Hypothesis Testing (Decision Making process)

The statistical method which is used to test the hypothesis about the population parameter on the basis of sample data.

Hypothesis

Assumption about the pop^n parameter which needs to test whether it is true or false, on the basis of sample data.

Example

① The mean score of students in NCIT is 72

② Mean height of Girls is less than mean height of Boys.

(Types)

Different steps in Hypothesis Testing

NULL hypothesis: No Difference hypothesis between population parameter and sample statistic. It is denoted by H_0.

According to Prof. R.A Fisher Null hypothesis is defined as the hypothesis which is tested for possible rejection under the assumption that it is true.

(Used μ_0, to calculate the test statistic)

μ

Eg: ① The mean score of students in NCIT is 72

NULL hypothesis: $H_0: \mu = 72$

② Mean height of girls is less than mean height of boys

$H_0: \mu_1 = \mu_2$

③ The Proportion of pass students in NCIT is greater than 0.6

H_0 & $P = 0.6$

Alternative Hypothesis:

The complementary hypothesis of Null hypothesis. In other words, If null hypothesis is rejected then another hypothesis is accepted, which is known as alternative hypothesis. It is denoted by H_1 or H_A or H_a

H_1 is always states against H_0

Example:

① The mean score obtained by the students in statistics is 72. (no difference)

$H_0 : \mu = 72$

Vs

$H_1 : \mu \neq 72$ (Difference) $\Rightarrow$ Two-tailed H_1

② Mean height of girls is less than mean height of boys

$H_0 : \mu_1 = \mu_2$

Vs

$H_1 : \mu_1 < \mu_2$ (Left-tailed H_1) (one-tailed H_1)

③ The proportion of pass students in NCIT is greater than 0.6

$H_0 : P = 0.6$

Vs

$H_1 : P > 0.6$ (Right-tailed H_1)

(one-tailed H_1)

Alternative hypothesis

Types of H_1

① Two - tailed H_1

② Right - tailed H_1] One - tailed H_1

③ left - tailed H_1]

Selection of Appropriate Statistical tool:

① The sample statistic of the popn parameter

② The sampling distribution of sample statistic.

Now, Test statistic

$$T = \frac{\text{Statistic} - \text{Parameter}}{\text{Standard error}}$$

Short notes

Errors in Hypothesis Testing

While making decision about to regret or accept H_0 in hypothesis testing, we may commit two types of errors..

(i) Type I error

(ii) Type II error

Type I error:

If H_0 is rejected when it is true than the error is called type I error. In other words, the rejection of true H_0 is called type I error.

The prob of type I error is denoted by α and is known as level of significance. It is also called size of Critical Region.

Type II error:

If H_0 is accepted when it is false, then the error is called type II error.

In other words, the acceptance of false H_0, is called type II error.

The probability of making type II error is denoted by β.

Errors in Hypothesis testing presented in table.

Statistical Decision	Actual situation	
	H_0 is true	H_0 is false
Reject H_0	Type I error (α)	correct decision ($1-\beta$) ← power of test
Accept H_0	Correct Decision ($1-\alpha$) ⇓ confidence level	Type II error (β)

Note:

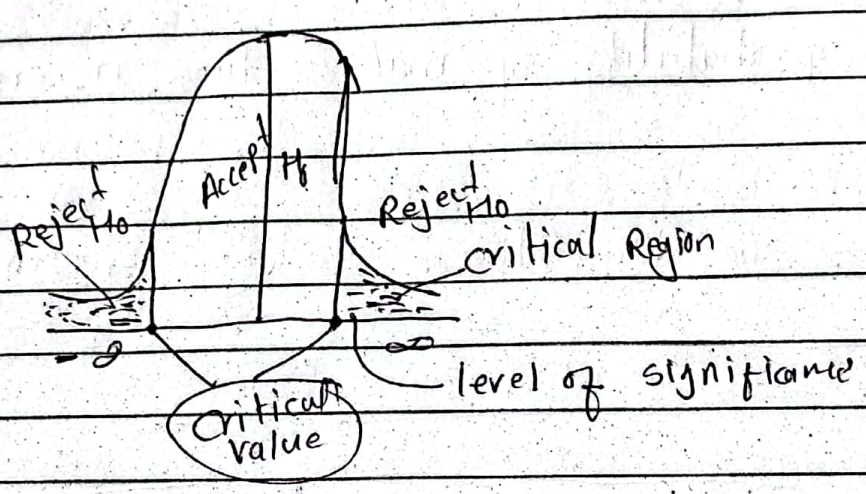
The impact of type II error is more dangerous or harmful than that of type I error.

