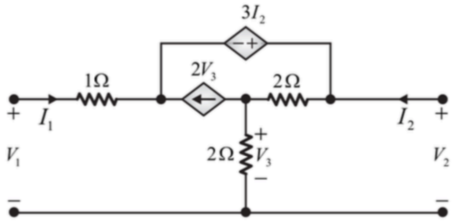
Chapter 5

2 Port Network

Lecture 3

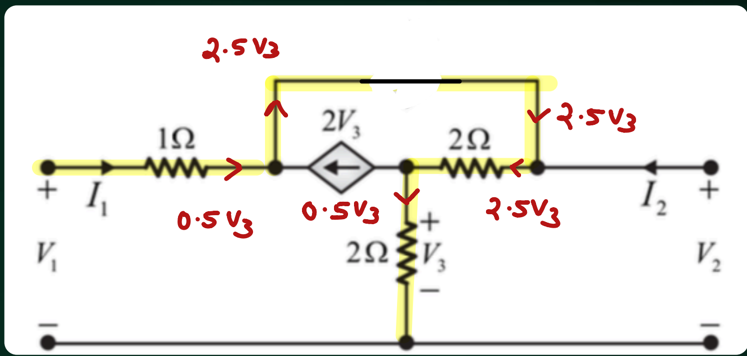
For the circuit shown below, the input resistance $R_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ is

[GATE IN 2008, IISc Bangalore]



(A) $-3\ \Omega$ (B) $2\ \Omega$

(C) $3\ \Omega$ (D) $13\ \Omega$



$I_1 = 0.5V_3$

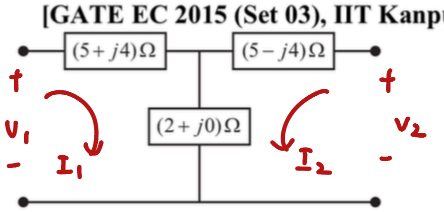
$V_1 = (0.5V_3)(1) + (2.5V_3)(2) + (0.5V_3)2$

$V_1 = 6.5V_3$

$\therefore \quad \frac{V_1}{I_1} = \frac{6.5V_3}{0.5V_3} = \underline{\underline{13}}\ \Omega$

The ABCD parameters of the following 2-port network are

[GATE EC 2015 (Set 03), IIT Kanpur]



(A) $\begin{bmatrix} 3.5+j2 & 20.5 \\ 20.5 & 3.5-j2 \end{bmatrix}$

(B) $\begin{bmatrix} 3.5+j2 & 30.5 \\ 0.5 & 3.5-j2 \end{bmatrix}$ ✓

(C) $\begin{bmatrix} 10 & 2+j0 \\ 2+j0 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 7+j4 & 0.5 \\ 30.5 & 7-j4 \end{bmatrix}$

$3.5^2+2^2 = 15.25$

$12.25+4-15.25$

$=1$

Method ①

$$V_1 = (7+j4)I_1 + 2I_2 \qquad ①$$

$$V_2 = 2I_1 + (7-j4)I_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$

$$\therefore B = \frac{V_1}{-I_2}\Big|_{V_2=0}$$

$$-2I_1 = (7-j4)I_2$$

$$\therefore I_1 = \frac{(7-j4)}{-2}I_2$$

Subs in ①

$$V_1 = \frac{(7+j4)(7-j4)}{-2}I_2 + 2I_2$$

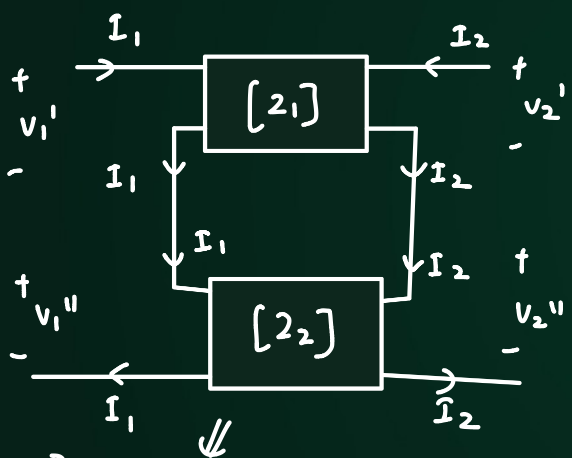
$$V_1 = \left(\frac{49+16}{-2}\right)I_2 + 2I_2 = -30.5I_2$$

