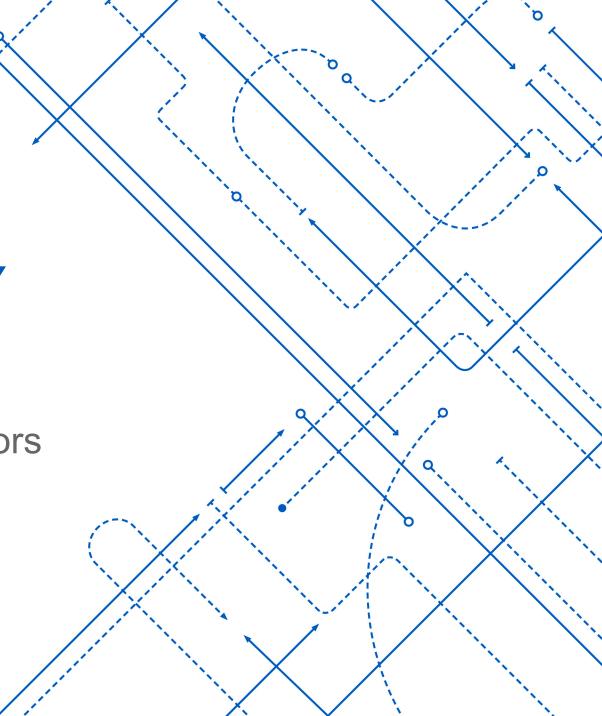


Relational Algebra – Derived Operators

Cheng-En Chuang

(Slides Adopted from Jan Chomicki and Ning Deng)





- 1. Intersection ∩
- 2. θ Join $\bowtie_{A \theta B}$
- 3. Natural Join θ
- 4. Quotient /
- 5. Linear Notation

- **1.** Intersection ∩
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Intersection ∩

- Takes two compatible relations
 - Returns tuples in **both** relations
- Properties
 - $arity(R_1 \cap R_2) = arity(R_1) = arity(R_2)$
 - $t \in R_1 \cap R_2$ iff $t \in R_1 \land t \in R_2$
- Intersection $R_1 \cap R_2$ is a derived operation
 - How to define it in terms of essential operators?
- Example
 - $\rho_{FirstName} (\pi_{FirstName}(Student) \cap \pi_{president}(Club))$
 - What is this query asking for?

Student

FirstName,	GPA,	SID
[James,	3.9,	1701]
[Jean,	3.9,	1702]
[John,	3.0,	1703]
[Mary,	4.0,	1801]
[Mike,	4.0,	1805]

Name
C1]
C2]
C3]

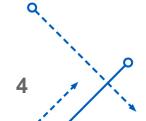
Club

MajorsIn

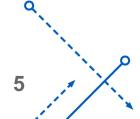
SID,	MID
[1701,	01]
[1701,	02]
[1805,	03]
[1801,	04]

Major

Name
CS]
EE]
Math]
ME]



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θ – Join $\bowtie_{A \theta B}$

- θ : A comparison operator $(=, \neq, <, >, \geq, \leq)$
 - A_1, \ldots, A_n : all attributes of R_1
 - B_1, \dots, B_k : all attributes of R_2
- θ Join of R_1 , R_2
 - Takes two relations, and pairs tuples
 - According to the join condition $A \theta B$
 - Property
 - $arity\left(R_1 \bowtie_{A_i \theta B_i} R_2\right) = arity(R_1) + arity(R_2)$
- \bullet How to define $R_1 \bowtie_{A_i \theta B_i} R_2$ in terms of essential operators?
- Example
 - Student $\bowtie_{FirstName \neq president} Club$
 - What is this query asking?

