0^+) + L \frac{d^2 i_2}{dt^2}(0^+) = 0$$

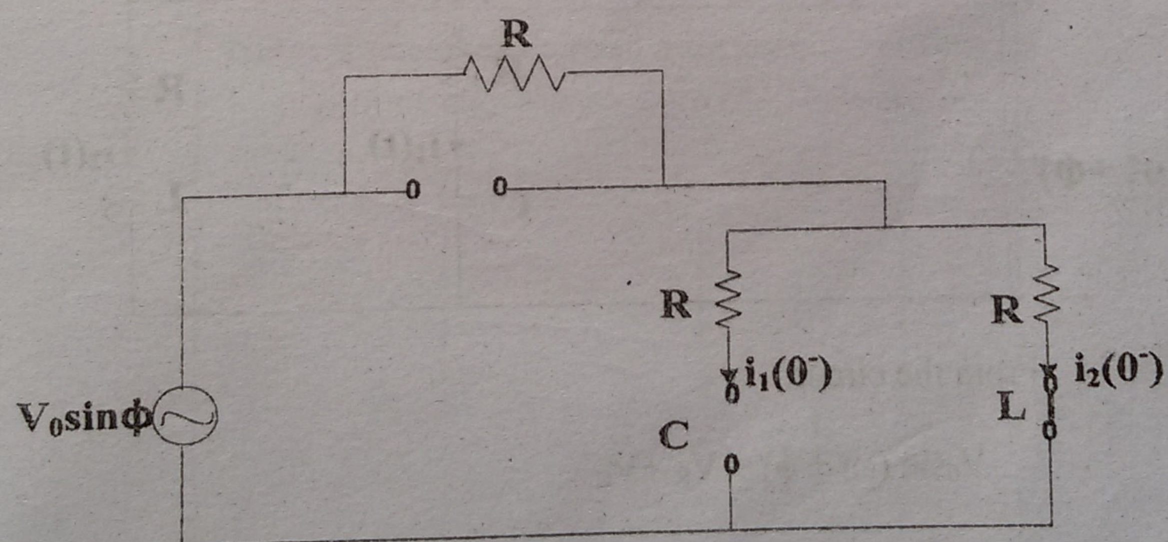
$$\text{Or,}\quad \frac{1}{C} * 0 - \frac{1}{C} * \frac{V_0}{R_1} + R2 * 0 + L \frac{d^2 i_2}{dt^2}(0^+) = 0$$

$$\frac{d^2 i_2}{dt^2}(0^+) = \frac{V_0}{LCR1} \ \textbf{Amp/sec}^2$$

Example 2.1: In the given circuit, switch K is closed at time $t = 0$, find $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$



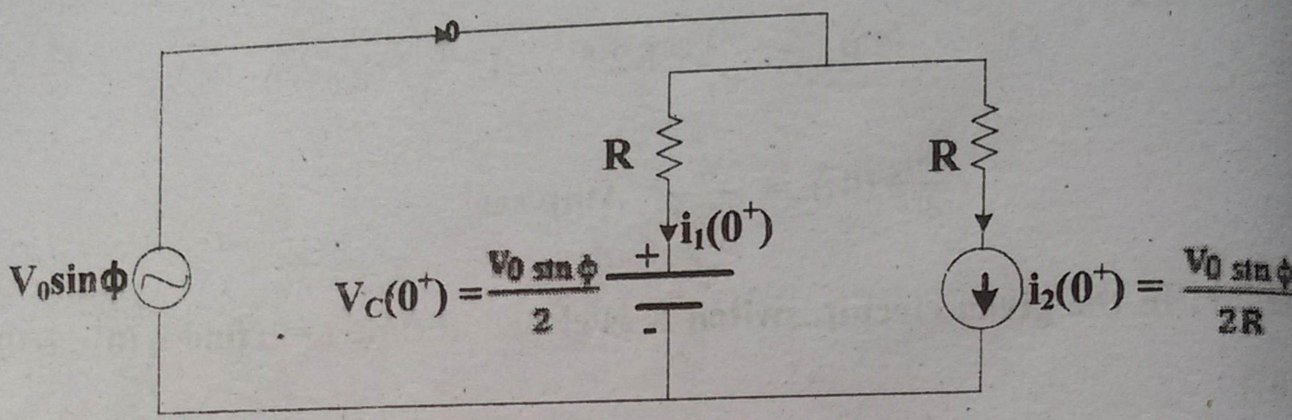
Solution: It's an energized condition, so at $t = 0^-$ inductor is replace with short circuit and capacitor with open circuit. Circuit at $t = 0^-$ is



