Chapter Twenty One

HOLOGRAPHY*

The electron microscope was to produce the interference figure between the object beam and the coherent background, that is to say the non-diffracted part of the illuminating beam. This interference pattern I called a ***hologram,*** *from the Greek word* ***holos*** *– the whole, because it contained the whole information. The hologram was then reconstructed with light, in an optical system which corrected the aberrations of the electron optics.*

—Dennis Gabor in his Nobel lecture December 11, 1971**

Important Milestones

1948	*Dennis Gabor discovered the principle of holography.*
1960	*The first successful operation of a laser device by Theodore Maiman.*
1962	*Off-axis technique of holography by Leith and Upatnieks.*
1962	*Denisyuk suggested the idea of three-dimensional holograms based on thick photoemulsion layers. His holograms can be reconstructed in ordinary sun light. These holograms are called Lippmann - Bragg holograms.*
1964	*Leith and Upatnieks pointed out that a multi-color image can be produced by a hologram recorded with three suitably chosen wavelengths.*
1969	*Benton invented 'Rainbow Holography' for display of holograms in white light. This was a vital step to make holography suitable for display applications.*

21.1 INTRODUCTION

A photograph represents a two-dimensional recording of a three-dimensional scene. What is recorded is the intensity distribution that prevailed at the plane of the photograph when it was exposed. The light sensitive medium is sensitive only to the intensity variations and hence while recording a photograph, the phase distribution which prevailed at the plane of the photograph is lost. Since only the intensity pattern has been recorded, the three-dimensional character (e.g., parallax) of the object scene is lost. Thus, one cannot change the perspective of the image in the photograph by viewing it from a different angle or one cannot refocus any unfocussed part of the image in the photograph. Holography is a method invented by Dennis Gabor in 1947, in which one not only records the amplitude but also the phase of the light wave; this is done by using interferometric techniques. Because of this, the image produced by the technique of holography has a true three-dimensional form. Thus, as with the object, one can change one's position and view a different perspective of the image or one can focus at different distances. The capability to produce images as true as the object itself is what is responsible for the wide popularity gained by holography.

The basic technique in holography is the following: In the recording of the hologram, one superimposes on the object wave another wave called the reference wave and the photo-

* A portion of this chapter is based on the unpublished lecture notes of Professor K. Thyagarajan.

** Dennis Gabor received the 1971 Nobel Prize in Physics for discovering the *principles of holography*; the original paper of Gabor appeared in 1948 [see Ref. 1]. Gabor's Nobel lecture entitled *Holography, 1948 – 1971* is non-mathematical and full of beautiful illustrations; it is reprinted in Ref. 2.

graphic plate is made to record the resulting interference pattern (see Fig. 21.1). The reference wave is usually a plane wave. This recorded interference pattern forms the hologram and (as will be shown) contains information not only about the amplitude but also about the phase of the object wave. Unlike a photograph, a hologram has little resemblance with the object; in fact, information about the object is coded into the hologram. To view the image, we again illuminate the hologram with another wave, called the **reconstruction wave** (which in most cases is identical to the reference wave used during the formation of the hologram); this process is termed as **reconstruction** (see Fig. 21.2). The reconstruction process leads, in general, to a virtual and a real image of the object scene. The virtual image has all the characteristics of the object like parallax, etc. Thus, one can move the position of the eye and look behind the objects or one can focus at different distances. The real image can be photographed without the aid of lenses just by placing a light sensitive medium at the position where the real image is formed. Figures 21.3(a), (b) and (c) represent the object, its hologram and the reconstructed image, respectively.

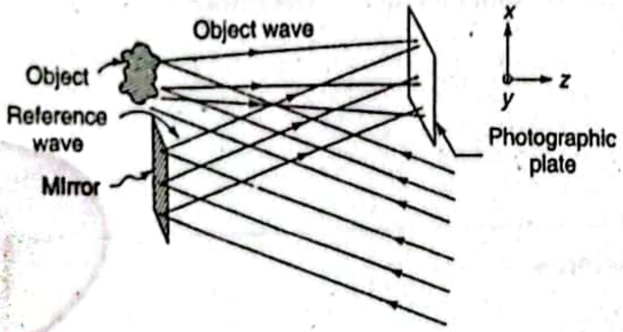

Fig. 21.1 Recording of a hologram.

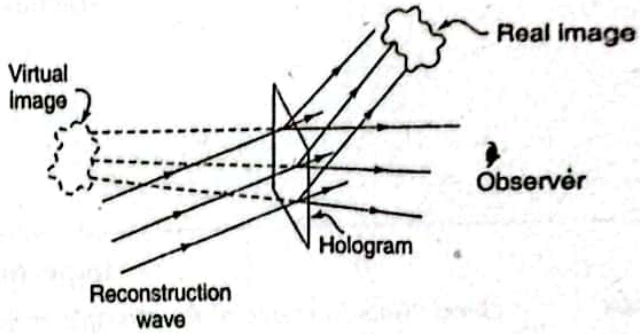

Fig. 21.2 Reconstruction process.

