9

DIFFRACTION

9.1 INTRODUCTION

It is a matter of common experience that the path of light entering a dark room through a hole in the window illuminated by sunlight is straight. Similarly, if an opaque obstacle is placed in the path of light, a sharp shadow is cast on the screen, indicating thereby that light travels in straight lines. Rectilinear propagation of light can be easily explained on the basis of Newton's corpuscular theory. But it has been observed that when a beam of light passes through a small opening (a small circular hole or a narrow slit) it spreads to some extant into the region of the geometrical shadow also. If light energy is propagated in the form of waves, then similar to sound waves, one would expect bending of a beam of light round the edges of an opaque obstacle or illumination of the geometrical shadow.

Each progressive wave, according to Huygens wave theory produces secondary waves, the envelope of which forms the secondary wavefront. In Fig. 9.1 (*a*), *S* is a source of monochromatic light and *MN* is a small aperture. *XY* is the screen placed in the path of light. *AB* is the illuminated portion of the screen and above *A* and below *B* is the region of the geometrical shadow. Considering *MN* as the primary wavefront, according to Huygens' construction, if secondary wavefronts are drawn, one would expect encroachment of light in the geometrical shadow. Thus, the shadows formed by small obstacles are not sharp. This bending of light round the edges of an obstacle or the encroachment of light within the geometrical shadow is called diffraction. Similarly, If an opaque obstacle *MN* is placed in the path of light [Fig. 9.1 (b), there should be illumination in the geometrical shadow region *AB* also. But the illumination in the geometrical shadow of an obstacle is not commonly observed because the light sources are not point sources and secondly the obstacles used are of very large size compared to the wavelength of light. If a shadow of an obstacle is cast by an extended source, say a frosted electric bulb, light from every point on the surface of the bulb forms its own diffraction pattern (bright and dark diffraction bands) and these overlap such that no single pattern can be identified. The term diffraction is referred to such problems in which one considers the resultant effect produced by a limited portion of a wavefront.

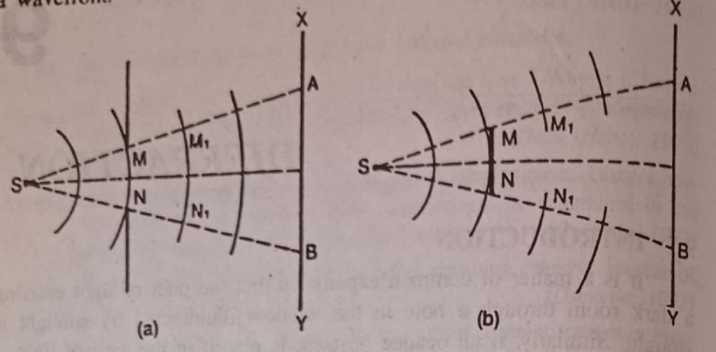

Fig. 9.1

Diffraction phenomena are part of our common experience. The luminous border that surrounds the profile of a mountain just before the sun rises behind it, the light streaks that one sees while looking at a strong source of light with half shut eyes and the coloured spectra (arranged in the form of a cross) that one sees while viewing a distant source of light through a fine piece of cloth are all examples of diffraction effects.

Augustin Jean Fresnel in 1815, combined in a striking manner Huygens wavelets with the principle of interference and could satisfactorily explain the bending of light round obstacles and also the rectilinear propagation of light.

9.2 FRESNEL'S ASSUMPTIONS

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors discussed below :-

In Fig. 9.2, S is a point source of monochromatic light and MN is a small aperture. XY is the screen and SO is perpendicular to XY. MCN is the incident spherical wavefront due to the point source S. To obtain the resultant effect at a point P on the screen, Fresnel assumed that (1) a wavefront can be divided into a large number of strips or zones called Fresnal's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from the various zones ; (2) the effect at a point due to any particular zone will depend on the distance of the point from the zone ; (3) the effect at P will also depend on the obliquity of the point with reference to the zone under consideration, *e.g.* due to the part of the wavefront at C, the

effect will be maximum at O and decreases with increasing obliquity., It is maximum in a direction radially outwards from C an it decreases in the opposite direction. The effect at a point due to the obliquity factor is proportional to $(1 + \cos\theta)$ where $\angle PCO = \theta$. Considering an elementary wavefront at C, the effect is maximum at O because $\theta = 0$ and $\cos\theta = 1$. Similarly, in a direction tangential to the primary wavefront at C (along CQ) the resultant effect is one half of that along

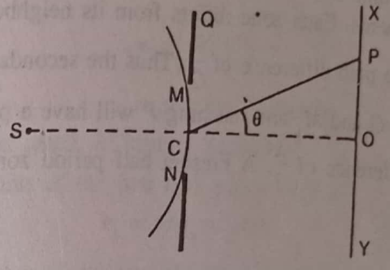

Fig. 9.2

CO because $\theta = 90°$ and $\cos 90 = 0$. In this direction CS, the resultant effect is zero since $\theta = 180°$ and $\cos 180 = -1$ and $1 + \cos 180 = 1 - 1 = 0$. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle viz., that if the secondary waves spread out in **all directions** from each point on the primary wavefront, they should give a wave travelling forward as well as backward. as the amplitude at the rear of the wave is zero there will evidently be no back wave.

9.3 RECTILINEAR PROPAGATION OF LIGHT

$ABCD$ is a plane wavefront perpendicular to the plane of the paper

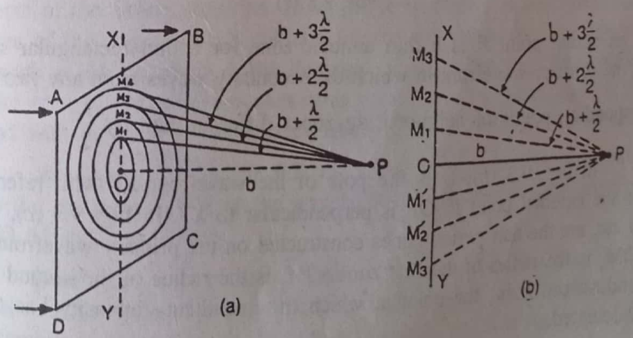

Fig. 9.3

$$\therefore \quad f_1 = \frac{r_1^2}{\lambda} \qquad ...(i)$$

$$f_2 = \frac{r_1^2}{3\lambda} \qquad ...(ii)$$

Also

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

$$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f_2}$$

Here $v_1 = 0.3$ m and $v_2 = 0.6$ m

$$\frac{1}{0.3} - \frac{1}{u} = \frac{\lambda}{r_1^2} \qquad ...(iii)$$

$$\frac{1}{0.06} - \frac{1}{u} = \frac{3\lambda}{r_1^2} \qquad ...(iv)$$

Multiplying equation (*iii*) by 3 and equating with (*iv*),

$$\frac{1}{0.1} - \frac{3}{u} = \frac{1}{0.06} - \frac{1}{u}$$

$$u = -0.3 \text{ m} \qquad ...(v)$$

Negative sign shows that the point source is to the left of the zone plate and its distance is 0.3 m.

Substituting the value of u and λ in equation (*iii*)

$$\frac{1}{0.3} + \frac{1}{0.3} = \frac{5 \times 10^{-7}}{r_1^2}$$

$$r_1 = 2.74 \times 10^{-4} \text{ m} \qquad ... (vi)$$

From equation (*i*)

$$f_1 = \frac{(2.74 \times 10^{-4})^2}{5 \times 10^{-7}} = 0.15 \text{ m}$$

Example 9.6. *A zone plate is made by arranging the radii of the circles which define the zones such that they are the same as the radii of newton's rings formed between a plane surface and the surface having radius of curvature 200 cm. Find the principal focal length of the zone plate.* [Delhi (Hons) 1992]

For Newton' rings,

radius of the n th ring,

$$r_n = \sqrt{n \lambda R}$$

$$r_1 = \sqrt{\lambda R} \qquad ...(i)$$

For a zone plate, the principal focal length

$$f_1 = \frac{r_1^2}{\lambda} \qquad ...(ii)$$

From (*i*) and (*ii*)

$$f_1 = \frac{\lambda R}{\lambda} = R$$

But $R = 200 \text{ cm} = 2 \text{ m}$

$$\therefore \quad f_1 = 2 \text{ m}$$

9.7 FRESNEL AND FRAUNHOFER DIFFRACTION

Diffraction phenomena can conveniently be divided into two groups viz, (*i*) Fresnel diffraction phenomena and (*ii*) Fraunhofer diffraction phenomena. In the Fresnel class of diffraction, the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction. In this case, the effect at a specific point on the screen due to the exposed incident wavefront is considered and no modification is made by lenses and mirrors. In such a case, the phenomenon observed on the screen is called Fresnel diffraction pattern. In the Fraunhofer class of diffraction phenomena, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction. Fraunhofer diffraction pattern can be easily observed in practice. The incoming light is rendered parallel with a lens and the diffracted beam is focussed on the screen with another lens. Observation of Fresnel diffraction phenomena do not require any lenses. Theoretical treatment of Fraunhofer diffraction phenomena is simpler. Fresnel class of diffraction phenomena are treated first in this chapter.

9.8 DIFFRACTION AT A CIRCULAR APERTURE

Let AB be a small aperture (say a pin hole) and S is a point source of monochromatic light. XY is a screen perpendicular to the plane of the paper and P is a point on the screen. SP is perpendicular to the screen. O is the centre of the aperture and r is the radius of the aperture. Let the distance of the source from the aperture be a ($SO = a$) and the distance of the screen from the aperture be b ($OP = b$). P_1OQ_1 is the incident spherical wavefront and with reference to the point P, O is the pole of the wavefront (Fig. 9.8). To consider the intensity at P, half period zones can be constructed with P as centre and radii $b+\frac{\lambda}{2}, b+\frac{2\lambda}{2}$ etc., on the exposed wavefront AOB. Depending on the distance of P from the aperture (*i.e.*, the distance b) the number of half period zones that can be constructed may be odd or even. If the distance a is such that only one half period zone can be constructed, then the intensity at P will be proportional to m_1^2 (where m_1 is the amplitude due to the first zone at P). On the other hand, if the whole of the wavefront is exposed to the point P, the resultant amplitude is $\frac{m_1}{2}$ or the intensity at P will be proportional to $\frac{m_1^2}{4}$. The position of the screen can be altered so as to construct 2, 3 or more half period zones for the same area of the aperture. If only 2 zones are exposed, the resultant amplitude at $P = m_1 - m_2$ (minimum) and if 3 zones are exposed, the amplitude $= m_1 - m_2 + m_3$ (maximum) and so on. Thus, by continuously altering the value of b, the point P becomes alternately bright and dark depending on whether odd or even number of zones are exposed by the aperture.

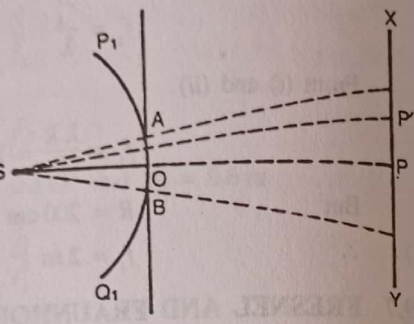

Fig 9.8

Now consider a point P' on the screen XY (Fig. 9.9) Join S to P'. The line SP' meets the wavefront at O'. O' is the pole of the wavefront

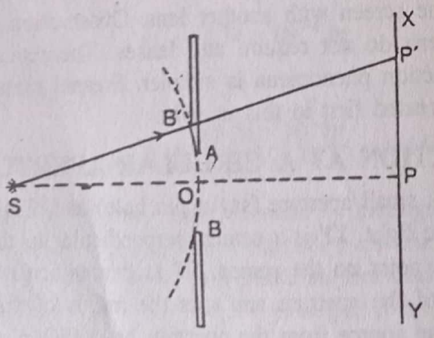

Fig. 9.9

with reference to the point P'. Construct half period zones with the point O' as the pole of the wavefront. The upper half of the wavefront is cut off by the obstacle. If the first two zones are cut off by the obstacle between the points O' and A and if only the 3 rd, 4 th and 5 th zones are exposed by the aperture AOB, then the intensity at P' will be maximum. Thus, if odd number of half period zones are exposed, point P' will be of maximum intensity and if even number of zones are exposed, the point P' will be of minimum intensity. As the distance of P' from P increases, the intensity of maxima and minima gradually decreases, because, with the point P' far removed from P, the most effective central half period zones are cut off by the obstacle between the points O' and A. With the outer zones, the obliquity increases with reference to the point P' and hence the intensity of maxima and minima also will be less. If the point P' happens to be of maximum intensity, then all the points lying on a circle of radius PP' on the screen will also be of maximum intensity. Thus, with a circular aperture, the diffraction pattern will be concentric bright and dark rings with the centre P bright or dark depending on the distance b. The width of the rings continuously decreases.

9.9 MATHEMATICAL TREATMENT OF DIFFRACTION AT A CIRCULAR APERTURE

In Fig. 9.10, S is a point source of monochromatic light, AB is the circular aperture and P is a point on the screen. O is the centre of the circular aperture. The line SOP is perpendicular to the circular aperture AB and the screen at P. The screen is perpendicular to the plane of the paper.

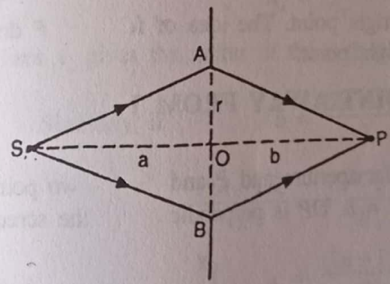

Fig. 9.10

Let δ be the path difference for the waves reaching P along the paths SAP and SOP.

$$SO = a; \quad OP = b; \quad OA = r$$

$$\delta = SA + AP - SOP$$

$$= (a^2+r^2)^{1/2} + (b+r^2)^{1/2} - (a+b)$$

$$= a\left(1+\frac{r^2}{a^2}\right)^{1/2} + b\left(1+\frac{r^2}{b^2}\right)^{1/2} - (a+b)$$

$$= a\left(1+\frac{r^2}{2\,a^2}\right)+b\left(1+\frac{r^2}{2\,b^2}\right) \quad a+b)$$

$$\delta = \frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$\therefore \quad \frac{1}{a}+\frac{1}{b} = \frac{2\delta}{r^2} \quad ...(i)$$

If the position of the screen is such that n full numbe alf period zones can be constructed on the aperture, then the path di nce

$$\delta = \frac{n\lambda}{2} \quad \text{or} \quad 2\delta = n\lambda$$

Substituting this value of 2δ in (i)

$$\frac{1}{a}+\frac{1}{b} = \frac{n\lambda}{r^2} \quad ...(ii)$$

The point P will be of maximum or minimum ir depending on whether n is odd or even. If the source is at infini ince (for an incident plane wavefront), then $a = \infty$ and

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2} \quad ...(iii)$$

If n is odd, P will be a bright point. The idea of fo P does not mean that if is always a bright point.

9.10 INTENSITY AT A POINT AWAY FROM T CENTRE

In Fig. 9.11, AB is a circular aperture and P and wo points on the screen. $PP' = x$ and $OP = b$. OP is perpendic the screen.

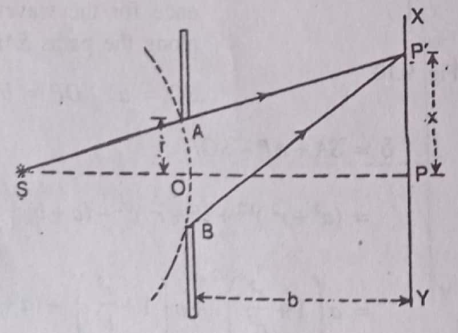

Fig. 9.11

Let r be the radius of the aperture. The path difference between the secondary waves from A and B and reaching P' can be given by

$$\delta = BP' - AP'$$

$$= \sqrt{b^2+(x+r)^2} - \sqrt{b^2+(x-r)^2}$$

$$= b\left(1+\frac{(x+r)^2}{2b^2}\right) - b\left(1+\frac{(x-r)^2}{2b^2}\right)$$

$$= b+\frac{(x+r)^2}{2b} - b - \frac{(x-r)^2}{2b^2}$$

$$= \frac{1}{2b}\left[(x+r)^2-(x-r)^2\right]$$

$$\delta = \frac{1}{2b}(4xr) = \frac{2rx}{b} \quad ...(i)$$

The point P' will be dark if the path difference $\delta = 2n\frac{\lambda}{2}$

(2 n means even number of zones).

$$2n\frac{\lambda}{2} = \frac{2rx_n}{2}$$

or

$$x_n = \frac{nb\lambda}{2r} \quad ...(ii)$$

where x_n gives the radius of the n th dark ring.

Similarly, if $\delta = \dfrac{(2n+1)\lambda}{2}$,

$$\frac{(2n+1)\lambda}{2} = = \frac{2rx_n}{b}$$

or

$$x_n = \frac{(2n+1)b\lambda}{4r} \quad ...(iii)$$

where x_n gives the radius of the n th bright ring.

The objective of a telescope consists of an achromatic convex lens and a circular aperture is fixed in front of the lens. Let the diameter of the aperture be D ($= 2r$). While viewing distant objects, the incident wavefront is plane and the diffraction pattern consists of a bright centre surrounded by dark and bright rings of gradually decreasing intensity. The radii of the dark rings is given by

$$x_n = \frac{nb\lambda}{2r} = \frac{nb\lambda}{D} \quad ...(iv)$$

The radius of the first dark ring is

$$x_1 = \frac{b\lambda}{D}$$

For an incident plane wavefront, $b = f$ the focal length of the objective.

$$\therefore \quad x_1 = \frac{f\lambda}{D}$$

The value of x_1 measures the distance of the first secondary minimum from the central bright maximum. However, according to Airy's theory, the radius of the first dark ring is given by

$$x_1 = \frac{1.22 f\lambda}{D} \quad ...(v)$$

It is interesting to note that the size of the central image depends on λ, the wavelength of light, f the focal length of the lens and D the diameter of the lens aperture.

Example 9.7. *A circular aperture of 1.2 mm diameter is illuminated by plane waves of monochromatic light. The diffracted light is received on a distant screen which is gradually moved towards the aperture. The centre of the circular patch of light first becomes dark when the screen is 30 cm from the aperture. Calculate the wavelength of light.*

(Rajasthan)

Diameter $= 1.2 \text{ mm} = 0.12 \text{ cm}$

Radius $= r = 0.06 \text{ cm}$

$b = 30 \text{ cm}$

Here $(b^2 + r^2) = (b+\lambda)^2$

$$30^2 + (0.06)^2 = (30+\lambda)^2$$

$$\lambda = \frac{(0.06)^2}{2 \times 30} \text{ approximately, neglecting } \lambda^2$$

$$= 0.00006 \text{cm} = \mathbf{6000\ Å}$$

Example 9.8. *A monochromatic beam of light on passing through a slit 1.6 mm falls on a screen held close to the slit. The screen is then gradually moved away and the middle of the patch of light on it becomes dark, when the screen is 50 cm from the slit. Calculate the wavelength of light.*

(Punjab)

Here the width of the slit $= 1.6 \text{ mm} = 0.16 \text{ cm}$

Half width $= r = 0.08 \text{ cm}$

Distance between the slit and the screen $= b = 50$ cm

Here $(b^2 + r^2) = (b+\lambda)^2$

$$\therefore \quad 50^2 + (0.08)^2 = (50+\lambda)^2$$

or $$\lambda = \frac{(0.08)^2}{2 \times 50} \text{ approximately(neglecting} \lambda^2)$$

$$= 0.000064 \text{cm} = 6400 \text{ Å}$$

9.11 DIFFRACTION AT AN OPAQUE CIRCULAR DISC

S is a point source of monochromatic light. CD is an opaque disc and MN is the screen. P is a point on the screen such that SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. XY is the incident spherical wavefront. EF is the geometrical shadow and P is the centre of the shadow. With reference to the point P, the wavefront can be divided into half period zones taking the centre of the disc (A) as the pole (Fig. 9.12). If one half period zone can be constructed on the surface of the disc, the rest of the zones are exposed to the point P and the resultant amplitude at $P = \frac{m_2}{2}$ approximately, where m_2 is the amplitude due to the second zone

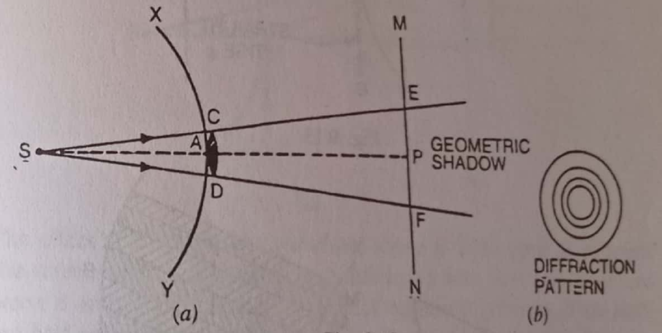

Fig. 9.12

$\left[m_2 - m_3 + m_4 - \ldots = \frac{m_2^2}{2} \text{ approximately}\right]$. Similarly, if two half period zones can be constructed on the surface of the disc, the resultant amplitude at P due to the exposed zones will be $\frac{m_3}{2}$ and so an. Thus, the point P will always be bright but the intensity at P decreases with increase in the diameter of the disc. That is, with a large diameter of the disc, the most effective central zones will be cut off by the disc and the exposed outer geometrical shadow and with monochromatic light, bright and dark bands (diffraction bands) are observed in the illuminated portion of the screen. However, with white light coloured bands will be observed and the bands of shorter wavelength are nearer the point P.

9.15 DIFFRACTION PATTERN DUE TO A NARROW SLIT

S is a narrow slit illuminated by monochromatic light. The length of the slit is perpendicular to the plane of the paper. AB is a rectangular aperture parallel to the slit, MN is the screen and P is a point on the screen such that SOP is perpendicular to the plane of the paper, XY is the incident

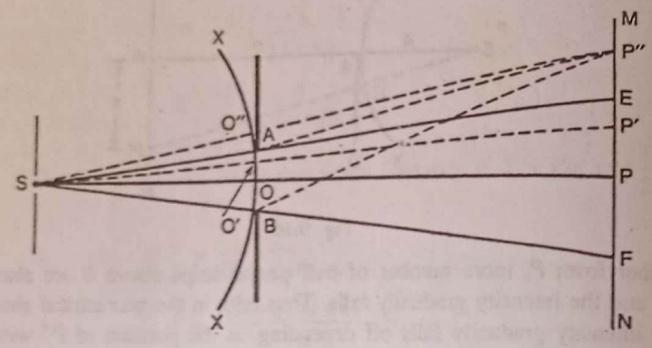

Fig. 9.18

cylindrical wavefront (Fig. 9.18). On the screen, EF is the illuminated portion and above E and below F is the region of the geometrical shadow.

If the slit AB is wide, then with reference to the point P, the cylindrical wavefront can be divided into a large number of half period strips and the resultant amplitude at P will be $\frac{m_1}{2}$ where m_1 is the amplitude due to the first half period strip. Thus, the point P will be illuminated. Even points very near P will be equally illuminated. If the wavefront is divided with reference to points nearer P, the number of half period strips above and below the new pole in the exposed portion of the wavefront will be quite large and hence this results in uniform illumination.

Now consider a point P' nearer the edge of the geometrical shadow (Fig. 9.18). Join S to P'. Here O' is the pole of the wavefront with reference to the point P'. If the wavefront is divided into half period strips, the number of half period strips between O' and B will be quite large and the illumination at P' due to the lower portion of the wavefront will be the same at all points near the edge of the geometrical shadow. But the intensity at P due to the exposed portion of the wavefront between

A and O' will depend on the number of half period strips present. If the number of half period elements is odd, the point P' will be of maximum intensity and if it is even the point will be of minimum intensity.

Let P'' be a point in the region of the geometrical shadow. Join S to P''. Here O'' is the pole of the wavefront with reference to the point P''. If the wavefront is divided into half period elements, then the upper half of the wavefront between X and O'' is cut off by the obstacle and only a portion between A and B is exposed to the point P''. If the number of half period elements exposed by AB is odd, then P'' will be of maximum intensity and if it is even it will be of minimum intensity. But as the most effective central half period strips between O'' and A are cut off, the intensity falls off rapidly in the region of the geometrical shadow and maxima and minima cannot be distinguished. The intensity distribution due to a wide aperture is shown in Fig. 9.19 (*b*).

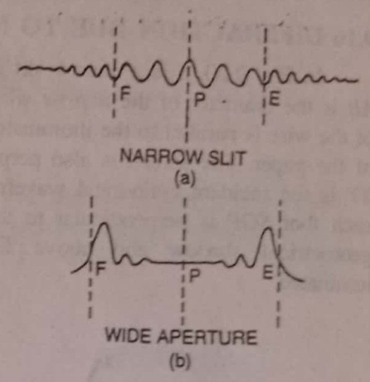

Fig. 9.19

On the other hand, if the slit is narrow, the intensity at the point P will depend on the number of half period strips that can be constructed on the exposed wavefront between A and B. If the number of half period strips is odd, the intensity at P will be maximum and if it is even the intensity at P will be minimum (Fig. 9.18) Thus, the point P can be bright or dark. If we consider a point P' in the illuminated portion EF of the screen, the intensity at P will depend on the number of half period strips that can be constructed between A and O' where O' is the pole of the wavefront with reference to the point P'. If the number of half period strips between A and O' is odd, P' will be a point of maximum intensity. Thus, between E and F alternate bright and dark bands will be observed and the point P may be bright or dark.

Now consider a point P'' in the region of the geometrical shadow (Fig. 9.18). O'' is the pole of the wavefront with reference to the point P'' and the intensity at P'' will depend on the number of half period strips exposed by the slit AB. The upper half of the wavefront above O'' is obstructed by the obstacle and even the most effective central half period strips between O'' and A are cut off by the obstacle. Thus, the intensity at P'' is far away from E, the maxima and minima become indistinguishin the region of the geometrical shadow. The intensity distribution due to Fresnel's diffraction at a straight edge is given in Fig. 9.17 on page 429.

9.22 FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

To obtain a Fraunhofer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

In Fig. 9.33, S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a. XY is the incident spherical wavefront. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. The screen is perpendicular to the

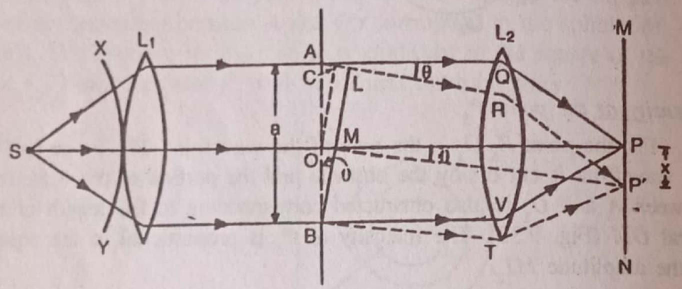

Fig. 9.33

plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wavefront is incident on the slit AB and each point on this wavefront is a source of secondary disturbance. The secondary waves travelling in the direction parallel to OP *viz.* AQ and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the waterfront travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

Now, consider the secondary waves travelling in the direction AR, inclined at an angle θ to the direction OP. All the secondary wave travelling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Draw OC and BL perpendicular to AR.

