

\int_0^x	\int_0^y
0	0

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Beta function: The function denoted by $B(m, n)$, where m and n are positive values [more than 0] and it

defined by $B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$ and it is known as Beta function. It is also called the first

Eulerian integral.

Given as $\int_0^1 x^{m-1} (1-x)^{n-1} dx$.

Gamma function: The function is denoted by $\Gamma(n)$ and it is given as $\int_0^\infty t^{n-1} e^{-xt} dt$.

and if defined by $\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt$ where $n > 0$ is known as gamma function. It is also known as second Eulerian integral.

$\int_0^\infty t^{n-1} e^{-xt} dt$

$$\frac{1}{\Gamma(n)} \int_0^\infty t^{n-1} e^{-xt} dt$$

calculus book

textbook of calculus
part 1

~~Soln:~~ we know that $\sin^2 \theta + \cos^2 \theta = 1$

$$B(m,n) = \int_0^{\pi/2} x^m (1-x)^n dx$$

$$\text{Let } x = \sin^2 \theta$$

x	0	1
θ	0	$\pi/2$

$$dx = 2\sin \theta \cos \theta d\theta$$

Putting this value in the eqn (1) we get,

$$B(m,n) = \int_0^{\pi/2} (\sin^2 \theta)^m (1 - \sin^2 \theta)^n = \int_0^{\pi/2} \sin^m \theta \cos^m \theta \cos^n \theta d\theta$$

$$= 2 \int_0^{\pi/2} \sin^{2m+2} \theta \cos^{2n+2} \theta \sin \theta \cos \theta d\theta$$

$$B(m,n) = 2 \int_0^{\pi/2} \sin^{2m+1} \theta \cos^{2n+1} \theta d\theta$$

$$\frac{B(m,n)}{2} = \int_0^{\pi/2} \sin \theta \cos^{2m+1} \theta d\theta$$

$$\text{Let, } 2m+1 = p \text{ and } 2n+1 = q \text{ and } m = \frac{p+1}{2}$$

And we know,

$$B(m,n) = \sqrt{m+n+1}$$

$$\frac{q+1}{2}$$

$\therefore B(m,n) = \sqrt{m+n+1}$

terence force

$$\frac{\sqrt{m \pi}}{2(m+n)} = \int_0^{\pi/2} \sin^{m-1} \theta \cos^{2n-1} \theta d\theta$$

$$\frac{\sqrt{P+1}}{2} \frac{\sqrt{Q+1}}{2} = \int_0^{\pi/2} \sin^P \theta \cos^Q \theta d\theta$$

$$\frac{\sqrt{P+1}}{2} \frac{\sqrt{Q+1}}{2} = \int_0^{\pi/2} \sin^P \theta \cos^Q \theta d\theta$$

$$\therefore \frac{\sqrt{P+1}}{2} \frac{\sqrt{Q+1}}{2} = \int_0^{\pi/2} \sin^P \theta \cos^Q \theta d\theta$$

$$(b) \quad \int_0^{\alpha} \frac{x^{m-1}}{1+x} dx$$

$$\text{Let, } x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$\text{Now, } \int_0^{\alpha} \frac{x^{m-1}}{1+x} dx = \int_0^{\pi/2} \frac{(\tan \theta)^{m-1} (\sec \theta)^2}{1+\tan^2 \theta} d\theta$$

$$= 2 \int_0^{\pi/2} \frac{\tan^{2n-2}}{\sec^{2\theta}} \cdot \tan \theta \sec^2 \theta d\theta$$

(from Q)

$$= 2 \int_0^{\pi/2} \tan^{2n-1} \theta d\theta$$

$$= 2 \int_0^{\pi/2} \frac{\sin 2n-1 \theta}{\cos^{2n-1} \theta} d\theta$$

$$= 2 \int_0^{\pi/2} \sin^{2n-1} \theta \cos^{(2n-1)} \theta d\theta$$

$$\Rightarrow 2 \int_0^{\frac{2n-1+t}{2}} \frac{-2n+t+1}{2} dt$$

$$= 2 \int_0^{\frac{2n-1+2t+2}{2}} \frac{2}{2} dt$$

$$\Rightarrow \frac{2n}{2} \int_0^{\frac{-2n+t+2}{2}} \frac{2}{2} dt$$

$$= \frac{2n}{2} \int_0^{\frac{1-n}{2}} \frac{2}{2} dt$$

$$= \frac{\sqrt{n}}{\pi} \int_0^{\frac{1-n}{2}} dt$$

$$= \frac{\sqrt{n}}{\pi} \frac{1-n}{2}$$

$$= \frac{1-n}{2\pi}$$

(Ans)

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$$\int_0^1 x^{m-1} (1-x^a)^n dx$$

$$\text{Let } x^a = \sin^2 \theta$$

$$x = \sin \theta$$

$$dx = \frac{2}{a} \sin^{-2} \theta \sin \theta \cos \theta d\theta$$

$$\boxed{\int_0^1 x^{m-1} (1-x^a)^n dx = \int_0^{\pi/2} \sin^{m-1} \theta \cos^{n-1} \theta \left(\frac{2}{a} \sin^{-2} \theta \sin \theta \cos \theta d\theta \right)}$$

$$\text{Now, } \int_0^1 x^{m-1} (1-x^a)^n dx = \int_0^{\pi/2} \sin^{m-1} \theta \cos^{n-1} \theta \left(\frac{2}{a} \sin^{-2} \theta \sin \theta \cos \theta d\theta \right)$$

$$= \int_0^{\pi/2} (\sin^{2m} \theta) \sin^n (\theta) \left(\frac{2}{a} \sin^{-2} \theta \sin^{-1} \theta \cos \theta d\theta \right)$$

