10

POLARIZATION

10.1 INTRODUCTION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion *i.e.*, whether the light waves are longitudinal or transverse, or whether the vibrations are linear, circular or torsional. The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

10.2 POLARIZATION OF TRANSVERSE WAVES

Let a rope AB be passed through two parallel slits S_1 and S_2. The rope is attached to a fixed point at B [Fig. 10.1(a)]. Hold the end A and

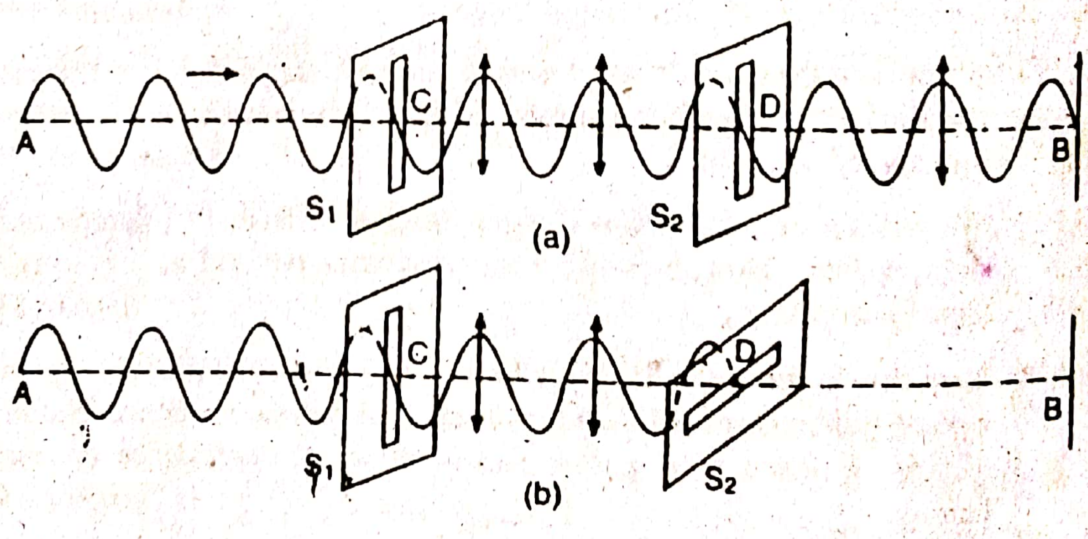

Fig. 10.1

move the rope up and down perpendicular to AB. A wave emerges along CD and it is due to transverse vibrations parallel to the slit S_1. The slit S_2 allows the wave to pass through it when it is parallel to S_1. It is observed that the slit S_2 does not allow the wave to pass through it when it is at right angles to the slit S_1 [Fig. 10.1(b)].

If the end A is moved in a circular manner, the rope will show circular motion up to the slit S_1. Beyond S_1, it will show only linear vibrations parallel to the slit S_1, because the slit S_1 will stop the other components. If S_1 and S_2 are at right angles to each other the rope will not show any vibration beyond S_2.

If longitudinal waves are set up by moving the rope forward and backward along the string, the waves will pass through S_1 and S_2 irrespective of their position.

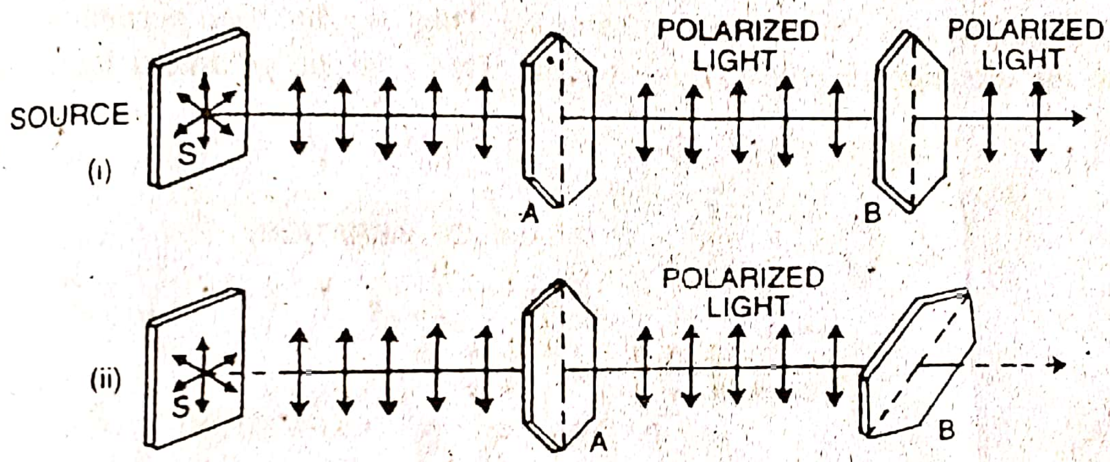

Fig. 10.2

A similar phenomenon has been observed in light when it passes through a tourmaline crystal.

Let light from a source S fall on a tourmaline crystal A which is cut parallel to its axis (Fig. 10.2). The crystal A will act as the slit S_1. The light is slightly coloured due to the natural colour of the crystal. On rotating the crystal A, no remarkable change is noticed. Now place the crystal B parallel to A.

(1) Rotate both the crystals together so that their axes are always parallel. No change is observed in the light coming out of B [Fig. 10.2 (*i*)].

(2) Keep the crystal A fixed and rotate the crystal B. The light transmitted through B becomes dimmer and dimmer. When B *is at right angles to* A, no light emerges out of B [Fig. 10.2 (*ii*)].

If the crystal B is further rotated, the intensity of light coming out of it gradually increases and is maximum again when the two crystals are parallel.

This experiment shows conclusively that light is not propagated as longitudinal or compressional waves. If we consider the propagation of light as a longitudinal wave motion then no extinction of light should occur when the crystal B is rotated.

It is clear that after passing through the crystal A, the light waves vibrate only in one direction. Therefore light coming out of the crystal A is said to be **polarized** because it has acquired the property of **one sidedness** with regard to the direction of the rays.

This experiment proves that light waves are transverse waves, otherwise light coming out of B could never be extinguished by simply rotating the crystal B.

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10.3 PLANE OF POLARIZATION

When ordinary light is passed through a tourmaline crystal, the light is polarized and vibrations are confined to only one direction perpendicular to the direction of propagation of light. This is plane polarized light and

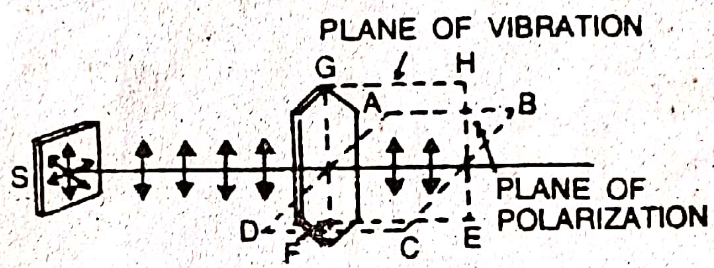

Fig. 10.3

it has acquired the property of one sidedness. The plane of polarization is that plane in which no vibrations occur. The plane $ABCD$ in Fig. 10.3 is the plane of polarization. The vibrations occur at right angles to the plane of polarization and the plane in which vibrations occur is known as plane of vibration. The plane $EFGH$ in Fig. 10.3 is the plane of vibration.

Ordinary light from a source has very large number of wavelengths. Moreover, the vibrations may be linear, circular or elliptical. From our idea of wave motion, circular or elliptical vibrations consist of two linear vibrations at right angles to each other and having a phase difference of $\frac{\pi}{2}$.

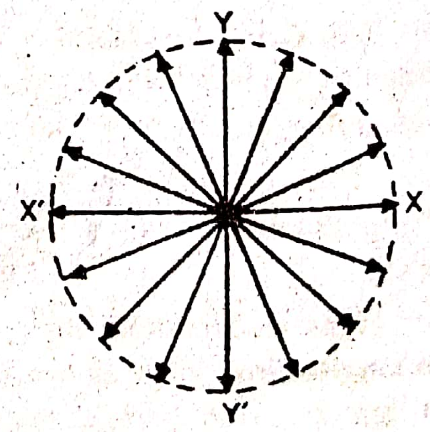

Fig. 10.4

Therefore any vibration can be resolved into two component vibrations at right angles to each other. As light waves are transverse waves the vibrations can be resolved into two planes xx' and yy'

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at right angles to each other and also perpendicular to the direction of propagation of light (Fig. 10.4).

In Fig. 10.5(*i*), the vibrations of the particles are represented parallel (arrow heads) and perpendicular to the plane of the paper (dots).

Fig. 10.5

In Fig. (10.3) (*ii*) the vibrations are shown only parallel to the plane of the paper. In Fig. (10.5) (*iii*) the vibrations are represented only perpendicular to the plane of the paper.

10.4 POLARIZATION BY REFLECTION

Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. Consider the light incident along the path *AB* on the glass surface (Fig. 10.6). Light is

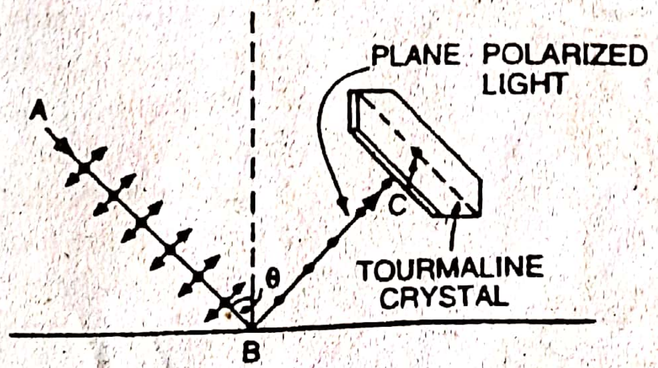

Fig. 10.6

reflected along *BC*. In the path of *BC*, place a tourmaline crystal and rotate it slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing angle. Similarly polarized light by reflection can be produced from water surface also.

