Enginering Physics

Bachelor of Engineering

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Chapter 1

WAVE AND OSCILLATIONS

1.1 SIMPLE HARMONIC MOTION

Any motion that repeats itself at regular intervals is called *periodic motion* or *harmonic motion.*

A particle may be said to execute a *simple harmonic motion (S.H.M.)* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

The differential equation of the simple harmonic motion is

$$\frac{d^2y}{dt} + \omega^2 y = 0 \qquad 1.1$$

where, y is the displacement of the particle of mass m.

$\omega = \sqrt{\frac{k}{m}}$ is often called angular velocity of object executing S.H.M..

k is a positive constant, called force constant.

The mechanical energy of the S.H.M. is constant and independent of time. The total energy E is proportional to square of the amplitude of the oscillation A.

i.e., $E = kA^2$ 1.2

1.2 EXAMPLES OF S.H.M.

i) The block spring system

The block spring system executes a simple harmonic motion with an angular frequency ω and time period of oscillation T as;

$$\left.\begin{aligned} \omega &= \sqrt{\frac{k}{m}} \\ T &= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \end{aligned}\right\} \qquad 1.3$$

ii) Simple pendulum

A simple pendulum is an idealized system in which a point mass is suspended at one end of an inextensible weightless string whose other

end is fixed in a rigid support. This fixed point is referred as the poin[t of]
suspension.

Within the small angle approximation, simple pendulum exec[utes]
S.H.M. with angular frequency,

$$\omega = \sqrt{\frac{g}{l}}$$

where, g is acceleration due to gravity

l is length of pendulum being measured from the point [of]
suspension to the centre of mass of the bob.

iii) Physical pendulum

A physical pendulum or compound pendulum or rigid pendulum i[s a]
rigid body, of whatever of shape, capable of oscillating about a horizon[tal]
axis passing through it.

The point in which vertical plane passing through the c. g. of [the]
pendulum meets the axis of rotation is called its centre of suspension a[nd]
the distance between the point of suspension and c. g. of the pendul[um]
measures the length of the pendulum.

The physical pendulum executes S.H.M. with time period,

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}}$$

i.e., $$T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}} = 2\pi\sqrt{\frac{L}{g}}$$

where, $I = m(k^2+l^2)$ is moment of inertia of the pendulum about t[he]
axis of suspension

k is the radius of gyration about an axis passing through the c. [g.]
of the pendulum

The equation (1.6) implies that the time period of the physical pendulu[m]
is same as that of the simple pendulum of the length $L = \frac{k^2}{l} + l$. Th[e]
length is, therefore, called the length of an equivalent simple pendulu[m]
or reduced length of the physical pendulum.

iv) Torsional pendulum

A heavy body like a disc or a cylinder, fastened at its mid-point to a fairl[y]
long and thin wire, suspended from a rigid support, con[stitutes a]
torsional pendulum.

[I]f the disc or the cylinder is Turned in its own plane i.e., in the
[h]orizontal plane, to twist the wire a little and then released, it executes
[t]orsional vibrations or oscillations about wire as axis. That's why, this
[p]endulum is called torsional pendulum.

The disc or the cylinder executes an angular S.H.M. with time period as;

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad 1.7$$

where, $C = \frac{\pi\eta R^4}{2L}$ is torsional couple per unit twist of the wire or
torsional constant.

R is the radius of the wire

L is the length of the wire

η is the modulus of rigidity of its material

The time period of a torsional pendulum remains unaffected even if the
amplitude be large, provided that the elastic limit of the suspension wire
is not exceeded.

1.3 FREE, FORCED, DAMPED AND RESONANT OSCILLATIONS

Free Oscillation

When a body capable of oscillation is displaced from its equilibrium
position and then left free, it begins to oscillate with a definite amplitude
and frequency. If the body is not restricted by any kind of friction, the
motion continues. Such oscillation is called free oscillation. *Example:*
When a bob of simple pendulum is displaced from its mean position and
left free, it executes a free oscillation.

Forced Oscillation

When a body is maintained in a state of oscillation by an external
periodic force of frequency other than the natural frequency of the body,
the oscillation is called forced oscillation. A body can be forced to
oscillate with any frequency depending upon that of the applied periodic
force. The oscillation dies out as soon as the applied force is removed.

Damped Oscillation

The oscillation whose amplitude goes on decreasing with time is called
damped oscillation. Practically all oscillations are damped. An oscillation
[w]hose amplitude remains constant with time is called undamped oscillation.

Resonant Oscillation

When a body is maintained in a state of oscillation by a periodic f having the same frequency as the natural frequency of the body, oscillation is called resonant oscillation. The phenomenon of produ resonant oscillation is called resonance.

1.4 QUALITY FACTOR

Mathematically, quality factor Q is defined as 2π times the rati energy store in the system to the energy cycle. It is sometimes ca figure of merit of the harmonic oscillator. The figure of merit is def as the ratio of the frequency at velocity resonance to the full bandw at half maximum power. It measures the sharpness of the resonance.

1.5 SOLVED EXAM QUESTIONS

1. **Define damped oscillation and derive the differential equa of damped oscillation of a mechanical system. Also deriv frequency of the oscillation.** [T.U. 2061 Baish

Solution:

Damped Oscillation:

The oscillation whose amplitude goes on decreasing with tin called damped oscillation. Practically all oscillations are dam

Energy of such a damped oscillator decreases with time.

Whenever a pendulum oscillates in the air, the energy of pendulum dissipated in each oscillation. After a long time vibration die out. The dissipative force is proportional to velo of particle at that time.

i.e., $F \propto \dfrac{dx}{dt}$

or, $F = -b\dfrac{dx}{dt}$

where, b is the damping constant. The differential equatio motion of in this case will be;

$$F = -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

or, $m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0$

i.e., $\dfrac{d^2x}{dt^2} + \dfrac{b}{m}\dfrac{dx}{dt} + \dfrac{k}{m}x = 0$

This is known as differential equation of damped oscillation of a mechanical system. The solution of this equation is

$$x = ae^{-\left(\frac{b}{2m}\right)t}\sin(\omega' t) \qquad 1.10$$

where, ω' is called damped angular frequency and given by;

$$\omega' = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} \qquad 1.11$$

Thus, the frequency of damped oscillation is;

$$f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} \qquad 1.12$$

The damped angular frequency ω' is less than natural angular frequency ω_0.

A damped oscillator has mass 250 g, spring constant 85 N/m and damping constant 70 g/s.

i) **How long does it take for the amplitude of the damped oscillator to drop to half of its initial value?**

ii) **How long does it take for the mechanical energy to drop to half of its initial value?**

Solution:

Here,

Mass of damped oscillator, $(m) = 250\ g = 0.25\ kg$

Spring constant, $(k) = 85\ Nm^{-1}$

Damping constant, $(b) = 70\ gs^{-1} = 0.07\ kgs^{-1}$

i) We have,

The amplitude variation of damped oscillator with time,

$$A = A_0e^{-\left(\frac{b}{2m}\right)t} \qquad 1.12$$

When amplitude of the damped oscillator drops to half of its initial value, *i.e.,* $A = \frac{A_0}{2}$, the equation (1.12) leads us to;

$$\frac{A_0}{2} = A_0e^{-\left(\frac{b}{2m}\right)t}$$

or, $\dfrac{1}{2} = e^{-\left(\frac{b}{2m}\right)t}$

or, $\ln\left(\dfrac{1}{2}\right) = -\left(\dfrac{b}{2m}\right)t$

or, $-0.693 = -\left(\dfrac{0.07}{2 \times 0.25}\right)t$

$\therefore\ t = 4.95\ s$

The damped oscillator takes 4.95 s for the amplitude to drop half of its initial value.

ii) We have,

At any point the mechanical energy of the damped oscillator be calculated using the expression,

$$E = E_0 e^{-\left(\frac{b}{m}\right)t}$$

When mechanical energy of the damped oscillator drops to half its initial value, *i.e.*, $E = \frac{E_0}{2}$, the equation (1.13) leads us to

$$\frac{1}{2} = e^{-\left(\frac{b}{m}\right)t}$$

$$\text{or,} \quad ln\left(\frac{1}{2}\right) = -\left(\frac{b}{m}\right)t$$

$$\text{or,} \quad -0.693 = -\frac{0.07}{0.25} \times t$$

$$\therefore \quad t = 2.48\ s$$

The damped oscillator takes 2.48 s for the mechanical energy to drop half of its initial value.

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3. **A uniform rod 1 m in length oscillates about a horizontal axis perpendicular to its length along the vertical plane. Find the position of the points about which the time period is minimum. If g = 9.8 ms^{-2}, Find the minimum period of oscillation.**
[T.U. 2061 Ash...]

Solution:

Let l_1 m be the distance from centre of suspension to the centre of gravity, then, $(1 - l_1)$ m be the distance from centre of oscillation to the centre of gravity.

The time period of pendulum will be minimum when its length is equal to its radius of gyration k.

$$i.e., \quad l_1 = k$$

$$\text{or,} \quad l_1^2 = k^2 = l_1(1 - l_1)$$

$$\text{or,} \quad 2l_1 = 1$$

$$\therefore \quad l_1 = 0.5\ m$$

Hence, the centre of suspension and centre of oscillation are at distance of 0.5 m from the centre of gravity.

The minimum time period of oscill...

$$T_{min} = 2\pi\sqrt{\frac{(k^2 + l_1^2)}{gl_1}} = 2\pi\sqrt{\frac{(l_1^2 + l_1^2)}{gl_1}} = 2\pi\sqrt{\frac{2l_1}{g}}$$

$$= 2\pi\sqrt{\frac{2 \times 0.5}{9.8}} = 2.01\ s$$

Define the terms: frequency and amplitude of the simple harmonic motion. Derive the expression for the time period of (i) physical pendulum and (ii) spring mass system. **[T.U. 2062 Baishakh]**

...olution:

Amplitude and frequency of the simple harmonic motion

The maximum displacement of the particle on either side of the equilibrium position of S.H.M. is called amplitude of S.H.M.

The number of the complete oscillations per second is called frequency of S.H.M.

Physical pendulum

A physical pendulum or compound pendulum or rigid pendulum is a rigid body, of whatever of shape, capable of oscillating about a horizontal axis passing through it.

The point in which vertical plane passing through the c.g. of the pendulum meets the axis of rotation is called its centre of suspension and the distance between the point of suspension and c.g. of the pendulum measures the length of the pendulum.

Figure: A physical pendulum

Figure depicts the vertical section of a rigid body, *i.e.*, a physical pendulum, free to rotate about a horizontal axis passing through the point called centre of suspension S. In its normal position of rest, its g. G, naturally lies vertically below S, the distance between S and G giving the length l of the pendulum.

When the pendulum is displaced through a small angle θ so that its c. g. takes up new position G', where $SG' = l$. The weight of the pendulum is acting vertically downward at G' and its reaction at the point of suspension S' constitutes the couple tending to bring back into its original position.

The moment of restoring couple $= -mgl\sin\theta$

i.e., $I\dfrac{d^2\theta}{dt^2} = -mgl\ sin\,\theta$

where, I is the moment of inertia of the pendulum about the of suspension and $\dfrac{d^2\theta}{dt^2}$ is the angular acceleration of the pendu

For a small angular displacement, we write,

$$\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\,\theta = 0$$

The pendulum thus executes a simple harmonic motion an time period is,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

The moment of inertia of the pendulum about an axis pas through S and perpendicular to its plane;

$$I = mk^2 + ml^2 = m(k^2 + l^2)$$

Thus,

$$T = 2\pi\sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}}$$

i.e., $T = 2\pi\sqrt{\dfrac{\frac{k^2}{l}+l}{g}} = 2\pi\sqrt{\dfrac{L}{g}}$

where, k is the radius of gyration about an axis passing thro the c. g. of the pendulum.

The equation (1.18) implies that the time period of the phy pendulum is same as that of the simple pendulum of the len $l = \frac{k^2}{l} + l$. This length is, therefore, called the length of equivalent simple pendulum or reduced length of the phys pendulum.

Spring Mass System

Consider a block of mass is oscillating at the end of a mass less spring as shown in figure. Assume that the net force acting on the block is that exerted by the spring which is obtained from the Hook's law.

i.e., $F = -kx$

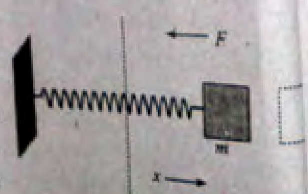

Figure: The restoring force in a sprin mass system

where, x is displacement from equilibrium position and k is force constant

From Newton's law,

$$a = \frac{F}{m} = -\frac{k}{m}x$$

i.e., $\dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$ 1.19

This equation shows that spring mass system executes a simple harmonic motion. Comparing equation (1.19) with differential equation of S.H.M., angular frequency ω is,

$$\omega = \sqrt{\frac{k}{m}}$$

The time period of spring mass system is,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad 1.20$$

Give the necessary theory of forced vibration and deduce the condition for amplitude and velocity resonance. Explain the sharpness of resonance and relate it with quality factor.

[T.U. 2063 Baishakh]

lution:

Forced Oscillation

When a body is maintained in a state of oscillation by an external periodic force of frequency other than the natural frequency of the body, the oscillation is called forced oscillation. A body can be forced to oscillate with any frequency depending upon that of the applied periodic force. The oscillation dies out as soon as the applied force is removed.

Assume that the applied force is sinusoidal and represented as;

$$F_{ext} = F_0\,sin\,\omega t$$

where, ω is applied angular frequency. Newton's second law in such a forced oscillator yields,

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\,sin\,\omega t \qquad 1.21$$

The energy dissipated by damping is balanced with the external energy. The solution of such differential equation is;

$$x = A_0 sin\,(\omega t + \phi_0) \qquad 1.22$$

where, ϕ_0 is the phase angle between the displacement x and the external force F_{ext}. On substituting the value of x into equation

(1.21), we obtain the amplitude of oscillation and phase angle ϕ_0 i
terms of frequency of external force ω and natural frequency ω_0 as

$$A_0 = \frac{F}{m\sqrt{(\omega^2-\omega_0^2)^2+\left(\frac{b\omega}{m}\right)^2}}$$

$$\phi_0 = \tan^{-1}\left\{\frac{m(\omega_0^2-\omega^2)}{\omega b}\right\}$$

Figure depicts that the amplitude becomes large when the appl frequency ω is near the natural frequency ω_0. When damping small, the amplitude near $= \omega_0$, will become very large. Th condition is known as resonance. Thus, when a periodic force frequency equal to natural frequency of a body, is applied, t body gains amplitude slowly and finally begins to vibrate w large amplitude. Hence resonance is defined as the phenomenon of setting a body into vibrations of its natural frequency by the application of an external periodic force of the same frequency. Such frequency is called resonant frequency. Resonance is also said to be state of vibration with maximum amplitude. This is also termed as amplitude resonance.

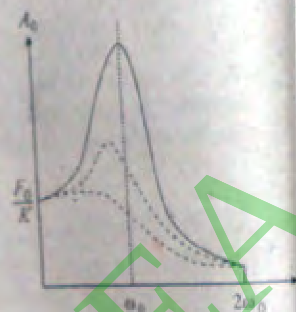

Figure: Amplitude as a function of app frequency

The Sharpness of the resonance refers to fall in amplitude w change in frequency on each side of maximum amplitude. As frequency of the applied force ω is increased or decreased fro resonant value ω_0, the value of the amplitude always decrea When the amplitude at resonance falls rapidly as the frequenc the applied force is changed slightly from its resonant value, resonance is said to be sharp.

The sharpness of resonance is inversely proportional to damping constant b and can be represented in terms of qu factor Q. Quality factor is defined as 2π times the ratio of en store in the system to the energy cycle. It is sometimes c

figure of merit of the harmonic oscillator. The figure of merit is defined as the ratio of the frequency at velocity resonance to the full bandwidth at half maximum power. It measures the sharpness of the resonance.

Quality factor, $Q = 2\pi\left(\frac{\omega_0}{\omega_1-\omega_2}\right)$

If ω_1 and ω_2 be the frequencies at which amplitude becomes $\frac{A_0}{\sqrt{2}}$, the quantity $(\omega_1-\omega_2)$ is called width of the resonance peak. A larger value of quality factor represents a system of high quality with narrow resonance peak.

6. **For a compound pendulum prove that both the length of point of suspension and point of oscillation from centre of gravity equal to radius of gyration.** **[T.U. 2063 Baishakh]**

Solution:

The time period of the compound pendulum is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is radius of gyration about centre of gravity and l is the distance of the point of suspension from centre of gravity. Squaring this equation, we obtain,

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2}{l}+l\right)$$

Differentiating with respect to l, we obtain,

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2}+1\right)$$

Obviously, T will be a maximum or a minimum when $\frac{dT}{dl} = 0$.

i.e., $l^2 = k^2$

$\therefore\ l = k$

If l' be the distance of the point of oscillation from centre of gravity, then,

$l^2 = k^2 = ll'$

i.e., $l = l'$

$\therefore\ l = k = l'$

Thus, the length of point of suspension and point of oscillation from the centre of gravity are equal to radius of gyration.

7. **A spring is hanging vertically and loaded with a mass of 100 g and allowed to oscillate. Calculate (i) time period of oscillation and (ii) frequency of oscillation if the spring is further loaded with 200 g producing an extension of 10 cm. [T.U. 2063 Baishakh]**

Solution:

When additional 200 g is loaded on spring, it produces the extension of 10 cm. Thus,

$$mg = kx$$

$$\text{or,} \quad k = \frac{mg}{x}$$

$$\therefore \quad k = 0.2\,kg \times \frac{9.8\,ms^{-2}}{0.1\,m} = 19.6\,N/m$$

i) Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1\,kg}{19.6\,N/m}} = 0.45\,s$$

ii) Frequency of oscillation,

$$f = \frac{1}{T} = \frac{1}{0.45\,s} = 2.22\,Hz$$

8. **What is S.H.M.? Discuss the theory of a simple spring mass system and derive an expression for its period and frequency. [T.U. 2064 Poush]**

Solution:

Simple Harmonic Motion [S.H.M.]

A particle may be said to execute a *simple harmonic motion [S.H.M.]* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

Spring mass system

See the solution of Q. No. 4 on page no. 7

9. **Show for a bar pendulum that the time period is minimum points of suspension and oscillation are equidistant from C.G. [T.U. 2064 Poush]**

Solution:

The time period of the compound pendulum is,

$$T = 2\pi\sqrt{\frac{(k^2 + l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$$

where, k is radius of gyration about centre of gravity and l is the distance of the point of suspension from centre of gravity. Squaring this equation, we obtain,

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2}{l} + l\right).$$

Differentiating with respect to l, we obtain,

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2} + 1\right)$$

Obviously, T will be a maximum or a minimum when $\frac{dT}{dl} = 0$.

$$\text{i.e.,} \quad l^2 = k^2$$

$$\therefore \quad l = k$$

The negative value of k is simply meaningless. Since $\frac{d^2T}{dl^2} > 0$, it is obvious that time period T is a minimum when $l = k$, *i.e.*, the time period of a compound pendulum is the minimum when its length is equal to its radius of gyration about an axis through its C.G.

10. **Explain the terms free vibration, damped vibration, forced vibration and resonance. Develop the differential equation of a particle executing damped vibrations in a medium. Explain the physical meaning of each term and eaeh constant in the equation. [T.U. 2065 Shrawan]**

Solution:

Free Vibration

When a body capable of vibration is displaced from its equilibrium position and then left free, it begins to oscillate with a definite amplitude and frequency. If the body is not restricted by any kind of friction, the motion continues. Such vibration is called free vibration. *Example:* When a bob of simple pendulum is displaced from its mean position and left free, it executes a free vibration.

Damped Vibration

The vibration whose amplitude goes on decreasing with time is called damped vibration. Practically all vibrations are damped. A vibration whose amplitude remains constant with time is called undamped vibration.

Forced Vibration

When a body is maintained in a state of vibration by an external periodic force of frequency other than the natural frequency of the body, the vibration is called forced vibration. A body can be forced to vibrate with any frequency depending upon that of the applied periodic force. The vibration dies out as soon as the applied force is removed.

Resonance

When a body is maintained in a state of vibration by a periodic force having the same frequency as the natural frequency of the body, the vibration is called resonant vibration. The phenomenon of producing resonant vibration is called resonance.

The differential equation of a particle executing damped vibrations in medium

The dissipative force of damped vibration is proportional to velocity of particle at that instant and velocity exponentially in the time,

i.e., $F = -b\frac{dx}{dt}$

where, b is damping constant and is measured in kg/s. The differential equation of motion for damped vibration will be,

$$F = -kx - b\frac{dx}{dt}$$

$$= m\frac{d^2x}{dt^2}$$

$$\text{or,}\quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\text{i.e.,}\quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

This is known as differential equation of damped oscillation of mechanical system. The solution of this equation is;

$$x = ae^{-\left(\frac{b}{2m}\right)t}\sin(\omega' t)$$

where, ω' is called damped angular frequency and given by;

$$\omega' = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)}$$

Thus, the frequency of damped oscillation is;

$$f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)}$$

The damped angular frequency ω' is less than natural angular frequency ω_0. For the exponential relation of displacement, the amplitude decreases to zero in long time. The time duration in which the amplitude drops to $\frac{1}{e}$ times of its initial value is called mean life time of oscillation.

11. What is compound pendulum? Deduce the expression for the time period of a compound pendulum and formulate the equivalent length of the sifmple pendulum. [T.U. 2065 Chaitra]

Solution: See the solution of Q. No. 4 on page no. 7

12. Differentiate between free oscillation and forced oscillation. [T.U. 2065 Kartik]

Solution:

Free Oscillation

When a body capable of oscillation is displaced from its equilibrium position and then left free, it begins to oscillate with a definite amplitude and frequency. If the body is not restricted by any kind of friction, the motion continues. Such oscillation is called free oscillation. *Example:* When a bob of simple pendulum is displaced from its mean position and left free, it executes a free oscillation.

Forced Oscillation

See the solution of Q. No. 5 on page no. 9

13. What is torsional pendulum? Find the time period for torsional pendulum. Also write its significance. [T.U. 2065 Kartik]

Solution:

Torsional pendulum

A heavy body like a disc or a cylinder, fastened at its mid-point to a fairly long and thin wire, suspended from a rigid support, constitutes a torsional pendulum.

If the disc or the cylinder is turned in its own plane *i.e.*, in the horizontal plane, to twist the wire a little and then released, it executes torsional vibrations or oscillations about wire as axis. That's why, this pendulum is called torsional pendulum.

Figure: A torsional pendulum

When the disc or the cylinder is turned through an angle θ, the suspension wire gets twisted through the same angle and this give rise to torsional couple $-C\theta$ in it, tending to bring it back into its original condition.

Here,

Torsional couple per unit twist of the wire or torsional constant is;

$$C = \frac{\pi\eta R^4}{2L} \qquad \ldots (i)$$

R is the radius of the wire.
L is the length of the wire.
η is the modulus of rigidity of its material.

If I be the moment of inertia of the disc about the wire as an axis, the couple acting on it is equal to $I\frac{d^2\theta}{dt^2}$, where, $\frac{d^2\theta}{dt^2}$ is the angular acceleration of disc. Thus, we can write,

$$I\frac{d^2\theta}{dt^2} = -C\theta$$

i.e., $$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$$

The disc or the cylinder executes an angular S.H.M. with time period as:

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Notice that there is no approximation whatever has been used to determine time period. The time period of a torsional pendulum remains unaffected *i.e., oscillations remain isochronous*, even if the amplitude be large. The elastic limit of the suspension wire, however, is not exceeded.

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14. A spring is hung vertically and loaded with a mass of 75 g and allowed to oscillate. Calculate (i) time period and (ii) frequency of oscillation, when the spring is further loaded with 100 g producing an extension of 5 cm. **[T.U. 2065 Kartik]**

Solution:

When additional 100 g is loaded on spring, it produces the extension of 5 cm. Thus,

$$mg = kx$$

or, $$k = \frac{mg}{x}$$

$$\therefore \quad k = 0.1\ kg \times \frac{9.8\ ms^{-2}}{0.05\ m} = 19.6\ Nm^{-1}$$

i) Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.075\ kg}{19.6\ Nm^{-1}}} = 0.39\ s$$

ii) Frequency of oscillation,

$$f = \frac{1}{T} = \frac{1}{0.39\ s} = 2.56\ Hz$$

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15. Show that there are four collinear points within compound pendulum having same time period. Give their physical significance. **[T.U. 2067 Ashadh]**

Solution:

The time period of the compound pendulum is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is the radius gyration and l is the length of pendulum from point of suspension to centre of gravity of the pendulum. If l' be the distance of the point of oscillation from centre of gravity, the time period of pendulum is;

$$T' = 2\pi\sqrt{\frac{\frac{k^2}{l'}+l'}{g}}$$

We have,

$$k^2 = ll'$$

Thus,

$$\frac{k^2}{l} = l'$$

and $$\frac{k^2}{l'} = l$$

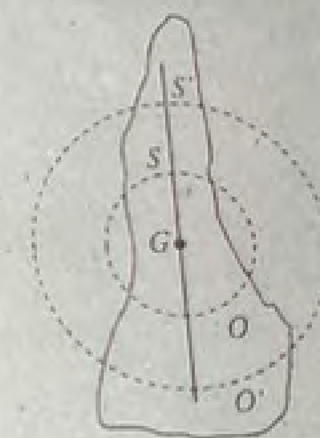

Figure: Centre of suspensions and oscillations in compound pendulum

This implies,

$$\frac{\frac{k^2}{l}+l}{g} = \frac{\frac{k^2}{l'}+l'}{g} = l + l'$$

$$\therefore \quad T = T'$$

i.e., the time period of the pendulum is same about the point of suspension S and point of oscillation O. This implies that these points are interchangeable.

There are two other points on either side of G about which the time period of the pendulum is same as S and O. If we sketch the two circles about centre G with the radii l and l' respectively, the SG produced will cut S and S' above and O and O' below G. From figure, We have,

$$SG = GO' = l$$

and $GO = GS' = \frac{k^2}{l} = l'$

Thus,

$$O'S' = O'G + GS' = l + l' = OS$$

Hence, there are four collinear points, i.e., S, S', O and O', colline with c. g. of pendulum G about which its time period is same.

16. **What is a torsional pendulum? Obtain an expression for its tim period and explain why, unlike a simple or a compoun pendulum the time period in this case remains unaffected even i the amplitude be large?** [T.U. 2067 Mangsi

Solution: See the solution of Q. No. 13 on page no. 15

17. **A meter stick suspended from one end swings as a physica pendulum (a) what is the period of oscillation? (b) What woul be the length of the simple pendulum that would have the sam period?** [T.U. 2067 Mangsi

Solution:

The moment of inertia of thin rod about an axis through one end

$$I = \frac{1}{3}ml^2$$

Since the $c.g.$ is at the centre,

$$l' = \frac{l}{2}$$

We have, time period of the pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

or, $T = 2\pi\sqrt{\frac{\frac{1}{3}ml^2}{mg\frac{l}{2}}}$

$\therefore$ $T = 2\pi\sqrt{\frac{2l}{3g}} = 2\pi\sqrt{\frac{2 \times 1\,m}{3 \times 9.8\,ms^{-2}}} = 1.64\,s$

A simple pendulum must have a length L in order to have sam time period of compound pendulum. Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

or, $L = \frac{gT^2}{4\pi^2} = \frac{9.8\,ms^{-2} \times (1.64\,s)^2}{4\pi^2}$

$\therefore$ $L = 0.67\,m$

18. **List the common pendulums in practice. Which of them is a physical pendulum and why? Show that point of suspension and point of oscillations are interchangeable.** [T.U. 2068 Shrawan]

Solution:

The common pendulums are;

i) Mass spring system
ii) Simple pendulum
iii) Compound pendulum
iv) Torsional pendulum
v) LC circuit
vi) Helmholtz resonator

A compound pendulum is a physical pendulum. It is a rigid body, of whatever of shape, capable of oscillating about a horizontal axis passing through it.

For the remaining part

See the solution of Q. No. 15 on page no. 17

19. **Explain the theory of simple mass spring system. Develop the relation for time period and frequency of two springs having spring constants K_1 and K_2 supporting a mass M between them on a frictionless horizontal table.** [T.U. 2068 Shrawan]

Solution:

Spring mass system

See the solution of Q. No. 4 on page no. 8

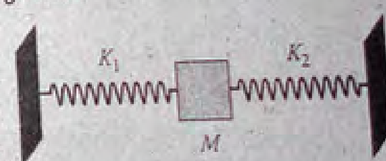

Figure: Two spring–coupled mass

Now, if the mass M is displaced to one side or the other through distance x, one spring gets extended and other compressed. Both of them exerting a restoring force on the mass in the same direction, tending to bring it back to its original position.

If F_1 and F_2 be the restoring forces due to the two springs respectively, the resultant restoring force,

$$F = F_1 + F_2 = -K_1x - K_2x = -(K_1 + K_2)x$$

This implies that the effecting force constant K is the combinations of two springs.

i.e., $K = (K_1 + +K_2)$

The time period of the oscillating mass M is, therefore,

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{M}{(K_1+K_2)}}\sqrt{\frac{m}{k}}$$

The electrical analogue of this system is a series arrangement two capacitors.

---

20. **A thin straight, uniform rod of length $l = 1\ m$ and mass $m = 1$ gm hangs from a pivot at one end. (i) What is its period for sm amplitude oscillation? (ii) What is the length of a simp pendulum that will have the same period? [T.U. 2064 Pou**

Solution:

The moment of inertia of thin rod about an axis through one end

$$I = \frac{1}{3}ml^2$$

Since the c. g. is at the centre,

$$l' = \frac{l}{2}$$

Time period of the pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\frac{1}{3}ml^2}{mg\frac{l}{2}}}$$

$$= 2\pi\sqrt{\frac{2l}{3g}} = 2\pi\sqrt{\frac{2\times 1\ m}{3\times 9.8\ ms^{-2}}}$$

$$= 1.64\ s$$

A simple pendulum must have a length L in order to have sa time period of compound pendulum. Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$i.e.,\quad L = \frac{gT^2}{4\pi^2} = \frac{9.8\ ms^{-2}\times(1.64\ s)^2}{4\pi^2} = 0.67\ m$$

The equivalent length of simple pendulum, $L = 0.67\ m$.

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21. **Define simple harmonic motion. Derive the relationship for ti period of (i) bar pendulum and (ii) torsional pendulum.[P.U. 20**

Solution:

Simple Harmonic Motion [S.H.M.]

See the solution of Q. No. 8 on page no. 12

Compound pendulum

See the solution of Q. No. 4 on page no. 7

Torsional pendulum

See the solution of Q. No. 13 on page no. 15

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22. **What is an elastic restoring force? Obtain an expression for the time period of a compound pendulum and show that the centers of suspension and oscillations are interchangeable. [P.U. 2004]**

Solution:

Elastic restoring force

An elastic restoring force is defined as property by the virtue of which elastic bodies regains its original shape after removing deforming force.

Compound pendulum

See the solution of Q. No. 4 on page no. 7

Interchangeability of point of suspension and point of oscillation

See the solution of Q. No. 15 on page no. 17

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23. **Define harmonic motion, elastic restoring force and Hook's law. Find an expression for time period of torsional pendulum.[P.U. 2005]**

Solution:

A particle may be said to execute a *harmonic motion* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

An elastic restoring force is defined as property by the virtue of which elastic bodies regains its original shape after removing deforming force.

A Hook's law states that "within an elastic limit, elastic stress is directly proportional to elastic strain."

Torsional pendulum

See the solution of Q. No. 13 on page no. 15

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24. **Define simple harmonic motion and state its characteristics. Describe with necessary theory how you will determine the value of the modulus of rigidity of a metal in laboratory by using torsional pendulum. [P.U. 2006]**

Solution:

Simple harmonic motion

See the solution of Q. No. 8 on page no. 12

Notice that there is no approximation whatever has been used to determine time period. The time period of a torsional pendulum remains unaffected *i.e., oscillations remain isochronous,* even if the amplitude are large. The elastic limit of the suspension wire however, is not exceeded.

Practically, if one regular body of given moment of inertia I_1 is placed on the disc. The time period of circular disc, T_1

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}} \quad \ldots \text{(ii)}$$

Time period of circular disk and ring is;

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{C}} \quad \ldots \text{(iii)}$$

Thus,

$$I_1 = \left(\frac{T_1^2}{T_2^2 - T_1^2}\right) I_2 \quad \ldots \text{(iv)}$$

This gives the moment of inertia of a circular disc.

The moment of inertia of circular ring,

$$I_2 = \frac{1}{2} m(r_1^2 + r_2^2) \quad \ldots \text{(v)}$$

where, m is the mass of a ring, r_1 is the radius of ring and r_2 is the radius of external ring.

Form the equations (i), (ii) and (iii), we obtain,

$$\eta = \frac{8\pi l I_2}{r^4(T_2^2 - T_1^2)} \quad \ldots \text{(vi)}$$

This is the required expression for the modulus of rigidity of a wire. Measuring time periods T_1 and T_2, moment of inertia I_2, length of suspension wire l and radius of suspension wire r, the modulus of rigidity can be determined.

---

25. **Differentiate between S.H.M. and periodic motion.** **[P.U. 2007]**

Solution:

A particle may be said to execute a *simple harmonic motion (S.H.M.)* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

Any motion that repeats itself at regular interval of time is said to be periodic motion.

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26. **What is torsional pendulum? Find the time period for torsional pendulum.** **[P.U. 2007]**

Solution: See the solution of Q. No. 13 on page no. 15

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27. **Define simple harmonic motion. Why are really S.H.M.'s rare? Describe with necessary theory how you will determine the value of the modulus of rigidity of a metal in laboratory by using torsional pendulum.** **[P.U. 2008]**

Solution:

Simple harmonic motion

See the solution of Q. No. 8 on page no. 12

Simple harmonic motion is based ideal assumptions, for example, in case of simple pendulum, the string attached to bob of pendulum is must be weightless and angle of displacement must be small during performing experiment. These assumptions are only ideal concept and in practice these are not possible.

Torsional pendulum

See the solution of Q. No. 13 on page no. 15

For the remaining part

See the solution of Q. No. 24 on page no. 21

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28. Define elastic restoring force. Derive an expression for the time period of a compound pendulum and prove that the centers of oscillation and suspension are interchangeable. [P.U. 2010]

Solution: See the solution of Q. No. 22 on page no. 21

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29. Define angular harmonic motion. Derive an expression for the time period of a compound pendulum and prove that the centers of oscillation and suspension are interchangeable. [P.U. 2011]

Solution:

Simple Harmonic Motion (S.H.M.)

See the solution of Q. No. 8 on page no. 12

For the remaining part

See the solution of Q. No. 28 on page no. 23

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30. **A spiral spring 3 m long from the ceiling. When a mass of 1 kg is suspended from the spring it lengthens by 40 cm. The mass is then pulled and released. Compute the frequency of oscillation.** **[P.U. 2002]**

Solution:

When 1 kg is loaded on spring, it produces the extension of 40 cm = 0.4 m. Thus,

$$mg = kx$$

$$\text{or,} \quad k = \frac{mg}{x}$$

$\therefore \quad k = 1\,kg \times \frac{9.8\,ms^{-2}}{0.4\,m} = 24.5\,N/m$

The frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{m}{k}} = \frac{1}{2\pi}\sqrt{\frac{1\,kg}{24.5\,Nm^{-1}}} = 3.21 \times 10^{-2}\,Hz$$

31. The moment of inertia of a disc used in a tutorial pendulum about the suspension wire is 0.2 kgm^2. It oscillates with a period of 2 s. Another disc is placed over the first one and the time period of the system becomes 2.5 s. Find the moment of inertia of the second disc about the wire. [P.U. 200

Solution:

Here,

Moment of inertia of first disc, $(I_1) = 0.2\,kg\,m^2$

Time periods, $(T_1) = 2\,s$

$(T_2) = 2.5\,s$

Moment of inertia of second disc, $(I_2) = ?$

We have,

$$I_1 = \left(\frac{T_1^2}{T_2^2 - T_1^2}\right) I_2$$

$$\text{or,}\quad I_2 = I_1\left(\frac{T_1^2}{T_2^2 - T_1^2}\right)^{-1} = 0.2\,kg\,m^2\left[\frac{(2\,s)^2}{(2.5\,s)^2-(2\,s)^2}\right]^{-1} = 0.11\,kg\,m^2$$

Moment of inertia of second disc, $(I_2) = 0.11\,kg\,m^2$

1.6 ADDITIONAL SOLVED PROBLEMS

1. A mass of 5 kg stretches a spring 0.5 m from its equilibrium position. The mass is removed and another body of mass 1 kg is hanged from the spring. What would be the period of motion if the spring is now stretched and released?

Solution:

When 5 kg is loaded on spring, it produces the extension of 0.5 m. Thus,

$$mg = kx$$

$$\therefore \quad k = \frac{mg}{x} = 5\,kg \times \frac{9.8\,ms^{-2}}{0.5\,m} = 96\,N/m$$

Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m'}{k}} = 2\pi\sqrt{\frac{1\,kg}{96\,Nm^{-1}}} = 0.64\,s$$

2. A simple pendulum of one meter length is hanged at one end. Considering oscillations to be of small displacements, find period of oscillations if mass of pendulum is 2.0 kg.

Solution:

Here,

Length of the pendulum, $(l) = 1\,m$

Mass of the pendulum, $(m) = 2\,kg$

We have,

Time period of the oscillation,

$$(T) = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9.8}}$$

$$= 2.01\,s$$

3. An oscillatory motion of a body is represented by $y = ae^{i\omega t}$, where y is the displacement in time t, a is its amplitude and ω is its angular frequency. Show that the motion is simple harmonic.

Solution:

Here,

$$y = ae^{iwt}$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = i\omega ae^{iwt}$$

Again,

$$\frac{d^2y}{dt^2} = -\omega^2 ae^{iwt} = -\omega^2 y$$

$$\text{i.e.,}\quad \frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is the differential equation of S.H.M. Hence, $y = ae^{iwt}$ represents a S.H.M.

4. Show that the velocity of simple harmonic oscillator at any instant leads the displacement by a phase angle of $\frac{\pi}{2}$. Hence find the general expression for the velocity of a simple harmonic oscillator.

Solution:

The displacement of a simple harmonic oscillator at any instant of time t can be written as;

$$y = a\sin(\omega t + \phi)$$

The velocity v is defined as rate of chance of displacement, *i.e.*,

$$v = \frac{dy}{dt} = a\omega\cos(\omega t + \phi) = a\omega\sin\left(\frac{\pi}{2} + \omega t + \phi\right)$$

Comparing the equation fro displacement velocity, the velocity of simple harmonic oscillator at any instant t leads the displacement by a phase difference $\frac{\pi}{2}$. However, the velocity varies simple harmonically with the same frequency.

As, $\sin(\omega t + \phi) = \frac{y}{a}$

or, $\cos(\omega t + \phi) = \sqrt{1 - \frac{y^2}{a^2}} = \frac{\sqrt{a^2 - y^2}}{a}$

Thus,

$$v = a\omega\cos(\omega t + \phi) = a\omega\frac{\sqrt{a^2 - y^2}}{a}$$

$$= \omega\sqrt{a^2 - y^2}$$

The velocity will be maximum if the displacement is zero.

The maximum velocity,

$$v_{max} = \omega a$$

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5. Two identical springs each of force constant k are connected in series. Calculate the force constant and time period of the system.

Solution:

Two identical springs are connected to a mass m as shown in the figure.

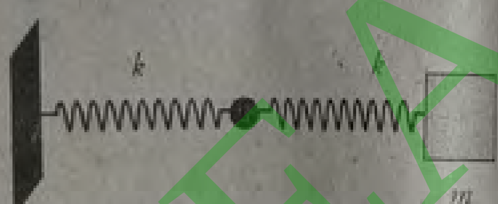

Figure: Two spring-coupled mass

Let x_1 and x_2 be the extensions produced in the two springs respectively as the mass m displaced outwards, the total extension,

$$x = x_1 + x_2$$

Since the restoring force due to each spring is same. We write, the restoring force,

$$F = -kx_1 = -kx_2$$

i.e., $x_1 = -\frac{F}{k} = x_2$

$\therefore$ $x = x_1 + x_2 = -\frac{F}{k} - \frac{F}{k} = -2\frac{F}{k}$

i.e., $F = -\frac{k}{2}x$

Indicating that the effective force constant of the combination of two identical springs $K = \frac{k}{2}$.

The time period of the oscillating mass system,

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{2m}{k}}$$

This is analogous to parallel combination of two capacitors of equal capacitance.

---

6. What is the frequency of a simple pendulum 2.0 m long? Assuming small amplitude, what would its frequency be in an elevator accelerating upward at the rate of 2 ms^{-2}? What would its frequency be in free fall?

Solution:

We have,

Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Thus, frequency of a simple pendulum,

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{9.8\ ms^{-2}}{2\ m}} = 0.35\ Hz$$

While moving elevator upwards with an acceleration a, the effective weight of the pendulum is $mg\left(1 + \frac{a}{g}\right) = m(g + a)$. In this case, the effective value of $g' = (g + a)$.

The frequency pendulum in the elevator,

$$f' = \frac{1}{2\pi}\sqrt{\frac{g'}{l}} = \frac{1}{2\pi}\sqrt{\frac{g + a}{l}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(9.8 + 2)\ ms^{-2}}{2\ m}}$$

$$= 0.39\ Hz$$

In the free fall of pendulum, the effective weight of the pendulum is $mg\left(1 - \frac{g}{g}\right) = 0$, i.e., the effective value of $g = g' = 0$. The time period of pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} = \infty$$

i.e., there is no oscillation of the pendulum at all. Its frequency therefore, zero.

7. **A uniform circular disc of radius R oscillates in a vertical plane about a horizontal axis. Find the distance of the axis of rotation from centre for which period is minimum. What is the value of time period?**

Solution:

Here, the circular disc oscillates as a compound pendulum of length l, whose time period is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}}$$

$$= 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is radius of gyration about an axis through its c. g., parallel to its axis of suspension. We know that, the time period of compound pendulum is minimum if its length is equal to its radius of gyration about its c. g., *i.e.*, $l = k$.

Therefore,

$$T = 2\pi\sqrt{\frac{\frac{k^2}{k}+k}{g}}$$

$$= 2\pi\sqrt{\frac{2k}{g}}$$

Since the moment of inertia of a disc about an axis perpendicular to its plane and passing through its centre,

$$I = \frac{1}{2}MR^2 = Mk^2$$

This implies,

$$k = \frac{R}{\sqrt{2}}$$

where, M is the mass of the circular disc and R is its radius.

Hence, the disc will oscillate with the minimum time period when the distance of the axis of rotation form its centre is $\frac{R}{\sqrt{2}}$.

The minimum time period,

$$T_{min} = 2\pi\sqrt{\frac{2\frac{R}{\sqrt{2}}}{g}}$$

$$= 2\pi\sqrt{\frac{1.41R}{g}}$$

8. **A uniform circular disc of diameter 20 *cm* vibrates about a horizontal axis perpendicular to its plane and at a distance of 5 *cm* from the centre. Calculate the time period of oscillation and the equivalent length of the simple pendulum.**

Solution:

Here, the circular disc oscillates as a compound pendulum of length l, whose time period is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is radius of gyration about an axis through its c. g., parallel to its axis of suspension. We know that, the time period of compound pendulum is minimum if its length is equal to its radius of gyration about its c. g., *i.e.*, $l = k$.

Therefore,

$$T = 2\pi\sqrt{\frac{\frac{k^2}{k}+k}{g}} = 2\pi\sqrt{\frac{2k}{g}}$$

Since the moment of inertia of a disc about an axis perpendicular to its plane and passing through its centre,

$$I = \frac{1}{2}MR^2 = Mk^2$$

This implies,

$$k = \frac{R}{\sqrt{2}}$$

where, M is the mass of the circular disc and R is its radius.

Hence, the disc will oscillate with the minimum time period when the distance of the axis of rotation form its centre is $\frac{R}{\sqrt{2}}$.

The minimum time period,

$$T_{min} = 2\pi\sqrt{\frac{2\frac{R}{\sqrt{2}}}{g}} = 2\pi\sqrt{\frac{1.41\,R}{g}} = 2\pi\sqrt{\frac{1.41\times 0.1\,m}{9.8\,ms^{-2}}} = 0.75\,s$$

To have the same time period, a simple pendulum must have a length L.

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Implies,

$$L = \frac{gT^2}{4\pi^2} = \frac{9.8\,ms^{-2}\times(0.75\,s)^2}{4\pi^2} = 0.14\,m$$

9. A solid sphere of radius 0.3 m executes torsional oscillations of time period $2\pi\sqrt{12}$ s at the end of a suspension wire whose upper end is fixed in a rigid support. If torsional constant of the wire be 6×10^{-3} Nm/rad, calculate the mass of the sphere.

Solution:

Here,

The moment of inertia of a solid sphere about any diameter or about any axis passing through its centre,

$$I = \frac{2}{5}MR^2$$

where, M is the mass of the solid sphere and R is its radius

Given that,

Torsional constant, $(C) = 6 \times 10^{-3}\ Nm/rad$

Radius of solid sphere, $(R) = 0.3$ m

Time period of oscillations, $(T) = 2\pi\sqrt{12}$ s

We have,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

or, $$2\pi\sqrt{12} = 2\pi\sqrt{\frac{I}{C}}$$

or, $$I = 12\,C$$

or, $$\frac{2}{5}MR^2 = 12\,C$$

$\therefore$ $$M = \frac{30\,C}{R^2} = \frac{30 \times 6 \times 10^{-3}\ Nm\ rad^{-1}}{(0.3)^2} = 2.0\ kg$$

The mass of the solid sphere, $(M) = 2.0$ kg

10. A wire has a torsional constant 2 Nm/rad. A disc of radius 5 cm and mass 100 gm is suspended at its centre. What is the frequency of torsional oscillations?

Solution:

Here,

The moment of inertia of a disc,

$$I = \frac{1}{2}MR^2$$

where, M is the mass of a disc and R is its radius

The time period of a torsional pendulum,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Thus, the frequency of the pendulum,

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{2C}{MR^2}}$$

Given that,

Torsional constant, $(C) = 2\ Nm/rad$

Radius of a disc, $(R) = 5\ cm = 0.05\ m$

Mass of a disc, $(M) = 100\ gm = 0.1\ kg$

$\therefore$ $$f = \frac{1}{2\pi}\sqrt{\frac{2 \times 2\ Nm/rad}{0.1\ kg \times (0.05\ m)^2}} = 20.13\ Hz$$

11. A mass of 1 kg is suspended from a spring of spring constant 25 N/m. If the undamped frequency is $\frac{2}{\sqrt{3}}$ times of the damped frequency, what will be the damping factor?

Solution:

We have,

Damped frequency,

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad \text{... (i)}$$

where, b is a damping factor and k is force constant of spring

In absence of damping, *i.e.*, when $b = 0$, the undamped frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{... (ii)}$$

Dividing equation (i) by equation (ii), we obtain,

$$\frac{f}{f'} = \frac{\sqrt{\frac{k}{m}}}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

or, $$\frac{2}{\sqrt{3}} = \sqrt{\frac{k}{k - \frac{b^2}{4}}} \qquad [\because m = 1\ kg]$$

or, $$\frac{4}{3} = \frac{25}{25 - \frac{b^2}{4}}$$

or, $$100 - b^2 = 75$$

$\therefore$ $$b = 5\ kg\ s^{-1}$$

Hence the damping factor is $5\ kg\ s^{-1}$.

Chapter 2

WAVE MOTION

2.1 WAVE AND WAVE MOTION

A *wave* is a continuous transfer of disturbance from one part of medium to another through successive vibrations of the particles of th medium about their mean positions. In wave motion, the energy an momentum are carried from one region to another region of th medium. If there is no transfer of energy, it is not a wave but a oscillation there is no transfer of energy.

Wave motion is a form of disturbance which travels through a mediu due to the repeated periodic motion of the particles of the medium abo their mean position.

A wave that can't travel without material medium is said to mechanical wave. E.g., water waves, earthquake, tsunami, sound wav etc. A wave that can travel without material medium is said to *electromagnetic wave*. E.g., light wave or radio signals.

A wave in which the particles of a medium vibrate about their mea position perpendicular to the direction of propagation of the wave is sai to be *transverse wave*. An electromagnetic wave is a transverse wave. wave in which the particles of a medium vibrate along the direction o propagation of the wave is said to be *longitudinal wave*. A sound wave is longitudinal wave.

2.2 WAVE CHARACTERISTICS

i) Wave motion is the disturbance travelling through a medium.
ii) When a disturbance is produced in a medium, the disturbed particles vibrate about their mean positions.
iii) Particles transfer their energy to neighboring particles through the disturbances but their net displacement over one period is zero.
iv) As the disturbance reaches to a particle, it starts to vibrate. The disturbance is communicated to the next neighbor a little later, so there is a phase difference in the vibratory motion of the consecutive particles.

v) The energy transference in the medium takes place with a constant speed, $v = f \times \lambda$ and depends on nature of the medium.
vi) The wave velocity is different form particle velocity of a medium.
vii) The wave motion is possible in a medium which possesses the property of elasticity and inertia.
viii) Particle velocity is a function of a time such that it depends in different points of a displacement whereas wave velocity is constant in a medium.
ix) Vibrating particles of the medium possess both kinetic and potential energies.

2.3 PROGRESSIVE WAVE

A wave that travels from one region of a medium to another is said to be progressive wave. In such wave, the disturbance travels forward and is transferred to neighboring particle after a certain time. All the particles vibrate in same amplitude and frequency but the vibration begins a little later than the particle immediately before it. A progressive wave may be transverse wave or longitudinal wave.

The equation of the progressive wave is,

$$y = a \sin\frac{2\pi}{\lambda}(vt - x) \qquad 2.1$$

If a wave is travelling from right to left, the wave equation is

$$y = a \sin\frac{2\pi}{\lambda}(vt + x) \qquad 2.2$$

where, v, a and λ are velocity, amplitude and wavelength of the progressive wave

The differential equation of wave motion is;

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} \qquad 2.3$$

2.4 RELATION BETWEEN PARTICLE VELOCITY AND WAVE VELOCITY

The equation of progressive wave motion,

$$y = a \sin\frac{2\pi}{\lambda}(vt - x)$$

where, y is the displacement of a particle of a medium at distance x from the origin at instant t, a is its amplitude and v is a wave velocity. Differentiating this expression for y with respect to t, we obtain particle velocity as,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos\frac{2\pi}{\lambda}(vt - x)$$

Again, differentiating the same expression for y with respect to x, w
obtain slope of the displacement curve as;

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda} a \cos\frac{2\pi}{\lambda}(vt - x)$$

An insertion of equation (2.5) into equation (2.4) leads us to;

$$U = \frac{dy}{dt} = -v\frac{dy}{dx}$$

i.e., particle velocity at a point = − (wave velocity) × (slope of displacement curve at that poin

2.5 ENERGY DENSITY, POWER AND INTENSITY OF A PROGRESSIV WAVE

The energy density ε of a plane progressive wave is defined as tota energy (sum of potential energy and kinetic energy) per unit volume o the medium through which wave is travelling.

i.e., $\varepsilon = 2\pi^2 \rho a^2 f^2$ 2.

where, ρ is density of medium and f is a frequency of wave

The rate of transfer of energy is defined as power P of wave.

i.e., $P = \frac{E}{t} = 2\pi^2 A v\rho a^2 f^2$ 2

where, A is the cross sectional area through which wave travels

The average power transferred across unit area perpendicular to the direction of energy flow, is called intensity I of a wave.

i.e., $I = 2\pi^2 v\rho a^2 f^2$ 2.

2.6 SOLVED EXAM QUESTIONS

1. **What is energy density of a wave? Write down its expression.**

Solution: **[T.U. 2061 Baishakh]**

The energy density ε of a plane progressive wave is defined as total energy (sum of potential energy and kinetic energy) per unit volume of the medium through which wave is travelling.

i.e., $\varepsilon = 2\pi^2 \rho a^2 f^2$

where, ρ is density of medium, a is amplitude and f is a frequency of wave.

2. **Derive an expression for velocity of wave in a stretched string.**

Solution: **[TU 2062 Baishakh]**

Consider an idealized uniform string which is perfectly flexible. Suppose that the pulse height is so small that the tension on the

string is not change by it. The pulse moves to the right with the speed v. Any small segment AB be treated asa circular arc of radius R. Let θ be the angel between the vertical line and radial line to A or B. The length of are AB is;

$$2\theta = \frac{Arc\ AB}{R}$$

i.e., $Arc\ AB = 2\theta R$

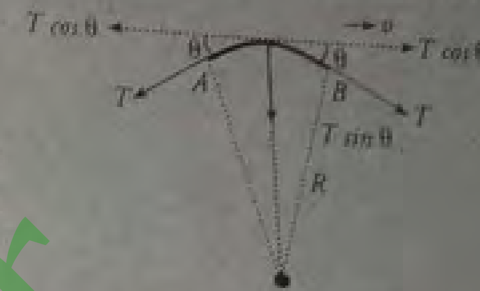

Figure: A small segment AB of stretched string that moves towards right with the speed v

If μ is the mass per unit length of the material of the string, the mass of the segment AB will be;

$$m = 2\mu R\theta$$

The tension T in the string must provide the centripetal force needed for circular motion. The tangents at these points A and B make the same angel as the normal to them make at the centre, from geometry. The horizontal component of this tension $T\cos\theta$ cancels because of the opposite directions. Therefore, the net force in the segment is $2T\sin\theta$ vertically downward. This force balances the centripetal force. Thus, we write;

$$2T\sin\theta = \frac{mv^2}{R}$$

Using the small angle approximation, we may write,

$$2T\theta = \frac{mv^2}{R} = \frac{(2\mu R\theta)v^2}{R}$$

i.e., $$v = \sqrt{\frac{T}{\mu}}$$

It is obvious that the velocity wave along a string depends only on the tension applied to the string and mass per unit length of the string. It is independent of shape and amplitude of the hump and the displacement initially produced in it, *i.e.*, the velocity of a wave along a string is quite independent of the actual wave form.

3. **Derive the differential equation of transverse wave of a stretched string with applied tension T and mass per unit length m. Also find the velocity of the wave propagating through the string.**

Solution: **[T.U. 2063 Baishakh]**

Consider a portion PQ of the string of length displaced slightly in the vertical plane into the position $P'Q'$ from its equilibrium position along the axis of x. The displacement is very small such that $PQ = P'Q' = \delta x$. The string is supposed to be perfectly flexible such that tension T is same at all points on it, both the positions, whether it is displaced and is not displaced. Assume that it is acting tangentially at P and Q to the end portions of PQ of the string as shown in figure. End points P and Q are inclined at angles θ and $\theta + d\theta$ respectively.

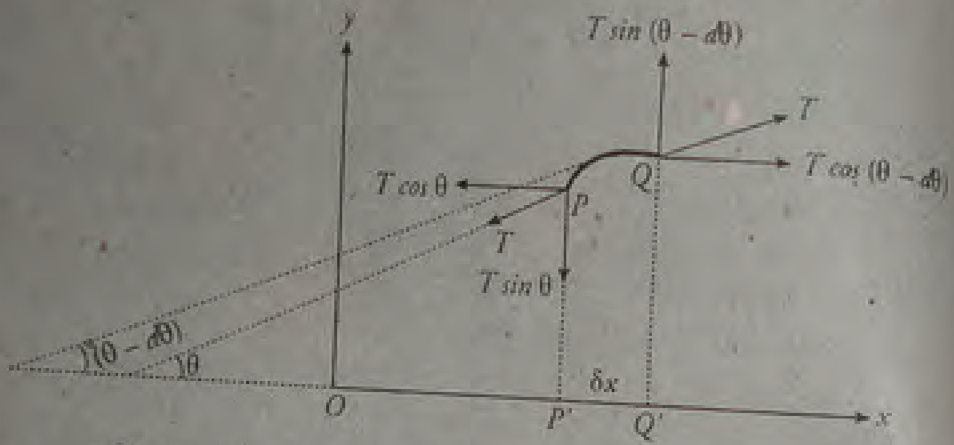

Figure: A small segment PQ of a transverse wave along a string

The horizontal and vertical components of the tension are also shown in the figure. The element δx of the string and its displacement being infinitesimally small, the horizontal and vertical components of T at P and Q are taken along horizontal and vertical line respectively, though of course in opposite directions of each other.

Thus, the resultant horizontal force on element δx of the string;

$$= T\cos(\theta - d\theta) - T\cos\theta = 0 \qquad [\because d\theta \text{ is very small}]$$

and the resultant vertical force on element δx of the string;

$$= T\sin(\theta - d\theta) - T\sin\theta = T\tan(\theta - d\theta) - T\tan\theta$$

$$[\because \theta \text{ and } (\theta - d\theta) \text{ are small}]$$

i.e., the resultant downward force on element δx of the string;

$$= T[\text{slope of the curve at } P - \text{slope of the curve at } Q]$$

$$= T\left[\frac{dy}{dx} - \left(\frac{dy}{dx} - \frac{d^2y}{dx^2}\delta x\right)\right]$$

If the acceleration of the element δx downwards be $\frac{d^2y}{dt^2}$, the downward force also equal to mass × acceleration $= m\delta x\frac{d^2y}{dt^2}$.

where, m is mass per unit length

Therefore, we write,

$$m\delta x\frac{d^2y}{dt^2} = T\left[\frac{dy}{dx} - \left(\frac{dy}{dx} - \frac{d^2y}{dx^2}\delta x\right)\right] = T\frac{d^2y}{dx^2}\delta x$$

i.e., $$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2}$$

This is the differential equation of transverse of a stretched string.

The velocity of wave propagating through the string is;

$$v = \sqrt{\frac{T}{m}}$$

It is obvious that the velocity wave along a string depends only on the tension applied to the string and mass per unit length of the string. It is independent of shape and amplitude of the hump and the displacement initially produced in it, *i.e.*, the velocity of a wave along a string is quite independent of the actual wave form.

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4. **What do you mean by particle velocity and wave velocity? Obtain the relationship between these two quantities. [T.U. 2064 Poush]**

Solution:

Particle velocity and wave velocity

The particle velocity of medium is defined as the rate of change of displacement y. The particle velocity is given by;

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda}a\cos\frac{2\pi}{\lambda}(vt - x)$$

The distance travelled by a wave in a second is called wave velocity. The wave velocity is given by;

$$v = f \times \lambda$$

The equation of progressive wave motion,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

where, y is the displacement of a particle of a medium at distance x from the origin at instant t, a is its amplitude and v is a wave velocity

Differentiating this expression for y with respect to t, we obtain particle velocity as,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda}a\cos\frac{2\pi}{\lambda}(vt - x)$$

Again, differentiating the same expression for y with respect to x, we obtain slope of the displacement curve as;

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda}a\cos\frac{2\pi}{\lambda}(vt - x) = -v\frac{dy}{dx}$$

$$i.e.,\ U = \frac{dy}{dt} = -v\frac{dy}{dx}$$

i.e., particle velocity at a point = − (wave velocity) × (slope of displacement curve at that poi[nt]

5. In the progressive wave show that the potential energy a[nd] kinetic energy of every particle will change with time, but t[he] average kinetic energy per unit volume and potential energy p[er] unit volume remains constant. [T.U. 2065 Shrawa[n]]

Solution:

In the plane progressive wave, there is a continuous transfer [of] energy in the direction of propagation. The energy is sum [of] kinetic energy and potential energy. For a particle executing th[e] simple harmonic motion, particle has maximum velocity at [its] equilibrium position and will be zero at extreme positions *i.e.*, since kinetic energy is proportional to square of velocity, it [is] maximum at mean position and zero at extreme poin[ts]. Consequently, particles have maximum potential energy a[t] extreme position and zero at mean position.

A work is said to be done against the acceleration of particle o[f] progressive wave if the particle move from its mean position to [a] distance y. For a small displacement dy, work done is;

$$dW = F\,dy$$

If ρ is the density of medium, the work done per unit volume is;

$$dw = \rho\, a_p\, dy$$

$$i.e.,\ dw = \rho\left\{\frac{4\pi^2 a v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt-x)\right\}dy$$

The total work done for displacement y is;

$$w = \int_0^y dw = \rho\frac{4\pi^2 v^2}{\lambda^2}\int_0^y a\sin\frac{2\pi}{\lambda}(vt-x)\,dy$$

The work done per unit volume is stored as potential energy per unit volume,

$$i.e.,\ V = \frac{4\pi^2\rho v^2}{\lambda^2}\frac{a^2}{2}\sin^2\frac{2\pi}{\lambda}(vt-x)$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(vt-x)$$

and kinetic energy per unit volume will become;

$$T = \frac{1}{2}\rho\left[\frac{2\pi a v}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x)\right]^2$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\cos^2\frac{2\pi}{\lambda}(vt-x)$$

The total energy per unit volume is;

$$E = T + V = \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\left\{\sin^2\frac{2\pi}{\lambda}(vt-x) + \cos^2\frac{2\pi}{\lambda}(vt-x)\right\}$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}$$

We have,

$$v = f\lambda$$

$$\therefore\ E = 2\pi^2 a^2\rho f^2$$

This shows that the average kinetic energy per unit volume and potential energy per unit volume are equal and is equal to half of total energy per unit volume.

6. Deduce analytically the velocity of transverse wave along a stretched string depends on the tension and mass per unit length of the string. [T.U. 2065 Shrawan]

Solution: Proceed as the solution of Q. No. 3 on page no. 35

7. Explain the term "wave motion". Show that for a plane progressive wave, on the average, half the energy is kinetic and half potential. [T.U. 2065 Chaitra]

Solution:

Wave motion

It is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean position.

In the plane progressive wave, there is a continuous transfer of energy in the direction of propagation. The energy is sum of kinetic energy and potential energy. For a particle executing the simple harmonic motion, particle has maximum velocity at its equilibrium position and will be zero at extreme positions *i.e.*, since kinetic energy is proportional to square of velocity, it is maximum at mean position and zero at extreme points. Consequently, particles have maximum potential energy at extreme position and zero at mean position.

A work is said to be done against the acceleration of particle of progressive wave if the particle move from its mean position to a distance y. For a small displacement dy, work done is;

$$dW = F\,dy$$

If ρ is the density of medium, the work done per unit volume is

$$dw = \rho\, a_p\, dy$$

i.e., $$dw = \rho\left\{\frac{4\pi^2 a v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt-x)\right\}dy$$

The total work done for displacement y is;

$$w = \int_0^y dw = \rho\frac{4\pi^2 v^2}{\lambda^2}\int_0^y a\sin\frac{2\pi}{\lambda}(vt-x)\,dy$$

The work done per unit volume is stored as potential energy per unit volume, i.e.,

$$V = \frac{4\pi^2\rho v^2}{\lambda^2}\frac{a^2}{2}\sin^2\frac{2\pi}{\lambda}(vt-x)$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(vt-x)$$

and kinetic energy per unit volume will become;

$$T = \frac{1}{2}\rho\left[\frac{2\pi a v}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x)\right]^2 = \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\cos^2\frac{2\pi}{\lambda}(vt-x)$$

The total energy per unit volume is;

$$E = T+V = \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\left\{\sin^2\frac{2\pi}{\lambda}(vt-x)+\cos^2\frac{2\pi}{\lambda}(vt-x)\right\}$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}$$

We have,

$$v = f\lambda$$

$$\therefore\ E = 2\pi^2 a^2\rho f^2$$

This shows that the average kinetic energy per unit volume and potential energy per unit volume are equal and is equal to half of total energy per unit volume.

8. **What is meant by superposition of waves? Point out the difference between transverse wave and mechanical wave and derive an expression for the speed of travelling wave in a stretched string.** [P.U. 2002]

Solution:

When two waves of same frequency having constant phase difference traverse in same region of the medium simultaneously meets and produce the resultant wave. Such phenomenon is called superposition of waves.

A transverse wave can travel without material medium whereas mechanical medium requires material medium to travel from one plane to another place.

An electromagnetic wave is an example of transverse wave. Sound is an example of mechanical wave.

Speed of travelling wave in a stretched string

Proceed as the solution of Q. No. 3 on page no. 35

9. Derive an expression for the velocity of transverse waves in a stretched string. [P.U. 2002]

Solution: See the solution of Q. No. 2 on page no. 34

10. Derive an expression for the velocity of transverse waves in a stretched string. [P.U. 2011]

Solution: Proceed as the solution of Q. No. 2 on page no. 34

2.6 ADDITIONAL SOLVED PROBLEMS

1. One end of a string is fixed. It hangs over a pulley and has a block of mass 2 kg attached to the other end. The horizontal part has a length of 1.6 m and mass 20 gm. What is the speed of the transverse pulse on the string?

Solution:

Here,

Mass of the block, $(m) = 2\ kg$

Tension on the string, $(T) = mg = 2\ kg \times 9.8\ ms^{-2} = 19.6\ N$

Since 1.6 m of a string has a mass of 20 gm,

Mass per unit length of the string,

$$\mu = \frac{20\times10^{-3}\ kg}{1.6\ m} = 1.25\times10^{-2}\ kgm^{-1}$$

We have,

Speed of transverse pulse on string,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6\ N}{1.25\times10^{-2}\ kgm^{-1}}} = 39.61\ ms^{-1}$$

2. A train of simple harmonic waves is travelling in a gas along the positive direction of an x-axis with an amplitude 2 cm, velocity 300 ms^{-1} and frequency 400 Hz. Calculate the displacement, particle velocity and particle acceleration at a distance of 4 cm

from the origin after an interval of 5 s. What will be the maximum speed of a particle?

Solution:

We have, the displacement of a particle,

$$y = a \sin\frac{2\pi}{\lambda}(vt - x)$$

where, x is the distance of a particle from the origin, after time t, a is amplitude of a wave, λ is its wavelength and v is its velocity.

Given that,

$a = 2\,cm = 0.02\,m$

$v = 300\,ms^{-1}$

$f = 400\,Hz$

At $t = 5\,s$

$x = 4\,cm = 0.04\,m$

Wavelength, $\lambda = \frac{v}{f} = \frac{300\,ms^{-1}}{400\,Hz} = 0.75\,m$

Now,

$$y = 0.02 \sin\frac{2\pi}{0.75}(300 \times 5 - 0.04) = 0.02 \sin 12566^\circ$$

$$= -1.12 \times 10^{-2}\,m$$

Hence, the distance of the particle at assistance of $0.04\,m$ from the origin, after an interval of $5\,s = -1.12 \times 10^{-2}\,m$.

