# Interference.

**Monocromatic sources of light:** Two sources of light are said to be monocromatic only when they emit light waves of same wavelength, i.e; same frequency, amplitude.

**Coherent sources:** Two sources are said to be coherent if there always exists a constant phase difference between the waves emitted by these sources.

In practice, it is not possible that two independent sources which are coherent. But, for experimental purpose, two virtual sources formed from a single source which act as coherent source.

**Slit:** A rectangular hole which width is more smaller than its length is called a slit.

**Interference:** When two waves of light of equal wavelength proceed in the same direction from the very narrow sources and superimpose at a point in a medium, at the instant of superposition according as the waves meet at the point in the same or opposite phase. This phenomenon is called interference.

When two waves meet in the same phase they produce brightness and in the opposite phase they produce darkness.

For example- a soap bubble with different colours like that of a rainbow, light reflecting from oil floating on water.

Interference
- Constructive interference.
- Destructive interference.

**Constructive interference:** When two light waves of same frequency meet at a point in the same phase with each other, then the interference is called constructive interference.

At this point intensity is maximum, this point is called bright point.

**Destructive interference:** When two light waves of same frequency meet at a point in the opposite phase with each other, then the interference is called destructive interference.
At this point intensity is minimum, this point is called dark point.

**Conditions of interference:**

When waves come together they can interfere constructively or destructively. To set up a stable and clear interference pattern, these conditions must be met:-

1. The two sources of wave should be coherent, which means they emit identical waves with a constant phase diffence.
2. The amplitude must be same. (and frequency)
3. The fringe wide should be as large as possible
4. The original source must be monocromatic. They should be of a single wavelength.
5. The propagation direction should be same.

**Principle of superposition:** The principle of superposition states that, when two waves of same kind meet at a point in space, the resultant displacement of that point is the vector sum of the displacements that two wave would separately produce at that point.

Q. What are coherent sources? Why are coherent sources required to produce interference of light?

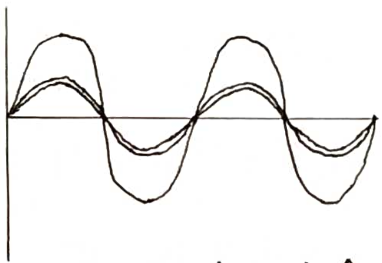
Two sources are said to be coherent if they emit light wave of the same wavelength, frequency, amplitude and a constant phase difference between each other.

Coherent sources are required to produce sustained interference pattern. Because then only we will have constant maximum and minimum intensity of light on screen, otherwise there will be a continuous fluctuation of intensity on screen, hence the intensity pattern will be lost.

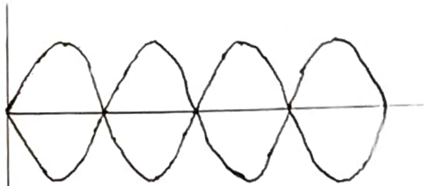
For example: A thin film of oil spread on water shows beautiful colours due to interference of light.

## Theory of interference:

Interference is the combination of two or more waves to form a composite wave, based on such Principle of superposition. The idea of the superposition Principle is shown in Fig.1



Constructive interference



Destructive interference.

Let us consider two waves,

$$E_1(x,t) = E_{0_1} \sin\{\omega t - (kx_1 + \varphi_1)\}$$
$$E_2(x,t) = E_{0_2} \sin\{\omega t - (kx_2 + \varphi_2)\}$$

The Principle of superposition of the two waves, the resultant wave is given by,

$$E = E_1(x,t) + E_2(x,t)$$

The interference is constructive if the amplitude of $E(x,t)$ is greater than the individual ones and the interference is destructive if the amplitude of $E(x,t)$ is smaller than the individual ones.

Again we know the wave equation in the form,

$$E(x,t) = E_0 \sin\{\omega t - (kx + \varphi)\} \qquad (1)$$

Where, $E_0$ is the amplitude of harmonic wave disturbance Propagated along the Positive axis.

