in the region of the geometrical shadow. The intensity distribution due to Fresnel's diffraction at a straight edge is given in Fig. 9.17 on page 429.

9.22 FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

To obtain a Fraunhofer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

In Fig. 9.33, S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a. XY is the incident spherical wavefront. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. The screen is perpendicular to the

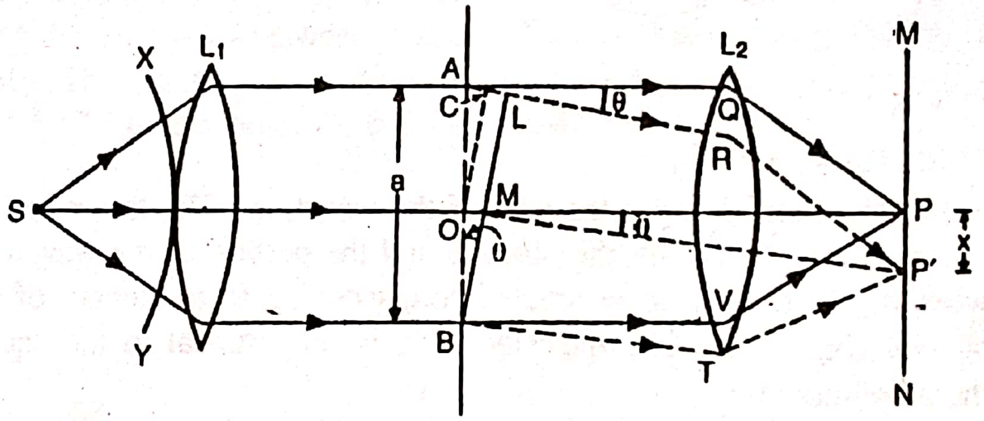

Fig. 9.33

plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wavefront is incident on the slit AB and each point on this wavefront is a source of secondary disturbance. The secondary waves travelling in the direction parallel to OP *viz.* AQ and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the waterfront travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

Now, consider the secondary waves travelling in the direction AR, inclined at an angle θ to the direction OP. All the secondary wave travelling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Draw OC and BL perpendicular to AR.

Then, in the ΔABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or $$AL = a\sin\theta$$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity. The whole wavefront can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\frac{\lambda}{2}$. Similarly for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\frac{\lambda}{2}$. Thus, destructive interference takes place and the point P' will be of minimum intensity. If the direction of the secondary waves is such that $AL = 2\lambda$, then also the point where they meet the screen will be of minimum intensity. This is so, because the secondary waves from the corresponding points of the lower half, differ in path by $\frac{\lambda}{2}$ and this again gives the position of minimum intensity. In general

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

where θ_n gives the direction of the n th minimum. Here n is an integer. If, however, the path difference is odd multiples of $\frac{\lambda}{2}$, the directions of the secondary maxima can be obtained. In this case,

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

or $$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$$

where $$n = 1, 2, 3 \text{ etc.}$$

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen is given in Fig. 9.34.

P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B

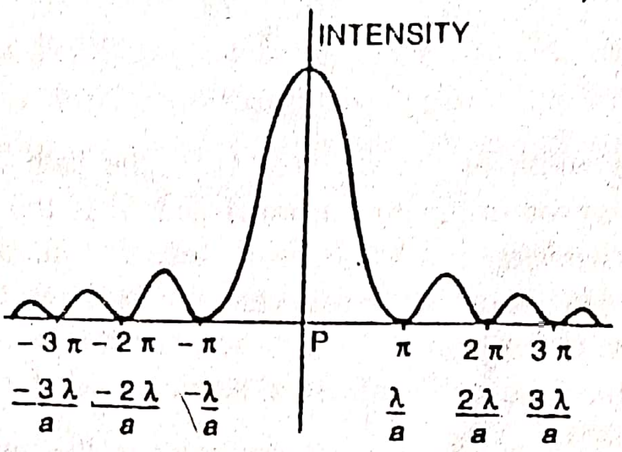

Fig. 9.34

is λ, 2λ etc., correspond to the positions of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards.