Particle velocity,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos\frac{2\pi}{\lambda}(vt - x)$$

$$= \frac{2\pi \times 300}{0.75} \times 0.02 \cos\frac{2\pi}{0.75}(300 \times 5 - 0.04)$$

$$= 41.67\,ms^{-1}$$

Particle acceleration,

$$\frac{d^2y}{dt^2} = -\left(\frac{2\pi v}{\lambda}\right)^2 a \sin\frac{2\pi}{\lambda}(vt - x)$$

$$= -\left(\frac{2\pi \times 300}{0.75}\right)^2 0.02 \sin\frac{2\pi}{0.75}(300 \times 5 - 0.04)$$

$$= 7.07 \times 10^4\,m\,s^{-2}$$

The particle speed will be maximum if $\cos\frac{2\pi}{\lambda}(vt - x) = 1$.

Thus, the maximum particle speed $= \frac{2\pi v}{\lambda} a = \frac{2\pi \times 300}{0.75} \times 0.02$

$$= 50.27\,ms^{-1}$$

5. A wave of frequency 500 Hz has a phase velocity 360 m/s. How far apart are two points 60° out of phase? What is the phase difference between two displacements at a certain point at times 10^{-3} s apart?

Solution:

We have, the displacement of a particle,

$$y = a \sin\frac{2\pi}{\lambda}(vt - x)$$

where, x is the distance of a particle from the origin, after time t, a is amplitude of a wave, λ is its wavelength and v is its velocity and $\frac{2\pi}{\lambda}(vt - x)$ is the phase angle of a point distance x from origin at any instant of time t.

Thus, phase angle of a point distance x_1 from origin at any instant of time $t = \frac{2\pi}{\lambda}(vt - x_1)$ phase angle of a point distance x_2 from origin at any instant of time $t = \frac{2\pi}{\lambda}(vt - x_2)$

The phase difference between two points;

$$= \frac{2\pi}{\lambda}(vt - x_1) - \frac{2\pi}{\lambda}(vt - x_2) = \frac{2\pi}{\lambda}(x_2 - x_1)$$

$$= \frac{2\pi v}{\lambda}\left(\frac{x_2 - x_1}{v}\right) = 2\pi f\left(\frac{x_2 - x_1}{v}\right) \qquad \left(\because f = \frac{v}{\lambda}\right)$$

Given that,

The phase difference between two points $= 60^\circ = \frac{\pi}{3}$ radians

i.e., $\frac{\pi}{3} = 2\pi f\left(\frac{x_2 - x_1}{v}\right)$

or, $\frac{1}{3} = 2 \times 500\left(\frac{x_2 - x_1}{360}\right)$

$\therefore\ x_2 - x_1 = 0.12\,m$

The two points are $0.12\,m$ apart.

Phase angle of a point distance x from origin at any instant of time $t_1 = \frac{2\pi}{\lambda}(vt_1 - x)$

Phase angle of a point distance x from origin at any instant of time $t_2 = \frac{2\pi}{\lambda}(vt_2 - x)$

In this case,

Phase difference,

$$\phi = \frac{2\pi}{\lambda}(vt_1 - x) - \frac{2\pi}{\lambda}(vt_2 - x) = 2\pi f(t_2 - t_1)$$

$$= 2\pi \times 500 \times 10^{-3} = \pi^c$$

4. The displacement equation of a transverse plane wave at an instant is given by;
$y = 0.3 \sin(3\pi t - 0.05\pi x)$
where, x and t are in meter and seconds
Calculate wave length frequency and velocity of the wave. Also calculate phase difference between two particles 0.25 m apart at same instant.

Solution:

Here,
The displacement equation of a transverse plane wave is;

$$y = 0.3 \sin(3\pi t - 0.05\pi x) \quad \ldots \text{(i)}$$

The general displacement equation of a transverse plane wave is,

$$y = a \sin\frac{2\pi}{\lambda}(vt - x) = a \sin\left(\frac{2\pi v}{\lambda}t - \frac{2\pi}{\lambda}x\right)$$

$$= a \sin\left(2\pi f t - \frac{2\pi}{\lambda}x\right) \quad \ldots \text{(ii)}$$

Comparing equations (i) and (ii), we obtain,

$$2\pi f = 3\pi$$

$$\therefore f = 1.5\ Hz$$

$$\frac{2\pi}{\lambda} = 0.05\pi$$

$$\therefore \lambda = 40\ m$$

Velocity of transverse wave,

$$v = f \times \lambda = 1.5\ Hz \times 40\ m = 60\ ms^{-1}$$

For the two particles 0.25 m apart, the phase difference,

$$\phi = \frac{2\pi}{\lambda}x = 0.05\,\pi \times 0.25 = (0.0125\pi)^c$$

5. Check whether the following expressions are a solution of one dimensional wave equation or not.
i) $y = x^2 + v^2t^2$
ii) $y = \sin 2x \cos vt$
iii) $y = 2 \sin x \cos vt$
iv) $y = (x - vt)^2$
v) $y = x^2 - v^2t^2$.

Solution:

i) Here,

$$y = x^2 + v^2t^2$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = 2v^2t$$

$$\text{or, } \frac{d^2y}{dt^2} = 2v^2$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2x$$

$$\text{or, } \frac{d^2y}{dx^2} = 2$$

Obviously,

$$\frac{d^2y}{dt^2} = 2v^2 = v^2\frac{d^2y}{dx^2}$$

Hence, $y = x^2 + v^2t^2$ is the solution of one dimensional wave equation, $\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$.

ii) Here,

$$y = \sin 2x \cos vt$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = v \sin 2x \sin vt$$

$$\text{or, } \frac{d^2y}{dt^2} = -v^2 \sin 2x \cos vt = -v^2y$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2 \cos 2x \cos vt$$

$$\text{or, } \frac{d^2y}{dx^2} = -4 \sin 2x \cos vt = -4y$$

Obviously,

$$\frac{d^2y}{dt^2} = -v^2y \neq v^2\frac{d^2y}{dx^2}$$

Hence, $y = \sin 2x \cos vt$ is the solution of one dimensional wave equation.

iii) Here,

$$y = 2 \sin x \cos vt$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = 2v \sin x \sin vt$$

$$\text{or, } \frac{d^2y}{dt^2} = -2v^2 \sin x \cos vt = -v^2y$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2\cos x \cos vt$$

or, $$\frac{d^2y}{dx^2} = -2\sin x \cos vt = -y$$

Obviously,

$$\frac{d^2y}{dt^2} = -v^2 y = v^2 \frac{d^2y}{dx^2}$$

Hence, $y = 2\sin x \cos vt$ is the solution of one dimensional wave equation, $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$.

iv) Here,

$$y = (x - vt)^2$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = -2xv + 2v^2 t$$

or, $$\frac{d^2y}{dt^2} = 2v^2$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2x - 2vt$$

or, $$\frac{d^2y}{dx^2} = 2$$

Obviously,

$$\frac{d^2y}{dt^2} = 2v^2 = v^2 \frac{d^2y}{dx^2}$$

Hence, $y = (x - vt)^2$ is the solution of one dimensional wave equation, $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$.

v) Here,

$$y = x^2 - v^2 t^2$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = -2v^2 t$$

or, $$\frac{d^2y}{dt^2} = -2v^2$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2x$$

or, $$\frac{d^2y}{dx^2} = 2$$

Obviously,

$$\frac{d^2y}{dt^2} = -2v^2 \neq v^2 \frac{d^2y}{dx^2}$$

Hence, $y = x^2 - v^2t^2$ is the solution of one dimensional wave equation.

6. If the intensity of a seismic wave is $10^6\ Wm^{-2}$ at 200 km from the source, what will be the intensity at 500 km the source?

Solution:

The intensity of wave decreases as square of the distance from the source. Therefore, at 500 km the intensity should be;

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$

i.e., $$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{200\ km}{500\ km}\right)^2 \times 10^6\ Wm^{-2} = 1.6 \times 10^5\ Wm^{-2}$$

7. A source of sound has a frequency of has a frequency of 256 Hz and amplitude of 0.50 cm. Calculate the energy flow across a square cm per sec. the velocity of sound in air is 330 m/s and its density is 1.29 kgm^{-3}.

Solution:

The total energy transferred in a square cm per second is;

$$I = \frac{E}{A \times t} = 2\pi^2 v\rho f^2 a^2$$

$$= 2\pi^2 \times 330\ ms^{-1} \times 1.29\ kg\ m^{-3} \times (256\ Hz)^2 (0.50 \times 10^{-2} m)^2$$

$$= 1.38 \times 10^4\ Jm^{-2}s^{-1}$$

8. The equation of a transverse wave travelling along a stretched string is given by;

$y = 10 \sin \pi (2t - 0.05\ x)$

where y and x are expressed in cm and t in seconds. Find the amplitude, velocity, and wavelength of the wave. What will be the maximum speed of a particle in the string?

Solution: Proceed as the solution of Q. No. 4 on page no. 44

9. By how much would intensity level at a given place change when intensity of sound produced by a source at that place is doubled?

Solution:

The intensity level of sound,

$$I_L = 10\, log_{10}\left(\frac{I}{I_0}\right)$$

For the intensity level of sound I_{L_1}, let I_1 be the initial intensity of sound. Thus,

$$I_{L_1} = 10\, log_{10}\left(\frac{I_1}{I_0}\right)$$

Given that, final intensity,

$$I_2 = 2I_1$$

Let final intensity level of sound be I_{L_2}. Thus, we write,

Final intensity level,

$$I_{L_2} = 10\, log_{10}\left(\frac{I_2}{I_0}\right) = 10\, log_{10}\left(\frac{2I_1}{I_0}\right)$$

$$= 10\, log_{10}2 + 10\, log_{10}\left(\frac{I_1}{I_0}\right)$$

$$= \left(3.01 + I_{L_1}\right) dB$$

Therefore, change in intensity level $= I_{L_2} - I_{L_1} = 3.01\, dB$

Chapter 3

ACOUSTICS PHENOMENA

3.1 ACOUSTICS OF BUILDINGS

A branch of physics or engineering that deal with process of production, transmission and reception of sound is called acoustics. Some of the important fields of acoustics are (i) design of acoustical instruments (ii) electro acoustics relating to method of sound production and recording (microphones, amplifiers, loudspeakers etc.) (iii) architecture acoustics dealing with the designs and construction of buildings, cinema halls, auditoriums, recording rooms in broadcasting stations.

Acoustics becomes very important while designing an auditorium hall for speech or music purposes. In designing an acoustically good building, the sound must be uniformly distributed inside the room with every single person hearing sound distinctly. An auditorium hall is said to be acoustically good if it satisfies the following conditions,

i) Uniform distribution of sound throughout auditorium hall with proper loudness and quality.
ii) The quality of music or speech should be unchanged.
iii) No echoes or resonances.
iv) No overlapping of syllables.
v) No external and internal noise disturbances.
vi) Having an optimum reverberation time.

Reverberation, loudness, focusing, echo, resonance, noise etc. are the factors affecting the acoustics of buildings.

3.2 REVERBERATION

Reverberation is the undue prolongation of sound in a particular space. A reverberation is created in an enclosed space causing a large number of reflections to build up and decay slowly as the sound is absorbed by walls and air.

The duration for which the sound can be heard after the source has ceased to produce sound is called reverberation time. It is found that the sound become inaudible when its intensity falls to one millionth of average intensity. The duration between the production of sound and inaudibility is called *time of reverberation*.

One can manipulate the reverberation time in an auditorium halls by varying the absorption of sound. The amount of absorption of sound depends upon the nature of materials and their surface area.

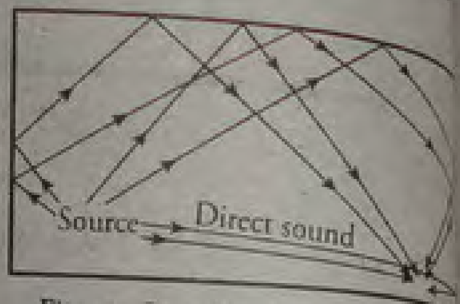

Figure: Reverberation phenomena

An *absorption coefficient* is the ratio of sound energy absorbed by the given surface to the sound energy absorbed by an equal area of a perfect absorber. Alternatively, an absorption coefficient of sound is defined as the reciprocal of the area which absorbs the same sound energy as absorbed by a unit area of a perfect absorber.

3.3 SABINE'S RELATION

W.C. Sabine (1980) found that the reverberation time is directly proportional to dimensions of rooms or halls and inversely proportional to absorption coefficient. He assumed that;

i) There is uniform distribution of sound inside the room or hall or auditorium hall.

ii) The energy is not lost in hall. The energy is lost only due to absorption of materials of the walls and ceiling and also due to escape through the windows and ventilations.

The Sabine's relations are;

$$T = \frac{0.158\,V}{A\alpha} \quad \text{[In S.I. units; velocity of sound in air} = 350\ m/s]$$

$$T = \frac{0.05\,V}{A\alpha} \quad \text{[In F.P.S.; velocity of sound in air} = 1120\ ft/s]$$

where, T = time of reverberation

V = dimension or volume of hall

A = area of walls

α = absorption coefficient of absorbing surface

Sabine established that there is an *optimum reverberation time* which is larger for large halls and it is also greater for music than that for speech. Below the optimum time, the intensity of sound is weak whereas above the optimum value the distinctness of syllables is hard.

3.4 ULTRASONIC WAVES

The sound waves having their frequencies lying between 20 Hz and 20 kHz can be detected by human ear and are called ***audible waves***. Audible sound waves originate in the vibrating strings (sonometer, violin, guitar, etc.), vibrating membrane (drums, loudspeaker, etc.) vibrating air column (flute, clarinet, etc.) and so forth.

The sound waves of frequency lower than audible range, *i.e.*, below 20 Hz, are called *infrasonics*. Such waves are produced by large vibrating bodies, e.g., earthquake waves, vibration of pendulums. These waves are insensitive to human ears.

The sound waves of frequency higher than audible range *i.e.*, above 20 kHz are called *ultrasonic waves*. Animals like dogs, birds and bats are sensitive to particular range of these waves. Bats are almost blind but can locate objects and navigate obstacles by producing ultrasonic waves and receiving echoes. These waves travel in air with same speed as that of audible waves; exhibit the properties of audible sound waves and some new phenomena. e.g., they travel in air with the velocity of 345 m/s at 25°C. Because of high frequency, their wavelength is extremely small. For a lower limit of frequency of 20 kHz, the wavelength of ultrasonic waves at room temperature is $\frac{345\ m/s}{20\ kHz} = 1.8\ cm$ and becomes much less at higher frequencies (0.35 cm at 1 MHz). This wavelength is comparable to X–ray wavelength (10^{-8} to $10^{-11}\,m$). Because of such a small wavelength, ultrasonic waves have great penetrating power. The ultrasonic waves are not electromagnetic waves.

Generation of Ultrasonic Waves

i) **Mechanical Generation**

It is divided into two groups. They are;

a) **Gas Driven**

These are simple devices to produce the ultrasonic waves of frequencies up to 30 kHz, such as Galton's whistle, siren, etc.

b) **Liquid Driven**

These are transducers type of devices which converts energy from one form to another, such as vibrating blade transducer and hydrodynamic oscillators.

ii) **Electrical Generation**

It is widely used for producing ultrasonic waves and is divided into two categories. They are;

a) **Piezoelectric Generators**

When certain crystals (such as Quartz, Tourmaline and Rochelle salt) subjected to pressure, they exhibit opposite electrical charges on their opposite faces resulting in potential difference. This phenomenon is known as *piezoelectric effect*. Conversely, if the opposite sides of these crystals are maintained at different electrical potentials by applying a voltage, then the crystal slice expands or contracts. This phenomenon is known as *converse piezoelectric effect* is utilized in production of ultrasonic waves of high frequency.

b) **Magnetostriction Oscillators**

Magnetostriction is the property by which ferromagnetic materials such as nickel, iron, cobalt, etc. in the form of rod changes in length when placed in magnetic field parallel to its length. This phenomenon can be used to produce ultrasonic waves.

Properties of Ultrasonic waves

i) These are the sound waves having frequencies greater than 20 kHz.
ii) Optical properties like reflection, refraction, diffraction, etc. are observed with ultrasonic waves.
iii) As the wavelength of ultrasonic waves is very small, the energy associated with these waves is enormous.
iv) Ultrasonic waves can penetrate through metals and other materials which are opaque to electromagnetic waves.
v) Ultrasonic waves produce cavitation effect when made to pass from some solids.
vi) Ultrasonic waves produce emulsion when applied at interface between two liquids.
vii) When ultrasonic waves meet surface of separation between two media, they undergo reflection.
viii) Ultrasonic waves of high frequencies produce chemical changes.
ix) Ultrasonic waves can be propagated through the elastic bodies such as liquid, solids, gases or vapors.
x) Ultrasonic luminescence is observed in liquids like water in certain special conditions.
xi) The velocity of ultrasonic waves depends on temperature of the medium through which they propagate.

Applications of Ultrasonic Waves

i) **Detection of flaws or cracks in metals**

Ultrasonic waves can be used in detecting cracks and cavities in metals. Due to the presence of cracks or cavit[...]nic waves propagating

through metals get reflected. The speed of these waves through the cavity in the metals will be different from that in solid region of the metals. Therefore, when ultrasonic waves pass through cracks or cavities inside it, a large amount of reflections occur. Some reflections even take place from bake surface of the metals. The reflected pulses from cavities and back surfaces are received by a receiving transducer and are amplified, and applied to one set plate of cathode ray oscilloscope (CRO). The transmitted signals and reflected signals form from the flaws and back surface of metal produced different peaks. The position of peaks on the time base of CRO determines the position of flaws from the surface of the metals.

ii) **Formation of alloys**

Various constituents of the alloys have different densities. The constituents of alloys can be mixed uniformly by beam of ultrasonic waves.

iii) **Soldering and metal cutting**

Ultrasonic waves can be used for drilling, cutting, soldering and room temperature welding of metals. To solder aluminum, ultrasonic waves along with electrical soldering iron are used.

iv) **In metallurgy**

The grain size can be refined and trapped gases can be removed by irradiating melt with ultrasonic waves during the process of cooling.

v) **Ultrasonic mixing**

A colloidal solution, an emulsion of two non-miscible liquids, can be mixed by simultaneously subjecting to ultrasonic radiation. Most of the emulsions like polishes, food products and pharmaceutical preparations are prepared by using ultrasonic mixing.

vi) **Effect on coagulation and crystallization process**

The suspended particles in liquids can be brought quite close to each other by ultrasonic waves, so that coagulation may occur. Also, the crystallization rate is improved by ultrasonic waves. When the crystallizing solution is kept for crystallization, the size of the can be made smaller and more uniform by the use of ultrasonic waves.

vii) **Signaling**

Due to the smaller wavelength, the ultrasonic waves can be concentrated into a sharp beam. Thus, these waves can be used in signaling in particular direction.

viii) In processing, testing and communications

In processing applications, ultrasonic vibrations at higher densities, is used to produce physical or chemical change in material.

The focused ultrasound is used for surgery. Echo-sounding techniques are used for testing materials. The high frequency elastic properties of materials can be measured by ultrasonic waves with great precision and give information about its structure.

The most important communications application is in ultrasonic delay lines, which are useful in radar and data handling system.

ix) SONAR (Sound Wave Navigation and Ranging)

SONAR is used for the detection of submarines, icebergs and other objects in the ocean. One can determine presence of submerged submarines, ships, icebergs, rocks or enemies aircrafts in the sky by this technique. In this technique, a sharp ultrasonic beam is directed in various directions. After reflection, it is picked by ultrasonic detector. The reflected waves are due to presence of some reflecting medium such as submarines, rocks, icebergs, etc. in the sea. The idea of the position of the body is obtained by the time interval between generation of ultrasonic waves and their return after reflection. If the body is moving there will be change in frequency of echo signals. It helps to determine the velocity of body and predicts its direction.

x) Depth of the sea

The ultrasonic waves are highly energetic and show a negligible diffraction effect. They can be used to determine depth of the sea. The time interval between sending and receiving ultrasonic waves from the sea surface is recorded. As the velocity of wave is known, depth of the sea can be estimated.

xi) Medical applications

The ultrasonic waves have been widely used for examining the shape and movement of organs within the body. The images of ultrasonic waves that are reflected from the boundaries of the organs are obtained and is known as ultrasound scanning. The ultrasonic waves reflected from moving objects like RBC exhibit a Doppler shift frequency that is used to measure the rate of flow of blood.

Ultrasonic waves are used by dentists for the proper extraction of broken teeth or decayed teeth.

xii) Biological use

Ultrasonic waves are used to lame smaller animals like rats, fish, frogs, etc. and kill bacteria. The reproductory power of yeast is loosed if they are exposed to ultrasonic waves.

xiii) Cleaning and clearing

Ultrasonic waves can be used for cleaning machine parts, utensils, washing clothes and removing dust and soot from chimneys.

3.5 SOLVED EXAM QUESTIONS

1. What are ultrasonic waves? How do they differ from ordinary sound? What are their uses? **[T.U. 2061 Baishakh]**

Solution:

Ultrasonic waves

The sound waves of frequency higher than audible range *i.e.*, above 20 *kHz*, are called ultrasonic waves. Animals like dogs, birds and bats are sensitive to particular range of these waves. Bats are almost blind but can locate objects and navigate obstacles by producing ultrasonic waves and receiving echoes.

The sound waves having their frequencies lying between 20 *Hz* and 20 *kHz* can be detected by human ear and are called *audible waves*. Audible sound waves originate in the vibrating strings (sonometer, violin, guitar, etc.), vibrating membrane (drums, loudspeaker, etc.) vibrating air column (flute, clarinet, etc.) and so forth.

Ultrasonic waves have lower wavelength than audible waves and is comparable to wavelength of X-rays.

Applications of Ultrasonic Waves

i) Detection of flaws or cracks in metals

Ultrasonic waves can be used in detecting cracks and cavities in metals. Due to the presence of cracks or cavities, ultrasonic waves propagating through metals get reflected. The speed of these waves through the cavity in the metals will be different from that in solid region of the metals. Therefore, when ultrasonic waves pass through cracks or cavities inside it, a large amount of reflections occur. Some reflections even take place from bake surface of the metals. The reflected pulses from cavities and back surfaces are received by a receiving transducer and are amplified, and applied to one set plate of cathode ray oscilloscope (CRO). The transmitted signals and reflected signals form from the flaws and back surface of metal produced different peaks. The position of peaks on the time base of CRO determines the position of flaws from the surface of the metals.

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The ultrasonic waves have been widely used for examining the shape and movement of organs within the body. The images of ultrasonic waves that are reflected from the boundaries of the organs are obtained and are known as ultrasound scanning. The ultrasonic waves reflected from moving objects like RBC exhibit a Doppler shift frequency that is used to measure the rate of flow of blood.

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Ultrasonic waves are used to lame smaller animals like rats, fish, frogs, etc. and kill bacteria. The reproductory power of yeast is loosed if they are exposed to ultrasonic waves.

xiii) **Cleaning and clearing**

Ultrasonic waves can be used for cleaning machine parts, utensils, washing clothes and removing dust and soot from chimneys.

2. **What is the reverberation time for a hall with length 12 m breadth 11 m and height 9 m if the coefficient of absorption of walls, ceiling and floor are 0.02, 0.04 and 0.08 respectively? [T.U. 2061 Baishakh]**

Solution:

Here,

The dimension of the hall $(l \times b \times h) = 12\,m \times 11\,m \times 9\,m$

Volume of the hall $(V) = 1188\,m^3$

Coefficient of absorption of walls $(\alpha_1) = 0.02$

Coefficient of absorption of ceiling $(\alpha_2) = 0.04$

Coefficient of absorption of floor $(\alpha_3) = 0.08$

Area of four walls, $(S_1) = 2h(l + b)$

$= 2 \times 9\,m(12\,m + 11\,m)$

$= 414\,m^2$

Area of ceiling (S_2) = Area of floor (S_3) = $l \times b = 12\,m \times 11\,m$

$= 132 m^2$

We know that,

The reverberation time of hall;

$$(T) = \frac{0.158\,V}{\sum_i \alpha_i S_i} = \frac{0.158 \times 1188}{0.02 \times 414 + 0.04 \times 132 + 0.08 \times 132} = 7.78\,s$$

3. **Define ultrasonic waves. Describe piezoelectric method for their production. How are these used for the distance measurement? [T.U. 2061 Ashwin]**

Solution:

Ultrasonic waves

See the solution of Q. No. 1 on page no. 55

These waves travel in air with same speed as that of audible wave exhibit the properties of audible sound waves and some new phenomena. E.g., they travel in air with the velocity of 345 m/s at 25°. Because of high frequency, their wavelength is extremely small. For a lower limit of frequency of 20 kHz, the wavelength of ultrasonic waves at room temperature is $\frac{345\ m/s}{20\ kHz} = 1.8\ cm$ and becomes much less at higher frequencies (0.35 cm at 1MHz)

Piezoelectric generation of ultrasonic waves

When certain crystals (such as Quartz, Tourmaline and Rochelle salt) subjected to pressure, they exhibit opposite electrical charges on their opposite faces resulting in potential difference. This phenomenon is known as piezoelectric effect. Conversely, if the opposite sides of these crystals are maintained at different electrical potentials by applying a voltage, then the crystal slice expands or contracts. This phenomenon is known as converse piezoelectric effect is utilized in production of ultrasonic waves of high frequency. If an electric field is applied across two faces of the crystal, compression or extension will occur across the other pair of faces. The magnitude of compression or extension is proportional to the potential difference between the faces. This effect is used to produce high frequency oscillations in quartz. Tin foil sheets cover the opposite faces of a slab of quartz crystal. The application of alternating potential difference of same frequency to foils produce oscillations in quartz crystal that can be communicated to liquid. If natural frequency of mechanical vibration of the quartz crystal coincides with that of the applied potential difference, resonance occurs and vibrations of large amplitude are set up in the quartz crystal and hence produce ultrasonic waves.

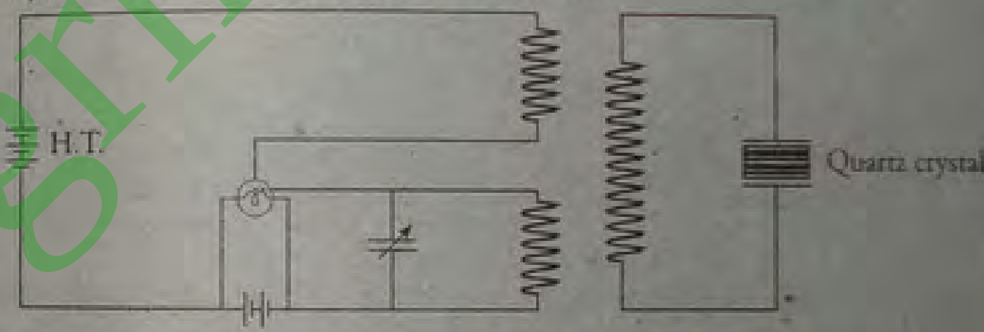

Figure: Piezoelectric generation of ultrasonic waves

The ultrasonic waves are highly energetic and show a negligible diffraction effect. They can be used to determine distance of objects. The time interval between sending and receiving ultrasonic waves from the particular object is recorded. As the velocity of wave is known, distance of an object can be estimated.

4. **Calculate the reverberation time of a small hall in BICC of 1500 m³ having seating capacity 120 persons when (i) the hall is empty and (ii) with full capacity of the audience for the following data.**

Surface	Areas	Coefficient of absorption
Plastered walls	112 m^2	0.03
Wooden floor	130 m^2	0.06
Plastered ceiling	170 m^2	0.04

Wooden doors	20 m²	0.06
Cushioned chairs	120	0.05
Audience	120	0.44

[T.U. 2063 Baishakh]

Solution:

Here,

Volume of BICC hall $(V) = 1500\ m^3$

i) Reverberation time of an empty hall,

$$T = \frac{0.158\,V}{\sum_i \alpha_i S_i}$$

$$= \frac{0.158 \times 1500}{0.03\times112+0.06\times130+0.04\times170+0.06\times20+0.05\times120}$$

$$= 9.42\ s$$

ii) Reverberation time with full capacity of audience,

$$T = \frac{0.158V}{\sum_i \alpha_i S_i}$$

$$= \frac{0.158\times1500}{0.03\times112+0.06\times130+0.04\times170+0.06\times20+0.05\times120+0.44\times120}$$

$$= 3.04\ s$$

The reverberation time is significantly lowers with audiences in hall.

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5. What do you understand by reverberation? Formulate Sabine reverberation equation and discuss its significance.[T.U. 2061 Poush]

Solution:

Reverberation

Reverberation is the undue prolongation of sound in a particular space. A reverberation is created in an enclosed space causing large number of reflections to build up and decay slowly as the sound is absorbed by walls and air.

Sabine's reverberation equation

W.C. Sabine found that the reverberation time is directly proportional to dimensions of rooms or halls and inversely proportional to absorption coefficient. He assumed that;

i) There is uniform distribution of sound inside the room/hall or auditorium hall.

ii) The energy is not lost in hall. The energy is lost only due to absorption of materials of the walls and ceiling and also due to escape through the windows and ventilations.

Assume that I is the average intensity of sound in a room at any given instant and δI is the fall in intensity due to absorption in a small interval of time δt. The fall in intensity is,

$$\delta I = -\alpha n I \delta t$$

where, α is the mean absorption coefficient of all absorbing surfaces in the room and n is number of reflections of sound per second

The negative sign indicates the loss in intensity as time elapses.

By statistical means, Jaeger proved that sound travels an average distance of $\frac{4V}{S}$ between successive reflections; where, V is the volume of the hall or auditorium and S is the total surface area of the absorbing surfaces in the hall or auditorium. Therefore, the time taken between two successive reflection is $\frac{4V}{Sv}$; where, v is the velocity of sound.

The number of reflection made $= \frac{t}{\frac{4V}{Sv}} = \frac{Sv}{4V}t$

Hence, the number of reflections per second would be,

$$n = \frac{Sv}{4V}$$

Thus, we can write,

$$\delta I = -\alpha \frac{Sv}{4V} I \delta t$$

$$\text{or,}\quad \frac{\delta I}{\delta t} = -\alpha \frac{Sv}{4V} I$$

In the limit, we can rearrange the terms to provide

$$\frac{dI}{I} = -\alpha \frac{Sv}{4V} dt$$

On integrating, we obtain,

$$ln\left(\frac{I_t}{I_0}\right) = -\alpha \frac{Sv}{4V} t$$

where, I_0 and I_t are intensity at $t = 0$ and $t = t$ respectively.

$$\text{i.e.,}\quad I_t = I_0 e^{-\alpha\frac{Sv}{4V}t}$$

By definition of reverberation time T,

$$I_t = \frac{I_0}{10^6}$$

$$\text{or,}\quad I_0 e^{-\alpha\frac{Sv}{4V}T} = \frac{I_0}{10^6}$$

$$\text{or,}\quad e^{-\alpha\frac{Sv}{4V}T} = 10^{-6}$$

$$\text{i.e.,}\quad T = \frac{6\,ln\,10 \times 4V}{\alpha S v}$$

The velocity of sound in air is 350 m/s. Thus, we obtain,

$$T = \frac{0.158\,V}{aS}$$

where, V is the volume of the hall or auditorium in m^3, S is th surface area in m^2 and α is the absorption coefficient of th material in the hall. The absorption coefficient for the standar building materials is given by Sabine's formula,

$$T = \frac{0.158\,V}{\sum_i a_i S_i}$$

The reverberation time is directly proportional to the volume o auditorium halls and inversely proportional to area of wall ceiling, floor, etc and absorption plus transmission through ope surfaces. In order to decrease the reverberation time, the walls i the auditorium are covered with high absorption coefficien materials. The surface area of walls is increased to decrease th reverberation time in good acoustic halls.

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6. **What is reverberation? Explain the causes of reverberation in theatre and how can it be reduced? Deriving the necessar equation for reverberation time, discuss the factors affecting th acoustics of buildings.** [T.U. 2065 Shrawa

Solution:

Reverberation

Reverberation is defined as the persistence of the sound even afte source is cut off. A reverberation is created in an enclosed spa causing a large number of reflections to build up and decay slow as the sound is absorbed by walls and air.

The reverberation is due to the multiple reflections on smoot surfaces of theatre and small number of openings like ventilation windows and doors. The reverberation can be reduced b increasing the absorption of sound in the theatre using soun absorbing materials such as using heavy curtains, hanging map and pictures in the walls, covering the wall and ceiling wit absorbing material like mineral wools, celotex, felt, flooring wi carpets, arranging seats having cushions at their backs whi provide a high degree of absorption in themselves. A large numb of audiences in the theatre significantly reduce the reverberation

Sabine's reverberation equation

See the solution of Q. No. 5 on page no. 60

The reverberation time is directly proportional to the volume of auditorium halls and inversely proportional to area of walls, ceiling, floor, etc and absorption plus transmission through open surfaces.

Factors affecting acoustics of building are;

i) control of reverberation
ii) shapes of walls and ceilings
iii) concave shapes and balconies
iv) floor plans with diverging side walls
iv) seats
v) loudness, resonance and noise.

---

7. **A lecture hall with a volume of 4500 m^3 is found to have a reverberation time of 1.5 s. What is the total absorbing power of all the surfaces in the hall? If the area of the sound absorbing surface is 1600 m^2, calculate the average absorption coefficient.** [T.U. 2065 Chaitra]

Solution:

Here,

Volume of the lecture hall $(V) = 4500\ m^3$

Reverberation time $(T) = 1.5\ s$

We have,

$$T = \frac{0.158\,V}{aS}$$

$$\text{or,}\quad \alpha S = \frac{0.158\,V}{T} = \frac{0.158 \times 4500}{1.5} = 474 \text{ S.I. units}$$

Hence, the absorbing power of all the surfaces of the hall is 474 S.I. units.

Area of absorbing surface $(S) = 1600\ m^2$

Thus,

$$\alpha \times 1600 = 474$$

$$\therefore \quad \alpha = 0.30$$

i.e., the average absorption coefficient = 0.30

---

8. **Define absorption coefficient. Derive Sabine's reverberation formula and also explain its importance in our daily life?** [T.U. 2065 Kartik]

Solution:

Absorption coefficient (α)

Absorption coefficient is defined as ratio of sound energy absorbed by the surface to that of the total sound energy incident on the surface. Its unit is open window units (OWU).

i.e., $\alpha = \dfrac{\text{Sound energy absorbed by the surface}}{\text{Total energy incident on the surface}}$

Sabine's reverberation formula

See the solution of Q. No. 5 on page no. 60

The reverberation time is directly proportional to the volume of auditorium halls and inversely proportional to area of walls, ceiling, floor, etc and absorption plus transmission through open surfaces. This relation provides the conditions for designing acoustically good auditorium halls and designs the rooms with pleasant talking environments.

9. **Define absorption coefficient of sound. Derive a relation between reverberation time and absorption coefficient for acoustically good hall.** [T.U. 2067 Ashadh]

Solution: See the solution of Q. No. 8 on page no. 63

10. *Derive a necessary equation for reverberation time and mention the factor affecting the acoustics of building.* [T.U. 2061 Baishakh]

Solution: See the solution of Q. No. 6 on page no. 62

11. **Distinguish between ultrasonic and infrasonic waves? Describe a method for the production of ultrasonic waves. Mention some important applications of ultrasonic waves.** [P.U. 2008]

Solution:

The sound waves of frequency lower than audible range, i.e. below 20 Hz, are called *infrasonics*. Such waves are produced by large vibrating bodies, e.g., earthquake waves, vibration of pendulums. These waves are insensitive to human ears.

The sound waves of frequency higher than audible range i.e. above 20 kHz, are called *ultrasonic waves*. Animals like dogs, birds and bats are sensitive to particular range of these waves. Bats are almost blind but can locate objects and navigate obstacles by producing ultrasonic waves and receiving echoes. These waves travel in air with same speed as that of audible waves; exhibit the properties of audible sound waves and some new phenomena. e.g. they travel in air with the velocity of 345 m/s at 25°. Because of high frequency, their wavelength is extremely small.

Piezoelectric generation of ultrasonic waves

See the solution of Q. No. 3 on page no. 58

Applications of Ultrasonic Waves

See the solution of Q. No. 1 on page no. 55

12. **Distinguish between ultrasonic and infrasonic waves? Describe a method for the production of ultrasonic waves. Discuss briefly the application of ultrasonic waves finding of focal length of the depth of sea and signaling.** [P.U. 2005 B]

Solution: See the solution of Q. No. 11 on page no. 64

3.6 ADDITIONAL SOLVED PROBLEMS

1. **A hall of floors is $(15 \times 30)\ m^2$ along with height of 6 m, in which 500 people occupy upholstered seat and wooden chairs. Optimum reverberation time for orchestral music is 1.36 s and absorption coefficient per person is 0.44 OWU.**
   i) **Calculate the coefficient of absorption to be provided by the walls, floor and ceiling when the hall is fully occupied.**
   ii) **Calculate the reverberation time if only the half upholster seats are occupied. (The absorption coefficient of chair is 0.02 OWU).**

Solution:

Here,

Volume of the hall $(V) = 15 \times 30 \times 6 = 2700\ m^3$

Optimum reverberation time $(T) = 1.36\ s$

i) We know that,

Sabine's relation for reverberation time,

$$T = \frac{0.158\,V}{aS}$$

$$\text{or, } aS = \frac{0.158\,V}{T} = \frac{0.158 \times 2700}{1.36}$$

$$= 313.68 \text{ S.I. units}$$

The absorption due to audience $= 500 \times 0.44 = 220$ S.I. units

Therefore, the absorption provided by walls, floor and ceiling

$= 313.68 - 220 = 93.68$ S.I. units

ii) When the hall is only half filled the absorption will be provided by vacant wooden seats in addition to the absorption by the audience.

Absorption by audience $= 250 \times 0.44$
$= 110$ S.I. units

Absorption by vacant wooden seats $= 250 \times 0.02$
$= 5$ S.I. units

$\therefore$ Total absorption of the hall $= 93.68 + 110 + 5$
$= 208.68$ S.I. units

The reverberation time,

$$T = \frac{0.158\,V}{aS} = \frac{0.158 \times 2700}{208.68} = 2.04\ s$$

---

2. Calculate the total absorption coefficient of auditorium hall with reverberation time 1.7 s whose volume is 1500 m^3.

Solution:

Here,

Reverberation time or auditorium hall $(T) = 1.7\ s$

Volume of the auditorium hall $(V) = 1500\ m^3$

We know that,

$$T = \frac{0.158\,V}{aS}$$

or, $$aS = \frac{0.158\,V}{T} = \frac{0.158 \times 1500}{1.7} = 139.41 \text{ S.I. units}$$

The total absorption coefficient of the auditorium hall is 139.41 S.I. units.

---

3. A piezoelectric crystal plate has a thickness of 1.6 mm. If the velocity of propagation of sound waves is 5700 m/s, calculate the fundamental frequency of the crystal.

Solution:

Here,

Thickness of the crystal plate $(t) = 1.6\ mm = 1.6 \times 10^{-3}\ m$

Velocity of sound waves $(v) = 5700\ m/s$

In the lowest mode of vibration, the distance between two faces of the crystal of thickness t will be $\frac{\lambda}{2}$. Therefore,

$$t = \frac{\lambda}{2}$$

i.e., $\lambda = 2t = 2 \times 1.6 \times 10^{-3}\ m = 3.2 \times 10^{-3}\ m$

We have,

$$f = \frac{v}{\lambda} = \frac{5700}{3.2 \times 10^{-3}} = 1781250\ Hz = 1.8\ MHz$$

Hence, the fundamental frequency of the crystal is 1.8 MHz.

---

4. The ultrasonic pulse echo method is employed to detect possible defects in a steel bar of thickness 45 cm if pulse arrival times are 30 and 80 microseconds, find the distance of the defect from the end of the bar at which the ultrasonic pulse enters the bar.

Solution

Here,

Thickness of the steel bar $= 45\ cm = 0.45\ m$

Echo times $(t_1) = 30\ \mu s = 30 \times 10^{-6}\ s$

$(t_2) = 80\ \mu s = 80 \times 10^{-6}\ s$

Assume that $x\ m$ is the distance of the possible defect in steel bar from the end of the bar at which ultrasonic pulse enters the bar. The pulse covers a distance of $2x$ in arriving back to the end after being reflected from the defect. Thus, we can write,

$$t_1 = \frac{2x}{v}$$

i.e., $$30 \times 10^{-6}\ s = \frac{2x}{v} \quad \text{... (i)}$$

The second pulse will arrive after being reflected from far end of the bar. Therefore, it covers a distance $2 \times 0.45\ m = 0.90\ m$ in $80 \times 10^{-6}\ s$. Thus,

$$80 \times 10^{-6}\ s = \frac{2 \times 0.45\ m}{v} \quad \text{... (ii)}$$

Dividing equation (i) by equation (ii), we obtain,

$$\frac{3}{8} = \frac{x}{0.45}$$

$$\therefore\ x = 0.17\ m$$

The distance of the flaw from nearest end is 0.17 m.

---

5. A quartz crystal with a thickness of 0.4 mm and a density of 2650 kgm^{-3} vibrates longitudinally producing ultrasonic waves. Find the fundamental frequency of vibration if the Young's modulus of quartz is $7.5 \times 10^{10} Nm^{-2}$.

Solution:

Here,

Thickness of the crystal $(t) = 0.4\ mm = 4 \times 10^{-4}\ m$

Density of the crystal $(\rho) = 2650\ kgm^{-3}$

Young's modulus of elasticity $(Y) = 7.5 \times 10^{10} Nm^{-2}$

We have,

Fundamental frequency of vibration $(f) = \frac{k}{2t}\sqrt{\frac{Y}{\rho}}$

$k = 1$ for the fundamental mode of vibration. Therefore,

$$f = \frac{1}{2\times4\times10^{-4}m}\sqrt{\frac{7.5\times10^{10}Nm^{-2}}{2650\ kgm^{-3}}}$$

$$= 6.65\times10^{6}Hz = 6.65\ MHz$$

Hence, the fundamental frequency of the vibration is 6.65 MHz.

6. **Calculate the fundamental frequency of quartz crystal of 3 mm thickness. Given that the density of the crystal is 2650 kgm^{-3} and Young's modulus of elasticity is $7.5\times10^{10}Nm^{-2}$.**

Solution: Proceed as solution of Q. No. 5 on page no. 67

7. **An ultrasonic generator has a quartz crystal vibrating at its fundamental frequency. The thickness of the crystal is 2.5 mm, density 2650 kgm^{-3} and Young modulus is $7.5\times10^{10}Nm^{-2}$. Calculate the fundamental frequency of vibration and second overtone.**

Solution: Proceed as solution of Q. No. 5 on page no. 67
[for the second overtone, $k = 3$]

Chapter 4

LENSES

4.1 SIGN CONVENTION

The distance of the object and image from the refracting surface is a vector quantity and these distances must be represented with proper signs.

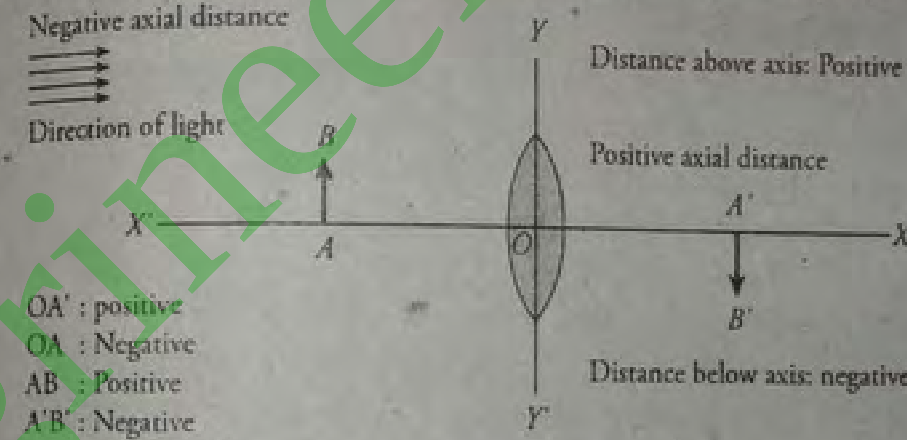

Figure: Sign convention

The above figure depicts the conventions of signs used in accordance with the convention of coordinate geometry. In general, the figures are drawn with the incident light travelling from left right. Considering the centre of the refracting surface at origin O and its axis along XX', distances measured to the left of O are taken as negative. Distances measured upward and normal to the X–axis are taken as positive whereas downward distances are taken as negative.

If we consider AB as an object in above figure, its distance OA is negative. If $A'B'$ represents an image of AB, its distance OA' is positive. The size of the object AB is positive while size of the image $A'B'$ is negative.

4.2 LENS TERMINOLOGY

A *lens* is a portion of a transparent medium bounded by two spherical surfaces or by one spherical surface and a plane surface. It is usually made of glass. The centres of two spherical surfaces which form the lens are known as *centres of curvature*. The distance of the refracting surface from the respective centre of curvature is called *radius of curvature*. The middle point of the lens from either side is said to be *optical centre*. The incident ray and emergent ray through the optical centre of lens are parallel to each other, *i.e.*, rays don't refract while passing through the optical centre. The imaginary line passing through the centres of curvature is said to be *principal axis*. A point on the principal axis, where rays of a parallel beam of light, also parallel to the principal axis actually converge or from where it appears to come from is said to be *principal focus*. A linear distance between optical centre to principal focus is known as *focal length*.

4.3 CARDINAL POINTS

Whenever refraction takes place through a thin lens, position and size of an image formed is determined by neglecting the thickness of less, as it is very small compared to distances of object and image from it. In case of thick lens or coaxial system of lenses, we cannot proceed with this assumption. The method of finding position and size of final image by considering refraction at each surface of a lens successively is extremely tedious and complex process. To overcome this difficulty, Gauss (1941) proved that the positions of certain specific points are known. These specific points are known as cardinal points. The pair of the points form six cardinal points of an optical system: two focal points, two principal points and two nodal points. All six points are situated on the optical axis of system and are conjugate to each other.

4.4 REFRACTION THROUGH A LENS

When light form the object O falls on the first reflecting surface of the lens, the image I' is formed at the distance v' from the reflecting surface.

Thus,

$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

where, R_1 is the radius of curvature of refracting surface

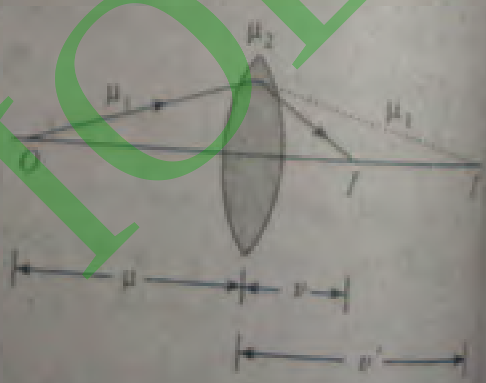

Figure: Refraction through a thin lens

The image I' acts as virtual object in medium of refracting index μ_2 for second refracting surface and final image I at distance v from the second refracting surface with radius of curvature R_2 such that;

$$\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \frac{\mu_1 - \mu_2}{R_2}$$

On simplifying, we obtain,

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, $\mu = \frac{\mu_2}{\mu_1}$ is the refractive index of material of lens

4.5 POWER OF LENS

Power of a lens is measure of its ability to produce convergence or divergence of a parallel beam of light. It is defined as the reciprocal of focal length expressed in meters. Thus,

$$\text{Power of a lens } (P) = \frac{1}{\text{Focal length in meters}}$$

The S.I. unit of power of lens is Diopter (D).

If two lenses of focal lengths f_1 and f_2 are in contact, then,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

where, P_1 and P_2 are power of two lenses and P is their equivalent power.

If two lenses are coaxially separated by a distance d, their equivalent power is;

$$P = P_1 + P_2 - dP_1P_2$$

4.6 CHROMATIC ABERRATION

The equations connecting object distance, image distance, focal length, refractive index, etc. are based on the assumption that angles made by the rays of light with axis are small. In practice, lenses are used to form images of points situated off axis as well. In general non paraxial rays of light from an object point do not meet at a single point after refraction through lens. The refractive index and hence the focal length of a lens are different for different wavelengths of light. Therefore, a number of colored images are formed by the lens for non-monochromatic light. These images even though formed by paraxial rays are at different position and are of different sizes.

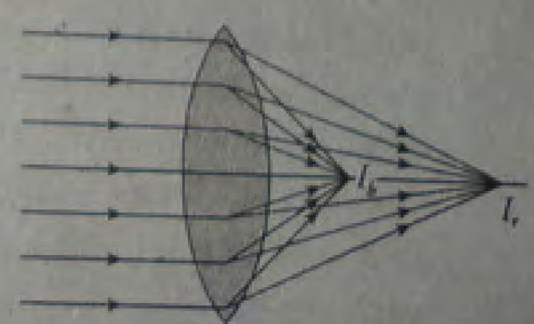

Figure: Chromatic aberration produced by a convex lens

The deviations from the actual size, shape and position of an image calculated by simple lens equations are called *aberrations* produced by a lens.

The aberrations produced by a variation of refractive index with wavelength of light are called *chromatic aberrations*. In other words, a single lens produces colored images of an object illuminated by white light; such defect is called chromatic aberration. Elimination of this defect in a system of lenses is called *achromatism*.

4.7 SOLVED EXAM QUESTIONS

1. What are common defects in the image produced by a single lens? Find the condition for achromatism of two lenses separated by distance. **[T.U. 2061 Ashwin]**

Solution:

The common defects in the image produced by a single lens are;

i) Chromatic Aberrations

ii) Monochromatic Aberrations

a) Spherical Aberrations

b) Coma

c) Astigmatism

d) Curvature of a field

e) Distortion

Conditions for Achromatism of two lenses

An elimination of chromatic aberration in a system of lenses is called *achromatism*. The achromatic combination is made by placing two lenses of different materials and suitable focal length in contact or separated by a finite distance.

Consider two lenses of focal lengths f_1 and f_2 separated by a distance d. The equivalent focal length F for the combination of two lenses is;

$$\frac{1}{F} = \frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

On differentiating, we obtain,

$$-\frac{dF}{F^2} = -\frac{df_1}{f_1^2}-\frac{df_2}{f_2^2}-d\left(\frac{df_1}{f_1^2 f_2}+\frac{df_2}{f_1 f_2^2}\right)$$

For achromatism,

$$dF = 0$$

For lenses of same material,

$$-\frac{df_1}{f_1} = -\frac{df_2}{f_2} = \omega$$

where, ω is dispersive power of the lens material. Thus,

$$\frac{\omega}{f_1}+\frac{\omega}{f_2}-d\left(\frac{\omega}{f_1 f_2}+\frac{\omega}{f_1 f_2}\right) = 0$$

$$\text{or, } \frac{1}{f_1}+\frac{1}{f_2} = \frac{2d}{f_1 f_2}$$

$$\therefore \quad d = \frac{f_1+f_2}{2}$$

Hence, two lenses must be separated by the distance equal to mean focal length of the two lenses for achromatic combination.

2. Two thin converging lenses of focal lengths 50 *cm* and 40 *cm* are placed co-axially 30 *cm* apart in air. Determine the positions of cardinal points. **[T.U. 2062 Baishakh]**

Solution:

Here,

$f_1 = 50\ cm$

$f_2 = 40\ cm$

$d = 30\ cm$

We have,

$$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{50\ cm \times 40\ cm}{50\ cm + 40\ cm - 30\ cm} = \frac{10}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 30 \times \frac{10}{3\times 40} = 2.5\ cm$$

Thus the first principal point P_1 is at a distance of 2.5 *cm* to the right of the first lens.

$$\beta = -d\frac{f}{f_1} = 30 \times \frac{10}{3\times 50} = 2\ cm$$

The second principal point P_2 is at distance 2 *cm* to the left of the second lens.

The first focal point F_1 is at a distance;

$$f_1 - f = \left(50-\frac{10}{3}\right) cm = 46.67\ cm \text{ from the first lens}$$

The second focal point F_2 is at a distance;

$$f - f_2 = \left(\frac{10}{3}-40\right) cm = -36.67\ cm \text{ from the second lens}$$

As the medium on the two sides of the lens system is the same, the nodal points N_1 and N_2 coincide with P_1 and P_2.

3. **What are cardinal points? Explain their significance with referenc to a co-axial lens system using ray diagram.** [T.U. 2063 Baishakh

Solution:

Whenever refraction takes place through a thin lens, position and size of an image formed is determined by neglecting the thicknes of less, as it is very small compared to distances of object and image from it. In case of thick lens or coaxial system of lenses, we cannot proceed with this assumption. The method of finding position and size of final image by considering refraction at each surface of a lens successively is extremely tedious and complex process. To overcome this difficulty, Gauss (1941) proved that the positions of certain specific points are known. These specific points are known as cardinal points. The pair of the point from six cardinal point of an optical system; two focal points, two principal points and two nodal points. All six points are situated on the optical axis of system and are conjugate to each other.

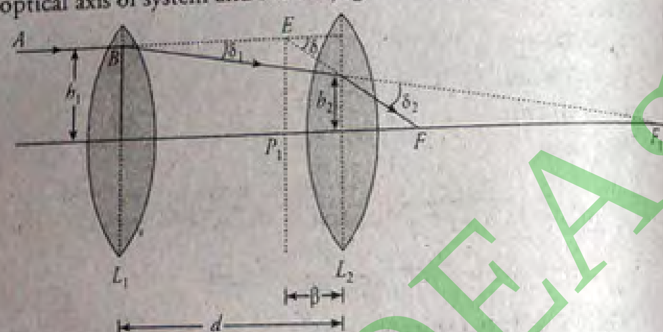

Figure: Co-axial lens system

4. **Two thin lenses of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 cm. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. Assume that both the lenses are of same material.** [T.U. 2063 Baishakh]

Solution:

Here,

Equivalent focal length $(F) = 50\ cm = 0.5\ m$

Two thin lenses of focal lengths f_1 and f_2 separated by a distance d. For no chromatic aberration, we write,

$$d = f_1 - f_2$$

For minimum spherical aberration, we can write,

$$d = \frac{f_1 + f_2}{2}$$

From these two equations, we obtain,

$$f_1 = 3f_2$$

$$d = 3f_2 - f_2 = 2f_2$$

The equivalent focal length for combination of two lenses is,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

or, $$\frac{1}{0.5} = \frac{1}{3f_2} + \frac{1}{f_2} - \frac{2f_2}{3f_2 f_2}$$

or, $$2 = \frac{2}{3f_2}$$

$$\therefore f_2 = \frac{1}{3}\ m = 0.33\ m$$

Substituting f_2, we obtain,

$$f_1 = 3 \times 0.33\ m = 0.99\ m$$

$$d = 2 \times 0.33\ m = 0.66\ m$$

5. **Two lenses of focal lengths 8 cm and 4 cm are placed at a distance apart. Calculate the position of the principal points to form an achromatic combination.** [T.U. 2064 Poush]

Solution:

Here,

$f_1 = 8\ cm$

$f_2 = 4\ cm$

$$d = \frac{f_1 + f_2}{2} = \frac{8\ cm + 4\ cm}{2} = 6\ cm$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{8\ cm \times 4\ cm}{8\ cm + 4\ cm - 6\ cm}$$

$$= \frac{16}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 6 \times \frac{16}{3 \times 4} = 8\ cm$$

$$\beta = -d\frac{f}{f_1} = -6 \times \frac{16}{3 \times 8} = -4\ cm$$

The principle points are 8 cm and −4 cm to form an achromatic combination.

6. **What are cardinal points in an optical system? Use a suitable diagram to explain the principal points.** **[T.U. 2064 Poush]**

Solution:

Cardinal Points

See the solution of Q. No. 3 on page no. 74

Consider a thick lens system or coaxial lens system. The system has two principal foci F_1 and F_2. The parallel ray of incident light after refraction intersect the principal axis at F_2. When incident ray and final emergent ray are produced, they intersect at a point H_2. A plane passing through H_2 and perpendicular to principal axis intersects the principal axis at P_2 is referred as second principal point. Similarly ray from F_1, after refraction passes parallel to the principal axis. The incident ray from F_1 and final emergent ray, if produced meet at H_1. A plane containing H_1 and perpendicular to principal axis is termed as first principal plane. The point P_1 in the principal axis is referred as first principal point.

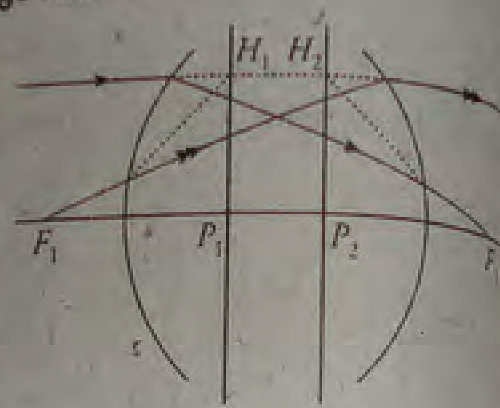

Figure: Principal points

7. **What are chromatic and monochromatic aberrations? List out different kinds of monochromatic aberrations. What is spherical aberration and discuss how it can be reduced to a minimum?** **[T.U. 2065 Shrawan]**

Solution:

The refractive index of the material of a lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths. Further, as the magnification of image is dependent of focal length of la lens, the size of the image is different for different wavelengths. The variation of the image distance from lens with refractive index measure axial or longitudinal chromatic aberration and variation in the size of the image measures lateral chromatic aberration.

The different kinds of monochromatic aberrations are;

a) Spherical Aberrations
b) Coma

c) Astigmatism
d) Curvature of a field
e) Distortion

The failure of rays to pass through a single point after reflection from a curved spherical surface is called spherical aberration. Spherical aberrations can be minimized in following ways.

i) Spherical aberrations can be minimized by using stops, which reduce the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, the amount of light passing through the lens is reduced and image appears less bright.
ii) Plano-convex lenses are used in optical instruments so as to reduce the spherical aberration. When the curved surface of the lens faces the incident or emergent light whichever is more parallel to the axis, the spherical aberration is minimum.
iii) Spherical aberration can be minimized by using two plano-convex lenses separated by a distance equal to the difference in their focal length. In this arrangement, the total deviation is equally shared by two lenses and spherical aberration is minimum.
iv) Spherical aberration for a convex lens is positive and that for a concave lens is negative. Thus, by a suitable combination of convex lens and concave lens, it can be made minimized.

8. **What are cardinal points of an optical system? Determine the equivalent focal length of a combination of two thin lenses separated by a finite distance. Hence find the position of two principal points.** **[T.U. 2065 Chaitra]**

Solution:

Cardinal Points

See the solution of Q. No. 3 on page no. 74

Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially. They are separated by a finite distance d. Consider a monochromatic ray AB parallel to principal axis is incident on lens L_1 at a height h_1. This ray is refracted towards BF_1 in absence of lens L_2. The F_1 is principal focus of lens L_1. The deviation produced by this lens is,

$$\delta_1 = \frac{h_1}{f_1}$$

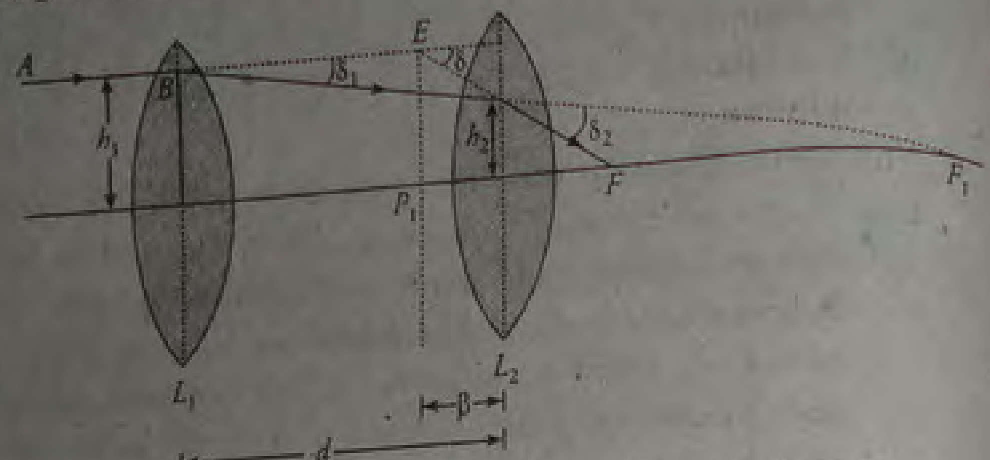

Figure: Refraction through the combination of two lenses

The ray BF_1 meets lens L_2 at point C, at the height h_2 from principal axis and gets emerged from this lens towards CF, where, F is the principal focus of the combination of two lenses L_1 and L_2. The deviation due to second lens is,

$$\delta_2 = \frac{h_2}{f_2}$$

where, f_2 is the focal length of the lens L_2

An incident ray AB is produced and final emergent ray FC will intersect at point. Thus, a thin lens placed at E, will produce same deviation as the lenses L_1 and L_2 separated at the distance d. An imaginary such lens of focal length, is called equivalent lens. The deviation produced by equivalent lens is;

$$\delta = \frac{h_1}{f}$$

From geometry, we write,

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}$$

Since ΔBL_1F_1 and ΔBL_2F_1 are similar. This provides the sides of these triangele have proportionate relation.

i.e., $$\frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$$

$$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d} \qquad (\because L_1L_2 = d)$$

$$\therefore \quad h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Thus,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 \cdot f_2}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

This provides the equivalent focal length of two thin lenses separated by the distance d.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{f_1 f_2}{\Delta}$$

where, $\Delta = -(f_1 + f_2 - d)$ and is known as optical separation or optical interval between the two lenses

It is numerically equal to the distance between the second principal focus of the first lens and first principal focus of the second lens.

---

9. **Two thin lenses of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 cm. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Assuming both the lenses are of same material, find the values of f_1, f_2 and d. [T.U. 2065 Chaitra]**

Solution: See the solution of Q. No. 4 on page no. 74

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10. **Two thin lenses having focal lengths 10 cm and 4 cm are coaxially separated by a distance of 5 cm. Find the equivalent focal length of the combination. Determine also the positions of the principal points. [T.U. 2068 Shrawan]**

Solution:

Here,

$f_1 = 10\ cm$

$f_2 = 4\ cm$

$d = 5\ cm$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{10\ cm \times 4\ cm}{10\ cm + 4\ cm - 6\ cm} = 5\ cm$$

$$\alpha = +d\frac{f}{f_2} = 5 \times \frac{5}{4} = 6.25\ cm$$

$$\beta = -d\frac{f}{f_1} = -5 \times \frac{5}{10} = -2.5\ cm$$

The equivalent focal length of combination is 5 cm. The principal points are at 6.25 cm from the right of the first lens and second lens is at the distance of 2.5 cm left from second lens.

11. **Two identical thin convex lenses of focal length 8 *cm* each are coaxial and 4 *cm* apart. Find the equivalent focal length and the positions of the principal points. Also find the position of the object for which image is formed at infinity.** **[P.U. 2002]**

Solution:

Here,

$f_1 = 8\ cm$

$f_2 = 4\ cm$

$$d = \frac{f_1 + f_2}{2} = \frac{8\ cm + 4\ cm}{2} = 6\ cm$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{8\ cm \times 4\ cm}{8\ cm + 4\ cm - 6\ cm} = \frac{16}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 6 \times \frac{16}{3 \times 4} = 8\ cm$$

$$\beta = -d\frac{f}{f_1} = -6 \times \frac{16}{3 \times 8} = -4\ cm$$

The equivalent focal length of combination is $\frac{16}{3}\ cm$. The principal points are at $8\ cm$ from the right of the first lens and second lens is at the distance of $4\ cm$ left from second lens.

For the final image to be formed at infinity,

$V = \infty$

$f = \frac{16}{3}\ cm$

Therefore,

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\therefore\ U = -f = -\frac{16}{3}\ cm$$

But,

$$u = U + \alpha = -\frac{16}{3} + 8 = \frac{8}{3}\ cm$$

Hence, the object is at a distance of $\frac{8}{3}\ cm$ to the left of the first lens.

12. **Define cardinal points. Derive the expression for combined focal length of two thin lenses placed at certain distance apart. [P.U. 2003]**

Solution: See the solution of Q. No. 8 on page no. 77

13. **What are cardinal points? Explain spherical and chromatic aberration in optical image. Obtain condition of achromatize for two thin lenses placed co-axially in contact.** **[P.U. 2004]**

Solution:

Cardinal Points

See the solution of Q. No. 3 on page no. 74

Spherical and Chromatic aberrations

The failure of rays to pass through a single point after reflection from a curved spherical surface is called spherical aberration. Spherical aberrations can be minimized by using stops, which reduce the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, the amount of light passing through the lens is reduced and image appears less bright. Spherical aberration can be minimized by using two plano–convex lenses separated by a distance equal to the difference in their focal length. In this arrangement, the total deviation is equally shared by two lenses and spherical aberration is minimum.

The refractive index of the material of a lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths. Further, as the magnification of image is dependent of focal length of la lens, the size of the image is different for different wavelengths. The variation of the image distance from lens with refractive index measure axial or longitudinal chromatic aberration and variation in the size of the image measures lateral chromatic aberration.