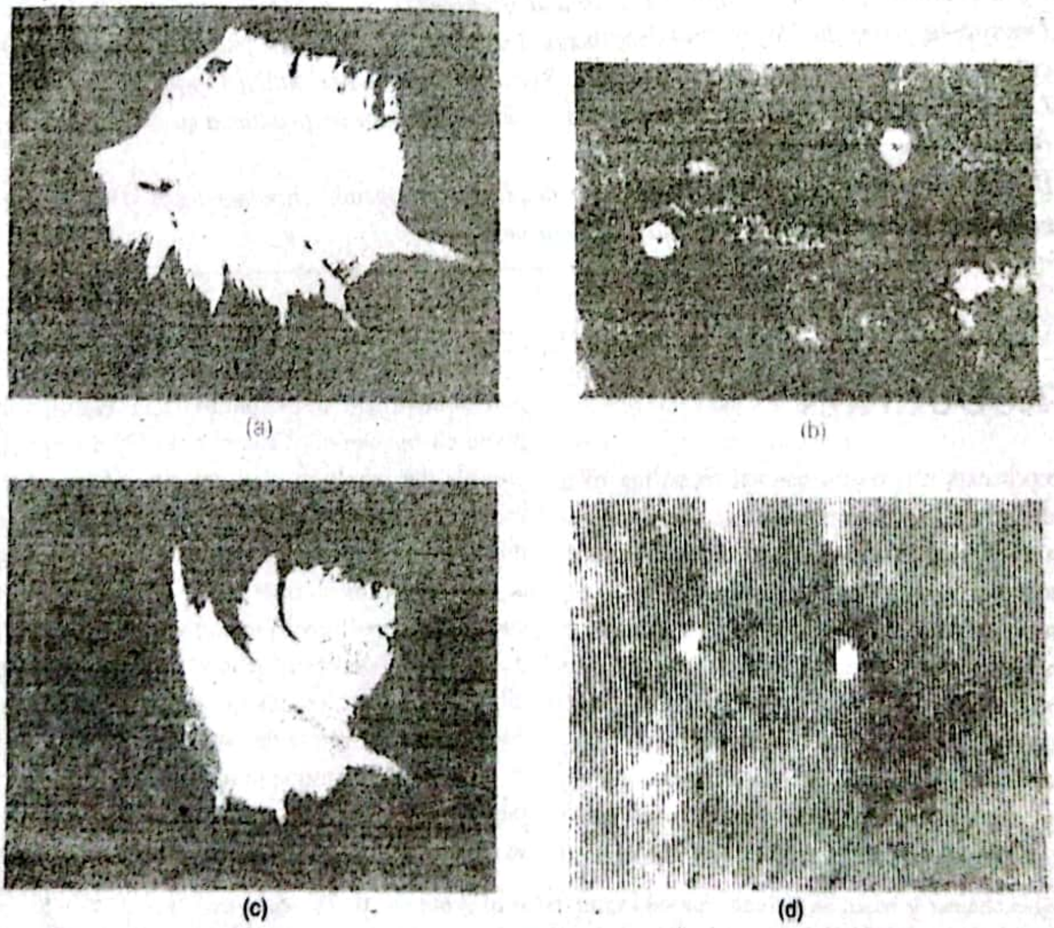

Fig. 21.3 (a) An ordinary photograph of an object. (b). The hologram of the object produced by a method similar to the one as shown in Fig. 21.1, (c) The reconstructed image as seen by an observer, (d) A magnified view of a small portion of the hologram shown in (b). [*Photographs courtesy: Professor R. S. Sirohi*].

21.2 THEORY

If the object is a point scatterer, then the object wave would just be $\frac{A}{r}\cos(kr - \omega t + \phi)$ where r represents the distance of the point of observation from the point scatterer and A represents a constant; $k = 2\pi/\lambda$. Any general object can be thought of as being made up of a large number of points and the composite wave reflected by the object would be vectorial sum of these. The fundamental problem in holography is the recording of this object wave, in particular, the phase distribution associated with it.

Let us consider the recording process. Let

$$O(x, y) = a(x, y)\cos[\phi(x, y) - \omega t] \tag{21.1}$$

represents the object wave (which, as mentioned earlier, is due to the superposition of waves from point scatterers on the object) in the plane of the photographic plate which is assumed to be $z = 0$ (see Fig. 21.1). We consider a plane reference wave and assume, for simplicity, that it is propagating in the x–z-plane inclined at an angle θ with the z-direction (see Fig. 21.1). Thus, the field associated with this plane wave would be given by

$$\begin{aligned} r(x, y, z) &= A\cos[\mathbf{k}\cdot\mathbf{r} - \omega t] \\ &= A\cos(kx\sin\theta + kz\cos\theta - \omega t) \end{aligned} \tag{21.2}$$

If $r(x, y)$ represents the field at the plane $z = 0$ due to this reference wave, then one can see that

$$\begin{aligned} r(x, y) &= A\cos[kx\sin\theta - \omega t] \\ &= A\cos[2\pi\alpha x - \omega t] \end{aligned} \tag{21.3}$$

where $\alpha = \sin\theta/\lambda$ is the spatial frequency (see Sec. 19.9). The above equation represents the field due to a plane wave inclined at an angle θ with the z-axis and as can be seen the phase varies linearly with x. Notice that there is no y-dependence because the plane wave has been assumed to have its propagation vector in the x–z-plane. Thus, the total field at the photographic plate (which is coincident with the plane $z = 0$) would be given by

$$u(x, y, t) = a(x, y)\cos[\phi(x, y) - \omega t] + A\cos[2\pi\alpha x - \omega t] \tag{21.4}$$

