Paver series

→ A series of the form $\sum_{n=0}^{\infty} a_n (n-c)^n = a_0 + a_1(n-c) + a_2(n-c)^2 + a_3(n-c)^3 + \cdots$ — equn ①

is called paver series where $a_0, a_1, a_2, a_3, \ldots$ are constant & c be tha centre of the series

In particular case of $c=0$, then equn ① becomes

$$\sum_{n=0}^{\infty} a_n n^n = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + \cdots$$

① $e^n = \sum_{n=0}^{\infty} \frac{n^n}{n!} = 1 + n + \frac{n^2}{2!} + \frac{n^3}{3!} + \cdots$

② $e^{-n} = \sum_{n=0}^{\infty} \frac{(-n)^n}{n!} = 1 - n + \frac{n^2}{2!} - \frac{n^3}{3!} + \cdots$

③ $\cos n = \sum_{n=0}^{\infty} (-1)^n \frac{n^{2n}}{(2n)!} = 1 - \frac{n^2}{2!} + \frac{n^4}{4!} - \frac{n^6}{6!} + \cdots$

odd func

④ $\sin n = \sum_{n=0}^{\infty} (-1)^n \frac{n^{2n+1}}{(2n+1)!} = n - \frac{n^3}{3!} + \frac{n^5}{5!} - \cdots$

⑤ $\frac{1}{1-x} = 1+x+x^2+x^3+ \cdots\cdots$

⑥ $\frac{1}{1+x} = 1-x+x^2-x^3+ \cdots\cdots$

⑦ $\frac{1}{(1-x)^2} = 1+2x+3x^2+4x^3+ \cdots\cdots$

⑧ $\frac{1}{(1+x)^2} = 1-2x+3x^2-4x^3+ \cdots\cdots$

Q) Solve: $y'-y=0$ (by power series method)

⇒ Soln,

here given diff equn is:

$$y'-y=0 \quad \text{----} ①$$

let,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots \quad ②$$

be the soln of ①

constant ko value nikalne.

Diff ② w.r.t 'x',

$$y' = 0 + a_1 + 2a_2x + 3a_3x^2 + \cdots$$
$$= a_1 + 2a_2x + 3a_3x^2 + \cdots\cdots$$

Substitude value of y & y' in ①,

$y'-y=0$

$\frac{dy}{dx} = y$

$\frac{dy}{y} = dx$

$\int \frac{1}{y}\, dy = \int dx$

~~log y = x + c~~

$\log y = x + \log c$

$\log\left(\frac{y}{c}\right) = x$

Antilog,

$\frac{y}{c} = e^x$

$y = ce^x$

$y = f(x)$

$$(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots) - (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots) = 0$$

$$(a_1 - a_0) + (2a_2 - a_1)x + (3a_3 - a_2)x^2 + (4a_4 - a_3)x^3 + \cdots = 0$$

$$(a_1 - a_0) + (2a_2 - a_1)x + (3a_3 - a_2)x^2 + (4a_4 - a_3)x^2 + \cdots = 0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3 + \cdots$$

Equating coeff. of like terms,

$$a_1 - a_0 = 0 \qquad 2a_2 - a_1 = 0 \qquad 3a_3 - a_2 = 0$$

$$a_1 = a_0 \qquad 2a_2 = a_1 \qquad 3a_3 = a_2$$

$$2a_2 = a_0 \qquad 3a_3 = \frac{a_0}{2}$$

$$a_2 = \frac{a_0}{2} \qquad a_3 = \frac{a_0}{6}$$

$$a_3 = \frac{a_0}{3\cdot 2}$$

$$4a_4 - a_3 = 0$$

$$4a_4 = a_3$$

$$4a_4 = \frac{a_0}{6}$$

$$a_4 = \frac{a_0}{24} = \frac{a_0}{4\cdot 3\cdot 2}$$

Then, from (2),

$$y = a_0 + a_0 x + \frac{a_0}{2}x^2 + \frac{a_0}{3\cdot 2}x^3 + \frac{a_0}{4\cdot 3\cdot 2}x^4 + \cdots$$

$$= a_0\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$$

$y = a_0 e^{n}$.