$$= \int_0^{\pi/2} \sin^{2m-2} \theta \cos^{2m} \theta \left(\frac{2}{a} \sin^{-2} \theta \sin^{-1} \theta \cos \theta d\theta \right)$$

$$= \frac{2}{a} \int_0^{\pi/2} \sin^{2m-2} \theta \cos^{2m} \theta \cos^{-2} \theta d\theta$$

$$= \frac{2}{a} \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2m+1} \theta d\theta$$

$$= \frac{2^{m+1}}{a} \int_0^{\pi/2} \frac{\sin^{2m+1} \theta}{2} d\theta$$

$$= 2 \int_0^{\pi/2} \frac{\sin^{2m+1} \theta}{a} d\theta$$

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$$= \frac{1}{a} \times \frac{\int_{m/n}^{\infty} x^n dx}{\int_{m/n+1}^{\infty} dx} \quad (2)$$

but

$$\int_{m/n}^{\infty} x^n dx$$

$$\int_0^1 \frac{3\pi}{32} \frac{x^5}{1-x} dx$$

Let $x = \sin^2 \theta$

$$dx = 2 \sin \theta \cos \theta d\theta \quad \text{and} \quad \theta = \arcsin x$$

obtains limit $\theta = \arcsin m/n$



$$= \frac{3\pi}{32} \int_0^1 \frac{x^5}{\sqrt{1-x^2}} dx \quad (\text{now})$$

$$= \frac{3\pi}{32} \int_0^{\arcsin m/n} \frac{\cos^2 \theta \cdot \sin^3 \theta \cdot 2 \sin \theta \cos \theta d\theta}{\sqrt{1-\sin^2 \theta}} \quad (\text{using } \cos \theta = \sin \theta)$$

$$= \frac{3\pi}{32} \int_0^{\arcsin m/n} \frac{\sin^6 \theta}{\cos \theta} d\theta \quad (\text{using } \sin^2 \theta = 1 - \cos^2 \theta)$$

$$= \frac{3\pi}{16} \int_0^{\arcsin m/n} \frac{\sin^5 \theta \cdot \sin \theta}{\cos \theta} d\theta \quad (\text{using } \sin^3 \theta = \sin^2 \theta \cdot \sin \theta)$$

$$= \frac{3\pi}{16} \int_0^{\arcsin m/n} \sin \theta \cdot \cos^3 \theta d\theta \quad (\text{using } \cos^2 \theta = 1 - \sin^2 \theta)$$

$$= \frac{3\pi}{16} \left[\frac{1 + \tan^2 \theta}{2} \right]_0^{\arcsin m/n} \quad \boxed{\text{Ans}}$$

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with respect to θ

$$\begin{aligned} & \frac{35}{16} \times \frac{\sqrt{4 - \tan^2 \theta}}{2 \sqrt{9 \tan^2 \theta + 6}} \\ &= \frac{35}{32} \times \frac{3 \sqrt{1 - \tan^2 \theta}}{\sqrt{9 \tan^2 \theta + 6}} = \frac{35}{32} \times \frac{3 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 6 \cos^2 \theta}} \\ &= \frac{35}{32} \times \frac{3 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 2 \cos^2 \theta} \sqrt{9 \sin^2 \theta + 6 \cos^2 \theta}} \\ &= \frac{35}{16} \times \frac{3 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 2 \cos^2 \theta} \sqrt{9 \sin^2 \theta + 6 \cos^2 \theta}} \\ &= \frac{35}{16} \times \frac{3 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 2 \cos^2 \theta} \times \frac{3}{2} \sqrt{2}} \\ &= \frac{35}{16} \times \frac{3 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 2 \cos^2 \theta} \times \frac{3}{2} \sqrt{2}} \end{aligned}$$

Integrating with respect to θ

$$\begin{aligned} & \int \frac{35}{16} \times \frac{3 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 2 \cos^2 \theta} \times \frac{3}{2} \sqrt{2}} d\theta \\ &= \frac{35}{16} \times \frac{1}{3} \int \frac{6 \cos^2 \theta}{\sqrt{9 \sin^2 \theta + 2 \cos^2 \theta} \times \sqrt{2}} d\theta \end{aligned}$$

~~2. Laplace transform of derivatives~~

(a)

Lst derivative's to transform function process (a)

$$\begin{aligned} L \int \frac{d f(t)}{dt} dt &= \int e^{-st} \frac{d f(t)}{dt} dt \\ &= \int e^{-st} f'(t) dt + s \int e^{-st} f(t) dt \\ &= -f(s) + Sf(s) \\ &\Rightarrow Sf(s) = f(s) + f(0) \end{aligned}$$

2nd derivative:

$$\begin{aligned} L \left[\frac{d^2 f(t)}{dt^2} \right] &= \int_0^\infty e^{-st} t^2 f(t) dt \\ &= \left[e^{-st} \frac{d f(t)}{dt} \right]_0 + s \int_0^\infty e^{-st} \cdot \frac{d^2 f(t)}{dt^2} \cdot dt \end{aligned}$$

$$\begin{aligned} &= -\frac{df(0)}{dt} + s \left[-f(0)e^{-st} + f'(s) \right] + s^2 f(s) \\ &= -f(0) - sf'(0) + s^2 f(s) \end{aligned}$$

~~✓~~ Laplace transformation: Replace transformation of function $f(t)$ is defined by $L \hat{f}(s) = \int_0^\infty e^{-st} f(t) dt$

$$L \hat{f}(t) = \int_0^\infty e^{-st} f(t) dt = f(s)$$

(C) ~~using~~ linear property of Laplace transform
transform. we take out $\sin 2t$ and $\cos 3t$.