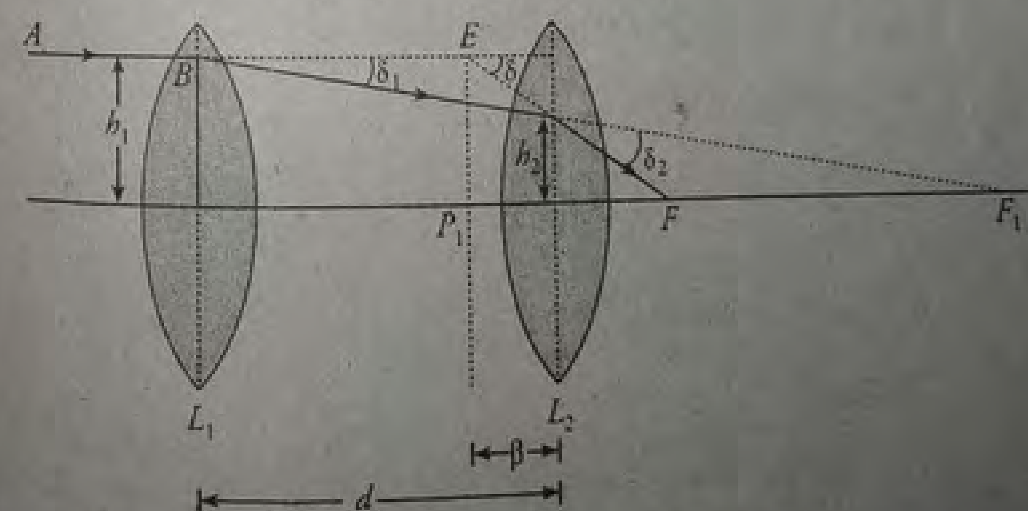

Figure: Refraction through the combination of two lenses

Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially. They are separated by a finite distance d. Consider a monochromatic ray AB parallel to principal axis is incident on lens L_1 at a height h_1. This ray is refracted towards BF_1 in absence of lens L_2. The F_1 is principal focus of lens L_1. The deviation produced by this lens is,

$$\delta_1 = \frac{h_1}{f_1}$$

The ray BF_1 meets lens L_2 at point C, at the height h_2 from principal axis and gets emerged from this lens towards CF, where F is the principal focusof the combination of two lenses L_1 and L_2. The deviation due to second lens is,

$$\delta_2 = \frac{h_2}{f_2}$$

where, f_2 is the focal length of the lens L_2

An incident ray AB is produced and final emergent ray FC will intersect at point. Thus, a thin lens placed at E, will produce same deviation as the lenses L_1 and L_2 separated at the distance d. An imaginary such lens of focal length, is called equivalent lens. The deviation produced by equivalent lens is;

$$\delta = \frac{h_1}{f}$$

From geometry, we write,

$$\delta = \delta_1 + \delta_2$$

or, $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}$$

Since ΔBL_1F_1 and ΔBL_2F_1 are similar. This provides the sides of these triangles have proportionate relation.

i.e., $$\frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$$

or, $$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d} \qquad (\because L_1L_2 = d)$$

$\therefore$ $$h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Thus,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 \cdot f_2}$$

$\therefore$ $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

This provides the equivalent focal length of two thin lenses separated by the distance d.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{f_1 f_2}{\Delta}$$

where, $\Delta = -(f_1 + f_2 - d)$ and is known as optical separation or optical interval between the two lenses.

14. Two thin convex lenses of focal lengths 20 cm and 40 cm are placed co-axially 20 cm apart. Find the position of principal points of the lens system. [P.U. 2004]

Solution:

Here,

$f_1 = 20\ cm$

$f_2 = 40\ cm$

$$d = \frac{f_1 + f_2}{2} = \frac{20\ cm + 40\ cm}{2} = 30\ cm$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

$$= \frac{20\ cm \times 40\ cm}{20\ cm + 40\ cm - 30\ cm} = \frac{8}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 30 \times \frac{8}{3 \times 40} = 2\ cm$$

$$\beta = -d\frac{f}{f_1} = -30 \times \frac{8}{3 \times 20} = -4\ cm$$

The equivalent focal length of combination is $\frac{8}{3}\ cm$. The principal points are at 2 cm from the right of the first lens and second lens is at the distance of 4 cm left from second lens.

15. Two thin lenses of focal lengths f_1 and f_2 seperated by a distance d have an equivalent focal length 50 cm. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. Assume that both the lenses are of same material. [P.U. 2005]

Solution: See the solution of Q. No. 4 on page no. 74

16. Define the cardinal points of a system of co-axial lines. Calculate the equivalent focal length and principal points of two thin co-axial lenses separated by a finite distance d. [P.U. 2007]

Solution: See the solution of Q. No. 8 on page no. 77

17. **What do you mean by the optical separation? Two lenses of focal lengths f_1 and f_2 are placed coaxially at a certain distance d apart in air. Derive an expression for the equivalent focal length of the combination and find the position of two principal planes.** **[P.U. 2008]**

Solution:

Optical separation is defined as the distance between the second principal focus of the first lens and first principal focus of the second lens.

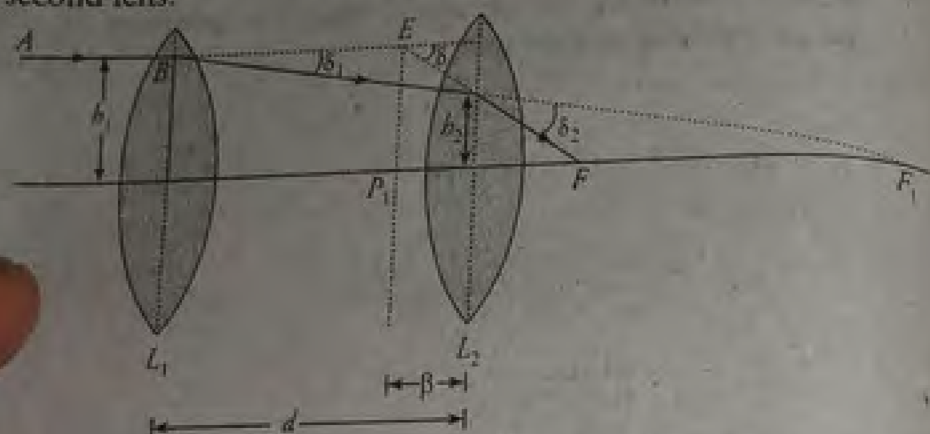

Figure: Refraction through the combination of two lenses

Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially. They are separated by a finite distance d. Consider a monochromatic ray AB parallel to principal axis is incident on lens L_1 at a height h_1 . This ray is refracted towards BF_1 in absence of lens L_2. The F_1 is principal focus of lens L_1. The deviation produced by this lens is,

$$\delta_1 = \frac{h_1}{f_1}$$

The ray BF_1 meets lens L_2 at point C, at the height h_2 from principal axis and gets emerged from this lens towards CF, where, F is the principal focus of the combination of two lenses L_1 and L_2. The deviation due to second lens is,

$$\delta_2 = \frac{h_2}{f_2}$$

where, f_2 is the focal length of the lens L_2 . An incident ray AB is produced and final emergent ray FC will intersect at point. Thus, a thin lens placed at E, will produce same deviation as the lenses L_1 and L_2 separated at the distance d. An imaginary such lens of focal length, is called equivalent lens. The deviation produced by equivalent lens is;

$$\delta = \frac{h_1}{f}$$

From geometry, we write,

$$\delta = \delta_1 + \delta_2$$

or, $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}$$

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i.e., $$\frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$$

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$$\therefore \quad h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Thus,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 \cdot f_2}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

This provides the equivalent focal length of two thin lenses separated by the distance d.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{f_1 f_2}{\Delta}$$

where, $\Delta = -(f_1 + f_2 - d)$ and is known as optical separation or optical interval between the two lenses.

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18. Two lenses of focal lengths f_1 and f_2 are placed coaxially at a certain distance d apart in air. Prove that $F = \frac{f_1 f_2}{\Delta}$, where, the symbols have their usual meaning. **[P.U. 2010]**

Solution: See the solution of Q. No. 8 on page no. 77

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19. **What are cardinal points? Derive the expression for combined focal length of two thin lenses placed at certain distance apart. [P.U. 2011]**

Solution: See the solution of Q. No. 8 on page no. 77

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20. Two thin lenses (same material) of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 *cm*. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. **[P.U. 2011]**

Solution: See the solution of Q. No. 4 on page no. 74

Chapter 5

FIBRE OPTICS

5.1 FIBRE OPTICS AND OPTICAL FIBRE

Fibre optics is branch of physics that deals with the propagation of ligh waves via optical fibre. In optical fibres, the light waves lunched in one end of the fibre and it is passed through the other end without any los of signals. The light waves passes through the optical fibre due to th total internal reflection of light. The light waves undergo total interna reflection more than hundred thousand times within the length of meter of the optical fibre.

An optical fibre is a dielectric wave guide that transmits light signals fro one place to another place. It consists of central core within whic propagation of electromagnetic field is confirmed and which surrounded by a cladding layer. The refractive index of the core is alwa greater than that of cladding layer. The cladding layer is surrounded by thin layer of buffer coating or jacket. The core and cladding layer a made up of pure silica glass, whereas buffer coating is made of plastics. cross sectional view of typical optical fibre is shown in figure below.

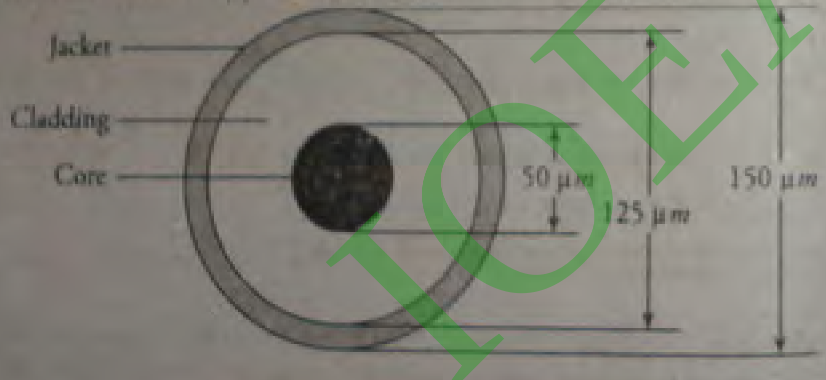

Figure: A cross sectional view of optical fibre

5.2 BASIC PRINCIPLE OF FIBRE OPTICS

The basic principle behind the propagation via optical fibre is tota internal reflection. The refractive index is one of the most importa optical parameters of a medium. It is defined as the optical density material with respect to vacuum. Mathematically,

$$\text{Refractive index of the medium } (\mu) = \frac{\text{Velocity of light in vaccum } (c)}{\text{Velocity of light in a medium } (v)}$$

The refractive index of vacuum is 1, that of air is 1.0002, that of water is 1.333, that of fused silica is 1.452 and that of crown glass is 1.517.

Whenever a light ray travels from an optically denser medium to optically rarer medium, it bends away from normal, *i.e.*, $i > r$. An angle of incidence in a denser medium is said to be *critical angle* C, if angle of refraction in a rarer medium is right angle. Whenever angle of incidence is greater than critical angle, incident ray does not refract. It reflects back in same medium. This phenomenon is called *total internal reflection*. At critical angle,

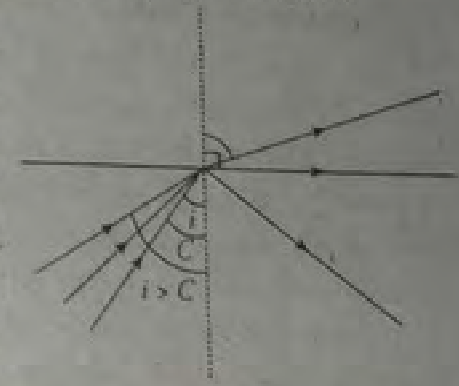

Figure: Critical angle (C) and total internal reflection

$$\mu_1 \sin C = \mu_2 \sin 90°$$

where, μ_1 and μ_2, $(\mu_1 > \mu_2)$ are refractive indices of denser and rarer media

$$\text{or,} \quad \sin C = \frac{\mu_2}{\mu_1}$$

$$\therefore \quad C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

This equation gives the value of critical angle.

5.3 CHARACTERISTICS OF OPTICAL FIBRES

The optical fibres have typical characteristics that make them highly attractive as a transmission medium. They offer the following characteristics;

i) **Potential bandwidth**

An optical fibre has enormous potential bandwidth, resulting from the use of optical carrier frequencies around $2 \times 10^{14}\ Hz$. With such a high frequency and bandwidth roughly equal to 10% of the carrier frequency.

ii) **Transmission loss**

Optical fibres have low transmission losses, as low as $0.1\ dB\ km^{-1}$.

iii) **Small size and weight**

The diameter of optical fibre is not greater than human hair.

iv) **Ruggedness and flexibility**

Optical fibre has a very high tensile strengths and possibility of being bent or twisted without damage.

v) **Immunity to electromagnetic interference**

It is an inherent characteristic of an optical fibre, viewed as a dielectric waveguide.

5.4 CLASSIFICATION OF OPTICAL FIBERS

Optical fibers are constructed to fulfill the requirements of particular purposes. The major requirement is in the area of bandwidth, *i.e.*, low medium, high medium or ultra high medium. In addition, a fibre may be constructed for its extremely low attenuation per kilometer. In some cases, tensile strength is the most desired characteristic. The optical fibres are classified into three major categories on the basis of *material, number of modes and refractive index profile*.

Based on the material used for preparation of optical fibres, they are classified into glass fibres and plastic fibres.

Glass fibres

In case of glass fibres, the core and cladding of optical fibres are prepared using glass. Silica is the basic raw material for the preparation of glass fibres. Silica has the refractive index of 1.458 at 850 *nm*. In order to increase or decrease or decrease the refractive index of Silica, doping materials such as germanium dioxide, phosphorus pentoxide, boron oxide, etc. are added. The addition of germanium dioxide or phosphorus pentoxide with silica increases the refractive index of silica whereas the addition of boron oxide decreases the refractive index of it. Examples of fiber compositions are as follows:

i) Germanium dioxide + Silica (Core); Silica (Cladding)
ii) Phosphorus pentoxide + Silica (Core); Silica (Cladding)
iii) Silica (Core); Phosphorus pentoxide + Silica (Cladding)

Plastic fibres

The plastic fibres are typically made up of plastics and they are of low cost. Although they exhibit considerably greater signal attenuation than glass fibers, the plastic fibres can be handled without special care due to its toughness and durability. Due to its high refractive index differences between core and cladding materials, plastic fibers yield high numerical aperture of 0.16 and large angle of acceptance up to 70°. A polystyrene core ($\mu_1 = 1.60$) and a polymethylmethacrylate cladding ($\mu_2 = 1.49$), a polymethylmethacrylate core ($\mu_1 = 1.49$) and a cladding made of its co-polymer ($\mu_2 = 1.40$), etc. are examples of plastic fibres.

Based on the refractive index profile, optical fibres are classified into step index optical fibres and graded index optical fibres.

Step Index optical fibres

The optical fibre in which refractive indices of core and cladding is constant. The refractive index of core μ_1 is slightly greater than that of cladding. Such optical fibre is called step index optical fibre. Therefore, there is noticeable boundary between the core and cladding. There is high transmission loss due to splitting of light singnals.

A step index fibre can be prepared single mode or multi–mode. Only one signal is passed through a single mode step index fibre, whereas more than one signal can be passed through multimode step index fibres.

In single mode step index fibre, the transmission is quicker as compared to other fibers. There is minimum dispersion. In this fibre, the bandwidth of information transmission is maximum and accuracy in reproducing pulses at the receiving end is high. However, the cost of fabrication is high in single mode step index fibre. The fabrication process is cumbersome as well. The source to fibre aperture is smallest as compared to other fibres. Due to the very small size of the central core, it is difficult to couple light in and out of such fibre. A highly directed light source such as a laser can couple light on the single mode step index fibres.

In multi–mode step index fibre, it is easy to couple light into and out of fibre due to large central core. The source to fibre aperture is relatively large. These fibres are economical. It is simple to fabricate. However, the transmission of information is slowest as compared to other fibres. In this fibre, the bandwidth of information transmission is minimum and distortion of pulse of light is maximum.

Graded Index optical fibres

In a graded index optical fibres, the refractive indices of the core vary with radial distances. The refractive index μ_1 is maximum at the centre of the core and it it minimum at the core–cladding interface μ_2. The refractive index decreases parabolically in proportion to the distance away from the centre of fibre to a constant value at the cladding. As compared to single mode fibre, the light can be easily coupled into and out of the fibre. However, as compared to multi–mode step index fibre, it is difficult to couple light into and out of the fibre. They are easier to construct as compared to single mode optical fibre. However, the construction is more difficult as compared to multi–mode step index optical fibre. The distortion of light pulse is more as compared to single mode optical fibre. However, the distortion is less as compared to multi–mode index optical fibre.

Based on the number of modes of propagation, optical fibres are classified into single mode optical fibres and multi–mode optical fibres. The number of modes of propagation through an optical fibre is;

$$N = \frac{d \times (NA)^2}{2\lambda}$$

where, d is the diameter of the optical fibre core, NA is the numerical aperture and λ is the wavelength of the light

Single mode optical fibres

If only one mode is passed through a fibre at a particular time, then it is said to be single mode optical fibre. The core diameter of the single mode optical fibre is nearly 8 to 9 μm. The core, cladding and sheath or jacket specification for a single mode optical fibre is 8.5/125/250 μm. The difference between the refractive indices of core and cladding is made very low. Due to this low difference between the refractive indices the critical angle at core cladding interface is very large. Therefore, the light rays that make very larger value of the angle of incidence at the core–cladding interface will pass through the fibre. So only one ray passes through the fibre, *i.e.*, the fundamental mode alone travels through the fibre. The single mode optical fibres are generally operated at 1300 nm and 1550 nm. The attenuation is low for single mode optical fibre at 1550 nm wavelength operation.

Multi–mode optical fibres

If more than one mode is passed through the optical fibre, it is said to be multi–mode optical fibre. A multi–mode optical fibres have core diameters of 50 μm or greater. A large number of signals are passed through the multimode fibres because of its large diameter. The multimode fibres are available in three different sizes. They are;

i) Core/Cladding/Jacket diameters ⇒ 50/125/ 250 μm
   Core/Cladding/Jacket diameters ⇒ 50/125/ 900 μm

ii) Core/Cladding/Jacket diameters ⇒ 62.5/125/ 250 μm
   Core/Cladding/Jacket diameters ⇒ 62.5/125/ 900 μm

iii) Core/Cladding/Jacket diameters ⇒ 100/140/ 250 μm

5.5 APPLICATIONS OF OPTICAL FIBRES

Fibre optics essentially deals with communication including voice signals, video signals and digital data. This is done by transmission of light through optical fibres. An optical fibre system consists of three basic parts: a light source, an optical fibre and a light detector. A source may be light emitting diodes [LED's]. An optical fibre transmits light waves. The

detector can be either an avalanche photo diode [APD] or positive intrinsic negative [PIN] diode. Basically, a fibre optic system converts an electrical signal to an infrared light signal. This signal is transmitted through an optical fibre. At the end of optical fibre, it is reconverted into an electrical signal.

The optical fibres are used in fabrication of fiberscope in endoscopy in medical sciences. Such fiberscopes are used in visualization of internal organs of human body. Optical fibres in combination of laser probe help to visualization of internal portion of human body and cauterization of tissues. Optical fibres are useful in industry as well. It can be used to examine welds, nozzles and combustion chambers inside aircraft engines which are inaccessible for observations in other available procedures.

5.6 ADVANTAGES OF OPTICAL FIBRES

The optical fibres have a lot of advantages over wireless or radio systems. That is why communication industries have introduced fibre optic systems significantly. The followings are the main advantages of optical fibres.

i) **Low attenuation**

Attenuation in an optical fibre is markedly lower than that of co–axial cable or twisted pair and is constant over a very wide range. Therefore, transmission within the wide range of distance is possible without repeaters, etc.

ii) **Smaller size and light weight**

Optical fibres are considerable thinner than co–axial cable or bundled twisted pair cable. Therefore, they occupy much less space.

iii) **Electromagnetic isolation**

Electromagnetic waves generated from electrical disturbances or electrical noise does not interfere with light signals. As a result, the system is not vulnerable to interference impulse noise or cross talk.

iv) **Physical connections**

No physical electrical connection is required between the senders and receives.

v) **Reliability**

Optical fibres are much more reliable because they can better withstand environmental conditions such as pollution radiation and salts produce no corrosion. Moreover, it is nominally affected by nuclear radiation. Its life is longer comparison to copper wire.

vi) Security and privacy

There is no interference in optical fibres and hence transmission is more secure and private because it is very difficult to tap into an optical fibre.

vii) Greater bandwidth

Bandwidth of optical fibres is higher than that of an equivalent wire transmission line.

viii) Isolation coating

As optical fibres are very good dielectrics, isolation coating is not required.

ix) Higher data rate

Data rate is much higher in an optical fibre and hence much more information can be carried out by each fibre in comparison to equivalent copper cables.

x) Lower cost

The cost per channel is lower than that of an equivalent wire cable system.

5.7 SOLVED EXAM QUESTIONS

1. What are the uses of optical fibres? [T.U. 2061 Baishakh]

Solution:

Uses of optical fibres

The use and demand for optical fiber has grown tremendously and optical–fibre applications are numerous. Telecommunication applications are widespread, ranging from global networks to desktop computers. These involve the transmission of voice, data or video over distances of less than a meter to hundreds of kilometers, using one of a few standard fibre designs in one of several cable designs.

Carriers use optical fiber to carry plain old telephone service across their nationwide networks. Local exchange carriers use fibre to carry this same service between central office switches at local levels, and sometimes as far as the neighborhood or individual home.

Optical fibre is also used extensively for transmission of data. Multinational firms need secure reliable systems to transfer data and financial information between buildings to the desktop terminals or computers and to transfer data around the world. Cable television companies also use fibre for delivery of digital video and data services. The high bandwidth provided by fibre makes it the perfect choice for transmitting broadband signals such as high–definition television (HDTV) telecasts.

Intelligent transportation systems, such as smart highways with intelligent traffic lights, automated tollbooths, and changeable message signs, also use fiber–optic–based telemetry systems.

Another important application for optical fibre is the biomedical industry. Fiber–optic systems are used in most modern telemedicine devices for transmission of digital diagnostic images. Other applications for optical fiber include space, military, automotive, and the industrial sector.

2. Give the reasons for attenuation and distortions of light through the optical fibres. [T.U. 2061 Ashwin]

Solution:

Attenuation

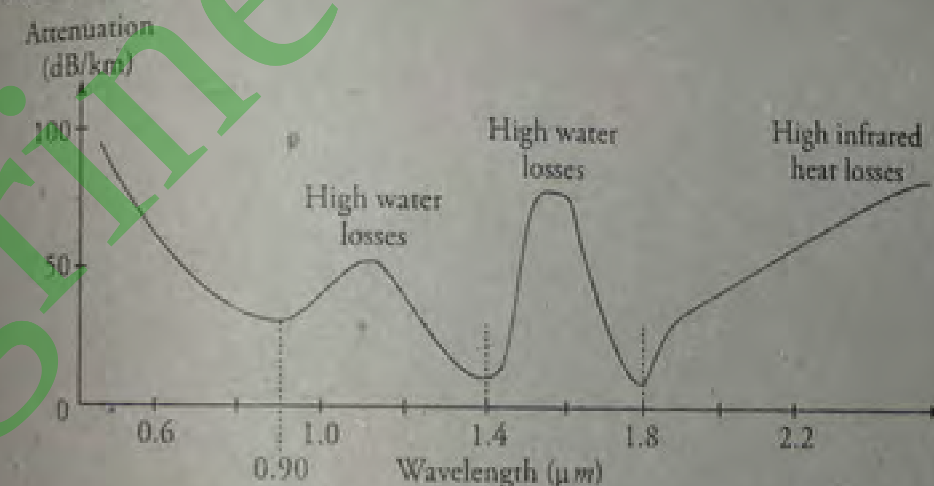

Figure: Typical attenuation versus wavelength curve of optical fibre

It is a decrease of magnitude of power of the light beam. It is represented in dB per unit length. The attenuation of optical signal with wavelength is shown in figure. Figure shows that there is low attenuation at 0.9 μm, 1.4 μm and 1.8 μm. These three wavelengths are called optical windows. The loss of signal at 0.9 μm is higher than losses at 1.4 μm and 1.8 μm. Thus, the devices have been designed to operate and exploit the low attenuation of 1.4 μm and 1.8 μm wavelengths. The attenuation in optical fibre may be due to absorption losses, scattering losses and radiation losses.

Distortion

In the transmission of optical signal, a pulse output is sometimes wider than input pulse; *i.e.*, pulse gets distorted as it moves through the fibre. The distortion of pulse is due to the dispersion effect which is measured in terms of nanoseconds per kilometer.

5. **Trace the diagram of step index and graded index optical fibres.** [T.U. 2062 Baishakh]

Solution:

The diagrams of step index optical fibre and graded index optical fibre are:

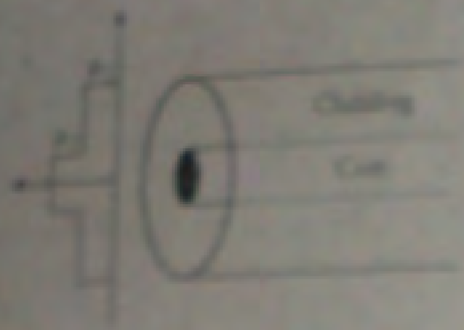

Figure: Construction of step index optical fibre

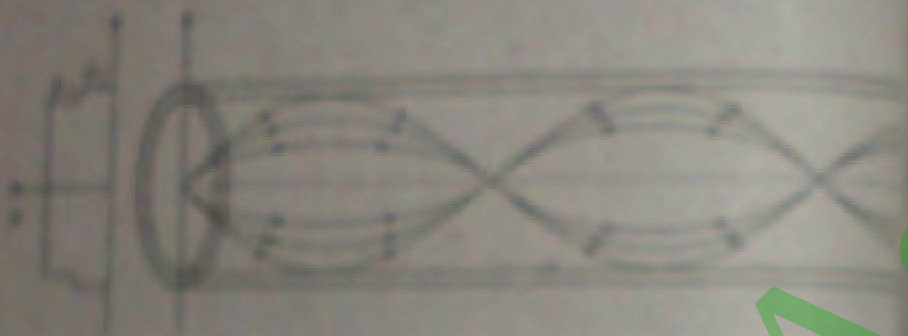

Figure: Construction and propagation signals in graded index optical fibre

6. **What is the principle behind functioning of an optical fibre? Describe the various types of optical fibres.** [T.U. 2062 Baishakh]

Solution:

Principle of fibre optics

The basic principle behind the propagation in the optical fibre is total internal reflection. The refractive index is one of the most important optical parameters of a medium. It is defined as the optical density of material with respect to vacuum.

Mathematically,

Refractive index of the medium,

$$(\mu) = \frac{\text{velocity of light in vacuum } (c)}{\text{velocity of light in a medium } (v)}$$

The refractive index of vacuum is 1, that of air is 1.00012, that of water is 1.333, that of fused silica is 1.452 and that of crown glass is 1.52

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$$N = \frac{d \times (NA)^2}{2\lambda}$$

where, d is the diameter of the optical fibre core, NA is the numerical aperture and λ is the wavelength of the light

Single mode optical fibres

If only one mode is passed through a fibre at a particular time, then it is said to be single mode optical fibre. The core diameter of the single mode optical fibre is nearly 8 to 9 μm. The core, cladding and sheath or jacket specification for a single mode optical fibre is 8.5/125/250 μm. The difference between the refractive indices of core and cladding is made very low. Due to this low difference between the refractive indices, the critical angle at core cladding interface is very large. Therefore, the light rays that make very larger value of the angle of incidence at the core–cladding interface will pass through the fibre. So only one ray passes through the fibre, *i.e.*, the fundamental mode alone travels through the fibre. The single mode optical fibres are generally operated at 1300 nm and 1550 nm. The attenuation is low for single mode optical fibre at 1550 nm wavelength operation.

Multi–mode optical fibres

If more than one mode is passed through the optical fibre, it is said to be multi–mode optical fibre. A multi–mode optical fibres have core diameters of 50 μm or greater. A large number of signals are passed through the multimode fibres because of its large diameter. The multimode fibres are available in three different sizes. They are;

i) Core/Cladding/Jacket diameters ⇒ 50/125/ 250 μm
   Core/Cladding/Jacket diameters ⇒ 50/125/ 900 μm

ii) Core/Cladding/Jacket diameters ⇒ 62.5/125/ 250 μm
   Core/Cladding/Jacket diameters ⇒ 62.5/125/ 900 μm

iii) Core/Cladding/Jacket diameters ⇒ 100/140/ 250 μm

5. The use of optical fibre is increasing day by day. What is the physics behind it? Discuss the advantages of optical fibres in modern communication technology. [T.U. 2064 Poush]

Solution:

The basic principle behind the propagation via optical fibre is total internal reflection. The refractive index is one of the most important optical parameters of a medium.

The optical fibres have typical characteristics that make them highly attractive as a transmission medium. An optical fibre has enormous potential bandwidth, resulting from the use of optical carrier frequencies around $2 \times 10^{14}\ Hz$. With such a high carrier frequency and bandwidth roughly equal to 10% of the carrier frequency. Optical fibres have low transmission losses, as low as $0.1\ dB\ km^{-1}$. The diameter of optical fibre is not greater than human hair. Optical fibre has a very high tensile strength and possibility of being bent or twisted without damage.

Advantages of optical fibres

The optical fibres have a lot of advantages over wireless or [...] systems. That is why communication industries have introduced fibre optic systems significantly. The followings are the [...] advantages of optical fibres.

i) **Low attenuation**

Attenuation in an optical fibre is markedly lower than that of co-axial cable or twisted pair and is constant over a very wide range. Therefore, transmission within the wide range of distance is possible without repeaters, etc.

ii) **Smaller size and light weight**

Optical fibres are considerable thinner than co–axial cable or bundled twisted pair cable. Therefore, they occupy much less [...]

iii) **Electromagnetic isolation**

Electromagnetic waves generated from electrical disturbance or electrical noise does not interfere with light signals. As a result, system is not vulnerable to interference impulse noise or cross [...]

iv) **Physical connections**

No physical electrical connection is required between the sender and receives.

v) **Reliability**

Optical fibres are much more reliable because they can withstand environmental conditions such as pollution, radiation and salts produce no corrosion. Moreover, it is nominally affected by nuclear radiation. Its life is longer comparison to copper [...]

vi) **Security and privacy**

There is no interference in optical fibres and hence transmission is more secure and private because it is very difficult to tap into an optical fibre.

vii) **Greater bandwidth**

Bandwidth of optical fibres is higher than that of an equivalent wire transmission line.

viii) **Isolation coating**

As optical fibres are very good dielectrics, isolation coating is not required.

ix) **Higher data rate**

Data rate is much higher in an optical fibre and hence much more information can be carried out by each fibre in comparison to equivalent copper cables.

x) **Lower cost**

The cost per channel is lower than that of an equivalent wire cable system.

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6. **What are mono-mode and multimode optical fibres? Differentiate between step index and graded index optical fibre. Also write down the applications of optical fibre in communication system.** [T.U. 2065 Shrawan]

Solution:

Mono–mode and multimode optical fibres

See the solution of Q. No. 4 on page no. 94

Differences between step index and graded index optical fibres

Step index optical fibres

See the solution of Q. No. 4 on page no. 94

Graded Index optical fibres

See the solution of Q. No. 4 on page no. 94

Applications of optical fibres

See the solution of Q. No. 1 on page no. 92

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7. **What is an optical fibre? How is it made? Write down the main differences between step index and graded index multimode optical fibres.** [T.U. 2065 Chaitra]

Solution:

Optical fibre

An optical fibre is a dielectric wave guide that transmits light signals from one place to another place. It consists of central core within which propagation of electromagnetic field is confirmed and which is surrounded by a cladding layer. The refractive index of the core is always greater than that of cladding layer. The cladding layer is surrounded by a thin layer of buffer coating or jacket. The core and cladding layer are made up of pure silica glass, whereas buffer coating is made of plastics. A cross sectional view of typical optical fibre is shown in figure.

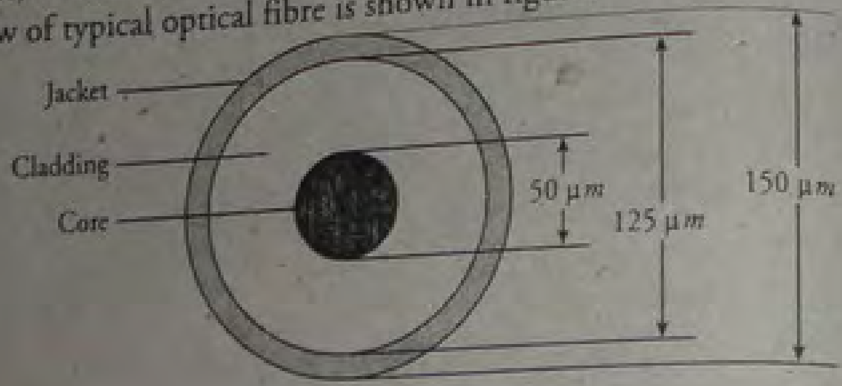

Figure: A cross sectional view of optical fibre

Differences between step index and graded index optical fibres

See the solution of Q. No. 4 on page no. 94

8. **What is an optical fibre? Explain the graded index optical fibre and also write the application of optical fibre in communication system as well as medical science.** **[T.U. 2065 Kartik]**

Solution:

Optical fibre

See the solution of Q. No. 7 on page no. 99

Graded Index optical fibres

See the solution of Q. No. 4 on page no. 94

Application in communications

Fibre optics essentially deals with communication including voice signals, video signals and digital data. This is done by transmission of light via optical fibres. An optical fibre system consists of three basic parts: a light source, an optical fibre and a light detector. A source may be light emitting diodes [LED's]. An optical fibre transmits light waves. The detector can be either an avalanche photo diode [APD] or positive intrinsic negative [PIN] diode. Basically, a fibre optic system converts an electrical signal to an infrared light signal. This signal is transmitted through an optical fibre. At the end of optical fibre, it is reconverted into an electrical signal.

Medical applications

The optical fibres are used in fabrication of fiberscope in endoscopy in medical sciences. Such fiberscopes are used in visualization of internal organs of human body. Optical fibres in combination of laser probe help to visualization of internal portion of human body and cauterization of tissues.

9. **Define acceptance angle of an optical fibre. Derive the relation for Numerical Aperture (NA) of the optical fibre. Also write down its significance.** **[T.U. 2067 Ashadh]**

Solution:

Acceptance angle of an optical fibre

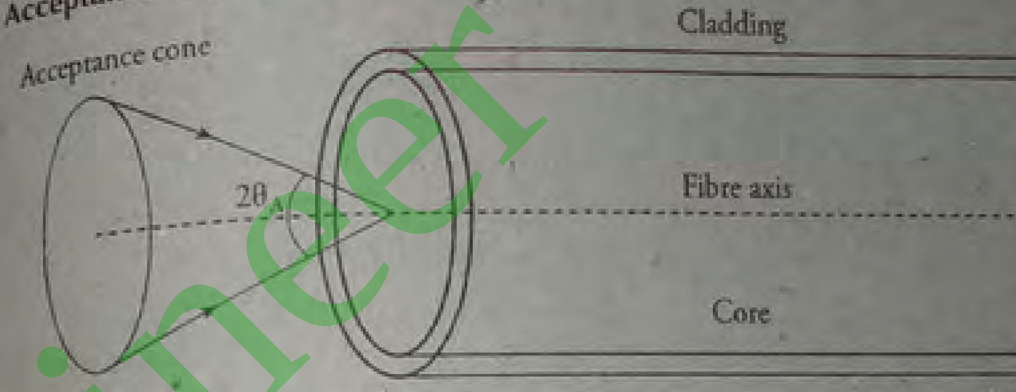

Figure: Acceptance angle and acceptance cone

The light signal rays that enter optical fibre within the angle $2\theta_A$ will be accepted by the fibre. The angle $2\theta_A$ is called acceptance angle. In 3D, it is an acceptance cone with semi–vertical angle θ_A. An accepatace cone is illustrated in figure.Light signals aimed at the optical fibre within this cone will be accepted and propagated to the far end. The larger the acceptance cone, easier the launching of signals into the optical fibre. It may be noted that the numerical aperture and acceptance angle are independent to optical fibre dimensions.

Numerical aperture (NA) of the optical fibre

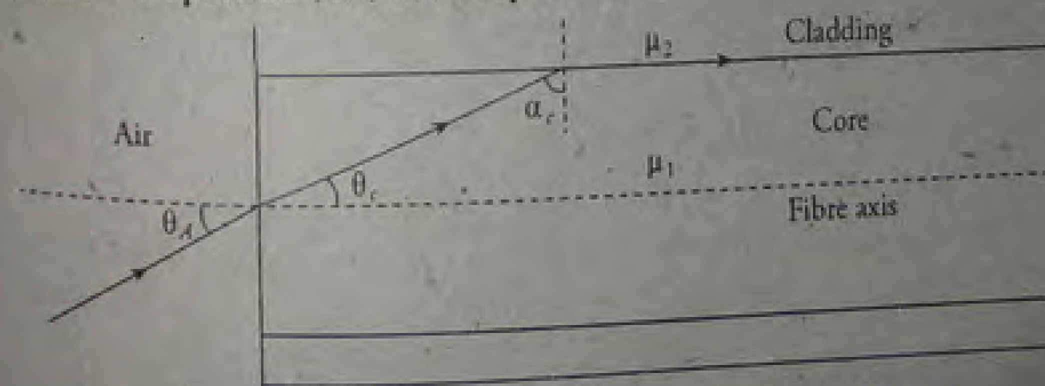

Figure: Propagation of a signal in a step index optical fibre

Consider a light signal that enters the fibre at angle θ_A with respect to fibre axis and strikes on core–cladding interface at angle θ_c as shown in the figure. According to Snell's law, we have,

$$\sin\alpha_c = \frac{\mu_2}{\mu_1}$$

Since,

$$\sin\alpha_c = \cos(90^\circ - \alpha_c) = \cos\theta_c$$

We obtain,

$$\cos\theta_c = \frac{\mu_2}{\mu_1}$$

or, $\sin\theta_c = \sqrt{1-\cos^2\theta_c} = \sqrt{1-\left(\frac{\mu_2}{\mu_1}\right)^2}$

If signal is lunched from air and μ_0 is the refractive index of air, the

$$\frac{\sin\theta_A}{\sin\theta_c} = \frac{\mu_1}{\mu_0}$$

Therefore,

$$\mu_0 \sin\theta_A = \mu_1 \sin\theta_c = \mu_1\sqrt{1-\left(\frac{\mu_2}{\mu_1}\right)^2} = \sqrt{\mu_1^2-\mu_2^2}$$

The term $\mu_0 \sin\theta_A$ is defined as numerical aperture (NA) of optical fibre. As $\mu_0 \approx 1$,

$$NA = \sin\theta_A = \sqrt{\mu_1^2-\mu_2^2}$$

Numerical aperture is a dimension less quantity. Its value ranges from 0.14 to 0.50. It is a measure of light signal gathering power of the fibre.

Furthermore, light signal propagation in an optical fibre associated with the parameter *"relative refractive index difference"* as well. It is defined by;

Relative refractive index difference,

$$\Delta = \frac{\mu_1-\mu_2}{\mu_1} = \frac{\mu_1^2-\mu_2^2}{\mu_1(\mu_1+\mu_2)} \cong \frac{(NA)^2}{2\mu_1^2} = \frac{(NA)^2}{2\mu_{core}^2}.$$

$$(\because \mu_1+\mu_2 \cong 2\mu_1)$$

Therefore,

$$NA = \mu_{core}\sqrt{2\Delta}$$

Numerical aperture is a measure of light signal gathering power of the fibre.

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10. **What is an optical fibre? Discuss its types. Derive the relation for numerical aperture (NA) in an optical fibre. [T.U. 2067 Mangsir]**

Solution:

Optical fibre

See the solution of Q. No. 7 on page no. 99

Types of optical fibre

See the solution of Q. No. 4 on page no. 94

Numerical aperture (NA) of the optical fibre

See the solution of Q. No. 9 on page no. 101

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11. **Write down the principle of optical fibre and show that numerical aperture (NA) = $\mu_{core}\sqrt{2\Delta}$, where symbols have their own meaning. [T.U. 2068 Shrawan]**

Solution:

Principle of optical fibre

See the solution of Q. No. 4 on page no. 94

At critical angle,

$$\mu_1 \sin C = \mu_2 \sin 90^\circ$$

where, μ_1 and μ_2, ($\mu_1 > \mu_2$) are refractive indices of denser and rarer media.

or, $\sin C = \frac{\mu_2}{\mu_1}$

$\therefore C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

This equation gives the value of critical angle.

Numerical aperture (NA) of the optical fibre

See the solution of Q. No. 9 on page no. 101

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12. **What is an optical fibre? Explain the type of physics behind its functioning. What are their types and explain the application of optical fibre in communications. [P.U. 2005]**

Solution:

Optical fibre

See the solution of Q. No. 7 on page no. 99

Principle of optical fibre

See the solution of Q. No. 4 on page no. 94

Physics of its functioning

Optical fibers are constructed to fulfill the requirements of particular purposes. The major requirement is in the area of bandwidth, *i.e.*, low medium, high medium or ultra high medium. In addition, a fibre may be constructed for its extremely low attenuation per kilometer. In some cases, tensile strength is the most desired characteristic. The optical fibres are classified into three major categories on the basis of *material, number of modes and refractive index profile.*

Types of optical fibre

See the solution of Q. No. 4 on page no. 94

Application in communications

See the solution of Q. No. 8 on page no. 100

13. What is optical fibre? Discuss its working principle. [P.U. 2011]

Solution: See the solution of Q. No. 1 and 4 on page no. 92 and 94

5.8 ADDITIONAL SOLVED PROBLEMS

1. A step index optical fibre has core index 1.43 and cladding index 1.4. Calculate critical angle, critical propagation angle and numerical aperture of the optical fibre.

Solution:

Here,

Refractive index of core, $(\mu_1) = 1.43$

Refractive index of cladding, $(\mu_2) = 1.4$

Now,

Critical angle,

$$C = sin^{-1}\left(\frac{\mu_2}{\mu_1}\right) = sin^{-1}\left(\frac{1.4}{1.43}\right) = 78.24°$$

Critical propagation angle,

$$\theta_c = cos^{-1}\left(\frac{\mu_2}{\mu_1}\right) = cos^{-1}\left(\frac{1.4}{1.43}\right) = 11.76$$

Numerical aperture,

$$NA = \mu_0 \sin\theta_A = \sqrt{\mu_1^2 - \mu_2^2} = \sqrt{(1.43)^2 - (1.4)^2} = 0.29$$

2. A step index fibre has a core of refractive index 1.55 and a cladding of refractive index 1.53 if the signal is lunched from a medium of refractive index 1.3. Find the numerical aperture and acceptance angle.

Solution:

Here,

Refractive index of core, $(\mu_1) = 1.43$

Refractive index of cladding, $(\mu_2) = 1.4$

Now,

Numerical aperture,

$$NA = \mu_0 \sin\theta_A = 1.3 \sin\theta_A = \sqrt{\mu_1^2 - \mu_2^2}$$

$$= \sqrt{(1.55)^2 - (1.53)^2} = 0.25$$

$$\sin\theta_A = \frac{0.25}{1.3} = 0.19$$

$$\therefore \theta_A = sin^{-1}(0.19) = 10.95°$$

Hence,

Acceptance angle, $2\theta_A = 21.90$

3. The refractive of indices of core and cladding of an optical fibre are 1.45 and 1.4 respectively. Calculate the numerical aperture, acceptance angle and Δ.

Solution:

Numerical aperture,

$$NA = \mu_0 \sin\theta_A = \sqrt{\mu_1^2 - \mu_2^2} = \sqrt{(1.45)^2 - (1.4)^2}$$

$$= 0.38$$

or, $1 \times \sin\theta_A = 0.38$

$\therefore \theta_A = 22.33°$

Acceptance angle,

$2\theta_A = 44.66°$

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = \frac{1.45 - 1.40}{1.45} = 0.034$$

4. Compute the numerical aperture and the acceptance angle of an optical fibre form the following data:

μ_1 (core) = 1.55 and μ_2 (cladding) = 1.5

Solution:

We have,

Numerical aperture,

$$NA = \mu_0 \sin\theta_A$$

$$= \sqrt{\mu_1^2 - \mu_2^2}$$

$$= \sqrt{(1.55)^2 - (1.50)^2}$$

$$= 0.39$$

$$\therefore \theta_A = sin^{-1}(0.39)$$

$$= 22.36°$$

Hence,

Acceptance angle,

$$2\theta_A = 2 \times 22.36°$$

$$= 45.92°$$

5. Compute the NA, acceptance angle and critical angle of optical fibre having refractive index of core $\mu_1 = 1.5$ and refractive index of cladding $\mu_1 = 1.45$.

Solution: Proceed as solution of Q. No. 4 on page no. 105

Chapter 6

INTERFERENCE

6.1 PRINCIPAL OF SUPERPOSITION

The principal of superposition states that the resultant displacement at any point and any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual wave trains if each were present alone. In the case of light wave, by displacement we mean the magnitude of electric field or magnetic field intensity.

6.2 SUPERPOSITION OF WAVES

Superposition of waves of equal phase and frequency

Let us assume that two sinusoidal waves of the same frequency are travelling together in a medium. The waves have the same phase, without any phase angle difference between them. Then the crest of one wave falls exactly on the crest of the other wave and so do the troughs. The resultant amplitude is got by adding the amplitudes of each wave point by point. The resultant amplitude is the sum of the individual amplitudes.

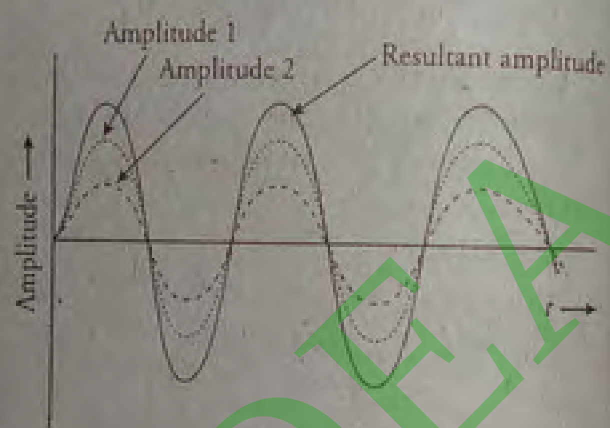

Figure: Superposition of waves of equal phase and frequency

i.e., $A = A_2 + A_2 + \dots\dots$

The resultant intensity is the square of the sum of the amplitudes

$$I = (A_1 + A_2 + A_3 + \dots\dots)^2 \qquad 6.1$$

Superposition of waves of constant phase difference

Let us consider two waves that have the same frequency but have certain constant phase angle difference between them. The two waves have a certain differential phase angle ϕ. In this case the crest of one

wave does not exactly coincide with the crest of the other wave. The resultant amplitude and intensity can be obtained by trigonometry.

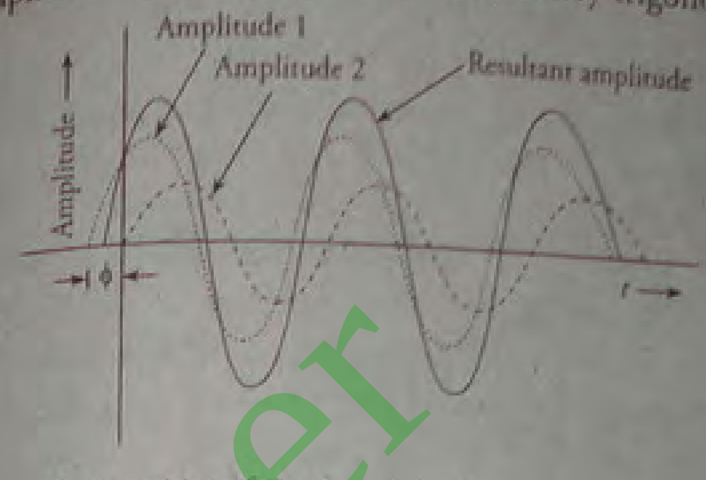

Figure: Superposition of two sine waves of constant phase difference

The two waves having the same frequency ($\omega = 2\pi f$) and a constant phase difference (ϕ) can be represented by the equations.

$$Y_1 = a \sin \omega t$$
$$Y_2 = b \sin(\omega t + \phi) \qquad 6.2$$

where, ϕ is the constant phase difference, a, b are the amplitudes and ω is the angular frequency of the waves

The resultant amplitude Y is given by;

$$\begin{aligned} Y = Y_1 + Y_2 &= a \sin \omega t + b \sin(\omega t + \phi) \\ &= a \sin \omega t + b(\sin \omega t \cos \phi + \cos \omega t \sin \phi) \\ &= a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi \\ &= (a + b \cos \phi) \sin \omega t + b \cos \omega t \sin \phi \end{aligned} \qquad 6.3$$

If R is the amplitude of the resultant wave and θ is the phase angle then,

$$\begin{aligned} Y &= R \sin(\omega t + \phi) \\ &= R(\sin \omega t \cos \theta + \cos \omega t \sin \theta) \\ &= R \cos \theta \sin \omega t + R \sin \theta \cos \omega t \end{aligned} \qquad 6.4$$

Comparing equations (1.3) and (1.4); we obtain,

$$R \cos \theta = a + b \cos \phi$$
$$R \sin \theta = b \sin \phi$$

or, $$R^2 = a^2 + b^2 + 2ab \cos \phi$$

$$\theta = \tan^{-1} \frac{b \sin \phi}{a + b \cos \phi} \qquad 6.5$$

Clearly, R is maximum when $\phi = 2n\pi$

and is minimum when $\phi = (2n + 1)\pi$

where, $n = 0, 1, 2, 3, \dots\dots$

When, ϕ is an even multiple of π we say that waves are in phase and when ϕ is an odd multiple of π, the waves are out of phase.

When the amplitude of waves are equal to a say, then,

$$I = 2a^2(1 + \cos\phi) = 4a^2 \cos^2\frac{\phi}{2} \qquad 6.6$$

A plot of I versus ϕ is shown in figure. Clearly, this reveals that the light distribution from the superposition of waves will consist of alternately bright and dark bands called interference fringes. Such fringes can be observed visually if projected on a screen or recorded photo-electrically. In the above discussion we have not considered travelling waves (*i.e.*, waves in which displacement is also a function of distance). If λ is the wavelength, then the change of phase that occurs over a distance λ is 2π. Thus, if the difference in phase between two waves arriving at a point is 2π, then difference in the path travelled by these waves is λ. Let the phase difference of two waves arriving at a point be δ and the corresponding path difference be x. For a path difference of λ, the phase difference $= 2\pi$. Therefore, for a path difference of x.

$$\text{Phase difference} = \delta = \frac{2\pi}{\lambda} \times x$$

$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

and

$$\text{Path difference} = x$$

$$= \frac{\lambda}{2\pi} \times \text{phase difference}$$

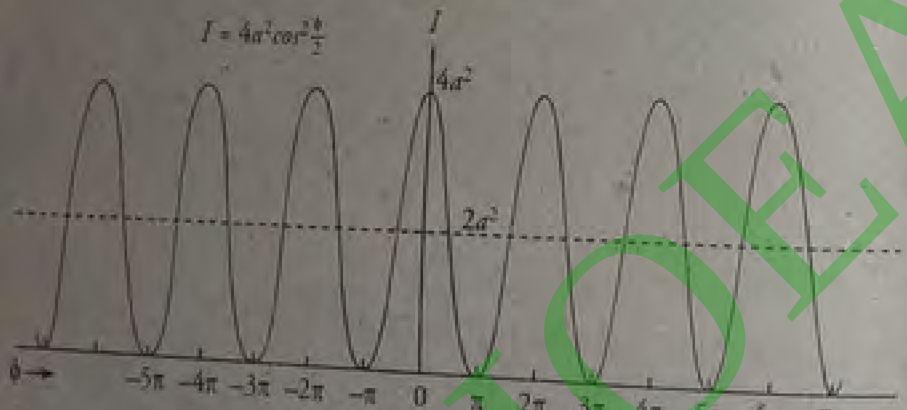

Figure: Intensity distribution for the interference fringes from two waves of same frequency and amplitude

6.3 SUPERPOSITION OF WAVES OF DIFFERENT FREQUENCIES

So far we have assumed that the waves have the same frequency. But light is never truly monochromatic. Many light sources emit quasimonochromatic light *i.e.*, light emitted will be predominantly of one frequency but will still contain other ranges of frequencies. When waves of different frequencies are superimposed, the result is more complicated.

6.4 SUPERPOSITION OF WAVES OF RANDOM PHASE DIFFERENCES

When waves having random phase differences between them superimpose, no discernible interference pattern is produced. The resultant intensity is got by adding the square of the individual amplitudes.

i.e.,
$$I = \sum_{i=1}^{N} A_i^2 = A_1^2 + A_2^2 + A_3^2 + \dots\dots \qquad 6.7$$

6.5 YOUNG'S DOUBLE SLIT EXPERIMENT

We have seen in the previous section that two waves with a constant phase difference will produce an interference pattern. Let us see how it can be realized in practice. Let us used two conventional light sources (like two sodium lamps) illuminating two pin holes as shown in figure. Then we will find that no interference pattern is observed on the screen. This can be understood from the following reasoning. In a conventional light source, light comes from a large number of independent atoms each atom emitting light for about 10^{-9} seconds *i.e.*, light emitted by an atom, is essentially a pulse lasting for only 10^{-9} seconds. Even if the atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently, light coming out from the holes S_1 and S_2 will have a fixed phase relationship for a period of about 10^{-9} seconds. Hence, the interference pattern will keep on changing every billionth of a second. The human eye can notice intensity changes which last at least for a tenth of a second and hence we will observe a uniform intensity over the screen. However, if we have a camera whose time of shutter can be made less than 10^{-9} sec, then the film will record an interference pattern. We can summarize the above argument by noting that light beams from two independent sources do not have a fixed phase relationship over a prolonged time period and hence, do not produce any stationary interference pattern.

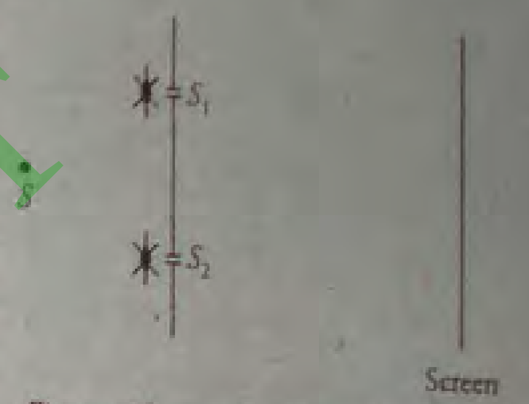

Figure: If two sodium lamps illuminate two pin holes S_1 and S_2 no interference pattern is observed on the screen

Thomas Young in 1802 devised an ingenious but simple method to lock the phase relationship between two sources. The trick lies in the division of a wave front into two. These two split wave fronts act as if they emanated from two sources having a fixed phase relationship and therefore, when these two waves were allowed to interfere, a stationary interference pattern was produced. In the actual experiment a light source illuminated a tiny pin hole S.

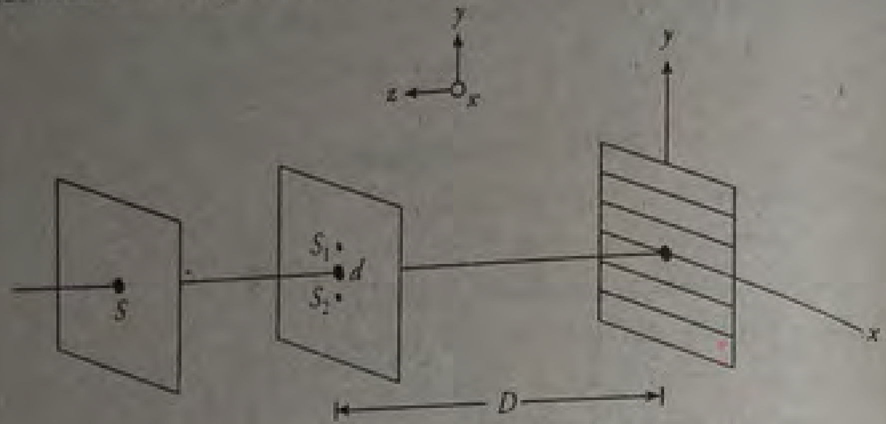

Figure: Young's arrangement to produce interference pattern

Light diverging from this pin hole fell on a barrier containing two rectangular apertures S_1 and S_2 which were very close to each other and were located equidistant from S. Spherical waves traveling from S_1 and S_2 were coherent and on the screen beautiful interference fringes could be obtained. In the centre screen, where the light waves from two slits have travelled through equal distances and where the path difference is zero, we have zeroth-order maximum. But maxima will occur whenever the path difference is one wavelength λ or an integral multiple of wavelength $n\lambda$. The integer n is called the order of interference.

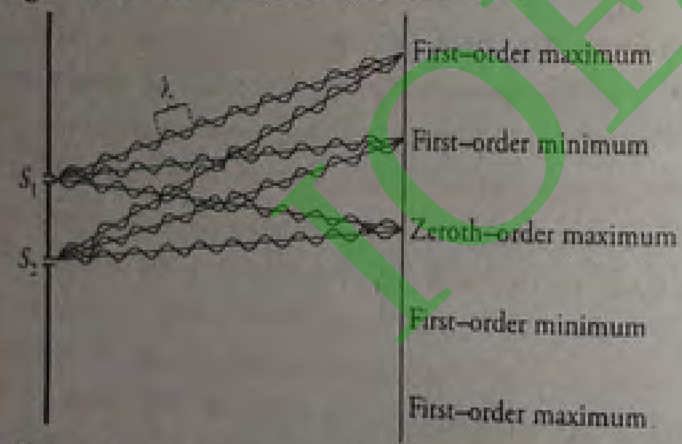

Figure: Maxima and minima in Young's double slit experiment

When the path difference is a multiple of $\left(n+\frac{1}{2}\right)\lambda$, we observe a dark fringe. In order to calculate the position of the maxima, we proceed as follows. Let d be the distance between the slits and D be the distance of the screen from the slits.

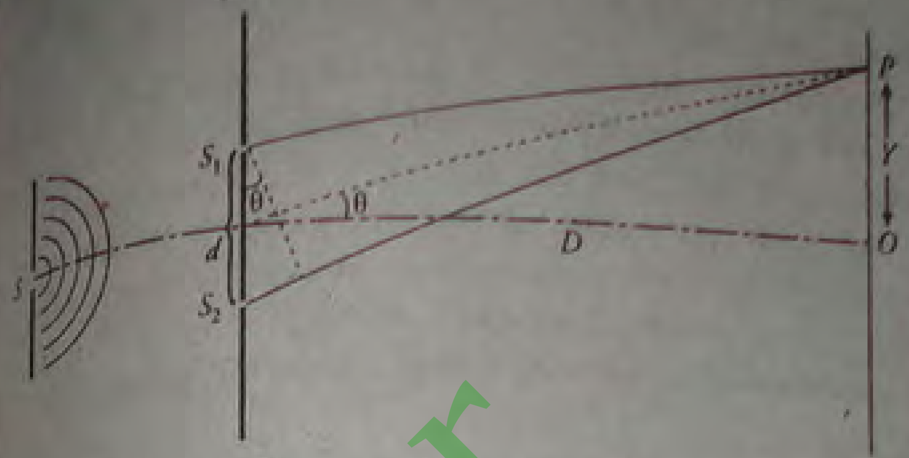

Figure: Path difference in Young's double slit experiment

Let P be the position of the maximum, then, the path difference between the two waves reaching P is;

$$d\sin\theta = n\lambda$$

or $$\sin\theta = \frac{n\lambda}{d};\ (n = 1, 2, 3, \ldots\ldots\ldots)$$

where, λ is the wavelength of light used and θ is the angle as shown in the figure. If Y is the distance of the point P from O, the centre of the screen, then we have,

$$Y = D\tan\theta$$

For small angles of θ,

$$Y = D\tan\theta = D\sin\theta$$

or $$Y = \frac{Dn\lambda}{d} = D\sin\theta$$

or $$\lambda = \frac{DY}{Dn} \qquad 6.8$$

Clearly, fringe width $= Y_{n+1} - Y_n = \beta = \frac{D\lambda}{d}$ 6.9

Hence, by measuring the distance between slits, the distance to the screen and the distance from the central fringe to some fringe on either side, the wavelength of light producing the interference pattern may be determined.

6.5 COHERENCE

An important concept associated with the idea of interference is coherence. Coherence means that two or more electromagnetic waves are in a fixed and predictable phase relationship to each other. In general the phase between two electromagnetic waves can vary from point to point (in space) or change from instant to instant (in time). There are thus two independent concepts of coherence namely temporal coherence and spatial coherence.

Temporal coherence

This type of coherence refers to the correlation between the field a... point and the field at the same point at a later time *i.e.*, the relati... between $E(x,y,z,t_1)$ and $E(x,y,z,t_2)$. If the phase difference betwe... the two fields is constant during the period normally covered... observations, the wave is said to have temporal coherence. If the pha... difference change many times and in an irregular way during the short... period of observation, the wave is said to be non coherent.

Spatial coherence

The waves at different points in space are said to be space coherent... they preserve a constant phase difference over any time t. This is possib... even when two beams are individually time incoherent, as long as a... phase change in one of the beams is accompanied by a simultaneo... equal phase change in the other beam (this is what happens in Youn... double slit experiment). With the ordinary light sources, this is possib... only if the two beams have been produced in the same part of the sourc... Time coherence is a characteristic of a single beam of light whereas spa... coherence concerns the relationship between two separate beams of ligh... Interference is a manifestation of coherence.

Light waves come in the form of wave trains because light is produc... during deexcitation of electrons in atoms. These wave trains are of fin... length. Each wave train contains only a limited number of waves. Th... length of the wave train Δs is called the *coherence length*. It is the produ... of the number of waves N contained in wave train and their wavelength... *i.e.*, $Ns = N\lambda$. Since velocity is defined as the distance travelled per un... of time, it takes a wave train of length Δs, as a certain length of time 'Δt'... to pass a given point.

$$\Delta t = \frac{\Delta s}{c}$$

where, c is the velocity of light

The length of time Δt is called the *coherence time*. The degree of tempor... coherence can be measured using a Michelson's interferometer.

It is clear from the above discussion that the important condition fo... observing interference is that the two sources should be coherent. Th... observations of interference are facilitated by reducing the separatio... between the sources of light producing interference. Further, in th... Young's double slit experiment the distance between two sources and th... screen should be large. The contrast between the bright and dark fring...

...improved by making equal the amplitudes of the light sources ...ducing interference. Further, the sources must be narrow and ...nochromatic. The concept of coherence is discussed in greater detail ...the chapter on lasers.

TYPES OF INTERFERENCE

The phenomenon of interference is divided into two classes depending ...the mode of production of interference. They are:

...Interference produced by the division of wavefront and

...Interference produced by the division of amplitude

...the first case the incident wavefront is divided into two parts by ...making use of the phenomenon of reflection, refraction or diffraction. The two parts of the wavefront travel unequal distances and reunite to ...produce interference fringes. Young's double slit experiment is a classic example for this. In Young's double slit experiment one uses two narrow ...to isolate beams from separate portions of the primary wavefront. In ...second case the amplitude of the incident light is divided into two ...parts either by parallel reflection or refraction. These light waves with ...divided amplitude reinforce after travelling different distances and ...produce interference. Newton's rings are an example for this type.

INTERFERENCE IN THIN FILMS

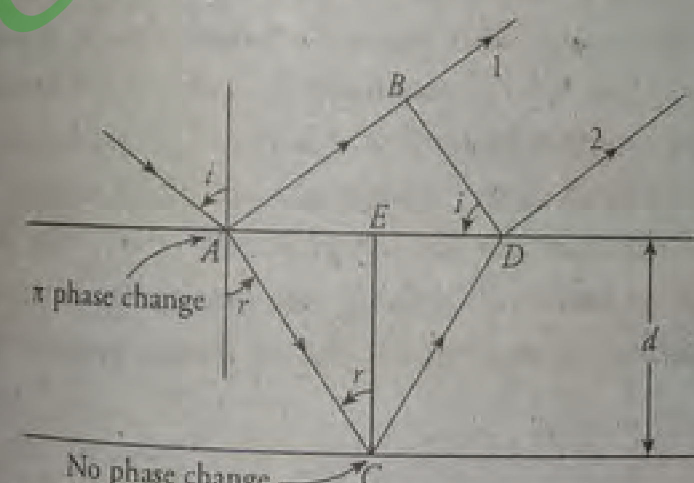

Figure: Interference in plane parallel films due to reflected light

...colours of thin films, soap bubbles and oil slicks can be explained as ...due to the phenomena of interference. In all these examples, the ...formation of interference pattern is by the division of amplitude. For ...example, if a plane wave falls on a thin film then the wave reflected from ...the upper surface interferes with the wave reflected from the lower ...surface. Such studies have many practical applications as provided by the ...example of production of non–reflecting coatings.

Interference in plane parallel films due to reflected light

Let us consider a plane parallel film as shown in the figure. Let light be incident at A. Part of the light is reflected toward B and the other part is refracted into the film towards C. This second part is reflected at C and emerges at D, and is parallel to the first part. At normal incidence, the path difference between rays 1 and 2 is twice the optical thickness of the film.

Path difference $= 2\mu d$

At oblique incidence the path difference is given by;

$$\text{Path difference} = \mu(AC + AD) - AB$$

$$= \frac{2\mu d}{\cos r} - AB$$

$$= \frac{2\mu d}{\cos r} - 2\mu d \tan r \cdot \sin r$$

Since $AB = AD \sin i = 2AE.\ \sin i = 2d \tan r \cdot \sin i = 2d \tan r \cdot \mu \cdot \sin r$

i.e., $$\text{Path difference} = 2\mu d\left\{\frac{1}{\cos r} - \tan r \cdot \sin r\right\}$$

$$= 2\mu d\left\{\frac{1 - \sin^2 r}{\cos r}\right\} = 2\mu d \cdot \cos r$$

where, μ is the refractive index of the medium between the surfaces

Since for air; $\mu = 1$, the path difference between rays 1 and 2 is given by;

Path difference $= 2d \cos r$

While calculating the path difference, the phase change that might occur during reflection has to be taken into account. Whenever light is reflected from an interface beyond which the medium has lower index of refraction, the reflected wave undergoes no phase change. When the medium beyond the interface has a higher refractive index there is phase change of π. The transmitted waves do not experience any phase change. Hence, the condition for maxima for the air film to appear bright is;

$$2\mu d \cos r + \frac{\lambda}{2} = n\lambda$$

or, $$2\mu d \cos r = n\lambda - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

where, $n = 1, 2, 3, \ldots\ldots\ldots$

The film will appear dark in the reflected light when

$$2\mu d \cos r + \frac{\lambda}{2} = (2n-1)n\lambda$$

or, $$2\mu d \cos r = n\lambda$$

where, $n = 1, 2, 3, \ldots\ldots\ldots$

Interference in plane parallel films due to transmitted light

Figure illustrates the geometry for observing interference in plane parallel films due to transmitted light. We have two transmitted rays CT and EU which are derived from the same point source and hence, are in a position to interfere. The effective path difference between these two rays is given by;

$$\text{Path difference} = \mu(CD + DE) - CP$$

But $$\mu = \frac{\sin i}{\sin r} = \frac{\frac{CP}{DE}}{\frac{QE}{CE}} = \frac{CP}{QE} \qquad [\because CP = \mu(QE)]$$

or, $$\text{Path difference} = \mu(CD + DQ + QE) - \mu(QE) = \mu(CD + DQ)$$

$$= \mu(ID + DQ) = \mu(QI) = 2\mu d \cos r$$

In this case it should be noted that, no phase change occurs when the rays are refracted unlike in the case of reflection. Hence, the condition for maxima is $2\mu d \cos r = n\lambda$ and the condition for minima is $2\mu d \cos r = (2n-1)n\lambda$.

