Mutual Information

Mutual Information

Mutual information is a measure of the mutual dependence of 2 variables.

In other words, the mutual information quantifies the "amount of information" gained about one variable through observing the other variable.

Measures linear and non-linear associations.

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\,p_Y(y)}\right)$$

- **I** is the mutual information
- p(X,Y) is the probability of X and Y occurring together
- p(X) is the probability of X
- p(Y) is the probability of Y

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log \left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\, p_Y(y)} \right)$$

- If X and Y are independent ➔ $p(X,Y) = p(X)p(Y)$ and then MI = 0
- If X is deterministic of Y ➔ MI tends to infinity.

Mutual Information in ML

p(X) is the probability of X

p(female) = 4 / 8 = 0.5

p(male) = 4 / 8 = 0.5

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information in ML

p(X) is the probability of X

p(female) = 4 / 8 = 0.5

p(male) = 4 / 8 = 0.5

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information in ML

p(Y) is the probability of Y

p(survived) = 4 / 8 = 0.5

p(died) = 4 / 8 = 0.5

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information in ML

p(Y) is the probability of Y

p(survived) = 4 / 8 = 0.5

p(died) = 4 / 8 = 0.5

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information in ML

p(X,Y) the joint probability

p(female and survived) = 3 / 8 = 0.375

p(female and died) = 1 / 8 = 0.152

p(male and survived) = 1 / 8 = 0.125

p(male and died) = 3 / 8 = 0.375

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information in ML

p(X,Y) the joint probability

p(female and survived) = 3 / 8 = 0.375

p(female and died) = 1 / 8 = 0.152

p(male and survived) = 1 / 8 = 0.125

p(male and died) = 3 / 8 = 0.375

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information in ML

p(X,Y) the joint probability

p(female and survived) = 3 / 8 = 0.375

p(female and died) = 1 / 8 = 0.152

p(male and survived) = 1 / 8 = 0.125

p(male and died) = 3 / 8 = 0.375

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	Yes
Male	Yes
Male	No
Male	No
Male	No

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\, p_Y(y)}\right)$$

$$p(\text{female, survived}) \times \log\left(\frac{p(\text{female, survived})}{p(\text{female}) \times p(\text{survived})}\right) +$$

$$p(\text{female, died}) \times \log\left(\frac{p(\text{female, died})}{p(\text{female}) \times p(\text{died})}\right) +$$

$$p(\text{male, survived}) \times \log\left(\frac{p(\text{male, survived})}{p(\text{male}) \times p(\text{survived})}\right) +$$

$$p(\text{male, died}) \times \log\left(\frac{p(\text{male, died})}{p(\text{male}) \times p(\text{died})}\right)$$

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log \left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\, p_Y(y)} \right)$$

p(female, survived) x log($\frac{\text{p(female, survived)}}{\text{p(female)} \times \text{p(survived)}}$) +

p(female, died) x log($\frac{\text{p(female, died)}}{\text{p(female)} \times \text{p(died)}}$) +

p(male, survived) x log($\frac{\text{p(male, survived)}}{\text{p(male)} \times \text{p(survived)}}$) +

p(male, died) x log($\frac{\text{p(male, died)}}{\text{p(male)} \times \text{p(died)}}$)

.375 x log($\frac{.375}{.5 \times .5}$) +

.125 x log($\frac{.125}{.5 \times .5}$) +

.125 x log($\frac{0.125}{.5 \times .5}$) +

.29 x log($\frac{.375}{.5 \times .5}$)

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\,p_Y(y)}\right)$$

$$p(\text{female, survived}) \times \log\left(\frac{p(\text{female, survived})}{p(\text{female}) \times p(\text{survived})}\right) +$$

$$p(\text{female, died}) \times \log\left(\frac{p(\text{female, died})}{p(\text{female}) \times p(\text{died})}\right) +$$

$$p(\text{male, survived}) \times \log\left(\frac{p(\text{male, survived})}{p(\text{male}) \times p(\text{survived})}\right) +$$

$$p(\text{male, died}) \times \log\left(\frac{p(\text{male, died})}{p(\text{male}) \times p(\text{died})}\right)$$