$$L \left[\sin 2t + 2 \cos 3t \right] = L \left[\frac{2}{2} \sin 2t \right] + L \left[2 \cos 3t \right]$$

Hence,
 $L \left[\sin 2t \right] \rightarrow \int_0^\infty e^{-st} \sin 2t dt$

$$\begin{aligned}
 & \int_0^\infty e^{-st} \left(\frac{e^{2it} - e^{-2it}}{1 + 2i} \right) dt = \int_0^\infty \frac{e^{2it} - e^{-2it}}{s + 2i} dt \\
 & \approx \frac{1}{2\pi} \int_0^\infty e^{-st + 2it} dt - \frac{1}{2\pi} \int_0^\infty e^{-st - 2it} dt \\
 & \approx \frac{1}{2\pi} \int_0^\infty e^{-(s-2i)t} dt - \frac{1}{2\pi} \int_0^\infty e^{-(s+2i)t} dt \\
 & \approx \frac{1}{2\pi} \left[\frac{e^{-(s-2i)t}}{-s+2i} - \frac{e^{-(s+2i)t}}{-s-2i} \right]_0^\infty \\
 & \approx \frac{1}{2\pi} \left[\frac{1}{(s-2i)} - \frac{1}{(s+2i)} \right] \\
 & \approx \frac{1}{2\pi i} \frac{4i}{s^2 + 4}
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty e^{-st} \left(\frac{e^{2it} + e^{-2it}}{1 - 2i} \right) dt = \int_0^\infty \frac{e^{2it} + e^{-2it}}{s - 2i} dt \\
 & \approx \frac{1}{2\pi} \int_0^\infty e^{-st + 2it} dt + \frac{1}{2\pi} \int_0^\infty e^{-st - 2it} dt \\
 & \approx \frac{1}{2\pi} \left[\frac{e^{-(s-2i)t}}{-s+2i} + \frac{e^{-(s+2i)t}}{-s-2i} \right]_0^\infty \\
 & \approx \frac{1}{2\pi} \left[\frac{1}{(s-2i)} + \frac{1}{(s+2i)} \right] \\
 & \approx \frac{1}{2\pi i} \frac{4i}{s^2 + 4}
 \end{aligned}$$

(b) Laplace transform of $\cosh t$

$$\begin{aligned}
 L_2 f(t) &= \int_0^\infty e^{-st} \cosh t dt \\
 &= \int_0^\infty e^{-st} \left(\frac{e^{t+2it} + e^{t-2it}}{2} \right) dt \\
 &= \frac{1}{2} \int_0^\infty e^{-st} (e^{(s+2i)t} + e^{(s-2i)t}) dt
 \end{aligned}$$

$$= \frac{1}{2} \int_0^\infty e^{-(s-1)t} dt + \frac{1}{2} \int_0^\infty e^{-st} - (s+1)^{-1} dt +$$

$$= \frac{1}{2} \left[\frac{e^{-(s-1)t}}{-(s-1)} \right]_0^\infty + \frac{1}{2} \left[\frac{e^{-st}}{-s} \right]_0^\infty + \frac{1}{2} \left[\frac{-e^{-(s+1)t}}{-(s+1)} \right]_0^\infty$$

$$= \frac{1}{2} \cdot \left[\frac{1}{s-1} \right]_0^\infty + \frac{1}{2} \left[\frac{1}{s} \right]_0^\infty + \frac{1}{2} \left[\frac{1}{s+1} \right]_0^\infty$$

$$= \frac{1}{2} \cdot \frac{1}{2(s-1)} \left[\frac{1}{(s-1)} \right]_0^\infty + \frac{1}{2} \left[\frac{1}{s} \right]_0^\infty + \frac{1}{2} \left[\frac{1}{(s+1)} \right]_0^\infty$$

$$= \frac{(s+1) + (s-1)}{2(s-1)(s+1)} \frac{1}{s} + \frac{(s+2 + s-2)}{2(s-1)(s+1)} \frac{1}{s}$$

$$= \frac{s+2 + s-2}{2(s-1)(s+1)} \frac{1}{s} = \frac{s}{2(s-1)(s+1)} \frac{1}{s} = \frac{s}{2(s^2-1)} \frac{1}{s} = \frac{1}{2(s^2-1)} \frac{1}{s} = \frac{1}{2s^2} \frac{1}{s} = \frac{1}{2s^3}$$

$$\therefore f(t) = \cos st \quad \text{for } s > 0 \quad \text{for non-negative values of } s$$

$$\therefore L[\cos st] = f(s) = \int_0^\infty e^{-st} \cos st dt$$

$$= \int_0^\infty e^{-st} \frac{1}{2} \cdot 2 \cos st dt + \int_0^\infty e^{-st} \cdot 0 dt = \frac{1}{2} \int_0^\infty e^{-st} \cdot 2 \cos st dt$$

$$= \frac{1}{2} \int_0^\infty e^{-st} (e^{iat} + e^{i\bar{a}t}) dt + f(s)$$

$$= \frac{1}{2} \int_0^\infty e^{-st+it} dt + \frac{1}{2} \int_0^\infty e^{-st-i\bar{a}t} dt$$