Thus, the conditions of maxima and minima in transmitted light are just the reverse of the condition for reflected light.

Interference in wedge shaped film

Let us consider two plane surfaces GH and G_1H_1 inclined at an angle α and enclosing a wedge shaped film. The thickness of the film increases from G to H as shown in the figure. Let μ be the refractive index of the material of the film. When this film is illuminated there is interference between two systems of rays, one reflected from the front surface and the other obtained by internal reflection at the back surface.

The path difference is given by;

$$\text{Path difference} = \mu(BC + CD) - BF$$

$$\text{Path difference} = \mu(BE + EC + CD) - \mu BE$$

$$\left[\because \sin i = \frac{BF}{BD};\ \sin r = \frac{BE}{BD};\ \mu = \frac{\sin i}{\sin r} \Rightarrow \mu = \frac{BF}{BE}\right]$$

$$\text{Path difference} = \mu(EC + CD) = \mu(EC + CP) = \mu EP$$

$$= 2\mu d \cos(r + \alpha)$$

Due to reflection an additional phase difference of $\frac{\lambda}{2}$ is introduced.

Hence,

$$\text{Path difference} = 2\mu d \cos(r + \alpha) + \frac{\lambda}{2}$$

For constructive interference

$$2\mu d \cos(r + \alpha) + \frac{\lambda}{2} = n\lambda$$

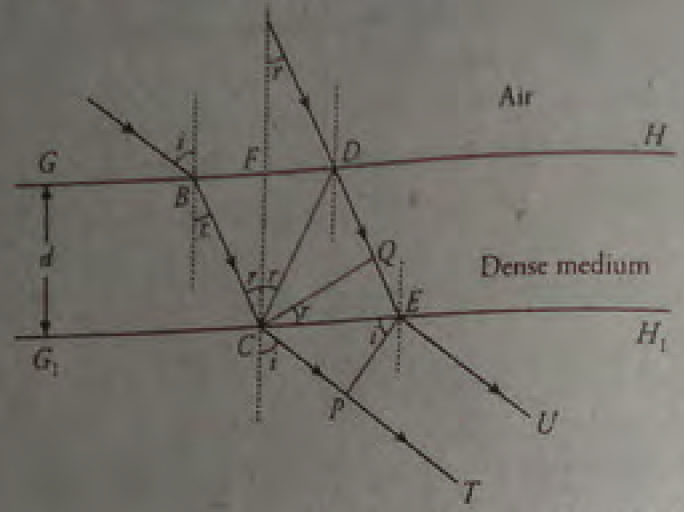

Figure: Interference in plane parallel films due to transmitted light

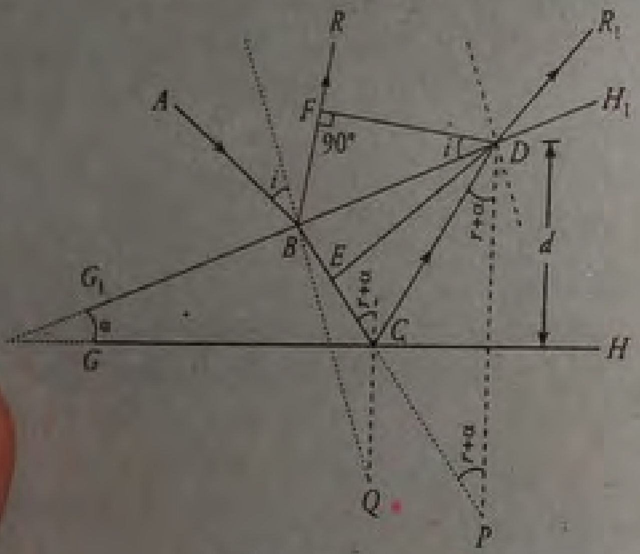

Figure: Interference in a wedge shaped film

or, $2\mu d\cos(r+\alpha) \quad = (2n-1)\frac{\lambda}{2}$

where, $n = 1, 2, 3, \ldots\ldots\ldots$

For destructive interference

$$2\mu d\cos(r+\alpha)+\frac{\lambda}{2} = (2n+1)n\lambda$$

or, $2\mu d\cos(r+\alpha) \quad = n\lambda$

where, $n = 0, 1, 2, 3, \ldots\ldots\ldots$

Spacing between two consecutive bright bands is obtained as follows.

For n^{th} maxima;

$$2\mu d\cos(r+\alpha) = (2n-1)\frac{\lambda}{2}$$

Let this band be obtained at a distance X_n from thin edge as shown in figure. For near normal incidence, $r = 0$. Assuming, $\mu = 1$,

From the figure,

$d \quad = X_n \tan\alpha$

$2X_n \tan\alpha\cos\alpha \quad = (2n-1)\frac{\lambda}{2}$

or, $2X_n \sin\alpha \quad = (2n-1)$

For $(n+1)^{th}$ maxima;

$2X_{n+1}\sin\alpha \quad = (2n+1)\frac{\lambda}{2}$

$2(X_{n+1}-X_n)\sin\alpha \quad = \lambda$

Fringe spacing;

$$\beta = X_{n+1}-X_n = \frac{\lambda}{2\sin\alpha} = \frac{\lambda}{2\alpha}$$

where, α is small and measured in radians

6.9 COLOURS OF THIN FILMS

The discussion of the interference due to parallel film and at a wedge should now enable us to understand as to why films appear coloured. To summarize, the incident light is split up by reflection at the top and bottom of the film. The split rays are in a position to interfere and interference of these rays is responsible for colours. Since the interference condition is a function of thickness of the film, the wavelength and the angle of refraction, different colours are observed at different positions of the eye. The colours for which the condition of maxima will be satisfied will be seen and others will be absent. It should be noted here that the conditions for maxima and minima in transmitted light are opposite to that of reflected light. Hence, the colours that are absent in reflected light will be present in transmitted light. The colours observed in transmitted and reflected light are complimentary.

6.10 NEWTON'S RINGS

When a plano–convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. If monochromatic light is allowed to fall normally and viewed as shown in the figure, then alternate dark and bright circular fringes are observed. The fringes are circular because the air film has a circular symmetry. Newton's rings are formed because of the interference between the waves reflected from the top and bottom surfaces of the air film formed between the plates as shown in the figure.

The path difference between these rays (*i.e.*, rays 1 and 2) is;

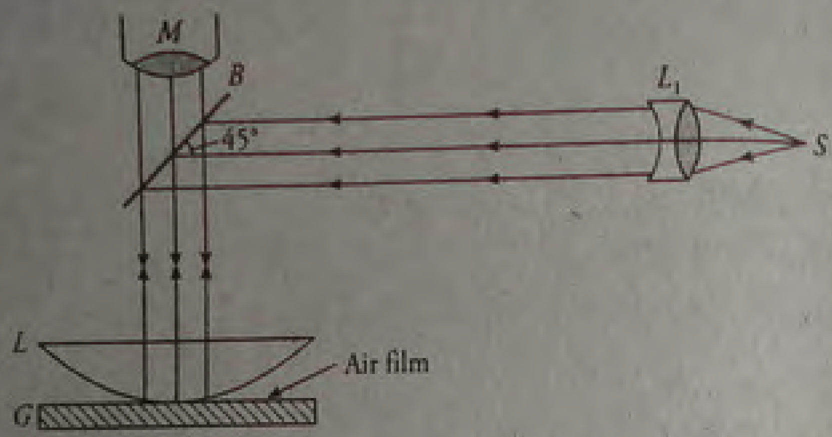

Figure: Experimental set up for viewing Newton's rings

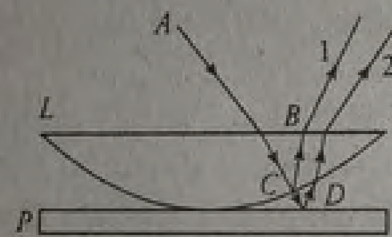

Figure: Interference in Newton's rings setup

$$2\mu d \cos r + \frac{\lambda}{2}$$

Since $r = 0, \mu = 1$

i.e., Path difference $= 2d + \frac{\lambda}{2}$

At the point of contact $d = 0$, the path difference is $\frac{\lambda}{2}$. Hence, the cent spot is dark. The condition for bright fringe is;

$$2d + \frac{\lambda}{2} = n\lambda$$

or, $\quad 2d = \frac{(2n-1)\lambda}{2}$

where, $n = 1, 2, 3, \ldots\ldots$

and the condition for dark fringe is;

$$2d + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or, $\quad 2d = n\lambda$

where, $n = 0, 1, 2, 3, \ldots\ldots$

Now, let us calculate the diameters of these fringes. Let LOL' be the le placed on the glass plate AB. The curved surface LOL' is part of a spherical surface with the centre at C. Let R be the radius of curvatu and r be the radius of Newton's ring corresponding to constant fi thickness d.

From the property of the circle,

i.e., $NP \times NQ = NO \times ND$

i.e., $r \times r = d(2R - d)$

$= 2Rd - d^2$

i.e., $r^2 = 2Rd$

or, $d = \frac{r^2}{2R}$

Thus, for a bright fringe

$$\frac{2r^2}{2R} = \frac{(2n-1)\lambda}{2}$$

or, $r^2 = \frac{(2n-1)\lambda R}{2}$

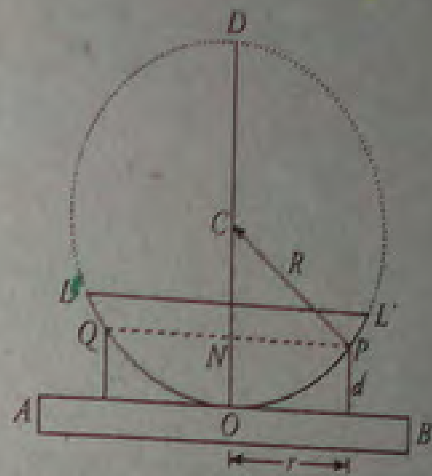

Figure: Calculation of diameter of Newton's ring

Replacing r by $\frac{D}{2}$ where D is the diameter, we get,

$$D_n = \sqrt{2\lambda R}\sqrt{2n-1}$$

Similarly, for a dark fringe,

$$\frac{2r^2}{2R} = n\lambda$$

or, $r^2 = n\lambda R$

or, $D_n^2 = 4n\lambda R$

or, $D_n = 2\sqrt{n\lambda R}$

Thus, the diameters of the rings are proportional to the square roots of the natural numbers.

By measuring the diameter of the Newton's rings, it is possible to calculate the wavelength of light as follows. We have for the diameter of the n^{th} dark fringe.

$$D_n^2 = 4n\lambda R$$

Similarly diameter for the $(n+p)^{th}$ dark fringe;

$$D^2 = 4(n+p)\lambda R$$

or, $D_{n+p}^2 - D_n^2 = 4pR\lambda$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

λ can be calculated using this formula.

Newton's rings set up could also be used to determine the refractive index of a liquid. First the experiment is performed when there is air film between the lens and the glass plate. The diameters of the n^{th} and $(n+p)^{th}$ fringes are determined. Then, we have,

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

Now, the liquid whose refractive index is to be determined is poured into the container without disturbing the entire arrangement. Again the diameter of the n^{th} and $(n+p)^{th}$ dark fringes are determined. Again, we have,

$$D'^2_{n+p} - D'^2_n = \frac{4p\lambda R}{\mu}$$

From the above equations;

$$\mu = \frac{D^2_{n+p} - D^2_n}{D'^2_{n+p} - D'^2_n}$$

6.11 SOLVED EXAM QUESTIONS

1. **What is the cause of colored image produce by the reflected light form thin films? Does it have anything to do with the colorful pattern of rainbow?** **[T.U. 2061 Baishakh]**

Solution:

When white light is incident on a thin film, the light which comes from any point from it will not include the colour, whose wavelength satisfies the relation,

$2\mu t \cos r = n\lambda$ in the reflected system.

Therefore, film will appear colored and color will depend upon the thickness and the angle of inclination. If r and t are constant, color will be uniform. In the case of oil on water, different colors are seen because r and t vary.

The colorful pattern of rainbow is not different from the above phenomenon. It's a dispersion of white light from the water droplets on the atmospheric air.

2. **A soap film of refractive index 1.33 is viewed at an angle of 35° to normal. It has thickness 5×10^{-7} cm. For what wavelength will the film be non-reflecting?** **[T.U. 2061 Baishakh]**

Solution:

Here,

Refractive index of soap film $(\mu) = 1.33$

Thickness of the film $(t) = 5 \times 10^{-7}$ cm

Let, i be an angle of incidence and r be an angle of refraction, then,

$$\mu = \frac{\sin i}{\sin r}$$

$$\text{or,}\quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 35°}{1.33} = 0.43$$

$$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.43)^2} = 0.90$$

Applying the relation;

$$2\mu t \cos r = n\lambda$$

For first order, $n = 1$;

$$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-7} \times 0.90 = 12 \times 10^{-7}\ m$$

This lies in the infrared (invisible) region.

For second order, $n = 2$;

$$\lambda_2 = 1.33 \times 5 \times 10^{-7} \times 0.90 = 6 \times 10^{-7}\ m$$

This lies in the visible region.

For third order, $n = 3$;

$$\lambda_3 = 2 \times 1.33 \times 5 \times 10^{-7} \times 0.30 = 4 \times 10^{-7}\ m$$

This lies in the visible region.

For fourth order, $n = 4$;

$$\lambda_4 = \frac{1}{2} \times 1.33 \times 5 \times 10^{-7} \times 0.90 = 3 \times 10^{-7}\ m$$

This lies in the ultraviolet (invisible) region.

Thus, for the wavelengths 6×10^{-7} m and 4×10^{-7} m, soap film is non reflecting.

3. **In the Newton's rings experiment the diameter of the tenth ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.** **[T.U. 2061 Ashwin]**

Solution:

We have,

For liquid medium, $(D_1^2) = \dfrac{4n\lambda R}{\mu}$

For air medium, $(D_2^2) = 4n\lambda R$

Dividing, we obtain,

$$\mu = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{1.4\ cm}{1.27\ cm}\right)^2 = 1.215$$

The refractive index of the liquid is 1.215.

4. **Derive the conditions of constructive and destructive interference using the mathematical theory for superposition of two waves of same frequency.** **[T.U. 2061 Ashwin]**

Solution:

Consider a narrow monochromatic source S and two pinholes A and B equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent sources. The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.

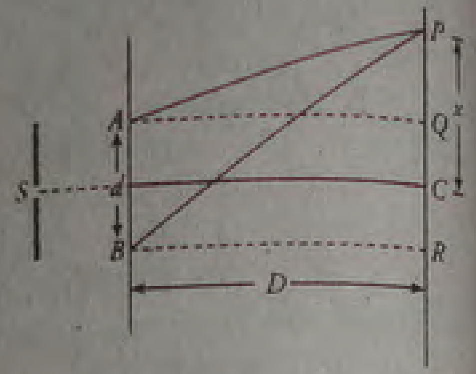

Figure: Interference fringes

Consider a point P at a distance x from C. The waves reach at the point P from A and B.

Here,

$$PQ = x - \frac{d}{2}$$

$$PR = x + \frac{d}{2}$$

Thus,

$$(BP)^2 - (AP)^2 = \left[D^2 - \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 - \left(x - \frac{d}{2}\right)^2\right]$$

$$= 2xd$$

$$\therefore \quad BP - AP = \frac{2xd}{BP + AP}$$

But,

$$BP \approx AP \approx D$$

Path difference $= BP - AP$

$$= \frac{xd}{D}$$

Phase difference $= \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$

If the path difference is a whole number multiple of wavelength of λ, the point P is bright, *i.e.*, Bright fringes appear. Therefore,

$$\frac{xd}{D} = n\lambda$$

where, $n = 0,1,2,3, \ldots\ldots\ldots$

$$\text{or,} \quad x = \frac{n\lambda D}{d} \quad \ldots \text{(i)}$$

This relation gives the distance of the bright fringes form the point C. At C, the path difference is zero and bright is fringe is formed.

When $n = 1$; $x_1 = \frac{\lambda D}{d}$

When $n = 2$; $x_2 = \frac{2\lambda D}{d}$

When $n = 3$; $x_3 = \frac{3\lambda D}{d}$

……… ……… ………

……… ……… ………

……… ……… ………

When $n = n$; $x_n = \frac{n\lambda D}{d}$

Thus, the distance between any two consecutive bright fringes is;

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$= \frac{\lambda D}{d} \quad \ldots \text{(ii)}$$

If the path difference is an odd multiple of half wavelength, the point P is dark, *i.e.*, dark fringes appear.

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$$

where, $n = 0,1,2,3, \ldots\ldots\ldots$

$$\text{or,} \quad x = \frac{(2n+1)\lambda D}{d} \quad \ldots \text{(iii)}$$

This relation gives the distances of dark fringes from the point C.

When $n = 1$; $x_1 = \frac{\lambda D}{2d}$

When $n = 2$; $x_2 = \frac{3\lambda D}{2d}$

When $n = 3$; $x_3 = \frac{5\lambda D}{2d}$

……… ……… ………

……… ……… ………

……… ……… ………

When $n = n$; $x_n = \frac{(2n+1)\lambda D}{2d}$

Thus, the distance between any two consecutive bright fringes is;

$$x_2 - x_1 = \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{d} \quad \ldots \text{(iv)}$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of C. All the fringes are of equal in width and are independent of the order of the fringe. However, the breadth of a bright or dark fringe is half of the fringe width.

5. What is interference of light? Derive the expression for the rad[ius] of dark and bright rings in Newton's ring experiment f[or] reflected light. [T.U. 2062 Baishakh]

Solution:

Interference of light

The phenomenon of interference is defined as the modification i[n] the distribution of light energy obtained by the superposition o[f] two or more light waves.

Newton's rings

Whenever a convex surface of a plano-convex lens is placed on a plane glass plate, a thin film is produced between two surfaces. The thickness of the air film so produced increases as we g[o] outwards from the point of contact. The loci of all the points corresponding to same thickness are circles concentric with the point of contact. The circular fringes so produced are known a[s] Newton's rings. The fringes produced with monochromatic ligh[t] are circular. The fringes are concentric circles with the point o[f] contact as the centre. When fringes are viewed with white light we observe colored fringes. With monochromatic light, bright a[nd] dark circular fringes are produced in the air film. Newton's rin[gs] can be formed due to reflected as well as transmitted light.

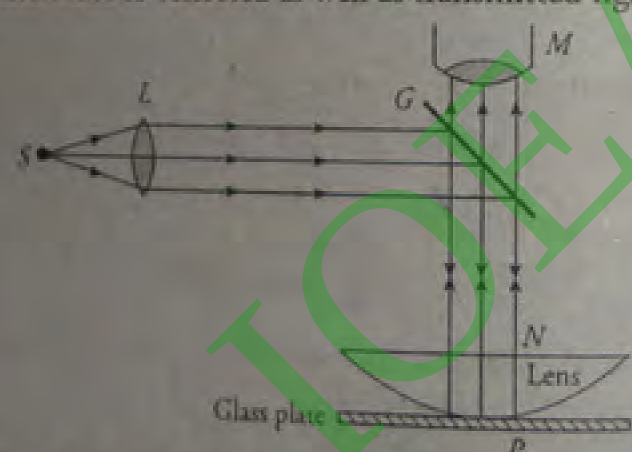

Figure: Experimental arrangement of Newton's ring experiment

Theory

A monochromatic light ray AB falls normally on air film at th[e] point B after passing through lens. Part of it is reflected fro[m] point B (rarer medium) on glass-air boundary and goes upwa[rd] along BF without phase reversal and partially refracts into air fil[m] along BC.

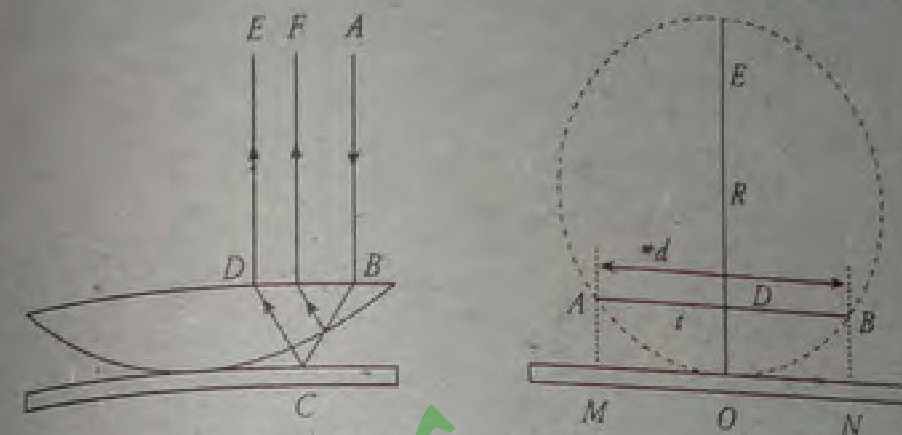

Figure: Newton's ring formation in reflected system and mathematical determination of Newton's ring diameter

At point C again reflection takes place on glass plate (denser medium). The reflected light goes along CDE with a phase difference of π or path difference $\frac{\lambda}{2}$. The two rays BF and CDE produce interference fringes depending upon their path difference. To obtain an expression for the diameter of a ring, consider the vertical section of AOB of plano-convex lens through the centre of curvature.

$$OD = t$$
$$AB = d$$

The radius of curvature of the lens is R. The lens is in contact with glass plate MON at O such that points A and B are equidistant from O. Complete the circle $AOBE$ and draw diameter. Draw BN and AM perpendicular to plane MN. The thickness of the air film will be zero at O, around O circular fringes will be obtained. The path differences between two rays are reflected from B and other reflected from C is $2\mu t \cos\gamma$. Since incidence is normal so angle of refraction is zero and refractive index of air, $\mu = 1$. Therefore,

Path difference $= 2t$

As one of the rays CDE suffers reflection on denser medium, an additional path difference $\frac{\lambda}{2}$ is introduced.

$\therefore$ Path difference $= 2t + \frac{\lambda}{2}$

The points A and B being equidistant form O will lie on bright ring of diameter AB if path difference is $n\lambda$.

i.e., $2t + \frac{\lambda}{2} = n\lambda$

$$2t = \frac{(2n-1)}{2}\lambda; \qquad n = 0, 1, 2, 3, \dots\dots$$

For dark rings,

$2t = n\lambda;$ $n = 0, 1, 2, 3, \dots\dots\dots$

From figure,

$AD \times DB = OD(2R - OD)$

Since,

$AD = DB = r$

$OD = BM = AM = t$

Thus,

$r^2 = t(2R - t)$

or, $r^2 = 2Rt$ $(\because 2R - t \approx 2R)$

$\therefore t = \frac{r^2}{2R}$

On substituting value of t for bright and dark rings, we obtain,

For bright rings,

$r^2 = \frac{(2n-1)\lambda R}{2}$

$\therefore r = \sqrt{\frac{(2n-1)\lambda R}{2}}$

For dark rings,

$r = \sqrt{n\lambda R}$

These relations are expressions for radii of bright and dark rings respectively.

---

6. An air wedge of angle 0.01 radians is illuminated by monochromatic light of wave length 600 nm falling normally on it. At what distance from the edge of the wedge, will the twentieth fringe be observed by reflected light. [T.U. 2063 Baishakh]

Solution:

Here,

Air wedge angle, $(\theta) = 0.01$ radians

Wavelength of monochromatic light, $(\lambda) = 600\ nm = 600 \times 10^{-9}\ m$

$(n) = 12$

We have,

$2t = n\lambda$

and $\theta = \frac{t}{x}$

or, $2\theta x = n\lambda$

$\therefore x = \frac{n\lambda}{2\theta} = \frac{12 \times 600 \times 10^{-9}}{2 \times 0.01} = 3 \times 10^{-3}\ m$

The 12^{th} fringe will be observed at the distance $3 \times 10^{-3}\ m$ form the edge of the wedge.

Newton's rings formed by sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of 40^{th} ring? [T.U. 2063 Baishakh]

Solution:

For dark rings, diameter of the ring is;

$D_n^2 = 4n\lambda R = (4 \times 40)\lambda R$ ... (i)

When the diameter of ring is doubled,

$D_2^2 = 4m\lambda R$

or, $(2D_n)^2 = 4m\lambda R$ ... (ii)

Dividing equation (i) by (ii), we obtain,

$\frac{1}{4} = \frac{40}{m}$

$\therefore m = 160$

The order of the required dark ring is 160.

How can we form the Newton's rings in reflected light? Show that the diameter of the dark rings are proportional to the square root of natural numbers and that of bright rings are proportional to square root of odd numbers in the reflected light. [T.U. 2064 Poush]

Solution:

Newton's rings

See the solution of Q. No. 5 on page no. 124

On substituting value of t for bright and dark rings, we obtain,

For bright rings,

$r^2 = \frac{(2n-1)\lambda R}{2}$

$\therefore r = \sqrt{\frac{(2n-1)\lambda R}{2}}$

or, $D = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$ ... (i)

For dark rings,

$r = \sqrt{n\lambda R}$

or, $D = 2\sqrt{n\lambda R}$ ... (ii)

Equations (i) and (ii) give the diameters of bright and dark rings respectively. The diameters of the dark rings are proportional to the square root of natural numbers and that of bright rings are proportional to square root of odd numbers in the reflected light.

9. **A screen containing two slits 0.1 mm apart is 1.20 m from t[he] viewing screen. Light of wavelength 500 nm, falls on the sl[its] from a distant source. Approximately how far apart will [the] interference fringes be on the screen?** [T.U. 2064 Pou[sh]]

Solution:

Here,

Slit separation, $(d) = 0.1\ mm = 0.1 \times 10^{-3} m$

Distance between slit and screen, $(D) = 1.20\ m$

Wavelength of light, $(\lambda) = 500\ nm = 500 \times 10^{-9} m$

Fringe width, $(\beta) = ?$

We have,

$$\beta = \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1.20}{0.1 \times 10^{-3}} = 6 \times 10^{-3}\ m$$

The interference fringes are $6 \times 10^{-3}\ m$ apart from each other.

---

10. **What are coherent sources? Show that the separation of t[wo] successive maxima or minima depends on the wavelength [of] light used, distance between the slit and the screen and the sl[it] width.** [T.U. 2065 Shraw[an]]

Solution:

Coherent sources

Two sources, which emit waves of the same frequency [or] wavelength, same amplitude and have zero phase different [or] constant phase difference at all the times are called *coherent sour[ces]*.

For the remaining part

See the solution of Q. No. 4 on page no. 121

The equations (iii) and (iv) shows that separation of two success[ive] maxima or minima depends on the wavelength of light us[ed,] distance between the slit and the screen and the slit width.

---

11. **What are Newton's rings? Determine the wavelength of light using Newton's rings method. Also explain the use [of] monochromatic ray of light in this method.** [T.U. 2065 Shraw[an]]

Solution:

Newton's rings

See the solution of Q. No. 5 on page no. 124

The figure depicts the experimental arrangement for determination of the wavelength of monochromatic light us[ing] Newton's rings. A monochromatic source of light S emits [li]ght

[r]ays that fall on lens L. The parallel beam of light from this lens is [r]eflected by the glass plate G inclined at an angle of 45° to the [ho]rizontal. A plano-convex lens is placed just below the glass plate [o]n the surface of another glass plate. Newton's rings are viewed [th]rough travelling microscope M focused on the air film. Circular [b]right and dark rings are seen with the central dark. With the help [o]f a travelling microscope, measure the diameter of the n^{th} dark [r]ing. Suppose the diameter of the n^{th} ring be D_n. Thus,

$$(D_n)^2 = 4n\lambda R \quad \ldots (i)$$

[M]easure the diameter of the $(n+m)^{th}$ dark ring. Its diameter is;

$$(D_{n+m})^2 = 4(n+m)\lambda R \quad \ldots (ii)$$

[S]ubtracting these equations we obtain,

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R$$

or, $$\lambda = \frac{(D_{m+n})^2 - (D_n)^2}{4mR} \quad \ldots (iii)$$

This relation determines the wavelength of monochromatic light. The radius of curvature of the lower surface of the lens is determined with the help of a spherometer but more accurately it can be determined by Boy's method. Hence the wavelength of a given monochromatic source of light can be determined.

Monochromatic light gives alternative bright and dark rings are formed. Diameter of such rings depends on the wavelength of light. No overlapping of alternative dark and bright rings takes place so that diameter can be accurately measured using travelling microscope. When white light is used, the diameter of the rings of the different colors will differ and colored rings are observed. Only the first few rings are clear and after that due to overlapping of the rings of different colors, rings cannot be viewed.

A soap film of refractive index $\frac{4}{3}$ and the thickness 20 μm is illuminated by white light incident at an angle of 49.6°. The light reflected by it is examined by a spectroscope in which is found dark band corresponding to a wavelength 500 nm. Calculate the order of interference of the dark band. [T.U. 2065 Shrawan]

[Solution:]

Here,

Refractive index of a soap film, $(\mu) = \frac{4}{3}$

Thickness of a soap film, $(t) = 20\ \mu m = 20 \times 10^{-6}\ m$

Angle of incidence, $(i) = 49.6°$
Wavelength of light, $(\lambda) = 500\ nm = 500 \times 10^{-9}\ m$

We have,

$$\mu = \frac{\sin i}{\sin r}$$

$$\text{or,}\quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 49.6°}{\frac{4}{3}} = 0.57$$

$$\therefore\quad \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.57)^2} = 0.82$$

Thus,

$$2\mu t \cos r = n\lambda$$

$$\text{or,}\quad n = \frac{2\mu t \cos r}{\lambda} = \frac{2\times 4\times 20\times 10^{-6}\times 0.82}{3\times 500\times 10^{-9}} \approx 87$$

The order of interference of dark band is 87.

---

13. **Newton's rings are formed by reflected light of wavelength 58 Å with a liquid between the plane and curved surfaces. If the diameter of the sixth bright ring is 3 mm and the radius of the curved surface is 100 cm, calculate the refractive index of the liquid.** **[T.U. 2065 Chaitra]**

Solution:

Here,
Wavelength of light, $(\lambda) = 5895$ Å $= 5895 \times 10^{-10}\ m$
Diameter of 6^{th} bright ring, $(D_6) = 3\ mm = 3\times 10^{-3}\ m$
Radius of curvature, $(R) = 100 cm = 1\ m$

We have,

$$D_n = 2\sqrt{\frac{(2n-1)\lambda R}{2\mu}}$$

$$\text{or,}\quad D_6 = 2\sqrt{\frac{(2\times 6-1)\lambda R}{2\mu}}$$

$$\text{or,}\quad 3\times 10^{-3} m = 2\sqrt{\frac{(2\times 6-1)5895\times 10^{-10}\times 1}{2\mu}}$$

$$\therefore\quad \mu = 1.20$$

The refractive index of a liquid is 1.20.

---

14. **In Newton's rings experiment the radius of the fourth and twelfth rings are 0.26 cm and 0.37 cm respectively. Find the diameter of the 24th dark ring.** **[T.U. 2065 Kartik]**

Solution:

Here,
Diameter of 4^{th} ring, $(D_4) = 0.26\ cm = 2.6\times 10^{-3}\ m$
Diameter of 12^{th} ring, $(D_{12}) = 0.37\ cm = 3.7\times 10^{-3}\ m$
$m = 12 - 4 = 8$

We have,

$$\lambda = \frac{(D_{m+n})^2-(D_n)^2}{4mR} = \frac{(D_{12})^2-(D_4)^2}{4\times 8R}$$

$$\text{or,}\quad \lambda R = \frac{\left(3.7\times 10^{-3}\right)^2-\left(2.6\times 10^{-3}\right)^2}{32}$$

$$\therefore\quad \lambda R = 2.17\times 10^{-7} \quad \ldots (i)$$

Now,

$$D_{24} = 2\sqrt{24\lambda R} = 2\sqrt{24\times 2.17\times 10^{-7}} = 4.56\times 10^{-3}\ m$$

The diameter of 24^{th} dark ring is $4.56\times 10^{-3}\ m$.

---

15. **Explain how interference fringes are formed by a thin wedge shaped film, when examined by normally reflected light. How will you estimate the difference of film thickness between two points?** **[T.U. 2067 Ashadh]**

Solution:

Consider two planes surfaces OM and ON inclined at an angle θ and enclosing a wedge shaped air film. The thickness of the air film increases from O to . When the air film is viewed with reflected monochromatic light, a system of equidistant interference fringes is observed which are parallel to the line of intersection of the two surfaces.

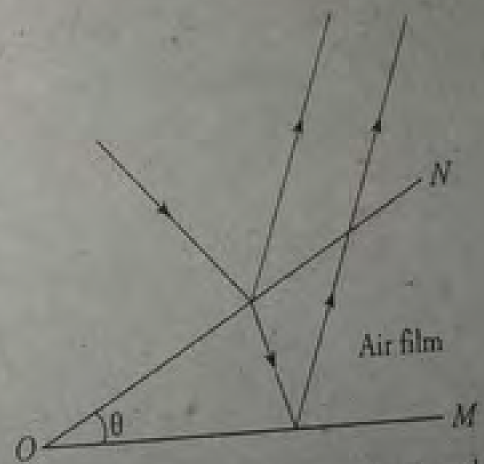

Figure: Interference fringe wedge shaped air film

The interfering rays do not enter the eyes parallel to each other but they appear to diverge from a point near the film. The effect is best observed when the angle of incidence is small.

We consider n^{th} bright fringe occurs at P_n. The thickness of the air film at P_n is P_nQ_n. As the angel of incidence is small,

$\cos r = 1$

Applying the relation for a bright fringe,

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

For air medium,

$\mu = 1$

and $P_nQ_n = t$

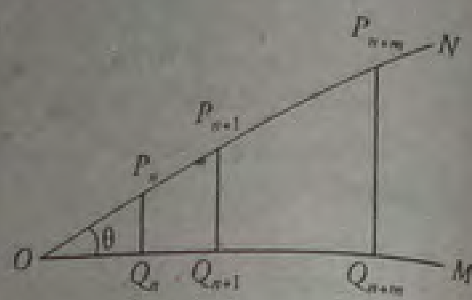

Figure: Probable position of fringes in interference of wedge film

Thus,

$$2P_nQ_n = (2n+1)\frac{\lambda}{2} \quad \ldots \text{(i)}$$

The next bright ring occurs at P_{n+1} position such that

$$2P_{n+1}Q_{n+1} = [2(n+1)+1]\frac{\lambda}{2}$$

$$= (2n+3)\frac{\lambda}{2} \quad \ldots \text{(ii)}$$

Subtracting, we obtain,

$$2P_{n+1}Q_{n+1} - 2P_nQ_n = \lambda$$

$$\therefore \quad P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}$$

Thus the next bright fringe will occur at the point where the thickness of the air film increase by $\frac{\lambda}{2}$. Suppose the $(n+m)^{th}$ bright ring is at P_{n+m}. There will be m bright fringes between P_n and P_{n+m} such that;

$$P_{n+m}Q_{n+m} - P_nQ_n = \frac{m\lambda}{2}$$

If the distance;

$$Q_nQ_{n+m} = x$$

$$\text{or,} \quad \theta = \frac{P_{n+m}Q_{n+m} - P_nQ_n}{Q_nQ_{n+m}}$$

$$= \frac{m\lambda}{2x}$$

$$\text{or,} \quad x = \frac{m\lambda}{2\theta}$$

Thus, the angle of inclination between OM and ON can be known. As x is the distance corresponding to m fringes, the fringe width is;

$$\beta = \frac{x}{m} = \frac{\lambda}{2\theta}$$

The interference fringe due to wedge shaped film is the basis for Newton's rings and Haidinger fringes. The result is applicable to determine the wavelength of incident light.

In Newton's ring arrangement of a source emitting two wavelengths 6×10^{-7} m and 5.9×10^{-7} m is used. It is found that n^{th} dark ring due to one wavelength coincides with $(n+1)^{th}$ dark ring due to other. Find the diameter of n^{th} dark ring if radius of curvature of lens is 0.9 m. [T.U. 2067 Ashadh]

Solution:

Here,

Radius of curvature of lens, $(R) = 0.9\ m$

Wavelengths of a light source, $(\lambda_1) = 6 \times 10^{-7}\ m$

$(\lambda_2) = 5.9 \times 10^{-7}\ m$

Diameter of n^{th} dark ring for λ_1 = Diameter of $(n+1)^{th}$ dark ring for λ_2

$$\text{or,} \quad (D_n)^2 = (D_{n+1})^2$$

$$\text{or,} \quad 4n\lambda_1 R = 4(n+1)\lambda_2 R$$

$$\text{or,} \quad 4n \times 6 \times 10^{-7}\ m = 4(n+1) \times 5.9 \times 10^{-7}\ m$$

$$\text{or,} \quad 24\ n = 23.6\ n + 23.6$$

$$\therefore \quad n = 59$$

The diameter of n^{th} dark ring,

$$(D_n)^2 = 4n\lambda_1 R$$

$$= 4 \times 59 \times 6 \times 10^{-7} \times 0.9$$

$$\therefore \quad D_n = 1.13 \times 10^{-2}\ m$$

The diameter of n^{th} dark ring is $1.13 \times 10^{-2}\ m$.

A plano-convex lens of radius 300 cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8^{th} dark ring in the transmitted system is 0.72 cm. Calculate the wavelength of light used. [T.U. 2067 Mangsir]

Solution:

Here,

Radius of curvature of a lens, $R = 300\ cm = 3\ m$

Diameter of the 8^{th} dark ring, $D_8 = 0.72\ cm = 7.2 \times 10^{-3}\ m$

For a transmitted light, we have,

$$D_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$$

$$\text{or,} \quad \lambda = \frac{(D_8)^2}{2(2 \times 8 - 1)R} = \frac{(7.2 \times 10^{-3})^2}{2(2\times8-1)\times 3}$$

$$\therefore \quad \lambda = 3756.52 \times 10^{-10}\ m$$

$$= 3756.52\ \text{Å}$$

The wavelength of monochromatic light = 3756.52 Å

18. Why are Newton's rings circular? Discuss and derive the necessary theory of Newton's ring experiment for transmitted light. [T.U. 2068 Shrawan]

Solution:

The Newton's rings are circular because the air film has a circular symmetry. In case of transmitted light, the interference fringes are produced such that fro bright rings,

$$2\mu t\cos\gamma = n\lambda$$

and for dark rings,

$$2\mu t\cos\gamma = (2n-1)\frac{\lambda}{2}$$

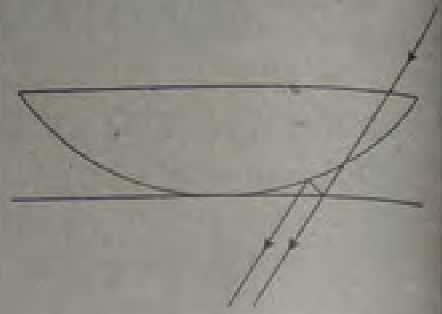

Figure: Newton's ring for transmitted light

For air film trapped between glass plate and plano-convex lens,

$\mu = 1$

Since angle γ is small, $\cos\gamma = 1$. Thus,

For bright rings,

$$2t = n\lambda$$

For dark rings,

$$2t = (2n-1)\frac{\lambda}{2}$$

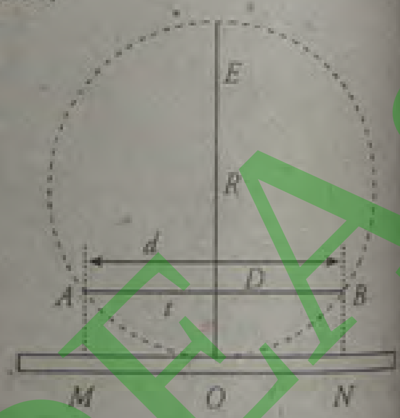

Figure: Mathematical determination of Newton's ring diameter

From figure,

$$AD \times DB = OD(2R - OD)$$

Since,

$$AD = DB = r$$

$$OD = BM = AM = t$$

Thus,

$$r^2 = t(2R - t)$$

or, $r^2 = 2Rt$ $\quad (\because 2R - t \approx 2R)$

$\therefore \quad t = \dfrac{r^2}{2R}$

On substituting value of t for bright and dark rings, we obtain,

For bright rings,

$$r^2 = n\lambda R$$

For dark rings,

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

When $n = 0$, for bright rings; $r = 0$. Therefore, central Newton's ring for transmitted light is bright, *i.e.*, just opposite to the ring pattern due to reflected light.

19. What is interference of light? Explain with necessary theory the Newton's ring method of measuring the wavelength of light. [P.U. 2003]

Solution:

Interference of light

See the solution of Q. No. 5 on page no. 124

For the remaining part

See the solution of Q. No. 11 on page no. 128

20. A soap film of refractive index $\frac{4}{3}$ and the thickness 1.5×10^{-4} *cm* is illuminated by white light incident at an angle of 60°. The light reflected by it is examined by a spectroscope in which is found dark band corresponding to a wavelength 5×10^{-5} *cm*. Calculate the order of interference of the dark band. [P.U. 2003]

Solution:

Here,

Refractive index of a soap film, $(\mu) = \dfrac{4}{3}$

Thickness of a soap film, $(t) = 1.5 \times 10^{-4}\ cm = 1.5 \times 10^{-6}\ m$

Angle of incidence, $(i) = 60°$

Wavelength of light, $(\lambda) = 5 \times 10^{-5}\ cm = 5 \times 10^{-7}\ m$

We have,

$$\mu = \frac{\sin i}{\sin r}$$

or, $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 60°}{\frac{4}{3}} = 0.65$

$\therefore \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.65)^2} = 0.76$

Thus,

$$2\mu t\cos r = n\lambda$$

or, $n = \dfrac{2\mu t\cos r}{\lambda}$

$$= \frac{2 \times 4 \times 1.5 \times 10^{-6} \times 0.76}{3 \times 5 \times 10^{-7}} \approx 54$$

The order of interference of dark band is 54.

21. Explain how Newton's rings are formed and describe with necessary theory, the method for the determination of wavelength of monochromatic light. Explain the blooming of lenses. [P.U. 2004]

Solution: See the solution of Q. No. 5 on page no. 124

Blooming of lenses

The process of coating a film on the lens is called *blooming*. A very thin coating on the lens surface can reduce reflections of light considerably. The amount of reflection of light at a boundary depends on the difference in refraction index between the two materials.

Ideally, the coating material should have a refractive index so that the amount of reflection at each surface is nearly equal. The destructive interference can occur nearly completely for one particular wavelength. The thickness of the film is chosen so that light reflecting from the front and rare surface of the film destructively interferes.

22. Light of wavelength 6.0×10^{-5} cm falls normally on the thin wedge shape film of refractive index 1.4, forming fringes that are 2 *mm* apart. Find the angle of the wedge. [P.U. 2004]

Solution:

Here,

Wavelength of light, $(\lambda) = 6.0 \times 10^{-5}\ cm = 6.0 \times 10^{-7}\ m$

Refractive index, $(\mu) = 1.4$

Fringe width, $(\beta) = 2\ mm = 2 \times 10^{-3} m$

We have,

$$\beta = \frac{\lambda}{2\theta\mu}$$

$$\text{or, } \theta = \frac{\lambda}{2\mu\beta}$$

$$= \frac{6.0 \times 10^{-7}}{2 \times 2 \times 10^{-3} \times 1.4}$$

$$= 1.07 \times 10^{-4}\ m$$

The angle of wedge is 1.07×10^{-4} m.

23. Explain the phenomenon of interference of light. Give the theory of the Newton's rings. How fringes can be used to find the wavelength of light? [P.U. 2005]

Solution: See the solution of Q. No. 5 and 11 on page no. 124 and 128

24. A soap film of refractive index 1.33 and the thickness 1.5×10^{-4} *cm* is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found dark band corresponding to a wavelength 5.63×10^{-5} *cm*. Calculate the order of interference of the dark band. [P.U. 2005]

Solution: Proceed as solution of Q. No. 20 on page no. 135

25. What are coherent sources? Explain the phenomenon of interference in thin films. [P.U. 2007]

Solution:

Coherent sources

Two sources are said to be coherent if they emit light waves of the same frequency, nearly of same amplitude and are always in phase with each other. It means that the two sources must emit radiation of the same wavelength. In actual practice, it is not possible to have two independent sources which are coherent. For experimental purposes, two virtual sources formed from a single source can act as coherent sources.

Interference in thin films

A familiar example of interference in thin film is beautiful colors produced by thin film of oil on the surface of water or thin film of soap bubble. Newton and Hooke observed and developed the interference phenomenon due to multiple reflections of thin transparent materials. Hooke observed such phenomenon in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings to explain phenomenon on the basis of interference between light reflected from the top and the bottom surface of an air film.

Consider a transparent film of thickness t and refractive index μ. An incident ray SA falls on the upper surface of the film is partially reflected along AT and partially refracted along AB. At point B, part of it is reflected along BC and finally emerges out along CQ. The differences in path between two rays AT and CQ can be determined. The normals CN and AM are drawn on AT and BC respectively. The angle of incidence is i and angle of refraction is r. CB is produced to meet AE produced at point P.

The optical path difference,

$$x = \mu(AB + BC) - AN$$

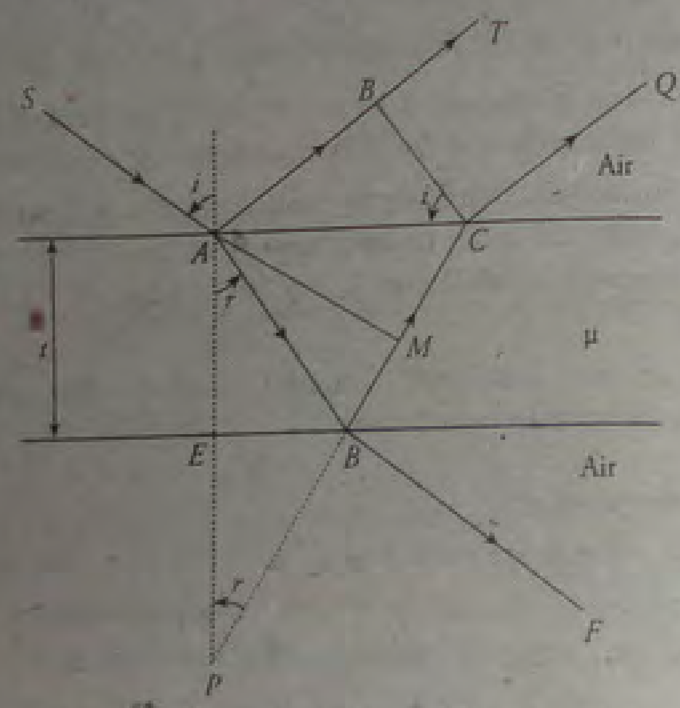

Figure: Interference in thin films due to reflected light

$$\mu = \frac{\sin i}{\sin r} = \frac{AN}{CM}$$

Thus,

$$x = \mu(AB + BC) - \mu CM = \mu(PC - CM) = \mu PM$$

In ΔAPM,

$$\cos r = \frac{PM}{AP}$$

or, $PM = AP\cos r = (AE + EP)\cos r = 2t\cos r$

$\therefore$ $x = 2\mu t\cos r$

This expression in the case of reflected light does not represent the actual path difference and only the apparent path difference. It has been established on the basis of electromagnetic theory that when light is reflected from the surface of an optically denser medium a phase change π equivalent to a path difference $\frac{\lambda}{2}$ occurs. Thus the actual path difference in this case is;

$$x = 2\mu t\cos r - \frac{\lambda}{2}$$

For a constructive interference, $x = n\lambda$; film appears bright.

$$n\lambda = 2\mu t\cos r - \frac{\lambda}{2}; \quad n = 0,1,2,3,\dots\dots\dots$$

For a destructive interference, $(2n+1)\frac{\lambda}{2}$; film appears dark.

$$(2n+1)\frac{\lambda}{2} = 2\mu t\cos r - \frac{\lambda}{2}$$

or, $2\mu t\cos r = (n+1)\lambda$

Since n is an integer only, therefore $(n+1)$ can be taken as n.

$\therefore$ $2\mu t\cos r = n\lambda; \quad n = 0,1,2,3,\dots\dots\dots$

The interference pattern will not be perfect because the intensities of the rays will not be same and their amplitudes are different. The amplitudes will depend on the amount of light reflected and transmitted through films.

A wedge shaped air film having an angle 45' 30" is illustrated by monochromatic light and fringes are observed normally. If the fringe width is 0.12 *cm* calculate the wavelength of light used.
[P.U. 2007]

Solution:

Here,

Angle of wedge shaped air film, $(\theta) = 45'30'' = 45.5 \times \frac{\pi}{2}$ radians

Fringe width, $(\beta) = 0.12\ cm = 1.2 \times 10^{-3}\ m$

We have,

$$\beta = \frac{\lambda}{2\theta}$$

or, $\lambda = 2\theta\beta = 2 \times \left(45.5 \times \frac{\pi}{2}\right) \times 1.2 \times 10^{-3} = 0.171\ m$

The wavelength of monochromatic source used is 0.171 m. This wavelength is extremely small compared to practical monochromatic lights. The information given in question has significant errors.

Explain the interference due to thin film and shows that the reflected and transmitted systems are complementary to each other.
[P.U. 2008]

Solution:

Interference in thin films

See the solution of Q. No. 25 on page no. 137

Consider a transparent film of thickness t and refractive index μ. An incident ray SA falls on the upper surface of the film is refracted along AB. At point B, part of it is reflected along BC and and partly refracted along BR . The light ray BC after reflection at C, finally emerges along DQ. Since the reflection takes place in rarer medium, no phase change occurs. The normals BM and DN are drawn on CD and BR respectively. The angle of incidence is i and angle of refraction is r. DC is produced to meet BP produced at point P. The optical path difference is,

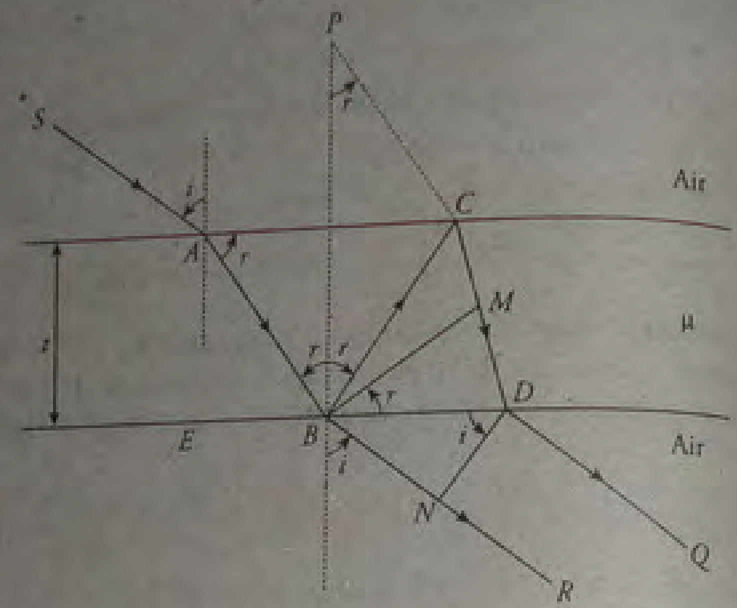

Figure: Interference due to transmitted light in thin films

$$x = \mu(BC + CD) - BN$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BN}{MD}$$

Thus,

$$x = \mu(AB + BC) - \mu CM = \mu(PC - CM) = \mu PM$$

In ΔBPM,

$$\cos r = \frac{PM}{BP}$$

or, $PM = BP\cos r = 2t\cos r$

$\therefore\ x = 2\mu t\cos r$

This relation gives path difference for transmitted light in thi[n] film interference.

For bright fringes, the path difference $x = n\lambda$

$\therefore\ 2\mu t\cos r = n\lambda;\ n = 0, 1, 2, 3, \dots\dots$ ... (iii)

For dark fringes, the path difference $x = (2n+1)\frac{\lambda}{2}$

$2\mu t\cos r = (2n+1)\frac{\lambda}{2};\ n = 0, 1, 2, 3, \dots\dots$ ... (iv)

The interference fringes obtained are less distinct in transmitt[ed] light because of unequal amplitude of transmitted light. Howev[er] when angle of incidence is nearly 45°, the fringes are more distin[ct]. The relation (i), (ii), (iii) and (iv) shows that the interference d[ue] to thin film and shows that the reflected and transmitted syste[m] are complementary to each other.

8. What do you mean by coherent sources of light? Write down the analytical treatment of interference and show that the distance between two consecutive dark and bright fringes is equal. [P.U. 2010]

Solution: See the solution of Q. No. 4 and 10 on page no. 124 and 128

9. A soap film 5×10^{-5} cm thick is viewed at an angle 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$). [P.U. 2010]

Solution:

Here,

Thickness of a soap film, $(t) = 5 \times 10^{-5}\ cm = 5 \times 10^{-7}\ m$

Angle of incidence, $(i) = 35°$

Refractive index, $(\mu) = 1.33$

$$\mu = \frac{\sin i}{\sin r}$$

or, $\sin r = \frac{\sin i}{\mu} = \frac{\sin 35°}{1.33} = 0.43$

or, $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.43)^2} = 0.90$

We have,

$$2\mu t\cos r = n\lambda$$

For the first order, $n = 1$

$$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-7} \times 0.90 = 11.97 \times 10^{-7}\ m$$

which lies in the infra-red region

For the second order, $n = 2$

$$\lambda_2 = 2 \times 1.33 \times 5 \times 10^{-7} \times \frac{0.90}{2} = 5.98 \times 10^{-7} m$$

which lies in visible region

For the third order, $n = 3$

$$\lambda_3 = 2 \times 1.33 \times 5 \times 10^{-7} \times \frac{0.90}{3}$$

$$= 3.99 \times 10^{-7} m$$

which lies in visible region

For the fourth order, $n = 4$

$$\lambda_2 = 2 \times 1.33 \times 5 \times 10^{-7} \times \frac{0.90}{4}$$

$$= 5.98 \times 10^{-7} m$$

which lies in ultra-violet region

Hence, the absent wavelengths in the reflected light are $5.98 \times 10^{-7} m$ and $3.99 \times 10^{-7} m$.

30. What is interference of light? Give the theory of Newton's rings in the case of reflected system and describe how the refractive index of liquid is determined by Newton's ring method.[P.U. 2011]

Solution: See the solution of Q. No. 5 on page no. 124

Suppose the diameter of the n^{th} ring be D_n. Thus,

$$(D_n)^2 = 4n\lambda R \quad \ldots \text{(i)}$$

Measure the diameter of the $(n+m)^{th}$ dark ring. Its diameter is;

$$(D_{n+m})^2 = 4(n+m)\lambda R \quad \ldots \text{(ii)}$$

Subtracting these equations we obtain,

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R \quad \ldots \text{(iii)}$$

The liquid is poured in the container C without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the n^{th} ring and $(n+m)^{th}$ ring are determined. Let μ be the refractive index of liquid. If D_n' and D_{n+m}' be the diameters of n^{th} ring and $(n+m)^{th}$ ring, then,

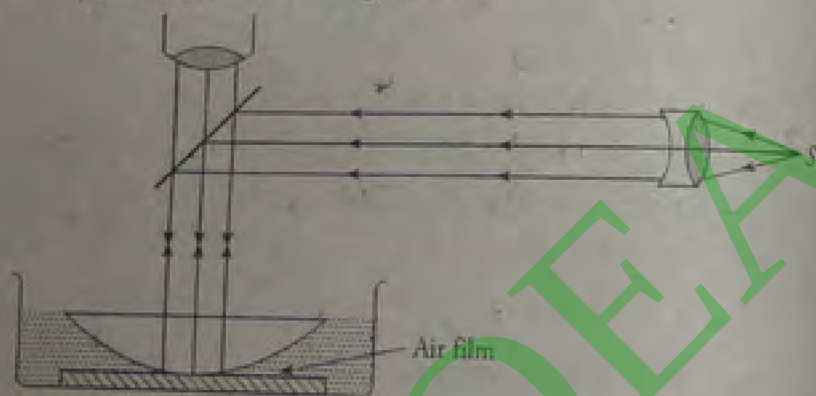

Figure: Determination of refractive index of a liquid using Newton's rings

$$(D_n')^2 = \frac{4n\lambda R}{\mu}$$

$$(D_{n+m}')^2 = \frac{4(n+m)\lambda R}{\mu}$$

Thus,

$$(D_{n+m}')^2 - (D_n')^2 = \frac{4m\lambda R}{2\mu}$$

$$\therefore \mu = \frac{4m\lambda R}{(D'_{m+n})^2 - (D_n')^2} \quad \ldots \text{(iv)}$$

Substituting equation (iii) into equation (iv), we obtain,

$$\mu = \frac{(D_{m+n})^2 - (D_n)^2}{(D'_{n+m})^2 - (D'_n)^2} \quad \ldots \text{(v)}$$

This relation determines the refractive index of a liquid.

31. In Young's two slit experiment, the separation of four bright fringes is 2.4 mm when the wavelength of light is 6×10^{-7} m. The distance from the slit to the screen is 1.0 m. Calculate the separation of the slits. [P.U. 2011]

Solution:

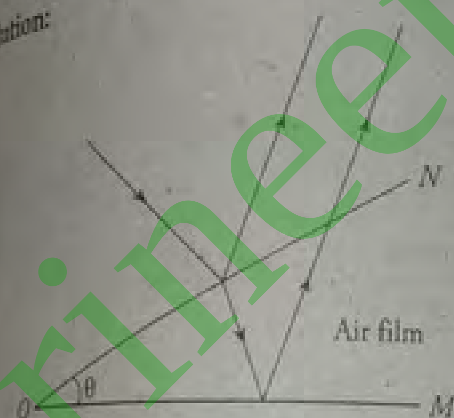

Figure: Interference fringe wedge shaped air film

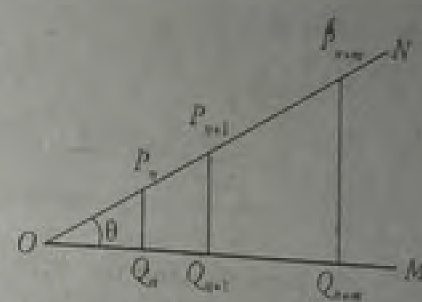

Figure: Probable position of fringes in interference of wedge film

Here,

The separation of four bright fringes, $(x_4) = 2.4\ mm = 2.4 \times 10^{-3} m$

Wavelength of light, $(\lambda) = 6 \times 10^{-7}\ m$

Distance from slit to screen, $(D) = 1.0\ m$

We have,

$$d = \frac{4\lambda D}{x_4} = \frac{4 \times 6 \times 10^{-7} \times 1.0}{2.4 \times 10^{-3}} = 10^{-3}\ m$$

The separation of the slits is $10^{-3}\ m$.

32. What is meant by blooming of lenses? Derive an expression for the interference in the thin films and wedge shape. [P.U. 2002]

Solution:

Blooming of lenses

See the solution of Q. No. 21 on page no. 136

Interference in thin films

See the solution of Q. No. 27 on page no. 139

Interference in the wedge shape

See the solution of Q. No. 15 on page no. 131

6.12 Additional Solved Problems

1. Two narrow and parallel slits 0.08 *cm* apart are illuminated by light of frequency 8×10^{-11} *KHz*. It is desired to have a fringe width of 6×10^{-4} *m*. Where should the screen be placed from the slits?

Solution:

$d = 0.08\ cm = 0.08 \times 10^{-2}\ m$

$\beta = 6 \times 10^{-4}\ m$

Frequency $(v) = 8 \times 10^{11}\ KHz$

i.e., $\lambda = \frac{c}{v} = \frac{3 \times 10^8}{8 \times 10^{11} \times 10^3}\ m$

$\lambda = ?$

From $\beta = \frac{\lambda D}{d}$

We have,

$$\therefore \quad D = \frac{\beta D}{\lambda} = \frac{6 \times 10^{-4} \times 0.08 \times 10^{-2} \times 8 \times 10^{14}}{3 \times 10^8} = 1.28\ m$$

2. In Young's double slit experiment, a source of light of wavelength 4200 Å is used to obtain interference fringes of width 0.64×10^{-2} *m*. What should be the wavelength of the light source to obtain fringes 0.46×10^{-2} *m* wide, if the distance between screen and the slits is reduced to half the initial value?

Solution:

In the first case;

$\lambda = 4200\ \text{Å} = 4200 \times 10^{-10}\ m$

$\beta = 0.64 \times 10^{-2}\ m$

$$\therefore \quad 0.64 \times 10^{-2} = \frac{4200 \times 10^{-10} \times D}{d} \qquad \text{... (i)}$$

In the second case;

$\beta = 0.46 \times 10^{-2}\ m$

$\lambda = ?$

$$\therefore \quad 0.46 \times 10^{-2} = \frac{\lambda \times \frac{D}{2}}{d} = \frac{2d}{\lambda d} \qquad \text{... (ii)}$$

Dividing equation (i) by (ii); we get,

$$\frac{0.64 \times 10^{-2}}{0.46 \times 10^{-2}} = \frac{4200 \times 10^{-10} \times D}{d} \times \frac{2d}{\lambda d}$$

$$\therefore \quad \lambda = \frac{4200 \times 10^{-10} \times 2 \times 0.46}{0.64}$$

$$= 6037.5\ \text{Å}$$

In Young's double slit experiment, the distance between the slits is 1 *mm*. The distance between the slit and the screen is 1 metre. The wavelength used in 5893 Å. Compare the intensity at a point distance 1 *mm* from the centre to that at its centre. Also find the minimum distance from the centre of a point where the intensity is half of that at the centre.

Solution:

Path difference at a point on the screen distance *y* from the central point is;

$$= \frac{Y \cdot d}{D}$$

Here,

$Y = 1\ mm = 1 \times 10^{-3}\ m$

$D = 1\ m$

$d = 1\ mm = 1 \times 10^{-3}\ m$

$$\therefore \quad \text{Path difference} = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{1} = 1 \times 10^{-6}\ m = \Delta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\Delta = \frac{10^{-6} \times 2\pi}{5893 \times 10^{-10}} = 3.394\pi \text{ radians}$$

$\therefore$ Ratio of intensity with the central maximum is;

$$\cos^2\frac{\delta}{2} = \cos^2(1.697\pi) = 0.3372$$

When the intensity is half of the maximum, if δ is the phase difference, we have,

$$\cos^2\frac{\delta}{2} = 0.5$$

$$\text{or,} \quad \frac{\delta}{2} = 45^\circ$$

$$\text{or,} \quad \delta = \frac{\pi}{2}$$

$$\text{Path difference} = \Delta = \delta\frac{\lambda}{2\pi}$$

$$= \frac{\pi}{2} \times \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$

Distance of the point on the screen from the centre;

$$(Y) = \Delta\frac{D}{d}$$

$$= \frac{\lambda}{4} \times \frac{1}{1 \times 10^{-3}}$$

$$= \frac{5893 \times 10^{-10}}{1 \times 10^{-3}}$$

$$= 1.473 \times 10^{-4}\ m$$

4. In a double slit experiment, fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other slit by another plate of glass of the same thickness but of refractive index 1.7. On doing so the central bright fringe shifts to the position originally occupied by the fifth bright fringe from the centre. Find the thickness of the glass plate.

Solution:

We have,

$$n\lambda = (\mu - \mu')t$$

Here,

$$n = 5$$
$$\mu - \mu' = 0.3$$
$$\lambda = 4800 \times 10^{-10}\ m$$
$$\therefore\ t = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8.0 \times 10^{-8}\ m$$

---

5. A drop of oil of volume 0.2 *c.c.* is dropped on a surface of tank of water of area 1 m^2. The film spreads uniformly over the whole surface. White light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band whose centre has a wavelength 5.5×10^{-5} *cm* in air. Find the refractive index of oil.

Solution:

The thickness of the film $(d) = \frac{0.2\ cm}{100 \times 100} = 2 \times 10^{-5}\ cm$

The film appears dark by reflected light for a wavelength λ given by the relation;

$$2\mu d \cos r = n\lambda$$

For normal incidence $r = 0$

$$\cos r = 1$$

Further $n = 1$ and $\lambda = 5.5 \times 10^{-5}\ cm$

$$\mu = \frac{n\lambda}{2t\cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1} = 1.375$$

---

6. A soap film 5×10^{-5} *cm* thick is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$).

Solution:

Let, i be the angle of incidence and r be the angle of refraction. Then,

$$\mu = \frac{\sin i}{\sin r}$$

or, $1.33 = \frac{\sin 35°}{\sin r}$

or, $r = 25.55°$

or, $\cos r = 0.90$

Applying the relation,

$$2\mu d \cos r = n\lambda$$

where, $d = 5.5 \times 10^{-5}\ cm$

i) For the first order, $n = 1$

$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-5} \times 0.90 = 12.0 \times 10^{-5}\ cm$

which lies in the infrared (invisible) region.

ii) For the first order, $n = 2$

$\lambda_2 = 1.33 \times 5 \times 10^{-5} \times 0.90 = 6.0 \times 10^{-5}\ cm$

which lies in the visible region.

iii) Similarly, taking $n = 3$

$\lambda_3 = 4.0 \times 10^{-5}\ cm$

which also lies in the visible region.

iv) If $n = 1$

$\lambda_4 = 3.0 \times 10^{-5}\ cm$

which lies in the ultraviolet (invisible) region.

Hence, absent wavelengths in the reflected light are $6.0 \times 10^{-5}\ cm$ and $4.0 \times 10^{-5}\ cm$.

---

7. Two glass plates enclose a wedge shaped air film, touching at one edge and separated by a wire of 0.05 *mm* diameter at a distance 15 *cm* from that edge. Calculate the fringe width. Monochromatic light of $\lambda = 6000$ Å from a broad source falls normally on the film.

