# What is the difference between a theorem, a lemma, and a corollary?



9/23/2008

I prepared the following handout for my Discrete Mathematics class (here's a pdf version).

**Definition** — a precise and unambiguous description of the meaning of a mathematical term. It characterizes the meaning of a word by giving all the properties and only those properties that must be true.

**Theorem** — a mathematical statement that is proved using rigorous mathematical reasoning. In a mathematical paper, the term theorem is often reserved for the most important results.

**Lemma** — a minor result whose sole purpose is to help in proving a theorem. It is a stepping stone on the path to proving a theorem. Very occasionally lemmas can take on a life of their own (Zorn's lemma, Urysohn's lemma, Burnside's lemma, Sperner's lemma).

**Corollary** — a result in which the (usually short) proof relies heavily on a given theorem (we often say that "this is a corollary of Theorem A").

**Proposition** — a proved and often interesting result, but generally less important than a theorem.

**Conjecture** — a statement that is unproved, but is believed to be true (Collatz conjecture, Goldbach conjecture, twin prime conjecture).

**Claim** — an assertion that is then proved. It is often used like an informal lemma.

**Axiom/Postulate** — a statement that is assumed to be true without proof. These are the basic building blocks from which all theorems are proved (Euclid's five postulates, Zermelo-Fraenkel axioms, Peano axioms).

**Identity** — a mathematical expression giving the equality of two (often variable) quantities ( trigonometric identities, Euler's identity).

**Paradox** — a statement that can be shown, using a given set of axioms and definitions, to be both true and false. Paradoxes are often used to show the inconsistencies in a flawed theory (Russell's paradox). The term paradox is often used informally to describe a surprising or counterintuitive result that follows from a given set of rules (Banach-Tarski paradox, Alabama paradox, Gabriel's horn).