CSE460/560 DATA MODELS AND QUERY LANGUAGES

Relational Algebra – Algebraic Laws of RA

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(Slides Adopted from Jan Chomicki and Ning Deng)

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Outline

1. Query Rewriting
2. Algebraic Laws

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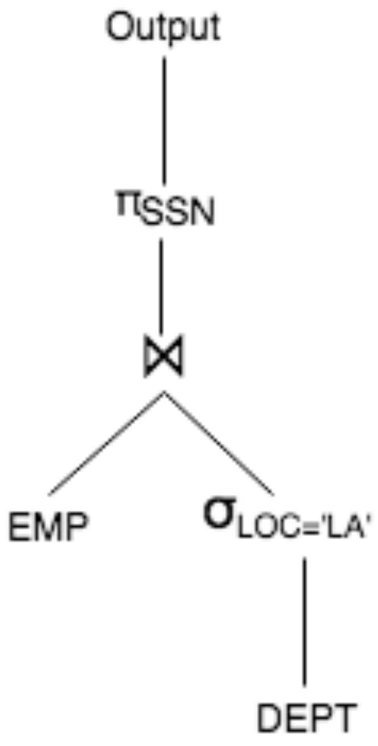
1. **Query Rewriting**
2. Algebraic Laws

Query Rewriting

- EMP(SSN, SAL, DNAME)
- DEPT(DNAME, LOC)
- Query
  - Find all the SSNs of the employees who work in a department located in LA
  - $\pi_{SSN}\left(\sigma_{EMP.DNAME=DEPT.DNAME \land DEPT.LOC='LA'}(EMP \times DEPT)\right)$
  - $\pi_{SSN}\left(\sigma_{EMP.DNAME=DEPT.DNAME}\left(\sigma_{DEPT.LOC='LA'}(EMP \times DEPT)\right)\right)$
  - $\pi_{SSN}\left(\sigma_{EMP.DNAME=DEPT.DNAME}\left(EMP \times \sigma_{DEPT.LOC='LA'}(DEPT)\right)\right)$
  - $\pi_{SSN}(EMP \bowtie \sigma_{DEPT.LOC='LA'}(DEPT))$
- 🍭What is the difference between these queries?

Query Rewriting (RA Tree)

- $\pi_{SSN}(EMP \bowtie \sigma_{DEPT.LOC='LA'}(DEPT))$



Outline

1. Query Rewriting
2. **Algebraic Laws**

Improving Query Plans

- Algebraic Laws (equivalence rules)
  - Commutative and Associative
  - Pushing down SELECTION
  - Pushing down PROJECT
  - …
- Cost based Optimization (We will not talk about this)

Commutative and Associative Operators

- $R \times S = S \times R; (R \times S) \times T = R \times (S \times T)$
- $R \bowtie S = S \bowtie R; (R \bowtie S) \bowtie T = R \bowtie (S \bowtie T)$
- $R \cup S = S \cup R; (R \cup S) \cup T = R \cup (S \cup T)$
- $R \cap S = S \cap R; (R \cap S) \cap T = R \cap (S \cap T)$
- How about θ-join?
  - R(a,b) , S(b,c), T(c,d)
  - $R \bowtie_{\theta} S = S \bowtie_{\theta} R$
  - $\left(R \bowtie_{\theta_1} S\right) \bowtie_{\theta_2 \wedge \theta_3} T = R \bowtie_{\theta_1 \wedge \theta_3} (S \bowtie_{\theta_2} T)$
    - θ_2 involves attributes only from S and T

Pushing Down SELECTION

- SELECTION σ tend to reduce the size relation
- Move the selections down the tree as far as they will go
  - Without changing what the expression does
- Splitting Laws
  - $\sigma_{C_1 \wedge C_2}(R) = \sigma_{C_1}(\sigma_{C_2}(R))$
  - $\sigma_{C_1 \vee C_2}(R) = \sigma_{C_1}(R) \cup \sigma_{C_2}(R)$
- Union and Difference
  - $\sigma_C(R \cup S) = \sigma_C(R) \cup \sigma_C(S)$
  - $\sigma_C(R - S) = \sigma_C(R) - \sigma_C(S)$

Pushing Down SELECTION

- Assumption: Relation *R* has all attributes mentioned in *C*
  - $\sigma_C(R \times S) = \sigma_C(R) \times S$
  - $\sigma_C(R \bowtie S) = \sigma_C(R) \bowtie S$
  - $\sigma_{C_1}\left(R \bowtie_{C_2} S\right) = \sigma_{C_1}(R) \bowtie_{C_2} S$
  - $\sigma_C(R \cap S) = \sigma_C(R) \cap \sigma_c(S)$

Pushing Down PROJECTION

- PROJECT π is generally less useful than pushing down SELECTION
  - Why?
- Principle
  - Pushing down the PROJECTION as long as it eliminates only attributes
    - That are neither used by any operator above on the RA tree
    - Nor are in the result of the entire expression

Pushing Down PROJECTION

- Cascading
  - $\pi_{A_1,\ldots,A_n}\left(\pi_{A_1,\ldots,A_{n+k}}(R)\right) = \pi_{A_1,\ldots,A_n}(R)$
- With SELECTION
  - $\pi_{A_1,\ldots,A_n}\left(\sigma_C(R)\right) = \sigma_c(\pi_{A_1,\ldots,A_n}(R))$
  - If attr(c) $\subseteq \{A_1, \ldots, A_n)$

Pushing Down PROJECTION

- With Cartesian Product
  - $\pi_{A_1,\dots,A_{n+k}}(S \times R) = \pi_{A_1,\dots,A_n}(S) \times \pi_{A_{n+1},\dots,A_{n+k}}(R)$
  - Where
    - $A_1, \dots, A_n$ come from *S*
    - $A_{n+1}, \dots, A_{n+k}$ come from *R*
- With Union
  - $\pi_{A_1,\dots,A_n}(R \cup S) = \pi_{A_1,\dots,A_n}(R) \cup \pi_{A_1,\dots,A_n}(S)$

Proving Laws

- $\forall R_1, R_2 : \sigma_C(R_1 - R_2) = \sigma_C(R_1) - \sigma_C(R_2)$
  - A tuple *t* in LHS iff
    - $t \in \pi_C(R_1 - R_2)$ iff
    - $t \in (R_1 - R_2) \wedge C(t)$ iff
    - $t \in R_1 \wedge t \notin R_2 \wedge C(t)$
  - A tuple *t* in RHS iff
    - $t \in \sigma_C(R_1) - \sigma(R_2)$ iff
    - $t \in \sigma_C(R_1) \wedge t \notin \sigma_C(R_2)$ iff
    - $t \in R_1 \wedge C(t) \wedge t \notin R_2 \vee \neg C(t)$ iff
    - $t \in R_1 \wedge t \notin R_2 \wedge C(t)$

Disproving Laws

- $\forall R_1, R_2 : \pi_A(R_1 - R_2) = \pi_A(R_1) - \pi_A(R_2)$
  - $R_1 = \{(1,2), (1,3)\}$
  - $R_2 = \{(1,2)\}$

Recommended Reading

Database Systems: The Complete Book
Chapter 16.1 – 16.2