Solution:

Here,

Fringe width $(\beta) = \frac{\lambda}{2\alpha}$

Clearly, $(\alpha) = \frac{0.05\ mm}{15\ cm} = \frac{0.005}{15}$ radians

$$(\beta) = \frac{\lambda}{2\alpha}$$
$$= \frac{6000 \times 10^{-5} \times 15}{2 \times 0.005} = 0.09\ cm$$

8. **An air wedge of angle 0.01 radians is illuminated by monochromatic light of 6000 Å falling normally on it. At what distance from the edge of the wedge, will the 10th fringe be observed by reflected light.**

Solution:

Here,

$\alpha = 0.01$ radians

$n = 10$

$\lambda = 6000 \times 10^{-10}\ m$

$$2d = \frac{(2n-1)\lambda}{2}$$

where, d is the thickness of wedge

But $\alpha = \frac{d}{x}$

or, $d = \alpha x$

$$\therefore\quad 2\alpha x = \frac{(2n-1)\lambda}{2}$$

Here, $n = 10$

$$x = \frac{(2n-1)\lambda}{4\alpha}$$

$$= \frac{19 \times 6000 \times 10^{-10}}{4 \times 0.01}\ m$$

$$= 2.85 \times 10^{-4}\ m$$

9. **A thin equiconvex lens of focal length 4 meters and refractive index 1.50 rests on and is in contact with an optical flat. Using light of wavelength 5460 Å, Newton's rings are viewed normally by reflection. What is the diameter of the 5th bright ring?**

Solution:

Here,

The diameter of the n^{th} bright ring is given by;

$$D = \sqrt{2(2n-1)\lambda R}$$

Here,

$n = 5$

$\lambda = 5460 \times 10^{-6}\ cm$

$f = 400\ cm$

$\mu = 1.50$

We have,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Here,

$R_1 = R_2 = R$

$$\therefore\quad \frac{1}{f} = (\mu - 1)\frac{2}{R}$$

$$i.e.,\quad \frac{1}{400} = (1.50 - 1)\frac{2}{R}$$

$$or,\quad R = 400\ cm$$

In Newton's ring experiment, the diameter of the 4th and 12th dark rings are 0.400 *cm* and 0.700 *cm* respectively. Find the diameter of the 20th dark ring.

Solution:

We have,

$$D^2_{n+p} - D^2_n = 4p\lambda R$$

Here,

$(n+p) = 12$

$n = 4$

$p = 12 - 4 = 8$

$$\therefore\quad D^2_{12} - D^2_4 = 4 \times 3 \times \lambda R \quad \text{... (i)}$$

$$D^2_{20} - D^2_4 = 4 \times 16 \times \lambda R \quad \text{... (ii)}$$

Dividing equation (ii) by (i); we get,

$$\frac{D^2_{20} - D^2_4}{D^2_{12} - D^2_4} = \frac{4 \times 16 \times \lambda R}{4 \times 8 \times \lambda R} = 2$$

$$or,\quad \frac{D^2_{20} - (0.4)^2}{(0.7)^2 - (0.4)^2} = 2$$

$$or,\quad D_{20} = 0.906\ cm$$

In a Newton's ring experiment the diameter of the 10th ring changes from 1.40 to 1.27 *cm* when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Solution:

When the liquid is used the diameter of the 10th ring is given by;

$$(D'_{10})^2 = \frac{4 \times 10 \times \lambda R}{\mu} \quad \text{... (i)}$$

For air medium

$$(D_{10})^2 = 4 \times 10 \times \lambda R \quad \text{... (ii)}$$

Dividing equation (i) by (ii); we get,

$$\mu = \frac{D^2_{10}}{D'^2_{10}} = 1.215$$

12. In a Newton's ring experiment the diameter of the 5th dark ring was 0.3 *cm* and the diameter of 25h ring was 0.8 *cm*, if the radius of curvature of the plano-convex lens is 100 *cm*, find the wavelength of the light used.

Solution:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Here,

$$D_{25} = 0.8\,cm$$
$$D_5 = 0.3\,cm$$
$$p = 25 - 5 = 20$$
and $$R = 100\,cm$$

$$\therefore \lambda = \frac{(0.8)^2 - (0.3)^2}{4 \times 20 \times 100} = 4.87 \times 10^{-5}\,cm$$

Chapter 7

DIFFRACTION

7.1 INTRODUCTION

Whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow and its intensity falls off rapidly. This deviation is extremely small when wavelength is small in comparison to the dimension of the obstacle. But the deviation becomes much more pronounced when the dimensions of the aperture or opaque disc are comparable with wavelength of light.

The phenomenon of bending of light around corners of an obstacle and their spreading into the geometrical shadow of an object is called diffraction.

According to the rectilinear propagation of light, light always travels in a straight line. But, whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow, thus violating the rectilinear propagation of light.

7.2 FRESNEL DIFFRACTION AND FRAUNHOFFER DIFFRACTION

Fresnel diffraction occurs when a source and screen are placed at a finite distance from an obstacle with a sharp edge. In this case, no lenses are used to make parallel rays. Thus, the incident wave front is either spherical or cylindrical in this diffraction.

Fraunhoffer diffraction occurs when source and screen placed at infinite distance. In this diffraction, incident wave front is a plane wave front. The incident rays are made parallel by some means.

7.3 TRANSMISSION GRATING AND REFLECTION GRATING

A diffraction grating consists of a large number of narrow slits side by side. The slits are separated by opaque spaces. When a wavefront is incident on grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such grating is called transmission grating.

The space between any two lines is transparent to the light and the lined portion is opaque to light. Such surfaces act as a transmission grating. If the lines are drawn on a silvered surface then light is reflected from the positions of the mirror in between any two lines and such surfaces act as reflection grating.

7.4 SOLVED EXAM QUESTIONS

1. **Define diffraction. How does the phenomenon of diffraction violate the rectilinear propagation of light? What are the advantages and disadvantages of spectrum produced by grating over the spectrum produced by a prism? [T.U. 2061 Baishakh]**

Solution:

Whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow and its intensity falls off rapidly. The deviation is extremely small when wavelength is small in comparison to the dimension of the obstacle. But the deviation becomes much more pronounced when the dimensions of the aperture or opaque disc are comparable with wavelength of light.

The phenomenon of bending of light around corners of an obstacle and their spreading into the geometrical shadow of an object is called *diffraction*.

According to the rectilinear propagation of light, light always travels in a straight line. But, whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow, thus violating the rectilinear propagation of light.

Advantages and disadvantages of grating spectrum and prism spectrum

With a grating, a number of spectra of different orders can be obtained on the two sides of the central maximum whereas with a prism only one spectrum can be obtained.

The spectra obtained with a grating are comparatively pure than those with a prism.

Knowing the grating element $(a + b)$ and measuring the diffraction single spectrum, the wavelength of any spectral line can be measured accurately. But in the case of a prism the angles of deviation are not directly related to the wavelength of the spectral line. The angles of deviation are dependent on the refractive index of the material of the prism, which depends on the wavelength of light.

With a grating, the diffraction angle for violet end of the spectrum is less than that for red. With a prism, the angle of deviation for the violet ray of light is more than that for red rays of light.

The intensities of the spectral lines with a grating are much less than with a prism. In a grating spectrum, most of the incident light energy is associated with the undispersed central bright maximum and the rest of the energy is distributed in the different order spectra on the two sides of the central maximum. But in a prism, most of the incident light energy is distributed in a single spectrum and hence brighter spectral lines are obtained.

The dispersive power of a grating is;

$$\frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$$

This is constant for particular order. Thus, the spectral lines are evenly distributed and spectrum obtained with a grating is said to be rational. The refractive index of the material of prism changes more rapidly at the violet end than at the red end of the spectrum.

The dispersive power of a prism is $\frac{d\mu}{(\mu-1)}$. It has higher value in the violet region of the spectrum than in the red region. Hence, there will be more spreading of the spectral lines towards the violet and the spectrum obtained with a prism is said to be irrational.

The resolving power of a grating is much higher than that of a prism. Hence the same two nearby spectral lines appear better resolved with a grating than a prism.

The spectra obtained with different gratings are identical because the dispersive power and resolving power of a grating do not depend on the nature of the material of the grating. But the spectra obtained with different prisms are never identical because both dispersive power and resolving power of a prism depend on the nature of the material of the prism.

2. **Define diffraction. Distinguish between Fresnel and Fraunhoffer diffraction. [T.U. 2062 Baishakh]**

Solution:

Diffraction

The phenomenon of bending of light around corners of an obstacle and their spreading into the geometrical shadow of an object is called diffraction.

Fresnel diffraction occurs when a source and screen are placed at a finite distance from an obstacle with a sharp edge. In this case, no lenses are used to make parallel rays. Thus, the incident wave front is either spherical or cylindrical in this diffraction.

Fraunhoffer diffraction occurs when source and screen placed at infinite distance. In this diffraction, incident wave front is a plane wave front. The incident rays are made parallel by some means.

3. What angular separation is produced between the spectral by a grating of 2000 lines per cm in the second order diffraction pattern when it is illuminated normally by a light that contains wavelengths 6000 Å and 6010 Å? [T.U. 2062 Baishakh]

Solution:

Here,

$$a+b=\frac{1}{N}=\frac{1}{2000}\,cm$$

For the second order diffraction with $\lambda_1 = 6000$ Å

$$(a+b)\sin\theta_1 = n\lambda_1$$

or, $\sin\theta_1 = \frac{n\lambda_1}{a+b} = 2\times 6000\,\text{Å}\times 2000\,cm^{-1}$

$= 2\times 6000\times 10^{-8} cm \times 2000\,cm^{-1} = 0.24$

$\therefore\ \theta_1 = \sin^{-1}(0.24) = 13.89°$

For the second order diffraction with $\lambda_2 = 6010$ Å

$$(a+b)\sin\theta_2 = n\lambda_2$$

or, $\sin\theta_2 = \frac{n\lambda_2}{a+b} = 2\times 6010\,\text{Å}\times 2000\,cm^{-1}$

$= 2\times 6010\times 10^{-8} cm \times 2000\,cm^{-1} = 0.2404$

$\therefore\ \theta_2 = \sin^{-1}(0.2404) = 13.91°$

Angular separation, $(\theta_2-\theta_1) = 13.91° - 13.89° = 0.02° = 1.2'$

Angular separation is produced between the spectral by a grating of 2000 lines per cm in the second order diffraction pattern when it is illuminated normally by a light that contains wavelengths 6000 Å and 6010 Å is 1.2′.

4. Derive an expression for the intensity distribution due to Fraunhoffer diffraction at a single slit and show that the intensity of the first subsidiary maxima is 4.5% of that of principal maxima. [T.U. 2063 Baishakh]

Solution:

Consider a collimated beam of monochromatic light of wavelength λ, produced by a point source S placed at the focus of a spherical lens L_1, incident normally on a narrow slit AB on a width a. The incident plane wavefront on the slit AB can be imagined to be divided into a large number of infinitesimally small strips. The path difference between the secondary waves emanating from the extreme points A and B is $a\sin\theta$ where a is width of the slit and $\measuredangle ABL = \theta$. For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can

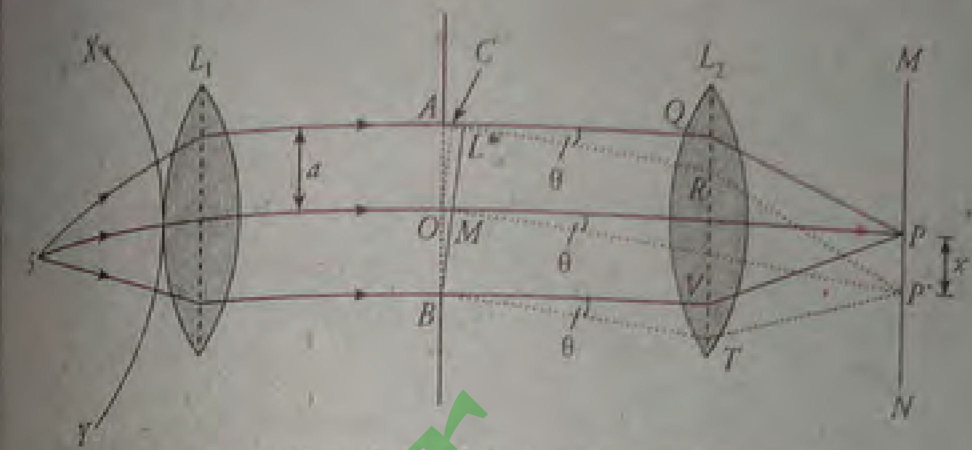

Figure: Fraunhoffer diffraction at a single slit

be taken to be the same. As one consider the secondary waves in a direction inclined at an angle θ from the point B upwards, the path difference changes and hence the phase difference also increases. Let α be the phase difference between the secondary waves from the points B and A of the slit. As the wavefront is divided into a large number of strips can be obtained by vector polygon method. Since the amplitudes are small and the phase difference increases infinitesimally small amounts from strip to strip. Thus, the vibration polygon coincides with the circular arc OP gives the direction of the initial vector and NM the direction of the final vector due to the secondary waves from A. K is the centre of the circular arc.

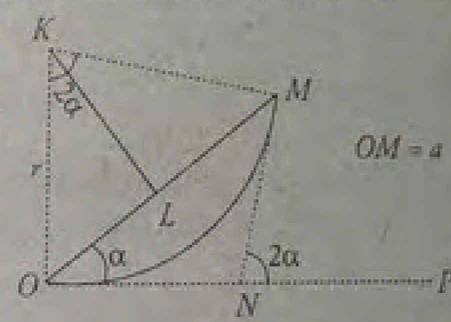

Figure: Intensity distribution for a circular wavefront

$\measuredangle MNP = 2\alpha$

$\measuredangle OKM = 2\alpha$

In the ΔOKL,

$OL = r\sin\alpha$

where, r is the radius of the circular arc

Chord $OM = 2OL = 2r\sin\alpha$

The length of the arc OM is proportional to the width of the slit.

Therefore, length of the arc $OM = ka$

where, k is a constant and a is the width of the slit

and $2\alpha = \frac{\text{arc}\,OM}{\text{radius}} = \frac{ka}{r}$

i.e., $2r = \frac{ka}{\alpha}$

Thus,

$$\text{Chord } OM = \frac{ka}{\alpha}\sin\alpha$$

or, $A \quad = \dfrac{\sin\alpha}{\alpha} ka$

$\therefore \quad A \quad = A_0 \dfrac{\sin\alpha}{\alpha}$

Thus, the resultant amplitude of vibration at appoint on the screen is $A_0 \dfrac{\sin\alpha}{\alpha}$ and intensity I at a point is;

$$I = A^2 = A_0^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

i.e., the intensity at point on the screen is proportional to $\left(\dfrac{\sin\alpha}{\alpha}\right)^2$

For the principal maxima, $\alpha \to 0$

$$\frac{\sin\alpha}{\alpha} = 1$$

Hence the intensity is maximum, the maximum intensity is;

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0$$

For the first secondary maxima, $\alpha = \dfrac{3\pi}{2}$

Thus,

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{-1}{\frac{3\pi}{2}}\right)^2 = 0.045\, I_0$$

i.e., the intensity of the first subsidiary maxima is 4.5 % of that of principal maxima.

---

5. **What do you understand by diffraction of light? Explain the distribution of intensity with a special reference of light in single slit.** **[T.U. 2064 Poush]**

Solution:

Diffraction of light

See the solution of Q. No. 2 on page no. 153

For the remaining part

See the solution of Q. No. 4 on page no. 154

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6. **A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 5000 lines per cm and the third order spectral line is found to be diffracted through 35°. Calculate the wavelength of light.** **[T.U. 2065 Shrawan]**

Solution:

Here,

$$a + b = \frac{1}{N} = \frac{1}{5000}\ cm$$

For the third order diffraction, $n = 3$

Thus,

$$(a+b)\sin\theta = n\lambda$$

or, $\lambda \quad = (a+b)\sin\theta$

$$= \frac{1}{3}\times\frac{1}{5000}\ cm \times \sin 35° = 3.824 \times 10^{-5}\ cm$$

$\therefore \lambda \quad = 3824 Å$

The wavelength of monochromatic light, $\lambda = 3824$ Å.

Distinguish between Franhoffer and Fresnel diffraction. **[T.U. 2065 Chaitra]**

Solution: See the solution of Q. No. 2 on page no. 153

What is the highest order spectrum which may be seen with monochromatic light of wavelength 559 *nm*, by means of diffraction grating with 15000 lines per inch? **[T.U. 2065 Kartik]**

Solution:

For the highest order spectrum,

$$\sin\theta_n = 1$$

Thus,

$$(a+b) = n\lambda$$

or, $n \quad = \dfrac{(a+b)}{\lambda} = \dfrac{1}{N\lambda} = \dfrac{1}{15000}\,inch \times \dfrac{1}{559\ nm}$

$$= \frac{1}{15000}\times 2.54\ cm \times \frac{1}{559\times10^{-9}\times 100\ cm}$$

$\therefore n \quad = 3$

The highest order spectrum which may be seen with monochromatic light of wavelength 559 *nm* is 3.

Show that the intensity of second order maxima of Fraunhoffer's single slit diffraction is $\left(\dfrac{2}{5\pi}\right)^2$ times the intensity of central maxima. **[T.U. 2066 Ashadh]**

Solution: See the solution of Q. No. 4 on page no. 154

For the first secondary maxima, $\alpha = \dfrac{5\pi}{2}$

Thus,

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{1}{\frac{5\pi}{2}}\right)^2 = \left(\frac{2}{5\pi}\right)^2 I_0$$

i.e., the intensity of second order maxima of Fraunhoffer's single slit diffraction is $\left(\dfrac{2}{5\pi}\right)^2$ times the intensity of central maxima.

10. What is diffraction light? Discuss the intensity distribution with special reference to diffraction of light in a single slit. [T.U. 2067 Mangsir]

Solution: See the solution of Q. No. 2 and 4 on page no. 153 and 154

11. Show that the intensity distribution pattern of Franuhoffer's single slit diffraction is $I_0\left[\frac{\sin\alpha}{\alpha}\right]^2$, where symbols carry their usual meanings. [T.U. 2067 Mangsir]

Solution: See the solution of Q. No. 4 on page no. 154

12. Define Fresnel's assumption of diffraction. Discuss the Fraunhoffer diffraction pattern at two slits. [P.U. 2002]

Solution:

Fresnel's assumption of diffraction

Fresnel assumed that a wavefront can be divided into a large number of strips or zones called Fresnel's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from various zones. The effect at a point due to any particular zone will depend on the distance of the point of the zone. The effect at any point will also depend on the obliquity of the point with reference to the zone under consideration.

Fraunhoffer diffraction pattern at two slits

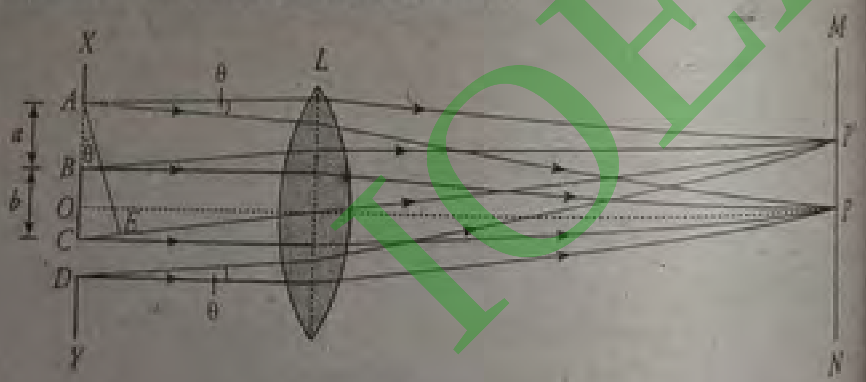

Figure: Fraunhoffer double slit diffraction

Consider two rectangular slits parallel to each other. The width of each slit is a and the width of the opaque portion is b. Let L be a collecting lens and MN is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wavefront be incident on the surface of

XY. All the secondary waves travelling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

The diffraction pattern has to be considered in two parts. The interference phenomenon due to the secondary waves emanating from the corresponding points of the two slits and the diffraction pattern due to the secondary waves from the two slits individually. For calculating the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ. Both the angles θ and ϕ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

Interference maxima and minima

Consider the secondary travelling in a direction at an angle θ with in initial direction. In the ΔACN,

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

or, $$CN = (a+b)\sin\theta \quad \ldots \text{(i)}$$

If the path difference is equal to odd multiples of $\frac{\lambda}{2}$, θ gives the direction of minima due to interference of the secondary waves from the two slits.

$$\therefore\ CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

or, $$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \quad \ldots \text{(ii)}$$

Substituting the non zero integral values of the n, we obtain corresponding directions of minima.

If the secondary waves travel in a direction θ' such that the path difference is even multiples of $\frac{\lambda}{2}$, then θ' gives the direction of the maxima due to interference of light waves emanating from the two slits.

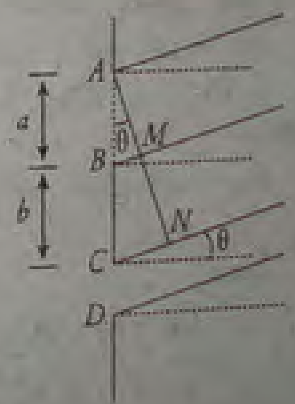

Figure: Sketch for the interference and diffraction maxima and minima

$$\therefore\ CN = (a+b)\sin\theta'_n = (2n)\frac{\lambda}{2} = n\lambda$$

$$\text{or,}\quad \sin\theta_n' = \frac{n\lambda}{(a+b)} \qquad \cdots \text{(iii)}$$

Substituting the non zero integral values of the n, we obtain corresponding directions of maxima.

It is found that the angular separation between any two consecutive minima or maxima is equal to $\frac{\lambda}{(a+b)}$. The angular separation is inversely proportional to $(a+b)$, the distance between two slits.

Diffraction maxima and minima

Consider secondary waves travelling in a direction inclined at an angle ϕ with the initial direction of the incident light. If the path difference BM is equal to λ the wavelength of light used, ϕ will give direction of diffraction minimum, *i.e.*, path difference between secondary waves emanating from extremities of a slit equal to λ. Considering the wavefornt on AB be made up of two halves the path difference between the corresponding points of the upper and the lower halves is equal to $\frac{\lambda}{2}$. The effect at P' due to wavefront incident on AB is zero. Similarly, for the same direction of the secondary waves, the effect at P' due to the wavefront incident on the slit CD is also zero. Thus, in general,

$$a\sin\phi_n = n\lambda \qquad \cdots \text{(iv)}$$

Substituting the non zero integral values of the n, we obtain corresponding directions of diffraction minima.

---

13. How many orders will be visible if the wavelength of the incident radiation is 5000 Å and the number of lines on the grating is 2620 per inch? [P.U. 2004]

Solution:

For the highest possible order spectrum,

$$\sin\theta_n = 1$$

Thus,

$$(a+b) = n\lambda$$

$$\text{or,}\quad n = \frac{(a+b)}{\lambda} = \frac{1}{N\lambda} = \frac{1}{2620}\,inch \times \frac{1}{5000\text{ Å}}$$

$$= \frac{1}{2620} \times 2.54\ cm \times \frac{1}{5000 \times 10^{-8}\ cm}$$

$$\therefore\quad n = 19$$

The highest order of spectrum that can be seen is 19.

A monochromatic light of wavelength 5890 Å is incident normally on a diffraction grating which has 6000 lines per cm. At what angle will the second order image be seen? Can you obtain the third order image with this grating? [P.U. 2005]

Solution:

Here,

$$a+b = \frac{1}{N} = \frac{1}{6000}\ cm$$

For the second order diffraction,

$$(a+b)\sin\theta = n\lambda$$

$$\text{or,}\quad \sin\theta = \frac{n\lambda}{(a+b)} = \frac{2\times 5890\text{ Å}}{1\ cm} \times 6000$$

$$= 2\times 5890 \times \frac{10^{-8}\,cm}{1\ cm} \times 6000 = 0.7068$$

$$\therefore\quad \theta = 45°$$

At 45°, second order image is seen.

For the third order spectrum, $n = 3$.

Thus,

$$(a+b)\sin\theta = 3\lambda$$

$$\text{or,}\quad \sin\theta = \frac{3\lambda}{(a+b)} = \frac{3\times 5890\text{ Å}}{1\ cm}\times 6000$$

$$= 3\times 5890 \times \frac{10^{-8}\,cm}{1\ cm}\times 6000 = 1.06$$

Since $-1 \le \sin\theta \le 1$, third order spectrum is not possible with this grating.

What is meant by diffraction of light? Derive an expression for the variation of intensity of diffraction pattern due to single slit. [P.U. 2007]

Solution: See the solution of Q. No. 2 and 4 on page no. 153 and 154

A diffraction grating used at normal incidence gives a line 6000 Å in a certain order superimposed on another line 4500 Å of the next higher order. If the angle of diffraction is 30°, how many lines are there in a *cm* in the grating? [P.U. 2010]

Solution:

Let n be order of spectrum observed due to diffraction grating with the incidence of light of wavelength $\lambda_1 = 6000$ Å and $(n+1)$ be next higher order spectrum observed with the incidence of light of wavelength $\lambda_2 = 4500$ Å.

For the n^{th} order diffraction with $\lambda_1 = 6000$ Å

$$(a+b)\sin 30^\circ = n \times 6000 \text{ Å}$$

For the $(n+1)^{th}$ order diffraction with $\lambda_2 = 4500$ Å

$$(a+b)\sin 30^\circ = (n+1) \times 4500 \text{ Å}$$

Dividing these equations, we obtain

$$1 = \frac{n}{(n+1)} \times \frac{60}{45}$$

$$\text{or,} \quad 45n + 45 = 60n$$

$$\therefore \quad n = 3$$

Thus,

$$(a+b)\sin 30^\circ = 3 \times 6000 \text{ Å} = 3 \times 6000 \times 10^{-10}\, m$$

$$\text{or,} \quad a+b = 3.60 \times 10^{-6}\, m$$

$$\therefore \quad N = \frac{1}{a+b} = 277778\, m^{-1} = 2777\, cm^{-1}$$

There are 2777 lines per cm of grating.

17. A plane grating has 1500 lines per inch. Find the angle of separation of the 5048 Å and 5016 Å lines of helium in the second order spectrum. [P.U. 2011]

Solution:

Here,

$$a+b = \frac{1}{N} = \frac{1}{1500}\ inch = \frac{2.54}{1500}\ cm$$

For the second order diffraction with $\lambda_1 = 5048$ Å $= 5048 \times 10^{-8} cm$

$$(a+b)\sin\theta_1 = n\lambda_1$$

$$\text{or,} \quad \sin\theta_1 = \frac{n\lambda_1}{a+b} = \frac{2 \times 5048 \times 10^{-8}\ cm}{2.54\ cm} \times 1500$$

$$= 5.962 \times 10^{-2}$$

$$\therefore \quad \theta_1 = \sin^{-1}(5.962 \times 10^{-2}) = 3.42^\circ$$

For the second order diffraction with $\lambda_2 = 5016$ Å $= 5016 \times 10^{-8} cm$

$$(a+b)\sin\theta_2 = n\lambda_2$$

$$\text{or,} \quad \sin\theta_2 = \frac{n\lambda_2}{a+b} = \frac{2 \times 5016 \times 10^{-8}\ cm}{2.54\ cm} \times 1500$$

$$= 5.924 \times 10^{-2}$$

$$\therefore \quad \theta_1 = \sin^{-1}(5.924 \times 10^{-2}) = 3.40^\circ$$

Chapter 8

POLARIZATION

INTRODUCTION

...rence and diffraction phenomena have shown that light is a form ...ve motion. These effects do not tell us about the type of wave ...e, i.e., whether the light is longitudinal wave or transverse wave. ...phenomenon of polarization has confirmed that light waves are ...rse wave. Polarization of light is the phenomenon due to which ...bration of light waves can be restricted in a particular plane. Such ...is called plane polarized light. The materials which polarize the ...rse wave are called polaroids. *Example:* Tourmaline crystal, Nichol ..., Calcite crystal, etc.

POLARIZED LIGHT

...ght which has acquired the property of one-sidedness is called ...ed light. The light which has same property in all directions is ...d unpolarized light.

...e vibrations take place only in one direction parallel to the plane ...gh the axis of a beam, light is said to be *plane polarized light.*

...a light polarizes due to the superposition of two waves of equal ...tude with orthogonal linear polarizations and in $\frac{\pi}{2}$ out of phase, ...ght is said to *circularly polarized light.*

...e two waves do not have equal amplitudes, the light is *elliptically ...ized light.*

DOUBLE REFRACTION

...n a ray of light is refracted by a crystal of calcite it gives two ...ted rays. This phenomenon is called double refraction. Calcite or ...nd spar is crystallized calcium carbonate and was found in large ...es in Iceland as very large transparent crystal crystals. It ...lizes in many forms and can e reduced by cleavage or breakage into ...mbohedron, bounded by six parallelograms.

...light rays refracted by these crystals, are called ordinary rays and ...ordinary rays. It is found that velocity of ordinary ray inside the crystal will be less compared to the velocity of light for the extraordinary ray. Moreover, the velocity of extraordinary ray is different in different direction because its refractive index varies with the angle of incidence. It has been found that both the rays are plane polarized. The vibrations of the ordinary ray are perpendicular to the principal section of the crystal while the vibrations of the extraordinary ray are in plane of the principal section of the crystal. Thus, the two rays are plane polarized, their vibrations being at right angles to each other.

8.4 NICOL PRISM

Nicol prism is an optical device used for producing and analyzing plane polarized light. When light is passed through a doubly refracting crystal it is split up into the ordinary ray and extraordinary ray. Both of these rays are plane polarized. One of these rays is cut off by total internal reflection. It is generally found that the ordinary ray is eliminated and only the extraordinary ray is transmitted through the prism.

Nicol prism is an optical device used for producing and analyzing plane polarized light. When light is passed through a doubly refracting crystal it is split up into the ordinary ray and extraordinary ray. Both of these rays are plane polarized. One of these rays is cut off by total internal reflection. It is generally found that the ordinary ray is eliminated and only the extraordinary ray is transmitted through the prism.

8.5 SPECIFIC ROTATION

Liquids containing an optically active substance (*Example:* sugar solution, camphor in alcohol, etc.) rotate the plane of the linearly polarized light. The angel through which the plane polarized light is rotated depends upon the thickness of the medium, concentration of the solution or density of the active substance in the solvent, wavelength of light and temperature.

The specific rotation is defined as the rotation produced by a decimeter (*i.e.*, 10 *cm*) long column of the liquid containing 1 gram of the active substance in one cubic centimeter solution. Thus,

$$S_\lambda^t = \frac{10\,\theta}{lC}$$

where, S_λ^t represents the specific rotation at temperature t°C for a wavelength λ, θ is angle of rotation, l is length of the solution in centimeter through which plane polarized light passes and C is concentration of the active substance in gm/cc in the solution.

8.6 QUARTER WAVE PLATE AND HALF WAVE PLATE

Quarter wave plate

A quarter wave plate is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optical axis. The incident plane polarized light is perpendicular to is surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_E respectively, then the path difference produced between the two rays is

For negative crystals, path difference $= (\mu_0 - \mu_E)t$

For negative crystals, path difference $= (\mu_E - \mu_0)t$

To produce a path difference of $\frac{\lambda}{4}$ in calcite crystal, we must have,

$$(\mu_0 - \mu_E)t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_0 - \mu_E)}$$

For quartz crystal,

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

When plane polarized light with plane of vibration is inclined at an angle 45° to the optical axis, is incident on a quarter wave plate the emergent light circularly polarized.

Half wave plate

A half wave plate is made from a doubly refracting uniaxial crystal of quartz or calcite with its refracting faces cut parallel to the optical axis. The thickness of the plate is such that the ordinary and extraordinary rays have a path difference $\frac{\lambda}{2}$ after passing through the crystal.

For negative crystals, path difference $= (\mu_0 - \mu_E)t$

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To produce a path difference of $\frac{\lambda}{2}$ in calcite crystal, we must have,

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$$t = \frac{\lambda}{2(\mu_0 - \mu_E)}$$

For quartz crystal,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

When plane polarized light with plane of vibration is inclined at an angle 45° to the optical axis, a path difference $\frac{\lambda}{2}$ is introduced between the extraordinary and ordinary rays. The emergent light is plane polarized and the direction of polarization of the linear incident light is rotated through 90°.

8.7 SOLVED EXAM QUESTIONS

1. **What phenomenon compels us to accept the wave nature of light? Give example.** **[T.U. 2061 Baishakh]**

Solution:

Interference and diffraction phenomena have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion, *i.e.*, whether the light is longitudinal wave or transverse wave. The phenomenon of polarization has confirmed that light waves are transverse wave. Polarization of light is the phenomenon due to which the vibration of light waves can be restricted in a particular plane. Such light is called plane polarized light. The materials which polarize the transverse wave are called polaroids. *Example:* Tourmaline crystal, Nichol prism, Calcite crystal, etc.

2. **What are quarter and half wave plates? Derive the expression for their thickness.** **[T.U. 2061 Baishakh]**

Solution:

Quarter wave plate

Quarter wave plate is a plate of doubly refracting uniaxial crystal of calcite of quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optical axis. The incident plane polarized light is perpendicular to is surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_E respectively, then the path difference introduced between the two rays is;

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$$t = \frac{\lambda}{4(\mu_0 - \mu_E)}$$

For quartz crystal,

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

When plane polarized light with plane of vibration is inclined at an angle 45° to the optical axis, is incident on a quarter wave plate the emergent light circularly polarized.

Half wave plate

Half wave plate is made from a doubly refracting uniaxial crystal of quartz or calcite with its refracting faces cut parallel to the optical axis. The thickness of the plate is such that the ordinary and extraordinary rays have a path difference $\frac{\lambda}{2}$ after passing through the crystal.

For negative crystals, path difference = $(\mu_0 - \mu_E)t$

For negative crystals, path difference = $(\mu_E - \mu_0)t$

To produce a path difference of $\frac{\lambda}{2}$ in calcite crystal, we must have,

$$(\mu_0 - \mu_E)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_0 - \mu_E)}$$

For quartz crystal,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

When plane polarized light with plane of vibration is inclined at an angle 45° to the optical axis, a path difference $\frac{\lambda}{2}$ is introduced between the extraordinary and ordinary rays. The emergent light is plane polarized and the direction of polarization of the linear incident light is rotated through 90°.

3. **Describe how a beam of plane polarized light may be produced by reflection and double refraction. Why is it not possible to polarized sound waves?** **[T.U. 2061 Ashwin]**

Solution:

Polarization by reflection

In general, it is found that at a single glass surface or any similar transparent medium, a small fraction of the incident light is reflected. For a glass, with refractive index 1.5 at the polarizing angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for perpendicular vibrations o... 85% is transmitted and 15% is reflected. Therefore, if we u... pile of plates and beam of ordinary light is incident at ... polarizing angle on the pile of plates, some of the vibratio... perpendicular to the plane of incidence are reflected by the f... plate and rest are transmitted through it. When this beam of li... is reflected by the second plate, again some of the vibratio... perpendicular to the plane of incidence are reflected by it and t... rest are transmitted. The process continues and when the bea... has traversed about 15 to 20 plates, the transmitted light ... completely free from the vibrations at right angles to the plane o... incidence and is having vibration only in the plane of incidenc... Thus, we obtain plane polarized by refraction with the help of piles of plates, the vibration being in the plane of incidence.

Polarization by double refraction

When a ray of light is refracted by a crystal of calcite it gives tw... refracted rays. This phenomenon is called double refraction. Calcite or Iceland spar is crystallized calcium carbonate and wa... found in large quantities in Iceland as very large transparent crysta... crystals. It crystallizes in many forms and can e reduced by cleavag... or breakage into a rhombohedron, bounded by six parallelograms. The light rays refracted by these crystals, are called ordinary ray and extraordinary rays. It is found that velocity of ordinary ra... inside the crystal will be less compared to the velocity of light fo... the extraordinary ray. Moreover, the velocity of extraordinary ray is different in different direction because its refractive index varies with the angle of incidence. It has been found that both the ray are plane polarized. The vibrations of the ordinary ray ar... perpendicular to the principal section of the crystal while th... vibrations of the extraordinary ray are in plane of the principa... section of the crystal. Thus, the two rays are plane polarized, the... vibrations being at right angles to each other.

Sound wave is a longitudinal wave and polarization phenomeno... is only shown by transverse wave. Hence, polarization is no... possible for sound waves.

4. **What are quarter wave and half wave plates? Derive the relation for them.** **[T.U. 2062 Baishakh]**

Solution: See the solution of Q. No. 2 on page no. 166

5. **What is specific rotation? Describe the constructive and working of Laurent's half shade polarimeter. Explain how will you use it to determine the specific rotation of sugar solution.** **[T.U. 2063 Baishakh]**

Solution:

Specific rotation

Liquids containing an optically active substance (*Example:* sugar solution, camphor in alcohol, etc.) rotate the plane of the linearly polarized light. The angel through which the plane polarized light is rotated depends upon the thickness of the medium, concentration of the solution or density of the active substance in the solvent, wavelength of light and temperature.

The specific rotation is defined as the rotation produced by a decimeter (*i.e.*, 10 *cm*) long column of the liquid containing 1 gram of the active substance in one cubic centimeter solution. Thus,

$$S_\lambda^t = \frac{10\,\theta}{lC}$$

where, S_λ^t represents the specific rotation at temperature t°C for a wavelength λ, θ is angle of rotation, l is length of the solution in centimeter through which plane polarized light passes and C is concentration of the active substance in gram per cubic centimeter in the solution.

Laurent's half shade polarimeter

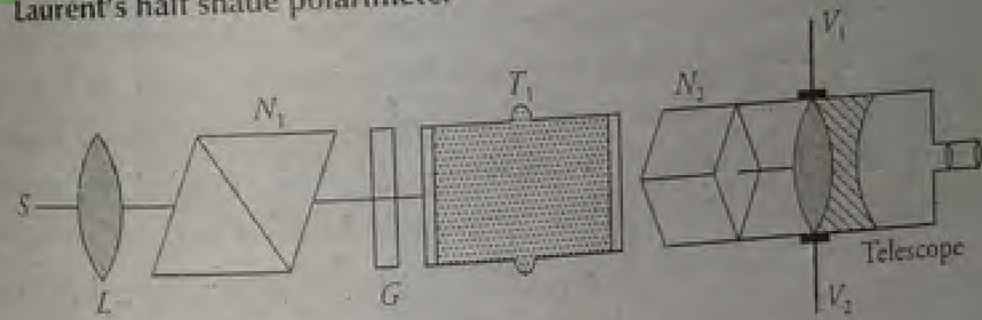


Figure: Laurent's half shade polarimeter

It consists of two Nicol prisms N_1 and N_2. N_1 is a polarizer and N_2 is an analyser. Behind N_1, there is a half plate of quartz Q which cover one half of the field of view, while the other half G is a glass plate. The glass plate absorbs the same amount of light as the quartz plate. T is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles, if present, will appear at the upper portion of the wide bore T_1 of the tube.

Light from the monochromatic source S is incident on the converging lens. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and other half passes through glass plate G. Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along AB.

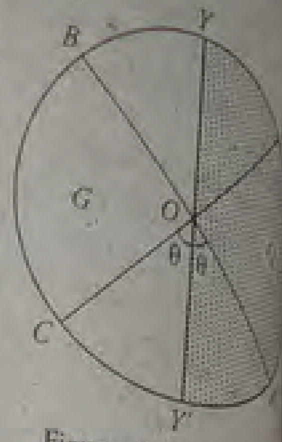


Figure: Half shade plate

Here, AB makes angle θ with Y'. On passing through the quartz plate Q, the beam is split up into ordinary and extraordinary components which travel along the same direction but of different speeds and emergence a phase difference of π or a path difference of $\frac{\lambda}{2}$ introduced between them. The vibration of the beam emerging out of the glass plate will be along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyzer N_2 has its principal plane of section along Y', i.e., along the direction which bisects the angle AOC, the amplitudes of light incident on the analyzer N_2 from both the halves will be equal. Therefore, the field of view will be equally bright. If the analyzer N_2 is rotated to the right of YY', then the right half will be brighter as compared to the left half. On the other hand, if the analyzer N_2 is rotated to the left of YY', the left half of brighter as compared to the right half. These three different cases are shown in figure.

To determine the specific rotation of an optically active substance, the analyzer N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube. The readings on the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, AB and CD are rotated in the clock-wise direction. Therefore, on the introduction of the tube containing the optically active substance

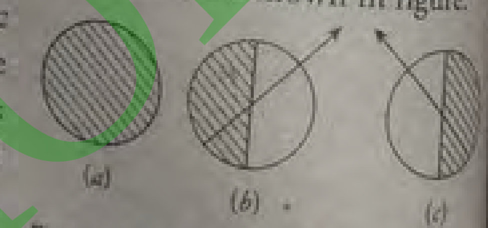


Figure: Field of view due to the rotation of analyzer in polarimeter

the field of view is not equally bright. The analyzer is rotated in the clockwise direction and is brought to a position so that the whole field of view is equally bright. The new positions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyzer has been rotated gives the angle through which the plane of vibrations of the incident beam has been rotated by optically active substance. In actual experiments, for various concentration of the optically active substance, the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation. The graph is straight line. Thus, from the relation,

$$S_\lambda^t = \frac{10\,\theta}{lC}$$

The specific rotation of the optically active substance is calculated.

Calculate the polarizing angle of light travelling from water (η = 1.33) to glass (η = 1.5). **[T.U. 2064 Poush]**

Solution:

Here,

Refractive index of water, $(\eta_1) = 1.33$

Refractive index of glass, $(\eta_2) = 1.5$

We have,

$$\theta_p = tan^{-1}\left(\frac{\eta_2}{\eta_1}\right) = tan^{-1}\left(\frac{1.5}{1.33}\right) = 48.44°$$

The polarizing angle of light travelling from water to glass is 48.44°.

What is double refraction in optics? Among the two refracted rays in double refraction phenomenon which one shows the refractive index varies with the angel of incidence. How do you define the +ve and –ve of crystals? **[T.U. 2065 Shrawan]**

Solution:

Double reflection in optics

See the solution of Q. No. 3 on page no. 167

Whenever a refractive index of an ordinary ray is greater than that of an extraordinary ray in double refraction, such crystal is said to be negative crystal. *Example:* Calcite crystal.

Whenever a refractive index of an extraordinary ray is greater than that of an ordinary ray in double refraction, such crystal is said to be positive crystal. *Example:* Quartz crystal.

8. **Calculate the thickness of a quarter wave plate and a half wave pl[ate] given that $\mu_E = 1.553, \mu_0 = 1.544$ and $\lambda = 5890$ Å.** [T.U. 2065 Cha[itra]]

Solution:

Here,

$\mu_E = 1.553$

$\mu_0 = 1.544$

$\lambda = 5890 \text{ Å} = 5890 \times 10^{-10}\, m$

For quarter wave plate,

$$t = \frac{\lambda}{4(\mu_E - \mu_0)} = \frac{5890 \times 10^{-10}}{4(1.553 - 1.544)} = 1.64 \times 10^{-5}\, m$$

For half wave plate,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)} = \frac{5890 \times 10^{-10}}{2(1.553 - 1.544)} = 3.27 \times 10^{-5}\, m$$

The thickness of quarter wave plate and half wave plate a[re] $1.64 \times 10^{-5}\, m$ and $3.27 \times 10^{-5}\, m$ respectively.

9. **What do you mean by plane polarized light? Explain the phenomenon of double refraction in crystal.** [T.U. 2065 Karti[k]]

Solution:

Plane polarized light

When vibrations take place only in one direction parallel to the plane through the axis of a beam, light is said to be *plane polarized*.

Double reflection in optics

See the solution of Q. No. 3 on page no. 167

10. **Calculate the thickness of quarter wave plate for light of wavelength 5893 Å. Given refractive indices of ordinary and extraordinary rays are 1.544 and 1.553 respectively.** [T.U. 2067 Ashadh]

Solution:

Here,

$\mu_E = 1.553$

$\mu_0 = 1.544$

$\lambda = 5893 \text{ Å} = 5893 \times 10^{-10}\, m$

For quarter wave plate,

$$t = \frac{\lambda}{4(\mu_E - \mu_0)} = \frac{5893 \times 10^{-10}}{4(1.553 - 1.544)} = 1.64 \times 10^{-5}\, m$$

For half wave plate,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)} = \frac{5890 \times 10^{-10}}{2(1.553 - 1.544)} = 3.27 \times 10^{-5}\, m$$

The thickness of quarter wave plate and half wave plate are $1.64 \times 10^{-5}\, m$ and $3.27 \times 10^{-5}\, m$ respectively.

A 20 cm long tube having sugar solution rotates the plane of polarization by 20°. If the specific rotation of sugar solution is 66°, calculate the strength of the solution. [T.U. 2068 Shrawan]

Solution:

Here,

Length of the tube, $(l) = 20\ cm$

Rotation of plane of polarization, $(\theta) = 20°$

Specific rotation, $(S'_\lambda) = 66°$

We have,

$$S'_\lambda = \frac{10\theta}{lC}$$

$$\text{or, } C = \frac{10\theta}{lS'_\lambda} = \frac{10 \times 20}{20 \times 66} = 0.15\ gm\ cm^{-3}$$

The strength of solution is $0.15\ gm\ cm^{-3}$.

A 200 mm long tube containing 52 cm^3 of sugar solution produces an optical rotation of 13° when placed in a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of solution. [P.U. 2002]

Solution:

Here,

Volume of the tube, $(V) = 52\ cm^3$

Length of the tube, $(l) = 200\ mm = 20\ cm$

Optical rotation, $(\theta) = 13°$

Specific rotation, $(S'_\lambda) = 66°$

We have,

$$S'_\lambda = \frac{10\,\theta}{lC}$$

$$\text{or, } C = \frac{10\,\theta}{lS'_\lambda} = \frac{10 \times 13}{20 \times 66} = 0.098\ gm\ cm^{-3}$$

Quantity of sugar in the solution,

$$M = CV = 52 \times 0.098 \doteq 5.10\ gm$$

A 20 cm long tube having sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar solution is 66°, calculate the strength of the solution. [P.U. 2003]

Solution:

Here,

Length of the tube, $(l) = 20\ cm$

Rotation of plane of polarization, $(\theta) = 11°$

Specific rotation, $(S_\lambda^t) = 66°$

We have,

$$S_\lambda^t = \frac{10\theta}{lC}$$

$$\text{or,} \quad C = \frac{10\theta}{lS_\lambda^t} = \frac{10 \times 11}{20 \times 66} = 0.083\ gm\ cm^{-3}$$

The strength of solution is $0.083\ gm\ cm^{-3}$.

14. What is meant by polarized and unpolarized light? Describe the construction of a Nicol prism and how it works as polarizer and analyzer. [P.U. 200…]

Solution:

The light which has acquired the property of one-sidedness is called *polarized light*. The light which has same property in all directions is called *unpolarized light*.

Nicol prism is an optical device used for producing and analyzing plane polarized light. When light is passed through a doubly refracting crystal it is split up into the ordinary ray and extraordinary ray. Both of these rays are plane polarized. One of these rays is cut off by total internal reflection. It is generally found that the ordinary ray is eliminated and only the extraordinary ray is transmitted through the prism.

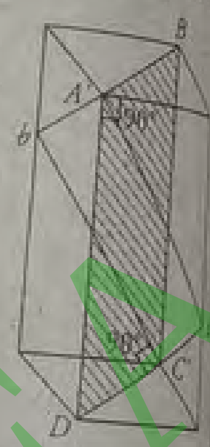


Figure: Nicol prism

A calcite crystal whose length is three time its breadth is taken. The end faces of crystal are cut down so as to reduce the angles at B and D from 71° in the principal section to a more acute angle of 68°. The crystal is then cut along the plane $A'bC'd$ perpendicular both to the principal section $A'BC'D$ and the end faces such that $A'C'$ makes an angle of 90° with the end faces $A'B$ and $C'D$. The two cut surfaces are ground, polished optically flat and then cemented together with Canada balsam, a transparent cement do that the crystal is just as transparent as it was previously to its having been sliced. The refractive index of Canada balsam μ_b has a value which lies midway between the refractive index of calcite's ordinary ray and that of extraordinary ray. The values of these for the sodium light of mean wavelength $\lambda = 5893Å$ are $\mu_o = 1.658$, $\mu_b = 1.55$ and $\mu_e = 1.486$. The sides of the prism are blackened … totally reflected rays.

The Nicol prism can be used as a polarizer as well as an analyzer. When two Nicol prisms are mounted coaxially, the first N_1 which produces plane polarized light is called the polarizer while the second N_2 which analyses the incoming light is called the analyzer. When unpolarized beam of light is incident on a Nicol prism N_1, the light emerging out of it is plane polarized and has vibrations parallel to its principal section. If now this light is made to pass through a second Nicol N_2 the principal section of which is parallel to the principal section of N_1, the light vibration in N_2 are parallel to its principal section and hence are completely transmitted. The intensity of the emergent beam is maximum.

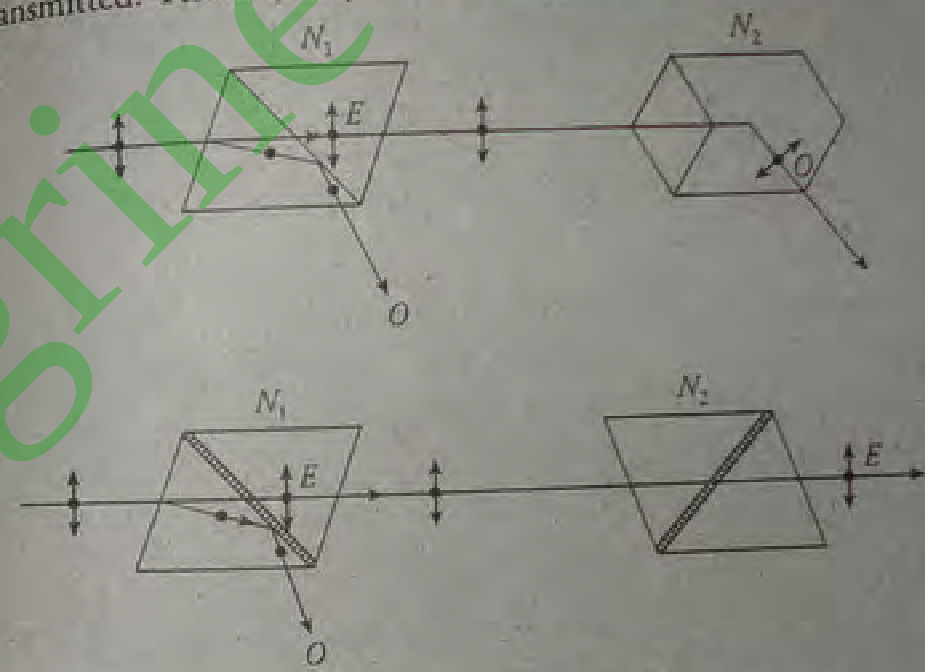


Figure: Nicol prisms as polarizer and analyzer

Now if the Nicol prism N_2 is rotated such that its principal section becomes perpendicular to that of N_1, then the vibrations of incident light in N_2 will be perpendicular to the principal section of N_2. These behave as ordinary vibrations for N_2 and are thus totally reflected and hence no light emerges from the second Nicol prism N_2. In this position the two Nicol prisms are said to be crossed. When the Nicol prism N_2 is further rotated the principal section of the two Nicol prisms again in parallel position. In this position the emergent ray is again transmitted through the analyzer Nicol prism N_2.

After rotation of 270° Nicol prisms N_1 and N_2 are again crossed position and no light is transmitted.

15. Explain plane polarized, circularly polarized and elliptically polarized light. [P.U. 2004]

Solution:

Plane polarized

See the solution of Q. No. 9 on page no. 172

When a light polarizes due to the superposition of two waves of equal amplitude with orthogonal linear polarizations and in $\frac{\pi}{2}$ out of phase, such light is said to be *circularly polarized light.*

When two waves do not have equal amplitudes, the light is *elliptically polarized.*

---

16. What do you understand by double refraction? Describe the construction and action of Nicol prism. [P.U. 2005]

Solution:

Double refraction

See the solution of Q. No. 3 on page no. 167

Nicol prism

See the solution of Q. No. 14 on page no. 174

If a ray light SM' is incident nearly parallel to BC' in the plane of the paper on the face B', it suffers double refraction and gives rise to the extraordinary ray ME and the ordinary ray MN. The extraordinary ray passes which is plane polarized and has vibrations in the plane of the paper. The ordinary ray which is also plane polarized suffers total internal reflection at the Canada balsam layer for nearly normal incidence.

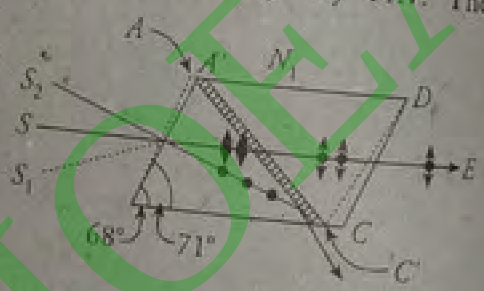


Figure: Working of Nicol prism

It is because Canada balsam is optically denser than calcite for the extraordinary and less dense for ordinary ray. The extraordinary ray is refracted through Canada balsam and is transmitted but the ordinary ray, moving from a denser calcite medium to the rarer Canada balsam medium, is totally reflected for angles of incidence greater than the critical angle.

If the incident ray makes an angel much smaller than $\measuredangle BMS$ with the surface $A'B$ the ordinary ray will strike the balsam layer at an angle less than the critical angle and hence will be transmitted.

If the incident ray makes angle greater than $\measuredangle BMS$ the extraordinary ray will become more and more parallel to the optical axis $A'Y$ and hence its refractive index will become nearly equal to that of calcite for the ordinary ray. This will then also suffer total internal reflection like the ordinary ray. Hence no light will emerge out of the Nicol's prism. A Nicol's prisms, therefore, cannot be used for highly convergent or divergent beams. The angle between the extreme rays of the incoming beam is limited to about 28°.

Chapter 9

LASER

9.1 LASER AND ITS CHARACTERISTICS

LASER stands for Light Amplification by Stimulated Emission [of] Radiation. A laser or an optical maser works on the principal of quantum theory of radiation. It is a highly concentrated, monochromatic coherent and unidirectional beam of light. The laser emerges out as a narrow beam which can travel over a distance without much loss in its intensity.

The main characteristics of laser are as follows:

i) The laser beam is coherent both spatially and temporally to the light.

ii) A laser beam is highly directional.

iii) It has high intensity and is monochromatic.

iv) It can be sharply focused.

9.2 SPONTANEOUS, STIMULATED AND INDUCED EMISSIONS

Whenever a particle is in excited state and no external radiation is present. After a time, particle will move of its own accord to its ground state emitting a photon of energy hf in the process, where, h is plank's constant and f is frequency. Such phenomenon is said to be spontaneous emission.

Whenever a photon of energy hf can stimulate a particle to move to ground state, during which an additional photon is emitted, whose energy is also hf. Such phenomenon is called stimulated emission.

Whenever a particle is initially in lower state which is raised to higher state by absorbing a photon energy hf. Such phenomenon is said to be induced emission.

9.3 ACTIVE MEDIUM, POPULATION INVERSION AND META-STABLE STATE

A medium in which light gets amplified is said to be active medium. Only small fraction of particular medium is responsible for stimulated emission. These are called active centers.

Under the ordinary conditions of thermal equilibrium, the number of particles in the higher energy state is considerably smaller than number of particles in lower energy state. The establishment of situation in which number of particles in higher energy level is greater th[an] ... of lower

energy level is called population inversion. The population inversion is achieved by exciting the medium with suitable form of energy. Such phenomenon is called pumping.

In general, life time of particle in excited state is very short, *i.e.*, 10^{-8} s. However, particles of some special elements can stay for longer time in the higher energy state where the life time is greater than 10^{-4} s. Such state is called meta-stable state.

9.4 APPLICATIONS OF LASER

i) Laser offers a wide opportunity to investigate the basic laws of interaction of atoms and molecules with electromagnetic wave of high intensity.

ii) Laser light is coherent, thus it can be used for measurement of distances based on interfereometric techniques.

iii) Laser light can be used in different communication system.

iv) It can be used in welding and cutting of materials.

v) It is used in eye surgery for the treatment of detached retina. It is widely used for treatment of dental decay, destruction of malignant tumors and treatment of skin diseases.

vi) It is used for automatic control of rockets and satellites. It can also be used for detection and destroy airplanes, missiles and tanks.

vii) It can be used in laser induced fusion process and isotopes separation.

9.5 SOLVED EXAM QUESTIONS

1. **Define the terms spontaneous emission, stimulated emission, meta-stable state and population inversion. How are these concepts related to the laser emission? [T.U. 2061 Baishakh]**

Solution:

Whenever a particle is in excited state and no external radiation is present. After a time, particle will move of its own accord to its ground state, emitting a photon of energy hf in the process, where, h is plank's constant and f is frequency. Such phenomenon is said to be spontaneous emission.

Whenever a photon of energy hf can stimulate a particle to move to ground state, during which an additional photon is emitted, whose energy is also hf. Such phenomenon is called stimulated emission.

In general, life time of particle in excited state is very short, *i.e.*, 10^{-8} s. However, particles of some special elements can stay for longer time in the higher energy state where the life time is greater than 10^{-4} s. Such state is called meta-stable state.

Under the ordinary conditions of thermal equilibrium number of particles in the higher energy state is considered smaller than number of particles in lower energy state. establishment of situation in which number of particles in h energy level is greater than that of lower energy level is c population inversion.

For the emission of laser we consider an assembly of particles special element such that each particle has lower energy state meta–stable state E_2 and higher energy state E_3 where energy difference between E_2 and E_3 is very small. The majority electrons go tho the higher energy state E_3 from the lower energy state E_1 by supplying photons carrying the energy $E_3 - E_1 =$ which is called optical pumping. The excited particles under nonradiative transition with a transfer of energy to lattice thermal motion and they go to the meta–stable state E_2. But the partic can stay in meta–stable state for the longer time so that number of particles reaching the meta–stable states more more as time increases and hence majority of atoms lie in meta stable state which is called population inversion.

When a photon carrying energy $E_2 - E_1 = hf$ incident in su group of atoms where population inversion takes place, it interac with one of the particles present in meta–stable state so th particle goes to the lower energy state with its emission of simil photon as that of incident photon. The two photons, one incide photon and other emitted photon, are available for the interactio of next two particles present in meta–stable state and so on, whi is called stimulated emission. Thus, finally, we can obtain a bea of photons which are perfectly monochromatic, coherent an parallel which is called laser.

2. **Define spontaneous and stimulated emissions. Explain the principles of generation of laser light.** **[T.U. 2061 Ashwin]**

Solution:

Spontaneous emission

Whenever a particle is in excited state and no external radiation is present, After a time, particle will move of its own accord to its ground state, emitting a photon of energy hf in the process. where, h is plank's constant and f is fr...y. Such phenomenon is said to be spontaneous emission.

Stimulated emission

Whenever a photon of energy hf can stimulate a particle to move to ground state, during which an additional photon is emitted, whose energy is also hf. Such phenomenon is called stimulated emission.

Principle of generation of laser light

For the emission of laser we consider an assembly of particles of special element such that each particle has lower energy state E_1, meta–stable state E_2 and higher energy state E_3 where energy difference between E_2 and E_3 is very small. The majority of electrons go tho the higher energy state E_3 from the lower energy state E_1 by supplying photons carrying the energy $E_3 - E_1 = hf'$ which is called optical pumping. The excited particles undergo non–radiative transition with a transfer of energy to lattice thermal motion and they go to the meta–stable state E_2. But the particles can stay in meta–stable state for the longer time so that the number of particles reaching the meta–stable states more and more as time increases and hence majority of atoms lie in meta–stable state which is called population inversion.

When a photon carrying energy $E_2 - E_1 = hf$ incident in such group of atoms where population inversion takes place, it interacts with one of the particles present in meta–stable state so that particle goes to the lower energy state with its emission of similar photon as that of incident photon. The two photons, one incident photon and other emitted photon, are available for the interaction of next two particles present in meta–stable state and so on, which is called stimulated emission. Thus, finally, we can obtain a beam of photons which are perfectly monochromatic, coherent and parallel which is called laser.

3. **What is Laser? Define the terms: optical pumping and population inversion. Why laser light in He–Ne laser is from neon and not from helium?** **[T.U. 2064 Poush]**

Solution:

Laser:

It stands for Light Amplification by Stimulated Emission of Radiation. A laser or an optical maser works on the principal of quantum theory of radiation. It is a highly concentrated, monochromatic coherent and unidirectional beam of light. The ...merges out as a narrow beam which can travel over a distance without much loss in its intensity. A laser beam is highly directional. It has high intensity and is monochromatic. It can be sharply focused.

Optical pumping

For the emission of laser we consider an assembly of particles of special element such that each particle has lower energy state in meta–stable state E_2 and higher energy state E_3 where energy difference between E_2 and E_3 is very small. The majority of electrons go tho the higher energy state E_3 from the lower energy state E_1 by supplying photons carrying the energy $E_3 - E_1 =$ … which is called optical pumping.

Population inversion

Under the ordinary conditions of thermal equilibrium, the number of particles in the higher energy state is considerably smaller than number of particles in lower energy state. The establishment of situation in which number of particles in higher energy level is greater than that of lower energy level is called population inversion.

In He–Ne laser, the purpose of the helium atoms is population inversion in the neon atoms. Thus, laser light in He–Ne laser is from neon and not from helium.

4. **What is laser? Why laser is different from ordinary ray of light? Also explain the terms: optical pumping, population inversion and stimulated emission.**

Solution:

Laser

See the solution of Q. No. 3 on page no. 181

Laser is a coherent source of light in which rays are highly monochromatic, all in same phase and almost parallel. A beam of laser can travel long distances without any significant loss of its intensity.

Optical pumping

See the solution of Q. No. 3 on page no. 181

Population inversion

See the solution of Q. No. 3 on page no. 181

Stimulated emission

Whenever a photon of energy hf can stimulate a particle to move to ground state, during which an additional photon is emitted, whose energy is also hf. Such phenomenon is called stimulated emission.

5. **Write down the properties of laser. Also explain the terms optical pumping, population inversion and stimulated emission. Write down the applications of laser.**

Solution:

Properties of Laser

The main properties of laser are as follows:

i) The laser beam is coherent both spatially and temporally to the light.
ii) A laser beam is highly directional.
iii) It has high intensity and is monochromatic.
iv) It can be sharply focused.

Optical pumping

See the solution of Q. No. 3 on page no. 181

Population inversion

See the solution of Q. No. 3 on page no. 181

Stimulated emission

See the solution of Q. No. 4 on page no. 182

Applications of Laser

The important applications of laser are as follows:

i) Laser offers a wide opportunity to investigate the basic laws of interaction of atoms and molecules with electromagnetic wave of high intensity.
ii) Laser light is coherent, thus it can be used for measurement of distances based on interfereometric techniques.
iii) Laser light can be used in different communication system.
iv) It can be used in welding and cutting of materials.
v) It is used in eye surgery for the treatment of detached retina. It is widely used for treatment of dental decay, destruction of malignant tumors and treatment of skin diseases.
vi) It is used for automatic control of rockets and satellites. It can also be used for detection and destroy airplanes, missiles and tanks.
vii) It can be used in laser induced fusion process and isotopes separation.

Chapter 10

ELECTRIC FORCE AND ELECTRIC FIELD

10.1 ELECTRIC FIELD

One way to explain electrostatic force between two charges is to say that each charge sets up an electric field in the space around it. The electrostatic force acting on any charge is then due to the electric field set up at its location by the other charge.

The electric field $\vec{E}$ at any point is defined in terms of electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there.

i.e., $$\vec{E} = \frac{\vec{F}}{q_0}$$

10.2 ELECTRIC FIELD LINES

Electric field lines provide a means for visualizing the direction and magnitude of electric fields. The electric field vector at any point is tangent to the field lines through that point. The density of the field lines in any region is proportional to the magnitude of the electric field in that region. Field lines originate on positive charges and terminate on negative charges.

10.3 COULOMB'S LAW

"The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them."

In mathematical terms;

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

Coulomb's law is able to explain:

a) The electrical forces that bind the electrons of an atom to its nucleus.
b) The forces that bind atoms together to form molecules.
c) The forces that binds and molecules together to form solids or liquids.

10.4 FIELD DUE TO A POINT CHARGE

The magnitude of the electric field $\vec{E}$ set up by a point charge q at distance r from the charge is;

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$ 10.3

The direction of $\vec{E}$ is away from the point charge is positive and toward it if charge is negative.

10.5 ELECTRIC DIPOLE

An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d. The electric dipole moment $\vec{p}$ is defined to have magnitude $p = qd$. The direction of $\vec{p}$ is from negative towards positive charge. An electric dipole experiences torque $\vec{\tau}$,

$$\vec{\tau} = \vec{p} \times \vec{E}$$ 10.4

The magnitude of torque depends on angle ϕ between $\vec{p}$ and $\vec{E}$. The potential energy U for an electric dipole is also depends on the orientation of $\vec{p}$ and $\vec{E}$.

$$U = -\vec{p}\cdot\vec{E}$$ 10.5

The magnitude of electric field set up by electric dipole at a distance r on the dipole axis is;

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{r^3}$$ 10.6

Where, r is the distance between the point and centre of the dipole

10.6 ELECTRIC QUADRUPOLE

The arrangement of four equal and opposite charges or two dipoles arrangement with certain electric field at any point is called quadrupole.

The electric field of a quadrupole along its axis is;

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^4}$$ 10.7

10.7 ELECTRIC FIELD DUE TO LINE OF CHARGE

The electric field due to charged rod is;

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int\frac{\cos\theta\, dx}{(x^2 + z^2)}$$ 10.8

If the rod is infinitely long, the electric intensity is;

$$E = \frac{\lambda}{4\pi\varepsilon_0 z}$$ 10.9

Where, z is a perpendicular bisector of a rod

For a finite rod of length l,

$$E = \frac{Q}{2\pi\varepsilon_0\sqrt{l^2+4z^2}}$$ 10.10

10.8 GAUSS'S LAW

It states that, "*total electric flux of an electric field passing through any ... surface is directly proportional to free charges enclosed by the surface.*"

i.e., $\varepsilon_0 = \oint \vec{E}\, d\vec{s} = q$

10.9 ELECTROSTATIC POTENTIAL ENERGY

The amount of work done required to bring a unit positive test ch... from infinity to any point in the electric field of another charge is ca... electric potential energy. The electric potential energy is;

$$U = \frac{qq_0}{4\pi\varepsilon_0 r}$$

For a system of changes;

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{i=1, j=1 \\ i \neq j}}^{nm} \frac{q_i q_j}{r_{ij}}$$

where, q_i and q_j are i^{th} and j^{th} charges at a distance r_{ij} of the given po...

10.10 ELECTRIC POTENTIAL

In an electric field due to a charge at all points, there will be fi... strength and if we place a unit positive charge at any point in the field, ... will experience a force along the direction of the electric field. If the uni... charge is moved, its potential energy will be changed at certain amoun... of work done in moving the charge between any points is a measure o... electric potential.