If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then

$$\sin\theta = \frac{x}{f} \quad ...(i)$$

where f is the focal length of the lens L_2

But, $$\sin\theta = \frac{\lambda}{a} \quad ...(ii)$$

$\therefore$ $$\frac{x}{f} = \frac{\lambda}{a}$$

or $$x = \frac{f\lambda}{a}$$

where x is the distance of the secondary minimum from the point P. Thus, the width of the central maximum = $2x$.

or $$2x = \frac{2f\lambda}{a} \quad ...(iii)$$

The width of the central maximum is proportional to λ, the wavelength of light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of the central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction bands are coloured. From equation (*ii*), if the width a of the slit is large, $\sin\theta$ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit, a is small and hence θ is large. This results a distinct diffraction maxima and minima on both the sides of P.

Example 9.9. *Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.*

Here $$\sin\theta = \frac{\lambda}{a}$$

where θ is half angular width of the central maximum.

$$a = 12 \times 10^{-5}\text{ cm}, \lambda = 6000\text{ Å} = 6 \times 10^{-5}\text{ cm}.$$

$$\therefore \quad \sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50$$

or $$\theta = 30^\circ$$

Example 9.10. *In Fraunhofer diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maximum, find the wavelength of light.* [*Delhi (Sub) 1977*]

In the case of Fraunhofer diffraction at a narrow rectangular aperture,

$$a\sin\theta = n\lambda$$

$$n = 1$$

$$\therefore \quad a\sin\theta = \lambda$$

$$\sin\theta = \frac{x}{D}$$

$$\therefore \quad \frac{ax}{D} = \lambda$$

$$\lambda = \frac{ax}{D}$$

Here

$$a = 0.2\text{ mm} = 0.02\text{ cm}$$

$$x = 5\text{ mm} = 0.5\text{ cm}$$

$$D = 2\text{ m} = 200\text{ cm}$$

$$\therefore \quad \lambda = \frac{0.02 \times 0.5}{200}$$

$$\lambda = 5 \times 10^{-5}\text{ cm}$$

$$\lambda = \mathbf{5000\text{ Å}}$$

Example 9.11. *Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.*

The first dark fringe is on either side of the central bright fringe.

Here $$n = \pm 1,\ D = 2\text{ m}$$

$$\lambda = 6000\text{ Å} = 6 \times 10^{-7}\text{ m}$$

$$\sin\theta = \frac{x}{D}$$

$$a = 0.30\text{ mm} = 3 \times 10^{-4}\text{ m}$$

$$a\sin\theta = n\lambda$$

$$\frac{ax}{D} = n\lambda$$

(a) $$x = \frac{n\lambda D}{a}$$

$$x = \pm\left[\frac{1 \times 6 \times 10^{-7} \times 2}{3 \times 10^{-4}}\right]$$

$$x = \pm 4 \times 10^{-3}\text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe,

$$y = 2x$$

$$= 2 \times 4 \times 10^{-3}$$

$$= 8 \times 10^{-3}\text{ m}$$

$$= 8\text{ mm}$$

Example 9.12. *A single slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2 m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light used.*

(*IAS*, 1990)

In the case of Fraunhofer diffraction at a narrow rectangular slit,

$$a\sin\theta = n\lambda$$

Here θ gives the directions of the minimum

$$n = 2$$

$$\lambda = ?$$

$$a = 0.14 \text{ mm} = 0.14 \times 10^{-3} \text{ m}$$

$$D = 2 \text{ m}$$

$$x = 1.6 \text{ cm} = 1.6 \times 10^{-2} \text{ m}$$

$$\sin\theta = \frac{x}{D} = \frac{n\lambda}{a}$$

$$\therefore \quad \lambda = \frac{xa}{nD}$$

$$= \frac{1.6 \times 10^{-2} \times 0.14 \times 10^{-3}}{2 \times 2}$$

$$= \mathbf{5.6 \times 10^{-7} \text{ m}}$$

$$= \mathbf{5600 \text{ Å}}$$

Example 9.13. *A screen is placed 2 m away from a narrow slit which is illuminated with light of wavelength 6000 Å. If the first minimum lies 5 mm on either side of the central maximum, calculate the slit width.*

(*Delhi, 1990*)

In the case of Fraunhofer diffraction at a narrow slit,

$$a \sin\theta = n\lambda$$

$$\sin\theta = \frac{x}{D}$$

$$\therefore \quad \frac{ax}{D} = n\lambda$$

Here, width of the slit = a = ?