From the circuit, $i_2(0^-) = i_2(0^+) = \frac{V_0 \sin}{2R}$

$$V_C(0^-) = V_C(0^-) = R * i_2(0^+) = \frac{V_0 \sin}{2}$$

Circuit at $t = 0^+$ is

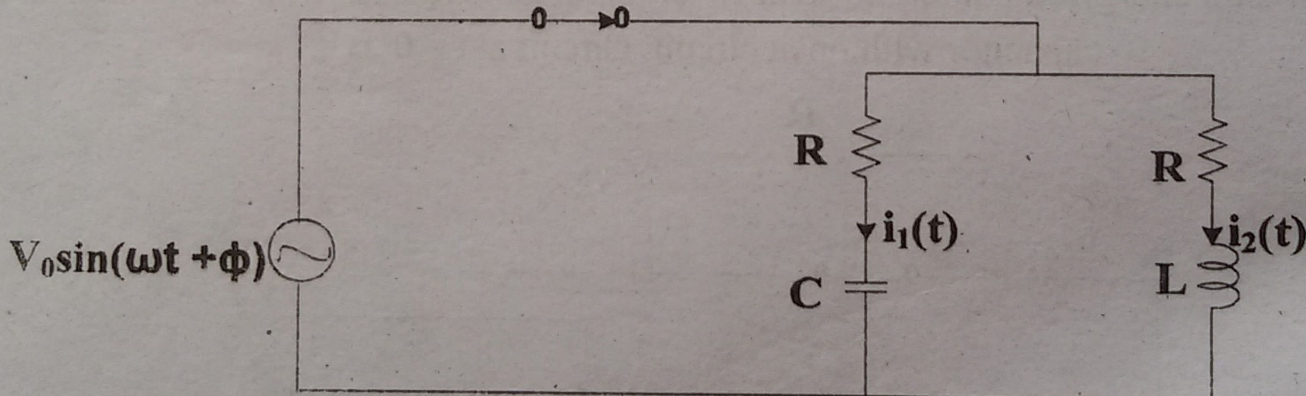


From the circuit,

$$i_1(0^+) = \frac{V_0 \sin - \frac{V_0 \sin}{2}}{R}$$

$$i_1(0^+) = \frac{V_0 \sin}{2R}$$

Now, to find $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$, apply KVL at $t > 0$ in the circuit



From the circuit,

$$V_0 \sin(\omega t + \phi) = V_R + V_C$$

Or, $V_0 \sin(\omega t + \phi) = i_1 * R + \frac{1}{C} \int i_1 dt$

Differential

Or, $V_0 * \omega \cos(\omega t + \phi) = R \frac{di_1}{dt} + \frac{1}{C} i_1$

Put $t = 0^+$

Or, $V_0 * \omega \cos(\omega * 0 + \phi) = R \frac{di_1}{dt}(0^+) + \frac{1}{C} i_1(0^+)$

Or, $V_0 * \omega \cos \phi = R \frac{di_1}{dt}(0^+) + \frac{1}{C} * \frac{V_0 \sin}{2R}$

$\frac{di_1}{dt}(0^+) = \frac{V_0 * \omega \cos - \frac{V_0 \sin}{2CR}}{R}$ **Amp/sec**

Again, from the circuit , (Outer loop)