The production of polarized light by glass is explained as follows. The vibrations of the incident light can be resolved into components parallel to the glass surface and perpendicular to the glass surface. Light due to the components parallel to the glass surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted.

Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

The polarized light has been analysed by using another mirror by Biot.

10.5 BIOTS POLARISCOPE

It consists of two glass plates M_1 and M_2 (Fig. 10.7). The glass plates are painted black on their back surfaces so as to avoid any reflection and this also helps in absorbing refracted light. A beam of unpolarized light AB is incident at an angle of about 57.5° on the first glass surface at B and is reflected along BC (Fig. 10.8). This light is again reflected at 57.5° by the second glass plate M_2 placed parallel to the first. The glass plate M_1 is known as the polarizer and M_2 as the analyser.

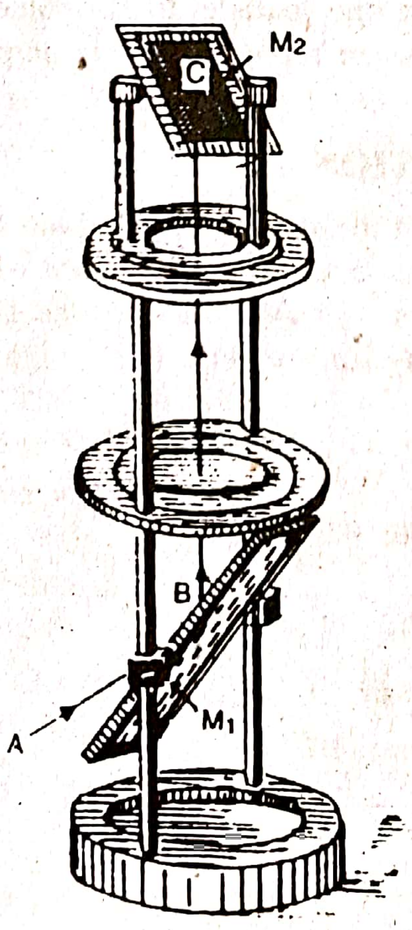

Fig. 10.7

When the upper plate M_2 is rotated about BC, the intensity of the reflected beam along CD decreases and becomes zero for 90° rotation of M_2. Remember, the rotation of the plate M_2 about BC, keeps the angle of incidence constant and it does not change with the rotation of M_2. Thus we find that light travelling along BC is plane polarized.

When the mirror M_2 is rotated further it is found that the intensity of CD becomes maximum at 180°, minimum at 270° and again maximum at 360°.

The above experiment proves that when light is incident at an angle of 57.5° on a glass surface, the reflected light consists of waves in which

the displacements are confined to a certain direction at right angles to the ray and we get polarized light by reflection.

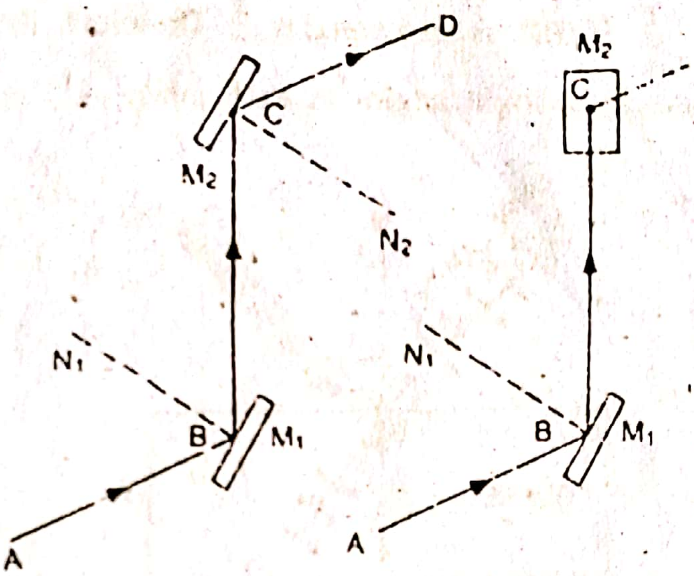

Fig. 10.8

10.6 BREWSTER'S LAW

In 1811, Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media.

He found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the **angle of polarization.**

He was able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and the refracted rays are perpendicular to each other.

Suppose, unpolarized light is inciden at an angle equal to the polarizing angle on the glass surface. It is reflected along *BC* and refracted along *BD* (Fig. 10.9).

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \qquad ...(i)$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i} \qquad ...(ii)$$

Comparing (*i*) and (*ii*)

$$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \qquad i = \frac{\pi}{2} - r, \quad \text{or} \quad i + r = \frac{\pi}{2}$$

As $i + r = \frac{\pi}{2}$, $\angle CBD$ is also equal to $\frac{\pi}{2}$. Therefore, the reflected and the refracted rays are at right angles to each other.

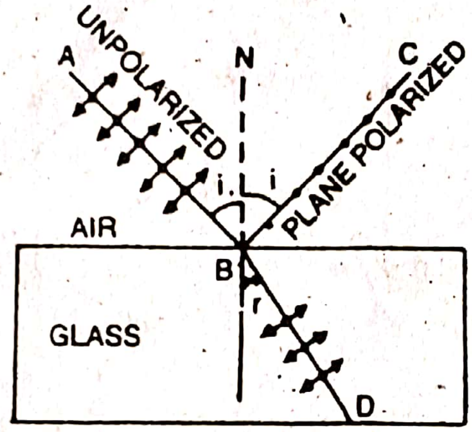

Fig. 10.9

From Brewster's law, it is clear that for crown glass of refractive index 1.52, the value of i is given by

$$i = \tan^{-1}(1.52) \quad \text{or} \quad i = 56.7^\circ$$

However, 57° is an approximate value for the polarizing angle for ordinary glass. For a refractive index of 1.7 the polarising angle is about 59.5° *i.e.*, the polarizing angle is not widely different for different glasses.

As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelengths. Therefore, polarization will be complete only for light of a particular wavelength at a time *i.e.*, for monochromatic light.

It is clear that the light vibrating in the plane of incidence is not reflected along BC [Fig. 10.9]. In the reflected beam the vibrations along BC cannot be observed, whereas vibrations at right angles to the plane of incidence can contribute for the resultant intensity. Thus, we get plane polarized light along BC. The refracted ray will have both the vibrations (*i*) in the plane of incidence and (*ii*) at right angles to the plane of incidence. But it is richer in vibrations in the plane of incidence. Hence it is partially plane-polarized.

10.7 BREWSTER WINDOW

One of the important applications of Brewster's law and Brewster's angle is in the design of a glass window that enables 100% transmission of light. Such a type of window is used in lasers and it is called a **Brewster window**.

When an ordinary beam of light is incident normally on a glass window, about 8% of light is lost by reflection on its two surfaces and about 92% intensity is transmitted. In the case of a gas laser filled with mirrors outside the windows, light travels through the window about a hundred times. In this way the intensity of the final beam is about 3×10^{-4} because $(0.92)^{100} \approx 3 \times 10^{-4}$. It means the transmitted beam has practically no intensity.

To overcome this difficulty, the window is tilted so that the light beam is incident at Brewster's angle. After about hundred transmissions, the final beam will be plane polarized.

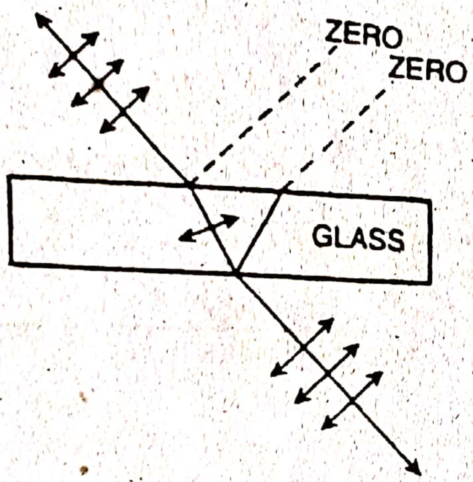

Fig. 10.10

The light component vibrating at right angles to the plane of incidence is reflected. After about 100 reflections at the Brewster window, the transmitted beam will have 50% of the intensity of the incident beam and it will be completely plane polarized. The net effect of this type of arrangement is that half the amount of light intensity has been discarded and the other half is completely retained. Brewster's windows are used in gas lasers.

10.8 POLARIZATION BY REFRACTION

It is found that at a single glass surface or any similar transparent medium, only a small fraction of the incident light is reflected.

For glass ($\mu = 1.5$) at the polarizing angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for the perpendicular vibrations only 85% is transmitted and 15% is reflected. Therefore, if we use a **pile of plates** and the beam of ordinary light is incident at the polarizing angle on the pile of plates, some of the vibrations perpendicular to the plane of incidence are reflected by the first plate and the rest are transmitted through it. When this beam of light is reflected by the second plate, again some of the vibrations perpendicular to the plane of incidence are reflected by it and the rest are transmitted. The process continues and when the beam has traversed about 15 or 20 plates, the transmitted light is completely free from the vibrations at right angles to the plane of incidence and is having vibrations only in the plane of incidence. Thus, we get plane-polarized light by refraction with the help of a pile of plates, the vibrations being in the plane of incidence as shown in Fig. 10.11.