The potential due to electric dipole is;

$$V = \frac{\vec{p} \cdot \vec{r}}{r^3} = \frac{\vec{p} \cdot \hat{r}}{r^2}$$ 10.14

The potential due to linear quadrupole is;

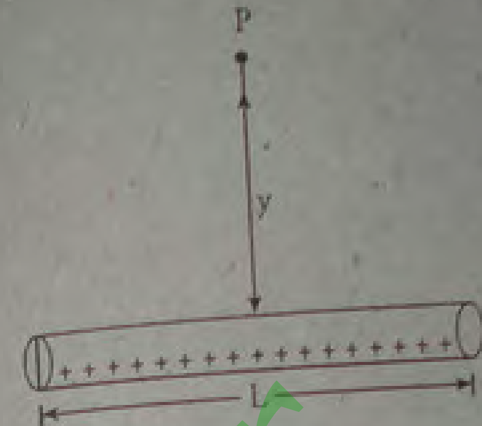
$$V = \frac{q}{4\pi\varepsilon_0 r^3}$$ 10.15

10.11 Solved Exam Questions

1. A thin non-conducting rod of finite length L carries a total charge of spread uniformly along it. Show that electric field intensity E at P on the perpendicular bisector in figure below is;

$$E = \frac{q}{2\pi\varepsilon_0 y} \frac{1}{\sqrt{L^2 + (2y)^2}}$$

[T.U. 2061 Ashwin]



Solution:

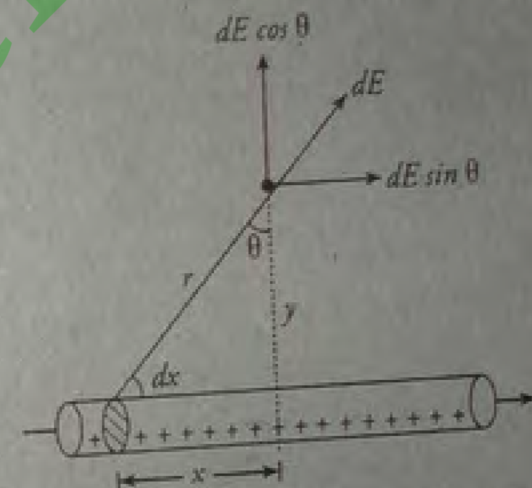


Figure: Electric field due to uniform rod

Consider a non-conducting rod uniformly distributed charge q with linear charge density λ ties along x–axis. To determine the electric field by the rod or at a point located perpendicular bisector of a rod or at a distance y, the rod is divided into small elements each of length dx. Consider one of the element at a distance x centre of the rod. It contains an element of charge given by;

$$dq = \lambda\, dx$$

This produces an electric field at P at a distance r is;

In case of resistivity;

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{x\, dx}{4\pi\varepsilon_0 (x^2 + y^2)} \quad \text{... (i)}$$

The resultant electric field due to charged rod is;

$$E = \int dE \cos\theta = \frac{\lambda}{4\pi\varepsilon_0} \int \frac{\cos\theta\, dx}{(x^2 + y^2)} \quad \text{... (ii)}$$

Since the charged rod is of finite length L, thus, we write,

$$E = \frac{\lambda y}{4\pi\varepsilon_0} \int_0^{\frac{L}{2}} \frac{dx}{(x^2 + y^2)^{\frac{3}{2}}}$$

Again putting;

$$x = y \tan\theta$$

or, $dx = y \sec^2 d\theta$

We obtain,

$$E = \frac{\lambda \frac{L}{2}}{2\pi\varepsilon_0 y\sqrt{\left(\frac{L}{2}\right)^2 + y^2}}$$

$$\therefore \quad E = \frac{q}{2\pi\varepsilon_0 y}\left[\frac{1}{L^2+(2y)^2}\right]^{\frac{1}{2}} \qquad \ldots \text{(iii)}$$

This is the electric intensity at P due to uniform charged rod of length L.

2. **What is an electric dipole? Deduce the expression for the potential due to dipole (i) at its axial line (ii) at its equatorial line.** [T.U. 2062 Baishakh]

Solution:

An electric dipole is a pair of electric charges of equal magnitude but opposite sign, separated by a finite distance.

Consider an electric dipole MN with charge $-q$ at M and $+q$ at B with separation $2d$ and O is the middle point of it.

Electric potential at P is to be determined at a distance r from the centre of dipole O and $\angle PON = \theta$. An arc with centre P and radius $OP = r$ is drawn to meet PN and PM produced at B and A. If dipole is very short; i.e., $2d = r$

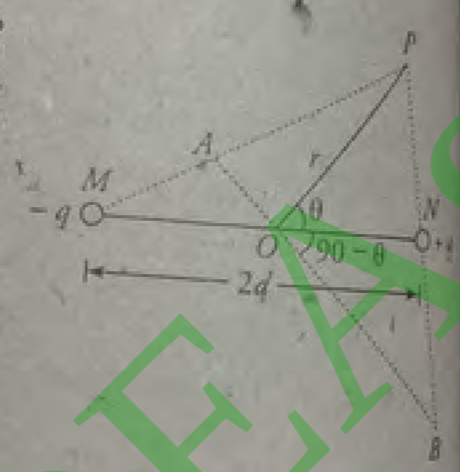


Figure: Electric dipole

Then, AB is nearly a straight line perpendicular to both PM and PB. Hence, we have,

$$AP = OP = BP = r$$
$$MA = BN$$

In ΔNOB,

$$BN = ON \sin(90° - \theta) = d\cos\theta$$

By definition of electric potential P due to dipole is;

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{PN} - \frac{q}{PM}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{PB - BN} - \frac{1}{PA + AM}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - d\cos\theta} - \frac{1}{r + d\cos\theta}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{2d\cos\theta}{r^2 - d^2\cos^2\theta}\right]$$

Since $r^2 \gg d^2$,

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad \ldots \text{(i)}$$

where, $p = 2qd$; electric dipole moment

The dipole moment is a vector whose direction is along MN from the negative to the positive side. Thus,

$$V = \frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{\vec{p}\cdot\hat{r}}{r^2} \qquad \ldots \text{(ii)}$$

For axial line, $\theta = 0°$, thus, electric potential is;

$$V = \frac{p}{4\pi\varepsilon_0 r^2} \qquad \ldots \text{(iii)}$$

and if $\theta = 90°$

$$V = 0$$

i.e., electrical potential at equatorial position is zero.

3. **What is an electric quadrupole? Show that the electric intensity on the axis of the short quadrupole for points a distance from its centre is given by $E = \frac{3Q}{2\pi\varepsilon_0 r^2}$; where, Q is the quadrupole moment of charge distribution.** [T.U. 2063 Baishakh]

Solution:

Electric quadrupole

The arrangement of four equal and opposite charges or two dipoles arrangement with certain electric field at any point is called *quadrupole*.

Consider a linear quadrupole AB of separation $2d$ and we have to determine the electric potential at P at a distance r from centre O of the quadrupole.

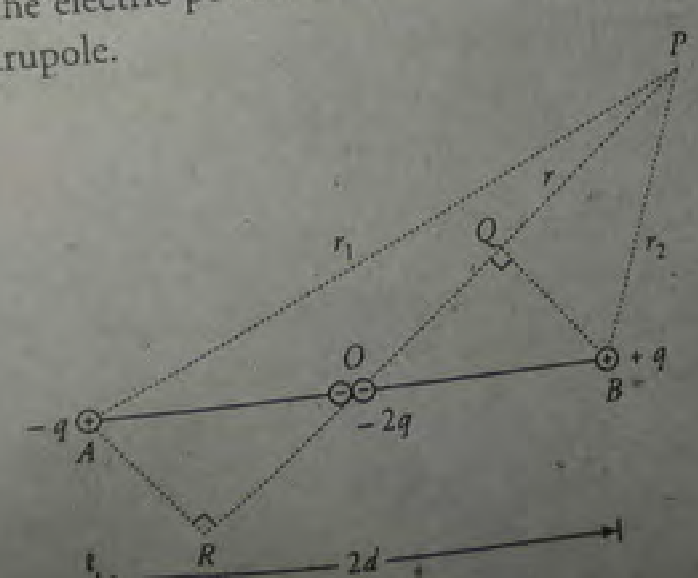


Figure: Linear quadrupole

The line PO is extended to R and BQ and RA perpendiculars are drawn so that;

$$\measuredangle BOP = \measuredangle AOR = \theta$$

In figure,

$$OP = r, AP = r_1, PB = r_2$$
$$RO = OQ = d \cos\theta$$
$$BQ = RA = d \sin\theta$$

and $r_1^2 = AR^2 + RP^2 = (d \sin\theta)^2 + (r + d \cos\theta)^2$

$$\therefore \quad r_1 = \sqrt{d^2 + r^2 + 2dr\cos\theta}$$

Similarly,

$$r_2 = \sqrt{d^2 + r^2 - 2dr\cos\theta}$$

The electric potential due to the electric quadrupole at P is,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{d^2+r^2+2dr\cos\theta}} + \frac{1}{\sqrt{d^2+r^2-2dr\cos\theta}} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right)^{-\frac{1}{2}} + \frac{1}{r}\left(1 + \frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right)^{-\frac{1}{2}} \ldots\right]$$

Applying binomial expansion, we obtain,

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[\left\{1 - \frac{1}{2}\left(\frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right) + \frac{3}{8}\left(\frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right)^2 + \ldots\right\}\right.$$
$$\left. + \left\{1 - \frac{1}{2}\left(\frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right) + \frac{3}{8}\left(\frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right)^2 + \ldots\right\} \ldots\right]$$

Neglecting higher order terms, we obtain,

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{d^2}{r^2} + \frac{6}{8}\left(\frac{2d\cos\theta}{r}\right)^2\right]$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_0}(3\cos^2\theta - 1) \quad \ldots (i)$$

This is the electric potential at any point P at an angle θ at a distance r from the centre of the quadrupole. For axial line, θ = ... we have,

$$V = \frac{2qd^2}{4\pi\varepsilon_0 r^3} = \frac{Q}{4\pi\varepsilon_0 r^3} \quad \ldots (ii)$$

where, $Q = 2qd^2$ is the electric quadrupole moment of the charge assembly

4. What is an electric quadrupole? Calculate the electric potential of a linear quadrupole of separation $2x$ at an axial distance R from its centre. [T.U. 2065 Shrawan]

Solution: See the solution of Q. No. 3 on page no. 189

5. What is an electric dipole? Derive an expression for electric field due to dipole at points on the (i) axis of the dipole and (ii) perpendicular bisector of dipole. [T.U. 2065 Chaitra]

Solution:

Electric dipole

An electric dipole is a pair of electric charges of equal magnitude q but of opposite charges, separated by a finite distance.

i) **Electric field along the axis of the dipole**

Consider an electric fields E_+ and E_- at any point P along the axial line of the dipole AB with separation $2d$.

According to principal of superposition, the resultant electric field at P at a distance r from centre of the dipole is;

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= \frac{q}{4\pi\varepsilon_0(r-d)^2} - \frac{q}{4\pi\varepsilon_0(r+d)^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4rd}{(r^2-d^2)^2}\right] \quad \ldots (i)$$

$$\therefore \quad E = \frac{2pr}{4\pi\varepsilon_0(r^2-d^2)^2}$$

where, $p = qd$: electric dipole moment

For a short dipole, $r \gg d$ thus, ... (ii)

$$E = \frac{p}{2\pi\varepsilon_0 r^3}$$

Hence, electric field drops $\frac{1}{r^3}$ for all distant point rapidly.

ii) **Electric field due to dipole at points on the perpendicular bisector of dipole**

Consider a point P which is equidistance from each charge $+q$ and $-q$.

In figure,

$$AP = BP(r^2 + d^2)^{\frac{1}{2}}$$

where, $OP = r$, the charges $+q$ and $-q$ set up electric fields $\vec{E}_+$ and $\vec{E}_-$ respectively, thus,

$$|\vec{E}_+| = |\vec{E}_-|$$

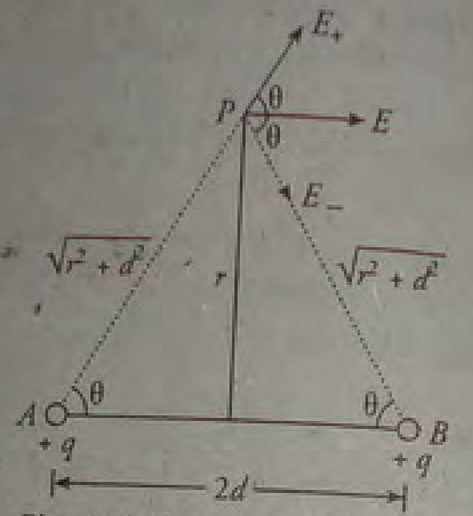


Figure: Electric field at equatorial line of a dipole

The total electric field at P is;

$$|\vec{E}| = |\vec{E}_+| + |\vec{E}_-|$$

$$= \frac{q}{4\pi\varepsilon_0(r^2+d^2)}\cos\theta + \frac{q}{4\pi\varepsilon_0(r^2+d^2)}\cos\theta$$

$$\text{or,} \quad E = \frac{2qd}{4\pi\varepsilon_0(r^2+d^2)^{\frac{3}{2}}}$$

$$\text{where, } \cos\theta = \frac{d}{(r^2+d^2)^{\frac{1}{2}}}$$

$$\therefore \quad E = \frac{P}{4\pi\varepsilon_0(r^2+d^2)^{\frac{3}{2}}} \quad \ldots \text{(iii)}$$

For short dipole, $r \gg d$ thus, we obtain,

$$E = \frac{P}{4\pi\varepsilon_0 r^3} \quad \ldots \text{(iv)}$$

The variation of electric field to $\frac{1}{r^3}$ is a characteristics result fo[r] electric dipole.

6. Derive an expression for the electric field at any point on the axi[s] of the short linear quadrupole. **[T.U. 2067 Ashadh]**

Solution:

The arrangement of two dipoles with certain electric field at an[y] point is called quadrupole.

The quadrupole moment is not vector but more complicate[d] quantity called tensor. It is represented by Q.

$$Q = 2qd^2 \quad \ldots \text{(i)}$$

where, q = charge of quadrupole of separation $2d$

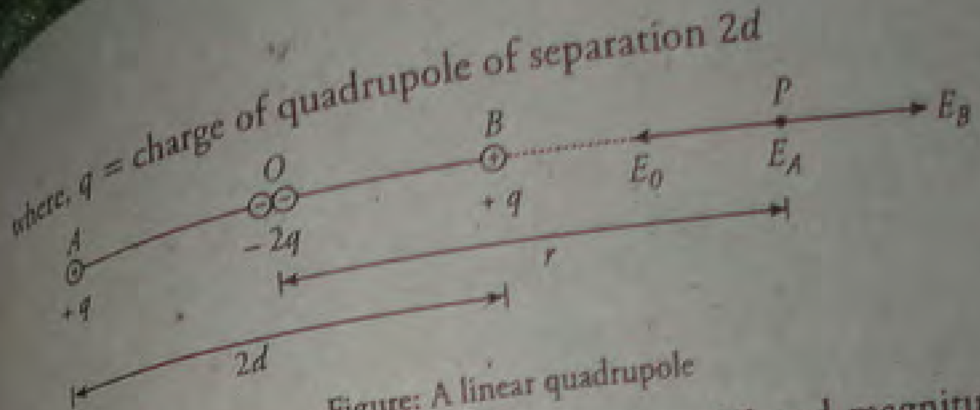


Figure: A linear quadrupole

Consider a linear quadrupole of separation $2d$ and magnitude of each charge q. The electric intensities at P be E_A, E_B and E_0 due to the charge at A, B and O respectively. The resultant field according to superposition principle is;

$$E = E_A + E_B - E_0$$

$$= \frac{q}{4\pi\varepsilon_0(r+d)^2} + \frac{q}{4\pi\varepsilon_0(r-d)^2} - \frac{2q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r+d)^2} + \frac{1}{(r-d)^2} - \frac{1}{r^2}\right]$$

$$= \frac{2qd^2}{4\pi\varepsilon_0}\left[\frac{3r^2-d^2}{r^2(r^2-d^2)^2}\right] \quad \ldots \text{(ii)}$$

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3r^2-d^2}{r^2(r^2-d^2)^2}\right]$$

This gives the magnitude of electric intensity of a quadrupole along its axis. For a short quadrupole $r \gg d$, then,

$$E = \frac{Q}{4\pi\varepsilon_0 r^4} \quad \ldots \text{(iii)}$$

Hence, the electric intensity of a quadrupole varies inversely with fourth power of the distance from its centre.

7. Derive an expression for the electric potential at any point on the axis of the uniformly charged disk. **[T.U. 2067 Ashadh]**

Solution:

Consider a charged ring of radius a and linear charge density λ be considered. O is the centre and P at a distance y from O.

The ring consists of a continuous distribution of charge rather than a set of discrete charge, so an elementary charge dq is taken in an elementary section dx.

i.e., $dq = \lambda\, dx$

The potential due to dq at a distance r is;

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda\, dx}{4\pi\varepsilon_0\sqrt{a^2+y^2}}$$

In this relation a, y, λ are constant for the system, thus integrating only for dx, we obtain,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{\sqrt{a^2+y^2}}\int_0^{2\pi a} dx$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_0\sqrt{a^2+y^2}}$$

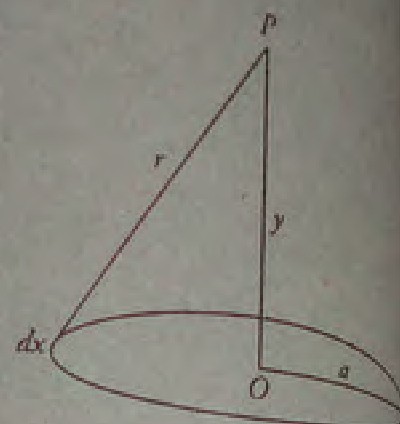


Figure: Potential of ring of charge

For a large value of y, the potential lends to the value of point charge.

8. **For a given short dipole, show that the electric potential at any point at a distance r is $V = \frac{P\cos\theta}{r^2}$ where, θ is the angle made by r to the dipole and P is its dipole moment. Using above relation, find an expression for resultant electric intensity at that point.**

[T.U. 2068 Shrawan]

Solution:

To determine an electric field at P, we will choose co–ordinate system with the origin at P, x–axis along r, y–axis perpendicular to r and is the plane containing $p = 2qd$. The z–axis perpendicular to x–axis and y–axis such that $dx = dr$ and $dy = r\,d\theta$. Thus,

$$E_r = E_x = -\frac{\partial V}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= \left(\frac{2p\cos\theta}{r^3}\right)$$

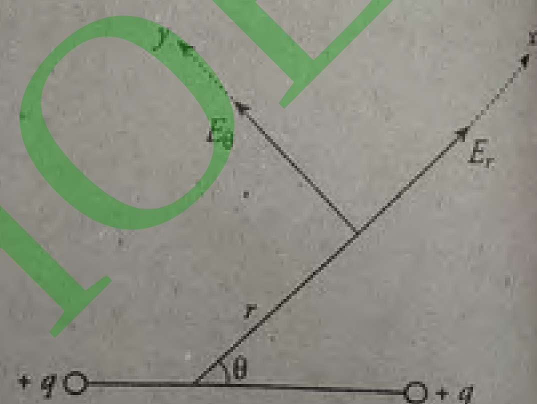


Figure: Electric dipole

or, $E_\theta = E_y = -\frac{\partial V}{\partial y}$

$$= -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= \frac{p\cos\theta}{r^3}$$

and $E_z = -\frac{\partial V}{\partial z} = 0$

It is because the potential at P remains unchanged along z–axis, if the dipole is rotated through small angle about AB axis. The resultant field will be;

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$\text{or, } E = \frac{p}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\therefore E = \frac{1}{r^3}\sqrt{3\cos^3\theta + 1}$$

This field will make an angle ϕ with r called a resultant direction which is;

$$\phi = \tan^{-1}\left(\frac{E_\theta}{E_r}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

In this relation, r and θ may be interpreted as the polar co–ordinates of P. For $\theta = 0°$, *i.e.*, along the axis electric field will be;

$$E_r = \frac{2p}{r^3}$$

For $\theta = 90°$, *i.e.*, along the equatorial line, we obtain,

$$E_\theta = \frac{2p}{r^3}$$

These relations give same values of fields.

Chapter 11

CAPACITORS

11.1 CAPACITORS AND CAPACITANCE

Any two conductors separated by an insulators or a vacuum from a capacitor. In most practical applications each conductor initially has zero net charge and electrons are transferred from one conductor to the other, this is called charging of a capacitor. The two conductors, after charging capacitor, have charges with equal magnitude and opposite sign and the net charge on the capacitor as a whole remains zero. A capacitor has charge Q or a charge Q is stored on the capacitor means that conductor at higher potential has charge $+Q$ and the conductor at lower potential has charge $-Q$.

The charge Q and the potential difference V for a capacitor are proportional to each other.

i.e., $Q = CV$ 11.1

where, C is proportionality constant called capacitance of a capacitor. Its value depends only on the geometry of the plates and not on their charges or potential difference. The capacitance of a capacitor is a measure of how much charge must be put on the plates to produce a certain potential difference between them. The greater the capacitance, the more charge is required.

The S.I. unit of capacitance is Coulomb per unit Volt or Farad (F).

11.2 PARALLEL PLATE CAPACITOR

The simplest form of capacitor consists of two parallel conducting plates, each with area A, separated by a distance d that is small comparison with their dimensions. When the plates are charged, the electric field is almost completely localized in the region between the plates. The field between such plates is essentially uniform and charges on the plates are uniformly distributed over their opposing surfaces.

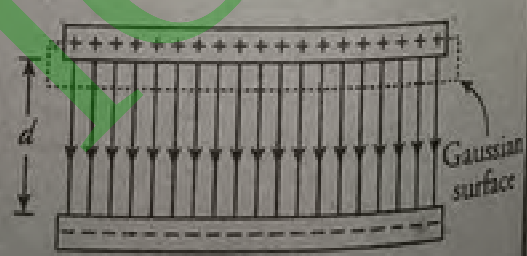


Figure: A charged parallel plate capacitor

The capacitance of parallel plate capacitor is;

$$C = \varepsilon_0 \frac{A}{d} \qquad 11.2$$

It depends upon geometrical factors, *i.e.,* the plate area and plate separation.

11.3 CYLINDRICAL CAPACITOR

Figure depicts the cross–section of a cylindrical capacitor of length L formed by two co–axial cylinders of radii a and b. We assume that $L \gg b$ so that we can neglect the fringing of electric field that occurs at the ends of the capacitor. Each plate contains a charge of magnitude Q.

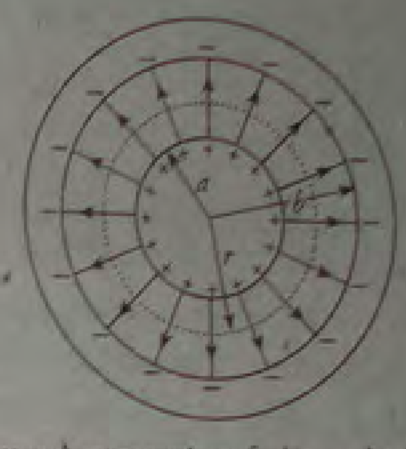


Figure: A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r

The total capacitance of such capacitor is;

$$C = 2\pi\varepsilon_0 \frac{L}{ln\left(\frac{b}{a}\right)} \qquad 11.3$$

The capacitance of cylindrical capacitors as well, depends only on geometrical factors.

11.4 SPHERICAL CAPACITOR

The spherical capacitor has the capacitance,

$$C = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right) \qquad 11.4$$

where, a and b are radii of concentric spherical shells of a spherical capacitors.

11.5 ISOLATED SPHERE

A capacitance of a single isolated spherical conductor of radius is;

$$C = 4\pi\varepsilon_0 R \qquad 11.5$$

11.6 CAPACITORS IN PARALLEL AND IN SERIES

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge stored in capacitors is the sum of charges stored on all the capacitors.

i.e., $Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$

$\therefore$ $Q = (C_1 + C_2 + C_3)V$

Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge Q and same potential difference as the capacitor.

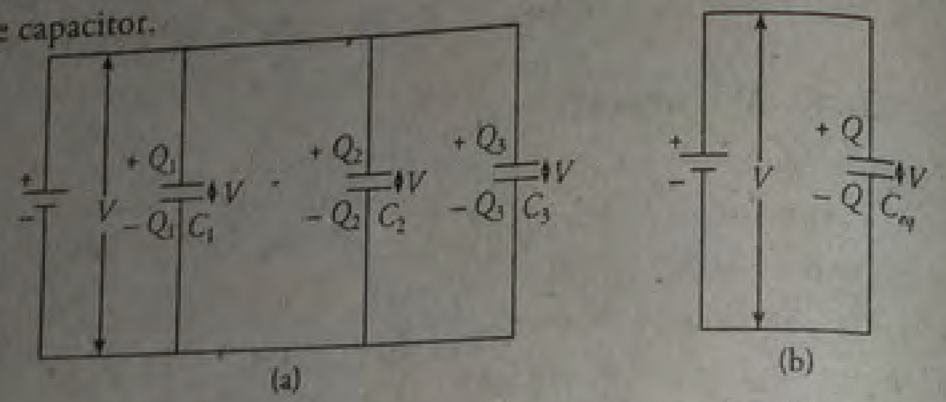


Figure: (a) Capacitors connected in parallel to battery (b) equivalent circuit diagram

The equivalent capacitance with same total charge Q and applied potential difference V as the combination is then,

$$C_{eq} = \frac{Q}{V} = C_1 + C_2 + C_3 \qquad 11.6$$

i.e., $C_{eq} = \sum_{i=1}^{n} C_i$; n capacitors in parallel 11.7

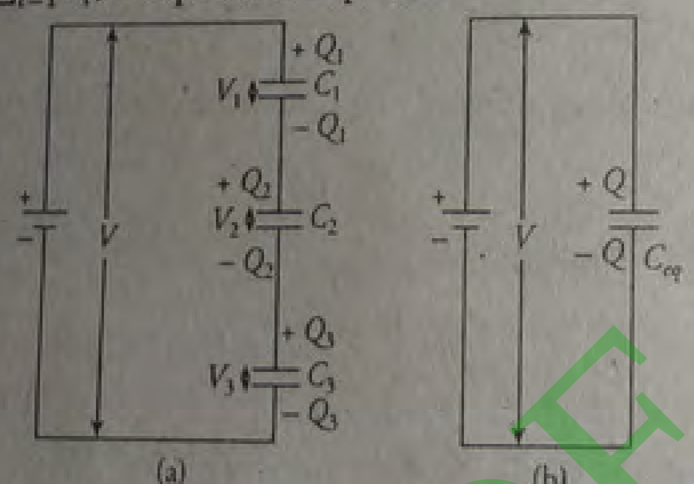


Figure: (a) Capacitors connected in series (b) equivalent circuit diagram

When a potential difference V is applied across several capacitors connected is series, the capacitors have identical charge Q, the sum of the potential difference across all the capacitors is equal to the applied potential difference V.

i.e., $V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$\therefore \quad V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

Capacitors that are connected in series can be replaced with an equivalent capacitor that has same charge Q and the same total potential difference V as the actual series capacitors. Thus,

$$C_{eq} = \frac{Q}{V}$$

$$= \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad 11.8$$

For n number of capacitors;

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_1} \qquad 11.9$$

i.e., the equivalent capacitance of a series of capacitors is always less than the least capacitance in series.

11.7 SOLVED EXAM QUESTIONS

1. **What is the capacitance of cylindrical capacitor made of two co-axial cylinders of length 2 *cm* and radii 2 *mm* and 2.1 *mm*? The space between the cylinders is filled with a medium of relative permittivity 7.8. [T.U. 2061 Baishakh]**

Solution:

Here,

Length of cylindrical capacitor $(L) = 2\ cm = 2 \times 10^{-2}\ m$

Radii of co–axial cylinders; $(a) = 2\ mm = 2 \times 10^{-2}\ m$

$(b) = 2.1\ mm = 2.1 \times 10^{-2}\ m$

Relative permittivity $(\varepsilon_r) = 7.8$

We have,

$$C = 4\pi\varepsilon \frac{L}{ln\left(\frac{b}{a}\right)} = 4\pi\varepsilon_r\varepsilon_0 \frac{L}{ln\left(\frac{b}{a}\right)}$$

$$= 4\pi \times 7.8 \times 8.85 \times 10^{-12} \times \frac{2 \times 10^{-2}}{ln\left(\frac{2.1 \times 10^{-2}}{2 \times 10^{-2}}\right)}$$

$\therefore \quad C = 3.56 \times 10^{-16}\ F$

The capacitance of cylindrical capacitor is $3.56 \times 10^{-16}\ F$.

2. **Define capacitance. Give a general method to calculate capacitance of a capacitor. Find the expression for the capacitance of a cylindrical capacitor.**

Solution:

The capacitance of a capacitor measure of how much charge must be put on the plates to produce a certain potential difference between them.

Mathematically;

Capacitance, $C = \frac{Q}{V}$

where, Q = charge stored in capacitor
V = applied potential difference

Its S.I. unit is Farad (F).

General method of calculating the capacitance

We assume that a charge Q on the plates of the capacitor. Th electric field $\vec{E}$ between the plates of a capacitor to the charge either plate can be calculated using Gauss's law.

i.e., $\oint \vec{E}\, d\vec{A} = \dfrac{Q}{\varepsilon_0}$ ... (i)

where, Q is the charge enclosed by a Gaussian surface. Gaussia surface will be such that it will enclose only one plate.

The potential difference V between the plates can be calculated b using equation.

$$V = \int_-^+ \vec{E}\, d\vec{s} \qquad \text{... (ii)}$$

where, V represents the difference $V_f - V_i$ and we will alway choose a path that follows as electric field lines from the negativ to the positive plates. For this path vector $\vec{E}$ and $d\vec{s}$ have opposi direction.

The capacitance of a capacitor is calculated using equation;

$$C = \frac{Q}{V} \qquad \text{... (iii)}$$

Capacitance of a cylindrical capacitor

Figure depicts a cross section of cylindrical capacitor of length L formed by two axial cylinders of radii a and b. We assume that $L \gg B$, so that we can neglect the fringing of electric field that occurs at the end of the cylinders. Each plates contains a charge of magnitude Q.

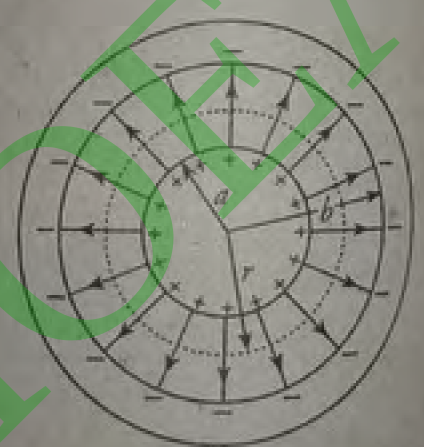


Figure: A cross section of a long cylindrica capacitor, showing a cylindrical Gaussia surface of radius r

Assume a Gaussian surface, we choose a cylinder of length L and radius r, closed by end cap Then, we write,

$$Q = \varepsilon_0 E A = \varepsilon_0 E(2\pi r L) \qquad \text{... (iv)}$$

where, $2\pi r L$ is the area of curved part of Gaussian surface. Ther is no flux through the end caps. Solving for E yields,

$$E = \frac{Q}{2\pi r \varepsilon_0 L} \qquad \text{... (v)}$$

The potential difference,

$$V = \int_-^+ E\, ds = \frac{Q}{2\pi r \varepsilon_0 L}\int_-^+ \frac{dr}{r} = \frac{Q}{2\pi r \varepsilon_0 L} ln\left(\frac{b}{a}\right) \qquad \text{... (vi)}$$

As integration is done radially inward, we choose $ds = -dr$.

$$C = \frac{Q}{V}$$

or, $$C = 2\pi\varepsilon_0 \frac{L}{ln\left(\frac{b}{a}\right)} \qquad \text{... (vii)}$$

We see that capacitance of a cylindrical capacitor depends only on geometrical factors.

3. **What is the capacitance of a spherical capacitor made of two concentric spheres of radii 6 and 6.01 *cm*? The space between them is filled with a medium with relative permittivity equal to 8.** **[T.U. 2062 Baishakh]**

Solution:

Radii of concentric spheres, $(a) = 6\,cm = 6 \times 10^{-2}\,m$

$(b) = 6.1\,cm = 6.1 \times 10^{-2}\,m$

Relative permittivity, $(\varepsilon_r) = 8$

We have,

$$C = 4\pi\varepsilon\left(\frac{ab}{b-a}\right) = 4\pi\varepsilon_r\varepsilon_0\left(\frac{ab}{b-a}\right)$$

$$= 4\pi \times 8 \times 8.85 \times 10^{-12}\left(\frac{6 \times 10^{-2} \times 6.1 \times 10^{-2}}{6.1 \times 10^{-2} - 6 \times 10^{-2}}\right)$$

$$= 4\pi \times 8 \times 8.85 \times 10^{-12} \times 3.66$$

$$\therefore \quad C = 3.26 \times 10^{-9}\,F$$

The capacitance of spherical capacitance is $3.26 \times 10^{-9}\,F$.

4. **Derive an expression of capacitance of co-axial cable used in a transmission line having inner radius a, outer radius b and length l. If $a = 0.1\,mm$ and $b = 0.60\,mm$, calculate the capacitance per meter for a cable. Assume that space between the conductors is filled with polystyrene of dielectric constant 2.6.**

Solution:

Capacitance of co–axial cable

See the solution of Q. No. 2 (third part only) on page no. 199

Here,

$a = 0.1\,mm = 0.1 \times 10^{-3}\,m$

$b = 0.6\ mm = 0.6 \times 10^{-3}\ m$
$l = 1\ m$
$\varepsilon_r = 2.6$
We have,

$$C = 2\pi\varepsilon \frac{l}{ln\left(\frac{b}{a}\right)} = 2\pi\varepsilon_r\varepsilon_0 \frac{l}{ln\left(\frac{b}{a}\right)}$$

$$= 2\pi \times 2.6 \times 8.85 \times 10^{-12} \times \frac{1}{ln\left(\frac{0.6 \times 10^{-3}}{0.1 \times 10^{-3}}\right)}$$

$$\therefore \quad C = 8 \times 10^{-11}\ F$$

The capacitance per meter for a cable is $8 \times 10^{-11}\ F$.

5. **Two dielectric slabs are inserted between the plates of parall plate capacitor. The thickness of slab with dielectric constant k and t_1 and of other slab having dielectric k_2 is t_2. The plates o capacitor are $t = t_1 + t_2$ apart with area of the plates being A. Fin the capacitance of new arrangement.** [T.U. 2064 Pous

Solution:

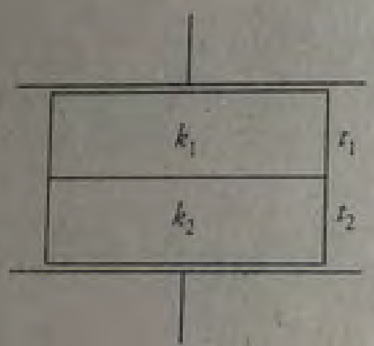


Figure: Parallel plate capacitor with two dielectrics inserted in it

Since $t = t_1 + t_2$, the parallel plate capacitor may be considered two capacitors with dielectric k_1 and k_2 in series. The capacitances are;

$$\left.\begin{aligned} C_1 &= \frac{k_1\varepsilon_0 A}{t_1} \\ C_2 &= \frac{k_2\varepsilon_0 A}{t_2} \end{aligned}\right\} \quad \ldots (i)$$

The combined capacitance is;

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{t_1}{k_1\varepsilon_0 A} + \frac{t_2}{k_2\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\left[\frac{t_1}{k_1} + \frac{t_2}{k_2}\right]$$

$$\therefore \quad C = \varepsilon_0 A\left[\frac{t_1}{k_1} + \frac{t_2}{k_2}\right]^{-1}$$

The capacitance of new arrangement is $C = \dfrac{\varepsilon_0 A}{\left[\frac{t_1}{k_1} + \frac{t_2}{k_2}\right]}$.

6. **Explain the principle of parallel plate capacitor and determines its capacitance.** [T.U. 2065 Kartik]

Solution:

When a capacitor is charged, its plates have charges of equal magnitude but opposite in signs, *i.e.*, $+Q$ and $-Q$.

The plates are equipotential surfaces; all points on a plate are at the same potential. Moreover, there is a potential difference between two plates. The capacitance is proportional to this potential difference.

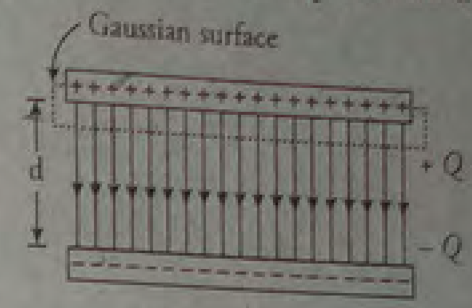


Figure: A charged parallel plate capacitor with a Gaussian surface that encloses charge on positive plate.

We assume that the plates of parallel plate capacitor are so large and so close together that we neglect the fringing of the electric field at the edges of the plates taking $\vec{E}$ to be constant throughout the region between the plates.

A Gaussian surface is drawn enclosing the positive charge Q on positive plate. We can write;

$$Q = \varepsilon_0 A \quad \ldots (i)$$

where, A is the area of the plate

The potential difference between two plates;

$$V = \int_{-}^{+} E\, ds = \int_0^d E\, ds = Ed \quad \ldots (ii)$$

The capacitance of parallel plate capacitor is;

$$C = \frac{Q}{V} = \frac{\varepsilon_0 EA}{Ed}$$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d} \quad \ldots (iii)$$

Thus, the capacitance does indeed depends only on geometrical factors, *i.e.*, capacitance depends on plate area A and plate separation d.

7. **A copper slab of thickness b is inserted into a parallel plate capacitor exactly half way between the plates. If the separation of the plate is d and of each of plate is A, show that the change in the capacitance is equal to $\dfrac{\varepsilon_0 Ab}{(d-b)d}$.** [T.U. 2067 Ashadh]

Solution:

The capacitance of capacitor (in air) is;

$$C_1 = \frac{\varepsilon_0 A}{d} \quad \dots \text{(i)}$$

where, A is area of plate and d is the plate separation

When the copper slab of thickness b is inserted, the capacitance of capacitor is;

$$C_2 = \frac{\varepsilon_0 A}{(d-b)} \quad \dots \text{(ii)}$$

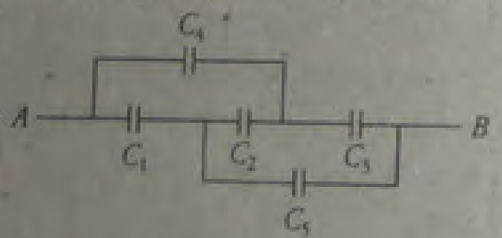
The change in capacitance of capacitor is;

$$C_2 - C_1 = \frac{\varepsilon_0 A}{(d-b)} - \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A - \varepsilon_0 A(d-b)}{(d-b)d}$$

$$\therefore \quad C_2 - C_1 = \frac{\varepsilon_0 Ab}{(d-b)d} \quad \dots \text{(iii)}$$

Hence, the change in capacitance of a capacitor is $\frac{\varepsilon_0 Ab}{(d-b)d}$.

8. Calculate the equivalent capacitance between A and B. Take $C_2 = 10\ \mu F$ and other capacitors are $4\ \mu F$ each. **[P.U. 2003]**



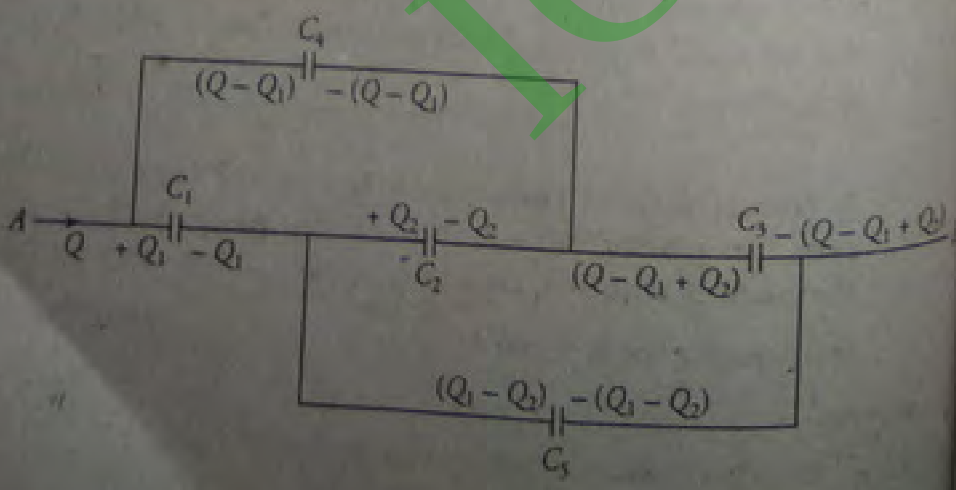
Solution:

Here,

$C_2 = 10\ \mu F$

$C_1 = C_3 = C_4 = C_5 = 4\ \mu F$

Let, the effective capacitance between points A and B. Assume that on applying a potential difference V between A and B, on effective capacitance be charged to Q. Let the charges across C_1 and C_2 be Q_1 and Q_2 respectively. The charges across various capacitors are shown in diagram below.



The potential difference across $(C_1 + C_5)$ must be equal to potential difference across $(C_4 + C_3)$

i.e., $V_1 + V_5 = V_4 + V_3$

or, $V_1 + V_5 = V$

$= V_4 + V_3 \quad \dots \text{(i)}$

or, $\frac{Q_1}{C_1} + \frac{Q_1 - Q_2}{C_5} = V$

or, $\frac{Q_1 + Q_1 - Q_2}{C_1} = V \quad (\because C_1 = C_5)$

or, $2Q_1 - Q_2 = C_1 V \quad \dots \text{(ii)}$

Again from equation (i); we obtain,

$V_4 + V_3 = V$

or, $\frac{Q - Q_1}{C_4} + \frac{Q - Q_1 + Q_2}{C_3} = V$

Since, $C_4 = C_3 = C_1$, then,

$\therefore \quad 2Q - 2Q_1 + Q_2 = VC_1 \quad \dots \text{(iii)}$

Adding equation (ii) and (iii); we have,

$2Q = 2C_1 V$

or, $\frac{Q}{V} = C_1$

i.e., $C = C_1 = 4\ \mu F$

The equivalent capacitance between A and B is $4\ \mu F$.

9. A storage capacitor has a capacitance $55\ \mu F$, is charged to 5.3 V, how many electrons are on its negative plates? **[P.U. 2004]**

Solution:

Here,

Capacitance of a capacitor $(C) = 55\ \mu F$

$= 55 \times 10^{-6}\ F$

Applied potential difference $(V) = 5.3\ V$

We have,

$Q = CV$

$= 55 \times 10^{-6} \times 5.3$

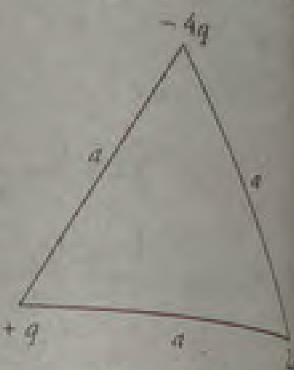
$\therefore \quad Q = 2.92 \times 10^{-4}\ C$

The number of electrons in its negative plate is;

$$n = \frac{2.92 \times 10^{-4}}{1.6 \times 10^{-19}} = 1.82 \times 10^{15}$$

10. Three charges are arranged as shown in figure. What is the mutual potential energy? Assume that $Q = 1.0 \times 10^{-7}$ C and a = 10 *cm*. [P.U. 200..]

Solution:



Here,

$Q = 1.0 \times 10^{-7}\ C$

$a = 10\ cm = 0.1\ m$

The mutual potential energy;

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{a} - \frac{8q^2}{a} + \frac{2q^2}{a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{10q^2}{a}\right]$$

$$= -\frac{1}{4\pi \times 8.85 \times 10^{-12}}\left[-\frac{10(1.0 \times 10^{-7})^2}{0.1}\right]$$

$\therefore\ U = 9.0 \times 10^{-3}\ J$

The mutual potential energy is $9.0 \times 10^{-3}\ J$.

11. **Explain the principle of parallel plate capacitor.** [P.U. 200..]

Solution:

When a capacitor is charged, its plates have equal magnitudes but opposite signs, *i.e.*, $+Q$ and $-Q$. The plates are equipotential surfaces; all points on a plate are at same electric potential. Moreover, there is a potential difference between the plates.

The charge Q and potential difference V for a capacitor are proportional to each other, *i.e.*,

$Q = CV$

where, C is proportionality constant, called capacitance of a capacitor.

12. **State Gauss's law in free space. How the law is modified if dielectric materials are present. Prove the relation $Q' = Q\left(1 - \frac{1}{k}\right)$ where symbols carry usual meaning.** [P.U. 2008]

Solution:

Gauss's law in free space may be stated as;

$$\varepsilon_0 \oint \vec{E}\, d\vec{A} = Q$$

$$i.e.,\ E_0 = \frac{Q}{\varepsilon_0 A} \quad \ldots \text{(i)}$$

In presence of dielectric materials, Gauss's law may be generalized as

$$\varepsilon_0 \oint k\vec{E}\, d\vec{A} = Q \quad \ldots \text{(ii)}$$

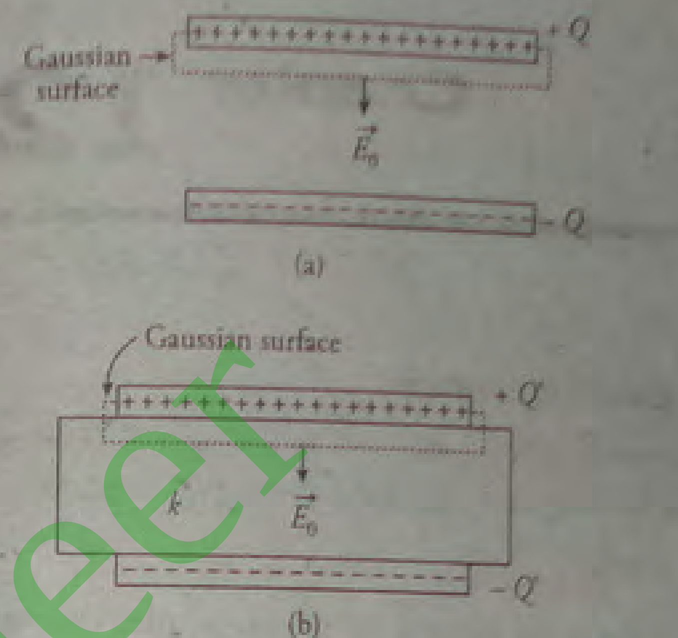


Figure: A parallel plate capacitor (a) without dielectric slab (b) with dielectric slab

Let Q and Q' be the magnitudes of charges on capacitor without and with dielectric materials.

The net charge enclosed by the Gaussian surface is $Q - Q'$, so Gauss's law, thus gives,

$$\varepsilon_0 \oint \vec{E}\, d\vec{A} = \varepsilon_0 EA = Q - Q'$$

$$\text{or,}\ E = \frac{Q - Q'}{\varepsilon_0 E} \quad \ldots \text{(iii)}$$

The effect of the dielectric is to weaken the original field E_0 by the fack E, we may write,

$$E = \frac{\varepsilon_0}{k} = \frac{Q}{k\varepsilon_0 A} \quad \ldots \text{(iv)}$$

Comparison of equations (iii) and (iv) gives;

$$Q - Q' = \frac{Q}{k}$$

$$\therefore\ Q' = Q\left(1 - \frac{1}{k}\right) \quad \ldots \text{(v)}$$

This is the required expression. This shows that magnitude Q' of the induced surface charge is less than that of free charge Q and is zero if no dielectric is present.

13. **State Gauss law in free space. How this law is modified if dielectric materials are present? Prove the relation $Q' = Q\left(1 - \frac{1}{k}\right)$ where the symbols carry usual meaning.** [P.U. 2010]

Solution: See the solution of Q. No. 12 on page no. 206

Chapter 12

DIRECT CURRENT

12.1 ELECTRIC CURRENT

An electric current is defined as the rate of flow of charge or amount of charges flowing in a unit time through a conductor.

i.e., $I = \frac{Q}{t}$

If dQ is the total amount of charge passing through the conductor in a small interval of time Δt, then average electric current will be;

$$I = \frac{dQ}{dt}$$

In S.I. units, the unit of current is Ampere.

12.2 CURRENT DENSITY

If an electric current I, flowing through a conductor, is uniformly distributed throughout the cross sectional area A, then current per unit area is known as current density (J).

i.e., $J = \frac{I}{A}$

Its unit is Ampere per square meter (Am^{-2}). It is a vector quantity and directed along vector area $\vec{A}$.

In terms of drift velocity $\vec{v}_d$,

$$\vec{J} = ne\vec{v}_d$$

where, n = number of free electrons per unit volume

e = charge of each electron

12.3 RESISTANCE, RESISTIVITY AND CONDUCTIVITY

Resistance

When electric charge in a conductor more due to the application of electric field, they collide with vibrating atoms at their mean position. This opposes the passes of electric charge through it, called resistance of conductor.

Mathematically;

$$R = \frac{V}{I}$$

where, V = potential difference

I = current through conductor

Its S.I. unit is Ohm (Ω).

Resistivity

The resistance R of a conductor is directly proportional to its length (l) and inversely proportional to cross sectional area (A).

i.e., $R \propto l$

and $R \propto \frac{1}{A}$

Combining these relations; we have,

$$R \propto \frac{l}{A}$$

$$\therefore \quad R = \rho\frac{l}{A} \qquad 12.4$$

where, ρ is called proportionality constant, called resistivity or specific resistance

When $l = 1\ m$ and $A = 1\ m^2$

Then,

$$R = \rho$$

Thus, resistivity of a material is numerically equal to the resistance of the material of unit length and unit cross sectional area. The S.I. unit of resistivity is Ohm–metre ($\Omega\ m$). Its value depends on temperature and nature of material.

Conductivity

The electrical conductivity of a material is defined as the reciprocal of resistivity.

$$\text{i.e.,} \quad \sigma = \frac{1}{\rho} = \frac{l}{RA} \qquad 12.5$$

Its S.I. unit is $\Omega^{-1}m^{-1}$ or Siemen m^{-1}.

12.4 Ohm's law:

It states that, *"the potential difference across any two points on a conductor is directly proportional to steady current through it, provided that its physical conditions remain unchanged."*

i.e., $V \propto I$

$$\text{or,} \quad V = RI \qquad 12.6$$

where, R is proportionality constant called resistance

12.5 SEMICONDUCTORS

Semiconductors are materials whose electronic properties are intermediate between metals and insulators. These electronic properties are determined by the crystal structure, bonding characteristics, energy bands, etc. The resistivity of semiconductors is in the range 10^{-2} Ω-cm to 10^{9} Ω-cm whereas that of metals and insulators is in the energy range of 10^{-6} Ω-cm and 10^{14} Ω-cm respectively.

An intrinsic semiconductor is a pure or undoped semiconductor (e.g. silicon (Si), Germanium (Ge), etc.) in which electrons and holes are produced by thermal agitation. The energy gap, *i.e.*, energy required to take an electron from the valance band to conduction band due to thermal activation and move into conduction band, thus becomes free mobile electrons.

Extrinsic semiconductor is a semiconductor in which carriers are produced by the addition of small quantities of group III or group V elements of periodic table to pure semiconductors.

The electrical conductivity of an intrinsic semiconductor can be significantly increased by adding a suitable impurity in a very small proportion during crystal growth. This process is called doping. When crystal is doped, the semiconductor is said to be extrinsic semiconductor.

If a very small quantity of pentavelent impurity like arsenic or phosphorous is introduced in germanium, it replaces an equal number of germanium atoms without changing the physical state of semiconductor. Each of four out of five valance electrons of impurity enters into covalent bonds with germanium while the fifth electrons is set to free to move from one atom to another. The impurity is called donor impurity as it donates electrons and crystal is called n–type semiconductor (donor semiconductor).

If a trivalent impurity like indium boron or gallium is added, the impurity atom displaces equal number of germanium atoms in the germanium crystal during its formation. In such case, only three out of four possible covalent bonds are formed while the fourth bond is not completed due to the deficiency of one electron. This incomplete bond acts like a hole. It has a tendency to accept one electron from neighbouring germanium atom to complete the fourth covalent bond. The transferred electron leaves behind a broken covalent bond *i.e.*, hole. Hence, a hole moves relative to the electron in an opposite direction of an electron when an electron moves from one bond to another. This

trivalent impurity is known as acceptor or p–type impurity and semiconductor containing such impurity is called p–type semiconductor (acceptor semiconductor).

12.6 SUPERCONDUCTORS

In 1911, Kamerlingh Onnes first observed that the resistivity of certain metals and alloys drop abruptly to zero when they are cooled to sufficiently low temperature. This phenomenon is called superconductivity and the substance exhibiting this phenomenon is called superconductors. For instance, the resistivity of pure mercury drops to zero, when it is cooled to temperature of about 4.2 K.

The temperature at which the electrical resistivity of substance becomes zero is called critical temperature or threshold temperature. Below the critical temperature substance is in superconducting state and above the critical temperature substance is in normal state.

12.7 SOLVED EXAM QUESTIONS

1. **How do resistivity of metals and semiconductors vary with temperature? Explain using formula for resistivity.**

[T.U. 2061 Baishakh]

Solution:

The resistance for most materials changes with temperature. For many materials including metals, the relation between resistance (R_θ) and temperature (θ) can be approximated through the relation;

$$R_\theta = R_0(1 + \alpha\theta)$$

where, R_0 and R_θ are the values of resistance of a material at 0°C and θ°C respectively such that the temperature difference is not too large and α is the temperature coefficient of resistance.

In case of resistivity;

$$\rho_\theta = \rho_0[1 + \alpha(\theta - \theta_0)]$$

where, ρ_0 corresponds to θ_0.

i.e., resistivity increases with the increase of temperature.

In case of semiconductors, the resistivity decreases with the increase of temperature.

i.e., $\rho_\theta = \rho_0[1 - \alpha(\theta - \theta_0)]$

2. **A wire with a resistance of 6.0 Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire assuming that the resistivity and density of material are unchanged.** [T.U. 2061 Ashwin]

Solution:

Here,

Resistance of original wire $(R_1) = 6\ \Omega$

Let, l be the length of original wire.

The new length of wire $(l') = 3l$

Resistance of longer wire $(R_2) = ?$

We have,

$$R_1 = \frac{\rho l}{A} \quad \ldots \text{(i)}$$

$$R_2 = \frac{\rho l'}{A'} \quad \ldots \text{(ii)}$$

where, A and A' are cross sectional area of original wire and new wire respectively

Dividing equation (i) by (ii); we obtain,

$$\frac{R_1}{R_2} = \frac{lA'}{l'A} = \frac{lA'}{3lA} = \frac{1}{3}\left(\frac{A'}{A}\right)$$

Since,

$$Al = A'l'$$

or, $$\frac{A'}{A} = \frac{l}{l'} = \frac{l}{3l} = \frac{1}{3}$$

Thus,

$$\frac{R_1}{R_2} = \frac{1}{9}$$

or, $R_2 = 9 \times 16\ \Omega = 144\ \Omega$

The resistance of longer wire is increased by 9 times of original wire, i.e., $R_2 = 144\ \Omega$.

---

3. **What is the cause of electrical resistance in conducting medium? Derive the relation for resistivity in terms of mean free path. Explain the variation of conductivity of metals and semiconductors.** [T.U. 2062 Baishakh]

Solution:

An electrical resistance in conducting medium is due to collision of mobile electrons with free electrons.

If an electron of mass m is applied in an electric field of magnitude E, the electron will experience an acceleration given by Newton's law:

$$a = \frac{F}{m} = \frac{eE}{m} \quad \ldots \text{(i)}$$

The nature of the oscillations experienced by conduction electrons is such that after a typical collision, each electron will completely lose its drift velocity. Each electron will then start off fresh after every encounter, moving off in random direction. In the average time τ between the collisions, the average electron will acquire a drift speed of $v_d = a\tau$.

Moreover, if we measure the drift speeds of all the electrons at any instant, we will find that their average drift speed is also $a\tau$.

Thus, at any instant, the electrons will have drift speed.

$$v_d = a\tau \quad \ldots \text{(ii)}$$

Then, equation (i) gives,

$$v_d = a\tau = \frac{eE\tau}{m} \quad \ldots \text{(iii)}$$

Since,

$$\vec{J} = ne\vec{v}_d \quad \ldots \text{(iv)}$$

where, n = number of carriers per unit volume

Combining equation (iii) with magnitude form of equation (iv) leads us to;

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

or, $$E = \left(\frac{m}{e^2 n\tau}\right) J \quad \ldots \text{(v)}$$

Comparing equation (v) with $E = \rho J$; we obtain,

$$\rho = \frac{m}{e^2 n\tau} = \frac{mv}{e^2 n\lambda} \quad \ldots \text{(vi)}$$

This is the required expression for resistivity of metals in terms of mean free path (λ).

The conductivity of metal is;

$$\sigma = \frac{1}{\rho} = \frac{mv}{e^2 n\lambda} \quad \ldots \text{(vii)}$$

i.e., conductivity of metal is directly proportional to mean free path and inversely proportional to average velocity of electrons.

The conductivity of semiconductor is;

$$\sigma_s = e(n\mu_e + p\mu_n) \quad \ldots \text{(viii)}$$

where, n and p are concentration of electrons and holes; μ_e and μ_n are mobilities of electrons and holes respectively.

$$\mu_e = \frac{e\tau_e}{m_e}$$

$$\mu_n = \frac{p\tau_n}{m_n}$$

The resistivity of metals increases with temperature. It is due to an increase in collision rate of carriers.

In case of semiconductor, concentration of carriers is small but increases with increase of temperature. This causes decrease resistivity with increasing temperature.

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4. **The resistance of a wire at 30°C is 3 Ω. At what temperature will the resistance be 5 Ω? The temperature coefficient of wire is $5.66 \times 10^{-3}\,C^{-1}$.** [T.U. 2062 Baishakh]

Solution:

Here,

Resistance of wire at 30° C $\quad (R) = 3\,\Omega$

Resistance of wire at T $\quad (R_T) = 5\,\Omega$

Temperature coefficient of wire $= 5.66 \times 10^{-3}\,C^{-1}$

We have,

$$R_T = R[1 + \alpha(T - 30°)]$$

$$\text{or, } 5 = 3[1 + 5.66 \times 10^{-3}(T - 30°)]$$

$$\text{or, } \left(\frac{5}{3} - 1\right)\frac{1}{5.66\times10^{-3}} = T - 30$$

$$\text{or, } 117.79 + 30 = T$$

$$\therefore T = 147.79°C$$

At the temperature of 147.79°C, the resistance will be 5 Ω.

---

5. **On the basis of mechanism of electrical conduction in metals and considering electrons as electron gas, show that the resistivity of metal is inversely proportional to number of free electrons per unit volume.** [T.U. 2063 Baishakh]

Solution: See the solution of Q. No. 3 on page no. 212

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6. **Express Ohm's law in terms of $\vec{J} = \sigma\vec{E}$ each symbol carries usual meaning.** [T.U. 2063 Baishakh]

Solution:

According to Ohm's law, potential difference (V) across the conductor is proportional to current flowing through it.

$$\text{i.e., } V = IR \qquad \text{... (i)}$$

where, R is resistance of conductor

$$\text{or, } I = \frac{V}{R} = \frac{E\cdot l}{\frac{\rho l}{A}} = \frac{E\cdot A}{\rho}$$

$$\text{or, } \frac{I}{A} = \frac{1}{\rho}\cdot E$$

$$\therefore J = \sigma E \qquad \text{... (ii)}$$

where $\frac{I}{A} = J$ and $\sigma = \frac{1}{\rho}$

In vector form;

$$\vec{J} = \sigma\vec{E}$$

The relation (ii) is the required expression.

---

7. **Use Ohm's law to prove $\vec{J} = \sigma\vec{E}$. The symbols carry their usual meaning.** [T.U. 2064 Poush]

Solution: See the solution of Q. No. 6 on page no. 214

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8. **State and explain Kirchhoff's law in electricity.** [T.U. 2064 Poush]

Solution:

Kirchhoff's current law states that, *"the sum of the currents entering any junction must be equal to the sum of currents leaving that junction."*

Let the currents I_3 and I_5 enter the junction whereas currents I_1, I_2 and I_4 leave it. Thus,

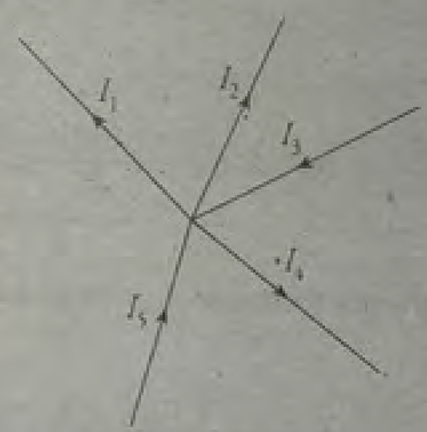

Figure: Sum of all currents to point in the circuit is zero

$$I_3 + I_5 = I_1 + I_2 + I_4$$

$$\text{or, } I_1 + I_2 + (-I_3) + I_4 + (-I_5) = 0$$

$$\text{i.e., } \sum I = 0$$

This law is based on non–accumulation of charges at any point in the circuit.

The Kirchhoff's voltage law states that, *"algebraic sum of potential difference around any closed path of a circuit must be zero."*

It is also stated as, *"algebraic sum of e.m.f.'s in any closed loop of a circuit must be equal to algebraic sum of current and resistance in the loop."*

9. If the copper wire is stretched to make it 0.1% longer, what is the percentage change in resistance?

Solution:

Here,

Let original length of copper wire $= l$

New length of copper wire $= l + 0.1\%$ of $l = (1.001)l$

Resistance of original wire $(R) = \frac{\rho l}{A}$

$$= \frac{\rho l^2}{Al} = \frac{\rho l^2}{V} \quad \dots \text{(i)}$$

Resistance of new wire $(R') = \frac{\rho l'^2}{V'}$

$$= \frac{\rho(1.001\, l)^2}{V'} \quad \dots \text{(ii)}$$

where, A is cross sectional area of original wire

V and V' are volume of original and new wires respectively

As the volume of wire remains constant;

$V = V'$

From equation (ii); we have,

$$R' = \frac{\rho l^2}{V}(1.001)^2 = \frac{\rho l^2}{V}(1.001)^2 = R \times 1.002$$

The percentage change in resistance;

$$= \left(\frac{R' - R}{R}\right) \times 100\% = \left(\frac{R(1.002) - R}{R}\right) \times 100\% = 0.2\%$$

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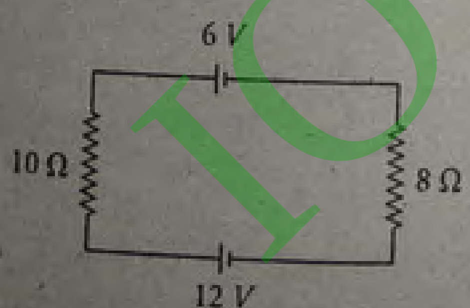
10. **A single loop circuit contains two resistors and two batteries as shown in figure.**



i) **Find the current in the circuit.**
ii) **What power is delivered to each resistor?**
iii) **What power is delivered by 12 V battery?** [T.U. 2065 Shrawan]

Solution:

Here,

$E_1 = 6\,V$
$E_2 = 12\,V$

$R_1 = 10\,\Omega$
$R_2 = 8\,\Omega$

Applying Kirchhoff's voltage law,

$$E_1 + IR_2 + (-E_2) + IR_1 = 0$$

or, $I = \frac{E_2 - E_1}{R_1 + R_2} = \frac{(12-6)}{10+8}$

$\therefore$ $I = \frac{1}{3} = 0.33\,A$

The current flowing through the circuit is $0.33\,A$.

The power delivered to the resistor R_1 is;

$$P_1 = IR_1^2 = \frac{1}{3}(12)^2 = 48\,W$$

The power delivered to the resistor R_2 is;

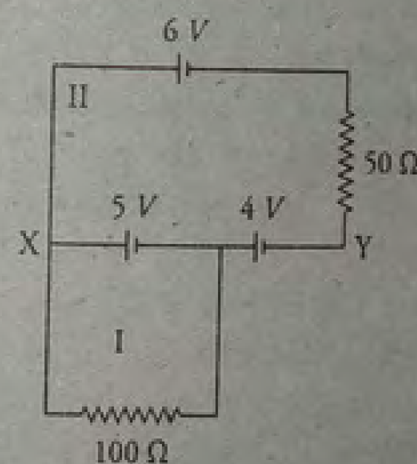
$$P_2 = IR_2^2 = \frac{1}{3}(18)^2 = 21.33\,W$$

The power delivered by 12 V batter is;

$$P = IV = \frac{1}{3} \times 12 = 4\,W$$

---

11. **In the given diagram, find the current in each resistor and potential difference X and Y.** [T.U. 2065 Chaitra]



Solution:

Here,

$E_1 = 6\,V$
$E_2 = 5\,V$
$E_3 = 4\,V$
$R_1 = 100\,\Omega$
$R_2 = 50\,\Omega$

Let I_1 and I_2 be the current through R_1 and R_2.

Applying Kirchhoff's voltage rule to loop (I); we obtain,

$E_1 = I_1 R_1$

$\therefore$ $I_1 = \frac{E_2}{R_1} = \frac{5}{100} = 0.05\,A$

Applying Kirchhoff's voltage rule to loop (II); we obtain,

$$-E_1 + E_2 + E_3 = I_2R_2$$

$$\text{or, } I_2 = \frac{-E_2 + E_2 + E_3}{R_2} = \frac{-6+4+5}{50}$$

$$\therefore I_2 = \frac{3}{50} = 0.06\, A$$

The potential difference across X and Y is $(4+5)V = 9\,V$.

---

12. Define the terms conductance and resistivity. Explain the atomic view of Ohm's law; also write down limitation of Ohm's law. **[T.U. 2065 Kartik]**

Solution:

The ratio of the current 'I' flowing through a conductor to potential difference across it, is called conductance (G), *i.e.*, the reciprocal of the resistance of a conductor is called conductance. The S.I. unit of conductance is ohm^{-1} or mho or Siemen (S).

The resistivity of a material is defined as resistance of a material of a unit length of a material of a unit length and unit cross sectional area. The S.I. unit of resistivity is Ohm meter ($\Omega\, m$).

Atomic view of Ohm's law

See the solution of Q. No. 3 on page no. 212

Limitation of Ohm's law

All the homogeneous materials, whether they are conductors like copper or semiconductors like pure silicon or silicon containing special impurities, obey Ohm's law within some range of values of the electric field. If the field is too strong, however, there are departures from Ohm's law in all cases.