The photographic plate responds only to the intensity which would be proportional to the time average of $[u(x, y, t)]^2$. Thus, the intensity pattern recorded by the photographic plate would be

$$\begin{aligned} I(x, y) &= \langle u^2(x, y, t)\rangle \\ &= \langle [a(x, y)\cos\{\phi(x, y) - \omega t\} \\ &\quad + A\cos(2\pi\alpha x - \omega t)]^2\rangle \end{aligned} \tag{21.5}$$

where the angular brackets denote time averaging (see Sec. 17.3). Thus,

$$\begin{aligned} I(x, y) &= a^2(x, y)\langle\cos^2\{\phi(x, y) - \omega t\}\rangle \\ &\quad + A^2\langle\cos^2(2\pi\alpha x - \omega t)\rangle \\ &\quad + 2a(x, y)A\langle\cos\{\phi(x, y) - \omega t\}\cos(2\pi\alpha x - \omega t)\rangle \end{aligned} \tag{21.6}$$

Since

$$\langle\cos^2[\phi(x, y) - \omega t]\rangle = \frac{1}{2} = \langle\cos^2(2\pi\alpha x - \omega t)\rangle \tag{21.7}$$

and

$$\begin{aligned} &\langle\cos[\phi(x, y)) - \omega t]\cos(2\pi\alpha x - \omega t)\rangle \\ &= \frac{1}{2}\langle\cos[\phi(x, y) + 2\pi\alpha x - 2\omega t]\rangle + \\ &\qquad \frac{1}{2}\langle\cos[\phi(x, y) - 2\pi\alpha x]\rangle \\ &= \frac{1}{2}\cos[\phi(x, y) - 2\pi\alpha x] \end{aligned} \tag{21.8}$$

Eq. (21.6) becomes

$$\begin{aligned} I(x, y) &= \frac{1}{2}a^2(x, y) + \frac{1}{2}A^2 \\ &\quad + Aa(x, y)\cos[\phi(x, y) - 2\pi\alpha x] \end{aligned} \tag{21.9}$$

From the above relation, it is obvious that the phase information of the object wave, which is contained in $\phi(x, y)$, is recorded in the intensity pattern.

When the photographic plate (which has recorded the above intensity pattern) is developed, one obtains a hologram [see Figs. 21.3(b) and (d)]. The transmittance of the hologram, i.e., the ratio of the transmitted field to the incident field, depends on $I(x, y)$. By a suitable developing process one can obtain a condition under which the amplitude transmittance would be linearly related to $I(x, y)$. Thus, in such a case if $R(x, y)$ represents the field of the reconstruction wave at the hologram plane, then the transmitted field would be given by

$$\begin{aligned} v(x, y) &= K\,R(x, y)\,I(x, y) \\ &= K\left[\frac{1}{2}a^2(x, y) + \frac{1}{2}A^2\right]R(x, y) \\ &\quad + K\,A\,a(x, y)\,R(x, y)\cos[\phi(x, y) - 2\pi\alpha x] \end{aligned} \tag{21.10}$$

where K is a constant. We consider the case when the reconstruction wave is identical to the reference wave $r(x, y)$ (see Fig. 21.2). In such a case we would obtain (omitting the constant K)

$$v(x, y) = \left[\frac{1}{2}a^2(x, y) + \frac{1}{2}A^2\right]A\cos(2\pi\alpha x - \omega t)$$

$$+ A^2 a(x, y)\cos[2\pi\alpha x - \omega t]\cos[\phi(x, y) - 2\pi\alpha x]$$

$$= \left[\frac{1}{2}a^2(x, y) + \frac{1}{2}A^2\right] A\cos(2\pi\alpha x - \omega t)$$

$$+ \frac{1}{2}A^2 a(x, y)\cos[\phi(x, y) - \omega t]$$

$$+ \frac{1}{2}A^2 a(x, y)\cos[4\pi\alpha x - \phi(x, y) - \omega t] \quad \textbf{(21.11)}$$

Equation (21.11) gives the transmitted field in the plane $z = 0$. We consider each of the three terms separately. The first term is nothing but the reconstruction wave itself whose amplitude is modulated due to the presence of the term $a^2(x, y)$. This part of the total field is traveling in the direction of the reconstructed wave. The second term is identical (within a constant term) to the RHS of Eq. (21.1) and hence represents the original object wave; this gives rise to a virtual image. Thus, the effect of viewing this wave is the same as viewing the object itself. The reconstructed object wave is traveling in the same direction as the original object wave.

To study the last term we first observe that in addition to the term $4\pi\alpha x$, the phase term $\phi(x, y)$ carries a negative sign. The negative sign represents the fact that the wave has a curvature opposite to that of the object wave. Thus, if the object wave is a diverging spherical wave then the last term represents a converging spherical wave. Thus, in contrast to the second term, this wave forms a real image of the object which can be photographed by simply placing a film (see Fig. 21.2).