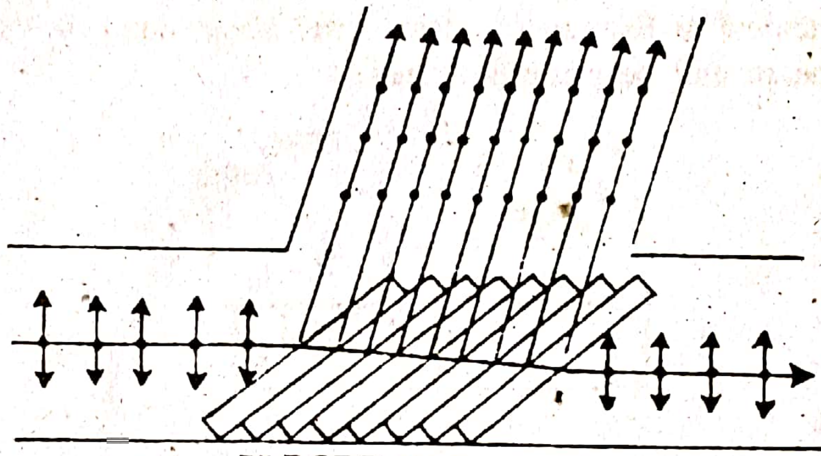

Fig. 10.11

The pile of plates consists of number of glass plates (microscope cover slips) and are supported in a tube of suitable size and are inclined at an angle of 32.5° to the axis of the tube. A beam of monochromatic light is allowed to fall on the pile of plates at the polarizing angle. The transmitted light is polarized perpendicular to the plane of incidence and can be examined by a similar pile of plates which works as an analyser.

Note. (*i*) If light is polarized perpendicular to the plane of incidence, it means vibrations are in the plane of incidence.

(*ii*) If light is polarized in the plane of incidence, it means vibrations are perpendicular to the plane of incidence.

10.9 MALUS LAW

When a beam of light, polarized by reflection at one plane surface is allowed to fall on the second plane surface at the polarizing angle the intensity of the twice reflected beam varies with the angle between the planes of the two surfaces. In the Biot's polariscope it was found that the intensity of the twice reflected beam is maximum when the two planes are parallel and zero when the two planes are at right angles to each other. The same is also true for the twice transmitted beam from the polarizer

It has been found that both the rays are plane polarized. The vibrations of the ordinary ray are perpendicular to the principal section of the crystal while the vibrations of the extraordinary ray are in the plane of the principal section of the crystal. Thus, the two rays are **plane polarised**, their vibrations being at right angles to each other.

Special Cases. (1) It should be remembered that a ray of light is not split up into ordinary and extraordinary components when it is incident on calcite parallel to its optic axis. In this case, the ordinary and the extraordinary rays travel along the same direction with the same velocity.

(2) When a ray of light is incident perpendicular to the optic axis on the calorie crystal, the ray of light is not split up into ordinary and extraordinary components. It means that the ordinary and the extraordinary rays travel in the same direction but with **different velocities**.

10.11 PRINCIPAL SECTION OF THE CRYSTAL

A plane which contains the optic axis and is perpendicular to the opposite faces of the crystal is called the **principal section** of the crystal. As a crystal has six faces, therefore, for every point there are three principal sections. A principal section always cuts the surface of a calcite crystal in a parallelogram with angles 109° and 71°

10.12 PRINCIPAL PLANE

A plane in the crystal drawn through the optic axis and the ordinary ray is defined as the principal plane of the ordinary ray, Similarly, a plane in the crystal drawn through the optic axis and the extraordinary ray is defined as the principal plane of the extraordinary ray. In general, the two planes do not coincide. In a particular case, when the plane of incidence is a principal section then the principal section of the crystal and the principal planes of the ordinary and the extraordinary rays coincide.

10.13 NICOL PRISM

It is an optical device used for producing and analysing plane polarized light. It was invented by William Nicol, in 1828, who was an expert in cutting and polishing gems and crystals. We have discussed that when a beam of light is transmitted through a calcite crystal, it breaks up into two rays : (1) the ordinary ray which has its vibrations perpendicular to the principal section of the crystal and (2) the extraordinary ray which has its vibrations parallel to the principal section.

The **nicol prism** is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that the ordinary ray is **eliminated** and only the **extraordinary ray is transmitted through the prism.**

A calcite crystal whose length is three times its breadth is taken. Let $A'BCDEFG'H$ represent such a crystal having A' and G' as its blunt corners and $A'CG'E$ is one of the principal sections with $\angle A'CG' = 70°$.

The faces $A'BCD$ and $EFG'H$ are ground in such a way that the angle ACG becomes = 68° instead of 71°. The crystal is then cut along the plane $AKGL$ as shown in Fig. 10.15. The two cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and the extraordinary rays for calcite.

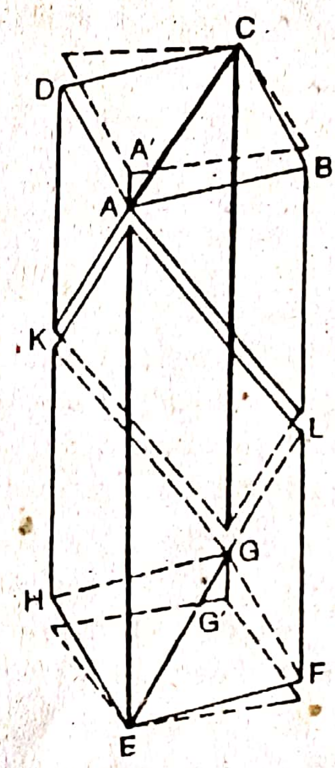

Fig. 10.15

Refractive index for the ordinary

$$\mu_o = 1.658$$

Refractive index for Canada balsam

$$\mu_B = 1.55$$

Refractive index for the extraordinary $\mu_E = 1.486$

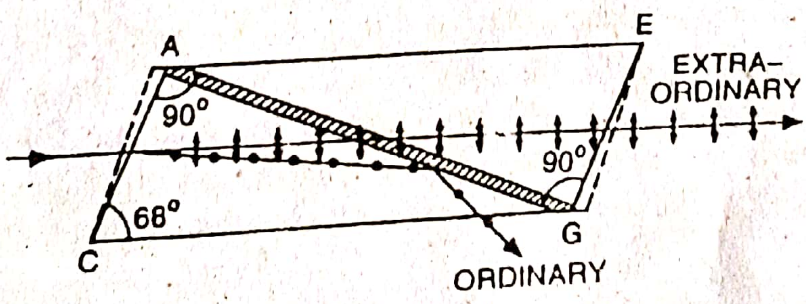

Fig. 10.16

In Fig. 10.16, the section $ACGE$ of the crystal is shown. The diagonal AC represents the Canada balsam layer in the plane $ALGK$ of Fig. 10.15.

It is clear that Canada balsam acts as a rarer medium for an ordinary ray and it acts as a denser medium for the extraordinary ray. Therefore, when the ordinary ray passes from a portion of the crystal into the layer of Canada balsam it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle, the ray is totally internally reflected and is not transmitted. The extraordinary ray is not affected and is therefore transmitted through the prism. The working of the prism is clear from the following cases :-

(1) Refractive index for ordinary ray with respect to Canada balsam

$$= \mu = \frac{1.658}{1.550}$$

$$\therefore \quad \sin\theta = \frac{1}{\mu} = \frac{1.550}{1.658}$$

$$\therefore \quad \theta = 69^\circ$$

If the angle of incidence for the ordinary ray is more than the critical angle, it is totally internally reflected and only the extraordinary ray passes through the nicol prism. Therefore, a ray of unpolarized light on passing through the nicol prism in this position becomes plane-polarized.

(2) If the angle of incidence is less than the critical angle for the ordinary ray, it is not reflected and is transmitted through the prism. In this position both the ordinary and the extraordinary rays are transmitted through the prism.

(3) The extraordinary ray also has a limit beyond which it is totally internally reflected by the Canada balsam surface. The refractive index for the extraordinary ray = 1.486 when the extraordinary ray is travelling at right angles to the direction of the optic axis. If the extraordinary ray travels along the optic axis, its refractive index is the same as that of the ordinary ray and it is equal to 1.658. Therefore, depending upon the direction of propagation of the extraordinary ray μ_e lies between 1.486 and 1.658. Therefore for a particular case μ_e may be more than 1.55 and the angle of incidence will be more than the critical angle. Then, the extraordinary ray will also be totally internally reflected at the Canada balsam layer. The sides of the nicol prism are coated with black paint to absorb the ordinary rays that are reflected towards the sides by the Canada balsam layer.

~~10.14~~ NICOL PRISM AS AN ANALYSER

Nicol prism can be used for the production and detection of plane-polarizer light.

When two nicol prisms P_1 and P_2 are placed adjacent to each other as shown in Fig. 10.17 (*i*), one of them acts as a polarizer and the other acts as an analyser. Fig. 10.17 (*i*) shows the position of two parallel nicols and only the extraordinary ray passes through both the prisms

If the second prism P_2 is gradually rotated, the intensity of the extraordinary ray decreases in accordance with Malus Law and when the two prisms are crossed [*i.e.*, when they are at right angles to each other, Fig. 10.16 (*ii*), then no light comes out of the second prism P_2. It means that light coming out of P_1 is plane polarized. When the polarized extraordinary ray enters the prism P_2 in this positions it acts as

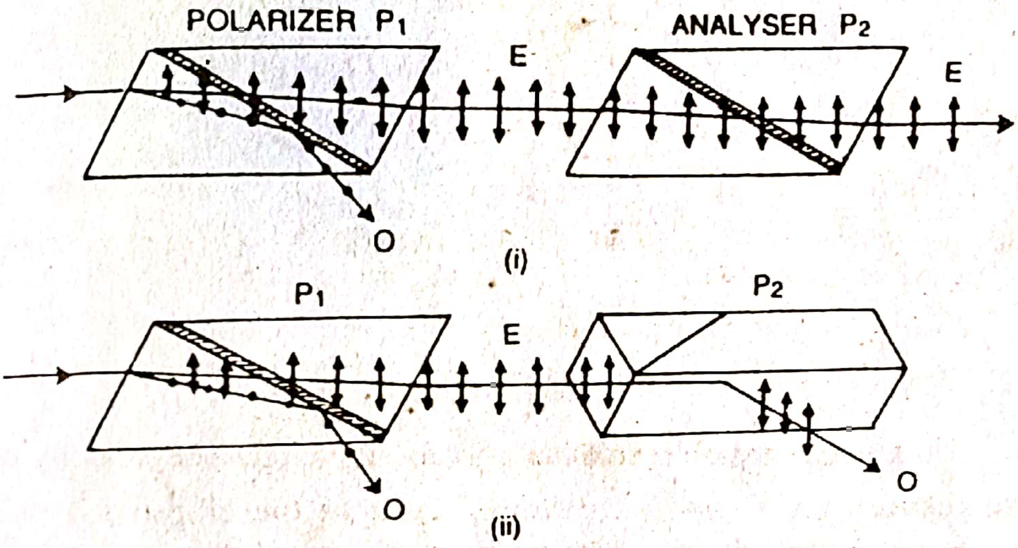

Fig. 10.17

an ordinary ray and is totally internally reflected by the Canada balsam layer and so no light comes out of P_2. Therefore, the prism P_1 produces plane-polarized light and the prism P_2 detects it.