Let,

$$\alpha(x,\varphi) = -(kx + \varphi)$$

From equation (1) we can write,

$$E(x,t) = E_0 \sin\{\omega t + \alpha(x,\varphi)\} \qquad (2)$$

Now we can write two wave equations such as,

$$E_1 = E_{01}\sin(\omega t + \alpha_1)$$
$$E_2 = E_{02}\sin(\omega t + \alpha_2)$$

Frequency and speed of the two waves have same and overlaping in space. The resultant disturbance is the linear superposition of these two waves. So,

$$E = E_1 + E_2$$
$$= E_{01}\sin(\omega t + \alpha_1) + E_{02}\sin(\omega t + \alpha_2)$$
$$= E_{01}(\cancel{\sin\omega t}\sin\omega t\cos\alpha_1 + \cos\omega t\sin\alpha_1) + E_{02}(\sin\omega t\cos\alpha_2 + \cos\omega t\sin\alpha_2).$$
$$= (E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2)\sin\omega t + (E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2)\cos\omega t.$$

Since the bracket quantities are constant in time. So,

Let, $E_0\cos\alpha = E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2$ ———— (3)

$E_0\sin\alpha = E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2$ ———— (4)

Squaring and adding equations (3) and (4) we get,

$$E_0^2\cos^2\alpha + E_0^2\sin^2\alpha = \cancel{E_{01}^2(\cos^2\alpha_1+\sin^2\alpha_1)+E_{02}^2(\cos^2\alpha_2+\sin^2\alpha_2)}$$
$$E_{01}^2\cos^2\alpha_1 + E_{02}^2\cos^2\alpha_2 + 2E_{01}E_{02}\cos\alpha_1\cos\alpha_2 + E_{01}^2\sin^2\alpha_1 + E_{02}^2\sin^2\alpha_2 + 2E_{01}E_{02}\sin\alpha_1 \cdot \sin\alpha_2$$

$$\Rightarrow E_0^2(\cos^2\alpha + \sin^2\alpha) = E_{01}^2(\cos^2\alpha_1 + \sin^2\alpha_1) + E_{02}^2(\cos^2\alpha_2 + \sin^2\alpha_2) + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$

$$\therefore E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \quad \text{———— (5)}$$

And dividing equation (4) by (3), we get,

$$\tan\alpha = \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2} \quad \text{———— (6)}$$

The equation (5) and (6) provided that they are satisfied for $E_0$ and $\alpha$.
The total disturbance,

$$E = E_0\cos\alpha\sin\omega t + E_0\sin\alpha\cos\omega t$$
$$\Rightarrow E = E_0\sin(\omega t + \alpha) \quad \text{———— (7)}$$

The equation (7) is the resultant wave of two waves.

This is the single disturbance resultant from the superposition of the two sinusoidal waves $E_1$ and $E_2$. The Composite wave (7) is the harmonic wave of the same frequency as the constituents although its amplitude and Phase are different.

From equ. (7) the term $2E_{01}E_{02}\cos(\alpha_2-\alpha_1)$ is Known as interference term and the difference in phase between two interfearing waves $E_1$ and $E_2$ is $(\alpha_2-\alpha_1)$.

Let, phase difference, $\delta = \alpha_2-\alpha_1$.

When, $\delta = 0, \pm 2\pi, \pm 4\pi, \dots\dots, 2n\pi$;

the resultant amplitude is maximum.

When, $\delta = \pm\pi, \pm 3\pi, \dots\dots, (2n+1)\pi$;

the resultant amplitude is minimum.

The waves are said to be inphase when crest overlaps crest. The waves are said to be outphase when trough overlaps crest.

Now, the Phase difference,

$$\delta = (kx_1+\varphi_1)-(kx_2+\varphi_2)$$

$$\Rightarrow \delta = k(x_1-x_2)+(\varphi_1-\varphi_2)$$

$$\Rightarrow \delta = \frac{2\pi}{\lambda}(x_1-x_2)+(\varphi_1-\varphi_2) \quad \text{———} \quad (8)$$

Where, $k=\frac{2\pi}{\lambda}$ is wave vector.

Here, $x_1$ and $x_2$ are the distance from the source of the two waves to the Point of observation and $\lambda$ is the wavelength of the Provaiding.