Then, in the ΔABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or

$$AL = a\sin\theta$$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity. The whole wavefront can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\frac{\lambda}{2}$. Similarly for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\frac{\lambda}{2}$. Thus, destructive interference takes place and the point P' will be of minimum intensity. If the direction of the secondary waves is such that $AL = 2\lambda$, then also the point where they meet the screen will be of minimum intensity. This is so, because the secondary waves from the corresponding points of the lower half, differ in path by $\frac{\lambda}{2}$ and this again gives the position of minimum intensity. In general

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

where θ_n gives the direction of the n th minimum. Here n is an integer.

If, however, the path difference is odd multiples of $\frac{\lambda}{2}$, the directions of the secondary maxima can be obtained. In this case,

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

or

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$$

where $n = 1, 2, 3$ etc.

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen is given in Fig. 9.34.

P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B

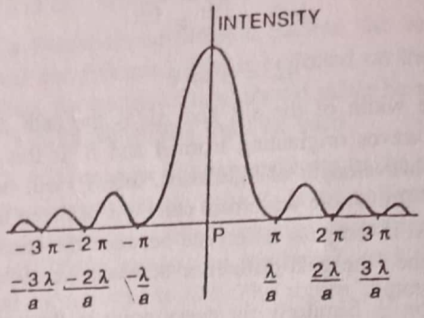

Fig. 9.34

is λ, 2λ etc., correspond to the positions of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards.

If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then

$$\sin\theta = \frac{x}{f} \qquad ...(i)$$

where f is the focal length of the lens L_2

But,
$$\sin\theta = \frac{\lambda}{a} \qquad ...(ii)$$

$$\therefore \qquad \frac{x}{f} = \frac{\lambda}{a}$$

or
$$x = \frac{f\lambda}{a}$$

where x is the distance of the secondary minimum from the point P. Thus, the width of the central maximum $= 2x$.

or
$$2x = \frac{2f\lambda}{a} \qquad ...(iii)$$

The width of the central maximum is proportional to λ, the wavelength of light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of the central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction

bands are coloured. From equation (ii), if the width a of the slit is large, $\sin\theta$ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit, a is small and hence θ is large. This results a distinct diffraction maxima and minima on both the sides of P.

Example 9.9. *Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.*

Here
$$\sin\theta = \frac{\lambda}{a}$$

where θ is half angular width of the central maximum.

$$a = 12 \times 10^{-5}\text{ cm}, \lambda = 6000\text{ Å} = 6 \times 10^{-5}\text{ cm}.$$

$$\therefore \qquad \sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50$$

or
$$\theta = 30^\circ$$

Example 9.10. *In Fraunhofer diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maximum, find the wavelength of light.* [Delhi (Sub) 1977]

In the case of Fraunhofer diffraction at a narrow rectangular aperture,

$$a\sin\theta = n\lambda$$

$$n = 1$$

$$\therefore \qquad a\sin\theta = \lambda$$

$$\sin\theta = \frac{x}{D}$$

$$\therefore \qquad \frac{ax}{D} = \lambda$$

$$\lambda = \frac{ax}{D}$$

Here
$$a = 0.2\text{ mm} = 0.02\text{ cm}$$
$$x = 5\text{ mm} = 0.5\text{ cm}$$
$$D = 2\text{ m} = 200\text{ cm}$$

$$\therefore \qquad \lambda = \frac{0.02 \times 0.5}{200}$$

$$\lambda = 5 \times 10^{-5}\text{ cm}$$

$$\lambda = 5000\text{ Å}$$

Example 9.11. *Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.*

The first dark fringe is on either side of the central bright fringe.

Here $n = \pm 1,\ D = 2\text{ m}$

$$\lambda = 6000\text{ Å} = 6 \times 10^{-7}\text{ m}$$

$$\sin\theta = \frac{x}{D}$$

$$a = 0.30\text{ mm} = 3 \times 10^{-4}\text{ m}$$

$$a \sin\theta = n\lambda$$

$$\frac{ax}{D} = n\lambda$$

(a)
$$x = \frac{n\lambda D}{a}$$

$$x = \pm\left[\frac{1 \times 6 \times 10^{-7} \times 2}{3 \times 10^{-4}}\right]$$

$$x = \pm 4 \times 10^{-3}\text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe,

$$y = 2x$$
$$= 2 \times 4 \times 10^{-3}$$
$$= 8 \times 10^{-3}\text{ m}$$
$$= 8\text{ mm}$$

Example 9.12 *A single slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2 m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light used.*

(IAS, 1990)

In the case of Fraunhofer diffraction at a narrow rectangular slit,

$$a \sin\theta = n\lambda$$

Here θ gives the directions of the minimum

$$n = 2$$
$$\lambda = ?$$

$$a = 0.14\text{ mm} = 0.14 \times 10^{-3}\text{ m}$$

$$D = 2\text{ m}$$

$$x = 1.6\text{ cm} = 1.6 \times 10^{-2}\text{ m}$$

$$\sin\theta = \frac{x}{D} = \frac{n\lambda}{a}$$

$$\therefore \quad \lambda = \frac{xa}{nD}$$

$$= \frac{1.6 \times 10^{-2} \times 0.14 \times 10^{-3}}{2 \times 2}$$

$$= 5.6 \times 10^{-7}\text{ m}$$

$$= 5600\text{ Å}$$

Example 9.13. *A screen is placed 2 m away from a narrow slit which is illuminated with light of wavelength 6000 Å. If the first minimum lies 5 mm on either side of the central maximum, calculate the slit width.*

(Delhi, 1990)

In the case of Fraunhofer diffraction at a narrow slit,

$$a \sin\theta = n\lambda$$

$$\sin\theta = \frac{x}{D}$$

$$\therefore \quad \frac{ax}{D} = n\lambda$$

Here, width of the slit = a = ?

$$x = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$$

$$D = 2\text{ m}$$

$$\lambda = 6000\text{ Å} = 6 \times 10^{-7}\text{ m}$$

$$n = 1$$

$$a = \left(\frac{n\lambda D}{x}\right)$$

$$a = \left(\frac{1 \times 6 \times 10^{-7} \times 2}{5 \times 10^{-3}}\right)$$

$$a = 2.4 \times 10^{-4}\text{ m}$$

$$a = 0.24\text{ mm}$$

Example 9.14. *Find the angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.*

(*Kanpur, 1990*)

Here $$\sin\theta = \frac{\lambda}{a}$$

where θ is the half angular width of the central maximum

$$a = 12 \times 10^{-5}\,\text{cm} = 12 \times 10^{-7}\,\text{m}$$

$$\lambda = 6000\,\text{Å} = 6 \times 10^{-7}\,\text{m}$$

$$\sin\theta = \frac{6 \times 10^{-7}}{12 \times 10^{-7}} = 0.5$$

$$\theta = 30°$$

Angular width of the central maximum,

$$2\theta = 60°$$

Example 9.15. *Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wavelength = 4890 Å.*

[*Kanpur, 1991*]

For minimum intensity

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{x_1}{f}, \quad n = 1$$

$$\frac{x_1}{f} = \frac{\lambda}{a}$$

Here $$\lambda = 4890\,\text{Å} = 4890 \times 10^{-10}\,\text{m}$$

$$a = 0.5\,\text{cm} = 5 \times 10^{-3}\,\text{m}$$

$$f = 40\,\text{cm} = 0.4\,\text{m}$$

$$x_1 = \frac{f\lambda}{a}$$

$$x_1 = \frac{0.4 \times 4890 \times 10^{-10}}{5 \times 10^{-3}}$$

$$x_1 = 3.912 \times 10^{-5}\,\text{m}$$

For secondary maximum

$$a\sin\theta_n = \frac{(2n+1)\lambda}{2}$$

For the first secondary maximum

$$n = 1$$

$$\sin\theta_n = \frac{x_2}{f}$$

$$\frac{x_2}{f} = \frac{3\lambda}{2a}$$

$$x_2 = \frac{3\lambda f}{2a}$$

$$x_2 = \frac{3 \times 4890 \times 10^{-10} \times 0.4}{2 \times 5 \times 10^{-3}}$$

$$x_2 = 5.868 \times 10^{-5}\,\text{m}$$

Difference, $$x_2 - x_1 = 5.868 \times 10^{-5} - 3.912 \times 10^{-5}$$

$$= 1.956 \times 10^{-5}\,\text{m}$$

$$= 1.596 \times 10^{-2}\,\text{mm}$$

9.23 INTENSITY DISTRIBUTION IN THE DIFFRACTION PATTERN DUE TO A SINGLE SLIT

The intensity variation in the diffraction pattern due to a single slit can be investigated as follows. The incident plane wavefront on the slit *AB* (Fig. 9.33) can be imagined to be divided into a large number of infinitesimally small strips. The path difference between the secondary waves emanating from the extreme points *A* and *B* is $a\sin\theta$ where *a* is the width of the slit and $\angle ABL = \theta$. For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can be taken to be the same. As one considers the secondary waves in a direction inclined

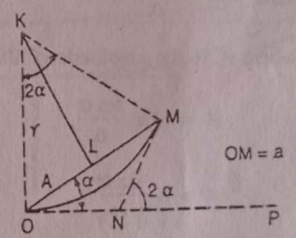

Fig. 9.35

at an angle θ from the point B upwards, the path difference changes and hence the phase difference also increases. Let α be the phase difference between the secondary waves from the points B and A of the slit (Fig. 9.27). As the wavefront is divided into a large number of strips,the resultant amplitude due to all the individual small strips can be obtained by the vector polygon method. Here, the amplitudes are small and the phase difference increases by infinetesimally small amounts from strip to strip. Thus, the vibration polygon coincides with the circular arc OM (Fig. 9.35). OP gives the direction of the initial vector and NM the direction of the final vector due to the secondary waves from A. K is the centre of the circular arc.

$$\angle MNP = 2\alpha$$

$$\therefore \quad \angle OKM = 2\alpha$$

In the Δ OKL

$$\sin\alpha = \frac{OL}{r}\ ; \ OL = r\sin\alpha$$

where r is the radius of the circular arc

$$\therefore \quad \text{Chord } OM = 2\,OL = 2r\sin\alpha \qquad \ldots (i)$$

The length of the arc OM is proportional to the width of the slit.

$\therefore$ Length of the arc $OM = Ka$

where K is a constant and a is the width of the slit.

$$\text{Also,} \quad 2\alpha = \frac{\text{Arc } OM}{\text{radius}} = \frac{Ka}{r}$$

$$\text{or} \quad 2r = \frac{Ka}{\alpha} \qquad \ldots (ii)$$

Substituting this value of 2r in equation (i)

$$\text{Chord } OM = \frac{Ka}{\alpha}\,.\sin\alpha$$

But, $OM = A$ where A is the amplitude of the resultant.

$$\therefore \quad A = (Ka)\frac{\sin\alpha}{\alpha}$$

$$A = A_0\frac{\sin\alpha}{\alpha} \qquad \ldots (iii)$$

Thus, the resultant amplitude of vibration at a point on the screen is given by $A_0\dfrac{\sin\alpha}{\alpha}$ and the intensity I at the point is given by

$$I = A^2 = A_0^2\frac{\sin^2\alpha}{\alpha^2} = I_0.\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \ldots (iv)$$

The intensity at any point on the screen is proportional to $\left(\dfrac{\sin\alpha}{\alpha}\right)^2$. A phase difference of 2π corresponds to a path difference of λ. Therefore a phase difference of 2α is given by

$$2\alpha = \frac{2\pi}{\lambda}\,.\,a\sin\theta \qquad \ldots (iv)$$

where $a\sin\theta$ is the path difference between the secondary waves from A and B (Fig. 9.35).

$$\alpha = \frac{\pi}{\lambda}\,.\,a\sin\theta \qquad \ldots (v)$$

Thus, the value of α depends on the angle of diffraction θ. The value of $\dfrac{\sin^2\alpha}{\alpha^2}$ for different values of θ gives the intensity at the point under consideration. Fig. 9.34 represents the intensity distribution. It is a graph of $\dfrac{\sin^2\alpha}{\alpha^2}$ (along the Y-axis), as a function of α or $\sin\theta$ (along the X-axis).

9.24 FRAUNHOFER DIFFRACTION AT A SINGLE SLIT (CALCULUS METHOD)

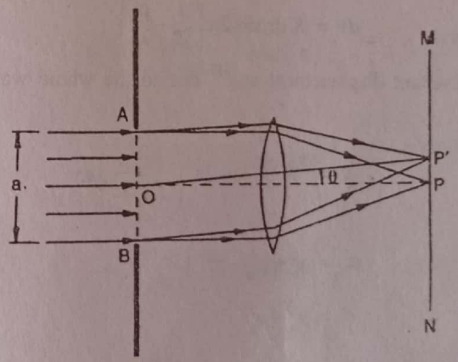

Fig. 9.36

Let a monochromatic parallel beam of light be incident on the slit AB of width a. The secondary waves travelling in the same direction as the incident light come to focus at the point P. The secondary waves travelling at an angle θ come to focus at P' (Fig. 9.36).

Consider the screen to be at a distance r from the slit. The centre of the slit O is the origin of coordinates. Consider a small element dz of the wavefront with coordinates (o, z). The coordinates of the point P' are (x_0, z_0) [Fig. 9.37]. The distance of the element from the point P' is ρ.

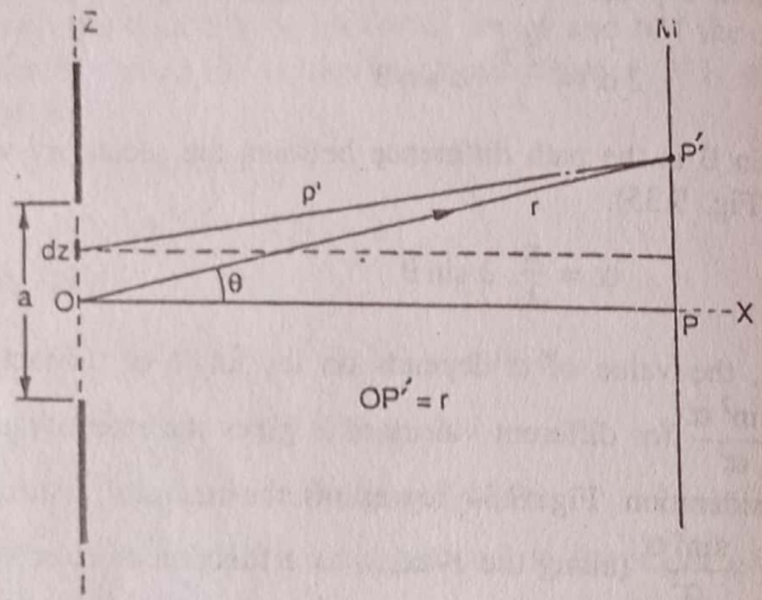

Fig. 9.37

The displacement at the point P' due to the element dz at any instant is given by,

$$dy = K\,dz \sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right) \quad ...(i)$$

The resultant displacement at P' due to the whole wavefront,

$$y = K\int_{-\frac{a}{2}}^{+\frac{a}{2}} \sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right) dz \quad ...(ii)$$

Also $$\rho^2 = x_0^2 + (z_0 - z)^2 \quad ...(iii)$$

$$r^2 = x_0^2 + z_0^2$$

or $$x_0^2 = r^2 - z_0^2$$

Substituting the value of x_0^2 in equation (iii)

$$\rho^2 = r^2 - z_0^2 + (z_0 - z)^2$$

$$\therefore \qquad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right) \qquad ...(vi)$$

The amplitude at P' is

$Ka\left(\frac{\sin\alpha}{\alpha}\right)$ and the intensity at P',

$$I' = K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)$$

$$I' = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right) \qquad ...(vii)$$

Here $I_0 = K^2a^2$ and is the value of the intensity at P, for $\alpha = 0$

$$\frac{\sin\alpha}{\alpha} = 1$$

$$\alpha \to 0$$

(*ii*) **Central Maximum.** For the point P on the screen (Fig. 9.27).

$$\theta = 0\,;$$

and hence $\qquad \alpha = 0\,;$

The value of $\frac{\sin\alpha}{\alpha}$ when $\alpha \to 0$ is equal to 1. Hence, the intensity

at $P = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0$ which is maximum.

(*ii*) **Secondary Maxima.** The directions of secondary maxima are given by the equation

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$$

Substituting this value of θ_n in equation (*v*) (page 457)

$$\alpha = \frac{\pi}{\lambda}\cdot\frac{a(2n+1)\lambda}{2a}$$

$$= \frac{(2n+1)\pi}{2} \qquad ...(viii)$$

Substituting $n = 1, 2, 3$ etc. in equation (*vii*), the values of α are given by

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ etc.}$$

(a) For the first secondary maximum

$$\alpha = \frac{3\pi}{2}$$

and $$I = I_0\cdot\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$= I_0\left[\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right]^2 = I_0\cdot\left[\frac{-1}{\frac{3\pi}{2}}\right]^2 = \frac{4I_0}{9\pi^2}$$

$$= \frac{I_0}{22}$$

(b) For the secondary maximum,

$$\alpha = \frac{5\pi}{2}$$

and $$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$= I_0\left[\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right]^2 = \left[\frac{1}{\frac{5\pi}{2}}\right]^2 = \frac{4I_0}{25\pi^2}$$

$$= \frac{I_0}{61}$$

Thus, the secondary maxima are of decreasing intensity and the directions of these maxima are obtained from the equation given above.

The intensity at P' is given by

$$I' = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)$$

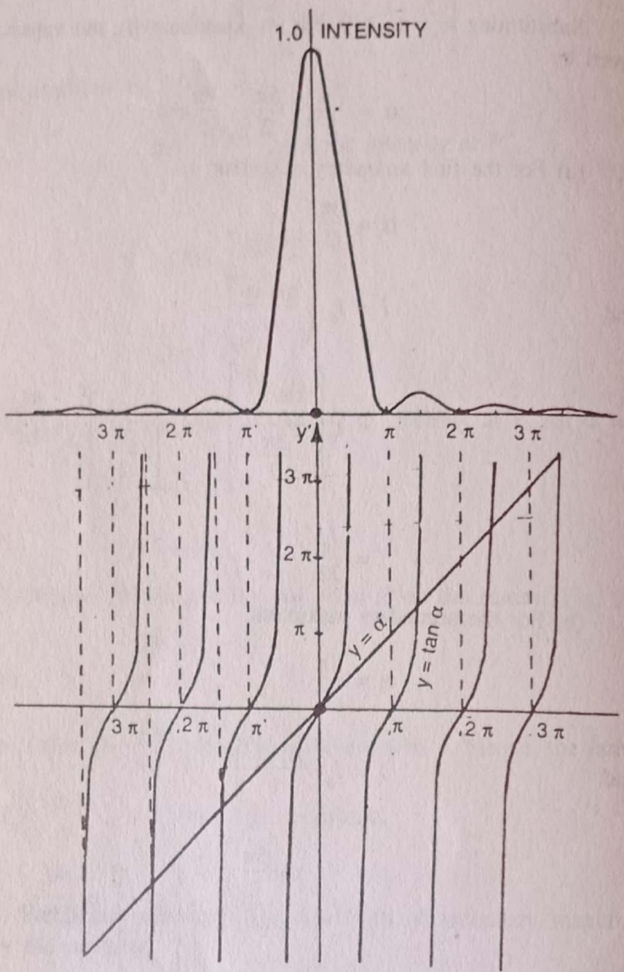

Fig. 9.38

$$dI' = I_0\left[\frac{\alpha^2\, 2\sin\alpha\cos\alpha - (\sin^2\alpha)\, 2\alpha}{\alpha^4}\right]d\alpha$$

For I' to be maximum

$$\frac{dI'}{d\alpha} = 0$$

$$\therefore \quad \alpha^2 (2\sin\alpha\cos\alpha) - (\sin^2\alpha)\, 2\alpha = 0$$

$$\tan\alpha = \alpha$$

If graphs are plotted for $y = \alpha$ and $y = \tan\alpha$ it will be found that the secondary maxima are not exactly midway between two minima. The positions of the secondary maxima are slightly towards the central maximum (Fig. 9.38).

(*iii*) **Secondary Minima.** The directions of the secondary minima are given by the equation

$$a\sin\theta = n\lambda$$

Substituting the value of $a\sin\theta$ in equation (v), (page 457)

$$\alpha = \frac{\pi}{\lambda}\cdot n\lambda = n\pi \qquad ...(ix)$$

Substituting $\quad n = 1, 2, 3$ etc. in equation (ix),

$$\alpha = \pi, 2\pi, 3\pi \text{ etc.}$$

When these values of α are substituted in the equation for the intensity viz.

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2; \quad I = 0$$

In Fig. 9.34, the positions of the secondary minima are shown for the values of

$$\alpha = \pi, 2\pi, 3\pi \text{ etc.}$$

$\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}$ etc. refer to the values of $\sin\theta$ for these positions.

9.25 FRAUNHOFER DIFFRACTION AT A CIRCULAR APERTURE

In Fig. 9.39, AB is a circular aperture of diameter d. C is the centre

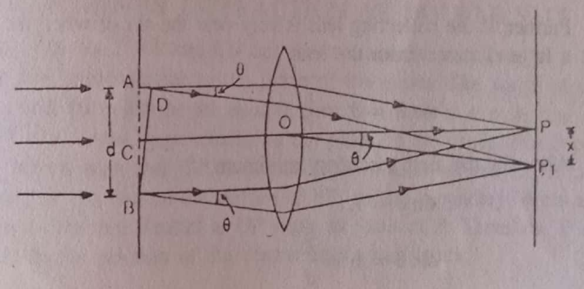

Fig. 9.39

of the aperture and P is a point on the screen. CP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. A plane wavefront is incident on the circular aperture. The secondary waves travelling in the direction CO come to focus at P. Therefore, P corresponds to the position of the central maximum. Here, all the secondary waves emanating from points equidistant from O travel the same distance before reaching P and hence they all reinforce one another. Now consider the secondary waves travelling in a direction inclined at an angle θ with the direction CP. All these secondary waves meet at P_1 on the screen. Let the distance PP_1 be x. The path difference between the secondary waves emanating from the points B and A (extremities of a diameter) is AD.

From the Δ ABD,

$$AD = d \sin\theta$$

Arguing as in Article 9.22, the point P_1 will be of minimum intensity if this path difference is equal to integral multiples of λ *i.e.*,

$$d \sin\theta_n = n\lambda \quad ...(i)$$

The point P_1 will be of maximum intensity if the path difference is equal to odd multiples of $\frac{\lambda}{2}$ *i.e.*,

$$d \sin\theta_n = \frac{(2n+1)\lambda}{2} \quad ...(ii)$$

If P_1 is a point of minimum intensity, then all the points at the same distance from P as P_1 and lying on a circle of radius x will be of minimum intensity. Thus, the diffraction pattern due to a circular aperture consists of a central bright disc called the Airy's disc, surrounded by alternate dark and bright concentric rings called the Airy's rings. The intensity of the dark rings is zero and that of the bright rings decreases gradually outwards from P.

Further, if the collecting lens is very near the slit or when the screen is at a large distance from the lens,

$$\sin\theta = \theta = \frac{x}{f} \quad ...(iii)$$

Also, for the first secondary minimum,

$$d \sin\theta = \lambda$$

$$\sin\theta = \theta = \frac{\lambda}{d} \quad ...(iv)$$

From equations (*iii*) and (*iv*)

$$\frac{x}{f} = \frac{\lambda}{d}$$

or

$$x = \frac{f\lambda}{d} \quad ...(v)$$

where x is the radius of the Airy's disc. But actually, the radius of the first dark ring is slightly more than that given by equation (*v*). According to Airy, it is given by

$$x = \frac{1.22 f\lambda}{d} \quad ...(vi)$$

The discussion of the intensity distribution of the bright and dark rings is similar to the one given for a rectangular slit. With increase in the diameter of the aperture, the radius of the central bright ring decreases.

Example 9.16. *In Fraunhofer diffraction pattern due to a single slit, the screen is at a distance of 100 cm from the slit and the slit is illuminated by monochromatic light of wavelength 5893 Å. The width of the slit is 0.1 mm. Calculate the separation between the central maximum and the first secondary minimum.* (*Mysore*)

For a rectangular slit,

$$x = \frac{f\lambda}{d}$$

Here $f = 100$ cm, $\lambda = 5893$ Å

$= 5893 \times 10^{-8}$ cm,

$d = 0.1$ mm $= 0.01$ cm, $x = ?$

$$\therefore \quad x = \frac{100 \times 5893 \times 10^{-8}}{0.01} = 0.5893 \text{ cm}$$

9.26 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT

In Fig. 9.40, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of the opaque portion is b. L is a collecting lens and MN is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wavefront be incident on the surface of XY. All the secondary waves travelling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

In this case, the diffraction pattern has to be considered in two parts, (*i*) the interference phenomenon due to the secondary waves emanating from th corresponding points of the two slits and (*ii*) the diffraction pattern due to the secondary waves from the two slits individually. For calculating

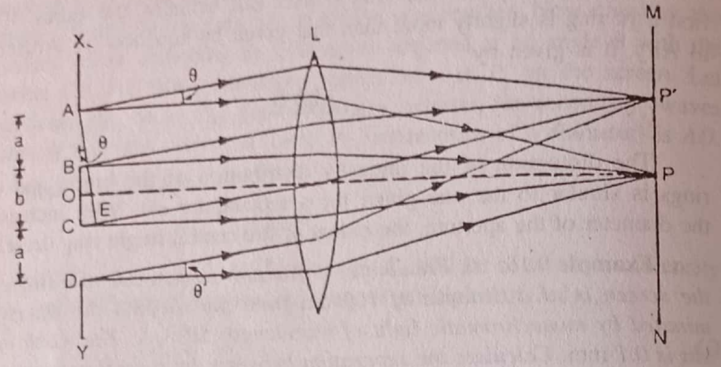

Fig. 9.40

the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ. Both the angles θ and ϕ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

(*i*) **Interference maxima and minima.** Consider the secondary waves travelling in a direction inclined at an angle θ with the initial direction.

In the Δ *ACN* (Fig. 9.41)

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

or

$$CN = (a+b)\sin\theta$$

If this path difference is equal to odd multiples of $\frac{\lambda}{2}$, θ gives the direction of minima due to interference of the secondary waves from the two slits.