Hence P_1 and P_2 are called the polarizer and the analyser respectively. The combination of P_1 and P_2 is called a polariscope.

10.15 HUYGENS EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTALS

Huygens explained the phenomenon of double refraction with the help of his principle of secondary wavelets. A point source of light in a double refracting medium is the origin of two wavefronts. For the ordinary ray, for which the velocity of light is the same in all directions the wavefront is spherical. For the extraordinary ray, the velocity varies with the direction and the wavefront is an ellipsoid of revolution. **The velocities of the ordinary and the extraordinary rays are the same along the optic axis.**

Consider a point source of light S in a calcite crystal [Fig. 10.18.(*a*)]. The sphere is the wave surface for the ordinary ray and the ellipsoid is the wave surface for the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as **negative crystals**. For crystals like quartz, which are known as positive crystals, index is maximum. **Therefore, the principal refractive index for the positive uniaxial crystal is the ratio of the velocity of light in vacuum to the minimum velocity of the extraordinary ray.**

For a negative crystal of calcite, $\mu_0 = 1.658$ and $\mu_E = 1.486$. Therefore, the ratio of the major to the minor axis of the wave surface of the extraordinary ray is 1.658 : 1.486.

For a positive crystal of quartz, $\mu_0 = 1.544$ and $\mu_E = 1.553$. Therefore, the ratio of the major to the minor axis of the wave surface of the extraordinary ray is, 1.553 : 1.544.

10.21 EXPERIMENTAL DETERMINATION OF REFRACTIVE INDEX

For determining the refractive index for the extraordinary ray a calcite crystal is cut in the form of a prism with the optic axis perpendicular to the refracting edge of the prism and perpendicular to the base *BC* [Fig. 10.27 (*a*)]. It can also be cut with the optic axis parallel to the

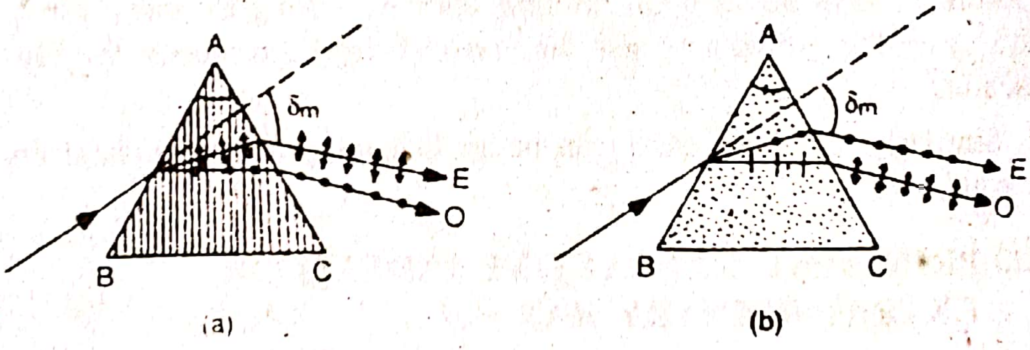

Fig. 10.27 (*a*) *Optic axis per perpendicular to the refracting edge*
(*b*) *Optic axis parallel to the refracting edge.*

refracting edge of the prism [Fig. 10.27 (*b*)]. The prism is placed on the spectrometer table and is adjusted for the minimum deviation position for the extraordinary rays. The angle of minimum deviation δ_m is determined and the principal refractive index for the extraordinary ray is calculated from the relation.

$$\mu_E = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

For a given wavelength, the ordinary and the extraordinary rays are separated while passing through the prism. Therefore, the angle of minimum deviation for the ordinary ray can be measured and thus its refractive index can be calculated.

10.22 DOUBLE IMAGE POLARIZING PRISMS

Nicol prism cannot be used with ultraviolet light on account of the Canada babam layer which absorbs these rays. Sometimes, it is also desirable to have both the ordinary and the extraordinary rays widely separated. For this purpose two prisms *viz.* (*i*) Rochon prism and (*ii*) Wollaston prism are used.

(1) **Rochon Prism.** It consists of two prisms *ABC* and *BCD* (of quartz or calcite) cut with their optic axes as shown in Fig. 10.28. The prism *ABC* is cut such that the optic axis is parallel to the face *AB* and the incident light. The prism *BCD* has the opticaxis perpendicular to the plane of incidence. Light incident normally on the face *AC* of the prism passes undeviated up to the boundary *BC*. In the prism *BCD*, the ordinary ray passes undeviated. If the prisms are made of quartz, the extraordinary ray is deviated as shown in Fig. 10.28. In the case of calcite, the extraordinary will be deviated to the other side. The prisms *ABC* and *BCD* are cemented together by glycerine or castor oil. Here, the ordinary emergent beam is achromatic whereas the extraordinary beam is chromatic.

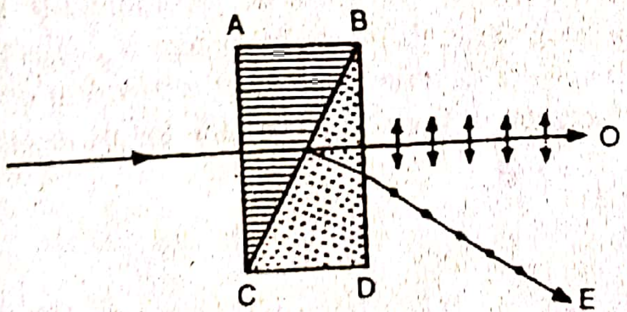

Fig. 10.28

(2) **Wollaston prism.** It consists of two prisms *ABC* and *BCD* of quartz or calcite cut with their optic axes as shown in Fig. 10.29. They are cemented together by glycerine or caster oil.

A ray of light is incident normally on the face *AC* of the prism *ABC*. The ordinary and the extraordinary rays travel along the same direction but with different speeds. After passing *BC* the ordinary ray behaves as the extra ordinary and the extra ordinary behaves as the ordinary while passing through the prism *BCD*. One ray is bent towards the normal while the other is bent away from the normal. In quartz $\mu_E > \mu_0$. Therefore, the ordinary ray while passing the boundary *BC* is refracted towards the normal as an extraordinary ray while the extraordinary ray is refracted away from the normal as an ordinary ray as shown in Fig. 10.29. If the prisms are made from calcite, the directions of the ordinary and the extraordinary rays are interchanged. While coming out of the face *BD* of the prism, the ordinary and the extraordinary rays are diverged. The prism

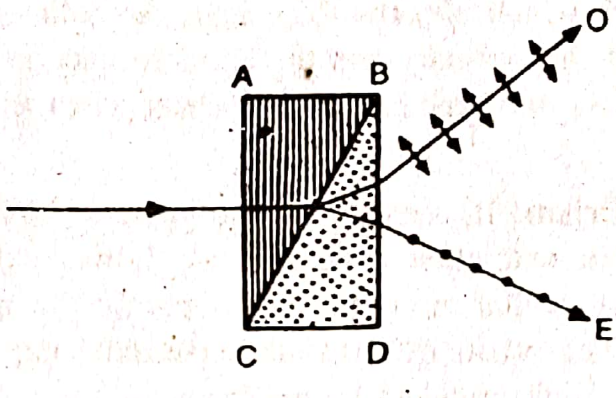

Fig. 10.29

is useful in determining the percentage of polarization in a partially polarized beam. Double image prisms are used in spectrophotometers and pyrometers.