6) $y'' + 9y = 0$ ↳ constant coeff

⇒ Soln,

here,

$y'' + 9y = 0$ —①

A differential equn of the form
$ay'' + by' + cy = R(n)$,
of the if of $R(n) = 0$,
it is homogenous 2^{nd} order differential equn and a, b, c are constant.

Let,

$y = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + a_4 n^4 + a_5 n^5 + \cdots$ —②

~~let~~ ② be the soln of $y'' + 9y = 0$

~~$y' =$~~ ①

$m^2 + 9 = 0$

$m = 0 \pm i3$

Now,

$y' = a_1 + 2a_2 n + 3a_3 n^2 + 4a_4 n^3 + 5a_5 n^4 + \cdots$

$y = (C_1 \cos 3n + C_2 \sin 3n) e^{0n}$

$y = C_1 \cos 3n + C_2 \sin 3n$

$y'' = 2a_2 + 6a_3 n + 12a_4 n^2 + 20a_5 n^3 + \cdots$

Substitute value of y, y'' in equn ①,

$(2a_2 + 6a_3 n + 12a_4 n^2 + 20a_5 n^3 + \cdots) + 9(a_0 + a_1 n + a_2 n^2 + \cdots) = 0$

$(2a_2 + 9a_0) + (6a_3 + 9a_1) n + (12a_4 + 9a_2) n^2 + (20a_5 + 9a_3) n^3 + \cdots = 0$

x power even $\quad$ x power odd $\quad$ even.

$(2a_2+9a_0)+(6a_3+9a_1)x+(12a_4+9a_2)x^2+(20a_5+9a_3)x^3+$
$\cdots = 0+0x+0x^2+0x^3+\cdots$

Equating coeff of like terms,

$2a_2+9a_0 \cancel{=} 0$ $\quad$ $6a_3+9a_1 \cancel{=} 0$ $\quad$ $12a_4+9a_2 \cancel{=} 0$

$2a_2 \cancel{=} -9a_0$ $\quad$ $a_3 = \cancel{\frac{-9a_1}{6}}$ $\quad$ $a_4 \cancel{=} \frac{-9a_2}{12}$

$a_2 = \frac{-9a_0}{2}$

even	odd
$2a_2+9a_0=0$	$6a_3+9a_1=0$
$a_2=\frac{-9a_0}{2}$	$a_3=\frac{-9a_1}{6}$
$12a_4+9a_2=0$	$20a_5+9a_3=0$
$a_4=\frac{-9a_2}{12}$	$a_5=\frac{-9a_3}{20}$
$=\frac{-9}{12}\cdot\frac{-9a_0}{2}$	$=\frac{-9}{20}\cdot\frac{-9a_1}{6}$
$=\frac{81}{24}a_0$	$=\frac{81}{120}a_1$

Na katne ok

Now, put in ②,

$$y=a_0+a_1x+\frac{-9a_0}{2}x^2+\frac{-9a_1}{6}x^3+\frac{81}{24}a_0x^4+$$
$$\frac{81}{120}a_1x^5+\cdots$$

$$=a_0\left(1-\frac{9}{2}x^2+\frac{81}{24}x^4+\cdots\right)+a_1\left(x-\frac{9}{6}x^3+\frac{81}{120}x^5+\cdots\right)$$

CamScanner

$$y = a_0\left(1 - \frac{9}{2}x^2 + \frac{81}{24}x^4 - - - -\right) + a_1\left(x - \frac{9}{6}x^3 + \frac{81}{120}x^5 - - -\right)$$

$$= a_0\left(1 - \frac{(3x)^2}{2} + \frac{(3x)^4}{24} - - -\right) + a_1\left(x - \frac{9}{6}x^3 + \frac{81}{120}x^5 - - -\right)$$

$$= a_0\left(1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - - - -\right) + \frac{a_1}{3}\left(x - \frac{9\times 3x^3}{6} + \frac{81\times 3}{120}x^5 \cdots\right)$$

$$= a_0\left(1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} \cdots\right) + \frac{a_1}{3}\left(x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} \cdots\right)$$

$$= a_0 \cos(3x) + \frac{a_1}{3}\sin(3x)$$

~~y=~~

Q) $y'' - 9y = 0$

$\Rightarrow$ Soln,

~~let~~ $y'' - 9y = 0$ —(1)

let,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots \quad (2)$$

~~Now~~, be the soln. of (1)