$V_0 \sin (\omega t + \phi) = V_R + V_L$

Or, $V_0 \sin (\omega t + \phi) = R * i_2 + L \frac{di_2}{dt}$

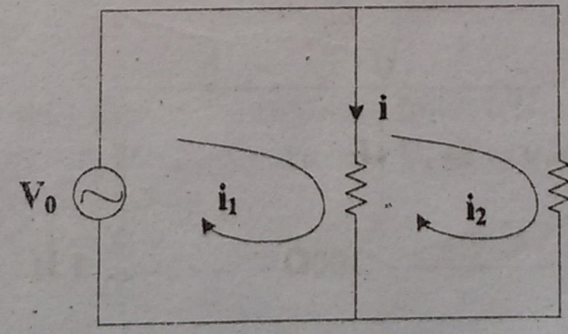
Put $t = 0^+$

Or, $V_0 \sin (\omega * 0 + \phi) = R * i_2 (0^+) + L \frac{di_2}{dt}(0^+)$

Or, $V_0 \sin \phi = R * \frac{V_0 \sin}{2R} + L \frac{di_2}{dt}(0^+)$

$\frac{di_2}{dt}(0^+) = \frac{V_0 \sin}{2L}$ **Amp/sec**

Note: Remember there is a difference between loop current and branch current. They are not same current.

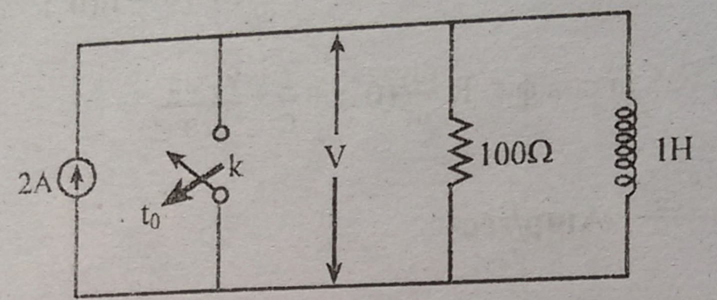


$i_1 - i_2 = i$

Here, $i_1 - i_2 = i$, as i_1 and i_2 are loop current where as 'i' is branch current.

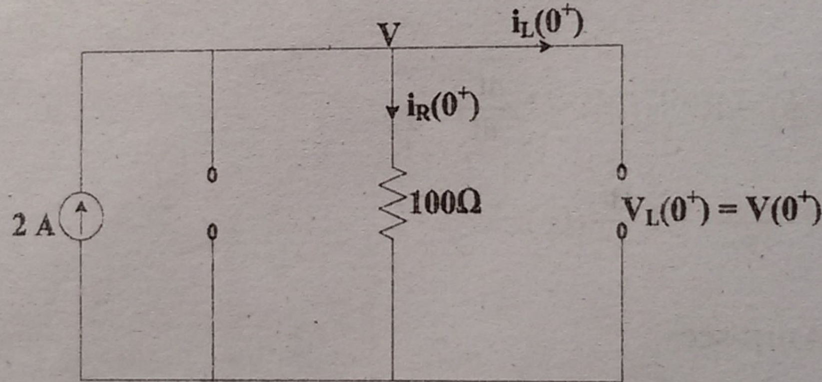
The above Example 2.1 is of branch current.

Example.2.2: In the given circuit, switch k is opened at $t = 0$, find the value of V, $\frac{dv}{dt}, \frac{d^2v}{dt^2}$ at $t = 0^+$.