$$\begin{aligned}
 &= \frac{1}{2} \int_0^\alpha e^{-(s-1)t} dt + \frac{1}{2} \int_0^\alpha e^{-(s+1)t} dt \\
 &= \frac{1}{2} \left[\frac{e^{-(s-1)t}}{-(s-1)} \right]_0^\alpha + \frac{1}{2} \left[\frac{e^{-(s+1)t}}{-(s+1)} \right]_0^\alpha \\
 &= \frac{1}{2} \cdot \left[\frac{e^{-t}}{-s+1} \right]_0^\alpha + \frac{1}{2} \cdot \left[\frac{e^{-t}}{-s-1} \right]_0^\alpha \\
 &= \frac{1}{2} \cdot \left[\frac{1}{2(s-1)} \right]_0^\alpha + \frac{1}{2} \cdot \left[\frac{1}{2(s+1)} \right]_0^\alpha \\
 &= \frac{(s+1) + (s-1)}{2(s^2-1)} \\
 &= \frac{s+2+s-2}{2(s-1)(s+1)} \\
 &= \frac{s}{2(s-1)(s+1)} \\
 &\quad + \frac{1}{2(s^2-1)} \cdot \frac{1}{(s+1)} \\
 &\quad - \frac{1}{2(s^2-1)} \cdot \frac{1}{(s-1)} \\
 &= \frac{s}{2(s-1)(s+1)} + \frac{1}{4(s-1)^2} - \frac{1}{4(s+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 &f(t) = \cos st \\
 L f(t) &= \int_0^\infty e^{-st} \cos st dt \\
 &\approx \int_0^\alpha e^{-st} \frac{1}{2} \cdot 2 \cos st dt \quad (\text{approximate integral}) \quad (1) \\
 &\approx \frac{1}{2} \int_0^\alpha e^{-st} (e^{iat} + e^{iat}) dt + (\text{cancel } i) \\
 &= \frac{1}{2} \int_0^\alpha e^{-st} \left[e^{iat} + \frac{1}{2} (e^{iat} - e^{-iat}) \right] dt
 \end{aligned}$$

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$$\begin{aligned} & \frac{1}{2} \int_0^{\alpha} e^{-c\alpha t} dt + \frac{1}{2} \int_0^{\alpha} e^{-c\alpha t} dt \\ &= \frac{1}{2} \left[\frac{e^{-c\alpha t}}{-c\alpha} \right]_0^{\alpha} + \left[\frac{e^{-c\alpha t} u}{-c\alpha} \right]_0^{\alpha} - c\alpha u \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \left[\frac{1}{c\alpha} (1 - e^{-c\alpha}) + \frac{1}{c\alpha} (e^{-c\alpha} - 1) \right] - c\alpha u \\ &= \frac{1}{2} \left[\frac{(e^{c\alpha} - 1)}{c\alpha^2} - \frac{(e^{c\alpha} - 1)}{c\alpha^2} \right] - c\alpha u \\ &= \frac{1}{2} \left[\frac{e^{c\alpha} - 1}{c\alpha^2} - \frac{e^{c\alpha} - 1}{c\alpha^2} \right] - c\alpha u \\ &= \frac{1}{2} \left[\frac{e^{c\alpha} - 1}{c\alpha^2} (1 + c\alpha - c\alpha) - c\alpha \right] + \frac{e^{c\alpha} - 1}{c\alpha^2} c\alpha u \end{aligned}$$

$$0 = \frac{1}{2} \left[\frac{e^{c\alpha} - 1}{c\alpha^2} (1 + c\alpha - c\alpha) - c\alpha \right] + \frac{e^{c\alpha} - 1}{c\alpha^2} c\alpha u$$

3. (a) The series $1 + \frac{\alpha\beta}{1-\alpha} x + \frac{\alpha\beta(1+\alpha)}{1-\alpha} x^2 + \dots$ is called hypergeometric series, which satisfies the recurrence relation of this hypergeometric series is

And the hypergeometric equation of this hypergeometric series is

$$x(v-x)y'' + [x - (c\alpha + \beta + 1)]y' - \alpha\beta y = 0$$

A_{ijk} is a mixed tensor
Addition, ~~contraction~~ multiplication and
contraction

Inverse of a diagonal matrix

2020
to obtain $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$

1(a) Even functions: ($f(x)$ is said
to be an even function if $f(-x)=f(x)$)

For Example, $f(x)=x^2$ $f(x)=\cos x$

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odd function: A function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$

(b) Fourier series: It is a mathematical way to represent non-trigonometric periodic function as an infinite sum of trigonometric function. A series choose each and every terms either of sine or cosine.

$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$
 $\therefore f(x) = a_0 + \sum_{m=1}^{\infty} \left[\frac{a_m}{m\pi} \cos mx + \frac{b_m}{m\pi} \sin mx \right]$
An even function can have no sine term in its Fourier series;

If the function $f(x)$ is even, i.e. $f(-x) = f(x)$
then $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$

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$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx + \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \\ \therefore a_0 = I_1 + I_2 \quad \text{(i)}$$

Now, $I_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$
if you put it in the diagram
so nothing happens (ii)

so nothing happens
because $dx = dy$
so nothing happens
nothing happens

$$\boxed{x \quad 0 \quad \theta \quad \pi}$$

nothing happens
 $\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^{\pi} f(-x) dx$

$$= -\frac{1}{\pi} \int_{\pi}^0 f(-y) dy \\ = -\frac{1}{\pi} \int_{\pi}^0 f(y) dy \\ = \frac{1}{\pi} \int_{-\pi}^0 f(y) dy$$

$\therefore a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx + \frac{1}{\pi} \int_{-\pi}^{\pi} f(-x) dx$