10.23 ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT

Let monochromatic light be incident on the nicol prism N_1 [Fig. 10.30 (*a*)]. After passing through the nicol prism N_2, it is plane-polarized and is incident normally on a uniaxial doubly refracting crystal *P* (calcite or quartz) whose faces have been cut parallel to the optic axis. The vibrations

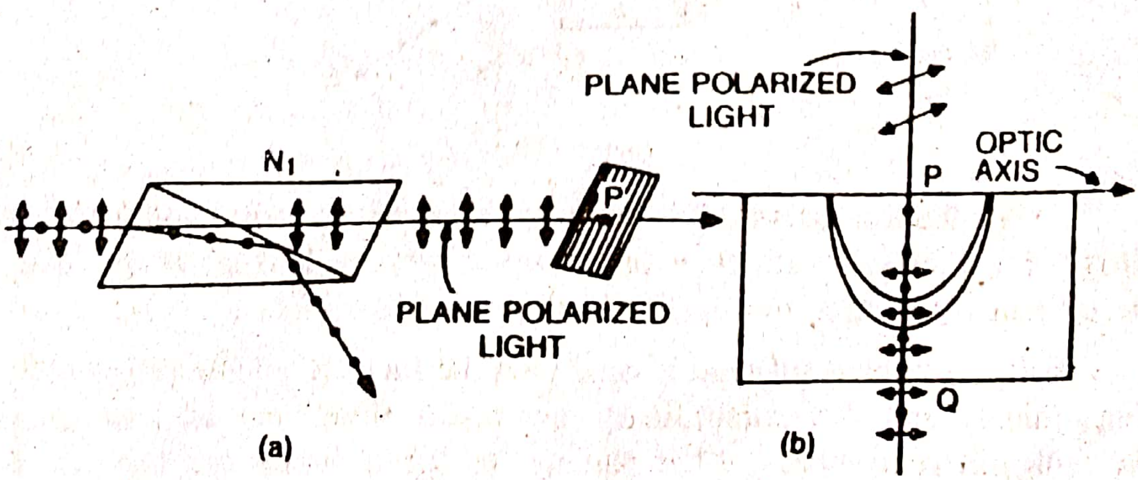

Fig. 10.30

of the plane-polarized light incident on the crystal are shown in Fig. 10.30 (*b*). The plane polarized light on entering the crystal is split up into two components, ordinary and extraordinary. Both the rays, in this case, travel along the same direction but, with different velocities. When the rays have travelled through the thickness d in the crystal, a phase difference δ is introduced between them.

Theory. Suppose the amplitude of the incident plane polarized light on the crystal is A and it makes an angle θ with the optic axis (Fig. 10.31). Therefore, the amplitude of the ordinary ray vibrating along PO is $A \sin\theta$ and the amplitude of the extraordinary ray vibrating along PE is $A \cos\theta$. Since a phase difference δ is introduced between the two rays, after passing thorough a thickness d of the crystal, the rays after comming out of the crystal can be represented in terms of two simple harmonic motions, at right angles to each other and having a phase difference.

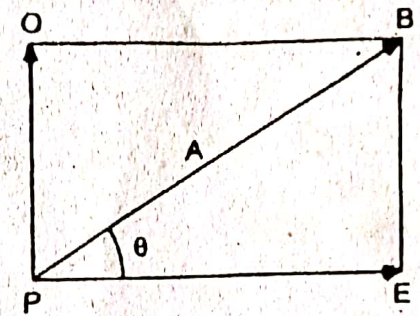

Fig. 10.31

$\therefore$ For the extraordinary ray,

$$x = A\cos\theta . \sin(\omega t + \delta)$$

For the ordinary ray,

$$y = A\sin\theta . \sin\omega t$$

Take, $\quad A\cos\theta = a$

and $\quad A\sin\theta = b$

$$x = a\sin(\omega t + \delta) \quad \text{...(i)}$$

$$y = b\sin\omega t \quad \text{...(ii)}$$

From (ii)

$$\frac{y}{b} = \sin\omega t$$

and

$$\cot\omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}} . \sin\delta$$

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}} . \sin\delta$$

Squaring and rearranging

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\delta = \sin^2\delta \qquad ...(iii)$$

This is the general equation of an ellipse.

Special Cases. (1) When $\delta = 0$ $\sin\delta = 0$ and $\cos\delta = 1$

From equation (*iii*)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a}-\frac{y}{b}\right)^2 = 0$$

or

$$y = \frac{bx}{a}$$

This is the equation of a straight line. Therefore, the emergent light will be plane polarized (Fig. 10.32).

(2) When $\delta = \frac{\pi}{2}$, $\cos\delta = 0, \sin\delta = 1$

From equation (*iii*)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

This represents the equation of a symmetrical ellipse. The emergent light in this case will be **elliptically polarized** provided $a \neq b$.

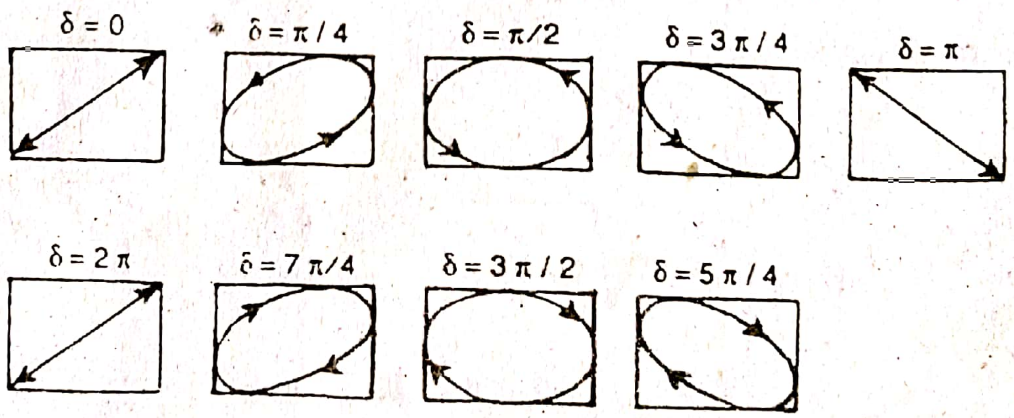

Fig. 10.32

(3) When $\delta = \frac{\pi}{2}$ and $a = b$

From equation (*iii*),

$$x^2 + y^2 = a^2$$

This represents the equation of circle of radius a. The emergent light will be **circularly polarized**. Here the vibrations of the incident plane-polarized light on the crystal make an angle of 45° with the direction of the optic axis.

(4) For $\delta = \pi/4$ or $7\pi/4$, the shape of the ellipse will be as shown in Fig. 10.32.

(5) For all other values of δ, the nature of vibrations will be as shown in Fig. 10.32.

10.24 QUARTER WAVE PLATE

It is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. The incident plane-polarized light is perpendicular to its surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_E respectively, then the path difference introduced between the two rays is given by :

For negative crystals, path difference $= (\mu_0 - \mu_E)t$

For positive crystals, path difference $= (\mu_E - \mu_0)t$

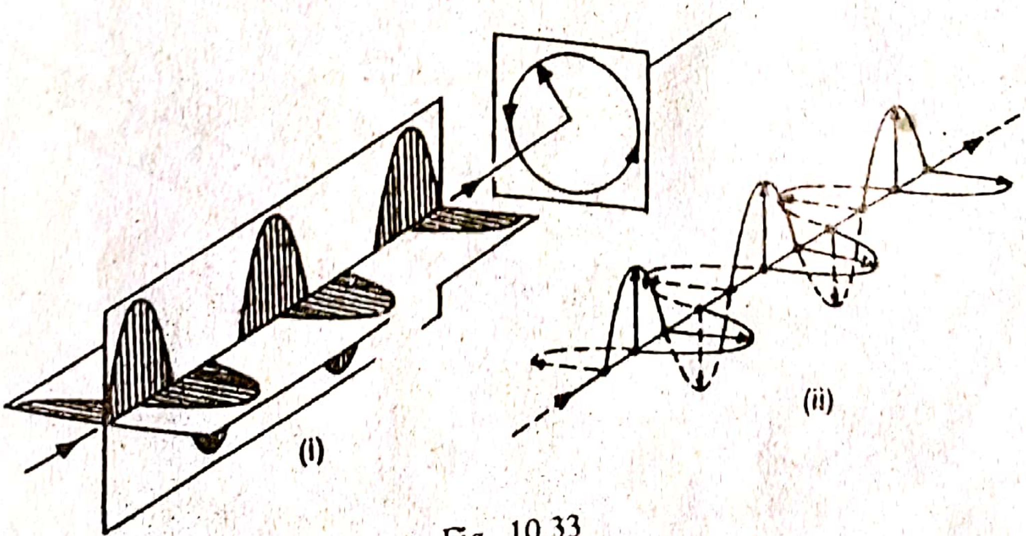

Fig. 10.33

To produce a path difference of $\frac{\lambda}{4}$, in calcite

$$(\mu_0 - \mu_E)t = \frac{\lambda}{4} \quad \ldots(i)$$

or

$$t = \frac{\lambda}{4(\mu_0 - \mu_E)}$$

For a half wave plate,

$$(\mu_0 - \mu_E)\, t = \frac{\lambda}{2}$$

$$\therefore \quad t = \frac{\lambda}{2(\mu_0 - \mu_E)}$$

$$t = \frac{5 \times 10^{-7}}{2(1.55 - 1.45)}$$

$$t = \mathbf{2.5 \times 10^{-5}\ m}$$

$$t = \mathbf{2.5 \times 10^{-2}\ mm}$$

Example 10.11. *Calculate the thickness of a double refracting plate capable of producing a path difference of $\frac{\lambda}{4}$ between extraordinary and ordinary waves.*

λ = 5890 Å, μ_0 = 1.53 and μ_E = 1.54 [*Kanpur, 1991*]

Here $$(\mu_E - \mu_0)\, t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

Here $$\lambda = 5890 \text{ Å} = 5.89 \times 10^{-7} \text{ m}$$

$$t = \frac{5.89 \times 10^{-7}}{4[1.54 - 1.53]}$$

$$t = \mathbf{1.47 \times 10^{-5}\ m}$$

$$t = \mathbf{1.47 \times 10^{-2}\ mm}$$

10.26 PRODUCTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARIZED LIGHT

(1) **Plane polarized light.** A beam of monochromatic light is passed through a nicol prism. While passing through the nicol prism, the beam is split up into extraordinary ray and ordinary ray. The ordinary ray is totally internally reflected back at the Canada balsam layer, while the extraordinary ray passes through the nicol prism. The emergent beam is plane polarized.

Circularly polarized light. To produce circularly polarized light, the two waves vibrating at right angles to each other and having the same amplitude and time period should have a phase difference of $\pi/2$ or a path difference of $\lambda/4$. For this purpose, a parallel beam of monochromatic light is allowed to fall on a nicol prism N_1 (Fig. 10.35). The beam after passing through the prism N_1, is plane polarized.