MI = 0.13

Mutual Information in ML

No association

p(X,Y) the joint probability

p(female and survived) = 2 / 8 = 0.25

p(female and died) = 2 / 8 = 0.25

p(male and survived) = 2 / 8 = 0.25

p(male and died) = 2 / 8 = 0.25

Gender	Survived
Female	Yes
Female	No
Female	Yes
Female	No
Male	Yes
Male	No
Male	Yes
Male	No

Mutual Information

$$I(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\, p_Y(y)}\right)$$

$$p(\text{female, survived}) \times \log\left(\frac{p(\text{female, survived})}{p(\text{female}) \times p(\text{survived})}\right) + \qquad .25 \times \log\left(\frac{.25}{.5 \times .5}\right) +$$

$$p(\text{female, died}) \times \log\left(\frac{p(\text{female, died})}{p(\text{female}) \times p(\text{died})}\right) + \qquad .25 \times \log\left(\frac{.25}{.5 \times .5}\right) +$$

$$p(\text{male, survived}) \times \log\left(\frac{p(\text{male, survived})}{p(\text{male}) \times p(\text{survived})}\right) + \qquad .25 \times \log\left(\frac{.25}{.5 \times .5}\right) +$$

$$p(\text{male, died}) \times \log\left(\frac{p(\text{male, died})}{p(\text{male}) \times p(\text{died})}\right) \qquad .25 \times \log\left(\frac{.25}{.5 \times .5}\right)$$

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\, p_Y(y)}\right)$$

$$p(\text{female, survived}) \times \log\left(\frac{p(\text{female, survived})}{p(\text{female}) \times p(\text{survived})}\right) +$$

$$p(\text{female, died}) \times \log\left(\frac{p(\text{female, died})}{p(\text{female}) \times p(\text{died})}\right) +$$

$$p(\text{male, survived}) \times \log\left(\frac{p(\text{male, survived})}{p(\text{male}) \times p(\text{survived})}\right) +$$

$$p(\text{male, died}) \times \log\left(\frac{p(\text{male, died})}{p(\text{male}) \times p(\text{died})}\right)$$

MI = 0.0

Mutual Information in ML

Deterministic

p(X,Y) the joint probability

p(female and survived) = 4 / 8 = 0.5

p(female and died) = 0 / 8 = 0

p(male and survived) = 0 / 8 = 0

p(male and died) = 4 / 8 = 0.5

Gender	Survived
Female	Yes
Female	Yes
Female	Yes
Female	Yes
Male	No
Male	No
Male	No
Male	No

Mutual Information

$$I(X;Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p_{(X,Y)}(x,y) \log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\, p_Y(y)}\right)$$

$$p(\text{female, survived}) \times \log\left(\frac{p(\text{female, survived})}{p(\text{female}) \times p(\text{survived})}\right) +$$

$$p(\text{female, died}) \times \log\left(\frac{p(\text{female, died})}{p(\text{female}) \times p(\text{died})}\right) +$$

$$p(\text{male, survived}) \times \log\left(\frac{p(\text{male, survived})}{p(\text{male}) \times p(\text{survived})}\right) +$$

$$p(\text{male, died}) \times \log\left(\frac{p(\text{male, died})}{p(\text{male}) \times p(\text{died})}\right)$$

$$.5 \times \log\left(\frac{.5}{.5 \times .5}\right) +$$

$$0 \times \log\left(\frac{0}{.5 \times .5}\right) +$$

$$0 \times \log\left(\frac{0}{.5 \times .5}\right) +$$

$$.5 \times \log\left(\frac{.5}{.5 \times .5}\right)$$

Mutual Information

$$\mathrm{I}(X;Y) = \sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p_{(X,Y)}(x,y)\log\left(\frac{p_{(X,Y)}(x,y)}{p_X(x)\,p_Y(y)}\right)$$

$$p(\text{female, survived}) \times \log\left(\frac{p(\text{female, survived})}{p(\text{female}) \times p(\text{survived})}\right) +$$

$$p(\text{female, died}) \times \log\left(\frac{p(\text{female, died})}{p(\text{female}) \times p(\text{died})}\right) +$$

$$p(\text{male, survived}) \times \log\left(\frac{p(\text{male, survived})}{p(\text{male}) \times p(\text{survived})}\right) +$$

$$p(\text{male, died}) \times \log\left(\frac{p(\text{male, died})}{p(\text{male}) \times p(\text{died})}\right)$$

MI = towards infinite

THANK YOU

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