Because (ii)

$$a_0 = 2 \times \frac{1}{\pi} \int_0^{\pi} f(x) dx \\ \text{because } \int_{-\pi}^{\pi} f(x) dx = \int_0^{\pi} f(x) dx + \int_{-\pi}^0 f(x) dx \\ \text{and } \int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^0 f(-x) dx + \int_0^{\pi} f(x) dx \\ \text{so } \int_{-\pi}^{\pi} f(x) dx = 2 \int_0^{\pi} f(x) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx + \frac{1}{\pi} \int_{-\pi}^{\pi} f(-x) dx \\ \text{putting, } x = -y \\ dx = -dy$$

$$\boxed{x \quad 0 \quad \theta \quad \pi}$$

$$\text{Now, } I_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos mx dx + \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(-y) \cos m(-y) (-dy)$$

$$= -\frac{1}{\pi} \int_{\pi}^{-\pi} f(y) \cos my dy$$

$$\therefore I_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos mx dx - \frac{1}{\pi} \int_{\pi}^{\pi} f(y) \cos my dy$$

Therefore, $\tan \frac{2}{\pi} \int_0^{\pi} f(y) \cos my dy$ (Q)

$$\text{And, } b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(-y) \sin m(-y) (-dy)$$

$$\therefore b_m = I_4 + I_5$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \sin my dy$$

$$\text{Now, } I_4 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx dx$$

$$= \frac{1}{\pi} \int_{\pi}^{-\pi} f(y) \sin my dy$$

$$= -\frac{1}{\pi} \int_{-\pi}^{\pi} f(y) \sin my dy$$

$$\therefore b_m = I_5 = -I_4$$

$$\therefore b_m = 0$$

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$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

$$= \frac{a_0}{2} + \int_0^{\pi} f(x) dx + \sum_{n=1}^{\infty} \left[\frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx \right] \text{ cosine}$$

$$= \frac{1}{\pi} \int_0^{\pi} f(x) dx + \sum_{n=1}^{\infty} \left[\frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx \right] \text{ cosine}$$

(a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$

Now $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$
 $= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx + \frac{1}{\pi} \int_{\pi}^{0} f(x) dx =$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx + \frac{1}{\pi} \int_{\pi}^{0} f(x) dx$$

$$= \frac{1}{\pi} [-\sin x]_{-\pi}^{\pi} + \frac{1}{\pi} [\sin x]_{\pi}^0$$

$$= 0 + 0 = 0$$

Now, $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$
 $= \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx \left(\frac{1}{2} + \int_0^x f(u) du \right) dx$
 $= \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx \cdot \frac{1}{2} dx + \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx \int_0^x f(u) du dx$
 $= \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx \cos nx dx + \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos nx \cos nx dx$

$$\begin{aligned}
&= -\frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} \cos(m\pi x) + \cos(m+1)\pi x \right] dx + \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin(m\pi x) dx \\
&\quad + \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{\sin(m\pi x)}{(m+1)} + \frac{\sin((m+1)\pi x)}{(m+1)} \right] dx \\
&= -\frac{1}{2\pi} \left[\frac{\sin(m\pi x)}{(m+1)} + \frac{\sin((m+1)\pi x)}{(m+1)} \right]_{-\pi}^{\pi} + \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\sin((m+1)\pi x)}{(m+1)} dx \\
&= -\frac{1}{2\pi} \left[\frac{\sin(m\pi)}{(m+1)} + \frac{\sin((m+1)\pi)}{(m+1)} \right] + \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\sin((m+1)\pi x)}{(m+1)} dx \\
&= -\frac{1}{2\pi} \times 0 + \frac{1}{2\pi} \times 0 = 0
\end{aligned}$$

$$bm = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \left(\frac{1}{2} \cos(m\pi x) + \cos(m+1)\pi x \right) \sin mx dx =$$

$$\begin{aligned}
&= \frac{1}{\pi} \int_{-\pi}^{\pi} \left(\frac{1}{2} \cos(m\pi x) \sin mx \right) dx + \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(m+1)\pi x \sin mx dx \\
&= \frac{1}{\pi} \int_{-\pi}^{\pi} \left[-\cos(m\pi x) \sin mx \right] dx + \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(m+1)\pi x \sin mx dx \\
&= \frac{1}{\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} \sin(m\pi x) \cos mx \right] dx + \frac{1}{\pi} \int_{-\pi}^{\pi} 2 \sin(m\pi x) \cos mx dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} \sin(m\pi x) \cos mx \right] dx + \frac{1}{2\pi} \int_{-\pi}^{\pi} 2 \sin(m\pi x) \cos mx dx \\
&= -\frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{1}{2} \sin((m+1)\pi x) \right] dx + \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin((m+1)\pi x) dx \\
&= -\frac{1}{2\pi} \left[\frac{1}{2} \sin((m+1)\pi x) \right]_{-\pi}^{\pi} + \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin((m+1)\pi x) dx \\
&= -\frac{1}{2\pi} \left[-\frac{\cos((m+1)\pi x)}{(m+1)} \right]_{-\pi}^{\pi} + \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{-\cos((m+1)\pi x)}{(m+1)} dx \\
&= \frac{1}{2\pi} \left[\frac{\cos((m+1)\pi x)}{(m+1)} \right]_{-\pi}^{\pi} + \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\cos((m+1)\pi x)}{(m+1)} dx
\end{aligned}$$

$$\begin{aligned}
 &= -\frac{1}{2\pi} \left[\int_{-\pi}^{\pi} \left(\frac{-\cos(n+1)x}{n+1} + \frac{-\cos(n-1)x}{n-1} \right) \left(\frac{-\cos((n+1)x)}{n+1} + \frac{-\cos((n-1)x)}{n-1} \right) dx \right] \\
 &\quad + \frac{1}{2\pi} \left[\left(\frac{-\cos(n+1)\pi}{n+1} + \frac{-\cos(n-1)\pi}{n-1} \right) - \left(\frac{-\cos(n+1)0}{n+1} + \right. \right. \\
 &\quad \left. \left. - \frac{-\cos(n-1)0}{n-1} \right) \right] \\
 &\quad \times \left[\frac{(n+1)(n-1)}{(1-\pi)(1+\pi)} + \right. \\
 &\quad \left. \left. \frac{(n+1)(n-1)}{(1-\pi)(1+\pi)} + \right. \right]
 \end{aligned}$$

