

TIME COMPLEXITY IS ANALYSED FOR:

- Very large input size
- Worst case scenario

Let's say we have the following example:

$$T(n) = 2n^2 + 3n + 1$$

First, step is to drop lower order terms:

$$T(n) = 2n^2 + \cancel{3n + 1}$$

Next, we drop the constant multiplier:

$$T(n) = \cancel{2}n^2 + \cancel{3n + 1}$$

... so we get:

$$= O(n^2)$$

Now, let's examine the rules for finding time complexities for the following algorithms:

```
1) Loop

for( i = 1; i<=n; i++ ){
    x=y+z;
}
```

... the loop takes constant time to execute ($x = y+z$)

The time complexity will be n times the time taken for the algorithm to execute... the time is a constant, and let's call it c : *constant time = c*

... so the time complexity will be c times n : cn

... in big O terms, we can neglect c and write the time complexity as $O(n)$.

2) Nested Loop

```
for( i = 1; i<=n; i++ ){  
    for( j = 1; j<=n; j++ ){  
        x=y+z;  
    }  
}
```

... as you can see, we have 2 loops that are nested.

The constant time for this algorithm to execute is $x=y+z$.

The top line is the outer loop (i), which will run n times.

The second line is the inner loop (j), which will run n times for each run of the outer loop.

```
for( i = 1; i<=n; i++ ){ // n times  
    for( j = 1; j<=n; j++ ){ // n times  
        x=y+z; // constant time  
    }  
}
```

The total run time will be $n * n * \text{constant time}$, which is...

$$= O(n^2)$$

3) Sequential Statements

i) $a = a + b;$ // constant time $= c_1$

ii) $\text{for}(i = 1; i \leq n; i++) \{$
 $x = y + z;$ $c_2 n$ $= \underbrace{c_1} + \underbrace{c_2 n} + \underbrace{c_3 n}$
 $\}$

$= O(n)$

iii) $\text{for}(j = 1; j \leq n; j++) \{$
 $c = d + e;$ $c_3 n$
 $\}$

4) If-else statements

```
if(condition)
{
  ---  $O(n)$ 
}
else ✓
{
  ---  $O(n^2)$ 
}
```

$= O(n^2)$

Let's say in the if condition is a loop whose time complexity is $O(n)$...

... and in the else statement is a nested for loop whose time complexity is $O(n)^2$...

... the else statement has a better time complexity, and thus the overall time complexity will be $O(n)^2$

Here is a chart of time complexities of various sorting algorithms:

Algorithm	Time Complexity		
	Best	Average	Worst
Selection Sort	$\Omega(n^2)$	$\Theta(n^2)$	$O(n^2)$
Bubble Sort	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$
Insertion Sort	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$
Heap Sort	$\Omega(n \log(n))$	$\Theta(n \log(n))$	$O(n \log(n))$
Quick Sort	$\Omega(n \log(n))$	$\Theta(n \log(n))$	$O(n^2)$
Merge Sort	$\Omega(n \log(n))$	$\Theta(n \log(n))$	$O(n \log(n))$
Bucket Sort	$\Omega(n+k)$	$\Theta(n+k)$	$O(n^2)$
Radix Sort	$\Omega(nk)$	$\Theta(nk)$	$O(nk)$

... it is highly recommended to memorize them, as they will come in handy.