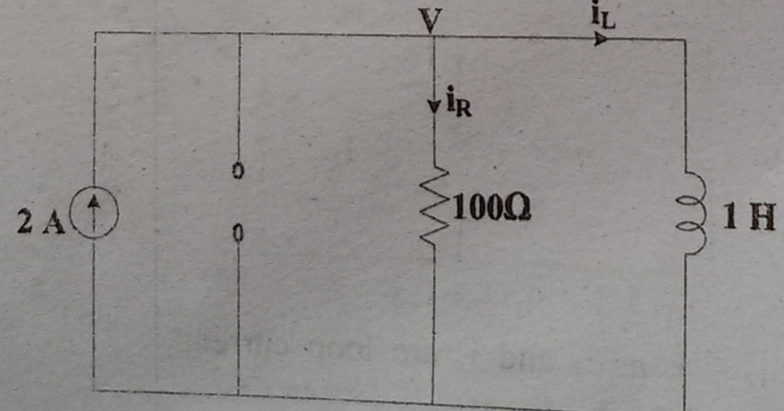
Solution:

It's a de-energized condition [since switch (ON at $t=0^-$) make short circuit and all current pass through it], so at $t = 0^+$ inductor is replace with open circuit.



$$V(0^+) = 200 \text{ Volt}$$

Now to find $\frac{dv}{dt}, \frac{d^2v}{dt^2}$ at $t = 0^+$ take circuit diagram at $t > 0$ is as shown below



Applying KCL at $t > 0$

$$2 = \frac{V}{100} + \frac{1}{1}\int V\,dt$$

Different w.r.t 't'

$$0 = \frac{1}{100}\frac{dv(t)}{dt} + V(t) \;\ldots\ldots\ldots\; (i)$$

Put $t = 0^+$

$$0 = \frac{1}{100}\frac{dv}{dt}(0^+) + V(0^+)$$

$$\frac{dv}{dt}(0^+) = -200 \times 100$$

$$\boldsymbol{\frac{dv}{dt}(0^+) = -2\times10^4 \text{ volt/sec}}$$

Again from equation (i)

$$0 = \frac{1}{100}\frac{dv}{dt} + v$$

Different w.r.t 't'

$$0 = \frac{1}{100}\frac{d^2v}{dt^2} + \frac{dv}{dt}$$

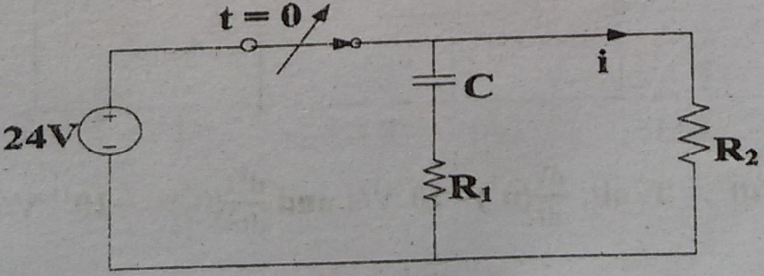
Put $t = 0^+$

$$\boldsymbol{\frac{d^2v}{dt^2}(0^+) = 2\times10^6 \text{ volt/sec}^2}$$

Note: Here student get confuse whether to use KVL at $t > 0$ 'OR' KCL at $t > 0$, it depends upon your own practice, But in my suggestion if there is pure parallel circuit along with current source like this example.2.2 use KCL at $t > 0$ and for other circuit use KVL at $t > 0$.

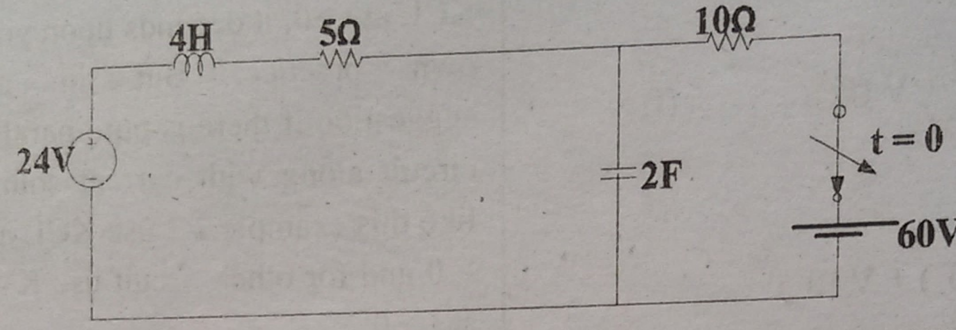
Problems:

Q.1: The circuit of figure below was under steady sate before the switch was opened at time $t = 0$. If $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C = 0.167F$, determine $V_C(0^-)$, $V_C(0^+)$ and $i(0^+)$.