$$\begin{aligned}
 &\approx -\frac{2}{2\pi} \left(-\frac{1}{n+1} + \frac{1}{n-1} \right) - \frac{2}{2\pi} \left(\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1} \right) \\
 &\approx \frac{1}{\pi} \left(\frac{1}{n+1} - \frac{(-1)^{n+1}}{n+1} \right) + \frac{1}{\pi} \left(\frac{1}{n-1} - \frac{(-1)^{n-1}}{n-1} \right) = \pi f(x)
 \end{aligned}$$

$$\begin{aligned}
 &\text{where } f(x) = \frac{1}{\pi} \left(\frac{1}{n+1} + \frac{(-1)^{n+1}}{n+1} \right) + \frac{1}{\pi} \left(\frac{1}{n-1} + \frac{(-1)^{n-1}}{n-1} \right) \\
 &\approx \frac{1}{\pi} \left(\frac{1}{n+1} + \frac{(-1)^{n+1}}{n+1} \right) + \frac{1}{\pi} \left(\frac{1}{n-1} + \frac{(-1)^{n-1}}{n-1} \right)
 \end{aligned}$$

Substituting the value

$$\begin{aligned}
 &\text{we have } f(x) = \frac{1}{\pi} \left(\frac{1}{n+1} + \frac{(-1)^{n+1}}{n+1} \right) + \frac{1}{\pi} \left(\frac{1}{n-1} + \frac{(-1)^{n-1}}{n-1} \right) \\
 &f(x) = 0 + 0 + \sum_{n=1}^{\infty} \left[\frac{1-(-1)^{n+1}}{n+1} + \frac{1-(-1)^{n-1}}{n-1} \right] \\
 &f(x) = \sum_{n=1}^{\infty} \left[\frac{1-(-1)^{n+1}}{n+1} \right] + \frac{1}{\pi} \left(\frac{1-(-1)^{n+1}}{n+1} \right)
 \end{aligned}$$

Since $\lim_{n \rightarrow \infty} (-1)^{n+1} = 0$ and $\lim_{n \rightarrow \infty} \frac{1}{n+1} = 0$,

$$\begin{aligned}
 &\cos(\theta) = \frac{\pi}{2} \left(\frac{1}{2} + \frac{1}{2} \right) + \frac{1}{\pi} \left(\frac{1}{2} \right) = \frac{1}{2} + \frac{1}{2\pi}
 \end{aligned}$$

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(2) Parseval's identity:
Statement: If the Fourier series for the function

for convergence uniformly on $[-\pi, \pi]$ then,

$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(x)^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2);$$

where, a_0, a_n and b_n called Fourier coefficients or constants.

Proof: The Fourier series for the function
in the interval $[-\pi, \pi]$ is given by,
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{nx}{\pi} + b_n \sin \frac{nx}{\pi}) \quad \text{(i)}$$

$$\text{where, } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \quad \text{and we have to find } a_n \text{ and } b_n.$$
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{nx}{\pi} dx + \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin \frac{nx}{\pi} dx$$
$$\text{and } b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin \frac{nx}{\pi} dx$$

Multiplying both sides of equation (i) and integrating between limits $-\pi$ to π we get,

$$\int_{-\pi}^{\pi} f(x)^2 dx = \int_{-\pi}^{\pi} \left(\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{nx}{\pi} + b_n \sin \frac{nx}{\pi}) \right)^2 dx$$

$$\int_0^{\pi} \sin^2 x dx = \frac{a^2}{2} \int_0^{\pi} \left(\sum_{n=1}^{\infty} (-1)^n \cos^{2n-1} x \right) dx$$

using successive differentiation of both sides

$$+ \sum_{n=1}^{\infty} \left(\text{Coef. of } \cos^{2n-1} x \right) \cos^{2n-1} x dx$$

$$= \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot \frac{d}{dx} \cos^{2n-1} x$$

$$= \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot (-1)^n \cdot 2n \cos^{2n-3} x$$

$$= \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot (-1)^n \cdot 2n \cdot \frac{d}{dx} \cos^{2n-3} x$$

differentiate again

$$= \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot (-1)^n \cdot 2n \cdot (-1)^n \cdot 2(2n-3) \cos^{2n-5}$$

$$\Rightarrow \int_0^{\pi} \sin^2 x dx = \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot (-1)^n \cdot 2n \cdot (-1)^n \cdot 2(2n-3)$$

$$\Rightarrow \int_0^{\pi} \sin^2 x dx = \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot (-1)^n \cdot 2n \cdot (-1)^n \cdot 2(2n-3) \cos^{2n-5}$$

$$\Rightarrow \int_0^{\pi} \sin^2 x dx = \frac{a^2}{2} + \sum_{n=1}^{\infty} \text{Coef. of } \cos^{2n-1} x \cdot (-1)^n \cdot 2n \cdot (-1)^n \cdot 2(2n-3) \cos^{2n-5}$$

differentiate again

2. (ii) Bessel's function: the equation
 $\frac{d^2y}{dx^2} + \frac{1}{x^2} y'' + \left(\frac{a^2}{x^2} - n^2\right)y = 0$ is called Bessel's differential equation of order n . It has two linearly independent solutions.

By solving this equation we get two linearly independent solutions which are called Bessel functions of order n .

$$J_m(r) = \sum_{n=0}^{\infty} \frac{(-1)^n (m)_n (\frac{r}{2})^{2n+m}}{n! (n+r)!}$$

called Bessel's function.