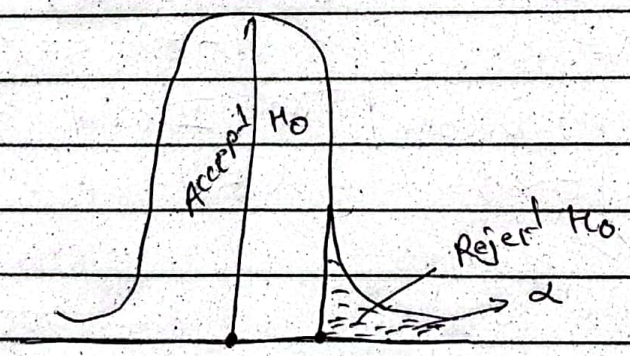
∴ For fixed α, we try to minimize the β. Since α and β cannot be minimized simultaneously.

Critical Region And Acceptance Region

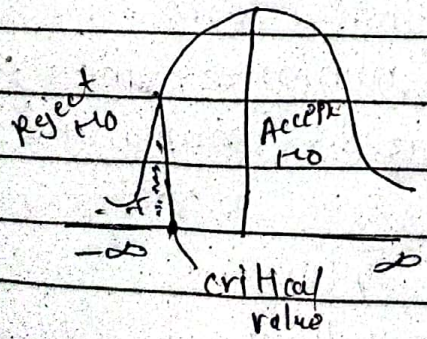
Sample space: The collection of all the possible values of test statistic

Critical Region : The region of Sample space where true H_0 is rejected is called critical region (Rejection Region). And the complementary region of critical region is Acceptance region





Right-sided Critical Region



② Left-sided Critical Region

One tailed test and two-tailed test

A hypothesis test is said to be one-tailed test if one one-tailed H_1 is used

Similarly:

A hypothesis test is said to be two-tailed test if two tailed H_1 is used.

Example:

$H_0 : \mu = \mu_0$, where μ_0 is assumed value of μ

Vs

$H_1 : \mu \neq \mu_0$ (Two-tailed test)

$H_1 : \mu > \mu_0$ (Right-tailed test (one-tailed test)

$H_1 : \mu < \mu_0$ (left-tailed test) one-tailed test.

General Procedures for hypothesis testing.

Step 1: Setting hypothesis: — Avoid

$H_0 : \mu = \mu_0$ ($\because \mu \geq \mu_0$ or $\mu \leq \mu_0$)

Vs

$H_1 : \mu \neq \mu_0$ (∴ two-tailed test)

$H_1 : \mu > \mu_0$ (one-tailed test)

$H_1 : \mu < \mu_0$ (one-tailed test)

Step 2: Identify the sample statistic and it's sampling distribution

Step 3: Test statistic

Under H_0, The test statistic is

$$T = \frac{\text{Statistic} - \text{Parameter (Assumed)}}{\text{Standard Error}}$$

$$= \frac{t - E(t)}{SE(t)}$$

Step 4: level of significance
Fixed $\alpha = 0.05$ otherwise given

Step 5: Obtain critical value (tabulated value)
Obtained for fixed α.

Step 6: Decision
① If calculated value is less than tabulated value, then accept H_0
② If calculated value is more than or equals to tabulated value, then we reject H_0.