To determine the effect of the term $4\pi\alpha x$, we consider the case when the object wave is also a plane wave traveling along the z-axis. For such a wave $\phi(x, y) = 0$ and the last term would represent a plane wave propagating along a direction $\theta' = \sin^{-1}(2\sin\theta)$. Thus the effect of the term $4\pi\alpha x$ is to rotate the direction of the wave. Hence the last term on the RHS of Eq. (21.11) represents the conjugate of the object wave propagating along a direction different from that of the reconstruction wave and the object wave, which forms a real image of the object. Since the waves represented by the three terms are propagating along different directions they separate after traversing a distance and enable the observer to view the virtual image without any disturbance.

A very interesting property possessed by holograms is that even if the hologram is broken up into different fragments, each separate fragment is capable of producing a complete virtual image of the object.* This property can be understood from the fact that for a diffusely reflecting object, each point of the object illuminates the complete hologram and consequently each point in the hologram receives waves from the complete object. But the resolution in the image decreases as the size of the fragment decreases. For non-diffusely reflecting objects or for transparencies, one makes use of an additional diffusing screen through which the object is illuminated.

Example 21.1 As an explicit example of the formation and reconstruction of a hologram, we consider the simple case when both the object wave and the reference wave are plane waves [see Fig. 21.4(a)]—a plane object wave corresponds to a single object point lying far away from the hologram. (a) Show that for such a case, the hologram consists of a series of Young's interference fringes having an intensity distribution of the $\cos^2$ type. (see also Fig. 14.11) (b) If we reconstruct the hologram with another plane wave [see Fig. 21.4(b)], then show that the transmitted light consists of a zero-order plane wave and two first-order plane waves; the two first-order waves correspond to the primary and conjugate waves.

Solution: (a) Consider a plane wave with its propagation vector lying in the x-z-plane and making an angle θ_1 with the z-axis. For such a wave, the field is of the form

$$A_1 \cos\,[kx\sin\theta_1 + kz\cos\theta_1 - \omega t]$$

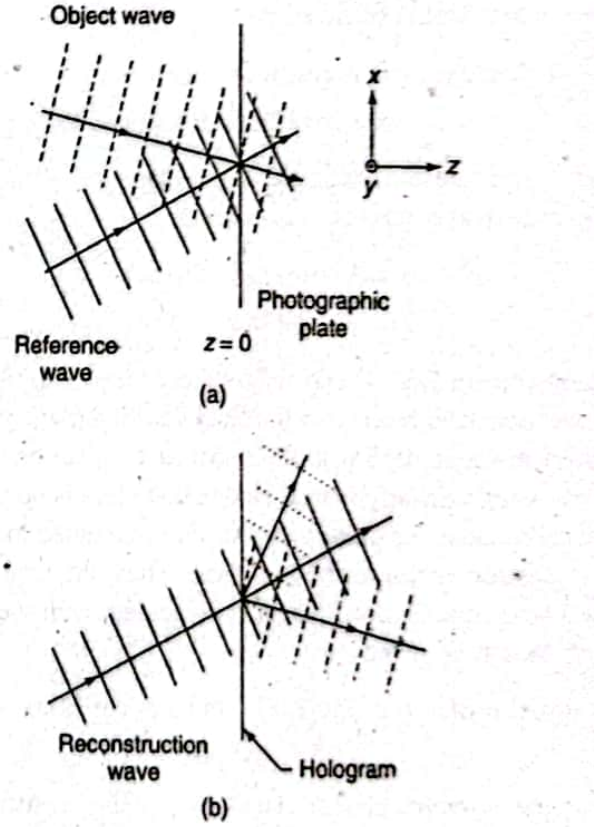

Fig. 21.4 (a) Formation of a hologram, when both the object wave and the reference wave are plane waves. (b) Reconstruction of the hologram with another plane wave.

* This property of a hologram exists only when the object is a diffuse scatterer such that the wave from each scattering point of the object reaches all parts of the hologram plate. There are cases where this does not hold good; for example, when a hologram of a transparency is to be recorded.

If the photographic film is assumed to coincide with the plane $z = 0$, then the field distribution on this plane would be given by

$$A_1 \cos [kx \sin \theta_1 - \omega t]$$

Similarly, the field (on the plane of the film) due to a plane wave making an angle θ_2 with the z-axis, will be given by

$$A_2 \cos [kx \sin \theta_2 - \omega t]$$

The resultant intensity distribution would be proportional to

$$\langle [A_1 \cos \{k x \sin \theta_1 - \omega t\} + A_2 \cos \{k x \sin \theta_2 - \omega t\}]^2 \rangle$$

$$= \frac{1}{2} A_1^2 + \frac{1}{2} A_2^2 + A_1 A_2 \cos [k x (\sin \theta_1 - \sin \theta_2)]$$

$$= \frac{1}{2} (A_1 - A_2)^2 + 2 A_1 A_2 \cos^2 \left[\frac{kx}{2} (\sin \theta_1 - \sin \theta_2)\right]$$

For $A_1 = A_2$, the above expression simplifies to

$$2A^2 \cos^2 \left[\frac{kx}{2} (\sin \theta_1 - \sin \theta_2)\right]$$

showing that the intensity remains constant along lines parallel to the y-axis with fringe spacing depending on the values of θ_1 and θ_2. Further, the intensity distribution is of the $\cos^2$ type (cf. Fig. 14.11).