2. (b) we know, $J_0(r) = \sum_{n=0}^{\infty} (-1)^n (n)_n r^n$

$$J_m(r) = \sum_{n=0}^{\infty} \frac{(-1)^n (m)_n r^{n+m}}{n! (n+r)!}$$

$$J'_m(r) = \sum_{n=0}^{\infty} (-1)^n (n+1)_n (m)_n r^{n+m-1} \cdot \frac{1}{(n+r)!}$$

$$J'_0(r) = \sum_{n=0}^{\infty} (-1)^n (n+1)_n r^{n+m}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)_n (m)_n r^{n+m}}{n! (n+r)!} + m \sum_{n=0}^{\infty} \frac{(-1)^n (m)_n r^{n+m}}{n! (n+r)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (n+2)_n (m)_n r^{n+m}}{n! (n+r)!} + m \sum_{n=0}^{\infty} \frac{(-1)^n (m)_n r^{n+m}}{n! (n+r)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (n+2)_n (m)_n r^{n+m}}{(n+2)! (n+r)!} + m \sum_{n=0}^{\infty} \frac{(-1)^n (m)_n r^{n+m}}{n! (n+r)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} (n+1)_n (m+1)_n r^{n+m+1}}{(n+2)! (n+r)!} + m \sum_{n=0}^{\infty} \frac{(-1)^n (m)_n r^{n+m}}{n! (n+r)!}$$

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$$\begin{aligned} & \frac{x \sum_{n=0}^{\infty} n! x^n (1-x)^{n+1} + (1-x) \sum_{n=0}^{\infty} n! x^n (1-x)^{n+1}}{x \sum_{n=0}^{\infty} n! x^n (1-x)^{n+1}} \\ &= \frac{(1-x)! + (1-x)^2 + (1-x)^3 + \dots}{(1-x)! + (1-x)^2 + (1-x)^3 + \dots} \\ &= x \sum_{n=0}^{\infty} \frac{(1-x)^{n+1} + (1-x)^{n+2} + \dots}{(1-x)! + (1-x)^2 + (1-x)^3 + \dots} \\ &= x \sum_{n=0}^{\infty} \frac{(1-x)^{n+1} (1-x)^{n+2} (1-x)^{n+3} \dots}{(1-x)! + (1-x)^2 + (1-x)^3 + \dots} \\ &= x \sum_{n=0}^{\infty} x^{n+1} (1-x)^{n+2} (1-x)^{n+3} \dots \end{aligned}$$

2. (iv) Degenerate (polynomial) of some second order

$$\text{Legendre differential equation, } (1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2n(n+1)y = 0$$

Solutions of constant n are called associated Legendre functions. Its solution are curve whose constants in unknowns

Legendre's associated equation has solutions which are called associated Legendre functions. It reduces to Legendre's equation when $n=0$.

The Legendre differential equation is

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0 \quad (i)$$

And the soln of this eqn is called Legendre's polynomials.

4. Laplace transform of $\sin st$

$$L[\sin st] = \int_0^\infty e^{-st} \sin st dt = \frac{b}{s^2 + b^2}$$

$$\begin{aligned} & \text{using } \int_0^\infty e^{-st} (C e^{st} - C e^{-st}) dt \\ & \text{we get } \int_0^\infty e^{-st} \sin st dt = \left[e^{-st} \sin st \right]_0^\infty \\ & = \frac{1}{2i} \int_0^\infty e^{-st} (\sin st + i \cos st) dt \end{aligned}$$

$$\begin{aligned} & = \frac{1}{2i} \int_0^\infty e^{-(s-2i)t} dt + \frac{1}{2i} \int_0^\infty e^{-(s+2i)t} dt \\ & = \frac{1}{2i} \left[\frac{e^{-(s-2i)t}}{s-2i} + \frac{e^{-(s+2i)t}}{s+2i} \right]_0^\infty \end{aligned}$$

$$\begin{aligned} & = \frac{1}{2i} \left[\left(\frac{1}{s-2i} - \frac{1}{s+2i} \right) - \left(\frac{1}{s+2i} - \frac{1}{s+2i} \right) \right] \\ & = \frac{1}{2i} \left[\frac{1}{s-2i} - \frac{1}{s+2i} \right] \end{aligned}$$

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Ques (a)

$$\sqrt{\frac{\pi}{2}} = \sqrt{\pi}$$

2019

we know that,

$$B(mn) = \frac{\sqrt{m} \sqrt{n}}{\sqrt{mn}}$$

and

$$B(mn) = 2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$$

we can write the equation (i) and (ii) as

$$\sqrt{m} \sqrt{n} = \int_{\frac{\pi}{2}}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$$

$$\sqrt{m+n} = 2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$$

Let, $\sin^{2m-1}\theta = t$

putting this value in eqn (3) we get,

$$\frac{\sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}}}{\sqrt{\frac{1}{2} + \frac{1}{2}}} = 2 \int_0^6 \sin^{2 \cdot \frac{1}{2}-1} \theta \cos^{2 \cdot \frac{1}{2}-1} \theta d\theta$$

for m=1, n=1

$$\frac{(\sqrt{\frac{1}{2}})^2}{(\sqrt{\frac{1}{2}})^2} = \frac{(\sqrt{\frac{1}{2}})^2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^0 \theta \cos^0 \theta d\theta}{(\sqrt{\frac{1}{2}})^2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^1 \theta \cos^1 \theta d\theta}$$

$$\frac{1}{1} = \frac{2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^0 \theta \cos^0 \theta d\theta}{2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^1 \theta \cos^1 \theta d\theta}$$

$$1 = \frac{2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^0 \theta \cos^0 \theta d\theta}{2 \int_{\frac{\pi}{2}}^{\pi/2} \sin^1 \theta \cos^1 \theta d\theta}$$

$$1 = \frac{2 \int_{\frac{\pi}{2}}^{\pi/2} 1 \cdot 1 d\theta}{2 \int_{\frac{\pi}{2}}^{\pi/2} (-\cos \theta) \cdot (-\sin \theta) d\theta}$$

$$\therefore \sqrt{\pi_{12}} = \sqrt{\pi}$$

(ii) $\int_0^\alpha e^{-x^2} dx = \sqrt{\pi_{12}}$

Here, $x^2 = z$ $x = \sqrt{z}$
 $dx = \frac{1}{2\sqrt{z}} dz$
 $\therefore dx = \frac{1}{2\sqrt{z}} dz$