$$x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

$$D = 2 \text{ m}$$

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$$

$$n = 1$$

$$a = \left(\frac{n\lambda D}{x}\right)$$

$$a = \left(\frac{1 \times 6 \times 10^{-7} \times 2}{5 \times 10^{-3}}\right)$$

$$a = \mathbf{2.4 \times 10^{-4} \text{ m}}$$

$$a = \mathbf{0.24 \text{ mm}}$$

Example 9.14. *Find the angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.*

(*Kanpur, 1990*)

Here $$\sin\theta = \frac{\lambda}{a}$$

where θ is the half angular width of the central maximum

$$a = 12 \times 10^{-5} \text{ cm} = 12 \times 10^{-7} \text{ m}$$

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$$

$$\sin\theta = \frac{6 \times 10^{-7}}{12 \times 10^{-7}} = 0.5$$

$$\theta = \mathbf{30^\circ}$$

Angular width of the central maximum,

$$2\theta = \mathbf{60^\circ}$$

Example 9.15. *Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wavelength = 4890 Å.* [*Kanpur, 1991*]

For minimum intensity

$$a \sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{x_1}{f}, \quad n = 1$$

$$\frac{x_1}{f} = \frac{\lambda}{a}$$

Here $$\lambda = 4890 \text{ Å} = 4890 \times 10^{-10} \text{ m}$$

$$a = 0.5 \text{ cm} = 5 \times 10^{-3} \text{ m}$$

$$f = 40 \text{ cm} = 0.4 \text{ m}$$

$$x_1 = \frac{f\lambda}{a}$$

$$x_1 = \frac{0.4 \times 4890 \times 10^{-10}}{5 \times 10^{-3}}$$

$$x_1 = \mathbf{3.912 \times 10^{-5} \text{ m}}$$

For secondary maximum

$$a \sin\theta_n = \frac{(2n+1)\lambda}{2}$$

For the first secondary maximum

$$n = 1$$

$$\sin \theta_n = \frac{x_2}{f}$$

$$\frac{x_2}{f} = \frac{3\lambda}{2a}$$

$$x_2 = \frac{3\lambda f}{2a}$$

$$x_2 = \frac{3 \times 4890 \times 10^{-10} \times 0.4}{2 \times 5 \times 10^{-3}}$$

$$x_2 = \mathbf{5.868 \times 10^{-5}\ m}$$

Difference,
$$x_2 - x_1 = 5.868 \times 10^{-5} - 3.912 \times 10^{-5}$$

$$= \mathbf{1.956 \times 10^{-5}\ m}$$

$$= \mathbf{1.596 \times 10^{-2}\ mm}$$

9.23 INTENSITY DISTRIBUTION IN THE DIFFRACTION PATTERN DUE TO A SINGLE SLIT

The intensity variation in the diffraction pattern due to a single slit can be investigated as follows. The incident plane wavefront on the slit AB (Fig. 9.33) can be imagined to be divided into a large number of infinitesimally small strips. The path difference between the secondary waves emanating from the extreme points A and B is $a \sin \theta$ where a is the width of the slit and $\angle ABL = \theta$. For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can be taken to be the same. As one considers the secondary waves in a direction inclined

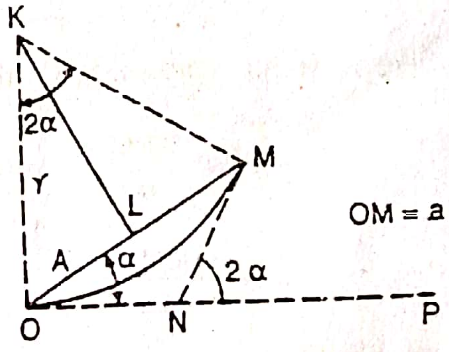

Fig. 9.35

at an angle θ from the point B upwards, the path difference changes and hence the phase difference also increases. Let α be the phase difference between the secondary waves from the points B and A of the slit (Fig. 9.27). As the wavefront is divided into a large number of strips,the resultant amplitude due to all the individual small strips can be obtained by the vector polygon method. Here, the amplitudes are small and the phase difference increases by infinetesimally small amounts from strip to strip. Thus, the vibration polygon coincides with the circular arc OM (Fig. 9.35). OP gives the direction of the initial vector and NM the direction of the final vector due to the secondary waves from A. K is the centre of the circular arc.