The nicol prism N_2 is placed at some distance from N_1 so that N_1 and N_2 are crossed. The field of view will be dark as viewed by the eye

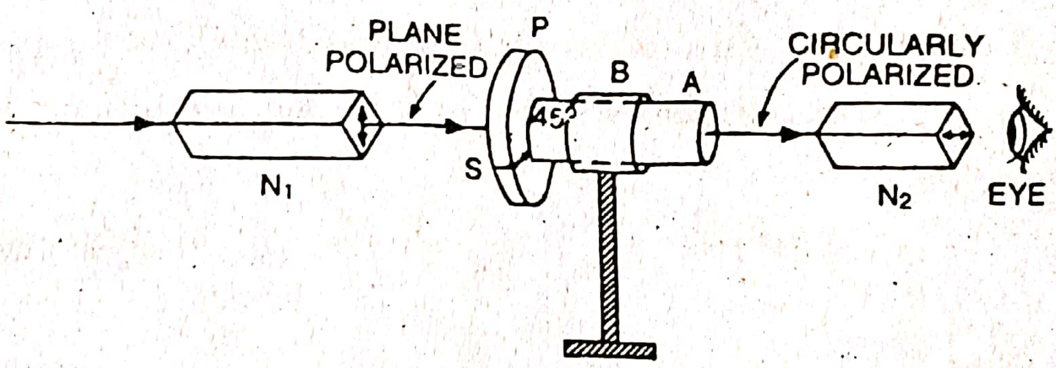

Fig. 10.35

in this position. A quarter wave plate P is mounted on a tube A. The tube A can rotate about the outer fixed tube B introduced between the nicol prisms N_1 and N_2. The plane polarized light from N_1 falls normally on P and the field of view may be bright. The quarter wave plate is rotated until the field of view is dark. Keeping P fixed, A is rotated such that the mark S on P coincides with zero mark on A. Afterwards, by rotating the quarter wave plate P, the mark S is made to coincide with 45° mark on A.

The quarter wave plate is in the desired position. In this case, the vibrations of the plane-polarized light falling on the quarter wave plate make an angle of 45° with the direction of the optic axis of the quarter wave plate. The polarized light is split up into two rectangular components (ordinary and extraordinary) having equal amplitude and time period and on coming out of the quarter wave plate, the beam is circularly polarized. If the nicol prism N_2 is rotated at this stage, the field of view is uniform in intensity similar to the ordinary light passing through the nicol prism.

(3) **Elliptically polarized light.** To produce elliptically polarized light, the two waves vibrating at right angles to each other and having unequal amplitudes should have a phase difference of $\pi / 2$, or a path difference of $\lambda / 4$. The arrangement of Fig. 10.35 can be used for this purpose. A parallel beam of monochromatic light is allowed to fall on the nicol prism N_1. The prisms N_1 and N_2 are crossed and the field of view is dark. A quarter wave plate is introduced between N_1 and N_2. The plane-polarized light from the nicol prism N_1 falls normally on the quarter wave plate. The field of view is illuminated and the light coming out of the quarter wave plate is elliptically polarized. (The only precaution in this case is that the vibrations of the plane-polarized light falling on the quarter wave plate should not make an angle of 45° with the optic axis, in which case, the light will be circularly polarized). When the nicol prism N_2 is rotated, it is observed that the intensity of illumination of the field of view varies between a maximum and minimum. This is just similar to the case when a beam consisting of a mixture of plane-polarized light and ordinary light is examined by a nicol prism.

10.27 DETECTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARIZED LIGHT

(1) **Plane-polarized light.** The beam is allowed to fall on a nicol prism. The nicol prism is rotated. If, on rotating the nicol prism, light is completely extinguished twice in each rotation of the nicol prism, the beam is plane-polarized.

(2) **Circularly polarized light.** The beam is allowed to fall on a nicol prism. The intensity of the beam remains uniform when the nicol prism is rotated. The beam, in this case, is either circularly polarized or unpolarized because both show the same result when allowed to pass through a rotating nicol prism.

To distinguish between the two, the beam is allowed to fall on a quarter wave plate and then on a nicol prism. If the beam is circularly polarized, after passing through the quarter wave plate, the ordinary and extraordinary rays will undergo a further path difference of $\frac{\lambda}{4}$. The beam after passing through the quarter wave plate becomes plane polarized. When the nicol prism is rotated, light is completely extinguished twice in each rotation of the nicol prism in this case. Therefore, the original beam is circularly polarized. On the other hand, when the beam after passing thought the quarter wave plate is not extinguished when the nicol prism is rotated, the original beam is unpolarized. To conclude, **if the original beam after passing through the quarter wave plate is extinguished twice in each rotation, when studied by a rotating nicol prism, it is circularly ploarized.**

The total path difference between the component vibrations is $\frac{\lambda}{2}$. Therefore the emergent light is plane polarised and its vibrations make an angle of 45° with the plane of incidence.

When an elliptically polarized light is passed through a Fresnel's rhomb, a further path difference of $\frac{\lambda}{4}$ is introduced between the component vibrations (parallel and perpendicular to the plane of incidence). The total path difference between the component vibrations is $\frac{\lambda}{2}$ and the emergent light is plane polarized.

Thus, Fresnel's rhomb behaves just similar to a quarter wave plate. A quarter wave plate is used only for light of a particular wavelength, whereas a Fresnel's rhomb can be used for light of all wavelengths.

10.31 OPTICAL ACTIVITY

When a polarizer and an analyser are crossed, no light emerges out of the analyser [Fig. 10.40 (*i*)]. When a quartz plate cut with its faces

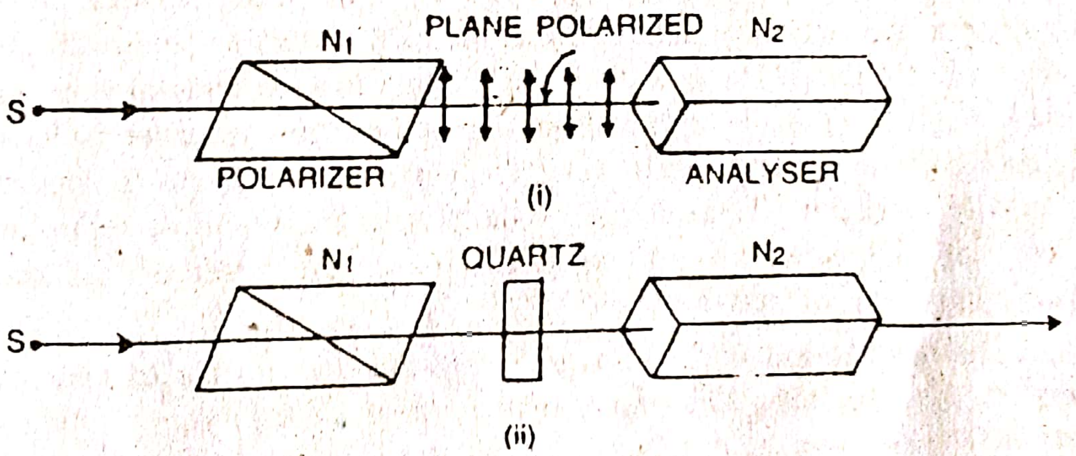

Fig. 10.40

parallel to the optic axis is introduced between N_1 and N_2 such that light falls normally upon the quartz plate, the light emerges out of N_2 [Fig. 10.40 (*ii*)].

The quartz plate turns the plane of vibration. The plane polarized light enters the quartz plate and its plane of vibration is gradually rotated as shown in Fig. 10.41.

The amount of rotation through which the plane of vibration is turned depends upon the thickness of the quartz plate and the wavelength of light. The action of turning the plane of vibration occurs inside the body of the plate and not on its surface. This phenomenon or the property of rotating the plane of vibration by certain crystals or substances is known as **optical**

activity and the substance is known as an optically active substance. It has been found that calcite does not produce any change in the plane of vibration of the plane polarised light. Therefore, it is not optically active.

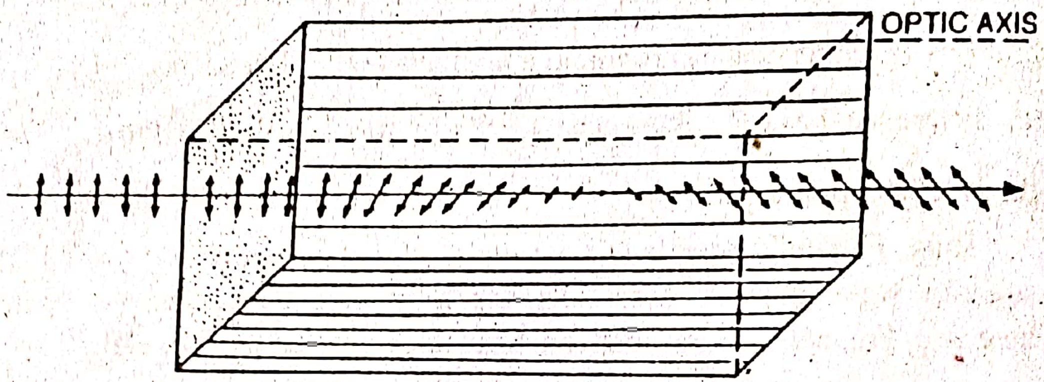

Fig. 10.41

Substances like sugar crystals, sugar solution, turpentine, sodium chlorate and cinnabar are optically active. Some of the substances rotate the plane of vibration to the right and they are called **dextro-rotatory** or **right handed**. Right handed rotation means that when the observer is looking towards light travelling towards him, the plane of vibration is rotated in a clockwise direction. The substances that rotate the plane of vibration to the left (anti-clockwise from the point of view of the observer) are known as **laevo-rotatory** or **left-handed**.