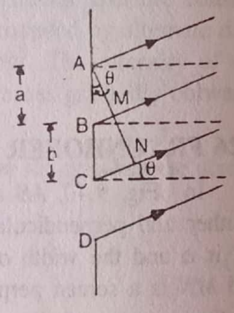

Fig. 9.41

$$\therefore \quad CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \quad ...(i)$$

Putting n = 1, 2, 3 etc., the values of $\theta_1, \theta_2, \theta_3$ etc., corresponding of minima can be obtained.

From equation (*i*)

$$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \quad ...(ii)$$

On the other hand, if the secondary waves travel in a direction θ′ such that the path difference is even multiples of $\frac{\lambda}{2}$, then θ′ gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$\therefore \quad CN = (a+b)\sin\theta'_n = 2n.\frac{\lambda}{2}$$

or

$$\sin\theta'_n = \frac{n\lambda}{(a+b)} \quad ...(iii)$$

Putting n = 1, 2, 3 etc., the values $\theta'_1, \theta'_2, \theta'_3$ etc., corresponding to the directions of the maxima can be obtained.

From equation (*ii*)

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and

$$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\therefore \quad \sin\theta_2 - \sin\theta_1 = \frac{\lambda}{a+b} \quad ...(iv)$$

Thus, the angular separation between any two consecutive minima (or maxima) is equal to $\frac{\lambda}{a+b}$. The angular separation is inversely proportional to $(a + b)$, the distance between the two slits.

(*ii*) **Diffraction maxima and minima.** Consider the secondary waves travelling in a direction inclined at an angle ϕ with the initial direction of the incident light.

If the path difference *BM* is equal to λ the wavelength of light used then ϕ will give the direction of diffraction minimum (Fig. 9.41). That is, the path difference between the secondary waves emanating from the extremities of a slit (*i.e.*, points *A* and *B*) is equal to λ. Considering the wavefront on *AB* to be made up of two halves, the path difference between the corresponding points of the upper and the lower halves is equal to $\frac{\lambda}{2}$. The effect at *P*′ due to the wavefront incident on *AB* is zero. Similarly for the same direction of the secondary waves, the effect at P' due to the wavefront incident on th slit CD is also zero. In general,

$$a \sin \phi_n = n\lambda \qquad \text{...(v)}$$

Putting $n = 1, 2$ etc., the values of ϕ_1, ϕ_2 etc., corresponding to the directions of diffraction minima can be obtained.

9.27 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT CALCULUS METHOD)

The intensity distribution due to Fraunhofer diffraction at double slit (two parallel slits) can be obtained by integrating the expression for dy (vide single slit) for both the slits.

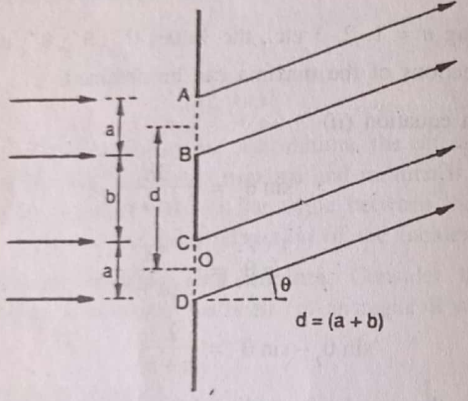

Fig. 9.42

Here

$$y = K\left[\int_{-\frac{a}{2}}^{+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]dz + \int_{d-\frac{a}{2}}^{d+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]dz\right]$$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)\right] - \frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]_{d-\frac{a}{2}}^{d+\frac{a}{2}}$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right) - \frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}+\frac{a\sin\theta}{2\lambda}\right) - \cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}-\frac{a\sin\theta}{2\lambda}\right)\right]$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right) + \frac{K\lambda}{\pi\sin\theta}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)\right]$$

But $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\right]$$

$$y = 2Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right)\cos\frac{\pi d\sin\theta}{\lambda}$$

Let $\dfrac{\pi d\sin\theta}{\lambda} = \beta$

$$\therefore \quad y = 2Ka\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta\,\sin 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right]$$

The intensity at a point P' is given by

$$I = 4K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

$$\therefore \quad I_0 = K^2a^2$$

$$\therefore \quad I = 4I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

The intensity of the central maximum = $4I_0$ when $\alpha = 0$ and $\beta = 0$.

In Fig. 9.43, the dotted curve represents the intensity distribution due to diffraction pattern due to double slit and the thick line curve represents the intensity distribution due to interference between the light from both the slits. The pattern consists of diffraction maxima within each diffraction maximum.

The intensity distribution due to Fraunhofer diffraction at two parallel slits is shown in Fig. 9.43. The full line represents equally spaced interference maxima and minima and the dotted curve represents the diffraction maxima and minima. In the region originally occupied by the

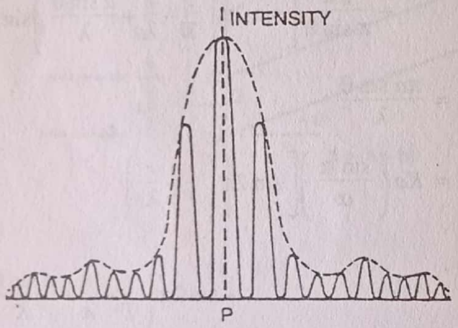

Fig. 9.43

central maximum of the single slit diffraction pattern, equally spaced interference maxima and minima are observed . The intensity of the central interference maximum is four times the intensity of the central maximum of the single slit diffraction pattern. The intensity of the other interference maxima on the two sides of the central maximum of the single slit diffraction pattern. The intensity of the other interference maxima on the two sides of the central maximum gradually decreases. In the region of the secondary maxima due to diffraction at a single slit, equally spaced interference maxima of low intensity are observed. The intensity

distribution shown in Fig. 9.43 corresponds to $2a = b$ where a is the width of each slit and b is the opaque spacing between the two slits. Thus, the pattern due to diffraction at a double slit consists of a diffraction pattern due to the individual slits of width a each and the interference maxima and minima of equal spacing. The spacing of the interference maxima and minima is dependent on the values of a and b.

9.28 DISTINCTION BETWEEN SINGLE SLIT AND DOUBLE SLIT DIFFRACTION PATTERNS

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima within the central maximum. The intensity of the central maximum in the diffraction pattern due to a double slit is four times that of the central maximum due to diffraction at a single slit. In the above arrangement, if one of the slits is covered with an opaque screen, the pattern observed is similar to the one observed with a single slit. The spacing of the diffraction maxima and minima depends on a, the width of the slit and the spacing of the interference maxima and minima depends on the value of a and b where b is the opaque spacing between the two slits. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.

9.29 MISSING ORDERS IN A DOUBLE SLIT DIFFRACTION PATTERN

In the diffraction pattern due to a double slit discussed earlier, the slit width is taken as a and the separation between the slits as b. If the slit width a is kept constant, the diffraction pattern remains the same. Keeping a constant, if the spacing b is altered the spacing between the interference maxima changes. Depending on the relative values of a and b certain orders of interference maxima will be missing in the resultant pattern.

The directions of interference maxima are given by the equation

$$(a+b)\sin\theta = n\lambda \qquad ...(i)$$

The direction of diffraction minima are given by the equation,

$$a\sin\theta = p\lambda \qquad ...(ii)$$

In equations (i) and (ii) n and p are integers. If the values of a and b are such that both the equations are satisfied simultaneously for the same values of θ, then the positions of certain interference maxima correspond to the diffraction minima at the same position on the screen.

(i) Let $a = b$

Then $2a \sin\theta = n\lambda$

and $a \sin\theta = p\lambda$

$\therefore \quad \frac{n}{p} = 2$

or $n = 2p$

If $p = 1, 2, 3$ etc.,

then $n = 2, 4, 6$ etc.

Thus, the orders 2, 4, 6 etc. of the interference maxima will be missing in the diffraction pattern. There will be three interference maxima in the central diffraction maximum

(ii) if $2a = b$

then $3a \sin\theta = n\lambda$

and $a \sin\theta = p\lambda$

$\therefore \quad \frac{n}{p} = 3$

or $n = 3p$

If $p = 1, 2, 3$ etc.

$n = 3, 6, 9$ etc.

Thus the orders 3, 6, 9 etc. of the interference maxima will be missing in the diffraction pattern. On both sides of the central maximum, the number of interference maxima is 2 and hence there will be five interference maxima in the central diffraction maximum. The position of the third interference maximum also corresponds to the first diffraction minimum.

(iii) If $a + b = a$

i.e., if $b = 0$

The two slits join and all the orders of the interference maxima will be missing. The diffraction pattern observed on the screen is similar to that due to a single slit of width equal to $2a$.

Example 9.17. *Deduce the missing orders for a double slit Fraunhofer diffraction pattern, if the slit widths are 0.16 mm and they are 0.8 mm apart.* [Berhampur (Hons.)]

The direction of interference maxima are given by the equation,

$$(a + b) \sin\theta = n\lambda \quad ...(i)$$

The directions of diffraction minima are given by

$$a \sin\theta = p\lambda \quad ...(ii)$$

$$\therefore \quad \frac{(a+b)}{a} = \frac{n}{p}$$

Here $a = 0.16$ mm $= 0.016$ cm

$b = 0.8$ mm $= 0.080$ cm

$$\therefore \quad \frac{0.016 + 0.080}{0.016} = \frac{n}{p}$$

$$\therefore \quad \frac{n}{p} = 6$$

$n = 6p$

For values of $p = 1, 2, 3$ etc.

$n = 6, 12, 18$ etc.

Thus the orders 6, 12, 18 etc. of the interference maxima will be missing in the diffraction pattern.

Example 9.18. *A diffraction phenomenon is observed using a double slit (illuminated with light of wavelength 5000 Å. The slit width is 0.02 mm and spacing between the two slits is 0.10 mm. The distance of the screen from the slits where the observation is made is 100 cm. Calculate (i) the distance between the central maximum and the first minimum of the fringe envelope and (ii) the distance between any two consecutive double slit dark fringes.* [IAS]

Here $a = 0.02$ mm $= 2 \times 10^{-5}$ m

$b = 0.1$ mm $= 10^{-4}$ m

$(a + b) = 1.2 \times 10^{-4}$ m

$\lambda = 5000$ Å $= 5 \times 10^{-7}$ m

$d = 100$ cm $= 1$ m

(i) The angular separation between the central maximum and the first minimum is

$$\sin\theta_1 = \theta_1 = \frac{\lambda}{2(a+b)}$$

and $\theta_1 = \frac{x_1}{D}$

$$\frac{x_1}{D} = \frac{\lambda}{2(a+b)}$$

$$x_1 = \frac{\lambda D}{2(a+b)}$$

$$x_1 = \frac{5\times 10^{-7}\times 1}{2(1.2\times 10^{-4})}$$

$$x_1 = \mathbf{2.08\times 10^{-3}\ m}$$

$$x_1 = \mathbf{2.08\ mm}$$

The distance between the central maximum and the first minimum is **2.08 mm,**

(*ii*)The angular separation between two consecutive dark fringes,

$$\sin\theta_2 - \sin\theta_1 = \theta_2 - \theta_1 = \theta = \frac{3\lambda}{2(a+b)} - \frac{\lambda}{2(a+b)}$$

$$\theta = \frac{\lambda}{(a+b)}$$

Also
$$\theta = \frac{x_2}{D} = \frac{\lambda}{(a+b)}$$

$$x_2 = \frac{\lambda D}{(a+b)}$$

$$x_2 = \frac{5\times 10^{-7}\times 1}{1.2\times 10^{-4}}$$

$$x_2 = \mathbf{4.16\times 10^{-3}\ m}$$

$$x_2 = \mathbf{4.16\ mm}$$

Example 9.19. *In double slit Fraunhofer diffraction, calculate the fringe spacing on a screen 50 cm away from the slits; if they are illuminated with blue light (λ = 4800 Å). Slit separation b = 0.1 mm and slit width a = 0.020 mm.*

What is the linear distance from the central maximum to the first minimum of the fringe envelope ? [*IAS, 1989*]

Here
$$a = 0.02\ \text{mm} = 2\times 10^{-5}\ \text{m}$$

$$b = 0.1\ \text{mm} = 10^{-4}\ \text{m}$$

$$(a+b) = 1.2\times 10^{-4}\ \text{m}$$

$$\lambda = 4800\ \text{Å} = 4.8\times 10^{-7}\ \text{m}$$

$$D = 50\ \text{cm} = 0.5\ \text{m}$$

(*i*) The angular separation between two consecutive fringes

$$\sin\theta_2 - \sin\theta_1 = \theta_2 - \theta_1 = \theta = \frac{3\lambda}{2(a+b)} - \frac{\lambda}{2(a+b)}$$

$$\theta = \frac{\lambda}{(a+b)}$$

Also
$$\theta = \frac{x}{D} = \frac{\lambda}{(a+b)}$$

$$x = \frac{\lambda D}{(a+b)}$$

$$x = \frac{4.8\times 10^{-7}\times 0.5}{1.2\times 10^{-4}}$$

$$x = \mathbf{2\times 10^{-3}\ m}$$

$$x = \mathbf{2\ mm}$$

Fringe spacing on the screen = 2mm

(*ii*) The angular separation between the central maximum and the first minimum is

$$\sin\theta_1 = \theta = \frac{\lambda}{2(a+b)}$$

$$\theta = \frac{x_1}{D} = \frac{\lambda}{2(a+b)}$$

$$x_1 = \frac{D\lambda}{2(a+b)}$$

$$x_1 = \frac{0.5\times 4.8\times 10^{-7}}{2\times 1.2\times 10^{-4}}$$

$$x_1 = 10^{-3}\ \text{m}$$

$$x_1 = \mathbf{1\ mm}$$

The distance between the central maximum and the first minimum is **1 mm.**

9.30 INTERFERENCE AND DIFFRACTION

It is clear from the double slit diffraction pattern that interference takes place between the secondary waves originating from the corresponding points of the two slits and also that the intensity of the interference maxima and minima is controlled by the amount of light reaching the screen due to diffraction at the individual slits. The resultant intensity at any point on the screen is obtained by multiplying the intensity function for the interference and the intensity function for the diffraction at the two slits. The values of the intensity functions are taken for the same direction of the secondary waves. But the interference of all the secondary waves originating from the whole wavefront is termed as diffraction. Hence the pattern obtained on the screen may be called an interference pattern or a diffraction pattern. The term interference may be used for those cases in which the resultant amplitude at a point is obtained by the superimposition of two or more beams. Diffraction can be defined as the phenomenon in which the resultant amplitude at a point on the screen is obtained by integrating the effect of infinitesimally small number of elements into which the whole wavefront can be divided. Thus, the resultant diffraction pattern obtained with a double slit can be taken as a combination of the effect of both interference and diffraction.

9.31 FRAUNHOFER DIFFRACTION AT N SLITS

Fraunhofer diffraction at two slits consists of diffraction maxima and minima governed by

$$\frac{\sin^2 \alpha}{\alpha^2}$$

and sharp interference maxima and minima, in each diffraction maximum governed by the $\cos^2 \beta$ term.

To derive an expression for the intensity distribution due to diffraction at N slits, the expression for dy has to be integrated for N slits.

For a single slit,

$$dy = K\int_{-\frac{a}{2}}^{+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right] dz$$

Let $\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)$ be equal to $\phi(z)$ (*i.e.* function of z)

For N slits

$$dy = K\left[\int_{-\frac{a}{2}}^{+\frac{a}{2}} \phi(z)\,dz + \int_{d-\frac{a}{2}}^{d+\frac{a}{2}} \phi(z)\,dz + \int_{2d-\frac{a}{2}}^{2d+\frac{a}{2}} \phi(z)\,dz + \ldots + \int_{(N-1)d-\frac{a}{2}}^{(N-1)d+\frac{a}{2}} \phi(z)\,dz\right]$$

On simplification

$$y = Ka\frac{\sin\alpha}{\alpha}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{2d\sin\theta}{\lambda}\right) + \ldots\ldots + \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{(N-1)\,d\sin\theta}{\lambda}\right)\right]$$

Here $\alpha = \frac{\pi a\sin\theta}{\lambda}$

For a general trigonometric summation

$$\sum_{p=0}^{p=n} \sin(x+pm) = \frac{\sin\left(x+\frac{nm}{2}\right)\sin\left[\left(\frac{n+1}{2}\right)m\right]}{\sin\left(\frac{m}{2}\right)}$$

Here $x = 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$

and $m = \frac{2\pi d\sin\theta}{\lambda}$

$$= 2\left[\frac{\pi d\sin\theta}{\lambda}\right] = 2\beta$$

$$\beta = \frac{\pi d\sin\theta}{\lambda}$$

$$n = (N-1)$$

$$y = \frac{Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin\left(x+\frac{(N-1)m}{2}\right)\sin\left(\frac{Nm}{2}\right)\right]}{\sin\left(\frac{m}{2}\right)}$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\frac{\sin\left(\frac{Nm}{2}\right)}{\sin\left(\frac{m}{2}\right)}\left[\sin\left(x+\frac{(N-1)\,m}{2}\right)\right]$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\frac{(\sin N\beta)}{\sin\beta}$$

$$\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{(N-1)\,d\sin\theta}{2\lambda}\right)\right] \quad ...(i)$$

The intensity at a point P'

$$I = K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) \quad ...(ii)$$

The maximum intensity, when $\alpha = 0$ and $\beta = 0$

$$I_0 = K^2a^2$$

$$\therefore \quad I = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) \quad ...(iii)$$

Since the expression $\frac{\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to a single slit. The additional factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference effects due to the secondary waves from the N slits.

The numerator will be zero when

$$N\beta = 0, \pi, 2\pi, 3\pi \ldots \text{etc} = k\pi$$

The denominator is also zero when

$$\beta = 0, \pi, 2\pi, 3\pi \ldots \text{etc.}$$

Since the quotient $\frac{0}{0}$ is indeterminate, therefore $N\beta = k\pi$ gives the condition for minimum intensity for all values of k other than

$$k = 0, N, 2N, 3N \text{ etc.}$$

The directions of principal maxima correspond to the values of $k = 0, N, 2N$ etc.

$$\therefore \quad N\beta = \frac{N\pi d\sin\theta}{\lambda}$$

or $$k\pi = \frac{N\pi d\sin\theta}{\lambda}$$

For the directions of principal maxima,

$$k = 0, 1N, 2N, 3N \text{ etc.} = nN$$

When $$n = 0, 1, 2, \ldots 3 \text{ etc.}$$

$$nN\pi = \frac{N\pi d\sin\theta}{\lambda}$$

$$d\sin\theta = n\lambda$$

Here $$n = 0, 1, 2, 3 \text{ etc.}$$

If the width of the slit is a and the width of the opaque spacing is b.

$$d = (a+b)$$

$$(a+b)\sin\theta = n\lambda$$

Putting $n = 1, 2, 3$ etc., the directions of principal maxima $\theta_1, \theta_2, \theta_3 \ldots\ldots$ etc. can be determined.

For values of k in between 0 and N, between N and $2N$, etc., there are $(N-1)$ secondary minima and $(N-2)$ secondary maxima.

The intensity distribution due to diffraction and N slits is shown in Fig. 9.46.

9.32 INTENSITY OF PRINCIPAL MAXIMA

In a diffraction grating there are about 6000 narrow slits in one cm. For values of $\beta = k\pi$ and $\beta = 0, \pi, 2\pi$ etc.

$$\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$$

It is indeterminate.

To find the value of this limit, the numerator and the denominator are differentiated

$$\therefore \quad \underset{\beta\to n\pi}{\text{Lt}}\frac{\sin N\beta}{\sin\beta} = \underset{\beta\to n\pi}{\text{Lt}}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$= \underset{\beta\to n\pi}{\text{Lt}}\frac{N\cos N\beta}{\cos\beta} = \pm N$$

Thus, the resultant amplitude is proportional to N and resultant intensity is proportional to N^2.

$$\therefore \quad I = N^2 I_0 \left(\frac{\sin^2 \alpha}{\alpha^2} \right)$$

These maxima are intense and are called principal maxima.

9.33 PLANE DIFFRACTION GRATING

A diffraction grating is an extremely useful device and in one of its forms it consists of a very large number of narrow slits side by side. The slits are separated by opaque spaces. When a wavefront is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called a transmission grating. The secondary waves from the positions of the slits interfere with one another, similar to the interference of waves in Young's experiment. Joseph Fraunhofer used the first gratings which consisted of a large number of parallel fine wires stretched on a frame. Now, gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a fine diamond point. The space in between any two lines is transparent to light and the lined portion is opaque to light. Such surfaces act as transmission gratings. If, on the other hand, the lines are drawn on a silvered surface (plane or concave) then light is reflected from the positions of the mirror in between any two lines and such surfaces act as reflection gratings.

If the spacing between the lines is of the order of the wave length of light, then an appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrum contain 10,000 lines per cm. Gratings, with originally ruled surfaces are only few. For practical purposes, replicas of the original grating are prepared. On the original grating surface a thin layer of collodion solution is poured and the solution is allowed to harden. Then, the film of collodion is removed from the grating surface and then fixed between two glass plates. This serves as a plane transmission grating. A large number of replicas are prepared in this way from a single original ruled surface.

9.34 THEORY OF THE PLANE TRANSMISSION GRATING

In Fig. 9.44, XY is the grating surface and MN is the screen, both perpendicular to the plane of the paper. The slits are all parallel to one another and perpendicular to the plane of the paper. Here AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing between any two consecutive slits is b. Let a plane wavefront be incident on the grating surface. Then all the secondary waves travelling in the same direction as that of the incident light will come to focus at the

point P on the screen. The screen is placed at the focal plane of the collecting lens. The point P where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

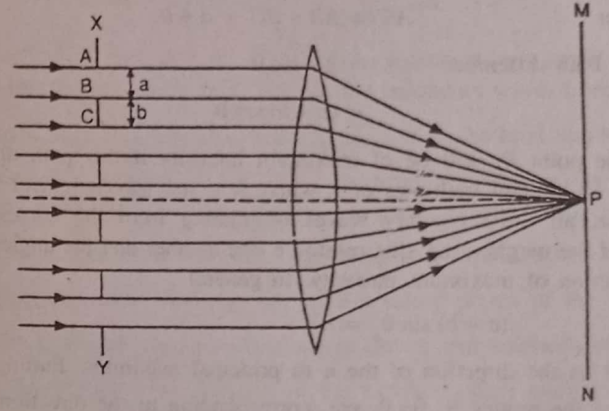

Fig. 9.44

Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light (Fig. 9.45). The collecting lens also is suitably rotated such that the axis of the lens is

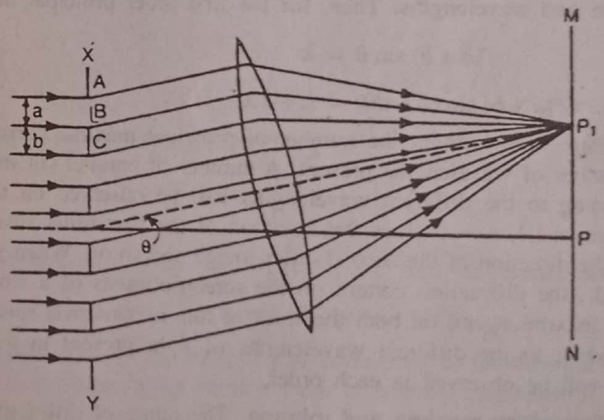

Fig. 9.45

parallel to the direction of the secondary waves. These secondary waves come to focus at the point P_1 on the screen. The intensity at P_1 will depend on the path difference between the secondary waves originating from the corresponding points A and C of two neighbouring slits. In Fig. 9.45, $AB = a$ and $BC = b$. The path difference between the secondary waves starting from A and C is equal to $AC \sin \theta$. (This will be clear from Fig. 9.41).

But $\qquad AC = AB + BC = a + b$

$\therefore$ Path difference $\qquad = AC \sin \theta$

$$= (a+b) \sin \theta$$

The point P_1 will be of maximum intensity if this path difference is equal to integral multiples of λ where λ is the wavelength of light. In this case, all the secondary waves originating from the corresponding points of the neighouring slits reinforce one another and the angle θ gives the direction of maximum intensity. In general

$$(a+b) \sin \theta_n = n\lambda \qquad ...(i)$$

where θ_n is the direction of the n th principal maximum. Putting $n = 1$, 2, 3 etc., the angles $\theta_1, \theta_2, \theta_3$ etc. corresponding to the directions of the principal maxima can be obtained.

If the incident light consists of more than one wavelength, the beam gets dispersed and the angles of diffraction for different wavelengths will be different. Let λ and $\lambda + d\lambda$ be two nearby wavelengths present in the incident light and θ and $(\theta + d\theta)$ be the angles of diffraction corresponding to these two wavelengths. Then, for the first order principal maxima

$$(a+b) \sin \theta = \lambda$$

and $\qquad (a+b) \sin (\theta + d\theta) = \lambda + d\lambda$

Thus, is any order, the number of principal maxima corresponds to the number of wavelengths present. A number of parallel slit images corresponding to the different wavelengths will be observed on the screen. In equation (i), $n = 1$ gives the direction of the first order image, $n = 2$ gives the direction of the second order image and so on. When white light is used, the diffraction pattern on the screen consists of a white central bright maximum and on both the sides of this maximum a spectrum corresponding to the different wavelengths of light present in the incident beam will be observed in each order.

