Chapter 3

DC Transients

Lecture 2

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Switching at $t=0$

$$v(t) = v(\infty) + \left(v(0^+) - v(\infty)\right) e^{-t/\tau} \quad ; \; t \geq 0$$

$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right) e^{-t/\tau} \quad ; \; t \geq 0$$

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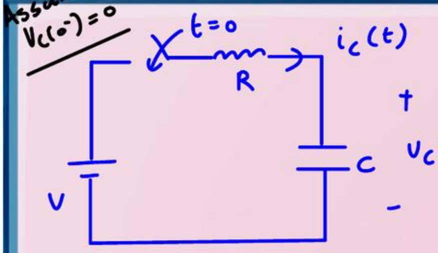
Assume $V_c(0^-)=0$



Nature of Response : Exponential

$V_c(t) \ni 0 \longrightarrow V$

$i_c(t) \ni \frac{V}{R} \to 0$

Note : $t=0^-$: ONLY TAKE AWAY

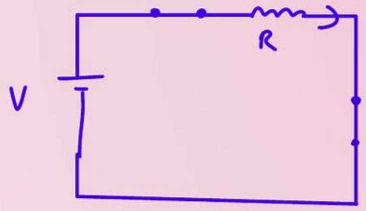
$V_c(0^-)$
(OR)
$I_L(0^-)$

Exp Rise from 0
$A(1-e^{-t/\tau})$

Exp fall to 0
$Be^{-t/\tau}$

Here $V_c(0^-) = 0V$

$t=0^+$



$i(0^+) = V/R$

$V_c(0^+) = 0V$

$t=\infty$: SS after switching

$t=\infty$

$i(\infty)=0$

$V_c(\infty)=V$

$$i_c(t) = i_c(\infty) + (i_c(0^+) - i_c(\infty))e^{-t/\tau}$$

$$= 0 + \left(\frac{V}{R} - 0\right)e^{-t/\tau}$$

$\tau = R_{eq}C$

Eq. R across C after SW

$\tau = RC$

$$\therefore \boxed{i_c(t) = \frac{V}{R}e^{-t/\tau} \; ; \; t \geq 0 \quad \tau = RC}$$

$$V_c(t) = V_c(\infty) + (V_c(0^+) - V_c(\infty))e^{-t/\tau}$$

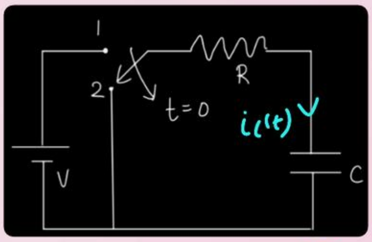
$$= V + (V-0)e^{-t/\tau}$$

$$V_c(t) = V(1-e^{-t/\tau}) \; ; \; t \geq 0$$

$\tau = RC$

Source Free Circuits

→ After switching



Note : For Source free circuits

$$\left.\begin{aligned} v(t) &= v(0^+)\,e^{-t/\tau} \\ i(t) &= i(0^+)\,e^{-t/\tau} \end{aligned}\right\} t \geq 0$$

$$V(\infty) = i(\infty) = 0$$

$$V(t) = \cancel{v(\infty)} + \left(v(0^+) - \cancel{v(\infty)}\right) e^{-t/\tau}$$

$t = 0^-$ → ss bfr sw → $C \to oc$

$V_c(0^-) = V$

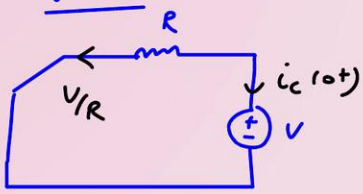
$$\therefore V_c(0^+) = V_c(0^-) = V$$

$$V_c(t) = V_c(0^+)\,e^{-t/\tau} \; ; \; t \geq 0$$

$$\boxed{V_c(t) = V e^{-t/\tau}} \; ; \; t \geq 0$$

$t = 0^+$

$t = 0^+$



$$i_c(t) = i_c(0^+)\,e^{-t/\tau} \; ; \; t \geq 0$$

$$\therefore i_c(0^+) = -V/R$$

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$$\therefore i_c(t) = -V/R \; e^{-t/\tau} \; ; \; t \geq 0$$

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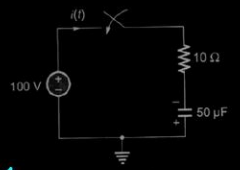
In the circuit shown below, the initial charge on the capacitor is 2.5 mC, with the voltage polarity as indicated. The switch is closed at time $t = 0$. The current $i(t)$ at a time t after the switch is closed is