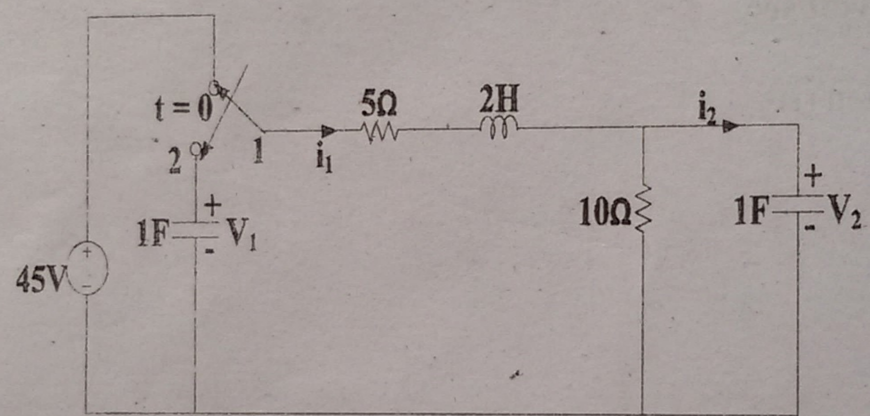
[Ans: $V_C(0^-)$=24V, $V_C(0^+)$=24, $i(0^+)$= 8A]

Q.2: The circuit of figure below was under steady state before the switch was opened at t = 0.Find the voltage and current for all four circuit elements at time $t = 0^+$



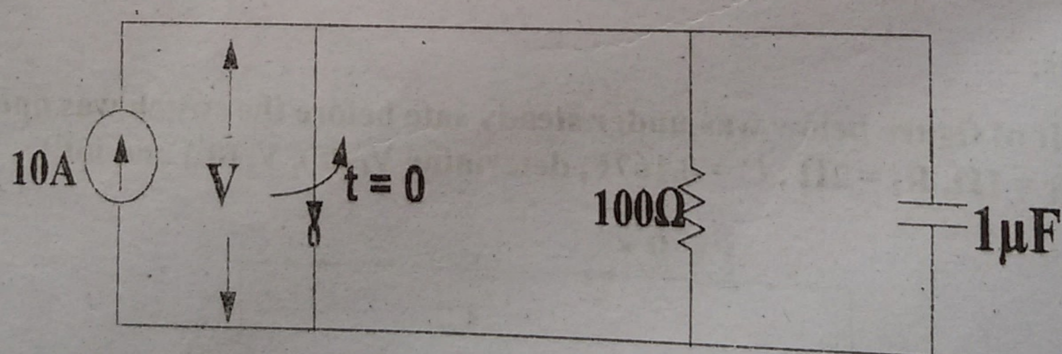
[Ans: Current: 3A,3A,3A,0A and Voltage:0V,15V,30V,0V]

Q.3: The switch was as position 1 under steady state. The switch was put to position 2 at t = 0. Find V_1, V_2, i_1 and i_2 at $t = 0^+$



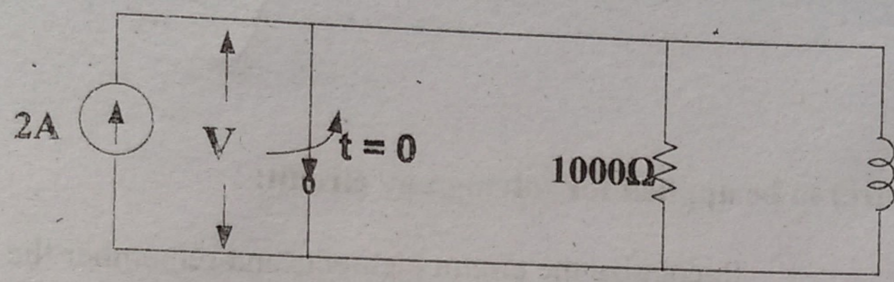
[Ans: $V_1(0^+)= 0V$, $V_2(0^+)= 30V$, $i_1(0^+)=3A$, $i_2(0^+) = 0A$]