If the waves are inphase $\varphi_1=\varphi_2$;

$$\therefore \delta = \frac{2\pi}{\lambda}(x_1-x_2) \quad \text{———} \quad (9)$$

We know,

$$n = \frac{c}{v} = \frac{\lambda_0}{\lambda}$$

$$\Rightarrow \frac{n}{\lambda_0} = \frac{1}{\lambda}$$

Now, from equ. (9) we get,

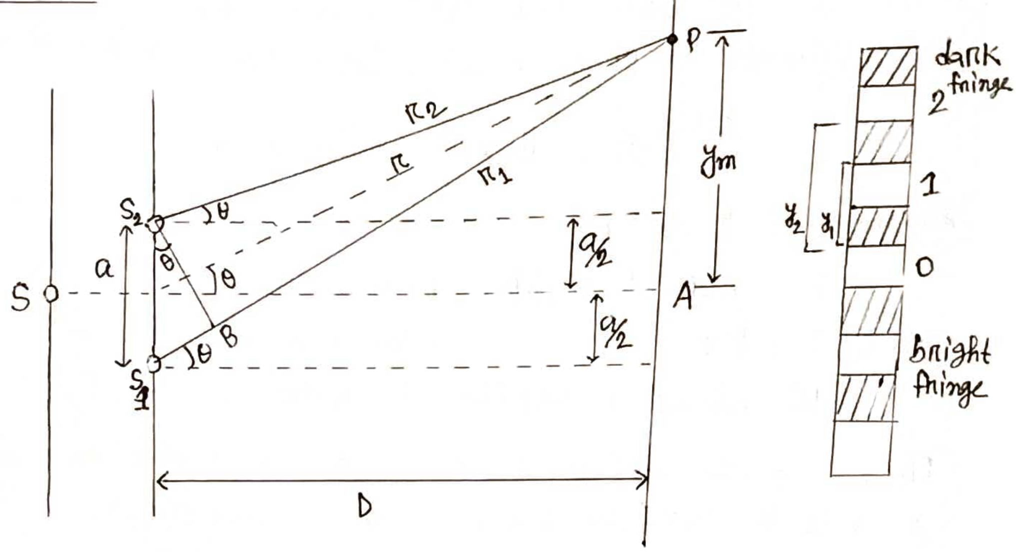
$$\delta = \frac{2\pi}{\lambda_0}n(x_1-x_2).$$

The quantity $n(x_1-x_2)$ is known as optical path difference and representing OPD or $\Delta$.

$$\therefore \delta = K_0 \Delta$$

Where, $K_0$ is the wave vector in vaccum.

## Young's experiment:




Let us consider a monochromatic source of light S emitting waves of wavelength $\lambda$ and two narrow pin holes $S_1$ and $S_2$ which is shown in fig. and $S_1$ and $S_2$ are equidistance from the source S. Here, $S_1$ and $S_2$ act as a coherent sources separated by the distance a. Let a screen be placed at a distance D from the coherent source. The Point A on the screen is equidistance from $S_1$ and $S_2$.

The light rays comes at the point A from the coherent source $S_1$ and $S_2$. Here, $S_1A = S_2A$, therefore, the path difference between the two wave which are comes from $S_1$ and $S_2$ is zero. Thus the point A has the maximum intensity is known as central maxima or the central fringe.

Consider a another point P at a distance $y_m$ from the central fringe A. The waves reached at the point P from the coherent sources $S_1$ and $S_2$. So, the path difference between the rays along $S_1P$ and $S_2P$ is;

$$S_1B = S_1P - S_2P$$

$$\Rightarrow S_1B = r_1 - r_2 \quad \text{——— (1)}$$

$$\sin\theta = \frac{S_1B}{a}$$
$$\Rightarrow S_1B = a\sin\theta.$$

From the fig. we can write,

$$S_1B = a\sin\theta \quad \text{———} (2)$$

From equ (1) and (2) we have,

$$a\sin\theta = r_1 - r_2 \quad \text{———} (3)$$

Now,

$$r_1^2 = D^2 + \left(y_m + \frac{a}{2}\right)^2$$
$$r_2^2 = D^2 + \left(y_m - a/2\right)^2$$

Using these two equations, we can write

$$\therefore r_1^2 - r_2^2 = \left(y_m + a/2\right)^2 - \left(y_m - a/2\right)^2$$
$$\Rightarrow (r_1 - r_2)(r_1 + r_2) = \left(y_m + a/2 + y_m - a/2\right)\left(y_m + a/2 - y_m + a/2\right)$$
$$= 2y_m \cdot 2 \cdot a/2$$
$$= 4 \cdot y_m \cdot a/2 \quad \text{———} (4)$$

Since the distance to the screen is much greater than the distance between the two source $S_1$ and $S_2$, the sum of $r_1$ and $r_2$ may be approximately by,

$$r_1 + r_2 = 2r = 2D.$$

So, from the equation (4);

$$(r_1 - r_2) \cdot 2D = 2y_m a$$
$$\Rightarrow r_1 - r_2 = \frac{y_m a}{D} \quad \text{———} (5)$$

From (3) and (5) we have,

$$a\sin\theta = \frac{y_m a}{D} \quad \text{———} (6)$$

(*) **For constructive interference (bright fringes):**

i.e; $a\sin\theta = m\lambda$ where, $m = 1, 2, 3, \dots, m$ is called the order number.