Now, $\int_0^\alpha e^{-x^2} dx = \int_0^{\sqrt{\alpha}} e^{-z} \frac{1}{2\sqrt{z}} dz$

$$= \int_0^{\sqrt{\alpha}} e^{-z} z^{-1/2} dz = \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{1/2 - 1} dz$$

$$= \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{-1/2} dz = \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{1/2 - 1} dz$$

$$= \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{-1/2} dz = \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{1/2 - 1} dz$$

$$= \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{-1/2} dz = \frac{1}{2} \int_0^{\sqrt{\alpha}} e^{-z} z^{1/2 - 1} dz$$

2. Take solution of Legendre diff eqn.

$$Y = C_0 \left[\frac{1 - m(m+1)}{z^m} x^{2m} + \frac{(m+1)(m+2)}{z^{m+1}} x^{2m+2} \right] + C_1 \left[x - e^{m+1} (m+2) x^{2m+1} + \frac{(m+1)(m+3)}{z^m} x^{2m+3} \right]$$

$$= C_0 [0] + C_1 \left[x - e^{m+1} (m+2) x^{2m+1} + \frac{(m+1)(m+3)}{z^m} x^{2m+3} \right]$$

$$= C_1 \left[x - e^{m+1} (m+2) x^{2m+1} + \frac{(m+1)(m+3)}{z^m} x^{2m+3} \right]$$

$$P_n(x) = \sum_{r=0}^{\infty} \frac{(x)^r (2n-2r)!}{(2n)! (n+r)!} \frac{(2n-2r)!}{(n-r)!}$$

when $n = N/2$ for n even

$$\text{and } N = \frac{n-1}{2} \text{ for } n \text{ odd}$$

$$P_m(x) = \frac{x^m!}{2^m m!} \left[x^{m-\frac{m(m-1)}{2}} x^{m-2} + \dots \right]$$

$$\text{For } P_3(x) \sum_{r=0}^1 \frac{(x)^r (2x^3 - 2x^0)!!}{(3!)x^0 (3+0)!!} \frac{x^8}{(8-2x^0)!!} - 2x^0$$

$$= \sum_{r=0}^1 \frac{1}{r!} \frac{x^{(m+6)!}}{8^0 x^{3!} (6+3)!!} \frac{0 = 16x^3 + 4x^6 + 16x^9}{(8+6)!!} - 2x^0$$

$$4. (a) f(z) = x^2 + xy + iy^2 + i(cx^2 + dy^2 + y^2)$$

where $c, d = \frac{1}{2} + \frac{1}{2}i$ and $r = (n^2 + dy^2 + y^2)$

$$\frac{\partial u}{\partial z} = \frac{\partial x + iy}{\partial z} = \frac{1}{2} \left(\frac{\partial x}{\partial z} + i \frac{\partial y}{\partial z} \right) = \frac{1}{2} \left(\frac{\partial x}{\partial z} + \frac{\partial y}{\partial z} \right) = \frac{1}{2} \left(\frac{\partial x}{\partial z} + \frac{\partial y}{\partial z} \right) = \frac{1}{2} \left(\frac{\partial x}{\partial z} + \frac{\partial y}{\partial z} \right)$$

2018

Top row (ii) goes up \sqrt{n} times
 1. (a) $\sqrt{n+1} = n\sqrt{n}$

we get, $\sqrt{n+1} = n\sqrt{n}$ $0 < \frac{v_n}{n\sqrt{n}} + \frac{v_n}{n\sqrt{n}}$

when, $n \neq 0$ we get, $v_n > 0$ v_n is positive

Now $\sqrt{2} \geq \sqrt{1} \geq 1$!

when $n=2$ we get,
 $\sqrt{3} \geq \sqrt{2}$, $\sqrt{2} \geq 1 \geq \frac{1}{2}$ $v_2 > v_1 > v_0$

when $n=3$ we get
 $\sqrt{4} \geq \sqrt{3} \geq \sqrt{2} \geq 1 \geq \frac{1}{2}$

$\sqrt{4} = 2\sqrt{3} = 3, 2, 1 < 3, 2$

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Therefore

$$\sqrt{n+1} \geq (n+1)! \geq n!$$

Hence

$$\sqrt{n+1} \geq n! = \ln z \text{ must proved}$$

$$P_0(x) = C_0 P_0(x) = 1$$

Similarly,

$$P_2(x) = \frac{1}{2}(5x^2 - 3x)$$

$$P_4(x) = C_4 x$$

$$P_6(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$so, P_8(x) = x$$

$$P_6(x) = \frac{1}{10}(123x^6 - 315x^4 + 105x^2)$$

We know from Legendre
polynomials,
 $\rightarrow 5$

$$x^4 = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$x^4 = \frac{8}{35} P_4(1) + \frac{1}{7} x^2 - \frac{3}{35}$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$x^3 = \frac{2}{5} P_3(x) \neq \frac{3}{5} x$$