Chapter 3

DC Transients

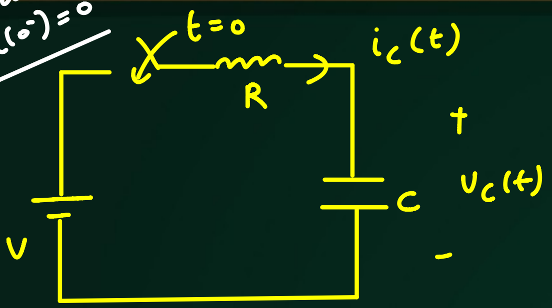
Lecture 2

Switching at $t=0$

$$v(t) = v(\infty) + \left(v(0^+) - v(\infty)\right) e^{-t/\tau} \quad ; \; t \geq 0$$

$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right) e^{-t/\tau} \quad ; \; t \geq 0$$

Assume $V_c(0^-) = 0$



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Nature of Response : Exponential

$V_c(t) \Rightarrow 0 \longrightarrow V$ ✓

$i_c(t) \Rightarrow \frac{V}{R} \rightarrow 0$ ✓

Note : $t = 0^-$: ONLY TAKE AWAY — $V_c(0^-)$ ✓ (OR) $I_L(0^-)$ ✓

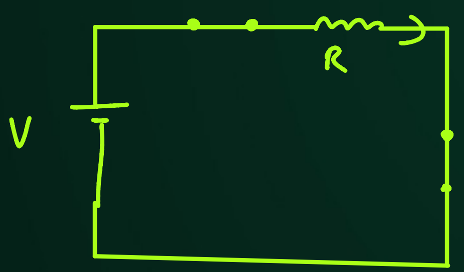
Exp Rise from 0
$A(1 - e^{-t/\tau})$

Exp fall to 0
$Be^{-t/\tau}$

Here $V_c(0^-) = 0V$

$t = 0^+$

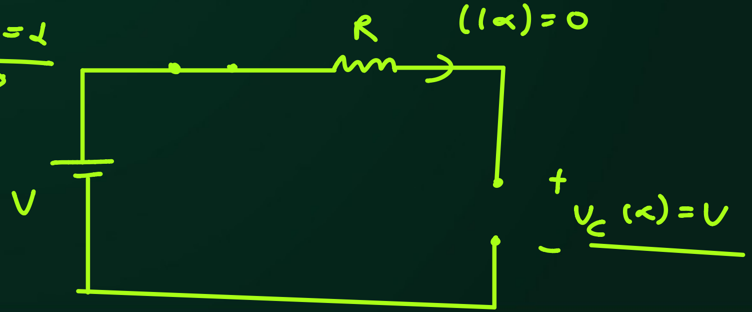
$i(0^+) = V/R$ ✓



$V_c(0^+) = 0V$ ✓

$t = \infty$: SS after switching

$t = \infty$



$$i_c(t) = i_c(\infty) + (i_c(0^+) - i_c(\infty))e^{-t/\tau}$$

$$= 0 + \left(\frac{V}{R} - 0\right)e^{-t/\tau}$$

$\tau = R_{eq} C$

Eq. R across C after SW

$\tau = RC$

$$\therefore \boxed{i_c(t) = \frac{V}{R} e^{-t/\tau} \; ; \quad t \geq 0 \qquad \tau = RC}$$

$$V_c(t) = V_c(\infty) + (V_c(0^+) - V_c(\infty))e^{-t/\tau}$$

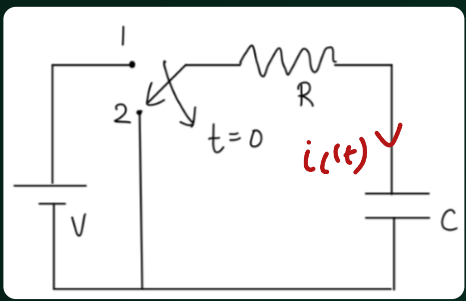
$$= V + (V - 0)e^{-t/\tau}$$

$$V_c(t) = V(1 - e^{-t/\tau}) \; ; \; t \geq 0$$

$\tau = RC$

Source Free Circuits

↳ After switching



Note : ⊛⊛ For source free circuits

$$v(t) = v(0^+)\,e^{-t/\tau}$$
$$i(t) = i(0^+)\,e^{-t/\tau}$$
$\Big\}\ t \geq 0$

$$v(\infty) = i(\infty) = 0$$

$$v(t) = \cancel{v(\infty)} + \left(v(0^+) - \cancel{v(\infty)}\right)e^{-t/\tau}$$