It has been found that some quartz crystals are dextro-rotatory while others are laevorotatory. One is the mirror image of the other in their orientation. The rotation of the plane of vibration in a solution depends upon the concentration of the optically active substance in the solution. This helps in finding the amount of cane sugar present in a sample of sugar solution.

10.32 FRESNEL'S EXPLANATION OF ROTATION

A linearly polarized light can be considered as a resultant of two circularly polarized vibrations rotating in opposite directions, with the same angular velocity. Fresnel assumed that a plane polarized light on entering a crystal along the optic axis is resolved into two circularly polarized vibrations rotating in opposite directions with the same angular velocity or frequency.

In a crystal like calcite, the two circularly polarized vibrations travel with the same angular velocity.

10.33 FRESNEL'S EXPERIMENT

Fresnel showed that linearly polarized light on entering an optically active crystal is resolved into two circularly polarized vibrations..He arranged a number of negative and positive optically active quartz prisms as shown in Fig. 10.44.

Two circularly polarized beams were observed, one rotating to the right (clockwise) and the other rotating to the left (anticlockwise). The optic axis is parallel to the base of each prism. When plane polarized light in incident normally on the first crystal surface (R), the two component circular vibrations (clockwise and anticlockwise, travel along the same direction with different speeds. When the beam is incident on the sblique

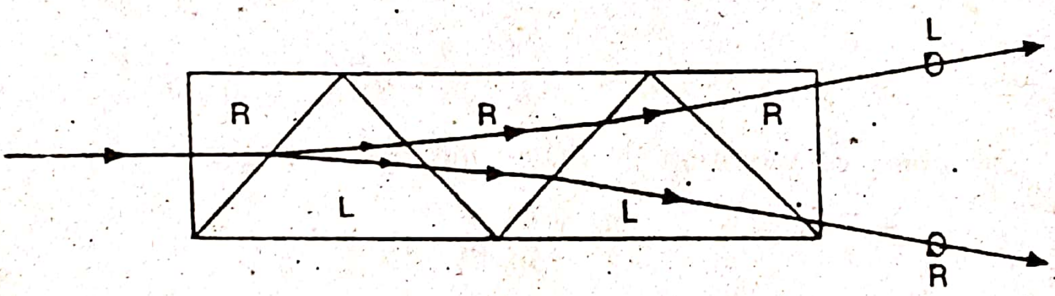

Fig. 10.44

surface of the second prism (L), the beam which was faster in the first prism becomes slower in the second prism and *vice versa*. Therefore; one beam is bent away from the normal and the other is bent towards the normal. The two beams are separated apart, while they travel through the prism L. Again at the boundary of the next prism (R), the speeds are interchanged and the beam that is bent towards the normal in prism L, is now bent away from the normal. Thus the two beams are separated more and more while passing through the arrangement. When the two beams emerge out, they are widely apart. When these beams are treated with a quarter wave plate and a nicol prism, both are found to be circularly polarized.

10.34 SPECIFIC ROTATION

Liquids containing an optically active substance *e.g.*, sugar solution, camphor in alcohol etc. rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon (1) the thickness of the medium (2) concentration of the souution or density of the active substance in the solvent (3) wavelength of light and (4) temperature.

The specific rotation is defined as the rotation produced by a decimetre (10 cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution. Therefore,

$$S_\lambda^t = \frac{10\theta}{lC}$$

where S_λ^t, represents the specific rotation at temperature t °C for a wavelength λ, θ is the angle of rotation, l is the length of the solution in cm through which the plane polarized light passes and C is the concentration of the active substance in g/cc in the solution.

The angle through which the plane of polarization is rotated by the optically active substance is determined with the help of a polarimeter. When this instrument is used to determine the quantity of sugar in a solution, it is known as a saccharimeter.

10.35 LAURENT'S HALF SHADE POLARIMETER

It consists of two nicol prisms N_1 and N_2. N_1 is a polarizer and N_2 is an analyser. Behind N_1, there is a half wave plate of quartz Q which covers one half of the field of view, while the other half G is a glass

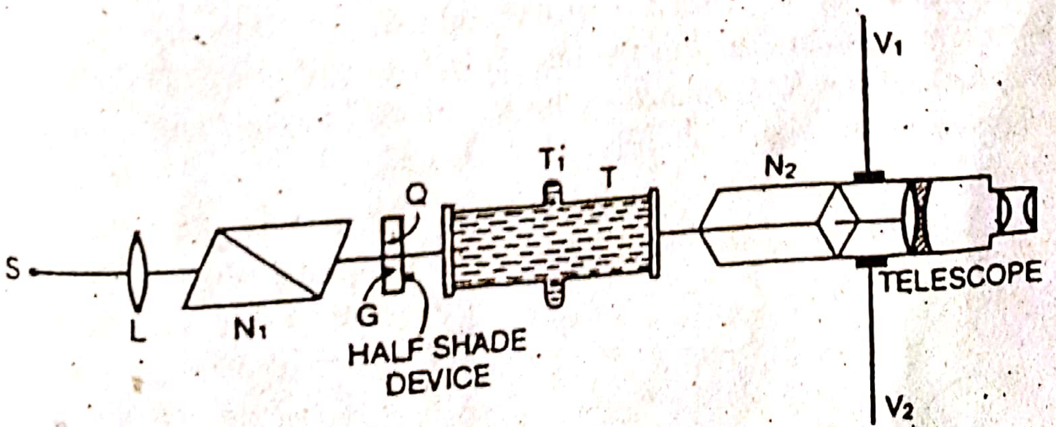

Fig. 10.45

plate. The glass plate G absorbs the same amount of light as the quartz plate Q. T is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles (if any) will appear at the upper portion of the wide bore T_1 of the tube.

Light from a monochromatic source S is incident on the converging lens L. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and the other half passes through the glass plate G. Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along AB (Fig. 10.46). Here AB makes an angle θ with YY'. On passing through the quartz plate Q, the beam is split up into ordinary and extraordinary components which travel along the same direction but with different speeds and on emergence a phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced between them.

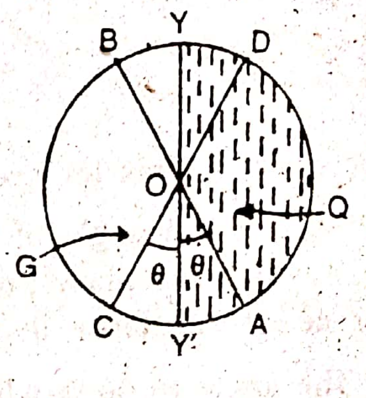

Fig. 10.46

The vibrations of the beam emerging out of quartz will be along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyser N_2 has its principal plane or section along YY' *i.e.*, along the direction which bisects the angle AOC, the amplitudes of light incident on the analyser N_2 from both the halves (*i.e.*, quartz half and glass half) will be equal. Therefore, the field of view will be equally bright [Fig. 10.47 (*i*)].

If the analyser N_2 is rotated to the right of YY', then the right half will be brighter as compared to the left half [Fig. 10.47 (*ii*)]. On the other hand, if the analyser N_2 is rotated to the left of YY', the left half is brighter as compared to the right half [Fig. 10.47 (*iii*)].

Therefore, to find the specific rotation of an optically active substance [say, sugar solution], the analyser N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube T. The readings of the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, AB and CD are rotated in the clockwise direction. Therefore, on the introduction of the tube containing the sugar solution, the field of view is not equally bright. The analyser is rotated in the clockwise direction and is brought to a position

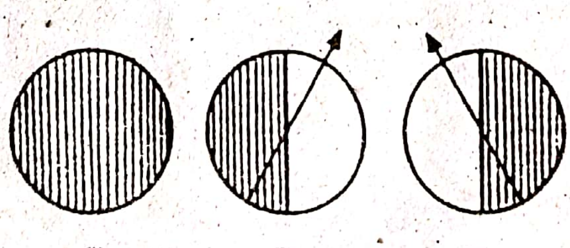

Fig. 10.47

so that the whole field of view is equally bright. The new positions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyser has been rotated gives the angle through which the plane of vibration of the incident beam has been rotated by the sugar solution. In the actual experiment, for various concentrations of the sugar solution, the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation θ. The graph is a straight line (Fig. 10.48).

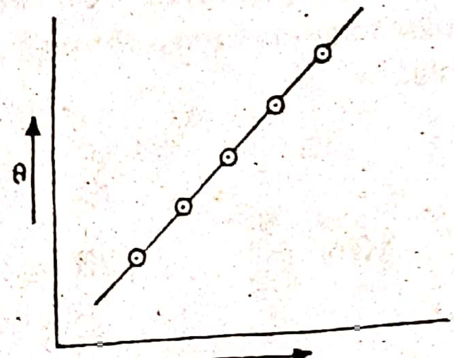

Fig. 10.48

Then from the relation

$S'_\lambda = \frac{10\,\theta}{lC}$, the specific rotation of the optically active substance is calculated.