Student

FirstName,	GPA,	SID
[James,	3.9,	1701]
[Jean,	3.9,	1702]
[John,	3.0,	1703]
[Mary,	4.0,	1801]
[Mike,	4.0,	1805]

Club

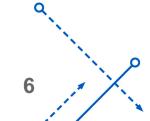
President,	Name
[James,	C1]
[Mary,	C2]
[Tom,	C3]

MajorsIn

SID,	MID
[1701,	01]
[1701,	02]
[1805,	03]
[1801,	04]

Major

MID,	Name		
[01,	CS]		
[02,	EE]		
[03,	Math		
[04,	ME]		



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Natural Join ⋈

- A useful join variant connects two relations
- A_1, \dots, A_n : all attributes of R_1
- $B_1, \dots B_k$: all attributes of R_2
- m: the number of attributes common to R_1 and R_2
- $R_1 \bowtie R_2$
 - Select from $R_1 \times R_2$ the tuples that agree on all common attributes
 - Project duplicated columns out
- Properties
 - $arity(R_1 \bowtie R_2) = arity(R_1) + arity(R_2) m$
- Example
 - *MajorsIn* ⋈ *Major*

Student

FirstName,	GPA,	SID
[James,	3.9,	1701]
[Jean,	3.9,	1702]
[John,	3.0,	1703]
[Mary,	4.0,	1801]
[Mike,	4.0,	1805]

Club

President,	Name
[James,	C1]
[Mary,	C2]
[Tom,	C3]

MajorsIn

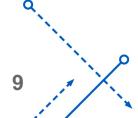
SID,	MID
[1701,	01]
[1701,	02]
[1805,	03]
[1801,	04]

Major

MID,	Nam	
[01,	CS]	
[02,	EE]	
[03,	Math	
[04,	ME]	



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Quotient

- $A_1, ..., A_{n+k}$: all attributes of R_1
- A_{n+1} , A_{n+k} : all attributes of R_2
- Quotient (Division) identifies all the attributes value in a relation
 - Found to be paired with all values from another relation
- Properties
 - $arity(R_1/R_2) = arity(R_1) arity(R_2) = n$
 - $t \in R_1/R_2$ iff for all $s \in R_2$ there is a $w \in R_1$ such that
 - $w[A_1, ..., A_n] = t[A_1, ..., A_n]$ and
 - $w[A_{n+1}, ..., A_{n+k}] = s[A_{n+1}, ..., A_{n+k}]$

Quotient

- Express queries with "all"
 - Which are students taking all the provided courses?

		S/C1	S/C2	S/C3
s		[James] [Mary]	[James] [Mary]	[James]
[James, [James, [Mary, [Mary,	OS] DS] DB] OS]	<u>Name</u> [John]	Name	Name
[John, [James, [James,	OS] DB] ML]	C1	C2	C3
Name,	Course	Course [OS]	Course [OS] [DB]	[OS] [DB] [ML]

Course



Quotient

- And yes, Quotient can be expressed in terms of essential operators
- $R_1/R_2 = \pi_{A_1,...,A_n}(R_1) \pi_{A_1,...,A_n}(\pi_{A_1,...,A_n}(R_1) \times R_2 R_1)$
- All possible candidates Unqualified Candidates
 - All possible candidates
 - $\pi_{A_1,\ldots,A_n}(R_1)$
 - Unqualified Candidates
 - All possible qualifying sub-conditions All qualifying sub-conditions
 - All possible qualifying sub-conditions
 - $\pi_{A_1 \dots, A_n}(R_1) \times R_2$
 - All qualifying sub-conditions
 - *R*₁

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Linear Notation

- Invent new names for intermediate relations
 - Assign them values that are RA expressions
- Renaming of attributes in schema of new relation implicitly
- Example

$$R_1(FirstName) := \pi_{FirstName}(Student) \cap \pi_{president}(Club)$$

 $R_2(FirstName) := \pi_{FirstName}(\sigma_{GPA>3.5}(Student))$
 $Ans(Name) := R_1 \cup R_2$

- What is this query asking?
 - What if we change the final expression to

$$Ans(Name) := R_1 \cap R_2$$



Recommended Reading

Database Systems: The Complete Book

Chapter 2.4, 5.2