(b) Before we calculate the transmitted field of the hologram, we first consider a narrow slit of width b being illuminated by a plane wave (see Fig. 21.5). Consider an element ds at a distance s from the center of the slit. Then the amplitude at a far away point P due to this element would be proportional to $\sin [k (r - s \sin \theta) - \omega t]\, ds$; here $k = 2\pi/\lambda$ and θ is defined in Fig. 21.5. Thus the total field in the direction θ would be given by

$$E = A \int_{-b/2}^{+b/2} \sin [k(r - s \sin \theta) - \omega t]\, ds \qquad (21.12)$$

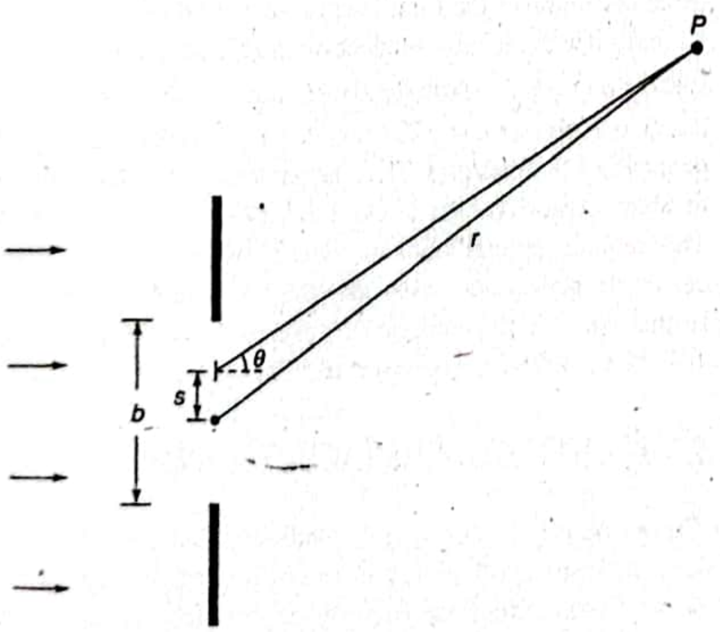

Fig. 21.5 A plane wave incident on a narrow slit of width b.

where A is a constant. The above integral can also be written as

$$E = A \int_{-b/2}^{+b/2} [\sin (kr - \omega t) \cos (ks \sin \theta) - \cos (kr - \omega t) \sin (ks \sin \theta)]\, ds$$

$$= 2 A \sin (kr - \omega t) \frac{\sin \left(\frac{kb}{2} \sin \theta\right)}{k \sin \theta}$$

where the second integral is zero because the integrand is an odd function of s. Thus,

$$E = Ab \sin (kr - \omega t) \frac{\sin \beta}{\beta} \qquad (21.13)$$

where

$$\beta = \frac{1}{2} kb \sin \theta = \frac{\pi b \sin \theta}{\lambda}$$

which is of the same form as obtained in Sec. 18.2. In the present case, the hologram has a $\cos^2 \alpha s$ type of variation in transmittance and hence the transmitted field will be of the form

$$E = A \int_{-b/2}^{+b/2} \cos^2 \alpha s \sin [kr - ks (\sin \theta - \sin \theta_i) - \omega t]\, ds \qquad (21.14)$$

where θ_i represents the angle of incidence of the illuminating plane wave. Thus,

$$E = \frac{1}{2} \int_{-b/2}^{+b/2} (1 + \cos 2 \alpha s) \times [\sin (k r - \omega t) \cos \{k s (\sin \theta - \sin \theta_i)\} - \cos (k r - \omega t) \sin \{k s (\sin \theta - \sin \theta_i)\}]\, ds$$

$$= \frac{1}{2} A \sin (kr - \omega t) \left[\int_{-b/2}^{+b/2} \cos \{ks(\sin \theta - \sin \theta_i)\}\, ds + \frac{1}{2} \int_{-b/2}^{+b/2} \cos \{ks (\sin \theta - \sin \theta_i + 2\alpha)\}\, ds + \frac{1}{2} \int_{-b/2}^{+b/2} \cos \{ks(\sin \theta - \sin \theta_i - 2\alpha)\}\, ds\right] \qquad (21.15)$$

The above integrations can easily be carried out. Thus, for example,

$$\int_{-b/2}^{+b/2} \cos \{ks (\sin \theta - \sin \theta_i + 2 \alpha)\}\, ds$$

$$= \frac{\sin \left[b \frac{k}{2} (\sin \theta - \sin \theta_i + 2\alpha)\right]}{\frac{k}{2} (\sin \theta - \sin \theta_i + 2\alpha)} \qquad (21.16)$$

which becomes more and more sharply peaked around $\sin\theta = \sin\theta_i - 2\alpha$ as $b \to \infty$, i.e., as the size of the hologram becomes larger. Thus, the three integrals in Eq. (21.15) in the limit of a large value of b give rise to three plane waves propagating along $\sin\theta = \sin\theta_i$, $\sin\theta = \sin\theta_i - 2\alpha$ and $\sin\theta = \sin\theta_i + 2\alpha$, which represent the zero-order and two first order waves.