$t = 0^-$ → ss bfr sw → $C \to OC$

$$v_c(0^-) = V \quad ⊛⊛$$

$$\therefore\ v_c(0^+) = v_c(0^-) = V$$

$$v_c(t) = v_c(0^+)\,e^{-t/\tau}\ ;\ t \geq 0$$

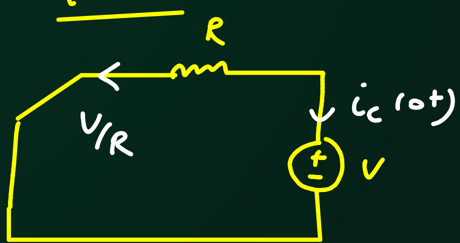
$$\boxed{v_c(t) = V e^{-t/\tau}}\ ;\ t \geq 0$$

$t = 0^+$

$$i_c(t) = i_c(0^+)\,e^{-t/\tau}\ ;\ t \geq 0$$

$$\therefore\ i_c(0^+) = -V/R$$

$t = 0^+$



$$\therefore\ i_C(t) = -V/R\, e^{-t/\tau}\ ;\ t \geq 0$$

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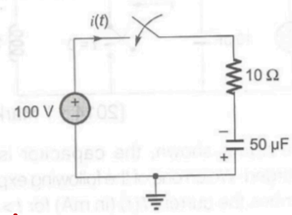
In the circuit shown below, the initial charge on the capacitor is 2.5 mC, with the voltage polarity as indicated. The switch is closed at time $t = 0$. The current $i(t)$ at a time t after the switch is closed is



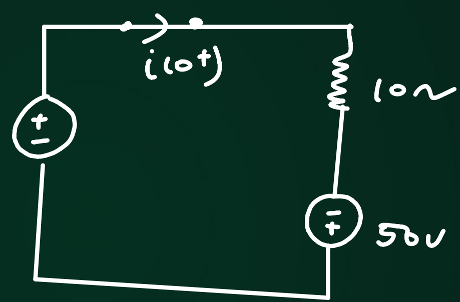
(a) $i(t) = 15 \exp(-2 \times 10^3 t)$ A
(b) $i(t) = 5 \exp(-2 \times 10^3 t)$ A
(c) $i(t) = 10 \exp(-2 \times 10^3 t)$ A
(d) $i(t) = -5 \exp(-2 \times 10^3 t)$ A

$t = 0^-$ $\quad q = CV \quad \therefore V = q/C$

$$\therefore V_c(0^-) = \frac{2.5\,mC}{50\,\mu F} = \frac{2500}{50}\,V = \underline{\underline{50V}}$$

$V_c(0^+) = V_c(0^-) = 50V$

$t = 0^+$



$$\therefore i(0^+) = \frac{100+50}{10} = \underline{15A}$$

$t = \infty$: SS after SW

Cap ⇒ OC $\quad \therefore i(\infty) = 0$

$\tau = R_{eq}C = (10)(50\mu) = 500\mu = \underline{0.5m}$

$$\therefore i(t) = \cancel{i(\infty)} + (i(0^+) - \cancel{i(\infty)})e^{-t/\tau} \;;\; t \geq 0$$

$$i(t) = i(0^+)e^{-t/\tau} \;;\; t \geq 0$$

$$i(t) = 15\,e^{-t/\tau} \;;\; t \geq 0$$

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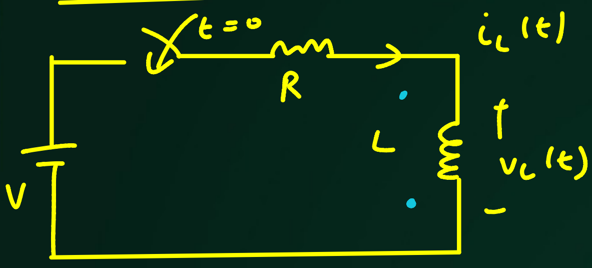
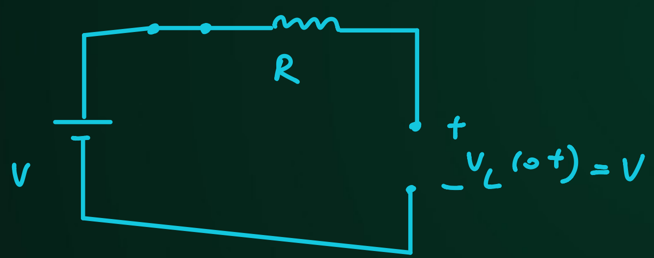
$i(t) = 15e^{-2000t} \; ; \; t \geq 0$