Now,

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots$$

put y'' & y in (1),

$$(2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots) - 9(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots) = 0$$

$$(2a_2+6a_3x+12a_4x^2+20a_5x^3+\cdots)-(9a_0+9a_1x+9a_2x^2+9a_3x^3+9a_4x^4+9a_5x^5\cdots)=0+0x+0x^2+0x^3$$

Equating coeff of like terms,

$$2a_2-9a_0=0$$
$$2a_2=9a_0$$
$$a_2=\frac{9}{2}a_0$$

$$6a_3\cancel{x}-9a_1\cancel{x}=0$$
$$a_3=\frac{9}{6}a_1$$

$$12a_4-9a_2=0$$
$$a_4=\frac{9a_2}{12}$$
$$=\frac{9}{12}\times\frac{9}{2}a_0$$
$$=\frac{81}{24}a_0$$

$$20a_5-9a_3=0$$
$$a_5=\frac{9}{20}a_3$$
$$=\frac{9}{20}\times\frac{9}{6}a_1$$
$$=\frac{81}{120}a_1$$

put values in ②,

$$y=a_0+a_1x+\frac{9}{2}a_0x^2+\frac{9}{6}a_1x^3+\frac{81}{24}a_0x^4+\frac{81}{120}a_1x^5+\cdots$$

$$=a_0\left(1+\frac{9}{2}x^2+\frac{81}{24}x^4+\cdots\right)+a_1\left(x+\frac{9}{6}x^3+\frac{81}{120}x^5\cdots\right.$$

$$y = a_0\left(1 + \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} \cdots\right) + \frac{a_1}{3}\left(x + \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} \cdots\right)$$

Multiple integral (Double & Triple Integral)

Triple Integral:-

Ex - 1.4

1)

iv) $\int_0^2 \int_1^3 \int_1^2 xy^2z \, dx\,dy\,dz$

$\Rightarrow$ soln,

$$\int_0^2 \int_1^3 \int_1^2 xy^2z \, dx\,dy\,dz$$

$$= \int_0^2 \int_1^3 y^2 z \left[\frac{x^2}{2}\right]_1^2 dy\,dz$$

$$= \int_0^2 \int_1^3 y^2 z \, \frac{4-1}{2} \, dy\,dz$$

$$= \frac{3}{2} \int_0^2 \left\{ z \left[\frac{y^3}{3}\right]_1^3 \right\} dz$$

$$= \frac{3}{2} \int_0^2 z \, \frac{26}{3} \, dz$$

$$= 13 \left[\frac{z^2}{2}\right]_0^2$$

$= 13\frac{4}{2}$

$= \cancel{39}.$ 26-11

2 v) $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 \, dz \, dy \, dx$

$x+y+z=a$

$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$

$= \cancel{\int_0^a \int_0^{a-x} \left[\frac{x^3}{3}\right]_0^{a-x-y}} dy\, dz$

$= \int_0^a \int_0^{a-x} x^2 [z]_0^{a-x-y} \, dy \, dx$

$x^2 \left[(a-x)y - \frac{y^2}{2}\right]_0^{a-x}$

$= \int_0^a \int_0^{a-x} x^2 (a-x-y) \, dy \, dx$

$= \int_0^a \int_0^{a-x} (x^2 a - x^3 - x^2 y) \, dy \, dx$

$= \int_0^a \left\{ x^2 a [y]_0^{a-x} - x^3 [y]_0^{a-x} - x^2 \left[\frac{y^2}{2}\right]_0^{a-x} \right\} dx$