Secondary maxima and minima. The angle of diffracting θ_n corresponding to the direction of the n th principal maximum is given by the equation

$$(a+b) \sin \theta_n = n\lambda$$

In this equation $(a+b)$ is called the **grating element**. Here a is the width of the slit and b is the width of the opaque portion. For a grating with 15,000 lines per inch the value of

$$(a+b) = \frac{2.54}{15000} \text{ cm}$$

Now, let the angle of diffraction be increased by a small amount $d\theta$ such that the path difference between the secondary waves from the points A and C (Fig. 9.45) increases by $\frac{\lambda}{N}$. Here N is the total number of lines on the grating surface. Then, the path difference between the secondary waves from the extreme points of the grating surface will be $\frac{\lambda}{N} \cdot N = \lambda$. Assuming the whole wavefront to be divided into two halves, the path difference between the corresponding points of the two halves will be $\frac{\lambda}{2}$ and all the secondary waves cancel one another's effect. Thus, $(\theta_n + d\theta)$ will give the direction of the first secondary minimum after the

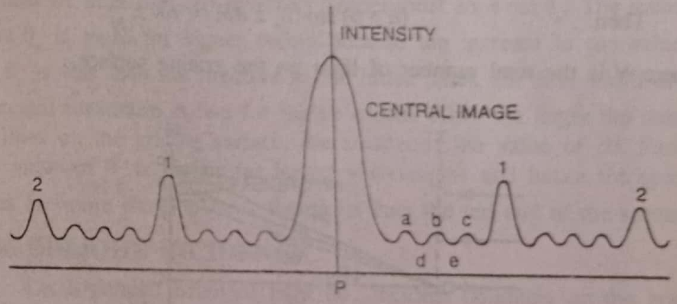

Fig. 9.46

n th primary maximum. Similarly, if the path difference between the secondary waves from the points A and C is $\frac{2\lambda}{N}, \frac{3\lambda}{N}$ etc. for gradually increasing values of $d\theta$, these angles correspond to the directions of 2 nd, 3 rd etc. secondary minima after the n th primary maximum. If the value is $\frac{2\lambda}{N}$, then the path difference between the secondary waves from the extreme points of the grating surface is $\frac{2\lambda}{N} \times N = 2\lambda$ and considering the wavefront to be divided into 4 portions, the concept of the 2 nd secondary

Further, if the value of a and θ_n are such that

$$a \sin\theta_n = \lambda \qquad ...(ii)$$

then, the effect of the wavefront from any particular slit will be zero. Considering each slit to be made up of two halves, the path difference between the secondary waves from the corresponding points will be $\frac{\lambda}{2}$ and they cancel one another's effect. If the two conditions given by equations (*i*) and (*ii*) are simultaneously satisfied, then dividing (*i*) by (*ii*)

$$\frac{(a+b)\sin\theta_n}{a\sin\theta_n} = \frac{n\lambda}{\lambda}$$

or

$$\frac{a+b}{a} = n \qquad ...(iii)$$

In equation (*iii*), the values of $n = 1, 2, 3$ etc. refer to the order of the principal maxima that are absent in the diffraction pattern.

(*i*) if $\quad \frac{a+b}{a} = 1;\ b = 0$

In this case, the first order spectrum will be absent and the resultant diffraction pattern is similar to that due to single slit,

(*ii*) if $\quad \frac{a+b}{a} = 2;\ a = b$

i.e., the width of the slit is equal to the width of the opaque spacing between any two consecutive slits. It this case, the second order spectrum will be absent.

9.38 OVERLAPPING OF SPECTRAL LINES

If the light incident on the grating surface consists of a large range of wavelengths, then the spectral lines of shorter wavelength and of higher order overlap on the spectral lines of longer wavelength and of lower order. Let the angle of diffraction θ be the same for (*i*) the spectral line of wavelength λ_1 in the first order, (*ii*) the spectral line of wavelength λ_2 in the second order and (*iii*) the spectral line of wavelength λ_3 in the third order. Then

$$(a+b)\sin\theta = 1\cdot\lambda_1 = 2\lambda_2 = 3\lambda_3 = \ldots$$

The red line of wavelength 7000 Å in the third order, the green line of wavelength 5250 Å in the fourth order and the violet line of wavelength 4200 Å in the fifth order are all formed at the same position of the screen because,

$$(a+b)\sin\theta = 3\times 7000\times 10^{-8}$$
$$= 4\times 5250\times 10^{-8}$$
$$= 5\times 4200\times 10^{-8}$$

Here $(a + b)$ is expressed in cm.

For the visible region of the spectrum, there is no overlapping of the spectral lines. The range of wavelengths for the visible part of the spectrum is 4000 Å to 7200 Å. Thus, the diffracting angle for the red end of the spectrum in the first order is less than the diffracting angle for the violet end of the spectrum in the second order. If, however, the observations are made with a photographic plate, the spectrum recorded may extend up to 2000 Å in the ultraviolet region. In this case, the spectral line corresponding to a wavelength of 4000 Å in the first order and a spectral line of wavelength 2000 Å in the second order overlap. Suitable fillers are used to absorb those wavelengths of the incident light which will overlap with the spectral lines in the region under investigation.

9.39 DETERMINATION OF WAVELENGTH OF A SPECTRAL LINE USING PLANE TRANSMISSION GRATING

In the laboratory, the grating spectrum of a given source of light is obtained by using a spectrometer. Initially all the adjustments of the spectrometer are made and it is adjusted for parallel rays by Schuster's method. The slit of the collimator is illuminated by monochromatic light (say light from a sodium lamp) and the position of the telescope is

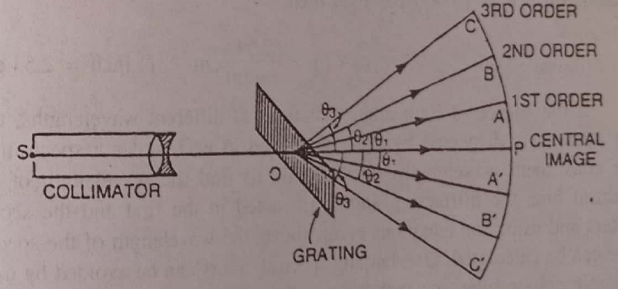

Fig. 9.50

adjusted such that the image of the slit is obtained at the position of the vertical cross-wire in the field of view of the telescope. Now the axes of the collimator and the telescope are in the same line. The position of the telescope is noted on the circular scale and 90° is added to this reading. The telescope is turned to this position. In this position the axis of the telescope is perpendicular to the axis of the collimator. The position of the telescope is fixed. The given transmission grating is mounted at the centre of the prism table such that the grating surface is perpendicular to the prism table. The prism table is suitably rotated such that the image of the slit reflected from the grating surface is obtained in the centre of the field of view of the telescope. This means that the parallel rays of light from the collimator are incident at an angle of 45° on the grating surface because the axes of the collimator and the telescope are perpendicular to each other. The reading of the prism table is noted and adding 45° to this reading the prism table is suitably rotated to the new position so that the grating surface is normal to the incident light.

If the wavelength of sodium light is to be determined, then the angles of diffraction θ_1 and θ_2 corresponding to the first and the second order principal maxima are determined (Fig. 9.50). *OA*, *OB* etc., give the directions of the telescope corresponding to the first and second order images. *A′*, *B′*, etc. refer to the positions of these images towards the left of the central maximum. The angles *AOA′* and *BOB′* are measured and half of these angles measure θ_1 and θ_2. Then

$$(a+b)\sin\theta_1 = 1\,\lambda \qquad ...(i)$$

and

$$(a+b)\sin\theta_2 = 2\,\lambda \qquad ...(ii)$$

Then the value of λ is calculated from equations (*i*) and (*ii*) and the mean value is taken. $(a + b)$ is the grating element and it is equal to the reciprocal of the number of lines per cm. If the number of lines on the grating surface is 15,000 per inch then

$$(a+b) = \frac{2.54}{15000}\ \text{cm} \qquad (1\ \text{inch} = 2.54\ \text{cm})$$

If the source of light emits radiations of different wavelengths, then the beam gets dispersed by the grating and in each order a spectrum of the constituent wavelengths is observed. To find the wavelength of any spectral line, the diffracting angles are noted in the first and the second orders and using the equations given above, the wavelength of the spectral line can be calculated, Overlapping spectral orders can be avoided by using suitable colour filters so that the wavelengths beyond the range of study are eliminated.

With a diffraction grating, the wavelength of a spectral line can be determined very accurately. The method involves only the accurate measurement of the angles of diffraction.

Take

$$\lambda = 6000\ \text{Å} = 6000 \times 10^{-8}\ \text{cm}$$

and

$$(a+b) = \frac{2.54}{15000}$$

Then from the equations,

$$(a+b)\sin\theta_1 = 1\lambda$$

and

$$(a+b)\sin\theta_2 = 2\lambda$$

$$\theta_1 = 20° - 45'$$

and

$$\theta_2 = 45° - 7'$$

As the angles are large they can be measured accurately with a properly calibrated spectrometer. The number of lines per inch (or cm), is given on the grating by the manufacturing company and hence $(a + b)$ can be calculated. As the method does not involve measurements of very small distances (as in the case of interference experiments) an accurate value of λ can be obtained.

Example 9.21. *Light is incident normally on a grating 0.5 cm wide with 2500 lines. Find the angles of diffraction for the principal maxima of the two sodium lines in the first order spectrum.*

$$\lambda_1 = 5890\ \text{Å} \quad \textit{and} \quad \lambda_2 = 5896\ \text{Å}.$$

Are the two lines resolved ? (*Punjab*)

$$(a+b)\sin\theta_n = n\lambda$$

$$\text{Width} = 0.5\ \text{cm}$$

$$\text{Total number of lines} = N = 2500\ \text{lines}$$

$$\text{Number of lines / cm} = N' = \frac{2500}{0.5} = 5000\ \text{lines / cm}$$

$$\text{Grating element } (a+b) = \frac{1}{N'} = \frac{1}{5000}\ \text{cm}$$

(1) For

$$\lambda_1 = 5890 \times 10^{-8}\ \text{cm}, \quad n = 1$$

$$(a+b)\sin\theta_1 = 1 \times \lambda_1$$

$$\frac{1}{5000}\sin\theta_1 = 5890 \times 10^{-8}$$

$$\sin\theta_1 = 0.2945$$

$$\theta_1 = 17.1°$$

(2) For

$$\lambda_2 = 5896 \times 10^{-8}\text{ cm},\ n = 1$$

$$(a+b)\sin\theta'_1 = 1 \times \lambda_2$$

$$\left(\frac{1}{5000}\right)\sin\theta'_1 = 5896 \times 10^{-8}$$

$$\sin\theta'_1 = 0.2948$$

$$\theta_1 = \mathbf{17.2°}$$

(3) The condition for just resolution is

$$\frac{\lambda}{d\lambda} = nN$$

Here

$$\lambda = 5890 \times 10^{-8}\text{ cm}$$

$$d\lambda = 6 \times 10^{-8}\text{ cm}$$

$$n = 1$$

$$N = ?$$

$$\frac{5890 \times 10^{-8}}{6 \times 10^{-8}} = 1 \times N$$

$$N = \mathbf{982}$$

As the total number of lines on the grating is 2500, the two lines will appear well resolved.

Example 9.22. *A parallel beam of monochromatic light is allowed to be incident normally on a plane grating having 1250 lines per cm and a second order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line.* *(Agra)*

$$(a+b)\sin\theta = n\lambda$$

Here

$$(a+b) = \frac{1}{1250},\quad \theta = 30°,\ \sin 30 = \tfrac{1}{2}$$

$$n = 2,\ \lambda = ?$$

$$\lambda = \frac{(a+b)\sin\theta}{n} = \frac{1 \times 1}{1250 \times 2 \times 2}$$

$$= \mathbf{2 \times 10^{-4}\ cm}$$

Example 9.23. *What is the highest order spectrum, which may be seen with monochromatic light of wavelength 6000 Å by means of a diffraction grating with 5000 lines/cm.* *[Delhi B.Sc. (Hons)]*

Here $(a+b)\sin\theta_n = n\lambda$

The maximum possible value of

$$\sin\theta_n = 1$$

$$(a+b) = n\lambda$$

Here

$$(a+b) = \frac{1}{5000}\text{ cm}$$

$$\lambda = 6000 \times 10^{-8}\text{ cm},\ n = ?$$

$$\frac{1}{5000} = n \times 6000 \times 10^{-8}$$

or

$$n = \mathbf{3.33}$$

The highest order of the spectrum that can be seen is 3.

Example 9.14. *A plane grating has 15000 lines per inch. Find the angle of separation of the 5048 Å and 5016 Å lines of helium in the second order spectrum.* *[Delhi B.Sc. (Hons)]*

Here

$$\lambda_1 = 5016\text{ Å} = 5016 \times 10^{-8}\text{ cm}$$

$$\lambda_2 = 5048\text{ Å} = 5048 \times 10^{-8}\text{ cm}$$

$$n = 2$$

$$(a+b) = \frac{2.54}{15000}\text{ cm}$$

Let θ_1 and θ_2 be the angles of diffraction for the second order for the wavelengths λ_1 and λ_2 respectively.

$$\therefore\quad (a+b)\sin\theta_1 = 2 \times \lambda_1;\quad (a+b)\sin\theta_2 = 2 \times \lambda_2$$

$$\sin\theta_1 = \frac{2\lambda_1}{(a+b)} = \frac{2 \times 5016 \times 10^{-8} \times 15000}{2.54}$$

$$= 0.5924$$

or

$$\theta_1 = 36° - 20'$$

$$\sin\theta_2 = \frac{2\lambda_2}{a+b} = \frac{2 \times 5048 \times 10^{-8} \times 15000}{2.54}$$

$$= 0.5962$$

or

$$\theta_2 = 36° - 36'$$

$$\therefore\ \text{Angle of separation, } (\theta_2 - \theta_1) = 16'$$

Example 9.25. *A plane transmission grating having 6000 lines/cm is used to obtain a spectrum of light from a sodium lamp in the second order. Calculate the angular separation between the two sodium lines whose wavelengths are 5890 Å and 5896 Å.* *[Bombay]*

$$(a+b)\sin\theta_1 = 2\lambda_1$$

$$(a+b)\sin\theta_2 = 2\lambda_2$$

Here $$a+b = \frac{1}{6000}$$

$$\lambda_1 = 5890\times 10^{-8}\text{ cm}$$

$$\lambda_2 = 5896\times 10^{-8}\text{ cm}$$

$\therefore$ $$\sin\theta_1 = 2\times 5890\times 10^{-8}\times 6000 = 0.7068$$

or $$\theta_1 = 44^\circ - 58'$$

and $$\sin\theta_2 = 2\times 5896\times 10^{-8}\times 6000 = 0.7075$$

or $$\theta_2 = 45^\circ - 2'$$

The angular separation $= \theta_2 - \theta_1$

$= 4$ minutes of an arc.

Example 9.26. *Light which is a mixture of two wavelengths 5000 Å and 5200 Å is incident normally on a plane transmission grating having 10,000 lines per cm. A lens of focal length 150 cm is used to observe the spectrum on a screen. Calculate the separation in cm of the two lines in the first order spectrum.*

Here $$a+b = \frac{1}{10000} = 10^{-4}\text{ cm}$$

$$\lambda_1 = 5000\text{ Å} = 5.0\times 10^{-5}\text{ cm}$$

$$\lambda_2 = 5200\text{ Å} = 5.2\times 10^{-5}\text{ cm}$$

$$n = 1$$

$$(a+b)\sin\theta_1 = n\lambda_1$$

$$\sin\theta_1 = \frac{n\lambda_1}{a+b} = \frac{1\times 5.0\times 10^{-5}}{10^{-4}} = 0.5$$

$$\theta_1 = 30^\circ$$

Similarly $$\sin\theta_2 = \frac{n\lambda_2}{a+b} = \frac{1\times 5.2\times 10^{-5}}{10^{-4}} = 0.52$$

or $$\theta_2 = 31.3^\circ$$

Further $$\tan\theta_1 = \frac{x_1}{f}$$

and $$\tan\theta_2 = \frac{x_2}{f}$$

$\therefore$ $$(x_2 - x_1) = f\,[\tan\theta_2 - \tan\theta_1]$$

$$= 150\,[0.6087 - 0.5774] = 150\times 0.0313$$

$$= \mathbf{4.695\text{ cm}}$$

Example 9.27. *Light of wavelength 5000 Å is incident normally on a plane transmission grating. Find the difference in the angles of deviation in the first and third order spectra. The number of lines per cm on the grating surface is 6000.*

Here $$\lambda = 5000\text{ Å} = 5\times 10^{-5}\text{ cm}$$

$$(a+b) = \frac{1}{6000}$$

For the first order,

$$(a+b)\sin\theta_1 = 1\times\lambda$$

$$\sin\theta_1 = \frac{\lambda}{a+b} = 5\times 10^{-5}\times 6000 = 0.30$$

or $$\theta_1 = 17.5^\circ$$

For the third order,

$$(a+b)\sin\theta_3 = 3\lambda$$

$$\sin\theta_3 = \frac{3\lambda}{a+b} = 3\times 5\times 10^{-5}\times 6000 = 0.90$$

$$\theta_3 = 64.2^\circ$$

$$\theta_3 - \theta_1 = 64.2 - 17.5 = \mathbf{46.7^\circ}$$

Example 9.28. *In a plane transmission grating the angle of diffraction for the second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Calculate the number of lines in one cm of the grating surface.* *(Delhi)*

$$(a+b)\sin\theta_n = n\lambda$$

Here $n = 2,\ \lambda = 5\times10^{-5}$ cm, $\theta_2 = 30^\circ$

$$\sin 30 = 0.5$$

$$(a+b) = \frac{n\lambda}{\sin\theta_2}$$

$$= \frac{2\times5\times10^{-5}}{0.5} = 10^{-3}\text{ cm}$$

$\therefore$ Number of lines per cm $= N'$

$$= \frac{1}{(a+b)} = \mathbf{1000}$$

Example 9.29. *How many orders will be visible if the wavelength of the incident radiation is 5000 Å and the number of lines on the grating is 2620 in one inch.* [Delhi, 1978]

Here $(a+b)\sin\theta_n = n\lambda$

The maximum possible value of

$$\sin\theta_n = 1$$

$\therefore$ $(a+b) = n\lambda$

Here, $(a+b) = \dfrac{2.54}{2620}$ cm

$$\lambda = 5000\text{ Å} = 5\times10^{-5}\text{ cm}$$

$$n = ?$$

$$n = \frac{(a+b)}{\lambda}$$

$$n = \frac{2.54}{2620\times5\times10^{-5}}$$

$$n > 19$$

The highest order of the spectrum that can be seen is **19**.

Example 9.30. *A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 5000 lines/cm and the second order spectral line is found to be diffracted through 30°. Calculate the wavelength of light.* [Delhi (Sub) 1978]

$$(a+b)\sin\theta = n\lambda$$

Here $(a+b) = \dfrac{1}{5000}$ cm

$$\theta = 30^\circ,\quad \sin 30^\circ = 0.5$$

$$n = 2$$

$$\lambda = ?$$

$$\lambda = \frac{(a+b)\sin\theta}{n}$$

$$= \frac{0.5}{5000\times2}$$

$$\lambda = \mathbf{5\times10^{-5}\ cm}$$

$$\lambda = \mathbf{5000\ Å}$$

Example 9.31. *A diffraction grating used at normal incidence gives a line, λ_1 = 6000 Å in a certain order superimposed on another line λ_1 = 4500 Å of the next higher order. If the angle of diffraction is 30°, how many lines are there in a cm in the grating.* [Delhi (Hons) 1977]

Here, $\lambda_1 = 6000\text{ Å} = 6\times10^{-5}$ cm

$$\lambda_2 = 4500\text{ Å} = 4.5\times10^{-5}\text{ cm}$$

$$\theta_1 = \theta_2 = 30^\circ$$

$\therefore$ $\sin\theta_1 = \sin\theta_2 = 0.5$

$$(a+b)\sin\theta_1 = n_1\lambda_1 \quad ...(i)$$

$$(a+b)\sin\theta_2 = (n_1+1)\lambda_2 \quad ...(ii)$$

Dividing (*ii*) by (*i*)

$$1 = \frac{(n_1+1)}{n_1}\times\frac{4.5\times10^{-5}}{6\times10^{-5}}$$

$\therefore$ $n_1 = 3$

From equation (*i*)

$$(a+b)\times0.5 = 3\times6\times10^{-5}$$

$$(a+b) = 36\times10^{-5}\text{ cm}$$

The number of lines / cm

$$= \frac{1}{(a+b)}$$

$$= \frac{1}{36 \times 10^{-5}}$$

$$= \mathbf{2778} \text{ lines / cm}$$

Example 9.32. *Light is incident normally on a grating of total ruled width 5×10^{-3} m with 2500 lines in all. Find the angular separation of the sodium lines in the first order spectrum. Wavelengths of lines are 589 and 589.6 nm. Can they be seen distinctly ?* [IAS, 1983]

Number of lines per metre

$$= \frac{2500}{5 \times 10^{-3}} = 5 \times 10^5$$

For $n = 1$

(i) $(a+b) \sin \theta_1 = n \lambda_1$

$$\frac{\sin \theta_1}{5 \times 10^5} = 1 \times 589 \times 10^{-9}$$

$$\sin \theta_1 = 0.2945$$

$$\theta_1 = 17.1275°$$

(ii) $(a+b) \sin \theta_2 = \lambda_2$

$$\frac{\sin \theta_2}{5 \times 10^5} = 589.6 \times 10^{-9}$$

$$\sin \theta_2 = 0.2948$$

$$\theta_2 = 17.1455$$

$$d\theta = \theta_2 - \theta_1 = 17.1455 - 17.1275 = 0.0180$$

But, as the separation is very small, the two lines cannot be seen distinctly.

Example 9.33. *Parallel been of light is incident normally on a diffraction grating having 6000 line/cm. Find the angular separation between the maxima for wavelengths 5890 Å and 5896 Å in the second order.* [IAS, 1986]

Here $n = 2$

$$\lambda_1 = 5890 \text{ Å} = 5890 \times 10^{-10} \text{ m}$$

$$\lambda_2 = 5896 \text{ Å} = 5896 \times 10^{-10} \text{ m}$$

Number of lines $= 6000$ per cm

$$= 6 \times 10^5 \text{ per metre}$$

$$\therefore \quad (a+b) = \frac{1}{6 \times 10^5} \text{ metre}$$

(i) $(a+b) \sin \theta_1 = n\lambda_1$

$$\sin \theta_1 = \frac{n\lambda_1}{(a+b)}$$

$$\sin \theta_1 = \frac{2 \times 5890 \times 10^{-10} \times 6 \times 10^5}{1}$$

$$\sin \theta_1 = 0.707$$

$$\theta_1 = 45°$$

(ii) $(a+b) \sin \theta_2 = n \lambda_2$

$$\sin \theta_2 = \frac{n\lambda_2}{(a+b)}$$

$$\sin \theta_2 = \frac{2 \times 5896 \times 10^{-10} \times 6 \times 10^5}{1}$$

$$\sin \theta_2 = 0.7075$$

$$\theta_2 = 45° - 2'$$

Angular separation

$$d\theta = \theta_2 - \theta_1 = 2' \text{ minutes of an arc}$$

Example 9.34. *Monochromatic light of wavelength 6.56×10^{-7} m falls normally on a grating 2 cm wide. The first order spectrum is produced at an angle of 18° – 15′ from the normal. Deduce the total number of lines on the grating.* [IAS, 1987]

Here $\lambda = 6.56 \times 10^{-7}$ m

$$\text{width} = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$n = 1$$

$$\theta_1 = 18° - 15', \ \sin \theta_1 = 0.3131$$

$$(a+b) \sin \theta_1 = n\lambda$$

$$(a+b) = \frac{n\lambda}{\sin \theta_1}$$

$$(a+b) = \frac{1 \times 6.56 \times 10^{-7}}{0.3131}$$

$$(a+b) = 20.95 \times 10^{-7} \text{ m}$$

Number of lines per metre

$$= \frac{1}{(a+b)}$$

$$= \frac{1}{20.95 \times 10^{-7}}$$

$$= 4.77 \times 10^5 \text{ per metre}$$

Total number of lines,

$$N = 4.773 \times 10^5 \times 2 \times 10^{-2}$$

$$N = \mathbf{9.546 \times 10^3}$$

$$N = \mathbf{9546}$$

Example 9.35. *How many orders will be visible if the wavelength of light is 5000 Å and the number of lines per inch on the grating is 2620 ?* [Lucknow, 1990]

Here $(a+b)\sin\theta_n = n\lambda$

The maximum possible value of

$$\sin\theta_n = 1$$

$$\therefore \quad (a+b) = n\lambda$$

Here $(a+b) = \frac{2.54}{2620}$ cm

$$(a+b) = 9.694 \times 10^{-4} \text{ cm}$$

$$(a+b) = 9.694 \times 10^{-6} \text{ m}$$

$$\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}$$

$$n = ?$$

$$n = \frac{(a+b)}{\lambda}$$

$$n = \frac{9.694 \times 10^{-6}}{5 \times 10^{-7}} = 19.388$$

$$n > 19$$

Number of orders visible in the spectrum = 19

Example 9.36. *How many orders will be observed by a grating having 4000 lines/cm, if it is stimulated by visible light is the range 4000 Å to 7000 Å.* [Kanpur, 1991]

Here $(a+b) = \left(\frac{1}{4000}\right)$ cm $= 2.5 \times 10^{-4}$ cm

$$(a+b) = 2.5 \times 10^{-6} \text{ m}$$

(1) $\lambda_1 = 4000 \text{ Å} = 4 \times 10^{-7}$ m

$$\sin\theta = 1$$

$$(a+b)\sin\theta = n_1\lambda_1$$

$$n_1 = \frac{(a+b)}{\lambda_1}$$

$$n_1 = \frac{2.5 \times 10^{-6}}{4 \times 10^{-7}} = 6.25$$

(2) $\lambda_2 = 7000 \text{ Å} = 7 \times 10^{-7}$ m

$$(a+b)\sin\theta = n_2\lambda_2$$

$$\sin\theta = 1$$

$$n_2 = \frac{(a+b)}{\lambda_2} = \frac{2.5 \times 10^{-6}}{7 \times 10^{-7}}$$

$$n_2 = 3.57$$

The order of the spectrum varies from 3 to 6 depending upon the wavelength of the visible range.

Example 9.37. *A diffraction grating used at normal incidence gives a green line, λ = 5400 Å in a certain order superimposed on the violet line, λ = 4500 Å of the next higher order. If the angle of diffraction is 10°, how many lines are there per centimetre in the grating ?* [Delhi,(Hons) 1992]

Here, grating element = (a + b)

$$(a+b)\sin\theta = n\lambda_1 \quad ...(i)$$

$$(a+b)\sin\theta = (n+1)\lambda_2 \quad ...(ii)$$

$$\therefore \quad n\lambda_1 = (n+1)\lambda_2$$

$$\lambda_1 = 5400 \text{ Å} = 5.4 \times 10^{-7} \text{ m}$$

$$\lambda_2 = 4050 \text{ Å} = 4.05 \times 10^{-7} \text{ m}$$

$$\therefore \quad n \times 5.4 \times 10^{-7} = (n+1) \times 4.05 \times 10^{-7}$$

$$n = 3$$

Also $\theta = 30°, \sin 30° = 0.5$

$$(a+b)\sin\theta = n\lambda_1$$

$$(a+b)\times 0.5 = 3\times 5.4\times 10^{-7}$$

$$(a+b) = 3.24\times 10^{-6}\text{ m}$$

$$(a+b) = 3.24\times 10^{-4}\text{ cm}$$

Number of lines per cm,

$$N = \frac{1}{(a+b)}$$

$$N = \frac{1}{3.24\times 10^{-4}}$$

$$N = \mathbf{3086\ lines/cm}$$

9.40 DISPERSIVE POWER OF A GRATING

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines. It can also be defined as the difference in the angle of diffraction per unit change in wavelength. The diffraction of the n th order principal maximum for a wavelength λ, is given by the equation,

$$(a+b)\sin\theta = n\lambda \qquad ...(i)$$

Differentiating this equation with respect to θ and λ [$(a+b)$ is constant and n is constant in a given order]

$$(a+b)\cos\theta\, d\theta = n\, d\lambda$$

or
$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

or
$$\frac{d\theta}{d\lambda} = \frac{nN'}{\cos\theta} \qquad ...(ii)$$

In equation (*ii*) $\frac{d\theta}{d\lambda}$ is the dispersive power, n is the order of the spectrum, N' is the number of lines per cm of the grating surface and θ is the angle of diffraction for the n tn order principal maximum of wavelength λ.