First order RL Circuit

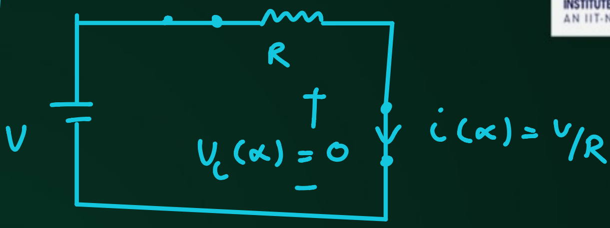


$t=0^-$ $\quad i_L(0^-) = 0\,A$

$t=0^+$ $\quad i_L(0^+) = i_L(0^-) = 0\,A$



$t=\infty$ → SS after SW



$i_L(t) \longrightarrow 0$ to V/R

$v_L(t) \longrightarrow V$ to 0

$$i_L(t) = \frac{V}{R}\left(1-e^{-t/\tau}\right) \; ; \; t \ge 0$$

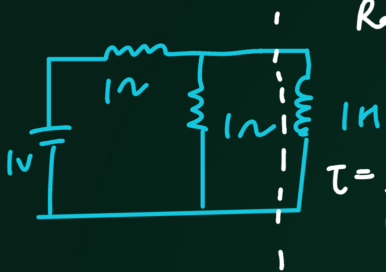
$$v_L(t) = V e^{-t/\tau} \; ; \; t \ge 0$$

$$\tau = \frac{L}{R_{eq}} = \frac{L}{R}$$

RL Circuit

Find Time Constant

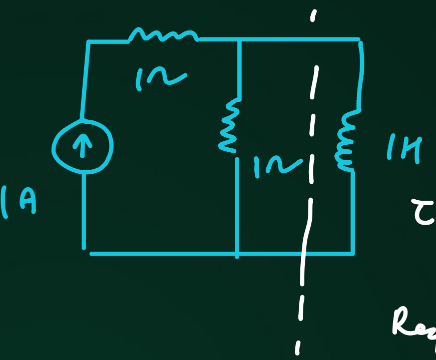
1)



$R_{eq} = 1\Omega \| 1\Omega$
$= 0.5\Omega$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{0.5} = 2\,\text{sec}$$

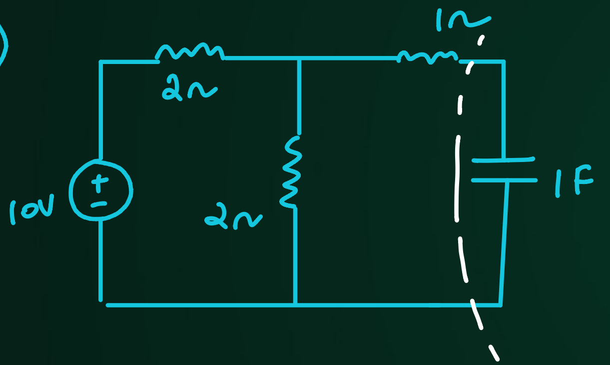
2)



$$\tau = \frac{L}{R_{eq}} = \frac{1}{1} = 1\,\text{sec}$$

$R_{eq} = 1\Omega$

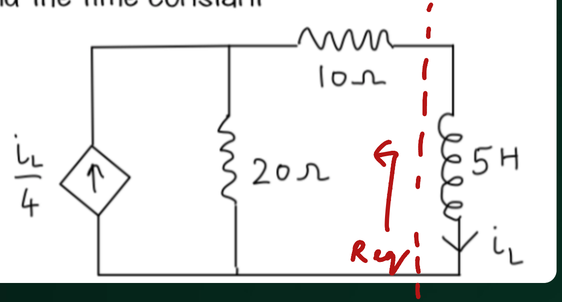
3)