$= \int_0^a x^2 a(a-x) - x^3(a-x) - \frac{x^2}{2}(a-x)^2 \, dx$

$= \int_0^a xa^2 - x^3 a - x^3 a + x^4 - \frac{x^2(a^2 - 2ax + x^2)}{2} \, dx$

$= \int_0^a xa^2 - 2x^3 a + x^4 - \frac{x^2 a^2 + 2ax^3 - x^4}{2} \, dx$

$= \frac{1}{2}\int_0^a 2a^2 x - 4x^3 a + 2x^4 - x^2 a + 2ax^3 - x^4 \, dx$

$= \frac{1}{2}\int_0^a 2a^2 x - 2x^3 a + x^4 - x^2 a \, dx$

$$= \frac{1}{2}\left[2a^2\left[\frac{x^2}{2}\right]_0^a - 2a\left[\frac{x^4}{4}\right]_0^a + \left[\frac{x^5}{5}\right]_0^a - a\left[\frac{x^3}{3}\right]_0^a\right]$$

$$= \frac{1}{2}\left(a^4 - \frac{a^5}{2} + \frac{a^5}{5} - \frac{a^4}{3}\right)$$

$$= \frac{1}{2}a^4 - \frac{a^5}{4} + \frac{a^5}{10} - \frac{a^4}{6}$$

$$= \frac{\cancel{4}}{60}\cancel{a} \quad \frac{1}{3}a^4 - \frac{3}{20}a^5$$

$$① \int (x+a)^{1/2}\,dx = \frac{(x+a)^{1/2+1}}{1/2+1}$$

$$② \int (a-x)^{1/2}\,dx = \frac{(a-x)^{3/2}}{-3/2}$$

$$③ \int 2x(x^2+a^2)^{1/2}\,dx = \frac{(x^2+a^2)^{1/2+1}}{1/2+1}$$

Q) Find the volume of hemisphere $x^2+y^2+z^2=a^2$ by triple integral.

=) soln,

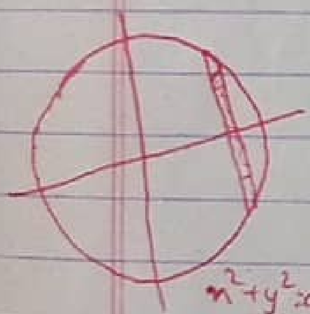
Here, the given sphere : $x^2+y^2+z^2=a^2$

$$x^2+y^2+z^2=a^2$$
$$z=\sqrt{a^2-x^2-y^2}$$

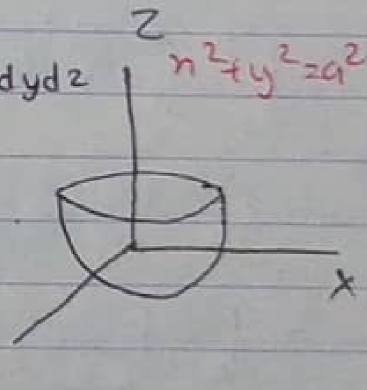
By defn,

$$V=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{0}^{\sqrt{a^2-x^2-y^2}} dz\,\underbrace{dy\,dx}_{\to\ xy\ \text{plane},\ z=0}$$

xy-plane ma circle



$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\left[z\right]_0^{\sqrt{a^2-x^2-y^2}} dy\,dz$$



$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\sqrt{a^2-x^2-y^2}\,dy\,dz$$

$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}(a^2-x^2-y^2)^{1/2}\,dy\,dz$$