From equation (*ii*), it is clear, that the dispersive power of the grating is (I) directly proportional to the order of the spectrum, (2) directly proportional to the number of lines per cm and (3) inversely proportional to cos θ. Thus, the angular spacing of any two spectral lines is double in the second order spectrum in comparison to the first order.

Secondly, the angular dispersion of the lines is more with a grating having larger number of lines per cm. Thirdly, the angular dispersion is minimum when $\theta = 0$. If the value of θ is not large the value of cos θ can be taken as unity approximately and the influence of the factor cos θ in the equation (*ii*) can be neglected.

Neglecting the influence of cos θ, it is clear that the angular dispersion of any two spectral lines (in a particular order) is directly proportional to the difference in wavelength between the two spectral lines. A spectrum of this type is called a normal spectrum.

If the linear spacing of two spectral lines of wavelengths λ and $\lambda + d\lambda$ is dx in the focal plane of the telescope objective or the photographic plate, then

$$dx = f\, d\theta$$

where f is the focal length of the objective. The linear dispersion

$$\frac{dx}{d\lambda} = f\frac{d\theta}{d\lambda} = \frac{f\cdot nN'}{\cos\theta} \qquad ...(iii)$$

or
$$dx = \frac{fnN'}{\cos\theta}\cdot d\lambda$$

The linear dispersion is useful in studying the photographs of a spectrum.

9.41 PRISM AND GRATING SPECTRA

For dispersing a given beam of light and for studying the resultant spectrum, a diffraction grating is mostly used instead of a prism.

The following points give broadly the distinction between the spectra obtained with a grating and a prism.

(*i*) With a grating, a number of spectra of different orders can be obtained on the two sides of the central maximum whereas with a prism only one spectrum can be obtained.

(*ii*) The spectra obtained with a grating are comparatively pure than those with a prism.

(*iii*) Knowing the grating element $(a + b)$ and measuring the diffracting single, the wavelength of any spectral line can be measured accurately. But in the case of a prism the angles of deviation are not directly related to the wavelength of the spectral line. The angles of deviation are dependent on the refractive index of the material of the prism, which depends on the wavelength of light.

$\therefore$ Resolving power of a prism

$$= t\frac{d\mu}{d\lambda}$$

Thus, the resolving power of a prism is (*i*) directly proportional to the length of the base and (*ii*) rate of change of refractive index with respect to wavelength for that particular material. The expression for resolving power given above is applicable only to spectral lines of equal intensity. If two spectral lines are of different intensities, then the value of $d\lambda$ *i.e.*, the difference in wavelength between the two lines must be higher so that the two lines appear as separate ones.

Example 9.46. *The refractive indices of a glass prism for the C and F lines are 1.6545 and 1.6635 respectively. The wavelengths of these two lines in the solar spectrum are 6563 Å and 5270 Å respectively. Calculate the length of the base of a 60° prism which is capable of resolving sodium lines of wavelengths 5890 Å and 5896 Å.* (*Vikram University*)

Resolving power $= \frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$

Here $\frac{d\mu}{d\lambda} = \frac{1.6635 - 1.6545}{(6563 - 5270)\,10^{-8}}$

$$\therefore \quad \frac{\lambda}{d\lambda} = t\left[\frac{1.6635 - 1.6545}{(6563 - 5270)\,10^{-8}}\right]$$

$$\lambda = 5893 \times 10^{-8}\ \text{cm}, \quad d\lambda = (5896 - 5890)\,10^{-8}$$

$$= 6 \times 10^{-8}\ \text{cm}$$

$$\therefore \quad t = \frac{5893 \times 10^{-8}}{6 \times 10^{-8}}\left[\frac{(6563 - 5270)\,10^{-8}}{1.6635 - 1.6545}\right]$$

$$= \mathbf{1.41\ cm}$$

Example 9.47. *Calculate the minimum thickness of the base of a prism which will just resolve the D_1 and D_2 lines of sodium. Given μ for wavelength 6563 Å = 1.6545 and for wavelength 5270 Å = 1.6635.* [*Bombay*]

In a prism,

Resolving power, $\frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$

Here $\frac{d\mu}{d\lambda} = \left[\frac{1.6635 - 1.6545}{(6563 - 5270) \times 10^{-8}}\right] = \left(\frac{0.0090}{1293 \times 10^{-8}}\right)$

and $\frac{\lambda}{d\lambda} = \frac{5893 \times 10^{-8}}{6 \times 10^{-8}} = \frac{5893}{6}$

$$\therefore \quad t = \frac{\left(\frac{\lambda}{d\lambda}\right)}{\left(\frac{d\mu}{d\lambda}\right)}$$

$$\therefore \quad t = \left(\frac{5893 \times 1293 \times 10^{-8}}{6 \times 0.0090}\right)\ \text{cm}$$

$$t = \mathbf{1.41\ cm}$$

9.59 RESOLVING POWER OF A PLANE DIFFRACTION GRATING

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighbouring line such that the two lines appear to be just resolved. Thus, the resolving power of a grating appear to be just resolved. Thus, the resolving power of a grating

$$= \frac{\lambda}{d\lambda}$$

In Fig. 9.74, *XY* is the grating surface and *MN* is the field of view of the telescope, P_1 is the *n* th primary maximum of a spectral line of wavelength λ at an angle of diffraction θ_n. P_2 is the *n* th primary maximum

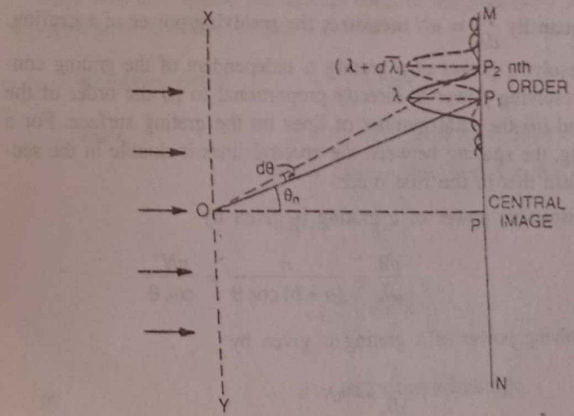

Fig. 9.74

of a second spectral line of wavelength $\lambda + d\lambda$ at a diffracting angle $\theta_n + d\theta$. P_1 and P_2 are the spectral lines in the n th order. These two spectral lines according to Rayleigh, will appear resolved if the position of P_2 also corresponds to the first minimum of P_1.

The direction of the n th primary maximum for a wavelength λ is given by

$$(a+b)\sin\theta_n = n\lambda \quad ...(i)$$

The direction of the n th primary maximum for a wavelength $(\lambda + d\lambda)$ is given by

$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda) \quad ...(ii)$$

The two lines will appear just resolved if the angle of diffraction $(\theta_n + d\theta)$ also corresponds to the direction of the first secondary minimum after the n th primary maximum at P_1 (corresponding to wavelength λ). This is possible if the extra path difference introduced is $\frac{\lambda}{N}$ Where N is the total number of lines on the grating surface.

$$\therefore \quad (a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N} \quad ...(iii)$$

Equating the right hand sides of equations (*ii*) and (*iii*)

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}; \quad n.\,d\lambda = \frac{\lambda}{N}$$

$$\frac{\lambda}{d\lambda} = nN \quad ...(iv)$$

The quantity $\frac{\lambda}{d\lambda} = nN$ **measures the resolving power of a grating.** Thus, the resolving power of a grating is independent of the grating constant. The resolving power is directly proportional to (*i*) the order of the spectrum and (*ii*) the total number of lines on the grating surface. For a given grating, the spacing between the spectral lines is double in the second order than that in the first order.

The dispersive power of a grating is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{nN'}{\cos\theta}$$

and the resolving power of a grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

where n is the order of the spectrum, N is the total number of lines of the grating. N' is the number of lines per cm on the grating surface. Here, θ gives the direction of the n th principal maximum corresponding to a wavelength λ. From the above equation, it is clear, that the dispersive power increases with increase in the number of lines per cm and the resolving power increases, with increase in the total number of lines on the grating surface (*i.e.*, the width of the grating surface). If N' is the same for two gratings, the dispersive power will be the same in the two cases but the one with larger width of the grating surface produces higher resolution of the spectral lines. With a grating having large width of the grating surface, the spectral lines are sharp and narrow.

High dispersive power refers to wide separation of the spectral lines whereas high resolving power refers to the ability of the instrument to show nearby spectral lines as separate ones.

Example 9.48. *What should be the minimum number of lines in a grating which will just resolve in the second order the lines whose wavelengths are 5890 Å and 5896 Å ?* (*Agra*)

Resolving power $= \frac{\lambda}{d\lambda} = nN$

Here, $n = 2$, $\lambda = 5890$ Å, $d\lambda = 5896 - 5890 = 6$ Å

$$\therefore \quad \frac{5890}{6} = 2N$$

or $N = \frac{5890}{6 \times 2}$ = **491 approximately.**

Example 9.49. *Calculate the minimum number of lines in a grating which will just resolve the sodium lines in the first order spectrum. The wavelengths are 5890 Å and 5896 Å.* (*Delhi*)

Resolving power $= \frac{\lambda}{d\lambda} = nN$

Here, $n = 1$, $\lambda = 5890$ Å, $= 5890 \times 10^{-8}$ cm

$d\lambda = 5896 - 5890 = 6$ Å $= 6 \times 10^{-8}$ cm

$$N = \frac{1}{n}\left[\frac{\lambda}{d\lambda}\right]$$

$$= \frac{1}{1}\left[\frac{5890}{6}\right]$$

or n = **982 approximately.**

Example 9.50. *Calculate the minimum number of lines per cm in a 2.5 cm wide grating which will just resolve the sodium lines (5890 Å and 5896 Å) in the second order spectrum.* [Delhi(Hons) 1976]

Let the total number of lines required on the grating be N.

$$\frac{\lambda}{d\lambda} = nN$$

Here $\lambda = 5890 \times 10^{-8}$ cm

$d\lambda = 6 \times 10^{-8}$ cm; $n = 2$

$N = ?$

$$N = \frac{\lambda}{n d\lambda}$$

$$N = \frac{5890 \times 10^{-8}}{2 \times 6 \times 10^{-8}} = \mathbf{491}$$

Width of the grating = 2.5 cm

Number of lines per cm

$$= \frac{491}{2.5} = \mathbf{196.4}$$

Example 9.51. *Calculate the least width of a plane diffraction grating having 500 lines/cm, which will just resolve in the second order the sodium lines of wavelengths 5890 Å and 5896 Å.* [Berhampur (Hons) 1987]

Let the total number of lines required on the grating be N.

$$\frac{\lambda}{d\lambda} = nN$$

Here $\lambda = 5890 \times 10^{-8}$ cm, $d\lambda = 6 \times 10^{-8}$ cm

$n = 2$, $N = ?$

$$N = \frac{\lambda}{n\, d\lambda}$$

$$N = \frac{5890 \times 10^{-8}}{2 \times 6 \times 10^{-8}}$$

$N = \mathbf{491}$

Least width of the grating

$$= \frac{491}{500} = \mathbf{0.982\ cm}$$

Example 9.52. *Examine if two spectral lines of wavelengths 5890 Å and 5896 Å can be clearly resolved in the (i) first order and (ii) second order by a diffraction grating 2 cm wide and having 425 lines/cm.* [Delhi B.Sc.(Hons.) 1985]

Total number of lines on the grating

$= 2 \times 425 = 850$

(1) For the first order

$$\frac{\lambda}{d\lambda} = nN. \quad \text{Here } n = 1$$

$$N = \frac{\lambda}{d\lambda} = \frac{5890 \times 10^{-8}}{6 \times 10^{-8}} = \mathbf{982\ lines}$$

As the total number of lines required for just resolution in the first order is 982 and the total number of lines on the grating is 850, the lines will not be resolved.

(2) For the second order

$$\frac{\lambda}{d\lambda} = nN, \quad n = 2$$

$$N = \frac{5890 \times 10^{-8}}{2 \times 6 \times 10^{-8}} = \mathbf{491}$$

As the total number of lines required is 491, and the given grating has a total of 850 lines, the lines will appear resolved in the second order.

Example 9.53. *Light is incident normally on a grating of total ruled width 5×10^{-3} m with 2500 lines in all. Calculate the angular separation of the two sodium lines in the first order spectrum. Can they be seen distinctly ?* [IAS, 1983]

(*i*) Here, $N = 2500$

Width of the ruling $= 5 \times 10^{-3}$ m

Number of lines/metre $= \frac{2500}{5 \times 10^{-3}} = 5 \times 10^5$

$$\therefore \quad (a+b) = \frac{1}{5 \times 10^5} = 2 \times 10^{-6} \text{ m}$$

$n = 1$

For the first order $\sin \theta_1 = \frac{n\lambda_1}{(a+b)} = \frac{\lambda_1}{(a+b)}$

$$\sin \theta_2 = \frac{n\lambda_2}{(a+b)} = \frac{\lambda_2}{(a+b)}$$

$$\lambda_1 = 5890 \times 10^{-10}\ \text{m}$$

$$\lambda_2 = 5896 \times 10^{-10}\ \text{m}$$

$$\sin\theta_1 = \frac{5890 \times 10^{-10}}{2 \times 10^{-6}}$$

$$\sin\theta_1 = 2945 \times 10^{-4} = 0.2945$$

$$\theta_1 = 17^\circ - 8'$$

$$\sin\theta_2 = \frac{5896 \times 10^{-10}}{2 \times 10^{-6}}$$

$$\sin\theta_2 = 0.2948$$

$$\theta_2 = 17^\circ - 9'$$

(*ii*) The resolving power of the grating,

$$\frac{\lambda}{d\lambda} = \frac{5890 \times 10^{-10}}{6 \times 10^{-10}}$$

$$= \mathbf{982}$$

As the total number of lines on the grating is 2500 which is more than 982, the lines can be see distinctly.

Example 9.54. *The wavelengths of sodium D lines are 589.593 µm and 588.996 µm. What is the minimum number of lines that a grating must have in order to resolve these lines in the first order spectrum.*

[*IAS, 1985*]

Here $\lambda_1 = 589.593 \times 10^{-6}\ \text{m}$

$$\lambda_2 = 588.996 \times 10^{-6}\ \text{m}$$

$$\Delta\lambda = \mathbf{0.597 \times 10^{-6}\ m}$$

$$\frac{\lambda}{\Delta\lambda} = nN$$

Here $n = 1$

$$\therefore \quad N = \frac{589.593 \times 10^{-6}}{0.597 \times 10^{-6}} = \mathbf{988}$$

Therefore, the minimum number of lines required for just resolution in the first order is **988**.

Example 9.55. *In the second order spectrum of a plane transmission grating, a certain spectral line appears at an angle of 10° while for another wavelength which is* 5×10^{-9} cm *is higher, the corresponding line is observed at an angle of 10° 3''. Find the wavelength of the two lines and the maximum grating width required to resolve them.*

(*sin 10° = 0.1736 and cos 10°* = (1.9848).

[*Delhi(Hons.1985*]

Dispersive power of the grating

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

$$(a+b) = \frac{n}{\cos\theta\left(\dfrac{d\theta}{d\lambda}\right)}$$

Here, $n = 2$

$$\cos\theta = \cos 10^\circ = 0.9848$$

$$d\theta = 3'' = \frac{3\pi}{60 \times 60 \times 180}\ \text{radian}$$

$$d\lambda = 50 \times 10^{-8}\ \text{cm}$$

$$\frac{d\theta}{d\lambda} = \frac{3\pi}{50 \times 60 \times 180 \times 50 \times 10^{-8}}$$

$$= \frac{3 \times 22 \times 10^{8}}{7 \times 60 \times 60 \times 180 \times 50}$$

$$= \frac{11000}{378}$$

$$\therefore \quad (a+b) = \frac{2 \times 378}{0.9848 \times 11000}$$

$$= 0.06979\ \text{cm}$$

$$(a+b)\sin\theta_1 = n\lambda_1$$

$$\lambda_1 = \frac{(a+b)\sin\theta_1}{n}$$

$$(a+b) = 0.06979$$

$$\sin\theta_1 = \sin 10^\circ = 0.1736$$

$$n = 2$$

$$\therefore \quad \lambda_1 = \frac{0.06179 \times 0.1736}{2}$$

$$= 6055 \times 10^{-8}\ \text{cm} = 6055\ \text{Å}$$

$$\lambda_2 = \lambda_1 + 50\ \text{Å} = 6055 + 50$$

$$= \mathbf{6105\ Å}$$

Let N be the total number of lines on the grating surface for the just resolution of the spectral lines.

$$\frac{\lambda}{d\lambda} = nN$$

$$\frac{6055 \times 10^{-8}}{50 \times 10^{-8}} = 2N$$

$$N = \frac{6055 \times 10^{-8}}{2 \times 50 \times 10^{-8}}$$

$$= 61 \text{ (approximately)}$$

Grating width $= N(a+b)$

$$= 61 \times 0.06979$$

$$= \mathbf{4.25\ cm}$$

Example 9.56. *A grating has 1000 lines ruled on it. In the region of wavelength λ = 6000 Å, find (i) the difference between two wavelengths that just appear separated in the first order and (ii) the resolving power in the second order spectrum.* (Delhi, 1990)

Here $N = 1000$

$$\lambda = 6000\ \text{Å} = 6 \times 10^{-7}\ \text{m}$$

(i) $\frac{\lambda}{d\lambda} = nN$

$$n = 1$$

$$\therefore \quad d\lambda = \frac{\lambda}{nN}$$

$$= \frac{6 \times 10^{-7}}{1 \times 1000}$$

$$= 6 \times 10^{-10}\ \text{m}$$

$$= 6\ \text{Å}$$

(ii) for the second order

Resolving power,

$$\frac{\lambda}{d\lambda} = nN$$

Here $n = 2$

$$\frac{\lambda}{d\lambda} = 2 \times 1000$$

$$= \mathbf{2000}$$

Example 9.57. *A transmission grating 4 cm long has 4000 lines/cm. Compute the resolving power of the grating for λ = 5900 Å in the first order spectrum. Will this grating separate the two lines of λ = 5890 Å and λ = 5896 Å which constitute the sodium line doublet ?* [Delhi(Hons) 1990]

Resolving power of a grating

$$\frac{\lambda}{d\lambda} = nN$$

Here, $n = 1$

and $N = 4000 \times 4 = 16000$ lines

$$\therefore \quad \frac{\lambda}{d\lambda} = 1 \times 16000$$

$$= \mathbf{16000}$$

For wavelengths 5890 Å and 5896 Å

Resolving power

$$\left(\frac{\lambda}{d\lambda}\right) = nN'$$

$$N' = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$$

$$= 1\left(\frac{5900}{6}\right)$$

$$N' = \mathbf{983}$$

As N is greater that N', the grating will separate the two spectral lines.

Example 9.58. *Examine if two spectral lines of wavelengths 5890 Å and 5896 Å can be clearly resolved in (1) the first order and (2) the second order by a diffraction grating 2 cm wide and having 425 lines per cm.* [Kanpur, 1990]

Total number of lines on the grating

$$= 2 \times 425 = 850$$

Here,

$$\lambda = 5890 \text{ Å} = 5890 \times 10^{-10} \text{ m}$$

$$d\lambda = 6 \text{ Å} = 6 \times 10^{-10} \text{ m}$$

(1) For the first order,

$$\frac{\lambda}{d\lambda} = nN$$

$$n = 1$$

$$N = \frac{\lambda}{d\lambda}$$

$$N = \frac{5890 \times 10^{-10}}{6 \times 10^{-10}}$$

$$N = \mathbf{982 \text{ lines}}$$

As the total number of lines required for just resolution in the first order is **982**, and the total number of lines on the grating is **850**, the lines will not be resolved.

(2) For the second order

$$n = 2$$

$$\frac{\lambda}{d\lambda} = nN = 2N$$

$$N = \frac{5890 \times 10^{-10}}{2 \times 6 \times 10^{-10}}$$

$$N = \mathbf{491}$$

As the total number of lines required is 491 and the given grating has a total of 850 lines, the lines will appear resolved in the second order.

Example 9.59. *In the second order spectrum of a plane diffraction grating, a certain spectral line appears at an angle of 10°, while another line of wavelength 5×10^{-9} cm higher appears at an angle 3" more. Find the wavelengths of the lines and the minimum grating width required to resolve them. Give sin 10° = 0.1736 and cos 10° = 0.9848.*

[Delhi(Hons) 1991]

Dispersive power of the grating

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

$$(a+b) = \frac{n}{\cos\theta\left(\frac{d\theta}{d\lambda}\right)}$$

Here,

$$n = 2$$

$$\cos\theta = \cos 10^\circ = 0.9848$$

$$d\theta = 3'' = \frac{3\pi}{60 \times 60 \times 180} \text{ radian}$$

$$d\lambda = 5 \times 10^{-8} \text{ cm} = 5 \times 10^{-11} \text{ m} = 0.5 \text{ Å}$$

$$\frac{d\theta}{d\lambda} = \frac{3\pi}{60 \times 60 \times 180 \times 5 \times 10^{-11}}$$

$$= 2.91 \times 10^5$$

$$\therefore \quad (a+b) = \frac{2}{0.9848 \times 2.91 \times 10^5}$$

$$= 6.978 \times 10^{-6} \text{ m}$$

$$(a+b)\sin\theta_1 = n\lambda_1$$

$$\lambda_1 = \frac{(a+b)\sin\theta_1}{n}$$

Here,

$$(a+b) = 6.978 \times 10^{-6} \text{ m}$$

$$\sin\theta_1 = \sin 10^\circ = 0.1736$$

$$n = 2$$

$$\lambda_1 = \frac{6.978 \times 10^{-6} \times 0.1736}{2}$$

$$= \mathbf{6057 \times 10^{-10} \text{ m}}$$

$$= \mathbf{6057 \text{ Å}}$$

$$\lambda_2 = 6057 + 0.5$$

$$= \mathbf{6057.5 \text{ Å}}$$

Let N be the total number of lines on the grating surface for the just resolution of the spectral lines

$$\frac{\lambda}{d\lambda} = nN$$

$$\frac{6057 \times 10^{-10}}{0.5 \times 10^{-10}} = 2 \times N$$

$$N = 6057$$

Grating width $= N(a+b)$

$$= 6057 \times 6.978 \times 10^{-6}$$

$$= \mathbf{4.226 \times 10^{-2}\ m}$$

$$= \mathbf{4.226\ cm}$$

Example 9.60. *Examine whether the D_1 and D_2 lines of sodium w be clearly separated in the*

(i) first order and (ii) second order by a one inch grating havin 300 *lines/cm.*

[λ for D_1 and D_2 lines are 5896 Å and 5890 Å] *[Delhi, 199*

Here total number of lines on the grating

$$= 2.54 \times 300 = 762$$

(1) For the first order,

Resolving power,

$$\frac{\lambda}{d\lambda} = nN$$

Here $n = 1,\quad \lambda = 5890\ \text{Å},\quad d\lambda = 6\ \text{Å}$

$$\therefore \quad \frac{5890}{6} = N$$

$$N = \mathbf{982}$$

As the number of lines required is more than the lines provided on the grating, the lines **will not** be resolved in the first order.

(ii) For the second order,

$$\frac{\lambda}{d\lambda} = nN$$

$$n = 2$$

$$\frac{5890}{6} = 2N$$

$$N = \mathbf{491}$$

As the number of lines required is less than the lines provided on the grating, the lines **will be resolved** in the second order.

9.60 MICHELSON'S STELLAR INTERFEROMETER

The smallest angular separation (θ) between two distant point sources for viewing the two images of the sources as separate with a telescope, is given by

$$\theta = \frac{1.22\,\lambda}{D} \quad ...(i)$$

where λ is the wavelength of light and D is the diameter of the objective of the telescope. Let the telescope objective be covered with a screen which is pierced with two parallel slits. Let the slit separation (d) be almost equal to the diameter of the objective of the telescope. A suitable value for $d = \frac{D}{1.22}$. Now let the telescope be directed towards a distant double star so that the line joining the two stars is perpendicular to the length of either slit. Interference fringes due to the double slit will be observed in the focal plane of the objective. The condition for the first disappearance of fringes is given by

$$\alpha = \frac{\lambda}{2d} = \frac{1.22\,\lambda}{2D} = \frac{\theta}{2} \quad ...(ii)$$

where α is the angular separation between the two stars when the first disappearance of the fringes takes place. Similarly, for values of α given by the multiples of $\frac{\lambda}{2d}$, disappearance of the fringes can be observed. If the double slit is avoided and the observations are made directly, the multiples can be ruled out. The angular separation α is half the angle θ, where θ is the minimum angle of resolution of the telescope objective.

The method employing the principle of double slit interference is used to measure the angular diameter of the disc of a star rather than the angular separation between two stars.

Michelson in 1920, successfully used this method to find the diameters of stars. The arrangement is known as Michelson's stellar interferometer (Fig. 9.75). It consists of four mirrors M_1, M_2, M_3 and M_4 arranged as shown in the figure. L is the objective of the telescope and the two slits are kept in the paths of light reflected from the mirrors M_3 and M_4. Let S_1 and S_2 be the ends of a diameter of the star. The paths of the rays of light from these two points S_1 and S_2 are shown in the figure. The mirrors M_1 and M_3 are parallel. Also mirrors M_2 and M_4 are parallel. The mirrors M_1 and M_2 are mounted on a girder and by sliding these mirrors, the distance D between the mirrors can be altered. The silvered faces of M_1 and M_2 (and M_3 and M_4) face each other. Interference fringes will be observed in the field of view of the telescope. The path difference between the rays of light from M_1 to L and M_2 to L is zero.