Example 21.2 Consider the formation of a hologram with a point object and a plane reference wave (see Fig. 14.13(a)). Choose the z-axis to be along the normal from the point source to the plane of the photograph, assumed to be coincident with the plane $z = 0$. For simplicity, assume the reference wave to fall normally on the photographic plate. Obtain the interference pattern recorded by the hologram.

Solution: Let the point source be situated at a distance d from the photographic plate. The field at any point $P(x, y, 0)$ on the photographic plate, due to waves emanating from the point object would be given by

$$O(x, y, z = 0, t) = \frac{A}{r}\cos(kr - \omega t) \tag{21.17}$$

where $r = (x^2 + y^2 + d^2)^{1/2}$ and A represents a constant. A plane wave traveling along a direction parallel to the z-axis would be given by

$$R(x, y, z, t) = B\cos(kz - \omega t) \tag{21.18}$$

Hence, the field due to the reference wave at the plane of the photographic plate ($z = 0$) would be

$$R(x, y, z = 0, t) = B\cos\omega t \tag{21.19}$$

Thus, the total field at the plane of the photographic plate would be

$$T(x, y, t) = O(x, y, z = 0, t) + R(x, y, z = 0, t)$$

$$= \frac{A}{r}\cos(kr - \omega t) + B\cos\omega t \tag{21.20}$$

The recorded intensity pattern would be

$$I(x, y) = \langle |T(x, y, t)|^2 \rangle$$

$$= \left\langle \left| \frac{A}{r}\cos(kr - \omega t) + B\cos\omega t \right|^2 \right\rangle \tag{21.21}$$

where, as before, angular brackets denote time averaging. Carrying out the above time averaging, we get

$$I(x, y) = \frac{A^2}{2r^2} + \frac{B^2}{2} + \frac{AB}{r}\cos kr \tag{21.22}$$

If we assume that $d \gg x, y$ (which is valid in most practical cases), we can write

$$r = (x^2 + y^2 + d^2)^{1/2} \approx d + \frac{x^2 + y^2}{2d} \tag{21.23}$$

Thus,

$$I(x, y) = \frac{A^2}{2d^2} + \frac{B^2}{2} + \frac{AB}{r}\cos\left[kd + \frac{k}{2d}(x^2 + y^2)\right] \tag{21.24}$$

The resultant fringe pattern is circular and centered at the origin (see Example 14.7). The hologram thus formed is essentially a zone plate with the transmittance varying sinusoidally in contrast to the Fresnel zone plate [see Fig. 14.13(b) and Sec. 20.3].

21.3 REQUIREMENTS

Since holography is essentially an interference phenomenon, certain coherence requirements have to be met with. In Chapter 17, we had introduced the notion of coherence length. Thus, if stable interference fringes are to be formed (so that they are recordable), the maximum path difference between the object wave and the reference wave should not exceed the coherence length. Further, the spatial coherence is important so that the waves scattered from different regions of the object could interfere with the reference beam.

During reconstruction, the reconstructed image depends both on the wavelength and the position of the reconstructing source. Hence if the resolution in the reconstructed image has to be good, the source must not be broad and must be emitting a narrow band of wavelengths. It may be worthwhile mentioning here that the reconstruction process has associated with it aberrations similar to that in the images formed by lenses. If the reconstruction source is of the same wavelength and is situated at the same relative position with respect to the hologram as the reference source, then the reconstructed image does not suffer from any aberrations.

Another critical requirement in making holograms is stability of the recording arrangement. Thus, the film, the object and any mirrors used in producing the reference beam must be motionless with respect to one another during exposure. One more requirement which is not so obvious (but is a necessity) is the resolution of the film. Two plane waves making angles $+\theta$ and $-\theta$ with the axis, produce an interference pattern with spacing $d = \frac{\lambda}{2\sin\theta}$. Assuming $\theta = 15°$ and $\lambda = 6328$ Å (He–Ne laser), one obtains $d = 1.222 \times 10^{-3}$ mm; thus the spatial frequency is 818 limes/mm. Thus the photographic plate should be able to record fringes as close as 0.1222×10^{-4} mm apart. This requires special kinds of material which tend to be exceedingly slow, thus taking the stability requirements even further. Some of the holographic materials are 649F Kodak or 10E 75 or 8E 75 Agfa–Gaevert films and plates.

21.4 SOME APPLICATIONS

The principle of holography finds applications in many diverse fields.* The ability to record information about the depth finds application in studying transient microscopic

* See, e.g., Refs. 3–12.

events. Thus, if one has to study some transient phenomenon which occurs in a certain volume, then using ordinary microscopic techniques it becomes difficult to first locate the position and make observation. If a hologram is recorded of the scene, then the event gets frozen into the hologram and hence one can focus through the depth of the reconstructed image and study the phenomenon at leisure.