$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}$$

Put $x = r\cos\theta$, $y = r\sin\theta$,

$x^2 + y^2 = r^2$

$dy\,dx = dx\,dy = r\,dr\,d\theta$

when $r = 0$ to $r = a$ &

$\theta = 0$ to $\theta = 2\pi$

Now,

$$\int_0^{2\pi}\int_0^{a} r\sqrt{a^2 - (r\cos\theta)^2 - (r\sin\theta)^2}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{a} r\sqrt{a^2 - r^2(\cos^2\theta + \sin^2\theta)}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{a} r\sqrt{a^2 - r^2}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{a} r(a^2 - r^2)^{1/2}\,dr\,d\theta$$

$$= \int_0^{2\pi}\frac{1}{-2}\int_0^{a} -2r(a^2 - r^2)^{1/2}\,dr\,d\theta$$

$$= \frac{-1}{2}\int_0^{2\pi}\left[\frac{(a^2 - r^2)^{3/2}}{3/2}\right]_0^{a} d\theta$$

$$= \frac{-1}{2}\int_0^{2\pi}\left[\frac{(a^2)^{3/2}}{3/2}\right] d\theta$$

$$= \frac{-1}{3}\int_0^{2\pi} -a^3\,d\theta$$

$$= \frac{+a^3}{3}\left[\theta\right]_0^{2\pi} = \frac{+2\pi a^3}{3}$$ //

$$\int e^{a+x}\,dx = e^{a+x}$$

$$\int e^{a+bx}\,dx = \frac{e^{a+bx}}{b}$$

2) ii) $\displaystyle\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$

$\Rightarrow$ Soln,

$$\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x}\left[e^{x+y+z}\right]_0^{x+y} dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x}\left(e^{x+y+x+y} - e^{x+y}\right) dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x}\left(e^{2x+2y} - e^{x+y}\right) dy\,dx$$

$$= \int_0^{\log 2}\left(\int_0^{x} e^{2x+2y}\,dy - \int_0^{x} e^{x+y}\,dy\right) dx$$

$$= \int_0^{\log 2}\left[\frac{e^{2x+2y}}{2}\right]_0^{x} - \left[e^{x+y}\right]_0^{x} dx$$

$$= \int_0^{\log 2}\left(\frac{e^{2x+2x} - e^{2x}}{2} - e^{2x} + e^{x}\right) dx$$

$$= \int_0^{\log 2}\left(\frac{e^{4x} - e^{2x}}{2} - e^{2x} + e^{x}\right) dx$$

$$= \frac{1}{2}\left\{\int_0^{\log 2} e^{4x}\,dx - \int_0^{\log 2} e^{2x}\,dx\right\} - \left[\frac{x^2}{2}\right]_0^{\log 2}$$

$$= \frac{1}{2}\left\{\left[\frac{e^{4x}}{4}\right]_0^{\log 2} - \left[\frac{e^{2x}}{2}\right]_0^{\log 2}\right\} - \frac{(\log 2)^2}{2}$$

$$= \frac{1}{2}\left\{\frac{e^{4\log 2} - e^{0}}{4} - \frac{e^{2\log 2} - e^{0}}{2}\right\} - \frac{(\log 2)^2}{2}$$

$$= \frac{1}{2}\left\{ \frac{e^{4\log 2} - 1 - 2e^{2\log 2} - 1}{4} - (\log 2)^2 \right\}$$

$$= \frac{1}{2}\left\{ \frac{16 - 1 - 2\times 4 - 1}{4} - \right.$$

$$= \frac{1}{2}\left\{ \frac{3}{2} - \right.$$

$$= \int_0^{\log 2} \left(\frac{e^{4x} - e^{2x}}{2} - e^{2x} + e^{x} \right) dx$$

$$= \int_0^{\log 2} \frac{e^{4x} - 3e^{2x} + 2e^{x}}{2}\, dx$$

$$= \frac{1}{2}\int_0^{\log 2} \left(e^{4x} - 3e^{2x} + 2e^{x} \right) dx$$

$$= \frac{1}{2}\left[\frac{e^{4x}}{4}\right]_0^{\log 2} - \frac{3}{2}\left[\frac{e^{2x}}{2}\right]_0^{\log 2} + \left[e^{x}\right]_0^{\log 2}$$

$$= \frac{e^{4\log 2} - 1}{8} - \frac{3(e^{2\log 2} - 1)}{4} + \frac{e^{\log 2} - e^{0}}{1}$$

$$= \frac{16-1}{8} - \frac{3(4-1)}{4} + 2 - 1$$

$$= \frac{15}{8} - \frac{9}{4} + 1$$

$$= 5/8 \;//$$

Dirichlet's theorem. Imp

\# Integration by using $\boxed{\text{Dirichlet's theorem}}$ of V is a region bounded by $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x+y+z \leq 1$ as shown in the figure. Then,

$$\iiint_V x^{l-1} y^{m-1} z^{n-1}\, dx\, dy\, dz = \frac{\Gamma l\, \Gamma m\, \Gamma n}{\Gamma l+m+n+1}$$