In the side figure, A is the point of incidence of the rays of light on the

10

POLARIZATION

10.1 INTRODUCTION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion *i.e.*, whether the light waves are longitudinal or transverse, or whether the vibrations are linear, circular or torsional. The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

10.2 POLARIZATION OF TRANSVERSE WAVES

Let a rope AB be passed through two parallel slits S_1 and S_2. The rope is attached to a fixed point at B [Fig. 10.1(*a*)]. Hold the end A and

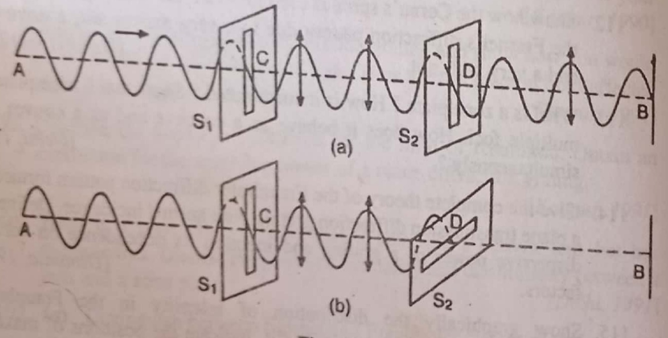

Fig. 10.1

move the rope up and down perpendicular to AB. A wave emerges along CD and it is due to transverse vibrations parallel to the slit S_1. The slit S_2 allows the wave to pass through it when it is parallel to S_1. It is observed that the slit S_2 does not allow the wave to pass through it when it is at right angles to the slit S_1 [Fig. 10.1(*b*)].

If the end A is moved in a circular manner, the rope will show circular motion up to the slit S_1. Beyond S_1, it will show only linear vibrations parallel to the slit S_1, because the slit S_1 will stop the other components. If S_1 and S_2 are at right angles to each other the rope will not show any vibration beyond S_2.

If longitudinal waves are set up by moving the rope forward and backward along the string, the waves will pass through S_1 and S_2 irrespective of their position.

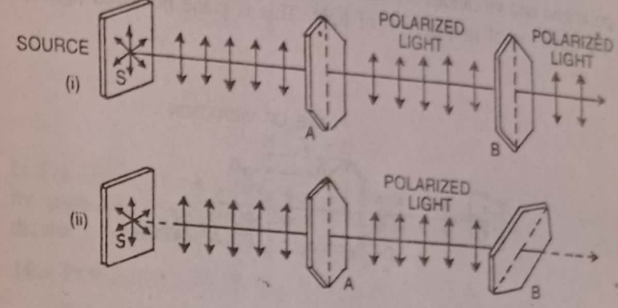

Fig. 10.2

A similar phenomenon has been observed in light when it passes through a tourmaline crystal.

Let light from a source S fall on a tourmaline crystal A which is cut parallel to its axis (Fig. 10.2). The crystal A will act as the slit S_1. The light is slightly coloured due to the natural colour of the crystal. On rotating the crystal A, no remarkable change is noticed. Now place the crystal B parallel to A.

(1) Rotate both the crystals together so that their axes are always parallel. No change is observed in the light coming out of B [Fig. 10.2 (*i*)].

(2) Keep the crystal A fixed and rotate the crystal B. The light transmitted through B becomes dimmer and dimmer. When B *is at right angles to A*, no light emerges out of B [Fig. 10.2 (*ii*)].

If the crystal B is further rotated, the intensity of light coming out of it gradually increases and is maximum again when the two crystals are parallel.

This experiment shows conclusively that light is not propagated as longitudinal or compressional waves. If we consider the propagation of light as a longitudinal wave motion then no extinction of light should occur when the crystal B is rotated.

It is clear that after passing through the crystal *A*, the light waves vibrate only in one direction. Therefore light coming out of the crystal *A* is said to be **polarized** because it has acquired the property of **one sidedness** with regard to the direction of the rays.

This experiment proves that light waves are transverse waves, otherwise light coming out of *B* could never be extinguished by simply rotating the crystal *B*.

10.3 PLANE OF POLARIZATION

When ordinary light is passed through a tourmaline crystal, the light is polarized and vibrations are confined to only one direction perpendicular to the direction of propagation of light. This is plane polarized light and

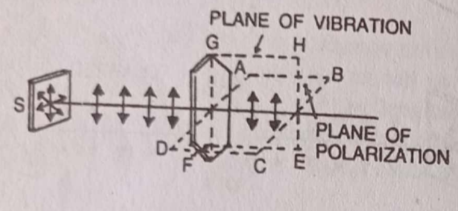

Fig. 10.3

it has acquired the property of one sidedness. The plane of polarization is that plane in which no vibrations occur. The plane *ABCD* in Fig. 10.3 is the plane of polarization. The vibrations occur at right angles to the plane of polarization and the plane in which vibrations occur is known as plane of vibration. The plane *EFGH* in Fig. 10.3 is the plane of vibration.

Ordinary light from a source has very large number of wavelengths. Moreover, the vibrations may be linear, circular or elliptical. From our idea of wave motion, circular or elliptical vibrations consist of two linear vibrations at right angles to each other and having a phase difference of $\frac{\pi}{2}$.

Therefore any vibration can be resolved into two component vibrations at right angles to each other. As light waves are transverse waves the vibrations can be resolved into two planes *xx′* and *yy′*

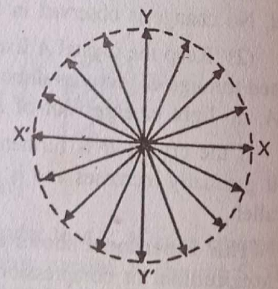

Fig. 10.4

at right angles to each other and also perpendicular to the direction of propagation of light (Fig. 10.4).

In Fig. 10.5(*i*), the vibrations of the particles are represented parallel (arrow heads) and perpendicular to the plane of the paper (dots).

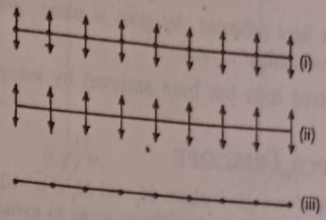

Fig. 10.5

In Fig. (10.3) (*ii*) the vibrations are shown only parallel to the plane of the paper. In Fig. (10.5) (*iii*) the vibrations are represented only perpendicular to the plane of the paper.

10.4 POLARIZATION BY REFLECTION

Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. Consider the light incident along the path *AB* on the glass surface (Fig. 10.6). Light is

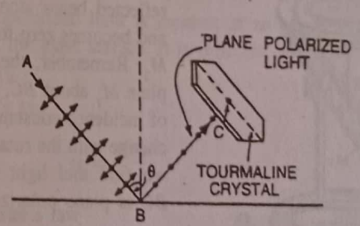

Fig. 10.6

reflected along *BC*. In the path of *BC*, place a tourmaline crystal and rotate it slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing angle. Similarly polarized light by reflection can be produced from water surface also.

The production of polarized light by glass is explained as follows. The vibrations of the incident light can be resolved into components parallel to the glass surface and perpendicular to the glass surface. Light due to the components parallel to the glass surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted.

Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

The polarized light has been analysed by using another mirror by Biot.

10.5 BIOTS POLARISCOPE

It consists of two glass plates M_1 and M_2 (Fig. 10.7). The glass plates are painted black on their back surfaces so as to avoid any reflection and this also helps in absorbing refracted light. A beam of unpolarized light AB is incident at an angle of about 57.5° on the first glass surface at B and is reflected along BC (Fig. 10.8). This light is again reflected at 57.5° by the second glass plate M_2 placed parallel to the first. The glass plate M_1 is known as the polarizer and M_2 as the analyser.

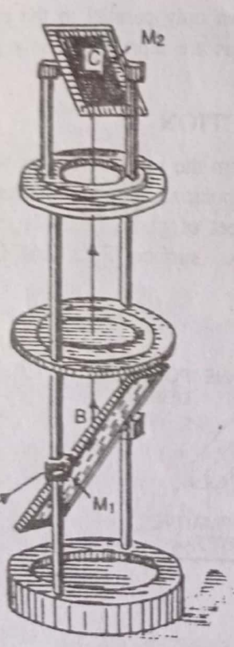

Fig. 10.7

When the upper plate M_2 is rotated about BC, the intensity of the reflected beam along CD decreases and becomes zero for 90° rotation of M_2. Remember, the rotation of the plate M_2 about BC, keeps the angle of incidence constant and it does not change with the rotation of M_2. Thus we find that light travelling along BC is plane polarized.

When the mirror M_2 is rotated further it is found that the intensity of CD becomes maximum at 180°, minimum at 270° and again maximum at 360°.

The above experiment proves that when light is incident at an angle of 57.5° on a glass surface, the reflected light consists of waves in which

the displacements are confined to a certain direction at right angles to the ray and we get polarized light by reflection.

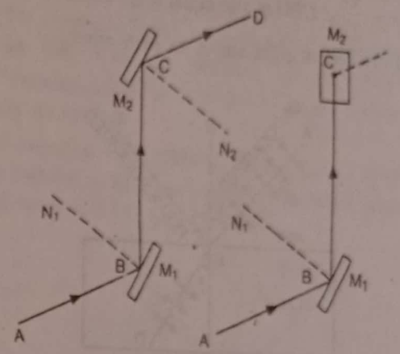

Fig. 10.8

10.6 BREWSTER'S LAW

In 1811, Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media.

He found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the **angle of polarization**.

He was able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and the refracted rays are perpendicular to each other.

Suppose, unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD (Fig. 10.9).

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \quad \text{...}(i)$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i} \quad \text{...}(ii)$$

Comparing (*i*) and (*ii*)

$$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \qquad i = \frac{\pi}{2} - r, \quad \text{or} \quad i + r = \frac{\pi}{2}$$

As $i + r = \frac{\pi}{2}$, $\angle CBD$ is also equal to $\frac{\pi}{2}$. Therefore, the reflected and the refracted rays are at right angles to each other.

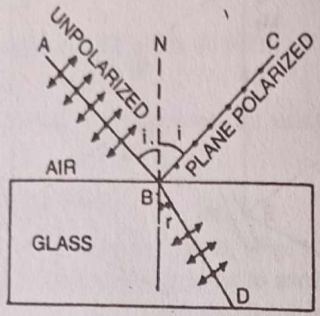

Fig. 10.9

From Brewster's law, it is clear that for crown glass of refractive index 1.52, the value of i is given by

$$i = \tan^{-1}(1.52) \quad \text{or} \quad i = 56.7^\circ$$

However, 57° is an approximate value for the polarizing angle for ordinary glass. For a refractive index of 1.7 the polarising angle is about 59.5° *i.e.*, the polarizing angle is not widely different for different glasses.

As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelengths. Therefore, polarization will be complete only for light of a particular wavelength at a time *i.e.*, for monochromatic light.

It is clear that the light vibrating in the plane of incidence is not reflected along BC [Fig. 10.9]. In the reflected beam the vibrations along BC cannot be observed, whereas vibrations at right angles to the plane of incidence can contribute for the resultant intensity. Thus, we get plane polarized light along BC. The refracted ray will have both the vibrations (*i*) in the plane of incidence and (*ii*) at right angles to the plane of incidence. But it is richer in vibrations in the plane of incidence. Hence it is partially plane-polarized.

10.7 BREWSTER WINDOW

One of the important applications of Brewster's law and Brewster's angle is in the design of a glass window that enables 100% transmission of light. Such a type of window is used in lasers and it is called a **Brewster window**,

When an ordinary beam of light is incident normally on a glass window, about 8% of light is lost by reflection on its two surfaces and about 92% intensity is transmitted. In the case of a gas laser filled with mirrors outside the windows, light travels through the window about a hundred times. In this way the intensity of the final beam is about 3×10^{-4} because $(0.92)^{100} \approx 3 \times 10^{-4}$. It means the transmitted beam has practically no intensity.

To overcome this difficulty, the window is tilted so that the light beam is incident at Brewster's angle. After about hundred transmissions, the final beam will be plane polarized.

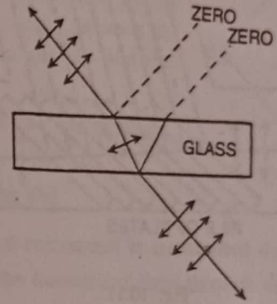

Fig. 10.10

The light component vibrating at right angles to the plane of incidence is reflected. After about 100 reflections at the Brewster window, the transmitted beam will have 50% of the intensity of the incident beam and it will be completely plane polarized. The net effect of this type of arrangement is that half the amount of light intensity has been discarded and the other half is completely retained. Brewster's windows are used in gas lasers.

10.8 POLARIZATION BY REFRACTION

It is found that at a single glass surface or any similar transparent medium, only a small fraction of the incident light is reflected.

For glass ($\mu = 1.5$) at the polarizing angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for the perpendicular vibrations only 85% is transmitted and 15% is reflected. Therefore, if we use a **pile of plates** and the beam of ordinary light is incident at the polarizing angle on the pile of plates, some of the vibrations perpendicular to the plane of incidence are reflected by the first plate and the rest are transmitted through it. When this beam of light is reflected by the second plate, again some of the vibrations perpendicular to the

It has been found that both the rays are plane polarized. The vibrations of the ordinary ray are perpendicular to the principal section of the crystal while the vibrations of the extraordinary ray are in the plane of the principal section of the crystal. Thus, the two rays are plane **polarised**, their vibrations being at right angles to each other.

Special Cases. (1) It should be remembered that a ray of light is not split up into ordinary and extraordinary components when it is incident on calcite parallel to its optic axis. In this case, the ordinary and the extraordinary rays travel along the same direction with the same velocity.

(2) When a ray of light is incident perpendicular to the optic axis on the calorie crystal, the ray of light is not split up into ordinary and extraordinary components. It means that the ordinary and the extraordinary rays travel in the same direction but with **different velocities**.

10.11 PRINCIPAL SECTION OF THE CRYSTAL

A plane which contains the optic axis and is perpendicular to the opposite faces of the crystal is called the **principal section** of the crystal. As a crystal has six faces, therefore, for every point there are three principal sections. A principal section always cuts the surface of a calcite crystal in a parallelogram with angles 109° and 71°

10.12 PRINCIPAL PLANE

A plane in the crystal drawn through the optic axis and the ordinary ray is defined as the principal plane of the ordinary ray, Similarly, a plane in the crystal drawn through the optic axis and the extraordinary ray is defined as the principal plane of the extraordinary ray. In general, the two planes do not coincide. In a particular case, when the plane of incidence is a principal section then the principal section of the crystal and the principal planes of the ordinary and the extraordinary rays coincide.

10.13 NICOL PRISM

It is an optical device used for producing and analysing plane polarized light. It was invented by William Nicol, in 1828, who was an expert in cutting and polishing gems and crystals. We have discussed that when a beam of light is transmitted through a calcite crystal, it breaks up into two rays : (1) the ordinary ray which has its vibrations perpendicular to the principal section of the crystal and (2) the extraordinary ray which has its vibrations parallel to the principal section.

The **nicol prism** is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that the ordinary ray is eliminated and, only the extraordinary ray is transmitted through the prism.

A calcite crystal whose length is three times its breadth is taken. Let $A'BCDEFG'H$ represent such a crystal having A' and G' as its blunt corners and $A'CG'E$ is one of the principal sections with $\angle A'CG' = 70°$.

The faces $A'BCD$ and $EFG'H$ are ground in such a way that the angle ACG becomes = 68° instead of 71°. The crystal is then cut along the plane $AKGL$ as shown in Fig. 10.15. The two cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and the extraordinary rays for calcite.

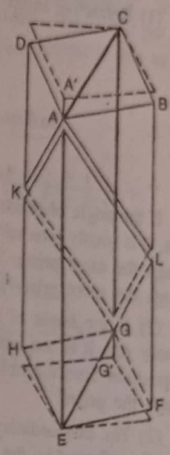

Fig. 10.15

Refractive index for the ordinary

$$\mu_o = 1.658$$

Refractive index for Canada balsam

$$\mu_B = 1.55$$

Refractive index for the extraordinary $\mu_E = 1.486$

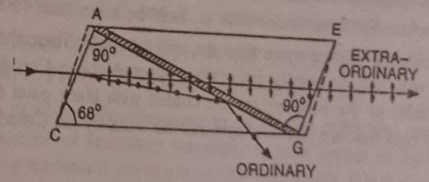

Fig. 10.16

In Fig. 10.16, the section $ACGE$ of the crystal is shown, The diagonal AC represents the Canada balsam layer in the plane $ALGK$ of Fig. 10.15.

It is clear that Canada balsam acts as a rarer medium for an ordinary ray and it acts as a denser medium for the extraordinary ray. Therefore, when the ordinary ray passes from a portion of the crystal into the layer of Canada balsam it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle, the ray is totally internally reflected and is not transmitted. The extraordinary ray is not affected and is therefore transmitted through the prism. The working of the prism is clear from the following cases :-

(1) Refractive index for ordinary ray with respect to Canada balsam

$$= \mu = \frac{1.658}{1.550}$$

$$\therefore \quad \sin\theta = \frac{1}{\mu} = \frac{1.550}{1.658}$$

$$\therefore \quad \theta = 69^\circ$$

If the angle of incidence for the ordinary ray is more than the critical angle, it is totally internally reflected and only the extraordinary ray passes through the nicol prism. Therefore, a ray of unpolarized light on passing through the nicol prism in this position becomes plane-polarized.

(2) If the angle of incidence is less than the critical angle for the ordinary ray, it is not reflected and is transmitted through the prism. In this position both the ordinary and the extraordinary rays are transmitted through the prism.

(3) The extraordinary ray also has a limit beyond which it is totally internally reflected by the Canada balsam surface. The refractive index for the extraordinary ray = 1.486 when the extraordinary ray is travelling at right angles to the direction of the optic axis. If the extraordinary ray travels along the optic axis, its refractive index is the same as that of the ordinary ray and it is equal to 1.658. Therefore, depending upon the direction of propagation of the extraordinary ray μ_e lies between 1.486 and 1.658. Therefore for a particular case μ_e may be more than 1.55 and the angle of incidence will be more than the critical angle. Then, the extraordinary ray will also be totally internally reflected at the Canada balsam layer. The sides of the nicol prism are coated with black paint to absorb the ordinary rays that are reflected towards the sides by the Canada balsam layer.

10.14 NICOL PRISM AS AN ANALYSER

Nicol prism can be used for the production and detection of plane-polarizer light.

When two nicol prisms P_1 and P_2 are placed adjacent to each other as shown in Fig. 10.17 (*i*), one of them acts as a polarizer and the other acts as an analyser. Fig. 10.17 (*i*) shows the position of two parallel nicols and only the extraordinary ray passes through both the prisms.

If the second prism P_2 is gradually rotated, the intensity of the extraordinary ray decreases in accordance with Malus Law and when

the two prisms are crossed [*i.e.*, when they are at right angles to each other, Fig. 10.16 (*ii*), then no light comes out of the second prism P_2. It means that light coming out of P_1 is plane polarized. When the polarized extraordinary ray enters the prism P_2 in this positions it acts as

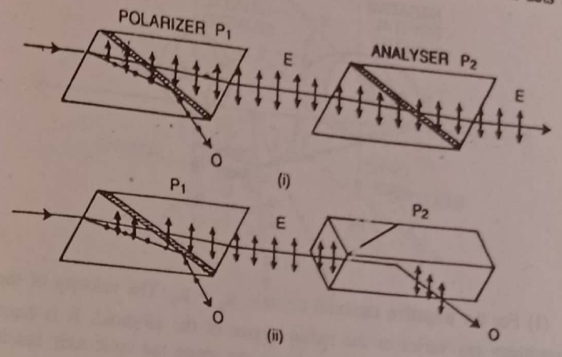

Fig. 10.17

an ordinary ray and is totally internally reflected by the Canada balsam layer and so no light comes out of P_2. Therefore, the prism P_1 produces plane-polarized light and the prism P_2 detects it.

Hence P_1 and P_2 are called the polarizer and the analyser respectively. The combination of P_1 and P_2 is called a polariscope.

10.15 HUYGENS EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTALS

Huygens explained the phenomenon of double refraction with the help of his principle of secondary wavelets. A point source of light in a double refracting medium is the origin of two wavefronts. For the ordinary ray, for which the velocity of light is the same in all directions the wavefront is spherical. For the extraordinary ray, the velocity varies with the direction and the wavefront is an ellipsoid of revolution. **The velocities of the ordinary and the extraordinary rays are the same along the optic axis.**

Consider a point source of light S in a calcite crystal [Fig. 10.18.(*a*)]. The sphere is the wave surface for the ordinary ray and the ellipsoid is the wave surface for the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as **negative crystals.** For crystals like quartz, which are known as positive crystals, the extraordinary wave surface lies within the ordinary wave surface [Fig. 10.18 (*b*)].

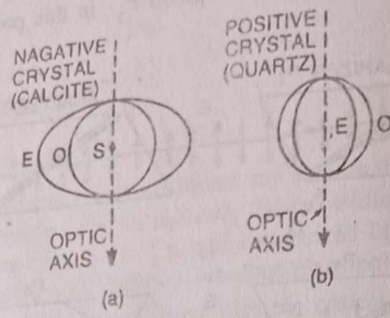

Fig. 10.18

(1) For the **negative uniaxial** crystals, $\mu_0 > \mu_E$. The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angles to the direction of the optic axis.

(2) For the **positive uniaxial** crystals $\mu_E > \mu_0$. The velocity of the extraordinary ray is least in a direction at right angles to the optic axis. It is maximum and is equal to the velocity of the ordinary ray along the optic axis. Hence, from Huygens' theory, the wavefronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wavefronts touch each other. The direction of the line joining these two points (Where the sphere and the ellipsoid touch each other) is the **optic axis**.

10.16 OPTIC AXIS IN THE PLANE OF INCIDENCE AND INCLINED TO THE CRYSTAL SURFACE

(*a*) **Oblique incidence.** *AB* is the incident plane wavefront of the rays falling obliquely on the surface *MN* of the negative crystal. The crystal is cut so that the optic axis is in the plane of incidence and is in the direction shown in Fig. 10.19. O_1 is the spherical secondary wavefront for the ordinary ray and E_1 is the ellipsoidal secondary wavefront for the extraordinary ray. *CP* is the tangent meeting the spherical wavefront at *P* and *CQ* is the tangent meeting the ellipsoidal wavefront at *Q*.

According to Huygens' construction, by the time the incident wave reaches from *B* to *C*, the ordinary ray travels the distance *AP* and the extraordinary ray travels the distance *AQ*. Suppose, the velocity of light

in air is V_a and the velocities of light for the ordinary ray along *AP* and the extraordinary ray along *AQ* are V_0 and V_e respectively. In this case,

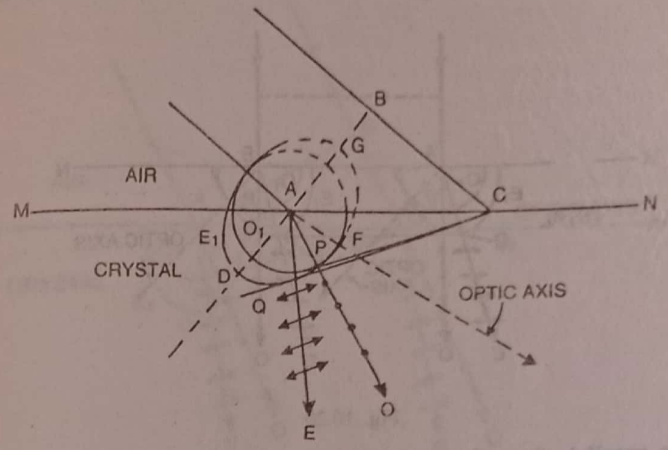

Fig. 10.19

$$\frac{BC}{V_a} = \frac{AP}{V_0} = \frac{AQ}{V_e} \qquad ...(i)$$

Therefore
$$AP = \frac{BC.V_0}{V_a} = \frac{BC}{\mu_0} \qquad ...(ii)$$

and
$$AQ = \frac{BC.V_e}{V_a} = \frac{BC}{\mu_e} \qquad ...(iii)$$

Here, μ_0 and μ_e are the refractive indices for the ordinary and the extraordinary rays along *AP* and *AQ* respectively. In Fig. 10.19, *CP* and *CQ* are the ordinary and the extraordinary refracted plane wavefronts respectively in the crystal. Therefore, the ordinary and the extraordinary rays travel with different velocities along different direction. Here, the semi-major axis of the ellipsoid is $\frac{BC}{\mu_E}$ and the semi-minor axis is $\frac{BC}{\mu_0}$, where μ_E is the principal refractive index for the extraordinary ray and

$$\mu_E < \mu_e < \mu_0$$

Note. The direction *AE* of the extraordinary ray is not perpendicular to the tangent *CQ*, whereas the direction *AO* of the ordinary ray is perpendicular to the tangent *CP*.

(*b*) **Normal incidence.** AB is the incident plane wavefront of the rays falling normally on the surface MN of the negative crystal (Fig. 10.20). The wavefront AB is parallel to the surface of the crystal and also

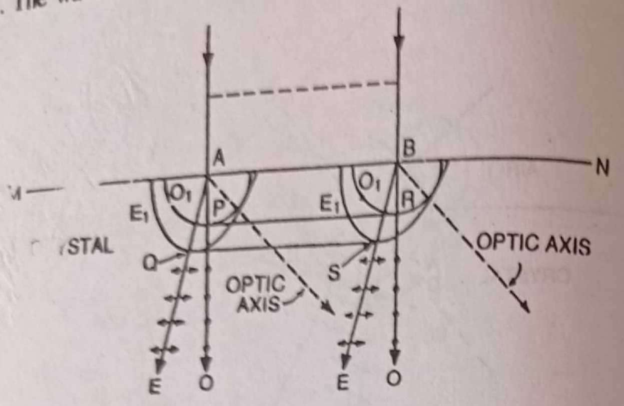

Fig. 10.20

remains parallel after refraction. PR is the refracted wavefront tangential to the spherical wavefront of the ordinary rays while QS is the refracted wavefront tangential to the ellipsoidal wavefront of extraordinary rays. The ordinary and the extraordinary refracted wave fronts are parallel. The ordinary and the extraordinary rays travel along different directions because AE is not perpendicular to QS while AO is perpendicular to PR.

10.17 OPTIC AXIS IN THE PLANE OF INCIDENCE AND PARALLEL TO THE CRYSTAL SURFACE

(*a*) **Oblique incidence.** AB is the incident plane wavefront of the rays falling obliquely on the surface MN of the negative crystal (Fig. 10.21). The optic axis is in the plane of incidence and parallel to the crystal surface. The spherical wavefront and the ellipsoidal wavefront originating from the point A touch each other along the line MN.