(a) $i(t) = 15 \exp(-2 \times 10^3 t)$ A ✓
(b) $i(t) = 5 \exp(-2 \times 10^3 t)$ A
(c) $i(t) = 10 \exp(-2 \times 10^3 t)$ A
(d) $i(t) = -5 \exp(-2 \times 10^3 t)$ A

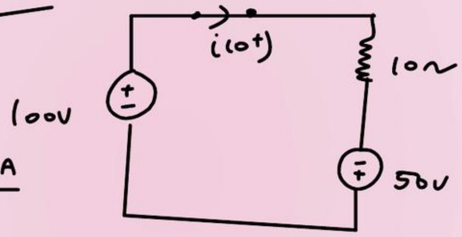
$t=0^-$ $\quad q = CV$

$\therefore V = q/C$

$\therefore V_c(0^-) = \dfrac{2.5mC}{50\mu F} = \dfrac{2500}{50} V = \underline{\underline{50V}}$

$V_c(0^+) = V_c(0^-) = 50V$

$\underline{t=0^+}$



$\therefore i(0^+) = \dfrac{100+50}{10} = \underline{15A}$

$\underline{t=\alpha}$: $\underline{SS \text{ after } SW}$

$\underline{Cap \Rightarrow OC}$ $\quad \therefore \underline{i(\alpha) = 0}$

$\tau = R_{eq}C = (10)(50\mu) = 500\mu = \underline{0.5m}$

$\therefore \quad i(t) = \cancel{i(\alpha)} + (i(0^+) - \cancel{i(\alpha)})e^{-t/\tau} \quad ; t \geq 0$

$i(t) = i(0^+)e^{-t/\tau} \quad ; t \geq 0$

$i(t) = 15\, e^{-t/\tau} \quad ; t \geq 0$

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$\underline{i(t) = 15\, e^{-2000t} \quad ; t \geq 0}$

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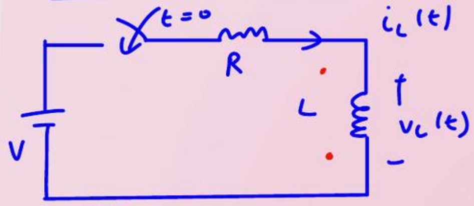
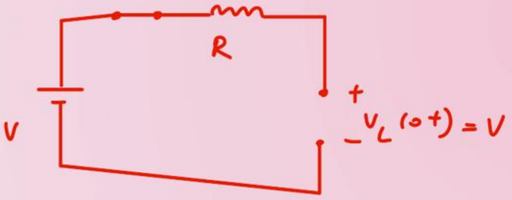
First order RL Circuit

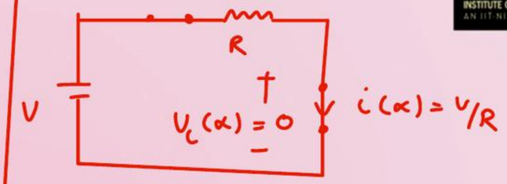


$t=0^-$ $\quad i_L(0^-) = 0\,A$

$t=0^+$ $\quad i_L(0^+) = i_L(0^-) = 0\,A$



$t=\infty$ → SS after SW



$i_L(t) \longrightarrow 0$ to V/R

$V_L(t) \longrightarrow V$ to 0

$$i_L(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right) \; ; \; t \ge 0$$

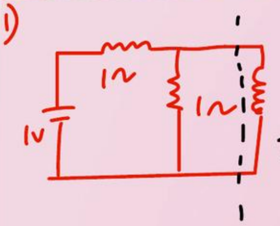
$$V_L(t) = V e^{-t/\tau} \; ; \; t \ge 0$$

$$\tau = \frac{L}{R_{eq}} = \frac{L}{R}$$

RL Circuit

Find Time Constant

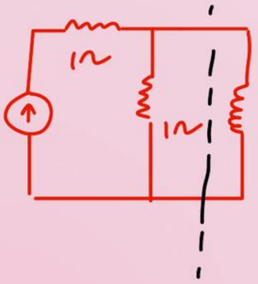
1)



$R_{eq} = 1\Omega \| 1\Omega = 0.5\Omega$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{0.5} = 2\,sec$$