Ex-1.4

6) Evaluate $\iiint_V x^2\, dx\, dy\, dz$ over the region V bounded by the places $x=0, y=0, z=0$ and $x+y+z=a$.

=> Soln,

Here, the given plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

put $\frac{x}{a}=u$; $\frac{y}{b}=v$

$x=au$; $\frac{dx}{du}=a$ $\qquad$ $y=bv$; ~~$dy=bdu$~~

$dx=a\,du$ $\qquad$ $dy=b\,dv$

~~$\frac{z}{c}=w$~~ ;

put $\frac{z}{c} = w$

$z = wc$

$\frac{dz}{dw} = c$

$dz = c\,dw$

Then,

$$\iiint_V x^2 \, dx\,dy\,dz = \iiint_V a^2 u^2 \, a\,du \, a\,dv \, a\,dw$$

$$= a^5 \iiint_V u^2 \, du\,dv\,dw$$ & plane $u+v+w=1$

$$= a^5 \iiint u^{3-1} v^{1-1} w^{1-1} \, du\,dv\,dw$$

$$= a^5 \left(\frac{\Gamma 3 \, \Gamma 1 \, \Gamma 1}{\Gamma 3+1+1+1} \right)$$

$$= a^5 \left(\frac{2 \times 1 \times 1}{\Gamma 6} \right)$$

$$= a^5 \frac{2}{5!}$$

$$= \frac{2a^5}{5!}$$

$$= \frac{a^5}{60} \text{ .//}$$

5 4 3 2 1

5×4×3

Eg:- 44, 45, 47

Power series

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$f(x) = \frac{x^2+4}{x-1}$$

- The singular point is $x=1$

$(x-1)f(x) = (x^2+4)$ is analytic (continious or defined)

① Regular (ordinary) and singular points of the differential equn. Consider the diff equn.

$$a(x)y'' + b(x)y' + c(x)y = 0$$

or, $\cancel{a(x)}y'' + \frac{b(x)}{a(x)}y' + c(x)y = 0$

or, $y'' + P(x)y' + q(x)y = 0$; where,

$$P(x) = \frac{b(x)}{a(x)} \quad \& \quad q(x) = \frac{c(x)}{a(x)}$$

If $P(x)$ & $q(x)$ are not analytic at $x = x_0$, then the point $x = x_0$ is called regular singular point if the function

$(x-x_0)p(x)$ & $(x-x_0)^2 q(x)$ are analytic.

but $(x-x_0)p(x)$ & $(x-x_0)^2 q(x)$ are not analytic, the point $x = x_0$ is said to be irregular singular point.

① $x^2 y'' + xy' + 3y = 0$

$y'' + \frac{1}{x} y' + \frac{3}{x^2} y = 0$, the point $x = 0$ is regular singular point.

Q) $3xy'' + 2y' + y = 0$ (Frobenius method)

→ soln,

Given diff eqn is

$3xy'' + 2y' + y = 0$ — ①

$y'' + \frac{2}{3x} y' + \frac{1}{3x} y = 0$ — ②

Comparing with

$$y'' + P(x)y' + q(x)y = 0$$

we get,

$$P(x) = \frac{2}{3x} \quad ; \quad q(x) = \frac{1}{3x}$$

The regular singular point is $x=0$

then,

By Frobenius method,

$$y = \left(\sum_{n=0}^{\infty} a_n x^n\right) x^r$$

$$= x^r \sum_{n=0}^{\infty} a_n x^n$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r} \quad \text{——— (2)}$$

$$y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1}$$

$$= \sum_{n=0}^{\infty} (n+r) a_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r-2}$$