Here,
$$\frac{BC}{V_a} = \frac{AP}{V_0} = \frac{AQ}{V_e} \quad ...(i)$$

where V_e is the velocity of the extraordinary ray along AQ.

$$\therefore \quad AP = \frac{BC \times V_0}{V_a} = \frac{BC}{\mu_0} \quad ...(ii)$$

and
$$AQ = \frac{BC \times V_e}{V_a} = \frac{BC}{\mu_e} \quad ...(iii)$$

where μ_e is the refractive index of the extraordinary ray along AQ. The ordinary and the extraordinary rays travel with different velocities along

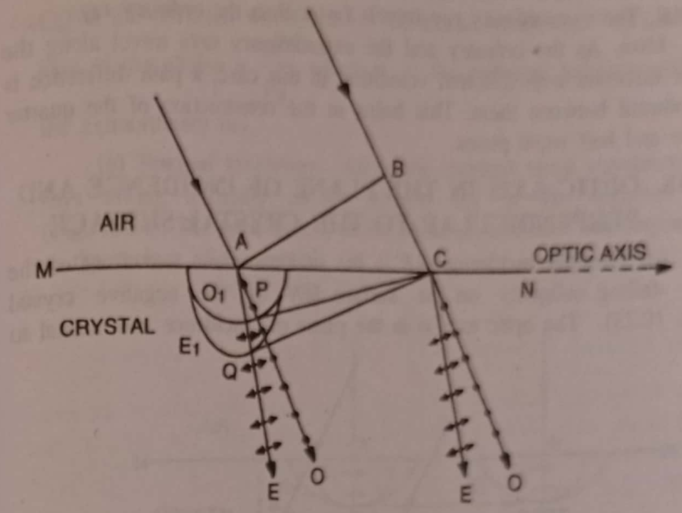

Fig. 10.21

different directions. If μ_E is the principal refractive index for the extraordinary ray, the semi-major axis of the ellipse is $\frac{BC}{\mu_E}$. Here $\mu_E < \mu_e < \mu_0$.

(*b*) **Normal Incidence.** The optic axis lies along MN and AB is the incident plane wavefront of the rays falling normally on the surface MN of the negative crystal (Fig. 10.22). PR is the refracted wavefront for the

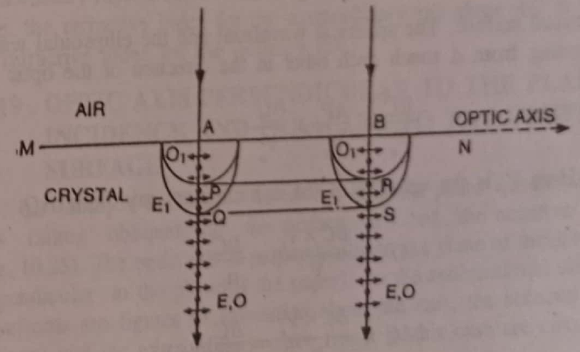

Fig. 10.22

ordinary ray and QS is the refracted wavefront for the extraordinary ray. The wavefronts PR and QS are parallel. The ordinary and the extraordinary rays travel along the same direction but with different velocities in the crystal. The extraordinary ray travels faster than the ordinary ray.

Uses. As the ordinary and the extraordinary rays travel along the same direction with different velocities, in this case, a path difference is introduced between them. This helps in the construction of the quarter wave and half wave plates.

10.18 OPTIC AXIS IN THE PLANE OF INCIDENCE AND PERPENDICULAR TO THE CRYSTAL SURFACE

(*a*) **Oblique incidence.** AB is the incident plane wavefront of the rays falling obliquely on the surface MN of the negative crystal (Fig. 10.23). The optic axis is in the plane of incidence and normal to

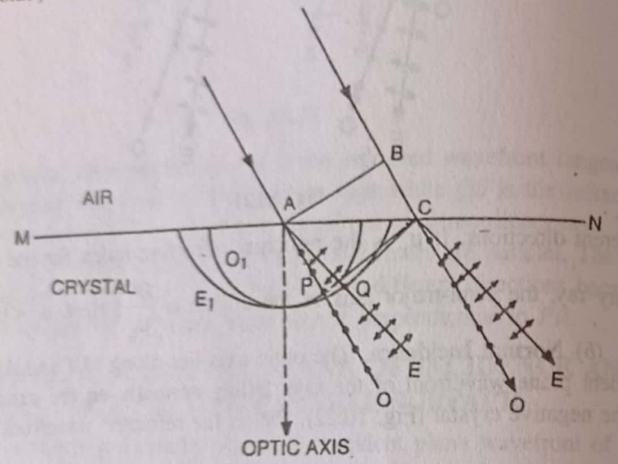

Fig. 10.23

the crystal surface. The spherical wavefront and the ellipsoidal wavefront originating from A touch each other in the direction of the optic axis.

Here
$$\frac{BC}{V_a} = \frac{AP}{V_0} = \frac{AQ}{V_e} \quad ...(i)$$

Here V_e is the velocity of the extraordinary ray along AQ.

$$\therefore \quad AP = \frac{BC \times V_0}{V_a} = \frac{BC}{\mu_0} \quad ...(ii)$$

and
$$AQ = \frac{BC \times V_e}{V_a} = \frac{BC}{\mu_e} \quad ...(iii)$$

where μ_e is the refractive index for the extraordinary ray along AQ. The ordinary and the extraordinary rays travel with different velocities along different directions. CP is the refracted wavefront for the ordinary ray and CQ is the refracted wavefront for the extraordinary ray. The semi-major axis of the ellipse $= \frac{BC}{\mu_E}$, where μ_E is the principal refractive index for the extraordinary ray.

(*b*) **Normal incidence.** AB is the incident plane wavefront of the rays falling normally on the surface MN of the negative crystal (Fig. 10.24). The optic axis is in the plane of incidence and perpendicular to the surface of the crystal.

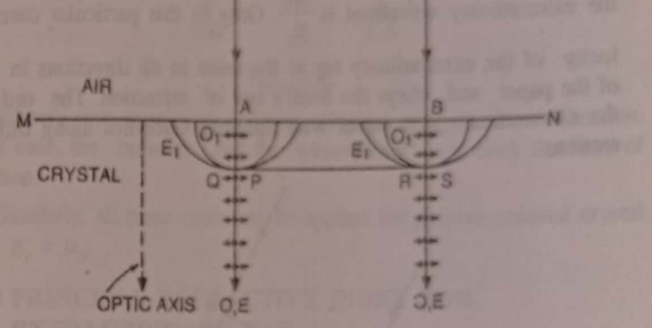

Fig. 10.24

The spherical and the ellipsoidal wavelets originating from the point A touch each other at P. There is no separation of the ordinary and the extraordinary rays. Both travel with the same velocity along the optic axis. Here, the refractive index for the extraordinary ray along AQ is equal to the refractive index for the ordinary ray.

10.19 OPTIC AXIS PERPENDICULAR TO THE PLANE OF INCIDENCE AND PARALLEL TO THE CRYSTAL SURFACE

(*a*) **Oblique incidence.** AB is the incident plane wavefront of the rays falling obliquely on the surface MN of the negative crystal (Fig. 10.25). The optic axis is perpendicular to the plane of incidence (*i.e.*, perpendicular to the plane of the paper). As the spherical and ellipsoidal wavefronts are figures of revolution about the axis, the sections for the wavefronts are circular. By ordinary and the extraordinary wave fronts in this case are circular. By the time the wave reaches from B to C, the extraordinary ray reaches Q and the ordinary ray reaches P.

Here
$$\frac{BC}{V_a} = \frac{AP}{V_0} = \frac{AQ}{V_E} \qquad ...(i)$$

$$AP = \frac{BC \times V_0}{V_a} = \frac{BC}{\mu_0} \qquad ...(ii)$$

and
$$AQ = \frac{BC \times V_E}{V_a} = \frac{BC}{\mu_E} \qquad ...(iii)$$

Therefore, the radius of the ordinary wavefront is $\frac{BC}{\mu_0}$ and that of the extraordinary wavefront is $\frac{BC}{\mu_E}$. Only in this particular case the velocity of the extraordinary ray is the same in all directions in the plane of the paper and obeys the Snell's law of refraction. The ordinary and the extraordinary rays travel with different velocities along different directions

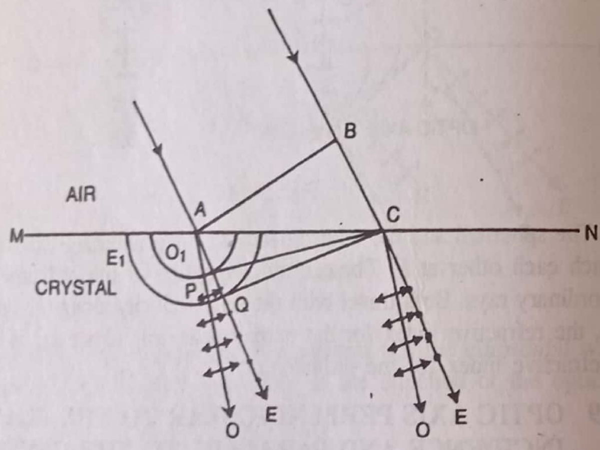

Fig. 10.25

as shown in Fig. 10.25. CP is the refracted wavefront for the ordinary ray and CQ is the refracted wavefront for the extraordinary ray.

(*b*) **Normal incidence.** In this case, the form of the wavefront for the ordinary and the extraordinary rays is shown in Fig. 10.26. The radius of the ordinary wavefront is $V_0 \times t$ and that of the extraordinary wavefront $V_E \times t$, where t is the time elapsed after the waves originated from the

point A. PR is the refracted wavefront for the ordinary rays and QS is the refracted wavefront for the extraordinary rays. The ordinary and the

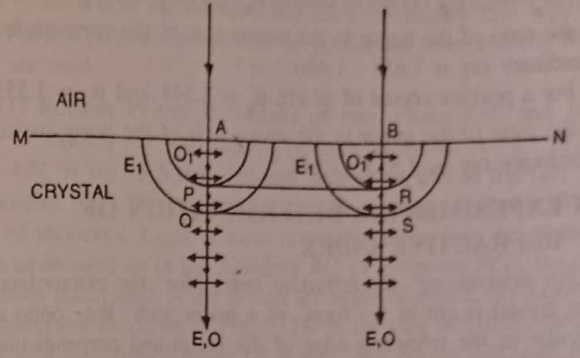

Fig. 10.26

extraordinary rays travel with different velocities along the same direction. In this case, the ordinary and the extraordinary rays obey the laws of refraction.

Similarly, all these cases can be applied for positive uniaxial crystal where $\mu_r > \mu_0$.

10.20 PRINCIPAL REFRACTIVE INDEX FOR EXTRAORDINARY RAY

The velocity of the extraordinary ray through a uniaxial crystal depends upon the direction of the ray. Therefore, the refractive index for the extraordinary ray is different along different directions. In the case of a negative crystal, the velocity of the extraordinary ray travelling perpendicular to the direction of the optic axis is maximum and the refractive index is minimum. This refractive index for the extraordinary ray is known as the **principal refractive index** (μ_E) and is defined as the ratio of the sine of the angle of incidence to the sine of the angle of refraction when the refracted ray travels perpendicular to the direction of the optic axis. This is also defined as the **ratio of the velocity in vacuum to the maximum velocity of the extraordinary ray.**

$$\mu_E = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of the extraordinary ray in a direction perpendicular to the optic axis}}$$

For a positive uniaxial crystal, the velocity of the extraordinary ray travelling perpendicular to the optic axis is minimum and the refractive index is maximum. **Therefore, the principal refractive index for the positive uniaxial crystal is the ratio of the velocity of light in vacuum to the minimum velocity of the extraordinary ray.**

For a negative crystal of calcite, $\mu_0 = 1.658$ and $\mu_E = 1.486$. Therefore, the ratio of the major to the minor axis of the wave surface of the extraordinary ray is 1.658 : 1.486.

For a positive crystal of quartz, $\mu_0 = 1.544$ and $\mu_E = 1.553$. Therefore, the ratio of the major to the minor axis of the wave surface of the extraordinary ray is, 1.553 : 1.544.

10.21 EXPERIMENTAL DETERMINATION OF REFRACTIVE INDEX

For determining the refractive index for the extraordinary ray a calcite crystal is cut in the form of a prism with the optic axis perpendicular to the refracting edge of the prism and perpendicular to the base BC [Fig. 10.27 (*a*)]. It can also be cut with the optic axis parallel to the

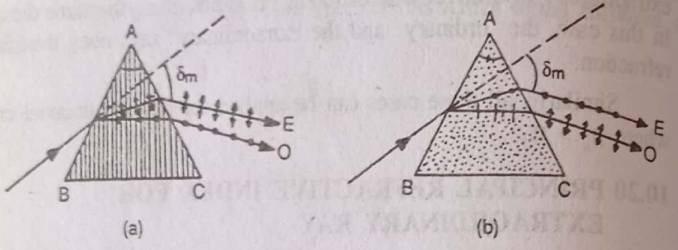

Fig. 10.27 (*a*) *Optic axis per perpendicular to the refracting edge.*
(*b*) *Optic axis parallel to the refracting edge.*

refracting edge of the prism [Fig. 10.27 (*b*)]. The prism is placed on the spectrometer table and is adjusted for the minimum deviation position for the extraordinary rays. The angle of minimum deviation δ_m is determined and the principal refractive index for the extraordinary ray is calculated from the relation.

$$\mu_E = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

For a given wavelength, the ordinary and the extraordinary rays are separated while passing through the prism. Therefore, the angle of minimum deviation for the ordinary ray can be measured and thus its refractive index can be calculated.

10.22 DOUBLE IMAGE POLARIZING PRISMS

Nicol prism cannot be used with ultraviolet light on account of the Canada balsam layer which absorbs these rays. Sometimes, it is also desirable to have both the ordinary and the extraordinary rays widely separated. For this purpose two prisms *viz.* (*i*) Rochon prism and (*ii*) Wollaston prism are used.

(1) Rochon Prism. It consists of two prisms ABC and BCD (of quartz or calcite) cut with their optic axes as shown in Fig. 10.28. The prism ABC is cut such that the optic axis is parallel to the face AB and the incident light. The prism BCD has the opticaxis perpendicular to the plane of incidence. Light incident normally on the face AC of the prism passes undeviated up to the boundary BC. In the prism BCD, the ordinary ray passes undeviated. If the prisms are made of quartz, the extraordinary ray is deviated as shown in Fig. 10.28. In the case of calcite, the extraordinary will be deviated to the other side. The prisms ABC and BCD are cemented together by glycerine or castor oil. Here, the ordinary emergent beam is achromatic whereas the extraordinary beam is chromatic.

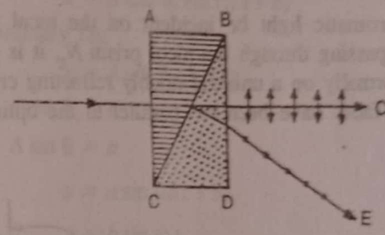

Fig. 10.28

(2) Wollaston prism. It consists of two prisms ABC and BCD of quartz or calcite cut with their optic axes as shown in Fig. 10.29. They are cemented together by glycerine or caster oil.

A ray of light is incident normally on the face AC of the prism ABC. The ordinary and the extraordinary rays travel along the same direction but with different speeds. After passing BC the ordinary ray behaves as the extra ordinary and the extra ordinary behaves as the ordinary while passing through the prism BCD. One ray is bent towards the normal while the other is bent away from the normal. In quartz $\mu_E > \mu_0$. Therefore, the ordinary ray while passing the boundary BC is refracted towards the normal as an extraordinary ray while the extraordinary ray is refracted away from the normal as an ordinary ray as shown in Fig. 10.29. If the prisms are made from calcite, the directions of the ordinary and the extraordinary rays are interchanged. While coming out of the face BD of the prism, the ordinary and the extraordinary rays are diverged. The prism

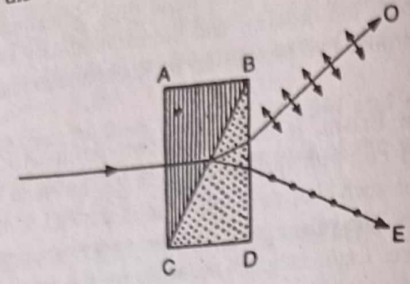

Fig. 10.29

is useful in determining the percentage of polarization in a partially polarized beam. Double image prisms are used in spectrophotometers and pyrometers.

10.23 ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT

Let monochromatic light be incident on the nicol prism N_1 [Fig. 10.30 (*a*)]. After passing through the nicol prism N_2, it is plane-polarized and is incident normally on a uniaxial doubly refracting crystal P (calcite or quartz) whose faces have been cut parallel to the optic axis. The vibrations

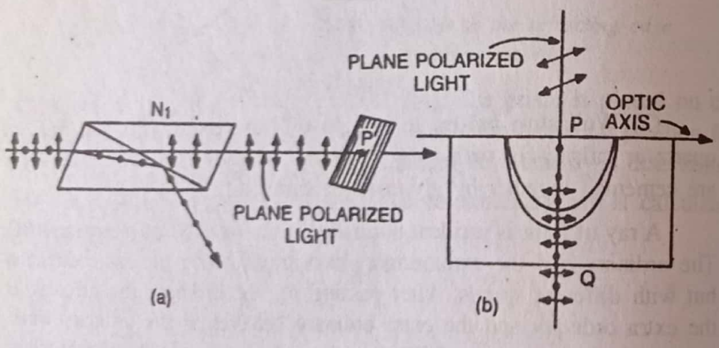

Fig. 10.30

of the plane-polarized light incident on the crystal are shown in Fig. 10.30 (*b*). The plane polarized light on entering the crystal is split up into two components, ordinary and extraordinary. Both the rays, in this case, travel along the same direction but, with different velocities. When the rays have

travelled through the thickness d in the crystal, a phase difference δ is introduced between them.

Theory. Suppose the amplitude of the incident plane polarized light on the crystal is A and it makes an angle θ with the optic axis (Fig. 10.31). Therefore, the amplitude of the ordinary ray vibrating along PO is $A \sin \theta$ and the amplitude of the extraordinary ray vibrating along PE is $A \cos \theta$. Since a phase difference δ is introduced between the two rays, after passing thorough a thickness d of the crystal, the rays after comming out of the crystal can be represented in terms of two simple harmonic motions, at right angles to each other and having a phase difference.

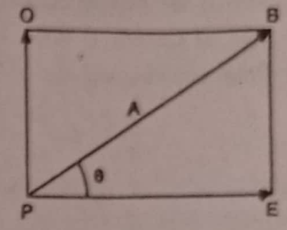

Fig. 10.31

$\therefore$ For the extraordinary ray,

$$x = A \cos \theta . \sin (\omega t + \delta)$$

For the ordinary ray,

$$y = A \sin \theta . \sin \omega t$$

Take, $\quad A \cos \theta = a$

and $\quad A \sin \theta = b$

$$x = a \sin (\omega t + \delta) \qquad \text{...(i)}$$

$$y = b \sin \omega t \qquad \text{...(ii)}$$

From (*ii*)

$$\frac{y}{b} = \sin \omega t$$

and

$$\cot \omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

$$\frac{x}{a} = \sin \omega t \cos \delta + \cos \omega t \sin \delta$$

$$\frac{x}{a} = \frac{y}{b} \cos \delta + \sqrt{1 - \frac{y^2}{b^2}} . \sin \delta$$

$$\frac{x}{a} - \frac{y}{b} \cos \delta = \sqrt{1 - \frac{y^2}{b^2}} . \sin \delta$$

Squaring and rearranging

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\delta = \sin^2\delta \qquad ...(iii)$$

This is the general equation of an ellipse.

Special Cases. (1) When $\delta = 0$ $\sin\delta = 0$ and $\cos\delta = 1$

From equation (*iii*)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a}-\frac{y}{b}\right)^2 = 0$$

or

$$y = \frac{bx}{a}$$

This is the equation of a straight line. Therefore, the emergent light will be plane polarized (Fig. 10.32).

(2) When $\delta = \frac{\pi}{2}$, $\cos\delta = 0$, $\sin\delta = 1$

From equation (*iii*)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

This represents the equation of a symmetrical ellipse. The emergent light in this case will be **elliptically polarized** provided $a \neq b$.

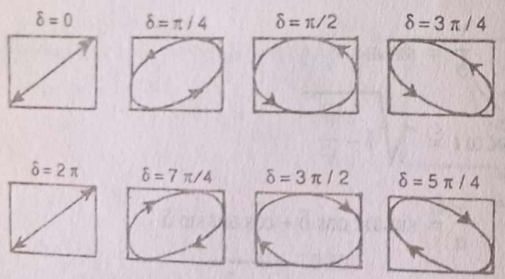

Fig. 10.32

(3) When $\delta = \frac{\pi}{2}$ and $a = b$

From equation (*iii*),

$$x^2 + y^2 = a^2$$

This represents the equation of circle of radius a. The emergent light will be **circularly polarized**. Here the vibrations of the incident plane-polarized light on the crystal make an angle of 45° with the direction of the optic axis.

(4) For $\delta = \pi/4$ or $7\pi/4$, the shape of the ellipse will be as shown in Fig. 10.32.

(5) For all other values of δ, the nature of vibrations will be as shown in Fig. 10.32.

10.24 QUARTER WAVE PLATE

It is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. The incident plane-polarized light is perpendicular to its surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_E respectively, then the path difference introduced between the two rays is given by :

For negative crystals, path difference $= (\mu_0 - \mu_E)\,t$

For positive crystals, path difference $= (\mu_E - \mu_0)\,t$

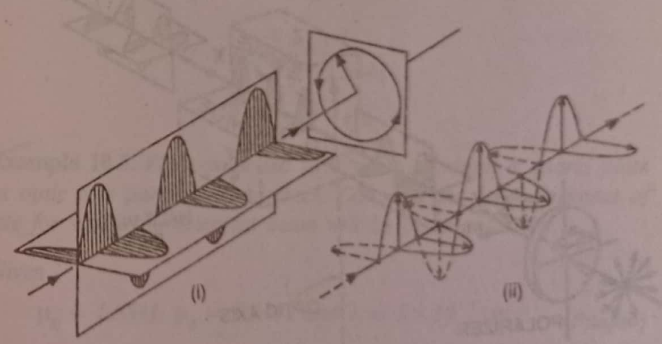

Fig. 10.33

To produce a path difference of $\frac{\lambda}{4}$, in calcite

$$(\mu_0 - \mu_E)\,t = \frac{\lambda}{4}$$

or

$$t = \frac{\lambda}{4(\mu_0 - \mu_E)} \qquad ...(i)$$

and in the case of quartz,

$$t = \frac{\lambda}{4(\mu_E - \mu_0)} \qquad ...(ii)$$

If the plane-polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quarter wave plate, the emergent light is circularly polarized (Fig. 10.33).

10.25 HALF WAVE PLATE

This plate is also made from a doubly refracting uniaxial crystal of quartz or calcite with its refracting faces cut parallel to the optic axis. The thickness of the plate is such that the ordinary and the extraordinary rays have a path difference $= \frac{\lambda}{2}$ after passing through the crystal.

For negative crystals, path difference $= (\mu_0 - \mu_E)\,t$

For positive crystals, path difference $= (\mu_E - \mu_0)\,t$

To produce a path difference of $\frac{\lambda}{2}$ in calcite,

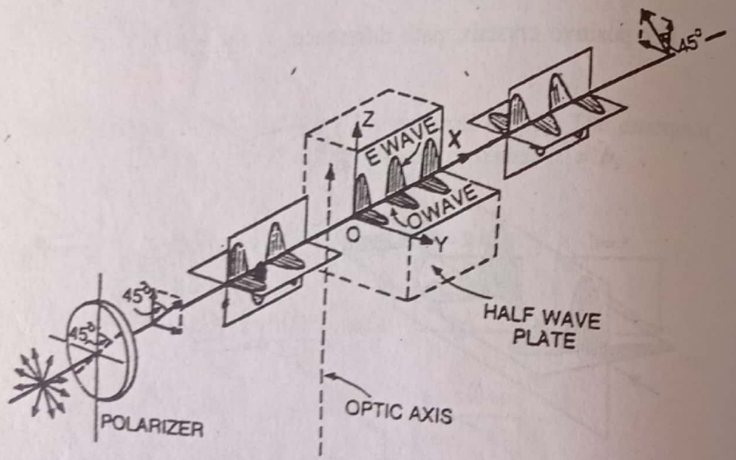

Fig. 10.34

$$(\mu_0 - \mu_E)\,t = \frac{\lambda}{2}$$

or

$$t = \frac{\lambda}{2(\mu_0 - \mu_E)} \qquad ...(i)$$

and in the case of quartz,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)} \qquad ...(ii)$$

When plane-polarized light is incident on a half-wave plate such that it makes an angle of 45° with the optic axis, a path difference of $\frac{\lambda}{2}$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90° as shown in Fig. 10.31. Thus, **a half wave plate rotates the azimuth of a beam of plane polarized light by 90°, provided the incident light makes an angle of 45° with the optic axis of the half wave plate.**

Example 10.2. *Calculate the thickness of a half wave plate of quartz for a wavelength of 5000 Å. Here $\mu_E = 1.553$ and $\mu_0 = 1.544$.* *(Delhi)*

For a half wave plate,

$$t = \frac{\lambda}{2[\mu_E - \mu_0]}$$

Here $\lambda = 5000\ \text{Å} = 5 \times 10^{-5}$ cm

$\mu_E = 1.553,\ \mu_0 = 1.544,\ t = ?$

$$\therefore \quad t = \frac{5 \times 10^{-5}}{2[1.553 - 1.544]}$$

or $$t = 2.78 \times 10^{-3}\ \text{cm}$$

Example 10.3. *Plane-polarized light passes through a quartz plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam will be plane-polarized.*

Given

$\mu_E = 1.5533,\ \mu_0 = 1.5442$ and $\lambda = 5 \times 10^{-5}$ cm *(Punjab)*

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$= \frac{5 \times 10^{-5}}{2(1.5533 - 1.5442)}$$

$$= 2.75 \times 10^{-3}\ \text{cm}$$

Example 10.4. *Calculate the thickness of (i) a quarter wave plate and (ii) a half wave plate given that*

$\mu_E = 1.553$ and $\mu_o = 1.544$ and $\lambda = 5000$ Å. [Delhi, 1976]

(i) For a quarter wave plate

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

$$= \frac{5000 \times 10^{-8}}{4[1.553 - 1.544]}$$

$$= \mathbf{1.39 \times 10^{-3}\ cm}$$

(ii) Fior a half wave plate

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$= \frac{5000 \times 10^{-8}}{2[1.553 - 1.544]}$$

$$= \mathbf{2.78 \times 10^{-3}\ cm}$$

Example 10.5. *Find the thickness of a quarter wave plate when the wavelength of light is 5890 Å. $\mu_E = 1.553$ and $\mu_0 = 1.544$.* [Delhi, 1977]

Here $$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

$$\lambda = 5890 \times 10^{-8}\ \text{cm},$$

$$\mu_E = 1.553, \quad \mu_0 = 1.544$$

$$\therefore \quad t = \frac{5890 \times 10^{-8}}{4(1.553 - 1.544)}$$

$$t = \frac{5890 \times 10^{-8}}{4 \times 0.009}$$

$$t = \mathbf{1.636 \times 10^{-3}\ cm}$$

Example 10.6. *Plane polarized light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and the extraordinary rays combine to form plane polarized light. Given,*

$$\mu_0 = 1.5442, \quad \mu_E = 1.5533$$

$$\lambda = 5 \times 10^{-5}\ \text{cm}$$ [Delhi, 1978]

Here, $$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$\lambda = 5 \times 10^{-5}\ \text{cm}$$

$$\mu_E = 1.5533$$

$$\mu_0 = 1.5442$$

$$t = \frac{5 \times 10^{-5}}{2(1.5533 - 1.5442)}$$

$$t = \mathbf{2.75 \times 10^{-3}\ cm}$$

Example 10.7. *Quartz has refractive indices 1.553 and 1.544. Calculate the thickness of the quarter wave plate for sodium light of wavelength 5890 Å.* [IAS, 1974]

For a quarter wave plate

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

Here $$\lambda = 5890\ \text{Å} = 5890 \times 10^{-10}\ \text{m}$$

$$\mu_E = 1.553$$

$$\mu_0 = 1.544$$

$$t = \frac{5890 \times 10^{-10}}{4[1.553 - 1.544]}$$

$$t = \mathbf{1.63 \times 10^{-5}\ m}$$

$$t = \mathbf{1.63 \times 10^{-2}\ mm}$$

Example 10.8. *A beam of linearly polarized light is changed into circularly polarized light by passing it through a sliced crystal of thickness 0.003 cm. Calculate the difference in refractive indices of the two rays in the crystal assuming this to be of minimum thickness that will produce the effect. The wavelength of light used in 6×10^{-7} m.* [IAS, 1989]

If the plane polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quarter wave plate, the emergent light is circularly polarized.