So, the equation becomes,

$$\frac{y_m a}{D} = m\lambda$$
$$\Rightarrow y_m = \frac{m\lambda D}{a} \quad \text{———} (7)$$

This equation gives the position of the $m^{th}$ bright fringe on the screen.

The distance for $m^{th}$ bright fringe, $y_m = m\frac{\lambda D}{a}$

The distance for $(m+1)^{th}$ bright fringe, $y_{m+1} = (m+1)\frac{\lambda D}{a}$.

The difference in the position of two constructive maxima is;

$$\Delta y = y_{m+1} - y_m$$
$$\therefore \Delta y = \frac{D\lambda}{a} \quad \text{———} (8)$$

(*) For destructive interference (dark fringes):

i.e; $a \sin\theta = (m+\frac{1}{2})\lambda$ where, $m = 1,2,3,\ldots\ldots$, m is called the order number.

So, $\frac{y_m a}{D} = (m+\frac{1}{2})\lambda$

$$\Rightarrow y_m = (m+1/2)\frac{\lambda D}{a} \qquad (9)$$

This equation gives the position of the mth dark fringe on the screen.

The distance for mth dark fringe $y_m = (m+1/2)\frac{\lambda D}{a}$.

The distance for $(m+1)^{th}$ dark fringe, $y_{m+1} = (m+1/2+1)\frac{\lambda D}{a}$.

So, the difference in the position of the two destructive,

$$\Delta y = y_{m+1} - y_m$$

$$\Delta y = \frac{\lambda D}{a} \qquad (10)$$

(*) Band width ($\beta$): The distance between any two consecutive bright or dark band's is called band width.

$$\Delta y = \frac{\lambda D}{a}$$

$$\therefore \beta = \frac{\lambda D}{a}.$$

Since, bright and dark fringe's are of same width, they are equi-spaced on either side of central maxima.

(*) Math: An Young's double slit experiment the separation of the slit is 1.9 mm and the fringe's spacing is 0.31 mm at a distance of 1 m from the slit. Calculate the wavelength of light.

Here,

$D = 1m = 1000 mm$

$a = 1.9 mm$.

$\Delta y/\beta = 0.31 mm$

$\lambda = ?$

We know that,

$$\Delta y/\beta = \frac{\lambda D}{a}$$

$$\Rightarrow \lambda = \frac{\beta . a}{D}$$

$$= \frac{0.31 \times 1.9}{1000}$$

$$= 5.89 \times 10^{-4} mm$$

$$= 5890 A^{\circ}$$

Ans.

**Math:** Green light of wavelength 5100 A° from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2m, find the slit separation.

Here, $\lambda = 5100 A^\circ$
$= 5100 \times 10^{-8} cm$
$D = 200 cm$
$\beta = \frac{2}{10} cm$
$= 1/5 cm.$
$a = ?$

We know that,
$$\beta = \frac{\lambda D}{a}$$
$$\Rightarrow a = \frac{\lambda D}{\beta}$$
$$= \frac{5100 \times 10^{-8} \times 200}{1/5}$$
$$= 0.051 cm$$
Ans.

The coherent sources are 0.18 mm apart and the fringes are observed on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of light.

Here,
The distance of the mth fringe from the central fringe is, $y_m = 10.8 mm$
$= \frac{10.8}{10} cm$
$= 1.08 cm.$

$D = 80 cm$
$a = 0.18 mm$
$= \frac{0.18}{10} cm$
$= 0.018 cm$
$m = 4$
$\lambda = ?$

We know that,
$$y_m = \frac{m \lambda D}{a}$$
$$\Rightarrow \lambda = \frac{y_m \cdot a}{m \cdot D}$$
$$= \frac{1.08 \times 0.018}{4 \times 80}$$
$$= 6.075 \times 10^{-5} cm$$
$$= \frac{6.075 \times 10^{-5}}{10^{-8}} A^\circ$$
$$= 6075 A^\circ$$
Ans.