Moreover, non–Ohmic conductors or non–linear resistances like diode value, triode value semi–conductors electrolytes, do not obey Ohm's law.

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13. What is the difference between intrinsic and extrinsic semiconductors? Derive the relation for conductivity of semiconductors. **[T.U. 2065 Kartik]**

Solution:

An intrinsic semiconductor is a pure or undoped semiconductor (silicon, germanium, etc.) in which electrons and holes are produced by thermal agitation.

Extrinsic semiconductor is a semiconductor in which carriers are produced by the addition of small quantities of group III or group V elements of periodic table to intrinsic semiconductors.

If n and p be the concentration of electrons and holes in a semiconductor at any finite temperature I, v_e and v_p be the drift velocities of electrons and holes, the total current density is:

$$J = J_e + J_p$$

Let μ_e and μ_n be the mobilities of electrons and holes, defined as drift velocities per unit applied electric field E,

$$i.e.,\ \mu_e = \frac{v_e}{E}$$

$$\text{or, } \mu_n = \frac{v_p}{E}$$

Thus,

$$J = nev_e + pev_p = ne\mu_e E + pe\mu_n E$$

$$\therefore J = eE[n\mu_e + p\mu_n] \qquad \ldots \text{(i)}$$

The conductivity of semiconductor is;

$$\sigma = \frac{J}{E} = e[n\mu_e + p\mu_n] \qquad \ldots \text{(ii)}$$

This gives the conductivity of semiconductor. In case of intrinsic semiconductors, $n = p$

$$\therefore \sigma = ne[\mu_e + \mu_n] \qquad \ldots \text{(iii)}$$

---

14. What is the drift velocity of the conductor electrons in a copper wire (molecular mass = 63.54 gm/mol, density = 8.96 gm/cm^3) with radius 900 μm when it has a uniform current 17 mA flowing through it? **[T.U. 2067 Ashadh]**

Solution:

Here,

Radius of wire $(r) = 900\,\mu m = 900 \times 10^{-6}\, m$

Molecular mass of copper $(M) = 63.54\, gm/mol$

Density of copper $(\rho) = 8.96\, gm/cm^3$

Current $(I) = 17\, mA = 17 \times 10^{-3}\, A$

Concentration of electron;

$$n = \frac{N_A\rho}{M} = \frac{6.023 \times 10^{23} \times 8.96}{63.54} = 8.49 \times 10^{22}\, cm^{-3}$$

$$= 8.49 \times 10^{28}\, m^{-3}$$

We have,

$$v_d = \frac{I}{neA} = \frac{I}{ne(\pi r^2)}$$

$$\text{or, } v_d = \frac{17 \times 10^{-3}}{8.49 \times 10^{28} \times 1.6 \times 10^{-19} \times 8\pi \times (900 \times 10^{-6})^2}$$

$\therefore \quad v_d = 4.93 \times 10^{-3}\ m/s$

The drift velocity of conduction electrons in a copper wire is $4.93 \times 10^{-3}\ ms^{-1}$.

15. **What is the average time between collisions of free electrons in a copper wire?**
(Atomic weight = 63 gm/mol, density = 9 gm/c.c., resistivity = 1.7 $\times 10^{-8}\ \Omega m$, $N_A = 6.02 \times 10^{23}\ mol^{-1}$) [T.U. 2067 Mangsir]

Solution:

Here,

Atomic weight of copper $(M) = 63\ gm/mol$

Density of copper $(\rho') = 9\ gm/c.c.$

Avogadro's number $(N_A) = 6.02 \times 10^{23}\ mol^{-1}$

Concentration of electrons;

$$n = \frac{N_A \rho'}{M} = \frac{6.02 \times 10^{23} \times 9}{63} = 8.6 \times 10^{22}\ cm^{-3}$$

$$= 8.6 \times 10^{28}\ m^{-3}$$

Resistivity $(\rho) = 1.7 \times 10^{-8}\ \Omega m$

We have,

$$\rho = \frac{m}{ne^2\tau}$$

$$\text{or, } \tau = \frac{m}{ne^2\rho} = \frac{9.1 \times 10^{-31}}{8.6 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-8}}$$

$\therefore \quad \tau = 2.43 \times 10^{-14}\ s$

The time between collisions of free electron is 2.43×10^{-14} seconds.

16. **Two copper wire of same length l and cross sectional area A and $2A$ are connected to a battery. What will be the ratio of drift velocities when the wires are in (a) series and (b) parallel?** [T.U. 2068 Shrawan]

Solution:

Let R_1 and R_2 be the resistances of wire each of length l and having cross sectional area A and $2A$ respectively. Thus, we can write;

$$R_1 = \rho\frac{l}{A}$$

$$\text{and } R_2 = \rho\frac{1}{2A} = \frac{1}{2}R_1$$

where, ρ is the resistivity of copper

The relation between drift velocity v_d is;

$$v_d = \frac{I}{neA}$$

In series combination;

Drift velocity of first and second wires are;

$$v_{d_1} = \frac{I}{neA} \quad \ldots \text{(i)}$$

$$v_{d_2} = \frac{I}{ne \cdot 2A} \quad \ldots \text{(ii)}$$

Dividing equation (i) by (ii); we obtain,

$$\frac{v_{d_1}}{v_{d_2}} = \frac{2}{1}$$

$\therefore \quad v_{d_1} : v_{d_2} = 2:1$

In parallel combination;

The currents through first and second wires are;

$$I_1 = \frac{V}{R_1}$$

$$\text{and } I_2 = \frac{V}{R_2} = \frac{2V}{R_1}$$

The drift velocities are;

$$v_{d_1} = \frac{I_1}{neA} = \frac{V}{neAR_1}$$

$$v_{d_2} = \frac{I_2}{ne(2A)} = \frac{2V}{2neAR_1} = \frac{V}{neAR_1}$$

Thus,

$$v_{d_1} : v_{d_2} = 1:1$$

The ratio of drift velocities in series combination is 2:1 and that of parallel combination is 1:1.

17. **What is the effect of temperature on semiconductor? Explain the nature of extrinsic and intrinsic semiconductor.** [P.U. 2002]

Solution:

The conductivity of a semiconductor depends strongly on temperature because thermal energy is required to excite electrons into the conduction band. Thus, the resistivity lowers with increase of temperature.

Intrinsic semiconductors have no impurity types of imperfection but may have defects. Germanium and silicon are two examples of intrinsic semiconductors. In these cases, there are four nearest neighbours to each atom and there are to electron bonds between each of the neighbours. The conduction occurs in an intrinsic semiconductor by thermal excitation of electrons from the valance to the conduction band. Electrons excited to the higher, predominantly empty conduction levels can move from atom to atom, thus contributing to conduction when field is applied to the semiconductor.

The electrical conductivity of intrinsic semiconductors can be significantly increased by insertions of fraction of impurities, *i.e.*, a simple way of increasing the conductivity of semiconductor crystals is done with the addition of controlled amount of contain impurities. For instance, the conductivity of silicon is increased a thousand times by addition of 10 parts per million of boron. The conduction that occurs then is called impurity conduction and impurities added are called dopants. When these foreign atoms are added into the semiconducting structure, the available quantum states are altered; one or more ne energy levels may appear in the bond structure. This introduces significant changes in the properties of the semiconductors. Such semiconductors are extrinsic semiconductors. Impurities or dopants frequently employed to silicon and germanium are elements of group III or group V of periodic table. The common dopants are boron, gallium, indium, phosphorous, arsenic, antimony, etc.

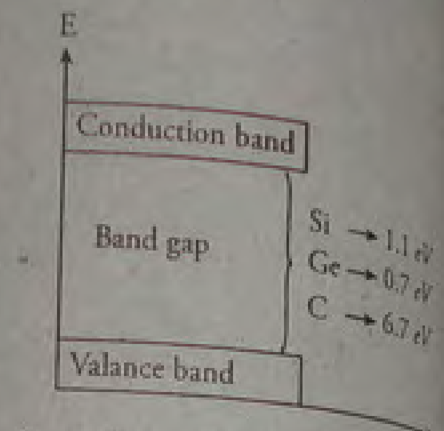
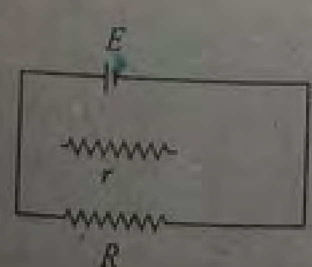


Figure: Energy bands in semiconductors

When small amount of dopants (one part in 10^7 approximately) of pentavalent impurity such as phosphorous added during crystal formation, the dopants lock into the crystal lattice since they are not greatly different in size from silicon atom and crystal is not unduly distorted.

18. In the given circuit, show that the power delivered to R, is maximum when load resistance r of the battery and this maximum power is $p = \frac{E^2}{4r}$. [P.U. 2002]



Solution:

Since output power of electrical circuit is;

$$P_{out} = I^2 R \quad \ldots \text{(i)}$$

and current flowing the circuit with a resistance R and e.m.f. E having internal resistance r of a battery is;

$$I = \frac{E}{(R+r)} \quad \ldots \text{(ii)}$$

Thus,

$$P_{out} = \left(\frac{E}{R+r}\right)^2 R$$

For a maximum powers, we must have,

$$\left(\frac{d\,P_{out}}{dR}\right) = 0$$

or, $$\frac{d}{dR}\left[\frac{E^2 R}{(R+r)^2}\right] = 0$$

or, $$\frac{(R+r)^2 \cdot E^2 - 2(R+r)E^2 \cdot R}{(R+r)^4} = 0$$

or, $$E^2[R^2 + 2Rr + r^2 - 2R^2 - 2Rr] = 0$$

or, $$-R^2 + r^2 = 0$$

$\therefore$ $$R = r$$

Since;

$$\frac{d^2 P_{out}}{dR^2} = -ve$$

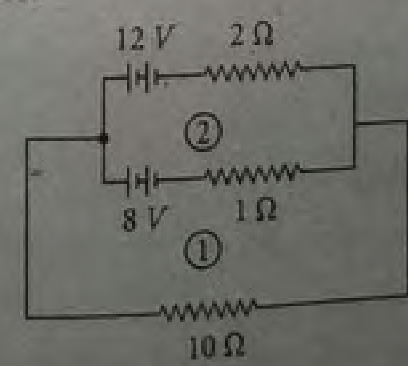
i.e., P_{out} will be maximum at $R = r$.

The maximum power,

$$P_{max} = \frac{E^2 r}{(r+r)^2} = \frac{E^2}{4r}$$

Hence, the maximum power is $\frac{E^2}{4r}$.

19. Use Kirchhoff's law to determine the value of currents flowing in each of the batteries. [P.U. 2002]



Solution:

Here,

$R_1 = 10\,\Omega \quad R_2 = 1\,\Omega \quad R_3 = 2\,\Omega$

$E_1 = 12\,V \quad E_2 = 8\,V$

Let I_1 and I_2 be the currents flowing through $12\,V$ and $8\,V$ batteries respectively. Applying Kirchhoff's current rule on the left junction,

$$I = I_1 + I_2 \quad \cdots \text{(i)}$$

Again, applying Kirchhoff's voltage rule for loop 1 and loop 2,

$$E_2 = I_2R_2 + IR_1$$

or, $8 = I_2 + 10\,I \quad \cdots$ (ii)

and $(12-8) = 2I_1 - I_2$

or, $4 = 2I_1 - I_2 \quad \cdots$ (iii)

Also, applying Kirchhoff's rule for outer loop,

$12 = 2I_1 - 10\,I$

or, $12 = I_1 - 5\,I \quad \cdots$ (iv)

From equations (i), (ii) and (iv); we obtain,

$I = (6 - 5\,I) + (8 - 10\,I)$

or, $16\,I = 14$

$\therefore \; I = \frac{7}{8}$ Amperes

From equation (ii); we have,

$I_2 = 8 - 10\,I = 8 - 10 \times \frac{7}{8} = 0.75\,A$

From equation (iv); we have,

$I_1 = 6 - 5\,I = 6 - 5 \times \frac{7}{8} = 1.625\,A$

The current through the batteries are $0.75\,A$ and $1.625\,A$ respectively.

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20. Define resistivity. Explain the atomic view of resistivity and show that $\rho = \frac{mv}{ne^2\lambda}$ where each symbol carries its usual meaning. [P.U. 2005]

Solution: See the solution of Q. No. 3 on page no. 212

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21. What are limitations of Ohm's law? State and explain the Kirchhoff's law? [P.U. 2007]

Solution:

Limitation of Ohm's law

See the solution of Q. No. 12 on page no. 218

Kirchhoff's law

See the solution of Q. No. 8 on page no. 215

Chapter 13

ELECTROMAGNETISM

13.1 MAGNETIC FLUX

Magnetic flux through any surface is defined as the number of magnetic lines of force crossing through that surface. It is numerically equals to product of area of surface (A) and normal component of a magnetic field (B)

i.e., Marnetic flux (ϕ) = BA

The magnetic flux is a scalar quantity. In S.I. units, it is measured in Weber (Wb) or Tesla per square meter (Tm^{-2}).

13.2 HALL EFFECT

Whenever a magnetic field is applied to the current carrying conductor, an electric field is set up in a direction perpendicular to both magnetic field and current. This effect is called Hall effect and developed field is called Hall field.

Consider a slab of material subjected to an external magnetic field $\vec{B}$ acting along z–direction and electric field $\vec{E}$ acting along y–direction so that current flows along x direction.

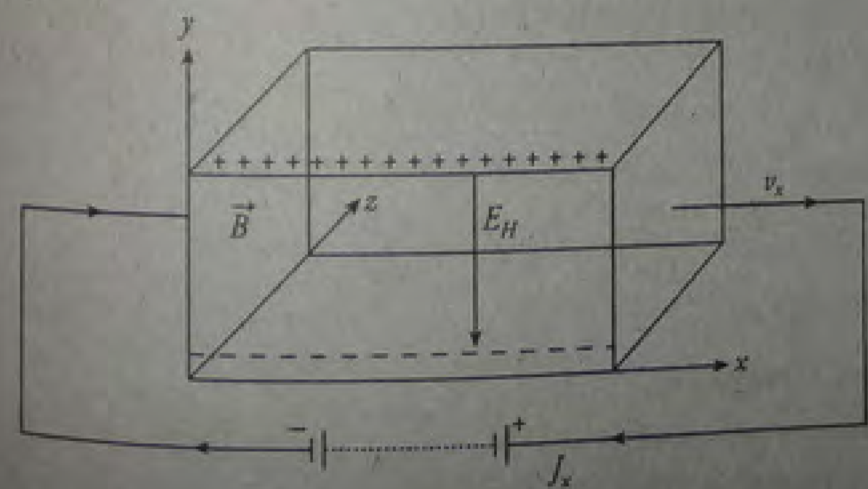


Figure: Hall effect

Under the influence of magnetic field, electron experiences force acting downward along y–axis so that lower surface collects the positive charge. This develops an electric field called Hall field (E_H). Ultimately, the force due to the Hall field cancels the Lorentz force and steady state is achieved.

The Lorentz force experienced by an electron of charge '$-e$' moving with velocity $\vec{v}$ in an electromagnetic field $\vec{E}$ and $\vec{B}$ is;

$$\vec{F} = (-e)[\vec{E} + \vec{v} \times \vec{B}]$$

At steady state;

$$F_y = 0$$

or, $$-e[E_H - v_x B] = 0$$

$\therefore$ $$E_H = v_x B$$

From Ohm's law,

$$I_x = n(-e)v_x \qquad 13.1$$

Hall coefficient (R_H) is defined by the relation;

$$R_H = \frac{E_H}{I_x B} = \frac{v_x B}{n(-e)Bv_x} = -\frac{1}{ne} \qquad 13.2$$

i.e., $$R_H = -\frac{1}{ne} \text{ (S.I)}$$

$$R_H = -\frac{1}{nec} \text{ (C.G.S.)} \qquad 13.3$$

These relations point out that Hall coefficient R_H gives sign and concentration of carriers. 13.4

The Hall mobility;

$$\mu = \frac{v_x}{E_x}$$

or, $$E_H = \mu E_x B \qquad 13.5$$

Thus, 13.6

$$R_H = \frac{E_H}{J_x B} = \frac{\mu E_x B}{\sigma E_x B} = \frac{\mu}{\sigma}$$

where, σ = electrical conductivity

i.e., $$\mu = \sigma R_H \text{ (S.I.)}$$

$$\mu = \sigma R_H c \text{ (C.G.S.)} \qquad 13.7$$

Hall angle, θ_H is defined by the relation; 13.8

$$\tan\theta_H = \frac{E_H}{E_x} = \frac{BJ_x}{neE_x} = \frac{nev_x B}{neE_x} = \frac{v_x B}{E_x}$$

Also, $$\tan\theta_H = \mu B \qquad 13.9$$

Applications of Hall effect

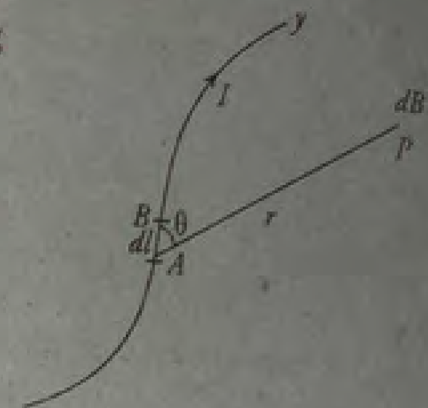
i) Determinations of
   a) nature of charge carriers
   b) concentration of carriers
   c) mobilities of holes and electrons
   d) nature of specimen
   e) power flow in EM waves

ii) Measurement of magnetic flux density.

13.3 BIOT AND SAVART LAW (LAPLACE'S LAW)

Biot and Savart law gives the magnitude of magnetic field $\vec{B}$ produced due to current flowing through a conductor.

Experiments show that magnitude of magnetic field $\vec{B}$ due to current carrying conductor is directly proportional to



$$dB \propto I$$

$$dB \propto dl$$

$$dB \propto \sin\theta$$

$$dB \propto \frac{1}{r^2}$$

or, $$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$dB = k\frac{I\,dl\sin\theta}{r^2} \qquad 13.10$$

where, k is the proportionality constant

In S.I. units;

$$k = \frac{\mu_0}{4\pi};\ \mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$$

$\therefore$ $$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \qquad 13.11$$

This is mathematical form of Biot and Savart law.

Applications

i) Magnetic field at the centre of narrow circular coil:

The total magnetic field due to narrow circular coil is;

$$B = \frac{\mu_0 NI}{2r} \qquad 13.12$$

The direction of B is given by right hand screw rule.

ii) Magnetic field along the axis of narrow circular coil;

$$B = \frac{\mu_0 NIa}{2r^2} \qquad 13.13$$

i.e., $$B = \frac{\mu_0 NIa^2}{2r^3} \qquad 13.14$$

or, $$B = \frac{\mu_0 NIa^2}{2(x^2 + a^2)^{\frac{3}{2}}} \qquad 13.15$$

iii) Magnetic field due to long straight conductor;

$$B = \frac{\mu_0 I}{2\pi a} \qquad 13.16$$

iv) Magnetic field inside the along solenoid;

$$B = \mu_0 nI = \frac{\mu_0 NI}{l}$$ 13.17

v) Magnetic field due to toroid;

$B = 0$ (at a point interior of toroid)

$B = 0$ (at a point exterior of toroid)

$B = \mu_0 nI$ (Inside toroid windings)

vi) Magnetic field due to Helmholtz coils;

$$B = 0.72 \times \frac{\mu_0 NI}{R}$$ 13.18

13.4 AMPERE'S CIRCUITAL LAW

'The line integral of magnetic field $\vec{B}$ around a closed loop is μ_0 times the net current enclosed by the loop.'

i.e., $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$ 13.19

13.5 GAUSS LAW IN MAGNETISM

For any closed surface, the number of lines entering the surface equals to the number of lines leaving the surface. Thus, Gauss law states that;

$$\oint \vec{B} \cdot d\vec{s} = 0$$ 13.20

This law is based on the fact that magnetic monopoles do not exist.

13.6 TWO PARALLEL CURRENT CARRYING CONDUCTORS

a) For like currents carrying conductors:

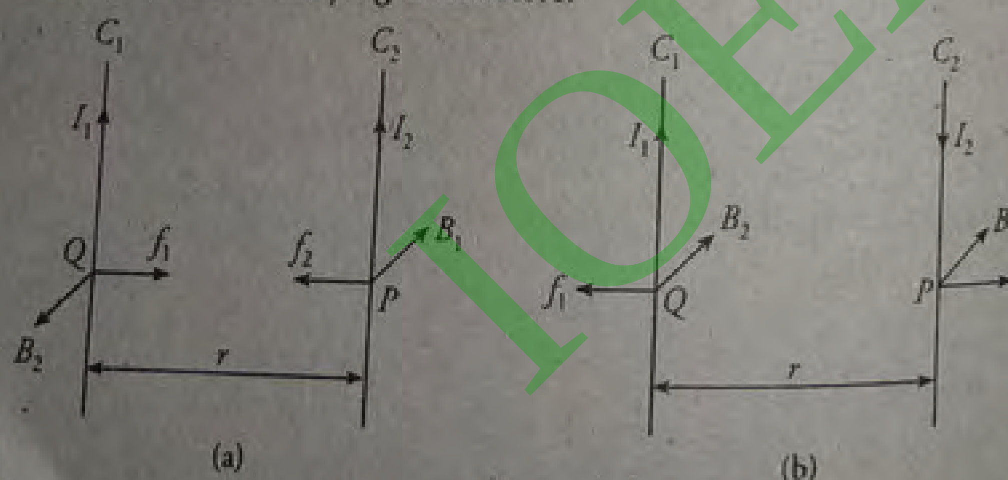


Figure: Parallel conductors with; (a) like currents
(b) unlike currents flowing through them

The magnetic field at the point P on the conductor C_2 due to the current I_1 in C_1 is;

$$B_1 = \frac{\mu_0 NI}{2\pi r}$$ 13.21

The force experienced by C_2;

$$F_2 = B_1 I_2 l_2 = \frac{\mu_0 I_1 I_2 l_2}{2\pi r}$$ 13.22

Force per unit length of C_2;

$$f_2 = \frac{F_2}{l_2} = \frac{\mu_0 I_1 I_2}{2\pi r}$$ 13.23

Similarly, force per unit length of C_1;

$$f_1 = \frac{\mu_0 I_1 I_2}{2\pi r}$$ 13.24

The forces are attractive in nature and forces per unit length of conductors are equal.

b) In case of unlike currents flowing through parallel conductor, repulsive forces develop in them.

13.7 SOLVED EXAM QUESTIONS

1. **Derive an expression for the magnetic flux density due to current carrying circular coil along its axis.** **[T.U. 2061 Baishakh]**

Solution:

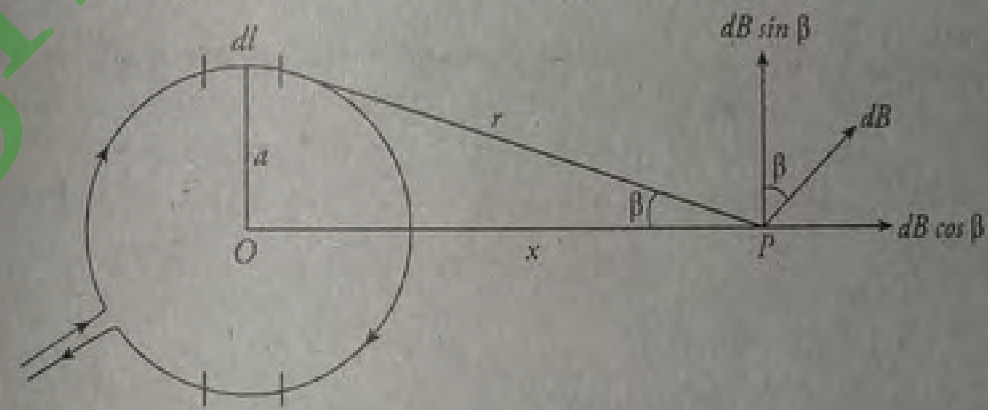


Figure: Magnetic field on the axial line of a circular current loop

Consider a narrow circular coil of radius 'a' in which current I is flowing. Let P be the field point at the distance x from the centre on the axial line. The magnitude of magnetic flux density dB at point P due to small element dl of the coil is;

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin \alpha}{r^2} \quad \ldots \text{(i)}$$

where, r is the distance of the point P from the element dl. Here, angle between dl and r is $90°(\alpha = 90°)$.

$$\therefore \quad dB = \frac{\mu_0 I dl}{4\pi r^2}$$

Suppose the r makes an angle β with x axis. Thus the small field dB has two components $dB \cos \beta$ (perpendicular to axis) and $dB \sin \beta$ (parallel to axis)

Considering diametrically opposite small element dl, the components perpendicular to axis cancel each other, while the components along the axis is $2dB\sin\beta$. Hence, the magnetic field due to one turn of the narrow coil circular coil is;

$$B = \int_0^{2\pi a} dB\sin\beta = \int_0^{2\pi a}\frac{\mu_0}{4\pi}\frac{Idl\sin\beta}{r^2} = \frac{\mu_0 I}{4\pi r^2}\sin\beta\int_0^{2\pi a} dl$$

$$= \frac{\mu_0 I}{4\pi r^2}\sin\beta\times 2\pi a = \frac{\mu_0 I a}{2r^2}\sin\beta$$

If narrow circular coil contains N turns;

$$B = \frac{\mu_0 I a N}{2r^2}\sin\beta$$

Since $\sin\beta = \frac{a}{r}$ and $r^2 = x^2 + a^2$

$$B = \frac{\mu_0 I a^2 N}{2r^3} = \frac{N\mu_0 I a^2}{2(x^2+a^2)^{\frac{3}{2}}} \qquad \ldots \text{(ii)}$$

This is the required expression. The direction of this field is given by right hand Screw rule.

---

2. **What is the value of Hall voltage between two sides of copper strip of width of 5 *mm* and thickness 3 *mm* when a downward magnetic field of flux density 0.9 *T* is applied normally to the strip carrying a current of 30 *A*? [T.U. 2061 Baishakh]**

Solution:

Here,

Width of strip $(b) = 5\ mm = 5\times 10^{-3}\ m$

Thickness of strip $(t) = 3\ mm = 3\times 10^{-3}\ m$

Magnetic flux density $(B) = 0.9\ T$

Current $(I) = 30\ A$

Hall voltage $(E_H) = ?$

We have,

Hall voltage;

$$V_H = \frac{BI}{net} = \frac{0.9\ T\times 30\ A}{n\times 1.6\times 10^{-19}\ C\times 3\times 10^{-3}\ m}$$

$$= \frac{1}{n}(5.63\times 10^{22})$$

For copper, number of electrons per unit volume,

$$n = 8.4\times 10^{28}\ m^{-3}$$

$$\therefore\ V_H = \frac{5.06\times 10^{17}}{8.4\times 10^{28}} = 6.69\times 10^{-7}\ V$$

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3. **Use the Biot and Savart law to calculate magnetic field on the axial line of a current carrying circular loop. [T.U. 2061 Ashwin]**

Solution: See the solution of Q. No. 1 on page no. 229

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4. **What is Hall effect? What conclusions can be drawn from this effect? Derive an expression for number density of charge carrier in terms of Hall effect. [T.U. 2061 Ashwin]**

Solution:

Whenever magnetic field is applied to the current carrying conductor an electric field is set up in a direction perpendicular to both magnetic field and current. This effect is called Hall effect and developed field is called Hall field.

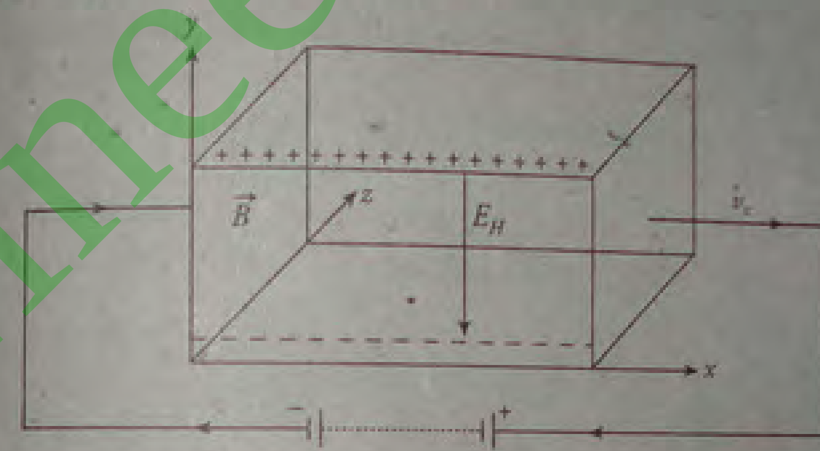

Figure: Hall effect

Hall effect is based on the principle of simple dynamics of charges moving in electromagnetic fields.

Consider a slab of material subjected to an external magnetic field $\vec{B}$ acting along z–direction and electric field $\vec{E}$ acting along y–direction so that current flows along x–direction.

Under the influence of magnetic field, electron experiences force acting y–axis so that lower surface collects the positive charge. This develops an electric field (E_H). Ultimately, the force due to the Hall field cancels Lorentz force and steady state is achieved.

The Lorentz force experienced by an electron of charge '$-e$' moving with velocity $\vec{v}$ in an electromagnetic field $\vec{E}$ and $\vec{B}$ is;

$$\vec{F} = -e[\vec{E} + \vec{v}\times\vec{B}]$$

At steady state; $F_y = 0$

or, $-e[E_H - v_x B] = 0$

$$\therefore\ E_x = v_x B \qquad \ldots \text{(i)}$$

From Ohm's law;

$$I_x = n(-e)v_x \qquad \dots (ii)$$

The Hall coefficient, R_H is defined by the relation:

$$R_H = \frac{E_H}{J_x B} = \frac{v_x B}{n(-e)v_x B} = -\frac{1}{ne}$$

$$\therefore \quad \left.\begin{aligned} R_H &= -\frac{1}{ne} \quad \text{(S.I.)} \\ &= -\frac{1}{nec} \quad \text{(C.G.S.)} \end{aligned}\right\} \qquad \dots (iii)$$

These relations point out that Hall coefficient R_H gives sign and concentrations of carriers.

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5. What is Hall effect? How the type of charge carriers can be identified with the help of Hall effect? Explain with figures. **[T.U. 2062 Baishakh]**

Solution: See the solution of Q. No. 4 on page no. 231

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6. What is the flux density at the centre of circular coil of radius 2 *cm* and with 20 turns carrying a current 10 A?[T.U. 2062 Baishakh]

Solution:

Here,

Radius of coil $(r) = 2\,cm = 2 \times 10^{-2}\,m$

Number of turns $(N) = 20$

Current, $(I) = 10\,A$

We have,

Flux density at the centre of the circular coil;

$$B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 20 \times 10}{2 \times 2 \times 10^{-2}} = 6.28 \times 10^{-3}\,T$$

---

7. State Ampere's theorem. Derive an expression for magnetic field due to a solenoid carrying current.

Solution:

Ampere's law states that *'the line integral of magnetic field $\vec{B}$ around a closed loop is μ_0 times net current enclosed by the loop.'*

$$i.e., \oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \dots (i)$$

In order to determine the magnetic field inside the solenoid using Ampere's law, we sketch a closed path PQRS as shown in figure. The line integral of $\vec{B}$ over a closed path PQRS is;

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{net current through loop PQRSP}$$

$$\text{or,} \quad \int_P^Q \vec{B} \cdot d\vec{l} + \int_Q^R \vec{B} \cdot d\vec{l} + \int_R^S \vec{B} \cdot d\vec{l} + \int_S^R \vec{B} \cdot d\vec{l} = \mu_0 nlI$$

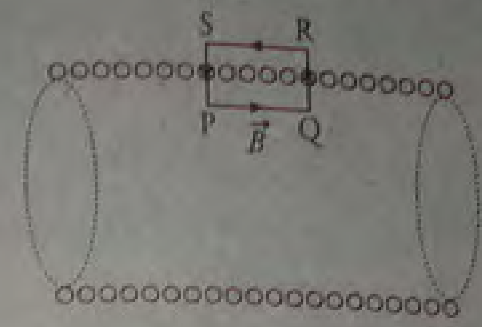

Figure: Magnetic field due to solenoid

Since,

$$\int_P^Q \vec{B} \cdot d\vec{l} = \int |\vec{B}||d\vec{l}| \cos 0° = Bl$$

$$\int_Q^R \vec{B} \cdot d\vec{l} = \int_S^P \vec{B} \cdot d\vec{l} = 0 \qquad (\because \vec{B} \perp d\vec{l})$$

$$\int_P^Q \vec{B} \cdot d\vec{l} = 0 \qquad (\because \text{Field outside the solenoid is zero})$$

Thus,

$$Bl = \mu_0 nlI$$

$$\therefore \quad B = \mu_0 nI$$

This is the required expression for magnetic field due to solenoid carrying current.

---

8. Compare Ampere's law with Biot and Savart law which is generally more useful for calculating $\vec{B}$ for a current carrying conductor. Calculate the magnetic field on the axial line of a long solenoid carrying current. **[T.U. 2065 Shrawan]**

Solution:

Ampere's law and Biot and Savart law gives the magnitude of magnetic field $\vec{B}$ produced due to current flowing through conductor. An amperean path must be chosen in case of Ampere's law *i.e.*, Ampere's law is applicable only if there is considerable symmetry. Thus, Biot and Savart law is generally more useful for calculating $\vec{B}$ for current carrying conductor.

Magnetic field on axial line of a long solenoid

See the solution of Q. No. 7 on page no. 232

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9. A rectangular copper strip 1.5 *cm* wide and 0.10 *cm* thick carries a current of 5 A. Find the Hall voltage for 1.2 T. Magnetic field applied in a perpendicular to the strip. **[T.U. 2065 Shrawan]**

Solution: Proceed as solution of Q. No. 2 on page no. 230

10. State Biot and Savart law. Use it to find magnetic field due to long straight current carrying conductor. [T.U. 2065 Chaitra]

Solution:

- Biot and Savart law states that, the magnitude of the field $d\vec{B}$ produced at point P at distance r by a current length element $Id\vec{l}$ turns out to be,

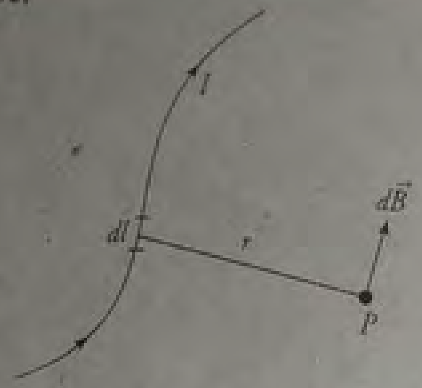

Figure: A current length element $Id\vec{l}$ produces a differential magnetic field $d\vec{B}$ at point P

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

where, θ = angle between the direction of $d\vec{l}$ and $\vec{r}$

μ_0 = permeability constant

$= 4\pi \times 10^{-7}\ TmA^{-1}$

Magnetic field due to long straight current carrying conductor

Figure illustrates the long current carrying conductor. The magnitude of the differential magnetic field at P by the current length element $Id\vec{l}$ located at a distance r from P is:

$$dB = \frac{\mu_0}{4\pi} = \frac{Idl\sin\theta}{r^2} \qquad \dots (i)$$

Now, consider a current length element in the lower half of the wire, one that is as far below P as $d\vec{l}$ is above P. The magnetic field produced by this current element has the same magnitude and direction as that from element $Id\vec{l}$. Further the magnetic field produced by the upper half of the exactly same as that produced by lower half. To find

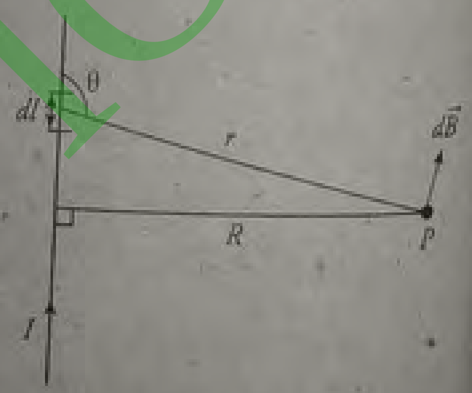

Figure: The magnetic field due to long current carrying conductor

the magnitude of the total magnetic field $\vec{B}$ at P, we need to multiply the result of our integration by 2.

$$i.e.,\ B = 2\int_0^\infty dB = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{\sin\theta\, dl}{r^2} \qquad \dots (ii)$$

The variables θ, l and r are not independent. They are related by:

$$r = \sqrt{l^2 + R^2}$$

$$\text{or, } \sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{l^2 + R^2}}$$

With these substitutions, equation (ii) becomes;

$$B = \frac{\mu_0 I}{2\pi}\int \frac{R\, dl}{(l^2 + R^2)^{\frac{3}{2}}}$$

We have,

$$\int \frac{R\, dl}{(l^2 + R^2)^{\frac{3}{2}}} = \frac{l}{R^2(l^2 + R^2)^{\frac{1}{2}}}$$

$$\text{or, } B = \frac{\mu_0 I}{2\pi R}\left[\frac{l}{(l^2 + R^2)^{\frac{1}{2}}}\right]_0^\infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi r} \qquad \dots (iii)$$

which is the required expression.

11. A copper strip 2 cm wide and 1.5 mm thick is placed in a magnetic field with field strength 2.6 Tesla. If a current of 145 A is set up in the strip, what Hall potential difference appears across the strip? The number of charges is 8.4×10^{28} m^{-3}. [T.U. 2065 Kartik]

Solution: Proceed as solution of Q. No. 2 on page no. 230

12. What is magnetic flux density? Derive an expression for magnetic flux density inside a long solenoid carrying current I, at a point near its centre. [T.U. 2065 Kartik]

Solution:

Magnetic flux density is defined as flux per unit area across an area at right angles to the magnetic field.

Magnetic flux density inside the solenoid

See the solution of Q. No. 7 on page no. 232

13. **A long straight wire of radius R carries a uniformly distributed current I. Calculate magnetic field inside and outside the wire.** **[T.U. 2067 Ashadh]**

Solution:

Figure illustrates the cross section of a long straight wire of radius R that carries a uniformly distributed current I directly out of the paper. The current is uniformly distributed over a cross section of the wire; the magnetic field $\vec{B}$ produced by the current must be cylindrically symmetrical. Thus, to determine the magnetic field at points inside the wire, we draw amperean loop of radius r. Ampere's law yields;

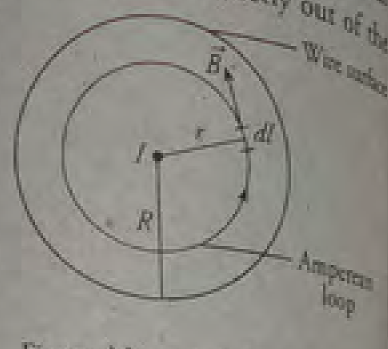

Figure: Magnetic field inside the wire

$$\oint \vec{B}\, d\vec{S} = \mu_0 \times \text{net current through the loop}$$

$$\text{or, } B\oint d\vec{S} = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$\text{or, } B(2\pi r) = \mu_0 I\frac{r^2}{R^2}$$

$$\therefore B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r \qquad \dots \text{(i)}$$

Hence, magnitude of $\vec{B}$ is proportional to r inside the wire. An amperean loop is drawn outside the wire to calculate field outside it. Ampere's law leads us to;

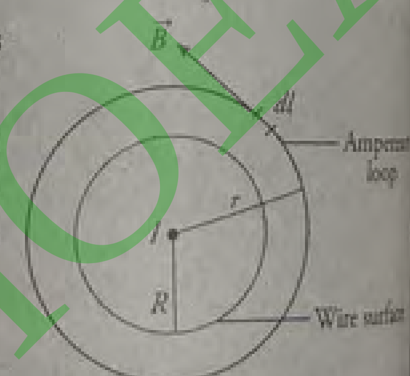

Figure: Magnetic field outside the wire

$$\oint \vec{B}\, d\vec{S} = \mu_0 \times I_{net}$$

$$\text{or, } B(2\pi r) = \mu_0 \times I$$

$$\therefore B = \frac{\mu_0 I}{2\pi r} \qquad \dots \text{(ii)}$$

This is the required expression for magnetic field outside long straight conductor.

14. **In a certain cyclotron, proton moves in a circular of radius 0.5 m. The magnitude of the magnetic field is 1.20 T. What is the oscillator frequency? What is the kinetic energy of the proton in eV?** **[T.U. 2067 Ashadh]**

Solution:

Here,

Radius $(r) = 0.5\ m$

Magnetic field $(B) = 1.05\ T$

Mass of a proton $(m) = 1.67 \times 10^{-27}\ kg$

Charge of a proton $(e) = 1.6 \times 10^{-19}\ C$

Oscillator frequency $(f) = ?$

Kinetic energy $(E) = ?$

We have,

$$f = \frac{eB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 1.02}{2\pi \times 1.67 \times 10^{-27}} = 1.56 \times 10^7\ Hz = 15.6\ MHz$$

When a proton of charge e and mass m circulating with velocity v, then we write,

$$evB = \frac{mv^2}{r}$$

$$\text{or, } v = \left(\frac{eB}{m}\right) r = \frac{1.6 \times 10^{-19} \times 1.02}{1.67 \times 10^{-27}} \times 0.5$$

$$= 4.89 \times 10^7\ ms^{-1}$$

Kinetic energy;

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.67 \times 10^{-27} \times (4.89 \times 10^7)^2$$

$$= 2.0 \times 10^{-12}\ J = \frac{2.0 \times 10^{-12}}{1.6 \times 10^{-19}}\ eV$$

$$= 1.25 \times 10^5\ eV$$

The oscillator frequency and kinetic energy of the proton are 15.6 MHz and $1.25 \times 10^5\ eV$ respectively.

15. **A circular coil having radius R and carries a current I. Calculate the magnetic flux density at an axial point and at what condition field will be minimum?** **[T.U. 2067 Mangsir]**

Solution: See the solution of Q. No. 1 on page no. 229

Special cases

a) At axial point, $x = 0$. Thus, from equation (ii) of Q. no. 1;

$$B = \frac{N\mu_0 I a^2}{2(a^2)^{\frac{3}{2}}} = \frac{N\mu_0 I}{2a} \qquad \dots \text{(iii)}$$

b) If $x \gg a$, $x^2 + a^2 \approx x^2$. Thus, from equation (ii);

$$B = \frac{N\mu_0 I a^2}{2x^3} \quad \cdots (iv)$$

i.e., if $x \gg a$, the magnetic field will be minimum.

16. A copper strip 2 *cm* wide and 10 *mm* thick is placed in a magnetic field of 1.5 *T*. If a current of 200 *A* is set up in the strip. Calculate Hall voltage and Hall mobility if the number of electrons per unit volume is $8.4 \times 10^{28}\ m^{-3}$ and resistivity is $1.72 \times 10^{-8}\ \Omega m$.
[T.U. 2068 Shrawan]

Solution:

Here,

Width of the strip	$(b) = 2\ cm = 2 \times 10^{-2}\ m$
Thickness of the strip	$(t) = 10\ mm = 10 \times 10^{-3}\ m$
Magnetic field	$(B) = 1.5\ T$
Concentration of electrons	$(n) = 8.4 \times 10^{28}\ m^{-3}$
Resistivity	$(\rho) = 1.72 \times 10^{-8}\ \Omega m$
Hall voltage	$(V_H) = ?$
Hall mobility	$(\mu) = ?$

We have,

Hall coefficient;

$$R_H = \frac{E_H}{J_i B}$$

$$\text{or,} \quad \frac{1}{ne} = \frac{\frac{V_H}{b}}{\frac{I}{bt} \cdot B}$$

$$\text{or,} \quad V_H = \frac{BI}{net}$$

$$= \frac{1.5 \times 200}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10 \times 10^{-3}}$$

$$\therefore \quad V_H = 2.33 \times 10^{-6}\ V = 2.23\ \mu V$$

Hall mobility;

$$\mu_H = \frac{R_H}{\rho} = \frac{1}{ne\rho}$$

$$= \frac{1}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.72 \times 10^{-8}}$$

$$\therefore \quad \mu_H = 231.2\ m^2V^{-1}s^{-1}$$

The required Hall voltage and Hall mobilities are $2.23\ \mu V$ and $231.2\ m^2V^{-1}s^{-1}$ respectively.

17. Show that the magnetic field due to a curve wire segment carrying current *I* and circular arc *R* is $\frac{\mu_0 I}{8R}$.

Solution:

Consider a curved wire segment carrying current I and consists of segments M'M, MN and NN'. The segments M'M and NN' are straight segments and segment MN is circular arc of radius R. Let B', B and B'' are magnetic fields at O due to the segments M'M, MN and NN' respectively. Since the angle between the angle between $d\vec{l}$ and $\vec{r}$ for straight lines MM' and NN'.

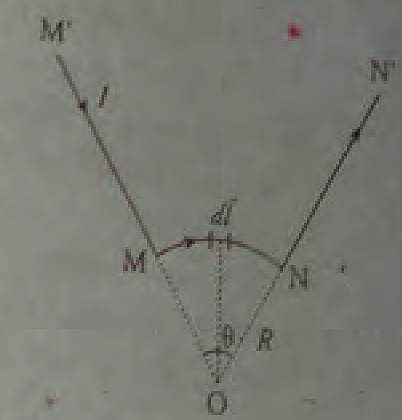

Figure: A curved wire segment carrying current *I*

$$\therefore \quad d\vec{l} \times \vec{r} = 0$$

Thus, the magnetic fields due to these sections are zero.

i.e., $B' = 0 = B''$

At every point on MN, $d\vec{l}$ is perpendicular to $\vec{r}$. According to Biot and Savart law, the magnetic field at O due to segment MN is;

$$dB = \frac{\mu_0 I\, dl \sin 90°}{4\pi R^2}$$

$$= \frac{\mu_0 I\, dl}{4\pi R^2}$$

For arc MN,

$$dl = R\, d\theta$$

$$\therefore \quad dB = \frac{\mu_0 I\, d\theta}{4\pi R}$$

$$\text{or,} \quad B = \int_0^{\frac{\pi}{2}} dB$$

$$= \int_0^{\frac{\pi}{2}} \frac{\mu_0 I\, d\theta}{4\pi R} = \frac{\mu_0 I}{4\pi R} \int_0^{\frac{\pi}{2}} d\theta$$

$$\therefore \quad B = \frac{\mu_0 I}{8R}$$

The total magnetic field at point O due to wire is;

$$B = \frac{\mu_0 I}{8R}$$

The direction of this field is determined by right hand Screw rule.

18. **State and explain the Biot and Savart law. Derive an expression for magnetic field near the straight conductor carrying current.** [P.U. 2003]

Solution:

Biot and Savart law gives the magnitude of the magnetic field $\vec{B}$ produced due to current flowing through a conductor.

Suppose dl be the small element of the conductor. There is a point P at a distance r from the centre of the element dl. Let dB is the magnetic flux density of the magnetic field at a point P due to small element dl, then according to Biot and Savart law,

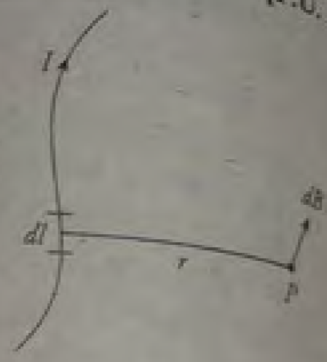

Figure: A current length element Idl produces a differential magnetic field $d\vec{B}$ at a point P

$$dB \propto I$$
$$dB \propto dl$$
$$dB \propto \sin\theta$$
$$dB \propto \frac{1}{r^2}$$

where, θ is the angle between μ and r. Combining these relations we obtain,

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

or, $$dB = k\frac{I\,dl\sin\theta}{r^2}$$

where, k is proportionality constant. In S.I. units,

$$k = \frac{\mu_0}{4\pi}$$

where, $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$ is called permeability of free space

Thus,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

This is mathematical expression of Biot and Savart law.

Magnetic field near the straight current carrying conductor

See the solution of Q. No. 10 on page no. 234

19. **State and explain Ampere's theorem. Derive and expression for the magnetic field due to solenoid carrying current.** [P.U. 2005]

Solution: See the solution of Q. No. 7 on page no. 232

State Biot and Savart law. Also calculate the magnetic flux density due to long straight conductor. [P.U. 2007]

Solution: See the solution of Q. No. 10 on page no. 234

State Ampere's law and use it to determine the magnetic flux density set up by long straight current carrying conductor. Calculate force between two parallel conductors carrying current in the same direction. [P.U. 2010]

Solution:

Ampere's law states that, "*the line integral of magnetic field $\vec{B}$ around a closed loop is μ_0 times the net current enclosed by the loop.*"

i.e., $\oint \vec{B}d\vec{l} = \mu_0 I$

where, I is the net current enclosed by a closed path

Consider a long straight conductor carries current I along its length. Let P be a point at a distance 'a' from the conductor. Every point at a same distance from it is identical and therefore, will have same magnitude of the magnetic field. Therefore, it is prudent to choose a circular loop centered at the wire and containing at point P. B at any point on the tangent to the loop and its direction is determined from the right hand Screw rule.

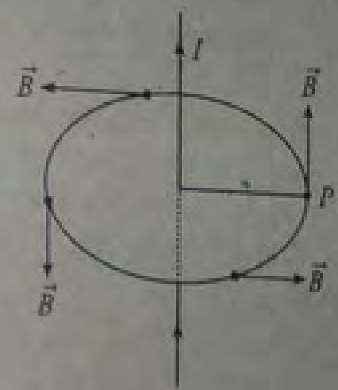

Figure: Magnetic field due to long straight conductor

Suppose the radius of the loop is 'a'.

From Ampere's law,

$$\oint \vec{B}d\vec{l} = \mu_0 I$$

or, $$\oint B\,dl = \mu_0 I \qquad (\because \vec{B} \parallel d\vec{l})$$

or, $$B \cdot 2\pi a = \mu_0 I$$

$$\therefore\ B = \frac{\mu_0 I}{2\pi a}$$

This is the required expression of magnetic field due to long straight conductor.

Suppose two long straight conductors carrying currents I_1 and I_2 in the same direction. If their separation is 'a', the magnetic field at any point on the second conductor due to current I_1 in the first conductor will be;

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

The force experienced by second conductor due to field B_1 on the length 'l' of the second wire will be;

$$F_2 = B_1 I_2 l = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

Force per unit length of second conductor;

$$f_2 = \frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

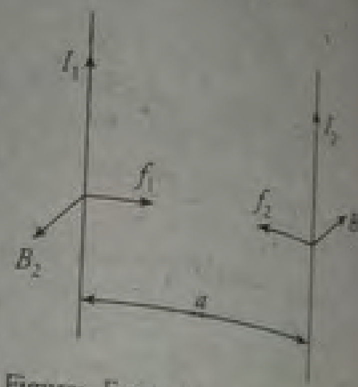

Figure: Forces between two parallel conductors

Similarly, force per unit length of first conductor;

$$f_1 = \frac{\mu_0 I_1 I_2}{2\pi a}$$

The forces are attractive in nature and forces per unit length of conductors are equal.

22. **A solenoid has length $L = 1.23\ m$ and inner diameter $d = 3.55\ cm$ and carries a current $I = 2.57\ A$. It consists of five closed pack layers, each with 850 turns along length L. What is the magnetic induction B at its centre?** [P.U. 2011]

Solution:

Here,

Length of solenoid $(L) = 1.23\ m$

Inner diameter $(d) = 3.55\ cm = 3.55 \times 10^{-2}\ m$

Total number of turns $(N) = 5 \times 850$ turns

Current $(I) = 2.57\ A$

Magnetic induction $(B) = ?$

We have,

Magnetic induction B at the centre of solenoid;

$$B = \mu_0 n I = \frac{\mu_0 N I}{L} = \frac{4\pi \times 10^{-7} \times 5 \times 850 \times 2.57}{1.23} = 1.12 \times 10^{-2} \text{ Tesla}$$

Chapter 14

ELECTROMAGNETIC INDUCTION

14.1 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

i) Whenever there is change in the magnetic flux through a closed circuit, there is an induced e.m.f.

ii) The magnitude of the induced e.m.f. is equal to the rate of change of magnetic flux associated with the circuit.

i.e., $\varepsilon \propto \frac{d\phi}{dt}$

$$\therefore \quad \varepsilon = -\frac{d\phi}{dt} \qquad 14.1$$

The negative sign shows that the induced e.m.f. has sign such that it always opposes the change in magnetic flux.

14.2 LENZ'S LAW

It states that, *"the direction of induced current in any circuit is such that it always opposes the cause producing it."*

Lenz's law is an example of conservation of energy. In inducing an e.m.f., we have to do work against the opposing forces produced by induced currents. This work done is transformed electrical energy. It is only possible to generate electrical energy or induced current at the cost of mechanical energy. Thus, energy is conserved.

14.3 EDDY CURRENTS

A self circulating current set up on a metallic piece placed in changing magnetic field is known as Eddy current. It causes loss of energy in the form of heat and are, in most of the cases, undesirable.

Eddy currents are used in;

i) electromagnetic damping
ii) induction heating
iii) electromagnetic brakes
iv) induction motors
v) energy meters

14.4 SELF INDUCTION AND MUTUAL INDUCTION

Whenever a magnetic flux associated with solenoid changes, an e.m.f. is induced in it. This phenomenon is called self induction. It is also known as *'back e.m.f.'*. Its direction is determined according to Lenz's law.

Whenever a current flows through one of the two neighbouring coils, an induced e.m.f. appears in second coil. This phenomenon is called *mutual induction*. A coil in which current flows, is called primary coil and a coil in which induced e.m.f. appears, is called *secondary coil*.

14.5 ENERGY STORED IN MAGNETIC FIELD

The rate at which work being done against self induced e.m.f. in an inductor (coil, solenoid or toroid);

$$P = IE \qquad 14.2$$

where, E is a self induced e.m.f. on an inductor while a varying current I flowing through it

i.e., $$E = -L\frac{dI}{dt} \qquad 14.3$$

Thus, $$P = LI\frac{dI}{dt} \qquad 14.4$$

We ignore the negative sign as power has magnitude only. The work done in time dt is;

$$dW = P\,dt$$

$$= LI\frac{dI}{dt}dt$$

$$\therefore \quad dW = LIdI \qquad 14.5$$

Total energy supplied during the time when the current in the inductor becomes I, starting from zero is;

$$W = \int_0^I LIdI$$

$$= L\frac{I^2}{2}\Bigg|_0^I$$

$$= \frac{1}{2}LI^2$$

$\therefore$ Energy stored in a magnetic field (or in an inductor) is;

$$U_B = \frac{1}{2}LI^2 \qquad 14.6$$

Energy stored per unit volume (in free space)

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0} \qquad 14.7$$

14.6 SOLVED EXAM QUESTIONS

1. **Define self induction? Calculate the coefficients of self induction for solenoid.** **[T.U. 2061 Baishakh]**

Solution:

Whenever a magnetic flux associated with a solenoid changes, an induced e.m.f. appears on it. This phenomenon is called *self induction*.

Consider a long solenoid of length l, cross–sectional area A and number of turns N. If a current I is flowing through it, the magnetic field inside the solenoid is;

$$B = \mu_0 nI \qquad \ldots \text{(i)}$$

where, $n = \frac{N}{l}$ = number of turns per unit length

The total magnetic flux linked with solenoid is;

$$\phi = N(BA)$$

or, $$LI = N(\mu_0 nIA)$$

or, $$LI = N\mu_0\frac{N}{l}IA$$

$$\therefore \quad L = \mu_0\frac{N^2}{l}A = \mu_0 n^2 Al \qquad \ldots \text{(ii)}$$

This gives the coefficient of self induction of solenoid and depends on the number of turns, length and cross–sectional area of solenoid.

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2. What is self inductance? Derive an expression for self inductance of a solenoid. [T.U. 2061 Ashwin]

Solution:

Self inductance

Whenever there is a unit rate of decrease of current through a coil, the induced e.m.f. appeared in that coil is said to be self inductance.

Self inductance of a solenoid

See the solution of Q. No. 1 on page no. 245

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3. **In LR circuit how does the current increase or decrease with time? Explain deriving the relation of current in terms of inductance and resistance in the circuit.**

Solution:

When the switch is moved towards a, the current in the resistor starts to rise. If the inductor were not present, the current would quickly rise to steady state value $\frac{\varepsilon}{R}$. The inductance of the inductor results in a back e.m.f., an inductor in the circuit opposes change in the current. If the e.m.f. in the circuit is increased so that current rises, the inductor opposes this change and hence it does not rise instantaneously. If the e.m.f. via battery is decreased, the presence of inductor results in a slow drop in the current rather than immediate drop.

When the switch is towards a, current flows anticlockwise direction. On applying Kirchhoff's rule to the circuit along anticlockwise direction;

$$\varepsilon = V_L + V_R$$

or, $$\varepsilon = L\frac{dI}{dt} + IR$$

or, $$L\frac{dI}{dt} + \left(I - \frac{\varepsilon}{R}\right)R = 0$$

or, $$\frac{dI}{dt} = -(I - I_0)\frac{R}{L}$$

where, $I_0 = \frac{\varepsilon}{R}$: maximum steady current

or, $$\frac{dI}{(I-I_0)} = -\frac{dt}{\left(\frac{L}{R}\right)}$$

On integrating, we obtain,

$$\int_0^I \frac{dI}{(I-I_0)} = -\frac{R}{L}\int_0^t dt$$

or, $$\ln(I - I_0)|_0^I = -\frac{R}{L}t\Big|_0^t$$

or, $$\frac{\ln(I-I_0)}{\ln(0-I_0)} = -\left[\frac{R}{L}t\right]$$

or, $$\frac{I-I_0}{-I_0} = e^{-\frac{R}{L}t}$$

or, $$I = I_0 - I_0 e^{-\frac{R}{L}t}$$

$\therefore$ $$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \qquad \dots (i)$$

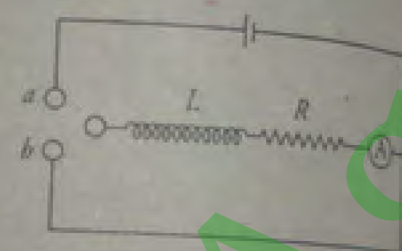

Figure: LR circuit

This equation shows the effect of inductor in the growth of current.

If $t = \frac{L}{R} = \tau_L$ (time constant), the equation (i) reduces to

$$I = I_0(1 - e^{-1}) = 0.63\, I_0$$

Hence, time constant τ_L is the time it takes the current in the circuit to reach about 63% of its final equilibrium value I_0.

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4. What is meant by self inductance? Obtain an expression for the inductance of a toroid (solenoid). **[T.U. 2065 Shrawan]**

Solution: See the solution of Q. No. 1 and 2 on page no. 245

What is displacement current? Prove that the displacement current density in a parallel plate capacitor can be written as $\vec{J_d} = \varepsilon_0 \frac{d\vec{E}}{dt}$. **[T.U. 2065 Kartik]**

Solution:

Consider conduction current I is passed through a capacitor with circular plates. The conductor current is not continuous across the capacitor gap because no charge is actually transported across this gap. However, we can imagine a current exists across the gas called displacement current.

Let the total charge stored by a capacitor is;

$$Q = CV = \varepsilon_0 A\left(\frac{V}{d}\right) = \varepsilon_0 AE \qquad \dots (i)$$

Differentiating equation(i) with time, we obtain displacement current I_d.

i.e., $$I_d = \frac{dQ}{dt} = C\frac{dV}{dt} \qquad \dots (ii)$$

$$= \varepsilon_0 A\frac{dE}{dt}$$

or, $$J_d = \frac{I_d}{A} = \varepsilon_0\frac{dE}{dt} \qquad \dots (iii)$$

This is the required expression for displacement current density.

Show that the displacement current in a parallel plate capacitor is given by $C\frac{dV}{dt}$. **[T.U. 2065 Chaitra]**

Solution: See the solution of Q. No. 5 on page no. 247

A variable field $10^{12}\ V/ms$ is applied to a parallel plate capacitor with circular plates of diameter 10 cm. Calculate induced magnetic field and displacement current. **[T.U. 2067 Mangsir]**

Solution:

Here,

$$\frac{dE}{dt} = 10^{12}\ Vm^{-1}s^{-1}$$

Diameter $(d) = 10\ cm$

Radius $(r) = 5\ cm = 5 \times 10^{-2}\ m$

We have,

$$\oint \vec{B}d\vec{l} = \mu_0\varepsilon_0\frac{d\phi}{dt}E$$

or, $$B \times 2\pi r = \mu_0\varepsilon_0 A\frac{d\vec{E}}{dt}$$

$$\text{or, } B = \frac{\mu_0\varepsilon_0\pi r^2}{2\pi r}\frac{dE}{dt} = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}$$

$$= \frac{4\pi\times10^{-7}\times8.85\times10^{-12}\times5\times10^{-2}}{2}\times10^{12}$$

$$\therefore B = 2.78\times10^{-7}\,T$$

Displacement current;

$$I_d = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0\pi r^2\frac{dE}{dt}$$

$$= 8.85\times10^{-12}\times\pi\times(5\times10^{-2})^2\times10^{12} = 6.95\times10^2\ \text{Amp.}$$

8. **An inductor L is connected to a battery of e.m.f. ε through a resistance R. Show that the potential difference across the inductance after time t is $V_L = \varepsilon e^{-\frac{R}{L}t}$.**
At what time is the potential difference across the inductance equals to that across the resistance such that $I = \frac{I_0}{2}$. [T.U. 2068 Shrawan]

Solution:

The growth current through RL circuit is;

$$I = I_0\left[1 - e^{-\frac{R}{L}t}\right]$$

The potential difference across the inductor is;

$$V_L = -L\frac{dI}{dt} = LI_0\left(-\frac{R}{L}\right)e^{-\frac{R}{L}t} = I_0Re^{-\frac{R}{L}t} = \varepsilon e^{-\frac{R}{L}t}$$

where, $\varepsilon = I_0R$

When the potential difference across the inductor is equal to that of resistance,

i.e., $V_L = V_R$

or, $\varepsilon e^{-\frac{R}{L}t} = IR$

or, $\frac{\varepsilon}{IR} = e^{\frac{R}{L}t}$

or, $\frac{I_0R}{IR} = e^{\frac{R}{L}t}$

or, $\frac{I_0}{I} = e^{\frac{R}{L}t}$

Since, $I = \frac{I_0}{2}$ then,

$2 = e^{\frac{R}{L}t}$

$$\therefore t = \frac{L}{R}\ln(2) = 0.693\left(\frac{L}{R}\right)\text{ sec.}$$

9. **State and explain Faraday's law of electromagnetic induction. Show that in induction, the mechanical energy is converted into electrical energy and finally into heat energy.[T.U. 2068 Shrawan]**

Solution:

Faraday's laws of electromagnetic induction

i) Whenever there is a change in magnetic flux through a closed circuit, there is an induced e.m.f.
The relative motion of a coil and a magnet causes the change in magnetic flux through coil, which generates induced e.m.f. in it. This results in current in the closed circuit containing the coil.

ii) The magnitude of induced e.m.f. is equal to rate of change of magnetic flux associated with the circuit.

i.e., $\varepsilon = -\frac{d\phi}{dt}$ ... (i)

The negative sign indicates that induced e.m.f. has sign such that it always opposes the change in magnetic flux.

For N–turns of coil, the induced e.m.f.

$\varepsilon = -N\frac{d\phi}{dt}$ ... (ii)

Consider a straight conductor (wire) of length l is moving through a uniform magnetic field B with constant velocity v such that $\vec{v}$ is perpendicular to $\vec{B}$. The electrons in the conductor experience a Lorentz force.

$\vec{F} = q(\vec{v}\times\vec{B}) = qvB$ ... (iii)

Under equilibrium condition;

$qE = qvB$

or, $E = vB$ ... (iv)

The corresponding potential difference is;

$V = El = vBl$ ... (v)

This shows that potential difference is maintained between the ends of the conductor as long as conductor continues to move through the uniform magnetic field.

The magnetic flux, at any instant is;

$\phi_B = BA = Blx$ where, $A = lx$ = area enclosed by the circuit

Thus,

$$\varepsilon = -\frac{d\phi_B}{dt} = -Bl\frac{dx}{dt} = -Blv$$

or, $IR = Blv$

$\therefore I = \frac{Blv}{R}$ ... (vi)

The rate at which the thermal energy appeared in the conductor is;

$$U_{magnetic} = \frac{1}{2}LI^2 = \frac{Q_m^2}{2C}\sin^2(\omega t + \phi)$$ 15.8

These are the expressions for frequency of electromagnetic oscillation, electric energy and magnetic energy in LC circuit.

15.2 LCR OSCILLATIONS (DAMPED OSCILLATIONS)

In LCR oscillations, the amplitude of oscillations decreases exponentially with time due to external resistive factors. According to Kirchhoff's law; we have,

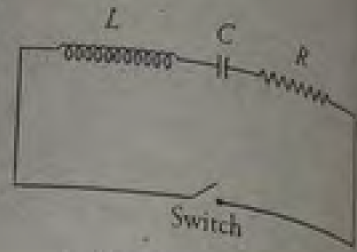

Figure: LCR circuit

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$$

$$\text{or,} \quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$ 15.9

This is differential equation for damped LCR oscillation and analogous to the mechanical damped harmonic oscillators,

$$\text{i.e.,} \quad m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx = 0$$ 15.10

The angular frequency of damped electromagnetic oscillation is;

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$ 15.11

The charge in damped oscillation is;

$$Q = Q_m e^{-\frac{R}{2L}t}\cos(\omega t + \phi)$$ 15.12

This is the solution of the differential equation (15.9).