One of the most promising applications of holography lies in the field of interferometry. The ability of the holographic process to release the object wave when reconstructed with a reconstruction wave allows us to perform interference between different waves which exist at different times. Thus, in the technique called **double exposure holographic interferometry**, the photographic plate is first partially exposed to the object wave and the reference wave. Then, the object is stressed and the photographic plate is again exposed along with the same reference wave. The photographic plate after development forms the hologram. When this hologram is illuminated with a reconstruction wave, then two object waves emerge from the hologram; one of them corresponds to the unstressed object and the other to the stressed object. Since the object waves themselves have been reconstructed, they interfere and produce interference fringes. These interference fringes are characteristic of the strain suffered by the body. A quantitative study of the fringe pattern produced in the body gives the distribution of strain in the object.

To understand the formation of the fringe pattern, we assume that the deformation of the object has been such as to alter only the phase distribution. Thus, if

$$O(x, y, t) = A(x, y) \cos [\phi(x, y) - \omega t] \qquad (21.25)$$

represents the object wave (in the hologram plane) when the object is unstressed [see Fig. 21.6(a)] and if $O'(x, y, t)$ represents the object wave when the object is stressed [see Fig. 21.6(b)] then we may write

$$O'(x, y, t) = A(x, y) \cos [\phi'(x, y) - \omega t] \qquad (21.26)$$

where the phase distribution has been assumed to change from $\phi(x, y)$ to $\phi'(x, y)$. On reconstruction, each of the above two object waves emerge from the hologram and what would be observed will be the intensity pattern due to interference of the two waves which would be given by*

$$\begin{aligned} I(x, y) &= \langle [A(x, y) \cos\{\phi(x, y) - \omega t\} \\ &\quad + A(x, y) \cos\{\phi'(x, y) - \omega t\}]^2 \rangle \\ &= A^2(x, y) + A^2(x, y) \cos [\phi'(x, y) - \phi(x, y)] \end{aligned} \qquad (21.27)$$

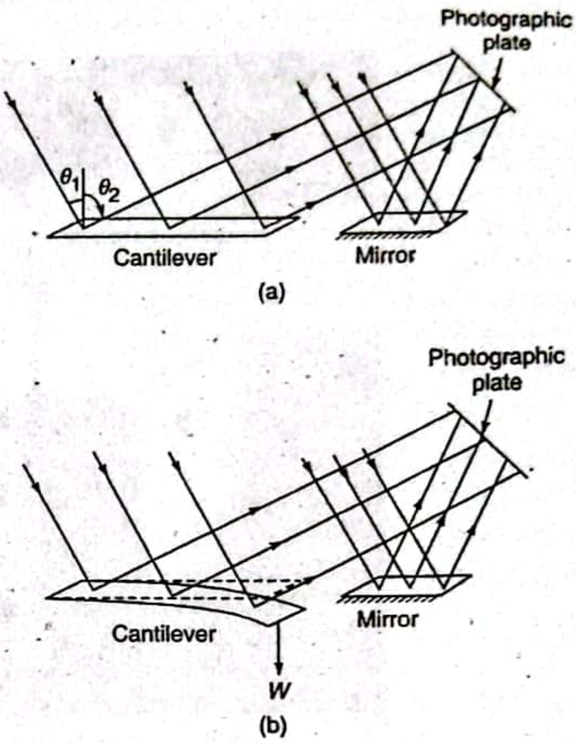

Fig. 21.6 (a) Recording of the unstressed object wave. (b) Recording of the stressed object wave on the same emulsion to produce the doubly exposed hologram.

Thus, whenever

$$\phi'(x, y) - \phi(x, y) = 2\, m\pi, \; m = 0, 1, 2, \ldots \qquad (21.28)$$

the two waves would interfere constructively and whenever,

$$\phi'(x, y) - \phi(x, y) = (2m + 1)\, \frac{\pi}{2}; \; m = 0, 1, 2, \ldots \qquad (21.29)$$

the two waves interfere destructively. Thus, depending on $[\phi'(x, y) - \phi(x, y)]$, one obtains, on reconstruction, the object superimposed with bright and dark fringes (see Fig. 21.7).

We will consider here a simple application of the above technique in the determination of the Young's modulus of a material. If we have a bar fixed at one end and loaded at the other and if it results in a displacement δ of the end of the bar, then one can show that**

$$\delta = \frac{W L^3}{3 Y I} \qquad (21.30)$$

where W is the load, L is the length of the bar, I is the moment of inertia of cross section which for a rectangular bar of

* The reconstruction process produces other wave components also but as was observed earlier, these components travel along different directions. Here we are concerned only with the object waves.