Q.4:In the given circuit switch is opened at t =0. Find the value of V, $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$ at $t = 0^+$



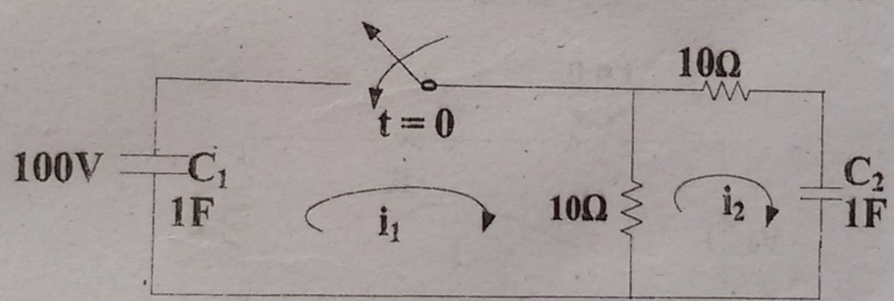
[Ans: $V(0^+)= 0$Volt, $\frac{dV}{dt}(0^+)= 10^7 V/s$ and $\frac{d^2t}{dt^2}(0^+)= -10^{11}\ V/s^2$]

Q.5: In the given circuit, Switch is opened at t =0. Find the values of V, $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$ at t = 0^+



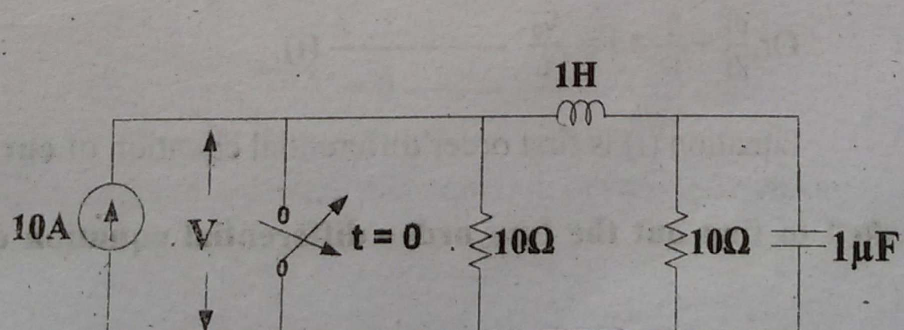
[Ans: $V(0^+)$= 2000Volt, $\frac{dV}{dt}(0^+)$= -2*10^6V/s and $\frac{d^2V}{dt^2}(0^+)$= 2*10^9 V/s^2]

Q.6: In the given circuit, C_1 is charged to 100Volts in the polarity shown. Switch is closed at t =0. Find the value of i_1, i_2, $\frac{di_1}{dt}$, $\frac{d^2i_1}{dt^2}$, $\frac{di_2}{dt}$ and $\frac{d^2i_2}{dt^2}$ at t = 0^+



[Ans: $i_1(0^+)$= 20A, $i_2(0^+)$= 10A, $\frac{di_1}{dt}(0^+)$= −5 A/s, $\frac{d^2i_1}{dt^2}(0^+)$= 1.3 A/s^2, $\frac{di_2}{dt}(0^+)$= −3 A/s, $\frac{d^2i_2}{dt^2}(0^+)$= 0.8A/s^2]

Q.7: In the given circuit, Switch is opened at t= 0.Find the values of V_1,V_2, $\frac{dV_1}{dt}$, $\frac{dV_2}{dt}$ at t = 0^+



[Ans: $V_1(0^+)$= 100Volt, $V_2(0^+)$= 0Volt, $\frac{dV_1}{dt}(0^+)$= 1000 V/s, $\frac{dV_2}{dt}(0^+)$= 0 V/s]