Substitute values of $y, y'' \& y''$ in (1)

$$\Rightarrow \sum_{n=0}^{\infty} \left[3\cancel{a}(n+r)(n+r-1) a_n x^{n+r-1} + 2(n+r) a_n x^{n+r-1} + a_n x^{n+r} \right.$$

$$\Rightarrow \sum_{n=0}^{\infty} \underbrace{\left[3(n+r)(n+r-1) + 2(n+r) \right] a_n x^{n+r-1}}_{1^{st}} + \underbrace{a_n x^{n+r}}_{2^{nd}} = 0 \quad \text{—— } \cancel{(4)}\ (3)$$

when $n=0$, Equating the coefficient of lower degree ~~power~~ i.e ~~of~~ x^{r-1}, ← coefficient of lowest degree compare garne.

$$(3r(r-1) + 2r) a_0 = 0$$
$$(3r^2 - 3r + 2r) a_0 = 0$$
$$(3r^2 - r) a_0 = 0 \quad ; a_0 \neq 0$$
$$\cancel{3r^2 a_0 - r a_0 = 0} \quad ; \quad \cancel{a_0 \neq 0}$$

$$3r^2 - r = 0$$
$$r = 0, \frac{1}{3}$$

when $n=1$, equating the coefficient of lower degree ~~of~~ x^r in 1^{st} term & $n=0$ in 2^{nd} terms.

$$[3(1+r)(r) + 2(1+r)] a_1 + a_0 = 0$$
$$[3r(1+r) + 2(1+r)] a_1 + a_0$$
$$(3r + 3r^2 + 2 + 2r) a_1 + a_0 = 0$$
$$(3r^2 + 5r + 2) a_1 \cancel{+a_0} + a_0 = 0$$

~~$a_1 \neq 0$ so,~~

$$(3r^2+5r+2=0)a_1 + a_0 = 0$$

$$(3r+2)(r+1)\,a_1 = -a_0$$

$$a_1 = \frac{-a_0}{(3r+2)(r+1)}$$

$n+1$

x^{n+r-1}

$= x^{n+r}$

Equating the coefficient of general term x^{n+r} for taking $n=n+1$ in 1^{st} term & $n=n$ in 2^{nd} term.

$$[3(n+r+1)(n+r)+2(n+r+1)]a_{n+1} + a_n = 0$$

$$[(n+r+1)(3n+3r+2)]a_{n+1} + a_n = 0$$

$$(n+r+1)(3n+3r+2)\,a_{n+1} + a_n = 0$$

$$\therefore a_{n+1} = \frac{-a_n}{(n+r+1)(3n+3r+2)}$$

general term

which is a general term.

Now, taking $n=0$,

$$a_1 = \frac{-a_0}{(3r+2)(r+1)}$$

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$n = 1,$

$$a_2 = \frac{-a_1}{(r+2)(3r+5)}$$

$n = 2,$

$$a_3 = \frac{-a_2}{(r+3)(3r+8)}, \ldots\ldots$$

Note :-

$$m^2 = 25$$
$$m = \pm 5$$
$$y = c_1 e^{5x} + c_2 e^{-5x}$$
$$= Ae^{5x} + Be^{-5x}$$
$$\boxed{y = A\, y_1(x) + B\, y_2(x)}$$

For $r = 0,$

$$a_1 = \frac{-a_0}{2(\cancel{-3})} = \frac{-a_0}{\cancel{6}2}$$

$$\cancel{a_0/6} \nearrow \frac{a_0}{20}$$

$$a_2 = \frac{-a_1}{2(5)} = \frac{-a_1}{10} = \frac{-\cancel{a}}{\cancel{10}} \times \frac{-a_0}{\cancel{6}} = \frac{a_0}{60}$$

$$a_3 = \frac{-a_2}{3(8)} = \frac{-a_2}{24} = \frac{-a_0}{480}$$