Example 10.14. *Determine the specific rotation of the given sample of sugar solution if the plane of polarization is turned through 13.2°. The length of the tube containing 10% sugar solution is 20 cm.*

Here,
$$\theta = 13.2^\circ$$
$$C = 10\% = 0.1\ \text{g/cm}^3$$
$$l = 20\ \text{cm}$$
$$S'_\lambda = \frac{10\times 13.2}{20\times 0.1} = 66^\circ$$

Example 10.15. *On introducing a polarimeter tube 25 cm long and containing sugar solution of unknown strength, it is found that the plane of polarization is rooted through 10°. Find the strength of the sugar solution in g/cm³. (Given that the specific rotation of sugar solution is 60° per decimetre per unit concentration)* (Rajasthan 1966)

Here,
$$\theta = 10^\circ$$
$$S = 60^\circ$$
$$l = 25\ \text{cm}$$
$$S = \frac{10\,\theta}{lC}$$

$$C = \frac{10\,\theta}{lS} = \frac{10 \times 10}{25 \times 60} = \frac{1}{15}$$

$$C = \mathbf{0.067\ g / cc}$$

Example 10.16. *Calculate the specific rotation if the plane of polarisation is turned through 26.4°, traversing 20 cm length of 20% sugar solution.* (Delhi)

$$S'_\lambda = \frac{10\,\theta}{lC}$$

Here $\theta = 26.4°, \quad l = 20\text{ cm}$

$$C = 20\% = 0.2\text{ g/cm}^3$$

$$\therefore \quad S'_\lambda = \frac{10 \times 26.4}{20 \times 0.2}$$

$$= \mathbf{66°}$$

Example 10.17. *A 20 cm long tube containing sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the strength of the solution.* [Kanpur]

Here $\theta = 11°$

$$l = 20\text{ cm}, \quad S = 60°$$

$$S = \frac{10\,\theta}{lC} \quad C = \frac{10\,\theta}{lS}$$

or

$$C = \frac{10 \times 11}{20 \times 66}$$

$$C = \mathbf{0.0833\ g / cm^3}$$

Example 10.18. *A 200 mm long tube and containing 48 cm³ of sugar solution produces an optical rotation of 11° when placed in a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of a solution.*

[Kanpur, 1990]

Here $\theta = 11°$

$$l = 200\text{ mm} = 20\text{ cm}$$

$$S = 66°$$

$$S = \frac{10\,\theta}{lC}$$

$$C = \frac{10\,\theta}{lS}$$

$$C = \frac{10 \times 11}{20 \times 66}$$

$$C = 0.0833\ g/cm^3$$

volume, $V = 48\ cm^3$

Hence mass of sugar in the solution

$$M = CV = 0.0833 \times 48 = \mathbf{4\ grams}$$

Example 10.19. *Calculate the specific rotation for sugar solution using the following data :*

Length of the tube = 20 cm

Volume of the tube = 120 cm³

Quantity of sugar dissolved = 6 g

Angle of rotation of the analyser for restoring equal Intensity = 6.6° [Kanpur, 1991]

$$\theta = 6.6^\circ$$

$$l = 20\ cm$$

$$C = \frac{6}{80}\ g/cm^3$$

$$S = \frac{10\,\theta}{lC}$$

$$S = \frac{10 \times 6.6 \times 120}{20 \times 6}$$

$$S = \mathbf{66^\circ}$$

10.36 BIQUARTZ

Instead of half shade plate, a biquartz plate is also used in polarimeters. If consists of two semi-circular plates of quartz each of thickness 3.75 mm. One half consists of right-handed optically active quartz, while the other is left-handed optically active quartz. If white light is used, yellow light is quenched by the biquartz plate and both the halves will have the **tint of passage.** This can be adjusted by rotating the analyser N_2 to a particular position. When the

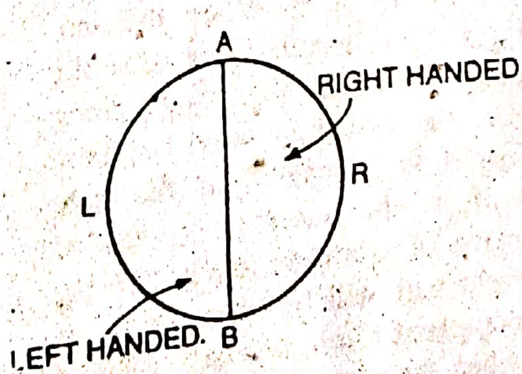

Fig. 10.49

and in the case of quartz,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)} \quad \text{...(ii)}$$

When plane-polarized light is incident on a half-wave plate such that it makes an angle of 45° with the optic axis, a path difference of $\frac{\lambda}{2}$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90° as shown in Fig. 10.31. Thus, **a half wave plate rotates the azimuth of a beam of plane polarized light by 90°, provided the incident light makes an angle of 45° with the optic axis of the half wave plate.**

Example 10.2. *Calculate the thickness of a half wave plate of quartz for a wavelength of 5000 Å. Here μ_E = 1.553 and μ_0 = 1.544.*

(Delhi)

For a half wave plate,

$$t = \frac{\lambda}{2[\mu_E - \mu_0]}$$

Here $\lambda = 5000 \text{ Å} = 5 \times 10^{-5}$ cm

$\mu_E = 1.553$, $\mu_0 = 1.544$, $t = ?$

$$\therefore \quad t = \frac{5 \times 10^{-5}}{2[1.553 - 1.544]}$$

or $\quad t = \mathbf{2.78 \times 10^{-3}}$ **cm**

Example 10.3. *Plane-polarized light passes through a quartz plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam will be plane-polarized.*

Given

$\mu_E = 1.5533$, $\mu_0 = 1.5442$ *and* $\lambda = 5 \times 10^{-5}$ cm *(Punjab)*

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$= \frac{5 \times 10^{-5}}{2(1.5533 - 1.5442)}$$

$$= \mathbf{2.75 \times 10^{-3}\ cm}$$

Example 10.4. *Calculate the thickness of (i) a quarter wave plate and (ii) a half wave plate given that*

$\mu_E = 1.553$ *and* $\mu_0 = 1.544$ *and* $\lambda = 5000$ Å. [Delhi, 1976]

(*i*) For a quarter wave plate

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

$$= \frac{5000 \times 10^{-8}}{4[1.553 - 1.544]}$$

$$= \mathbf{1.39 \times 10^{-3}\ cm}$$

(*ii*) Fior a half wave plate

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$= \frac{5000 \times 10^{-8}}{2[1.553 - 1.544]}$$

$$= \mathbf{2.78 \times 10^{-3}\ cm}$$

Example 10.5. *Find the thickness of a quarter wave plate when the wavelength of light is 5890 Å.* $\mu_E = 1.553$ *and* $\mu_0 = 1.544$.

[Delhi, 1977]

Here $$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

$$\lambda = 5890 \times 10^{-8} \text{ cm},$$

$$\mu_E = 1.553, \quad \mu_0 = 1.544$$

$$\therefore \quad t = \frac{5890 \times 10^{-8}}{4(1.553 - 1.544)}$$

$$t = \frac{5890 \times 10^{-8}}{4 \times 0.009}$$

$$t = \mathbf{1.636 \times 10^{-3}\ cm}$$

Example 10.6. *Plane polarized light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and the extraordinary rays combine to form plane polarized light. Given,*

$$\mu_0 = 1.5442, \quad \mu_E = 1.5533$$

$$\lambda = 5 \times 10^{-5} \text{ cm}$$

[Delhi, 1978]

Here, $$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$\lambda = 5 \times 10^{-5} \text{ cm}$$

$$\mu_E = 1.5533$$

$$\mu_0 = 1.5442$$

$$t = \frac{5 \times 10^{-5}}{2(1.5533 - 1.5442)}$$

$$t = \mathbf{2.75 \times 10^{-2} \text{ cm}}$$

Example 10.7. *Quartz has refractive indices 1.553 and 1.544. Calculate the thickness of the quarter wave plate for sodium light of wavelength 5890 Å.* [*IAS, 1974*]

For a quarter wave plate

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

Here $$\lambda = 5890 \text{ Å} = 5890 \times 10^{-10} \text{ m}$$

$$\mu_E = 1.553$$

$$\mu_0 = 1.544$$

$$t = \frac{5890 \times 10^{-10}}{4[1.553 - 1.544]}$$

$$t = \mathbf{1.63 \times 10^{-5} \text{ m}}$$

$$t = \mathbf{1.63 \times 10^{-2} \text{ mm}}$$

Example 10.8. *A beam of linearly polarized light is changed into circularly polarized light by passing it through a sliced crystal of thickness 0.003* cm. *Calculate the difference in refractive indices of the two rays in the crystal assuming this to be of minimum thickness that will produce the effect. The wavelength of light used in 6×10^{-7}* m. [*IAS, 1989*]

If the plane polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quarter wave plate, the emergent light is circularly polarized.

Here, $$t = 0.003 \text{ cm} = 3 \times 10^{-5} \text{ m}$$

$$\lambda = 6 \times 10^{-7} \text{ m}$$

$$(\mu_E - \mu_o)\,t = \frac{\lambda}{4}$$

$$(\mu_E - \mu_o) = \frac{\lambda}{4t}$$

$$= \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-5}}$$

$$= 5 \times 10^{-3}$$

Example 10.9. *For calcite $\mu_o = 1.658$ and $\mu_E = 1.486$ for sodium light. Calculate the minimum thickness of the quarter wave plate for calcite.*

$\lambda = 5893$ Å [*Kanpur, 1990*]

Here $\lambda = 5893 \text{ Å} = 5.893 \times 10^{-7}$ m

$$\mu_E = 1.486$$

$$\mu_o = 1.658$$

$$t = ?$$

$$(\mu_o - \mu_E)\,t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_o - \mu_E)}$$

$$t = \frac{5.893 \times 10^{-7}}{4\,[1.658 - 1.486]}$$

$$t = \mathbf{8.56 \times 10^{-7}\ m}$$

$$t = \mathbf{8.56 \times 10^{-4}\ mm}$$

Example 10.10. *Calculate the thickness of a half wave plate for light of wavelength 5000 Å.*

[$\mu_o = 1.55$ and $\mu_E = 1.45$] [*Lucknow, 1990*]

Here $\mu_0 = 1.55, \quad \mu_E = 1.45$

$$\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}$$

As $\mu_o > \mu_E$, it is a negative crystal

$\therefore$ Path difference $= (\mu_0 - \mu_E)\,t$