15.3 FORCED OSCILLATIONS

Whenever we apply external frequency by means of ac frequency generator to the LCR oscillations, such type of oscillation is called to be *forced oscillation.*

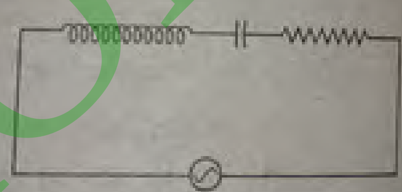
Figure: LCR circuit with an AC source

Using the principle of conservation;

$$L\frac{dI}{dt} + \frac{Q}{C} + IR = E_0\cos\omega_d t$$

$$\text{or,} \quad \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = \frac{E_0}{L}\cos\omega_d t$$ 15.13

This is the required differential equation for forced LCR oscillation and analogous to forced mechanical oscillation.

$$\therefore \quad I = \left(\frac{dQ}{dt}\right) = Q_0\omega_d\cos(\omega_d t - \phi) \qquad \text{... (iv)}$$

where, ϕ is the phase constant of the current

$$Q_0 = \frac{E_0}{L\left[\left(\omega_0^2 - \omega_d^2\right)^2 + \left(\frac{R\omega_d}{L}\right)^2\right]^{\frac{1}{2}}}$$

$$I_0 = \frac{\omega_d E_0}{L\left[\left(\omega_0^2 - \omega_d^2\right)^2 + \left(\frac{R\omega_d}{L}\right)^2\right]^{\frac{1}{2}}} = \frac{E_0}{Z} \qquad \text{... (v)}$$

where, $Z = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2} = \sqrt{R^2 + (X_L - X_C)^2}$ is an impedence of circuit

At resonance,

$$X_L = X_C$$

Thus,

$$I_0 = \frac{E}{R} \qquad \text{... (vi)}$$

The current amplitude I in a series LCR circuit driven by a sinusoidal external e.m.f. is maximum $\left(i.e., I = \frac{E_0}{R}\right)$ when the driving angular frequency ω_d equals the natural frequency ω of the circuit (*i.e.,* at resonance). Then $X_C = X_L$, $\phi = 0$ and the current is in the phase with e.m.f.

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6. **Explain what is meant by LC oscillation? Derive a differential equation for LC free oscillation. Give its solution.[T.U. 2065 Shrawan]**

Solution: See the solution of Q. No. 1 on page no. 253

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7. **What are free and damped electromagnetic oscillations? Deduce the frequency of oscillations and hence show charge distribution graphically. [T.U. 2065 Chaitra]**

Solution:

Free electromagnetic oscillation

See the solution of Q. No. 2 on page no. 254

Damped oscillation

See the solution of Q. No. 3 on page no. 255

Variation of charge distribution

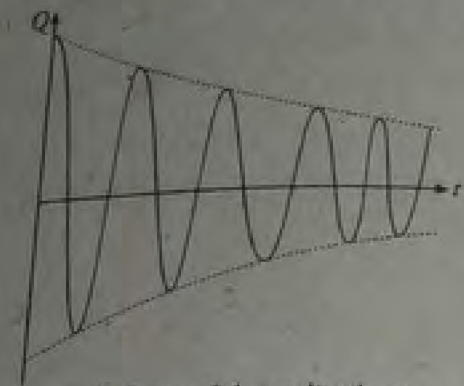

Figure: Variation of charge distribution

8. **What is meant by LC oscillation? Derive the differential equation of LC circuit and calculate the frequency of oscillation.** [T.U. 2065 Kartik]

Solution: See the solution of Q. No. 1 on page no. 253

9. **Derive the differential equation of damped harmonic oscillation in LCR circuit the equation and find the damped frequency of oscillation and explain its significance.** [T.U. 2067 Ashadh]

Solution: See the solution of Q. No. 3 on page no. 255

Significance

The equation (iii) of question number 3 electromagnetic counterpart of damped block spring oscillator and gives displacement of it.

The equation (iv) describes sinusoidal oscillation (the cosine function) with an exponentially decaying amplitude $Qe^{-\frac{R}{L}\cdot t}$.

The angular frequency ω' of damped harmonic oscillations is always less than angular frequency $\omega = \frac{1}{\sqrt{LC}}$ of the undamped oscillation.

10. **Derive the differential equation of a forced oscillation of LCR circuit with an ac source and find expression for current amplitude.** [T.U. 2067 Mangsir]

Solution: *See the solution of Q. No. 5 on page no. 256*

11. **A circuit containing a 25 mH inductor, 4 Ω resistor and 40 μF capacitor has an initial charge of 3 μC. What time is required to decrease charge on capacitor by e times?** [T.U. 2061 Baishakh]

Solution:

Here,

Inductance $(L) = 25\ mH = 25 \times 10^{-3}\ H$

Capacitance $(C) = 40\ \mu F = 40 \times 10^{-6}\ F$

Resistance $(R) = 4\ \Omega$

Initial charge $(Q_0) = 3\ \mu C = 3 \times 10^{-6}\ C$

We have,

$$Q = Q_0 e^{-\frac{R}{2L}\cdot t}$$

At $Q = \frac{Q_0}{e}$,

$$\frac{Q_0}{e} = Q_0 e^{-\frac{R}{2L}\cdot t}$$

or, $e^{-1} = e^{-\frac{R}{2L}\cdot t}$

$$\therefore\ t = \frac{2L}{R} = \frac{2 \times 25 \times 10^{-3}}{4} = 0.0125\ s$$

In 0.0125 seconds, the charge on capacitor is decreased by e times.

12. **An RLC series circuit has 5 Ω resistor, 20 μF capacitor and 1.0 H inductor. The oscillating e.m.f. has peak value 30 V. At what angular frequency will the current amplitude have its maximum value and what is this maximum value?** [T.U. 2061 Ashwin]

Solution:

At resonance, (*i.e.*, for inductive reactance equals to capacitive reactance, the current amplitude is maximum).

$X_L = X_C$

or, $\omega L = \frac{1}{\omega C}$

or, $2\pi f_R L = \frac{1}{2\pi f_R C}$

or, $f_R^2 = \frac{1}{4\pi^2 LC}$

i.e., $f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1.0\ H \times 20 \times 10^{-6}\ F}} = 35.59\ Hz$

At frequency of 35.59 Hz, current amplitude has its maximum amplitude.

The maximum current amplitude is;

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R} = \frac{30\ V}{5\ \Omega} = 6\ Amp.$$

13. What is the phase shift between the current and voltage in a circuit containing 20 Ω resistance, 30 mH inductance and 100 μF capacitance for 50 Hz A.C.? [T.U. 2062 Baishakh]

Solution:

Here,

Resistance $(R) = 20\,\Omega$

Inductance $(L) = 30\,mH = 30 \times 10^{-3}\,H$

Capacitance $(C) = 100\,\mu F = 100 \times 10^{-6}\,F$

AC frequency $(f) = 50\,Hz$

Capacitive reactance;

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$$

Inductive reactance;

$$X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 30 \times 10^{-3} = 9.42\,\Omega$$

The phase shift;

$$\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$= tan^{-1}\left(\frac{9.42 - 31.83}{20}\right)$$

$$= -48.25° = -0.842 \text{ radians}$$

The negative phase constant is consistent with the fact that load is mainly capacitive.

---

14. At $t = 0$, a 40 mH inductor is placed in series with a resistor of 3Ω and a charged capacitor $C = 4.8\,\mu F$. Explain how circuit will oscillate. Determine frequency of oscillation. What is the time required for the charge amplitude to drop to one third of its starting value? [T.U. 2063 Baishakh]

Solution:

Here,

Inductance $(L) = 40\,mH = 40 \times 10^{-3}\,H$

Resistance $(R) = 3\,\Omega$

Capacitance $(C) = 4.8\,\mu F = 4.8 \times 10^{-6}\,F$

Now,

$$\frac{4L}{C} = \frac{4 \times 40 \times 10^{-3}}{4.8 \times 10^{-6}} = 3.33 \times 10^4$$

or, $R^2 = 9$

Since $R^2 < \frac{4L}{C}$, the circuit is oscillatory.

$$\omega = \frac{1}{\sqrt{LC}}$$

i.e., $$\frac{d^2x}{dt^2} + \frac{b}{L}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t \qquad 15.14$$

The solution of equation (15.13) is;

$$Q = Q_0 \sin(\omega_d t - \phi) \qquad 15.15$$

or, $$I = \frac{dQ}{dt} = Q_0\omega_t \cos(\omega_d t - \phi)$$

$$I = I_0 \cos(\omega_d t - \phi) \qquad 15.16$$

15.4 SOLVED EXAM QUESTIONS

1. Set up a differential equation for the LC circuit with initial charge Q_0. Derive an expression for the frequency. How is the system similar to spring mass system? [T.U. 2061 Baishakh]

Solution:

A circuit containing capacitor and inductor forms an electromagnetic oscillator in which current varies sinusoidally with time.

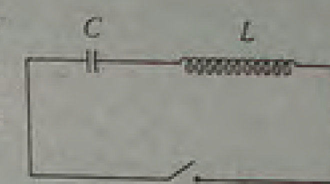

Figure: LC circuit

The total energy at any instant of time in an oscillating circuit is sum of electric and magnetic energies.

i.e., $U = U_{electric} + U_{magnetic}$

$$\therefore U = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 \qquad \text{... (i)}$$

If we assume that the circuit resistances to be zero, no energy is dissipated and total energy remains constant time even though current I and charge Q vary.

i.e., $$\frac{dU}{dt} = 0$$

or, $$\frac{d}{dt}\left[\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right] = 0$$

or, $$\frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

or, $$LI\frac{d^2Q}{dt^2} + \frac{Q}{C}I = 0$$

$$\therefore \frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \qquad \text{... (ii)}$$

This is the differential equation for the LC oscillation. Its solution is;

$$Q = Q_0 \cos(\omega t + \phi) \qquad \text{... (iii)}$$

where, Q_0 = initial charge
ω = angular electromagnetic frequency

$$\frac{dQ}{dt} = -\omega Q_0 \sin(\omega t + \phi) \quad \text{... (iv)}$$

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_0 \cos(\omega t + \phi) \quad \text{... (v)}$$

An insertion of equations (iii) and (v) in equation (ii); we obtain,

$$-\omega^2 Q_0 \cos(\omega t + \phi) + \frac{Q_0}{LC}\cos(\omega t + \phi) = 0$$

or, $\omega^2 = \frac{1}{LC}$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}} \quad \text{... (vi)}$$

Equation (vi) gives angular frequency of electromagnetic oscillation. The frequency of oscillation,

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{... (vii)}$$

Time period of oscillation,

$$T = \frac{1}{f} = 2\pi\sqrt{LC} \quad \text{... (viii)}$$

The differential equation (ii) is analogous to differential equation of spring mass system,

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \text{... (ix)}$$

The solutions of electromagnetic oscillator and mechanical oscillator differential equation are analogous.

2. **What are electromagnetic oscillations? Compare EM oscillation with mechanical oscillation. Derive the expression for time period of electromagnetic free oscillation.** **[T.U. 2061 Ashwin]**

Solution:

In a LC circuit, the charge, current and potential difference do not decay exponentially with time but vary sinusoidally. The resulting oscillations of the capacitor's electric field and the inductor's magnetic field are said to be electromagnetic oscillations. Such a circuit is said to oscillate.

The LC oscillation and blocks–spring system are analogous to each other. Two kinds of energies are involved in block–spring system. One is potential energy $\frac{1}{2}kx^2$ of the extended or compressed spring; other is kinetic energy $\frac{1}{2}mv^2$ of the moving block.

Two kinds of energies are involved in LC oscillations. (*i.e.*, electrical energy, $\frac{1}{2}\frac{1}{C}Q^2$ and magnetic energy, $\frac{1}{2}LI^2$). The mechanical energies of the block–spring system are analogous to electromagnetic energies of LC oscillator.

The angular frequencies are;

$\omega = \sqrt{\frac{k}{m}}$: block–spring system

$\omega = \frac{1}{\sqrt{LC}}$: LC oscillator

Expression for time period of free oscillation

See the solution of Q. No. 1 on page no. 253

Derive the equation for damped oscillator of LCR circuit and write down the oscillatory solution. **[T.U. 2062 Baishakh]**

Solution:

A circuit containing inductance capacitance and resistance is called LCR circuit. With a resistance R present, the total electromagnetic energy U of the circuit is no longer constant; instead, it decreases with time as energy is transferred to thermal energy in the resistance. Because of this loss of energy, the oscillations of charge, current and potential difference decrease in amplitude and the oscillations are said to be damped.

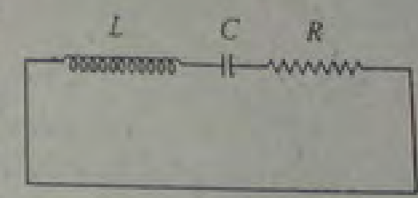

Figure: LCR circuit

The total electromagnetic energy U is;

$$U = U_{\text{electric}} + U_{\text{magnetic}}$$

$$= \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 \quad \text{... (i)}$$

This total energy decrease as energy is transferred to thermal energy. The rate of that transfer is;

$$\frac{dU}{dt} = -I^2R \quad \text{... (ii)}$$

The negative sign indicates that U decreases. By differentiating equation (i); we obtain,

$$\frac{dU}{dt} = LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R$$

Since, $\frac{dQ}{dt} = I;\ \frac{d^2Q}{dt^2} = \frac{dI}{dt}$

Thus,

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0 \quad \cdots \text{(iii)}$$

which is the differential equation for damped LCR oscillations.

The solution of equation (iii) is;

$$Q=Q_0e^{-\frac{R}{2L}t}\cos(\omega' t+\phi) \quad \cdots \text{(iv)}$$

where, $\omega'=\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$ ... (v)

4. What is free electromagnetic oscillation? Compare with the simple harmonic motion. Derive the expression for simple harmonic motion. [T.U. 2063 Baishakh]

Solution: See the solution of Q. No. 2 on page no. 254

5. Derive a relation for current that flows in a LCR circuit with sinusoidally varying e.m.f. Find the condition for the current resonance. [T.U. 2064 Poush]

Solution:

Whenever the natural angular frequency ω of a circuit many be, forced oscillation of charge, current and potential difference in the circuit always occur at a driving angular frequency ω_d.

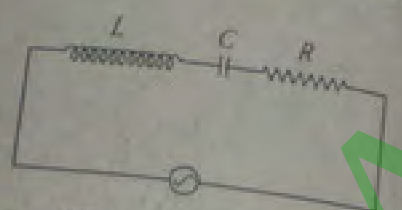

Figure: LCR circuit with an AC generator

A series LCR circuit may be set into forced oscillation at a driving angular frequency ω_d by an external alternating e.m.f.

$$E=E_0\cos\omega_d t \quad \cdots \text{(i)}$$

By the principle of conservation of energy

$$L\frac{dI}{dt}+\frac{Q}{C}+IR=E_0\cos\omega_d t \quad \cdots \text{(ii)}$$

As, $I=\frac{dQ}{dt}$ and $\frac{dI}{dt}=\frac{d^2Q}{dt^2}$

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E_0\cos\omega_d t$$

$$\therefore \quad \frac{d^2Q}{dt^2}+\frac{R}{L}\frac{dQ}{dt}+\frac{L}{C}Q=\frac{E_0}{L}\cos\omega_d t \quad \cdots \text{(iii)}$$

The solution of this differential equation is;

$$Q=Q_0\sin(\omega_d t-\phi)$$

$$U_{electric}=U_{magnetic}$$

$$\text{or, } \frac{1}{2}LI_0^2=\frac{1}{2}\frac{Q_0^2}{C}$$

$$\text{or, } U=\frac{1}{2}\frac{Q_0^2}{C}=\frac{1}{2}L\left(\frac{E}{R}\right)^2$$

$$=\frac{1}{2}\times 5\times\left[\frac{100}{10}\right]^2$$

$$U=250\,J$$

The energy stored in magnetic field $(U)=250\,J$.

Discuss the oscillations of LC circuit. Point out the similarities of LC oscillator with oscillating spring mass system without friction. Calculate the frequency of oscillation. [P.U. 2008]

Solution: See the solution of Q. No. 1 on page no. 253

What is meant by forced oscillation? Compare the electromagnetic oscillation with mass spring system. Obtain the differential equation of LC oscillation and calculate frequency of oscillation. [P.U. 2010]

Solution:

Forced oscillation

See the solution of Q. No. 5 on page no. 256

Comparison of electromagnetic oscillation

See the solution of Q. No. 2 on page no. 254

LC oscillation

See the solution of Q. No. 1 on page no. 253

In LC circuit having $L=10\,mH$ and $C=2\,\mu F$, what is the maximum value of current in the inductance if the condenser is initially charged to 50 V? [P.U. 2010]

Solution:

For maximum current,

$$U_{electric}=U_{magnetic}$$

$$\text{or, } \frac{1}{2}\frac{Q_0^2}{C}=\frac{1}{2}LI_0^2$$

$$\text{or, } \frac{1}{2}CV^2=\frac{1}{2}LI_0^2$$

$$\therefore \quad I_0=V\sqrt{\frac{C}{L}}=50\times\sqrt{\frac{2\,\mu F}{10\,mH}}$$

$$= 50 \times \sqrt{\frac{2 \times 10^{-6} F}{10 \times 10^{-3} H}}$$

$$= 0.707 \text{ Ampere}$$

The maximum value of current in inductance $(I_0) = 0.707$ Ampere.

29. Develop a differential equation of undamped LC oscillator and find an expression for the frequency of oscillation. [P.U. 2011]

Solution: See the solution of Q. No. 1 on page no. 253

30. Compare e.m.f. oscillation with mass spring system performing S.H.M. [P.U. 2011]

Solution: See the solution of Q. No. 2 on page no. 254

Chapter 16

ELECTROMAGNETIC WAVES

16.1 MAXWELL'S EQUATIONS

The differential and integral forms of Maxwell's equation are;

$$\left.\begin{aligned}\nabla \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \\ \int \vec{E} \cdot d\vec{s} &= \frac{q}{\varepsilon_0}\end{aligned}\right\} \quad 16.1$$

$$\left.\begin{aligned}\nabla \cdot \vec{B} &= 0 \\ \int \vec{B} \cdot d\vec{s} &= 0\end{aligned}\right\} \quad 16.2$$

$$\left.\begin{aligned}\nabla \times \vec{E} &= -\frac{\partial \overline{B}}{\varepsilon_0} \\ \int \vec{E} \cdot d\vec{l} &= -\frac{\partial \phi_B}{\partial t}\end{aligned}\right\} \quad 16.3$$

$$\left.\begin{aligned}\nabla \times \vec{E} &= \vec{J} + \varepsilon_0 \frac{\partial \overline{E}}{\partial t} \\ \int \vec{H} \cdot d\vec{l} &= I + \varepsilon_0 \frac{\partial \phi_E}{\partial t}\end{aligned}\right\} \quad 16.4$$

where, $\vec{H} = \dfrac{\overline{B}}{\mu_0}$

The equation (16.1) corresponds to Gauss's law of electro–statistics, equation (16.2) corresponds to Gauss's law of magneto–statistics, equation (16.3) corresponds to Faraday's law of electromagnetic induction and equation (16.4) corresponds to Ampere's circuital law.

16.2 POYNTING VECTOR $(\vec{S})$

It is defined as energy crossing per unit area per second in a plane electromagnetic wave.

i.e., $$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = (\vec{E} \times \vec{H}) \quad 16.5$$

The continuity equation in terms of Poynting vector is;

$$\nabla \cdot \vec{S} + \frac{\partial u}{\partial t} = 0 \quad 16.6$$

where, μ is total energy density in electromagnetic field

The physical meaning of the equation (16.6) is that time rate of change of electromagnetic energy with a certain volume plus time rate of energy flowing out through boundary surface is equal to the power transferred to electromagnetic field.

This is the statement of conservation of electromagnetism and known as Poynting theorem.

16.3 SOLVED EXAM QUESTIONS

1. Write down the Maxwell's equations in differential form and develop wave equation. [T.U. 2061 Baishakh]

Solution:

The Maxwell's equations are;

$$\left.\begin{aligned}\nabla\cdot\vec{E}&=\frac{\rho}{\varepsilon_0}\\ \nabla\cdot\vec{B}&=0\\ \nabla\times\vec{E}&=-\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H}&=\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\}\quad\ldots\text{(i)}$$

In free space, $\rho=0, \vec{J}=0$. The Maxwell's equation, thus, in free space are;

$$\left.\begin{aligned}\nabla\cdot\vec{E}&=0\\ \nabla\cdot\vec{B}&=0\\ \nabla\times\vec{E}&=-\mu_0\frac{\partial\vec{H}}{\partial t}\\ \nabla\times\vec{H}&=\varepsilon_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\}\quad\ldots\text{(ii)}$$

Taking curl of third equation of (ii); we have,

$$-\nabla^2\vec{E}=-\mu_0\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

or, $$-\nabla^2\vec{E}=-\mu_0\frac{\partial}{\partial t}\left[\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right]$$

or, $$\nabla^2\vec{E}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\therefore\quad \nabla^2\vec{E}-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=0\quad\ldots\text{(iii)}$$

or, $$f=\frac{1}{2\pi\sqrt{LC}}$$

$$\therefore\quad f=\frac{1}{2\pi\left[40\times10^{-3}\times4.8\times10^{-6}\right]^{\frac{1}{2}}}=363.22\ Hz$$

When charge amplitude drops to $\frac{1}{3}$ of its initial value,

$$Q=\frac{1}{3}Q_0=Q_0e^{-\frac{R}{2L}\cdot t}$$

or, $$\frac{1}{3}=e^{-\frac{R}{2L}\cdot t}$$

or, $$3=e^{\frac{R}{2L}\cdot t}$$

or, $$ln(3)=\frac{R}{2L}\cdot t$$

$$\therefore\quad t=\frac{2L}{R}ln(3)=\frac{2\times40\times10^{-3}}{3}ln(3)=0.029\text{ seconds}$$

The charge amplitude drops to $\frac{1}{3}$ of its initial value in 0.029 sec.

15. At $t=0$, a 40 mH inductor is placed in series with a resistor of 3Ω and a charged capacitor $C=4.8\,\mu F$. Show that this circuit will oscillate and find the frequency of oscillation. What is time required for the charge amplitude to drop to half its starting value? [T.U. 2064 Poush]

Solution: Proceed as Q. No. 14 on page no. 260

16. Consider an RLC circuit in which $R=7.6\,\Omega, L=2.2\,mH$ and $c=1.8\,\mu F$. Calculate the frequency of the damped oscillation of the circuit. [T.U. 2065 Shrawan]

Solution:

Here,

Resistance $(R)=7.6\,\Omega$

Inductance $(L)=2.2\,mH=2.2\times10^{-3}\,H$

Capacitance $(C)=1.8\,\mu F=1.8\times10^{-6}\,F$

The frequency of damped oscillation of RLC circuit;

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

$$=\frac{1}{2\pi}\left[\sqrt{\frac{1}{2.2\times10^{-3}\times1.8\times10^{-6}}-\left(\frac{7.6}{2.2\times10^{-3}}\right)^2}\right]$$

$$\therefore\quad f=2.51\times10^{8}\ Hz=251\ MHz$$

17. **You are given an inductor of 1 mH. If you are asked to make it oscillate with a frequency of 1 MHz, how can you make such an oscillating device?** [T.U. 2065 Chaitra]

Solution:

Here,

Inductance $(L) = 1\ mH = 1 \times 10^{-3}\ H$

Frequency $(f) = 1\ MHz = 1 \times 10^{6}\ Hz$

The frequency of LC oscillation;

$$f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

$$\text{or,}\quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(1\times10^6)^2 \times 1 \times 10^{-3}} = 2.5 \times 10^{-11}\ F$$

On combining a capacitor of capacitance $2.51 \times 10^{-11}\ F$ with the given inductor of 1 mH, we can make oscillations of 1 MHz.

18. **What should be the capacitance of a capacitor in a tuned circuit of frequency 10 MHz having an inductance of 0.01 mH? The resistance of circuit is negligible.**

Solution:

Here,

Frequency $(f) = 10\ MHz = 10 \times 10^{6}\ Hz$

Inductance $(L) = 0.01\ mH = 0.01 \times 10^{-3}\ H$

The frequency of LC oscillation;

$$f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

$$\text{or,}\quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(10^7)^2 \times 0.01 \times 10^{-3}}$$

$$= 2.5 \times 10^{-13}\ F$$

The capacitance of capacitor $2.5 \times 10^{-13}\ F$.

19. **Compare electromagnetic oscillation with the oscillation of spring mass system. Obtain differential equation for LCR damped oscillation. Write and explain the solution of the equation.** [P.U. 2003]

Solution:

Comparison of electromagnetic oscillation and spring mass system

See the solution of Q. No. 2 on page no. 254

LCR damped oscillation

See the solution of Q. No. 3 on page no. 255

20. **A circuit has $L = 10\ mH$ and $C = 10\ \mu F$. How much resistance should be added to circuit so that the frequency of oscillation will be 1% less that of free LC oscillation?** [P.U. 2003]

Solution:

The frequency of LC oscillation;

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The frequency of damped LCR oscillation;

$$f' = \frac{1}{2\pi}\left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}}$$

By question;

$$\frac{f-f'}{f} = \frac{1}{100}$$

$$\text{or,}\quad 1-\frac{f'}{f} = \frac{1}{100}$$

$$\text{or,}\quad 1-\frac{\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}}{\frac{1}{\sqrt{LC}}} = \frac{1}{100}$$

$$\text{or,}\quad 1-\frac{R^2C}{4L} = \frac{99}{100}$$

$$\text{or,}\quad \frac{R^2C}{4L} = \frac{1}{100}$$

$$\text{or,}\quad \frac{R^2C}{L} = \frac{1}{25}$$

$$\text{or,}\quad R^2 = \frac{1}{25}\frac{L}{C} = \frac{1}{25}\frac{10\ mH}{10\ \mu F} = \frac{1}{25}\times\frac{10\times10^{-3}\ H}{10\times10^{-6}\ F}$$

$$\therefore\quad R = 6.33\ \Omega$$

A resistance of $6.33\ \Omega$ should be added to a circuit so that the frequency of oscillation will be 1% less than that of free LC oscillation.

21. **Obtain the differential equation that describes oscillation of a resistance-less LC circuit.** [P.U. 2004]

Solution: See the solution of Q. No. 1 on page no. 253

22. In an oscillating LC circuit, $L = 1.10\ mH$ and $C = 4.0\ \mu F$. If the maximum charge on the capacitor is $3.0\ \mu C$. Find the value of maximum current. [P.U. 2004]

Solution:

Here,

Inductance $(L) = 1.10\ mH = 1.10 \times 10^{-3}\ H$

Capacitance $(R) = 4.0\ \mu F = 4 \times 10^{-6}\ F$

Charge on capacitor $(Q_0) = 3.0\ \mu C = 3 \times 10^{-6}\ C$

For maximum current,

$$U_{electric} = U_{magnetic}$$

or, $$\frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}LI_0^2$$

or, $$I_0^2 = \frac{Q_0^2}{LC} = \frac{(3\times10^{-6})^2}{1.10\times10^{-3}\times4\times10^{-6}} = 2.05\times10^{-3}$$

$\therefore$ $I_0 = 4.52 \times 10^{-2}$ Amp.

The value of maximum current $(I_0) = 4.52 \times 10^{-2}$ Ampere.

23. Compare the electromagnetic oscillation with spring mass system performing simple harmonic motion. Develop a differential equation of damped oscillation on LCR circuit and find the expression for the frequency of oscillation. [P.U. 2005]

Solution: See the solution of Q. No. 2 and 3 on page no. 254 and 255

24. What is meant by LC oscillation? Derive the differential equation of an LC circuit and also calculate frequency of LC oscillation. [P.U. 2007]

Solution: See the solution of Q. No. 1 on page no. 253

25. A coil has an inductor of 5 H and a resistance 20 Ω. If a 100 V e.m.f. is applied, what energy is stored in magnetic field after the current i.e., built up to its maximum value $\frac{E}{R}$. [P.U. 2007]

Solution:

Here,

Inductance $(L) = 5\ H$

Resistance $(R) = 2.0\ \Omega$

E.m.f. $(E) = 100\ V$

For the maximum current, we have,

Equation (iii) is comparable to propagation of wave equation.

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad \ldots \text{(iv)}$$

We can say that propagation of electric field $\vec{E}$ is;

$$v = \frac{1}{(\mu_0\varepsilon_0)^{\frac{1}{2}}}$$

Since, $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$ and $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

$$\therefore\ v = \frac{1}{(4\pi\times10^{-7}\ Hm^{-1}\times 8.85\times10^{-12}\ Fm^{-1})^{\frac{1}{2}}} \approx 3\times10^{8}\ ms^{-1}$$

i.e., electric field travels with velocity of light.

Again, taking the curl of fourth equation of (ii); we obtain,

$$-\nabla^2\vec{H} = -\varepsilon_0\mu_0\frac{\partial^2\vec{H}}{\partial t^2}$$

i.e., $$\nabla^2\vec{H} = -\varepsilon_0\mu_0\frac{\partial^2\vec{H}}{\partial t^2} = 0 \quad \ldots \text{(v)}$$

Comparing with equation (iv); we get,

$$v = \frac{1}{(\mu_0\varepsilon_0)^{\frac{1}{2}}} \approx 3\times10^{8}\ ms^{-1}$$

Hence, electric and magnetic fields travel with the velocity of light and equations (iii) and (v) represents wave equation for electromagnetic wave.

2. What is continuity equation? [T.U. 2061 Baishakh]

Solution:

Continuity equation states that, *"the total current flowing out of some volume must be equal to the rate of decrease of charge within the volume assuming that charge neither be created nor destroyed."*

i.e., $\nabla^2\vec{J} + \frac{\partial\rho}{\partial t} = 0$

3. Show that Poynting vector of electromagnetic field may be written as $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$ [T.U. 2061 Ashwin]

Solution:

The Maxwell's equations are;

$$\left.\begin{aligned}\nabla\cdot\vec{E}&=\frac{\rho}{\varepsilon_0}\\ \nabla\cdot\vec{B}&=0\\ \nabla\times\vec{E}&=-\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H}&=\vec{j}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\}\quad\ldots\text{(i)}$$

Multiplying third equation by $\vec{H}$ and fourth equation by $\vec{E}$; we obtain,

$$\vec{H}(\nabla\times\vec{E})=\vec{H}\left(-\frac{\partial\vec{B}}{\partial t}\right)\quad\ldots\text{(ii)}$$

$$\vec{E}(\nabla\times\vec{H})=\vec{E}\left(\vec{j}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\quad\ldots\text{(iii)}$$

Subtracting equation (iii) from (iv); we obtain,

$$\vec{H}(\nabla\times\vec{E})-\vec{E}(\nabla\times\vec{H})=-\vec{H}\frac{\partial(\mu_0\vec{H})}{\partial t}-\vec{E}\cdot\vec{j}-\varepsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t}$$

or, $\nabla(\vec{E}\times\vec{H})$, $=-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu_0H^2\right)-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0E^2\right)-\vec{E}\cdot\vec{j}$

or, $\nabla\cdot\vec{S}$ $=-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu_0H^2+\frac{1}{2}\varepsilon_0E^2\right)-\vec{E}\cdot\vec{j}$

$\therefore$ $\nabla\cdot\vec{S}$ $=-\frac{\partial}{\partial t}(u_m+u_E)-\vec{E}\cdot\vec{j}$

$$=-\frac{\partial u}{\partial t}-\vec{E}\cdot\vec{j}\quad\ldots\text{(iv)}$$

where, $u=u_m+u_E=\frac{1}{2}\mu_0H^2+\frac{1}{2}\varepsilon_0E^2$; total energy density of electromagnetic field

$\vec{S}=\vec{E}\times\vec{H}$ is called Poynting vector of electromagnetic field

i.e., $\vec{S}=\frac{1}{\mu_0}(\vec{E}\times\vec{B})$

where, $\vec{H}=\frac{\vec{B}}{\mu_0}$

This is the required expression of Poynting vector.

4. Prove that the amplitudes of electric and magnetic field vectors due to electromagnetic field vectors due to electromagnetic wave are related by $E=eB$, where symbols carry their usual meaning. **[T.U. 2061 Ashwin]**

Solution:

Consider a plane wave is propagating in x–direction. The electric field and magnetic field are acting along y–direction and z–direction. Then we have wave equations;

$$\left.\begin{aligned}\frac{\partial^2E_y}{\partial x^2}&=\mu_0\varepsilon_0\frac{\partial^2E_y}{\partial t^2}\\ \frac{\partial^2B_z}{\partial x^2}&=\mu_0\varepsilon_0\frac{\partial^2B_z}{\partial t^2}\end{aligned}\right\}\quad\ldots\text{(i)}$$

The solutions are;

$$\left.\begin{aligned}E_y&=E_0\sin(kx-\omega t)\\ B_z&=B_0\sin(kx-\omega t)\end{aligned}\right\}\quad\ldots\text{(ii)}$$

where, E_0 and B_0 are amplitudes of electric and magnetic fields

From Faraday's law of electromagnetic induction, we have,

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

or, $\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ E_x&E_y&E_z\end{vmatrix}=-\frac{\partial}{\partial t}(B_x\hat{\imath}+B_y\hat{\jmath}+B_z\hat{k})$

or, $\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_x}{\partial y}\right)\hat{\imath}+\left(\frac{\partial E_z}{\partial z}-\frac{\partial E_z}{\partial x}\right)\hat{\jmath}+\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)\hat{k}=\left(-\frac{\partial B_x}{\partial t}\right)\hat{\imath}$

$$+\left(-\frac{\partial B_y}{\partial t}\right)\hat{\jmath}+\left(-\frac{\partial B_z}{\partial t}\right)\hat{k}$$

Comparing coefficients of $\hat{\imath},\hat{\jmath},\hat{k}$ on both sides; we obtain,

$$\left.\begin{aligned}\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}&=-\frac{\partial B_x}{\partial t}\\ \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}&=-\frac{\partial B_y}{\partial t}\\ \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}&=-\frac{\partial B_z}{\partial t}\end{aligned}\right\}\quad\ldots\text{(iii)}$$

Since, electric field is acting along y–direction and magnetic field is acting along z–direction.

$E_x=E_z=0$

and $B_x=B_y=0$

Thus,

$$-\frac{\partial E_y}{\partial z}=-\frac{\partial B_x}{\partial t}$$

$$\therefore\ \frac{\partial E_y}{\partial z}=\frac{\partial B_z}{\partial t}\quad\ldots\text{(iv)}$$

$$\frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}\quad\ldots\text{(v)}$$

Differentiating equation (ii); we get,

$$\left.\begin{aligned}\frac{\partial E_y}{\partial x} &= kE_0 \sin(kx - \omega t) \\ \frac{\partial E_z}{\partial t} &= -\omega B_0 \sin(kx - \omega t)\end{aligned}\right\} \quad \ldots \text{(vi)}$$

An insertion of equation (vi) on equation (v) gives,

$$kE_0 \sin(kx - \omega t) = \omega B_0 \sin(kx - \omega t)$$

or, $kE = \omega B$

or, $E = \frac{\omega}{k} B$

or, $\frac{E_0}{B_0} = \frac{\omega}{k} = c$

$\therefore \quad E = cB \quad \ldots$ (vii)

where, $c = \frac{\omega}{k}$: velocity of light

Equation (vii) is the required expression.

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5. Prove that Poynting vector may be written as $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

[T.U. 2062 Baishakh]

Solution: See the solution of Q. No. 3 on page no. 269

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6. What is displacement current? Why it is necessary and what is its significance?

[T.U. 2062 Baishakh]

Solution:

Maxwell concluded that Ampere's circuital law;

$$\nabla \times \vec{H} = \vec{J} \quad \ldots \text{(i)}$$

Equation (i) is incomplete. The contradiction of Ampere's circuital law aroused with the equation of continuity.

$$\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t} = 0 \quad \ldots \text{(ii)}$$

could be removed by converting the equation of continuity into a form whose divergence is zero by adding something $\vec{J}$ in equation (iii)

Maxwell replaced $\vec{J}$ in equation (i) by $\vec{J} + \frac{\partial \vec{D}}{\partial t}$. Thus, Ampere's circuital law becomes,

$$\left.\begin{aligned}\nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \text{i.e., } \nabla \times \vec{B} &= \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\end{aligned}\right\} \quad \ldots \text{(iii)}$$

The term $\vec{D}$ is called displacement current density since it arises when the electric displacement changes with time. The introduction of displacement current is one of Maxwell's major contribution without which the electromagnetic theory would have been impossible.

The displacement current $\frac{\partial D}{\partial t}$ to the total current, is a solenoidal current of which divergence is zero. The introduction of displacement current is necessary in order to apply the Ampere's circuital law for magnetic field due to non-stationary currents.

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7. What are peak values of electric and magnetic field vectors if the intensity of electromagnetic wave is 0.03 Wm^{-2}?

Solution:

Here,

Intensity of electromagnetic wave, $(S) = \frac{P}{A} = 0.03\ Wm^{-2}$

Peak value of electric field, (E_0) =?

Peak value of magnetic field, (B_0) =?

Now,

$$S = \frac{E_0 B_0}{2\mu_0} = \frac{cB_0^2}{2\mu_0}$$

$$\therefore \quad B_0 = \sqrt{\frac{2\mu_0 S}{c}} = \sqrt{\frac{2\times 4\pi\times 10^{-7}\times 0.03}{3\times 10^8}} = 1.59 \times 10^{-8}\ T$$

$$E_0 = cB_0 = 3 \times 10^8 \times 1.59 \times 10^{-8} = 4.77\ Vm^{-1}$$

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8. What are Maxwell's equations in differential form? Use them to obtain the electromagnetic wave equation for $\vec{E}$ and $\vec{B}$ in homogeneous isotropic non-conducting medium. Also find their solutions.

Solution:

The Maxwell's equations in differential form;

$$\left.\begin{aligned}\nabla \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\end{aligned}\right\} \quad \ldots \text{(i)}$$

In a homogeneous isotropic non–conducting medium,

Charge density, $\rho = 0$

Current density, $\vec{J} = 0$

Maxwell's equations in this medium,

$$\left.\begin{aligned} \nabla \cdot \vec{E} &= 0 \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\mu \frac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{H} &= \varepsilon \frac{\partial \vec{E}}{\partial t} \end{aligned}\right\} \quad \ldots \text{(ii)}$$

Taking curl of third equation of (ii); we obtain,

$$-\nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$\text{or,} \quad \nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{i.e.,} \quad \nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \ldots \text{(iii)}$$

Taking curl of fourth equation of (ii); we obtain,

$$-\nabla^2 \vec{H} = \varepsilon \frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\text{or,} \quad -\nabla^2 \vec{H} = \varepsilon\mu \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{i.e.,} \quad \nabla^2 \vec{H} = -\mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \ldots \text{(iv)}$$

The standard equation of travelling wave is;

$$\nabla^2 \mu - \frac{1}{v^2}\frac{\partial^2 \mu}{\partial t^2} = 0 \quad \ldots \text{(v)}$$

Comparison of equations (iii) and (iv) with equation (v) leads to;

$$v = \frac{1}{\mu\varepsilon} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$$\therefore \quad \frac{c}{v} = \eta = \sqrt{\mu_r \varepsilon_r}\text{: refractive index}$$

$$\text{or,} \quad \eta^2 = \mu_r \varepsilon_r$$

For non–magnetic medium,

$$\mu_r = 1$$

Thus,

$$\eta^2 = \varepsilon_r$$

Electromagnetic field exists in isotropic non–conducting medium with the velocity $\frac{c}{\sqrt{\mu_r \varepsilon_r}}$.

It is obvious that $v = c$, velocity of electromagnetic wave in non–conducting medium is less than free space.

The solutions of equation (iii) and (iv) are;

$$\left.\begin{aligned} \vec{E} &= E_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} \\ \vec{H} &= H_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} \end{aligned}\right\} \quad \ldots \text{(vi)}$$

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9. In electromagnetic wave prove the relation $\frac{E}{B} = c$, where symbols carry their usual meaning. **[T.U. 2064 Poush]**

Solution: See the solution of Q. No. 4 on page no. 270

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10. Write down the Maxwell's equation in integral form. Convert them into differential form.

Solution:

The Maxwell's equations in integral form;

$$\left.\begin{aligned} \int \vec{E}\cdot d\vec{S} &= \frac{q}{\varepsilon_0} \\ \int \vec{B}\cdot d\vec{S} &= 0 \\ \int \vec{E}\cdot d\vec{l} &= -\mu \frac{\partial \phi_B}{\partial t} \\ \int \vec{H}\cdot d\vec{l} &= I + \varepsilon_0 \frac{\partial \phi_E}{\partial t} \end{aligned}\right\} \quad \ldots \text{(i)}$$

The first equation of (i), represents Gauss law of electrostatics. Applying Gauss divergence theorem on it, we obtain,

$$\int (\nabla \cdot \vec{E}) dv = \int \frac{\rho}{\varepsilon_0} dv$$

$$\therefore \quad \nabla \vec{E} = \frac{\rho}{\varepsilon_0} \quad \ldots \text{(ii)}$$

This relates the electric field and charge density in electrostatic field. The second equation of (i) represents Gauss law of magnetostatics. It implies that magnetic force is solenoidal and isolated magnetic pole is not known. Applying Gauss divergence theorem on equation;

$$\int \vec{B}\cdot d\vec{S} = 0$$

We obtain,

$$\int (\nabla \cdot \vec{B})\, d\vec{v} = 0$$

$$\nabla \cdot \vec{B} = 0 \quad \ldots \text{(iii)}$$

The third equation of (i) represents Faradays law of electromagnetic induction. Applying Stoke's theorem, we obtain,

$$\int(\nabla\cdot\vec{E})\,d\vec{s} = -\frac{\partial}{\partial t}\int\vec{B}\,d\vec{s}$$

$$\therefore\quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \cdots \text{(iv)}$$

The fourth equation of (i) represents Ampere's circuital law. Applying Stokes theorem on it, we obtain,

$$\int(\nabla\cdot\vec{H})\,d\vec{s} = \int\vec{j}\,d\vec{s} + \varepsilon_0\frac{\partial}{\partial t}\int\vec{E}\,d\vec{s}$$

$$\therefore\quad \nabla\times\vec{H} = \vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \cdots \text{(v)}$$

This relation confirms that magnetic field is set up if electric field changes with time. Equations (i) to (v) are Maxwell's equations in differential form.

11. **What is Poynting vector? Prove the relation $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, where symbols carry their usual meaning.** [T.U. 2065 Shrawan]

Solution: See the solution of Q. No. 3 on page no. 269

12. **Do Maxwell's allow for existence of magnetic monopoles? Explain.** [T.U. 2065 Shrawan]

Solution:

The Maxwell's equations are;

$$\left.\begin{aligned}\nabla\cdot\vec{E} &= \frac{\rho}{\varepsilon_0}\\ \nabla\cdot\vec{B} &= 0\\ \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H} &= \vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\} \qquad \cdots \text{(i)}$$

The second equation of (i) represents Gauss law of magnetostatics. It implies that magnetic lines of force are solenoidal and isolated pole does not exist, *i.e.*, Maxwell do not allow for existence of magnetic monopoles.

13. **Derive electromagnetic wave equations for electric and magnetic fields. Give their plane wave solutions.** [T.U. 2065 Shrawan]

Solution: See the solution of Q. No. 1 on page no. 268

The plane wave solutions are;

$$\left.\begin{aligned}\vec{E} &= E_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)}\\ \vec{H} &= H_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)}\end{aligned}\right\}$$

14. **What is displacement current? Why Maxwell's modification is necessary in Ampere's law of magnetism?** [T.U. 2065 Chaitra]

Solution: See the solution of Q. No. 6 on page no. 272

15. **Explain what is Maxwell's equation? Write Maxwell's equations in free space and find the electromagnetic equations for electric and magnetic fields. Also provide their plane wave solutions.** [T.U. 2065 Chaitra]

Solution: See the solution of Q. No. 13 on page no. 276

16. **Using Maxwell's equation, prove that $\frac{E}{B} = c$.** [T.U. 2065 Kartik]

Solution: See the solution of Q. No. 4 on page no. 270

17. **Define Poynting vector and show that $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$, where symbols carry usual meanings.** [T.U. 2065 Kartik]

Solution: See the solution of Q. No. 3 on page no. 269

18. **Derive the Maxwell's equation in differential form. Also, explain their physical significance.** [T.U. 2065 Kartik]

Solution: See the solution of Q. No. 10 on page no. 275

19. **Define Poynting vector. Prove that $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$, where symbols have their usual meanings.** [T.U. 2067 Ashadh]

Solution: See the solution of Q. No. 3 on page no. 269

20. **Using Maxwell's equations in free space, derive electromagnetic wave equations for $\vec{E}$ and $\vec{B}$. Find their plane wave equations.** [T.U. 2067 Mangsir]

Solution: See the solution of Q. No. 13 on page no. 276

21. **Starting from Maxwell's equations in free space, obtain differential equations for electromagnetic waves. Find the plane wave solutions.** [T.U. 2068 Shrawan]

Solution: See the solution of Q. No. 13 on page no. 276

22. **Derive expression for energy stored per unit volume in electric and magnetic fields.** [T.U. 2061 Ashwin]

Solution:

The total normal electric lines of forces coming out from the closed surface is $\frac{q}{\varepsilon_0}$, where, q is charge enclosed by the surface and ε_0 is the permittivity in free space. According to Gauss law of electrostatics,

$$\int \vec{E}\,d\vec{s} = \frac{q}{\varepsilon_0}$$

$$\text{or,}\quad EA = \frac{q}{\varepsilon_0}$$

$$\text{or,}\quad \frac{V}{d} = \frac{q}{\varepsilon_0 A}$$

$$\text{or,}\quad \frac{\varepsilon_0 A}{d} = \frac{q}{V} = C$$

where, $C = \frac{q}{V}$

$$\therefore\quad C = \frac{\varepsilon_0 A}{d}$$

The potential;

$$V = \frac{W}{q}$$

$$\text{or,}\quad dW = V\,dq$$

On integrating,

$$W = \int_0^q V\,dq = \frac{1}{C}\int_0^q q\,dq = \frac{q^2}{2C} = \frac{V^2C}{2}$$

$$= \frac{1}{2}(Ed)^2\frac{\varepsilon_0 A}{d} = \frac{1}{2}\varepsilon_0 E^2(Ad) = \frac{1}{2}\varepsilon_0 E^2\,(\text{Volume})$$

$$\text{or,}\quad \frac{W}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

$$\text{i.e.,}\quad u_e = \frac{1}{2}\varepsilon_0 E^2$$

This gives energy density, i.e., the energy per unit volume in an electric field.

23. **Prove the charge conservation theorem $\nabla\cdot\vec{j} + \frac{\partial\rho}{\partial t} = 0$, each symbol carries usual meaning.** [T.U. 2063 Baishakh]

Solution:

The Ampere's circuital law;

$$\nabla\times\vec{H} = \vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\text{or,}\quad \nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

The divergence on both sides,

$$\nabla\cdot(\nabla\times\vec{B}) = \mu_0(\nabla\cdot\vec{j}) + \mu_0\varepsilon_0\frac{\partial}{\partial t}(\nabla E)$$

$$\text{or,}\quad 0 = \mu_0(\nabla\cdot\vec{j}) + \mu_0\varepsilon_0\frac{\partial}{\partial t}\left(\frac{\rho}{\varepsilon_0}\right)$$

$$\text{or,}\quad 0 = \mu_0\left[\nabla\cdot\vec{j} + \frac{\partial\rho}{\partial t}\right]$$

$$\text{or,}\quad \nabla\cdot\vec{j} + \frac{\partial\rho}{\partial t} = 0 \qquad \ldots(i)$$

$$\text{i.e.,}\quad \nabla\cdot\vec{j} = -\frac{\partial\rho}{\partial t}$$

where, equation (i) is called charge conservation theorem and it states as 'total charge of any system remains constant if no external charges are brought into it.'

24. **With the help of Maxwell's equation of electromagnetism, prove that $\nabla\vec{j} = -\frac{\partial\rho}{\partial t}$.** [T.U. 2064 Poush]

Solution: See the solution of Q. No. 23 on page no. 278

25. **Write Maxwell's equations in integral and differential form. Show that $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$.** [P.U. 2002]

Solution: See the solution of Q. No. 8 and 10 on page no. 273 and 275

26. Prove that $\frac{\varepsilon_0}{B_0} = c$, where E_0 and B_0 are amplitudes wave and c is velocity of light. [P.U. 2002]

Solution: See the solution of Q. No. 4 on page no. 270

27. Show the charge conservation theorem $\frac{\partial\rho}{\partial t} + \nabla\vec{j} = 0$. [P.U. 2002]

Solution: See the solution of Q. No. 23 on page no. 278

28. For a travelling wave prove that $C = \frac{\varepsilon_0}{B_0}$, where the terms have their usual meaning. [P.U. 2004]

Solution: See the solution of Q. No. 4 on page no. 270

29. Explain the Poynting vector and show that $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$ where symbols have their usual meaning. [P.U. 2004]

Solution: See the solution of Q. No. 3 on page no. 269

30. Show that $\nabla \vec{D} = \rho$

where, $\vec{D}$ = electric displacement vector

ρ = free energy density

[P.U. 2004]

Solution:

The total electric lines of force closed surface is $\frac{q}{\varepsilon_0}$, where q is the charge enclosed by the surface and ε_0 is the permittivity of free space. The electric field is defined as normal flux per unit area.

$$\text{i.e., } E = \frac{d\phi}{dS\cos\theta}$$

$$\text{or, } d\phi = E\, dS\, \cos\theta$$

$$\therefore \phi = \int \vec{E}.d\vec{S}$$

$$\text{But } \int \vec{E}.d\vec{S} = \int E.dS\cos\theta$$

$$= \int \frac{q}{4\pi\varepsilon_0 r^2} dS\cos\theta$$

$$= \frac{q}{4\pi\varepsilon_0}\int d\Omega$$

$$= \frac{q}{\varepsilon_0}\frac{1}{4\pi} \times 4\pi = \frac{q}{\varepsilon_0}$$

Thus,

$$\int \vec{E}.d\vec{S} = \int \frac{\rho\, dv}{\varepsilon_0}$$

Applying Gauss divergence theorem,

$$\int (\nabla \cdot \vec{E}) dv = \int \frac{\rho}{\varepsilon_0} dv$$

$$\text{or, } \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\text{or, } \nabla(\varepsilon_0 \vec{E}) = \rho$$

$$\therefore \nabla \vec{D} = \rho$$

where, $\vec{D} = \varepsilon_0 \vec{E}$

which is the required expression

31. **What is meant by displacement current? Write is significance.** [P.U. 2005 B]

Solution: See the solution of Q. No. 6 on page no. 272

32. **Write down the Maxwell's equation in integral form and convert them in differential form.** [P.U. 2005 B]

Solution: See the solution of Q. No. 10 on page no. 275

33. With the help of electromagnetic waves equation, prove that $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, where c is the velocity of light. [P.U. 2005 B]

Solution: See the solution of Q. No. 1 on page no. 268

34. Show that charge conservation theorem $\nabla \vec{j} + \frac{\partial \rho}{\partial t} = 0$. [P.U. 2007]

Solution: See the solution of Q. No. 23 on page no. 278

35. **Write down the Maxwell's equation? Prove that:**

$$\nabla \times \vec{B} = \mu_0\left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

[P.U. 2007]

Solution:

The Maxwell's equation in differential form

See the solution of Q. No. 1 on page no. 268

According to Amperes' circuital law;

$$\int \vec{B}.d\vec{l} = \mu_0 I$$

Applying stokes law, we obtain,

$$\int (\nabla \times \vec{B})\, d\vec{S} = \mu_0 \int \vec{j} \cdot d\vec{S}$$

$$\text{or, } \nabla \times \vec{B} = \mu_0 \cdot \vec{j} \quad \text{... (i)}$$

Taking divergence on both sides, we have,

$$\nabla(\nabla \times \vec{B}) = \mu_0 \cdot \nabla \vec{j}$$

$$\text{or, } \nabla \vec{j} = 0 \quad \text{... (ii)}$$

Equation (ii) contradicts continuity equation

$$\nabla \vec{j} = -\frac{\partial \rho}{\partial t} \quad \text{... (iii)}$$

Maxwell suggested that definition of total current density is incomplete and advised to add something to $\vec{j}$. Let it be $\vec{j}'$. Then, equation (i) becomes;

$$\nabla \times \vec{B} = \mu_0(\vec{j} + \vec{j}')$$

Taking divergence on both sides, we obtain,

$$\nabla(\nabla \times \vec{B}) = \nabla(\vec{j} + \vec{j}')$$

$$= \nabla \vec{j} + \nabla \vec{j}'$$

$$\text{or, } 0$$

$$\therefore \nabla \vec{j}' = -\nabla \vec{j} = \frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial t}(\nabla \cdot \vec{D}) \quad [\because \nabla \vec{D} = \rho]$$

$$= \nabla\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\text{or, } \nabla\left(\vec{j}' - \frac{\partial \vec{D}}{\partial t}\right) = 0$$

i.e., $\vec{J}' - \frac{\partial \vec{D}}{\partial t} = 0$

Thus, the equation (i) takes the form,

$$\nabla \times \vec{B} = \mu_0(\vec{J} + \vec{J}') = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$\therefore \quad \nabla \times \vec{B} = \mu_0\left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \qquad \text{... (iv)}$$

Since, $\vec{J}' = \frac{\partial \vec{D}}{\partial t}$ arises due to the variation of electric displacement $\vec{D}$ with time, it is termed as displacement current density.

36. *Write down the basic equation of electromagnetism known as Maxwell's equations in differential form and convert them into integral form.* [P.U. 2008]

Solution:

The basic equations of electromagnetism are:

$$\nabla \times \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \text{... (i)}$$

$$\text{or,} \quad \nabla \times \vec{E} = 0 \qquad \text{... (ii)}$$

$$\text{or,} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{... (iii)}$$

$$\text{or,} \quad \nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \text{... (iv)}$$

Equation (i) represents Gauss law of electrostatics; on integrating, we obtain,

$$\int(\nabla \cdot \vec{E})d\vec{v} = \int \frac{\rho}{\varepsilon_0} d\vec{v}$$

Applying Gauss divergence theorem, we have,

$$\int \vec{E}\, d\vec{S} = \frac{q}{\varepsilon_0} \qquad \text{... (v)}$$

i.e., the total electric displacement through the surface enclosing a volume in equal to the total charge within the volume.

Equation (ii) represents Gauss law of magnetostatics. On integrating, we obtain,

$$\int(\nabla \cdot \vec{B})d\vec{v} = 0$$

$$\text{or,} \quad \int \vec{B}\, d\vec{S} = 0 \qquad \text{... (vi)}$$

This equation implies that magnetic lines of forces are solenoidal and magnetic monopoles do not exist.

Equation (iii) represents Faradays la of electromagnetic induction. On integrating, we obtain,

$$\int(\nabla \cdot \vec{E})d\vec{S} = -\frac{\partial}{\partial t}\int \vec{B}\, d\vec{S}$$

$$\text{or,} \quad \int(\nabla \times \vec{E})d\vec{S} = -\frac{\partial \vec{B}}{\partial t} d\vec{S}$$

Applying Stokes theorem, we have,

$$\vec{E}\, d\vec{l} = -\frac{\partial \phi_B}{\partial t} \qquad \text{... (vii)}$$

This signifies that the electromotive force around the closed path is equal to the time derivative of magnetic displacement through any surface bounded by the path.

Equation (iv) represents Ampere's circuital law. On integrating, we obtain,

$$\int(\nabla \times \vec{H})d\vec{S} = \int \vec{J} + d\vec{S} + \varepsilon_0 \frac{\partial}{\partial t}\int \vec{E} d\vec{S}$$

$$\text{or,} \quad \vec{H}\, d\vec{l} = I + \varepsilon_0 \frac{\partial \phi_E}{\partial t} \qquad \text{... (viii)}$$

This signifies that magnetomotive force around the closed path is the sum of conduction current and time derivative of electric displacement through any surface bounded by the path.

37. Prove the relation $\frac{E_m}{B_m} = c$, where E_m and B_m are amplitudes of electromagnetic wave and c is velocity of light. [P.U. 2008]

Solution: See the solution of Q. No. 4 on page no. 270

38. What is displacement current? Write down the significance of displacement current. [P.U. 2010]

Solution: See the solution of Q. No. 6 on page no. 272

39. State and prove the equation of continuity. [P.U. 2010]

Solution: See the solution of Q. No. 23 on page no. 278

40. With the help of electromagnetic wave equation. Prove that $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, where c is velocity of light. [P.U. 2010]

Solution: See the solution of Q. No. 1 on page no. 268

41. Write Maxwell's equations both in integral and differential forms and point out the physical significance of each of the equation.

Solution: See the solution of Q. No. 36 and 10 on page no. 282 and 275

42. Define Maxwell's equation in differential form.

Solution:

The Maxwell's equations in differential form,

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{... (i)}$$

$$\text{or, } \nabla \cdot \vec{B} = 0 \quad \text{... (ii)}$$

$$\text{or, } \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{... (iii)}$$

$$\text{or, } \nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{... (iv)}$$

Derivation of equation (i)

See the solution of Q. No. 30 on page no. 280

Derivation of equation (ii)

The flux of magnetic induction $\vec{B}$ across any closed surface is always zero.

$$\text{i.e., } \int \vec{B} \cdot d\vec{S} = 0$$

$$\text{or, } \int (\nabla \cdot \vec{B})\, dv = 0$$

For any arbitrary volume, the integrand should vanish.

$$\text{i.e., } \nabla \cdot \vec{B} = 0$$

Derivation of equation (iii)

We have,

According to Faraday's la of electromagnetic induction, induced e.m.f. in a closed loop is:

$$\varepsilon = -\frac{\partial \phi}{\partial t} \quad \text{... (v)}$$

The negative sign is due to Lenz's law. e.m.f. ε can also be found by calculating the work done in carrying a unit charge completely around the loop.

$$\text{i.e., } \varepsilon = \int \vec{E} \cdot d\vec{l} \quad \text{... (vi)}$$

$$\text{or, } -\frac{\partial \phi}{\partial t} = \int \vec{E} \cdot d\vec{l}$$

$$\text{or, } -\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{S} = \int \vec{E} \cdot d\vec{l}$$

$$\text{or, } -\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{S} = \int (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\therefore \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{... (vii)}$$

This is known as Faradays law of electromagnetic induction.

Derivation of equation (iv)

See the solution of Q. No. 35 on page no. 281

Write Maxwell's equation in integral form and convert them into differential form. [P.U. 2003]

Solution: See the solution of Q. No. 10 on page no. 275

With the help of electromagnetic wave equation, prove that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, where, c is the velocity of light, ε_0 is permittivity and μ_0 is permeability. [P.U. 2003]

Solution: See the solution of Q. No. 4 on page no. 270

Prove that the relation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

where, ρ = charge density

$\vec{J}$ = current [P.U. 2003]

Solution: See the solution of Q. No. 23 on page no. 278

Chapter 17

PHOTONS AND MATTER WAVES

17.1 PHOTONS AND QUANTIZATION ENERGY

Einstein (1905) postulated that electromagnetic radiation (light) consists of particles called photons. Each photon carries energy $E = h\nu$ and has linear momentum $p = \frac{h}{\lambda}$, where, ν, λ and h are frequency of the light, wavelength of the light wave and plank's constant.

Further,

$$p = \frac{h}{\lambda} = \frac{h}{\frac{c}{\nu}} = \frac{h\nu}{c} = \frac{h}{c}\cdot\frac{E}{h} = \frac{E}{c}$$

where, $h = 6.63 \times 10^{-34}\ Js$ 17.1

The energy of light wave is quantized, *i.e.*, it is an integral multiple of energy of a single photon. If the incident light consists of n photons, the total energy is;

$$E = nh\nu; \quad 0, 1, 2, \dots\dots\dots \tag{17.2}$$

Whenever the values of a property are restricted to discrete multiples, it is said to be quantized. Thus, the energy of photon is quantized.

17.2 de BROGLIE'S HYPOTHESIS

de Broglie (1923) suggested that material particles, just like photons, can have wave like aspects.

Consider a photon of frequency ν, its energy is;

$$E = h\nu = mc^2 \tag{17.3}$$

where, m is the mass of photon particle. The photon travels with velocity of light, its momentum is;

$$p = mc \tag{17.4}$$

The insertion of equation **(17.4)** into equation **(17.3)** gives;

$$\frac{h\nu}{c} = p$$

or, $\lambda = \frac{h}{p}$

where, $\lambda = \frac{c}{\nu}$ 17.5

de Broglie assumed that equation **(17.5)** holds good for material particles like electrons and is known as de Broglie relation.

Consider an electron of mass m and velocity v is accelerated from rest through a potential difference V, the work done on the electron eV is converted into the kinetic energy of the electron.

i.e., $\frac{1}{2}mv^2 = eV$

or, $v = \sqrt{\frac{2eV}{m}}$ 17.6

and its corresponding momentum;

$$p = \sqrt{2meV} \tag{17.7}$$

The de Broglie equation implies that;

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \tag{17.8}$$

Ignoring relativistic consideration;

$$\lambda = \frac{1.23}{\sqrt{V}}\ nm \tag{17.9}$$

17.3 HEISENBERG'S UNCERTAINTY PRINCIPLE

Heisenberg's uncertainty principle states that, "*it is impossible to determine precisely and simultaneously two complementary variables to arbitrary accuracy.*" Such pairs of variables are canonically conjugate variables. Position q and momentum p, energy E and time t, angular momentum τ and angular displacement θ, etc. are canonically conjugate variables. The Heisenberg's uncertainty principle for position and momentum is;

$$\Delta q \Delta p \geq \frac{\hbar}{2};\ \hbar = \frac{h}{2\pi} \tag{17.10}$$

This uncertainty principle usually describes one or more of the following statements.

i) It is impossible to predict states in which position and momentum are simultaneously well localized.

ii) It is impossible to measure position and momentum simultaneously.

iii) It is impossible to measure position without disturbing momentum and vice versa.

17.4 SCHRODINGER WAVE EQUATION

The time dependent Schrodinger equation is;

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \qquad 17.11$$

where, H = Hamiltonian of the system

The independent Schrodinger equation is;

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \qquad 17.12$$

where, E and V are total energy and potential energy of the particle

17.5 TUNNELING EFFECT

The phenomenon of transmission of a particle through a potential barrier of finite width and height even when its energy is less than the barrier height is called quantum mechanical tunneling effect. It is also known as barrier penetration. It is entirely due to the wave nature of particles. The particles cross the barrier without going over the top as if it has been passed through a tunnel in the barrier.

17.6 SOLVED EXAM QUESTIONS

1. **Derive the Schrodinger time independent wave equation. What is the physical significance of wave functions?[T.U. 2067 Mangsir]**

Solution:

For a plane monochromatic wave;

$$\psi(r,t) = Ae^{i(\vec{k}.\vec{x} - \omega t)}$$

where, $k = \frac{2\pi}{\lambda}$ is a wave vector

On differentiating with respect to time, we obtain,

$$\frac{\partial\psi}{\partial t} = (-i\omega)Ae^{i(\vec{k}.\vec{x} - \omega t)}$$

Further,

$$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)^2Ae^{i(\vec{k}.\vec{x} - \omega t)} = -\omega^2Ae^{i(\vec{k}.\vec{x} - \omega t)} = -\omega^2\psi(\vec{r},t)$$

According to Maxwell's wave equation;

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = \frac{1}{v^2}(-\omega^2)\psi(\vec{r},t) = -\frac{\omega^2}{v^2}\psi(\vec{r},t)$$

$$= -\frac{(2\pi\nu)^2}{v^2}\psi(\vec{r},t) \qquad [\because \omega = 2\pi\nu]$$

$$= -\frac{\left(2\pi\frac{v}{\lambda}\right)^2}{v^2}\psi(\vec{r},t) \qquad [\because v = \nu\times\lambda]$$

$$= -\frac{4\pi^2}{\lambda^2}\psi(\vec{r},t)$$

Since, $\lambda = \frac{h}{p} = \frac{h}{mv}$

Thus,

$$\nabla^2\psi = -\frac{4\pi^2m^2v^2}{h^2}\psi(\vec{r},t) = -\frac{2m(E-V)}{\hbar^2}\psi(\vec{r},t)$$

where, $\hbar = \frac{h}{2\pi}$; $\frac{1}{2}mv^2 = T = (E-V)$

E and V are total energy and potential energy respectively

Thus,

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

This is called Schrodinger time independent wave equation.

Physical Interpretation of Wave Function

The probability that a particle will be found at a given place in space at a given instant of time is characterized by the wave function $\psi(\vec{r},t)$. As $\psi(\vec{r},t)$ is a complex variable, it can't have a direct physical meaning. The only quantity having physical meaning is square of its amplitude $\psi(\vec{r},t)$.

i.e., $|\psi(\vec{r},t)|^2 = \psi^*(\vec{r},t)\cdot\psi(\vec{r},t)$

This gives the particle probability density.

2. **Derive the expression for the energy of the particle in a one dimensional infinite deep potential well. [T.U. 2068 Shrawan]**

Solution:

The particle of mass m is in one dimensional infinite square well potential characterized by;

$$V(x) = \begin{cases} 0; & -L < x < L \\ \infty; & |x| \geq L \end{cases}$$

The time independent Schrodinger equation for $V(x) = 0(-L < x < L)$ can be written as; ... (i)

$$\frac{d^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

where, $\psi(x)$ represents the wave function ... (ii)

i.e., $\psi''_{(x)}{}^2 + K^2\psi(x) = 0$ ... (iii)

where, $k^2 = \frac{2mE}{\hbar^2}$

The general solution of this equation will be;

$$\psi(x) = A\cos kx + B\sin kx \qquad \cdots \text{(iv)}$$

Applying boundary conditions;

$$\psi(x)|_{x=-L} = 0$$

and $\psi(x)|_{x=L} = 0$

We obtain,

$$\left.\begin{array}{l} A\cos kL - B\sin kL = 0 \\ A\cos kL + B\sin kL = 0 \end{array}\right\} \qquad \cdots \text{(v)}$$

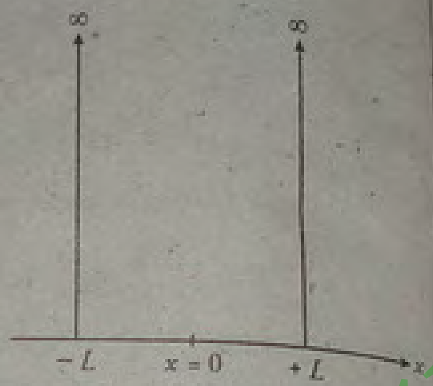
Adding and subtracting these equations; we have,

$$2A\cos kL = 0$$

or, $k = \frac{n\pi}{2L}; n = 1, 3, 5, \dots\dots\dots$

and $2B\sin kL = 0$

$$k = \frac{n\pi}{2L}; n = 0, 2, 4, \dots\dots\dots$$



Thus for both even and odd 'n',

$$k = \frac{n\pi}{2L}$$

or, $k^2 = \frac{n^2\pi^2}{4L^2}$

or, $\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{4L^2}$

$$\therefore \quad E_n = \frac{n^2\pi^2\hbar^2}{8mL^2} \qquad \cdots \text{(vi)}$$

This gives the energy spectrum of the particle inside the infinite square well potential. It is found that energy spectrum is discrete.