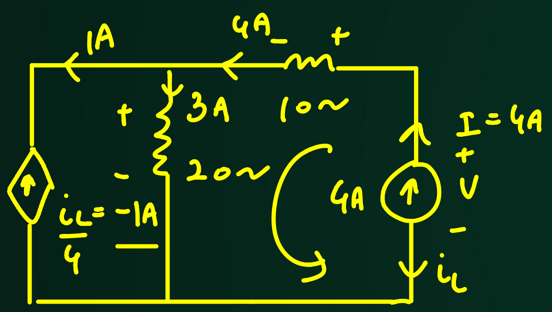
$$\tau = R_{eq}C = 2(1) = 2\,\text{sec}$$

$$R_{eq} = (2\|2) + 1 = 2$$

Q) Find the time constant



$$\tau = \frac{L}{R_{eq}} = \frac{5}{25} = 0.2\ sec$$

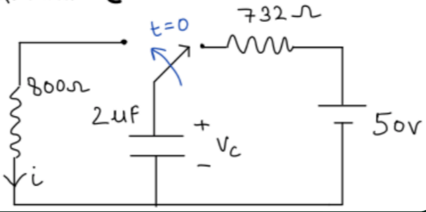


$$R_{eq} = \frac{V}{I} = \frac{V}{4} = \frac{100}{4} = 25\Omega$$

$$i_L = -4A$$

$$V = (4)(10) + (3)(20) = 100V$$

Q) Find $V_c(t)$ for $t \geq 0$



$V_c(t) = V_c(0^+)\, e^{-t/\tau}$; $t \geq 0$ (Source Free)

$V_c(0^-) = V_c(0^+)$

$t = 0^-$: $V_c(0^-)$: SS bfr sw

$\therefore V_c(0^-) = 50V$

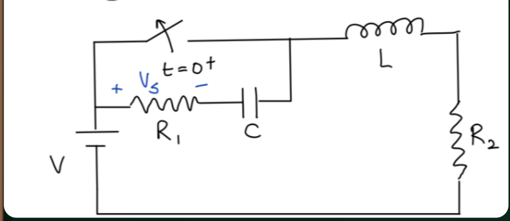
$V_c(t) = 50\, e^{-t/\tau}$; $t \geq 0$

$\tau = R_{eq} C$

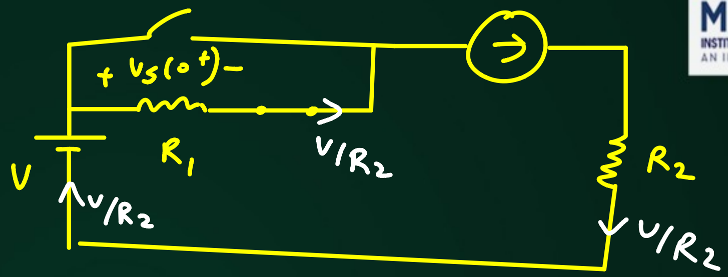
$= (0.8k)(2\mu) = 1.6mS$

$V_c(t) = 50\, e^{-\frac{1000t}{1.6}}$; $t \geq 0$

Q) Find $V_s(0^+)$



$t = 0^+$

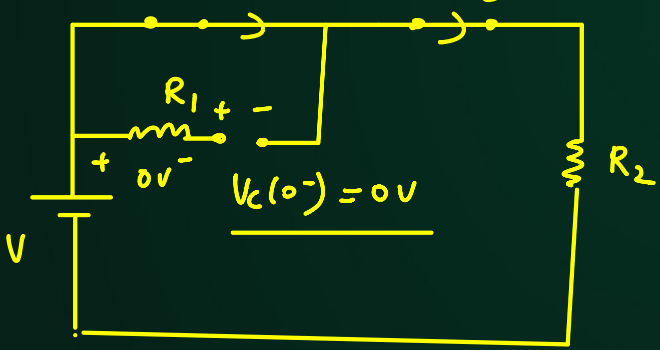


$t=0^-$ < $V_c(0^-)$, $I_L(0^-)$

$t=0^-$: SS bfr sw

$i_L(0^-)$

$i_L(0^-) = V/R_2$



$V_c(0^-) = 0\,V$

$$V_S(0^+) = \left(\frac{V}{R_2}\right)(R_1)$$