** See, e.g., Ref. 13, p. 75.

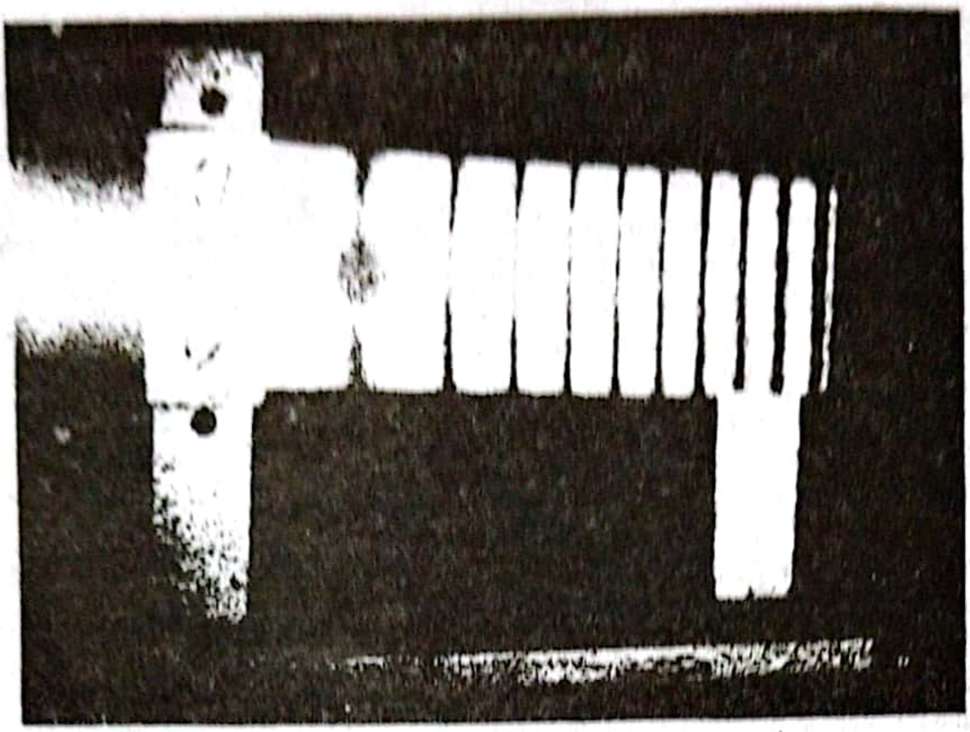

Fig. 21.7 Interference fringes produced in the measurement of Young's modulus using double exposure interferometry. [*Photograph courtesy: Professor R. S. Sirohi*].

width a and thickness b, is given by $I = ab^3/12$; Y represents the Young's modulus of the material of the rod. Thus if we could determine δ for a given load, then Y can be determined from Eq. (21.30).

We will first determine an expression for $(\phi' - \phi)$. In Fig. 21.6, we have shown the undisplaced and displaced positions of the cantilever illuminated by a laser light along a direction making an angle θ_1 with the z-axis. We observe the cantilever along a direction making an angle θ_2 with the z-axis. The phase change when the cantilever undergoes a displacement δ as shown in Fig. 21.6(b) would be

$$\phi' - \phi = \frac{2\pi}{\lambda}(\delta\cos\theta_1 + \delta\cos\theta_2)$$

$$= \frac{2\pi}{\lambda}\delta(\cos\theta_1 + \cos\theta_2) \qquad (21.31)$$

If there are N fringes over the length L of the cantilever, then since a phase difference of 2π corresponds to one fringe [see Eq. (21.28)] we can write

$$\frac{2\pi}{\lambda}\delta(\cos\theta_1 + \cos\theta_2) = N\,.\,2\pi$$

or

$$\delta = \frac{N\lambda}{(\cos\theta_1 + \cos\theta_2)}$$

Thus by measuring N, θ_1 and θ_2 and knowing λ, δ can be determined. Figure 21.7 shows the reconstruction of a double exposed hologram of an aluminum strip of width 4 cm, thickness 0.2 cm and of length 12 cm. From the number of fringes formed, one can calculate the Young's modulus (see Problem 21.3).

Summary

- The basic technique in holography is the following : In the recording of the hologram, one superimposes on the object wave another wave called the reference wave and the photographic plate is made to record the resulting interference pattern. The reference wave is usually a plane wave. This recorded interference pattern forms the hologram and contains information not only about the amplitude but also about the phase of the object wave. To view the image, we again illuminate the hologram with another wave, called the reconstruction wave. The reconstruction process leads, in general, to a virtual and a real image of the object scene. The virtual image has all the characteristics of the object like parallax, etc.
- If the object wave and the reference wave are plane waves, the hologram consists of a series of Young's interference fringes.
- For a point object and a plane reference wave, the hologram is very similar to a zone plate with the transmittance varying sinusoidally in contrast to the Fresnel zone plate.

Problems

21.1 Consider the reconstruction of the hologram as formed in the configuration of Example 21.2, by a plane wave traveling along a direction parallel to the z-axis. Show the formation of a virtual and a real image.

21.2 In continuation of Example 21.2, calculate the interference pattern when the incident plane wave makes an angle θ with the z-axis [see Fig. 14. 13]. Assume $B \approx A/d$.

$$\left[\textbf{Ans. } 4B^2 \cos^2 \left\{kd - kx \sin\theta + \frac{k}{2d}(x^2 + y^2)\right\}\right]$$

21.3 Figure 21.7 corresponds to the reconstruction of a doubly exposed hologram, the objects corresponding to the unstrained and strained positions of an aluminum bar of width 4 cm, thickness 0.2 cm and length 12 cm. If the strained position corresponds to a load of 1 gm force applied at the end of the bar, calculate the Young's modulus of aluminum. Assume $\theta_1 \approx \theta_2 \approx 0$; assume $\lambda = 6328$ Å.

[Hint: N represents the number of fringes produced over the length of the cantilever.]

[Ans. 0.7×10^{11} **N/m^2]**

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