Here, $$t = 0.003\ \text{cm} = 3 \times 10^{-5}\ \text{m}$$

$$\lambda = 6 \times 10^{-7}\ \text{m}$$

$$(\mu_E - \mu_0)\, t = \frac{\lambda}{4}$$

$$(\mu_E - \mu_0) = \frac{\lambda}{4t}$$

$$= \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-5}}$$

$$= 5 \times 10^{-3}$$

Example 10.9. *For calcite $\mu_o = 1.658$ and $\mu_E = 1.486$ for sodium light. Calculate the minimum thickness of the quarter wave plate for calcite.* [Kanpur, 1990]

$$\lambda = 5893 \text{ Å}$$

Here

$$\lambda = 5893 \text{ Å} = 5.893 \times 10^{-7} \text{ m}$$

$$\mu_E = 1.486$$

$$\mu_0 = 1.658$$

$$t = ?$$

$$(\mu_0 - \mu_E)\, t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_0 - \mu_E)}$$

$$t = \frac{5.893 \times 10^{-7}}{4[1.658 - 1.486]}$$

$$t = \mathbf{8.56 \times 10^{-7}\ m}$$

$$t = \mathbf{8.56 \times 10^{-4}\ mm}$$

Example 10.10. *Calculate the thickness of a half wave plate for light of wavelength 5000 Å.*

[$\mu_0 = 1.55$ and $\mu_E = 1.45$] [Lucknow, 1990]

Here $\mu_0 = 1.55$, $\mu_E = 1.45$

$$\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}$$

As $\mu_0 > \mu_E$, it is a negative crystal

$\therefore$ Path difference $= (\mu_0 - \mu_E)\, t$

For a half wave plate,

$$(\mu_0 - \mu_E)\, t = \frac{\lambda}{2}$$

$$\therefore \quad t = \frac{\lambda}{2(\mu_0 - \mu_E)}$$

$$t = \frac{5 \times 10^{-7}}{2(1.55 - 1.45)}$$

$$t = \mathbf{2.5 \times 10^{-5}\ m}$$

$$t = \mathbf{2.5 \times 10^{-2}\ mm}$$

Example 10.11. *Calculate the thickness of a double refracting plate capable of producing a path difference of $\frac{\lambda}{4}$ between extraordinary and ordinary waves.*

$\lambda = 5890$ Å, $\mu_0 = 1.53$ and $\mu_E = 1.54$ [Kanpur, 1991]

Here $(\mu_E - \mu_0)\, t = \frac{\lambda}{4}$

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

Here $\lambda = 5890 \text{ Å} = 5.89 \times 10^{-7}$ m

$$t = \frac{5.89 \times 10^{-7}}{4[1.54 - 1.53]}$$

$$t = \mathbf{1.47 \times 10^{-5}\ m}$$

$$t = \mathbf{1.47 \times 10^{-2}\ mm}$$

10.26 PRODUCTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARIZED LIGHT

(1) **Plane polarized light.** A beam of monochromatic light is passed through a nicol prism. While passing through the nicol prism, the beam is split up into extraordinary ray and ordinary ray. The ordinary ray is totally internally reflected back at the Canada balsam layer, while the extraordinary ray passes through the nicol prism. The emergent beam is plane polarized.

and as glass windows in trains and aeroplanes. In aeroplanes, one of the polaroids is fixed while the other can be rotated to control the amount of light coming inside.

10.30 FRESNEL'S RHOMB

Fresnel constructed a rhomb of glass whose angles are 54° and 126° as shown in Fig. 10.38, based upon the fact that a phase difference of $\frac{\pi}{4}$ is introduced between the component vibrations (parallel and perpendicular to the plane of incidence) when light is totally internally reflected back at glass-air interface when the angle of incidence is 54°.

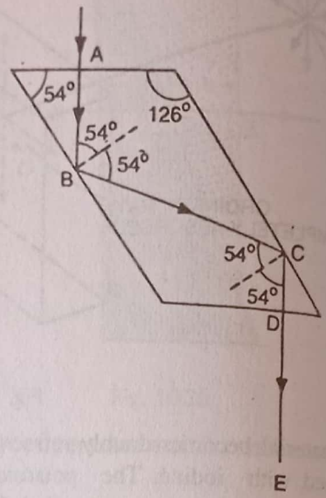

Fig. 10.39

A ray of light enters normally at one end of the rhomb and is totally internally reflected at the point B along BC. The angle of incidence at B is 54°, which is more than the critical angle of glass. Let the incident light be plane polarized and let the vibrations make an angle of 45° with the plane of incidence. Its components (*i*) parallel to the plane of incidence and (*ii*) perpendicular to the plane of incidence are equal. These components after reflection at the point B undergo a phase difference of $\frac{\pi}{4}$ or a path difference of $\frac{\lambda}{8}$. A further phase difference of $\frac{\pi}{4}$ or a path difference of $\frac{\lambda}{8}$ is introduced between the components when the ray BC. is totally internally reflected back along CD. Therefore the final emergent ray DE has two components, vibrating at right angles to each other and they have a path difference of $\frac{\lambda}{4}$. Therefore, the emergent light DE is circularly polarized. Fresnel's rhomb works similar to a quarter wave plate.

If the light entering the Fresnel's rhomb is circularly polarized, a further path difference of $\frac{\lambda}{4}$ is introduced between the component vibrations.

The total path difference between the component vibrations is $\frac{\lambda}{2}$. Therefore the emergent light is plane polarised and its vibrations make an angle of 45° with the plane of incidence.

When an elliptically polarized light is passed through a Fresnel's rhomb, a further path difference of $\frac{\lambda}{4}$ is introduced between the component vibrations (parallel and perpendicular to the plane of incidence). The total path difference between the component vibrations is $\frac{\lambda}{2}$ and the emergent light is plane polarized.

Thus, Fresnel's rhomb behaves just similar to a quarter wave plate. A quarter wave plate is used only for light of a particular wavelength, whereas a Fresnel's rhomb can be used for light of all wavelengths.

10.31 OPTICAL ACTIVITY

When a polarizer and an analyser are crossed, no light emerges out of the analyser [Fig. 10.40 (*i*)]. When a quartz plate cut with its faces

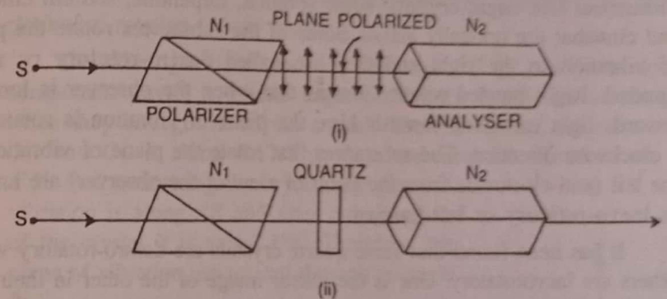

Fig. 10.40

parallel to the optic axis is introduced between N_1 and N_2 such that light falls normally upon the quartz plate, the light emerges out of N_2 [Fig. 10.40 (*ii*)].

The quartz plate turns the plane of vibration. The plane polarized light enters the quartz plate and its plane of vibration is gradually rotated as shown in Fig. 10.41.

The amount of rotation through which the plane of vibration is turned depends upon the thickness of the quartz plate and the wavelength of light. The action of turning the plane of vibration occurs inside the body of the plate and not on its surface. This phenomenon or the property of rotating the plane of vibration by certain crystals or substances is known as optical

activity and the substance is known as an optically active substance. It has been found that calcite does not produce any change in the plane of vibration of the plane polarised light. Therefore, it is not optically active.

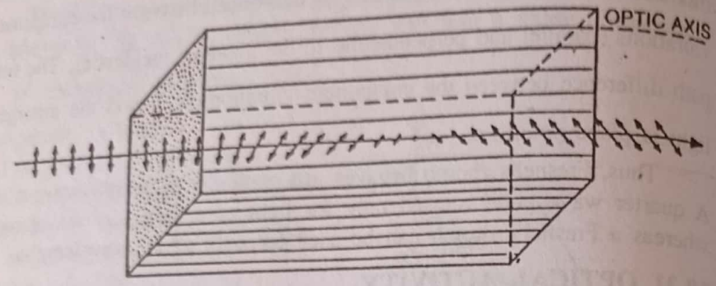

Fig. 10.41

Substances like sugar crystals, sugar solution, turpentine, sodium chlorate and cinnabar are optically active. Some of the substances rotate the plane of vibration to the right and they are called **dextro-rotatory** or **right handed**. Right handed rotation means that when the observer is looking towards light travelling towards him, the plane of vibration is rotated in a clockwise direction. The substances that rotate the plane of vibration to the left (anti-clockwise from the point of view of the observer) are known as **laevo-rotatory** or **left-handed**.

It has been found that some quartz crystals are dextro-rotatory while others are laevorotatory. One is the mirror image of the other in their orientation. The rotation of the plane of vibration in a solution depends upon the concentration of the optically active substance in the solution. This helps in finding the amount of cane sugar present in a sample of sugar solution.

10.32 FRESNEL'S EXPLANATION OF ROTATION

A linearly polarized light can be considered as a resultant of two circularly polarized vibrations rotating in opposite directions, with the same angular velocity. Fresnel assumed that a plane polarized light on entering a crystal along the optic axis is resolved into two circularly polarized vibrations rotating in opposite directions with the same angular velocity or frequency.

In a crystal like calcite, the two circularly polarized vibrations travel with the same angular velocity.

In Fig. 10.42, *OL* is the circularly polarised vector rotating in the anti-clockwise direction and *OR* is the circularly polarized vector rotating in the clockwise direction. The resultant vector of *OR* and *OL* is *OA*. According to Fresnel, when lineraly polarised light enters a crystal of calcite along the optic axis, the circularly polarized vibrations, rotating in opposite directions, have the same velocity. The resultant vibration will be along *AB*. Thus, crystals like calcite do not rotate the plane of vibration.

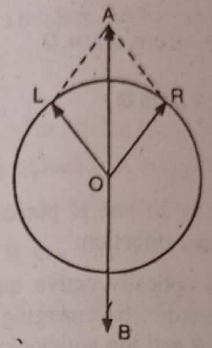

Fig. 10.42

In the case of quartz, the linearly polarized light, on entering the crystal is resolved into two circularly polarized vibrations rotating in opposite directions. In the case of a right-handed optically active crystal, the clockwise rotation travels faster while in a left-handed optically active crystal the anti-clockwise rotation travels faster.

Considering a right-handed quartz crystal (Fig. 10.43) the clockwise component travels a greater angle δ than the anticlockwise component when they emerge out of the crystal. The resultant of these two vectors *OR* and *OL* is along *OA'*. Therefore, the resultant vibrations are along *A'B'*. Before entering the crystal, the plane of vibration is along *AB* and after emerging out of the crystal it is along *A'B'*. Therefore, the plane of vibration has rotated through an angle $\frac{\delta}{2}$. The angle, through which the plane of vibration is rotated, depends upon the thickness of the crystal.

Analytical Treatment for Calcite. Circularly polarised light is the resultant of two rectangular components having a phase difference of $\frac{\pi}{2}$.

For clockwise circular vibrations,

$$x_1 = a \cos \omega t$$

$$x_2 = a \sin \omega t$$

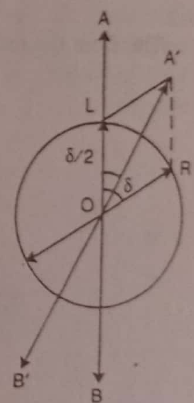

Fig. 10.43

For anti-clockwise circular vibrations,

$$x_2 = -a\cos\omega t$$

$$y_2 = a\sin\omega t$$

Therefore, the resultant vibrations,

along the x-axis, $\quad X = x_1 + x_2 = a\cos\omega t - a\cos\omega t = 0$

and along the y-axis, $\quad Y = y_1 + y_2 = a\sin\omega t + a\sin\omega t$

$$= 2a\sin\omega t$$

Thus, the resultant vibration has an amplitude $2a$ and is plane polarized. The plane of vibration is along the original direction.

For Quartz. In the case of right-handed optically active quartz crystal, the clockwise vibration travels faster. Therefore, on emerging out of the crystal, the clockwise vibrations start from R and the anticlockwise vibrations start from L. The phase difference between them $= \delta$.

For clockwise vibrations,

$$x_1 = a\cos(\omega t + \delta)$$

$$x_1 = a\sin(\omega t + \delta)$$

For anti-clockwise vibrations,

$$x_2 = -a\cos\omega t$$

$$y_2 = a\sin\omega t$$

Therefore the resultant displacements along the two axes are,

$$X = x_1 + x_2$$

$$= a\cos(\omega t + \delta) - a\cos\omega t$$

$$= 2a\sin\frac{\delta}{2}\sin\left(\omega t + \frac{\delta}{2}\right) \quad \text{...(i)}$$

$$Y = y_1 + y_2$$

$$= a\sin(\omega t + \delta) + a\sin\omega t$$

$$= 2a\cos\frac{\delta}{2}\sin\left(\omega t + \frac{\delta}{2}\right) \quad \text{...(ii)}$$

The resultant vibrations along the X-axis and Y-axis have the same phase. Therefore, the resultant vibration is plane polarized and it makes

an angle $\frac{\delta}{2}$ with the original direction. Therefore, the plane of vibration has rotated through an angle $\frac{\delta}{2}$ on passing through the crystal.

Dividing (i) by (ii),

$$\tan\frac{\delta}{2} = \frac{X}{Y}$$

Also taking the refractive index of clockwise vibrations $= \mu_R$ and the anticlockwise vibration $= \mu_L$, the optical path difference in passing through a thickness d of the crystal $= (\mu_L - \mu_R)\,d$.

If the wavelengths of light $= \lambda$,

then, phase difference $\quad \delta = \frac{2\pi}{\lambda} \times (\text{path difference})$

$$\delta = \frac{2\pi}{\lambda}(\mu_L - \mu_R)\,d$$

The plane of vibration is rotated through an angle

$$\frac{\delta}{2} = \frac{\pi}{\lambda}\left(\mu_L - \mu_R\right)d$$

In the case of left-handed optically active crystals,

$$\frac{\delta}{2} = \frac{\pi}{\lambda}(\mu_R - \mu_L)\,d$$

Example 10.3. *The rotation in the plane of polarization ($\lambda = 5893$ Å) in a certain substance is 10°/cm. Calculate the difference between the refractive indices for right and left circularly polarized light in the substance.* *(Mysore)*

Here $\quad \delta = \frac{2\pi}{\lambda}\left(\mu_R - \mu_L\right)d$

$$\mu_R - \mu_L = \left(\frac{\delta}{d}\right)\left(\frac{\lambda}{2\pi}\right)$$

Here $\quad \frac{\delta}{d} = 10°/\text{cm} = \frac{10 \times 2\pi}{360}$

$$= \frac{\pi}{36}\ \text{radian/cm}$$

$$\lambda = 5893 \times 10^{-8}\ \text{cm}$$

$$\therefore \quad \mu_R - \mu_L = \frac{\pi}{36} \times \frac{5893 \times 10^{-8}}{2\pi} = 8.185 \times 10^{-7}$$

10.33 FRESNEL'S EXPERIMENT

Fresnel showed that linearly polarized light on entering an optically active crystal is resolved into two circularly polarized vibrations. He arranged a number of negative and positive optically active quartz prisms as shown in Fig. 10.44.

Two circularly polarized beams were observed, one rotating to the right (clockwise) and the other rotating to the left (anticlockwise). The optic axis is parallel to the base of each prism. When plane polarized light in incident normally on the first crystal surface (*R*), the two component circular vibrations (clockwise and anticlockwise, travel along the same direction with different speeds. When the beam is incident on the sblique

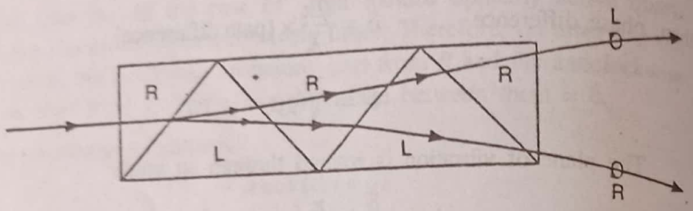

Fig. 10.44

surface of the second prism (*L*), the beam which was faster in the first prism becomes slower in the second prism and *vice versa*. Therefore, one beam is bent away from the normal and the other is bent towards the normal. The two beams are separated apart, while they travel through the prism *L*. Again at the boundary of the next prism (*R*), the speeds are interchanged and the beam that is bent towards the normal in prism *L*, is now bent away from the normal. Thus the two beams are separated more and more while passing through the arrangement. When the two beams emerge out, they are widely apart. When these beams are treated with a quarter wave plate and a nicol prism, both are found to be circularly polarized.

10.34 SPECIFIC ROTATION

Liquids containing an optically active substance *e.g.*, sugar solution, camphor in alcohol etc. rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon (1) the thickness of the medium (2) concentration of the souution or density of the active substance in the solvent (3) wavelength of light and (4) temperature.

The specific rotation is defined as the rotation produced by a decimetre (10 cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution. Therefore,

$$S_\lambda^t = \frac{10\,\theta}{l\,C}$$

where S_λ^t, represents the specific rotation at temperature t °C for a wavelength λ, θ is the angle of rotation, l is the length of the solution in cm through which the plane polarized light passes and C is the concentration of the active substance in g/cc in the solution.

The angle through which the plane of polarization is rotated by the optically active substance is determined with the help of a polarimeter. When this instrument is used to determine the quantity of sugar in a solution, it is known as a saccharimeter.

10.35 LAURENT'S HALF SHADE POLARIMETER

It consists of two nicol prisms N_1 and N_2. N_1 is a polarizer and N_2 is an analyser. Behind N_1, there is a half wave plate of quartz *Q* which covers one half of the field of view, while the other half *G* is a glass

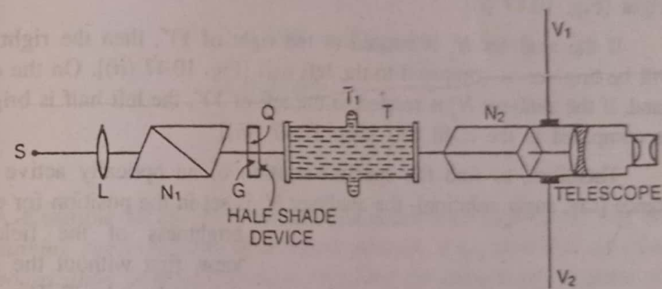

Fig. 10.45

plate. The glass plate *G* absorbs the same amount of light as the quartz plate *Q*. *T* is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles (if any) will appear at the upper portion of the wide bore T_1 of the tube.

Light from a monochromatic source S is incident on the converging lens L. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and the other half passes through the glass plate G. Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along AB (Fig. 10.46). Here AB makes an angle θ with YY'. On passing through the quartz plate Q, the beam is split up into ordinary and extraordinary components which travel along the same direction but with different speeds and on emergence a phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced between them.

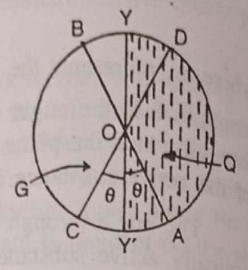

Fig. 10.46

The vibrations of the beam emerging out of quartz will be along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyser N_2 has its principal plane or section along YY' *i.e.*, along the direction which bisects the angle AOC, the amplitudes of light incident on the analyser N_2 from both the halves (*i.e.*, quartz half and glass half) will be equal. Therefore, the field of view will be equally bright [Fig. 10.47 (*i*)].

If the analyser N_2 is rotated to the right of YY', then the right half will be brighter as compared to the left half [Fig. 10.47 (*ii*)]. On the other hand, if the analyser N_2 is rotated to the left of YY', the left half is brighter as compared to the right half [Fig. 10.47 (*iii*)].

Therefore, to find the specific rotation of an optically active substance [say, sugar solution], the analyser N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube T. The readings of the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, AB and CD are rotated in the clockwise direction. Therefore, on the introduction of the tube containing the sugar solution, the field of view is not equally bright. The analyser is rotated in the clockwise direction and is brought to a position

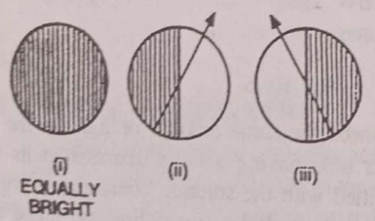

Fig. 10.47

so that the whole field of view is equally bright. The new positions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyser has been rotated gives the angle through which the plane of vibration of the incident beam has been rotated by the sugar solution. In the actual experiment, for various concentrations of the sugar solution, the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation θ. The graph is a straight line (Fig. 10.48).

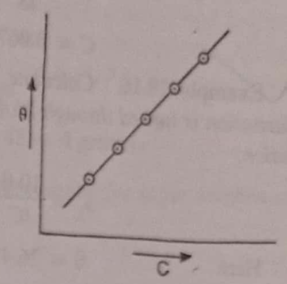

Fig. 10.48

Then from the relation

$$S_\lambda^t = \frac{10\,\theta}{lC},$$ the specific rotation of the optically active substance is calculated.

Example 10.14. *Determine the specific rotation of the given sample of sugar solution if the plane of polarization is turned through 13.2°. The length of the tube containing 10% sugar solution is 20 cm.*

Here,

$$\theta = 13.2°$$

$$C = 10\% = 0.1\ \text{g/cm}^3$$

$$l = 20\ \text{cm}$$

$$S_\lambda^t = \frac{10 \times 13.2}{20 \times 0.1} = 66°$$

Example 10.15. *On introducing a polarimeter tube 25 cm long and containing sugar solution of unknown strength, it is found that the plane of polarization is rooted through 10°. Find the strength of the sugar solution in g/cm³. (Given that the specific rotation of sugar solution is 60° per decimetre per unit concentration)* (*Rajasthan 1966*)

Here,

$$\theta = 10°$$

$$S = 60°$$

$$l = 25\ \text{cm}$$

$$S = \frac{10\,\theta}{lC}$$

$$C = \frac{10\,\theta}{lS} = \frac{10\times 10}{25\times 60} = \frac{1}{15}$$

$$C = 0.067 \text{ g / cc}$$

Example 10.16. *Calculate the specific rotation if the plane of polarisation is turned through 26.4°, traversing 20 cm length of 20% sugar solution.* (Delhi)

$$S_\lambda^t = \frac{10\,\theta}{lC}$$

Here $\theta = 26.4°$, $l = 20$ cm

$C = 20\% = 0.2 \text{ g / cm}^3$

$$\therefore \quad S_\lambda^t = \frac{10\times 26.4}{20\times 0.2}$$

$$= 66°$$

Example 10.17. *A 20 cm long tube containing sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the strength of the solution.* [Kanpur]

Here $\theta = 11°$

$l = 20$ cm, $S = 60°$

$$S = \frac{10\theta}{lC} \qquad C = \frac{10\,\theta}{lS}$$

or

$$C = \frac{10\times 11}{20\times 66}$$

$$C = 0.0833 \text{ g / cm}^3$$

Example 10.18. *A 200 mm long tube and containing 48 cm³ of sugar solution produces an optical rotation of 11° when placed in a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of a solution.* [Kanpur, 1990]

Here $\theta = 11°$

$l = 200 \text{ mm} = 20 \text{ cm}$

$S = 66°$

$$S = \frac{10\,\theta}{lC}$$

$$\therefore \quad C = \frac{10\,\theta}{lS}$$

$$C = \frac{10\times 11}{20\times 66}$$

$$C = 0.0833 \text{ g / cm}^3$$

volume, $V = 48 \text{ cm}^3$

Hence mass of sugar in the solution

$$M = CV = 0.0833\times 48 = 4 \text{ grams}$$

Example 10.19. *Calculate the specific rotation for sugar solution using the following data :*

Length of the tube = 20 cm

Volume of the tube = 120 cm³

Quantity of sugar dissolved = 6 g

Angle of rotation of the analyser for restoring equal intensity = 6.6° [Kanpur, 1991]

$\theta = 6.6°$

$l = 20$ cm

$$C = \frac{6}{80} \text{ g / cm}^3$$

$$S = \frac{10\,\theta}{lC}$$

$$S = \frac{10\times 6.6\times 120}{20\times 6}$$

$$S = 66°$$

10.36 BIQUARTZ

Instead of half shade plate, a biquartz plate is also used in polarimeters. If consists of two semi-circular plates of quartz each of thickness 3.75 mm. One half consists of right-handed optically active quartz, while the other is left-handed optically active quartz. If white light is used, yellow light is quenched by the biquartz plate and both the halves will have the **tint of passage.** This can be adjusted by rotating the analyser N_2 to a particular position. When the

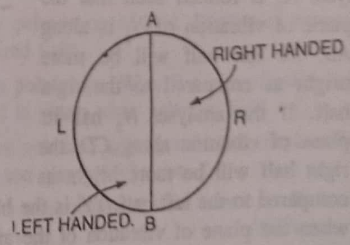

Fig. 10.49