$$B = \frac{V_1}{-I_2} = 30.5$$

Interconnection of 2 port Networks

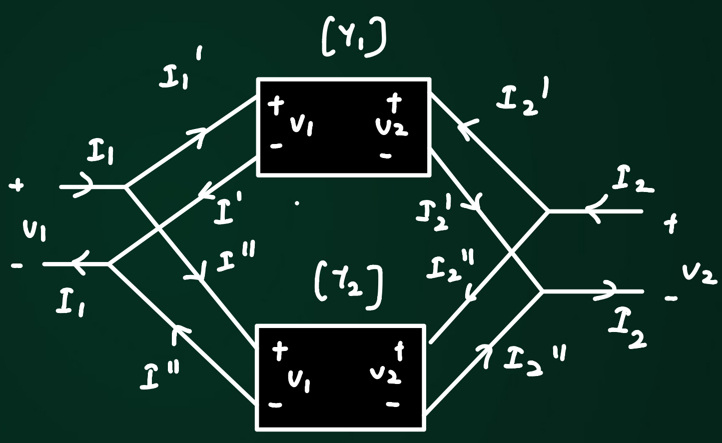
Rules

1. Series $\longrightarrow$ Z parameters are added
2. Parallel $\longrightarrow$ Y parameters are added
3. Cascade $\longrightarrow$ ABCD parameters are multiplied.



$$Z = [Z_1] + [Z_2]$$

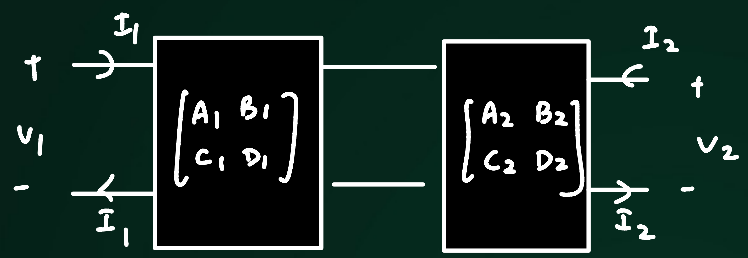
Series Interconnection



$$\therefore [Y] = [Y_1] + [Y_2]$$

Parallel Interconnection

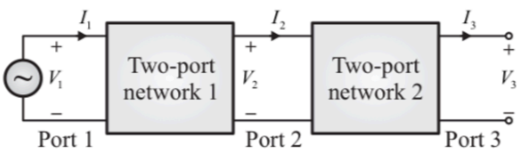
Cascade Interconnection



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Two passive two-port networks are connected in cascade as shown in figure. A voltage source is connected at port 1.

[GATE EE 2017 (Set - 01), IIT Roorkee]



Given :

$$V_1 = A_1V_2 + B_1I_2 \qquad I_1 = C_1V_2 + D_1I_2$$

$$V_2 = A_2V_3 + B_2I_3 \qquad I_2 = C_2V_3 + D_2I_3$$

A_1, B_1, C_1, D_1, A_2, B_2, C_2 and D_2 are the generalized circuit constants. If the Thevenin equivalent circuit at port 3 consists of a voltage source V_T and an impedance Z_T connected in series, then

(A) $V_T = \dfrac{V_1}{A_1A_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$

(B) $V_T = \dfrac{V_1}{A_1A_2 + B_1C_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1A_2}$

(C) $V_T = \dfrac{V_1}{A_1 + A_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1 + A_2}$

(D) $V_T = \dfrac{V_1}{A_1A_2 + B_1C_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}}\begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

V_T : Thevenin voltage

$V_T = V_3 \big|_{I_3 = 0}$

$R_{th} = \dfrac{V_3}{-I_3}\Big|_{V_1 = 0}$

When $I_3 = 0$

$\therefore$ $V_1 = AV_3 + BI_3$ ①

$I_1 = CV_3 + DI_3$

$V_3 = \dfrac{V_1}{A}$ or $V_3 = \dfrac{C}{I_1}$

$V_3 = \dfrac{V_1}{A_1A_2 + B_1C_2} = V_T$

From ① when $V_1 = 0$

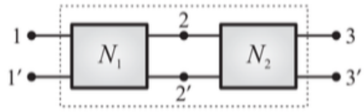
$$AV_3 = -BI_3$$

$$\therefore \frac{V_3}{-I_3} = R_m = \frac{B}{A} = \frac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$$