$$\angle MNP = 2\alpha$$

$$\therefore \quad \angle OKM = 2\alpha$$

In the Δ OKL

$$\sin\alpha = \frac{OL}{r} \;;\; OL = r\sin\alpha$$

where r is the radius of the circular arc

$$\therefore \quad \text{Chord } OM = 2\,OL = 2r\sin\alpha \qquad \ldots (i)$$

The length of the arc OM is proportional to the width of the slit.

$\therefore$ Length of the arc $OM = Ka$

where K is a constant and a is the width of the slit.

Also,
$$2\alpha = \frac{\text{Arc } OM}{\text{radius}} = \frac{Ka}{r}$$

or
$$2r = \frac{Ka}{\alpha} \qquad \ldots (ii)$$

Substituting this value of 2r in equation (i)

$$\text{Chord } OM = \frac{Ka}{\alpha}.\sin\alpha$$

But, $OM = A$ where A is the amplitude of the resultant.

$$\therefore \quad A = (Ka)\frac{\sin\alpha}{\alpha}$$

$$A = A_0\frac{\sin\alpha}{\alpha} \qquad \ldots(iii)$$

Thus, the resultant amplitude of vibration at a point on the screen is given by $A_0\dfrac{\sin\alpha}{\alpha}$ and the intensity I at the point is given by

$$I = A^2 = A_0^2 \frac{\sin^2 \alpha}{\alpha^2} = I_0 . \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad ...(iv)$$

The intensity at any point on the screen is proportional to $\left(\frac{\sin \alpha}{\alpha} \right)^2$. A phase difference of 2π corresponds to a path difference of λ. Therefore a phase difference of 2α is given by

$$2 \alpha = \frac{2 \pi}{\lambda} . a \sin \theta \quad ...(iv)$$

where $a \sin \theta$ is the path difference between the secondary waves from A and B (Fig. 9.35).

$$\alpha = \frac{\pi}{\lambda} . a \sin \theta \quad ...(v)$$

Thus, the value of α depends on the angle of diffraction θ. The value of $\frac{\sin^2 \alpha}{\alpha^2}$ for different values of θ gives the intensity at the point under consideration. Fig. 9.34 represents the intensity distribution. It is a graph of $\frac{\sin^2 \alpha}{\alpha^2}$ (along the Y-axis), as a function of α or $\sin \theta$ (along the X-axis).

9.24 FRAUNHOFER DIFFRACTION AT A SINGLE SLIT (CALCULUS METHOD)

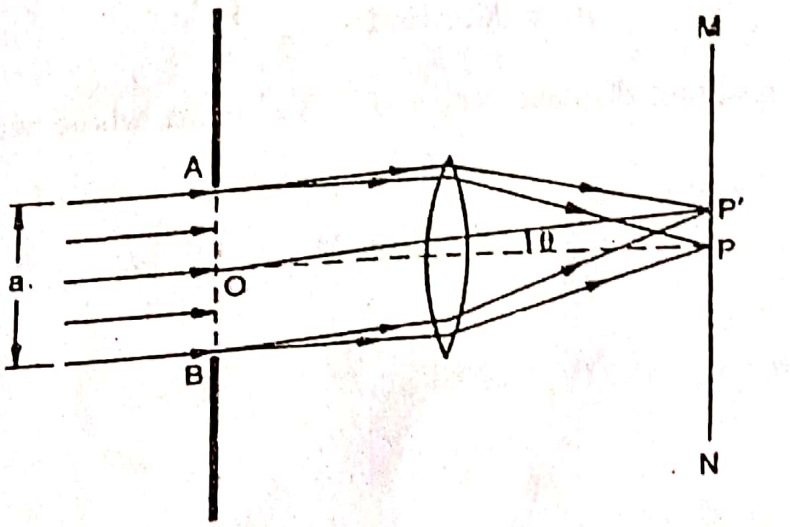

Fig. 9.36

Let a monochromatic parallel beam of light be incident on the slit AB of width a. The secondary waves travelling in the same direction as