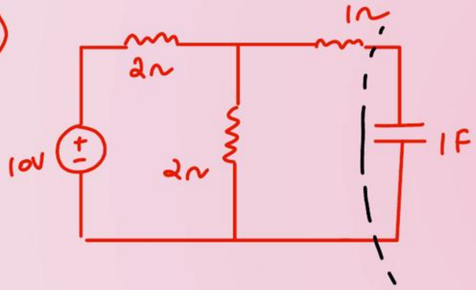
2)



$$\tau = \frac{L}{R_{eq}} = \frac{1}{1} = 1\,sec$$

$R_{eq} = 1\Omega$

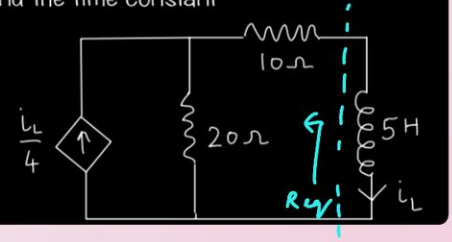
3)



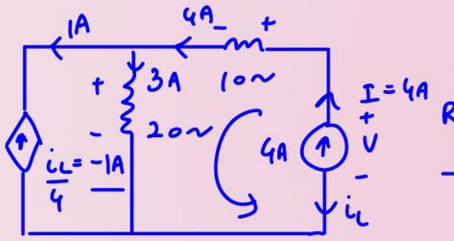
$$\tau = R_{eq}C = 2(1) = 2\,sec$$

$$R_{eq} = (2 \| 2) + 1 = 2$$

Q) Find the time constant



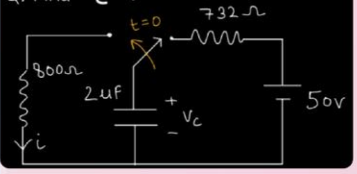
$$\tau = \frac{L}{R_{eq}} = \frac{5}{25} = 0.2\,sec$$



$$R_{eq} = \frac{V}{I} = \frac{V}{4} = \frac{100}{4} = 25\Omega$$

$i_L = -4A$ $\quad V = (4)(10) + (3)(20) = 100\,V$

Q) Find $V_c(t)$ for $t \geq 0$



$V_c(t) = V_c(0^+)\, e^{-t/\tau}$; $t \geq 0$ (Source Free)

$V_c(0^-) = V_c(0^+)$

$t = 0^-$: $V_c(0^-)$: SS bfr SW

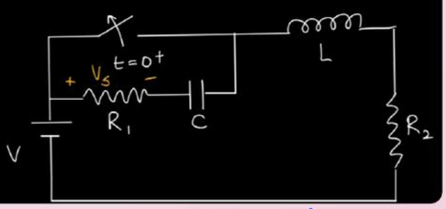
$\therefore V_c(0^-) = 50V$

$V_c(t) = 50\, e^{-t/\tau}$; $t \geq 0$ $\quad \tau = R_{eq}C$

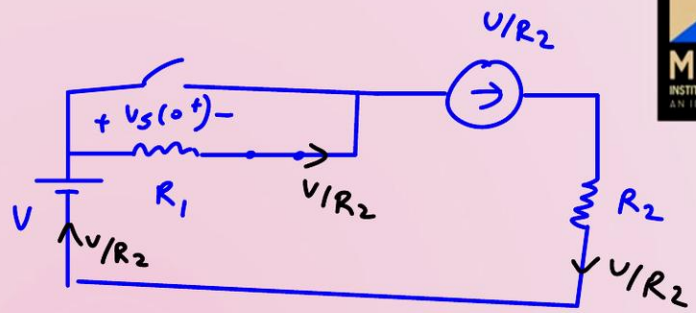
$= (0.8k)(2u) = 1.6ms$

$V_c(t) = 50\, e^{-\frac{1000t}{1.6}}$; $t \geq 0$

Q) Find $V_S(0^+)$

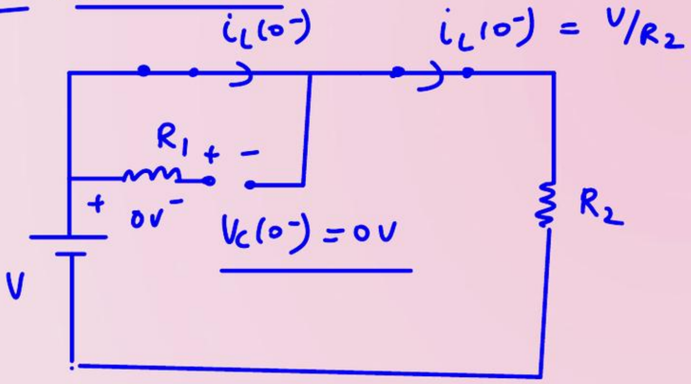


$t = 0^+$



$t = 0^- < \begin{matrix} V_c(0^-) \\ I_L(0^-) \end{matrix}$

$t = 0^-$: SS bfr SW



$i_L(0^-) = V/R_2$

$V_c(0^-) = 0V$

$V_S(0^+) = \left(\frac{V}{R_2}\right)(R_1)$

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