The connection of two 2-port networks is shown in the figure. The ABCD parameters of N_1 and N_2 networks are given as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{N_1} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \text{and}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{N_2} = \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix}$$



The ABCD parameters of the combined 2-port network are

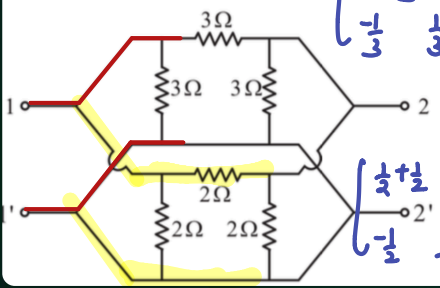
(A) $\begin{bmatrix} 2 & 5 \\ 0.2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 0.5 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 2 \\ 0.5 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ 0.5 & 5 \end{bmatrix}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 \\ 0.2 & 1 \end{bmatrix} \quad (A)$$

In the h-parameter model of the 2-port network given in the figure shown, the value of h_{22} (in S) is ________.



$$\begin{bmatrix} \frac{1}{3}+\frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{1}{3}+\frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

Rule : Y parameters can be added

$$Y = \begin{bmatrix} \frac{2}{3}+1 & -\frac{5}{6} \\ -\frac{5}{6} & \frac{2}{3}+1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 5/3 & -5/6 \\ -5/6 & 5/3 \end{bmatrix}$$

$$I_1 = 5/3\, V_1 - 5/6\, V_2 \quad (1)$$

$$I_2 = -5/6\, V_1 + 5/3\, V_2 \quad (2)$$

$$\frac{\cancel{5}}{6} V_2 = \frac{\cancel{5}}{3} V_1$$

$$\underline{V_1 = 0.5 V_2}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

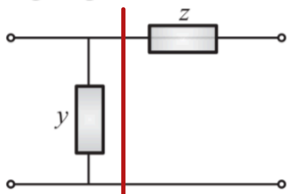
$$\therefore h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Subs in (2)

$$I_2 = -\frac{5}{6}\left(\frac{1}{2}V_2\right) + \frac{5}{3}V_2 = -\frac{5}{12}V_2 + \frac{20}{12}V_2 = \frac{15}{12}V_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{5}{4}$$

Which one of the following is the transmission matrix for the network shown in the figure given below?



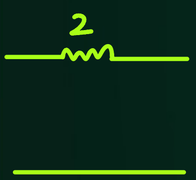
(A) $\begin{bmatrix} 1 & 1+yz \\ y & z \end{bmatrix}$ (B) $\begin{bmatrix} 1+yz & z \\ y & 1 \end{bmatrix}$

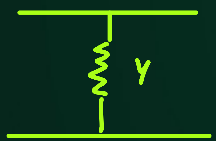
(C) $\begin{bmatrix} 1 & z \\ y & 1+yz \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1+yz \\ z & y \end{bmatrix}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

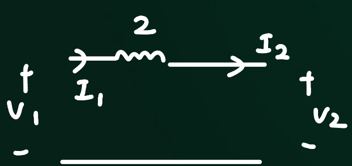
$$= \underline{\begin{bmatrix} 1 & 2 \\ Y & 1+Y2 \end{bmatrix}}$$

<u>ABCD</u>





$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$I_1 = I_2$ (1)

$V_1 - 2I_2 = V_2$

$V_1 = V_2 + 2I_2$ (2)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

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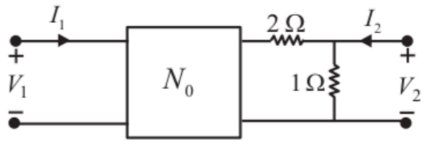
In the arrangement of figure given below

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 30 & 23 \\ 13 & 10 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

ABCD parameter of Network N_0 is



(A) $\begin{bmatrix} 7 & -9 \\ 3 & -4 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 9 \\ 3 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} -7 & 9 \\ 3 & -4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\begin{bmatrix} 30 & 23 \\ 13 & 10 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 23 \\ 13 & 10 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \underline{3a+b} & \underline{2a+b} \\ \underline{3c+d} & \underline{2c+d} \end{bmatrix}$$

$a = 30 - 23 = 7$ $b = 9$ $\begin{bmatrix} 7 & 9 \\ 3 & 4 \end{bmatrix}$ (B)

$c = 13 - 10 = 3$ $d = 4$