Matrix

Boolean matrix operation.

The matrix whose entry is either 0 or 1 is called Boolean matrix

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

a) Join ($\vee$)

let $A = [a_{ij}]$

$B = [b_{ij}]$ be the two matrix then the join of A and B is 0 to 1 matrix where the entry $(i,j)^{th} = a_{ij} \vee b_{ij}$

example:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{2\times 3}$$

$$A \cup B = \begin{bmatrix} 0\vee 1 & 1\vee 1 & 0\vee 0 \\ 0\vee 0 & 0\vee 1 & 1\vee 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

b) Meet ($\wedge$)

let $A = [a_{ij}]$

$B = [b_{ij}]$ be the two matrix then the meet of A and B is 0 to 1 matrix where the entry $(i,j)^{th} = a_{ij} \wedge b_{ij}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{2\times 3}$$

$$A \cap B = \begin{bmatrix} 0\wedge 1 & 1\wedge 1 & 0\wedge 0 \\ 0\wedge 0 & 0\wedge 1 & 1\wedge 1 \end{bmatrix}$$

$$A \cap B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Boolean product of two matrix ($\odot$)

ex: $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$A \odot B = \begin{bmatrix} (0\wedge 1) \vee (1\wedge 0) \vee (0 \wedge 1) & (0\wedge 1)\vee(1\wedge 1)\vee(0\wedge 1) \end{bmatrix}$$

exa:

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}_{3\times 2}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{2\times 3}$

$$A \odot B = \begin{bmatrix} (1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(0\wedge 1) \\ (0\wedge 1)\vee(1\wedge 0) & (0\wedge 1)\vee(1\wedge 1) & (0\wedge 0)\vee(1\wedge 1) \\ (1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(0\wedge 1) \end{bmatrix}$$

$$A \odot B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times 3}$$

V.V.I

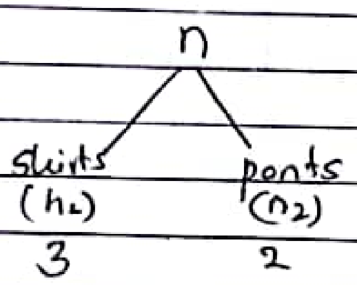
Basic of counting:

Suppose a procedure can be broken down into Sequence of two task If there are n_1 ways of doing first task and n_2 ways of doing Second task then there are $n_1 \times n_2$ ways to do the procedure.

Example:

Suppose You have 3 shirts and 4 pair of pants how many different outfit can you create by Selecting 1 shirt and 1 pair of pant.

shirts (3)	pants (2)
white (W)	Black (B)
Red (R)	Bb (Blue
Black (B)	



total outputs $= n_1 \times n_2$

$= 3 \times 2$.

An extended version of product rule states that if a procedure is carried out by performing the task $t_1, t_2, \ldots t_n$ in a Sequence. if each task t_i can be done in n_i ways then there are $n_1 \times n_2 \times n_3 \ldots n_n$ ways to perform the procedure

Suppose you want to create a password using a combination of 4 uppercase letters and 2 digits How many different passwords can you create?
solution.

The total length of password = 6

total digit
0-9
=> 10

total Alphabets
= 26

$26 \times 26 \times 26 \times 26 \times 10 \times 10$

(b) repetition of character and digit not allowed

$26 \times 25 \times 24 \times 23 \times 10 \times 9$
=>

(Q2) How many strings of 8 uppercase english letters are there

a) If letters can be repeated
b) If letters cannot be repeated
c) that starts with X and letters can be repeated
d) that starts with X and letters cannot be repeated.
e) that starts with 'N' and letter can be repeated
f) that starts with 'BO' and letter can be repeated
h) that starts or ends with 'BO' if letters can be repeated
i) that starts or ends with 'BO' if letter cannot be repeated

Sum rule:

If a task can be done ~~in~~ either in n_1 way or n_2 ways where none of the set n_1 way is same as any of set n_2 ways then there are $n_1 + n_2$ ways to do the task

example:

Suppose you want to chose a tshirt to wear from a selection of 5 different tshirts additionally you want choose a pair of pants from a selection of 3 different pants. How many different outfit choices do you have?

Q How many bit strings of either start with 1 bit or ends with two '00' bits

Solution

- Bit strings that starts with '1'

$|A| = 1*2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7.$

- Bit strings that ends with '00'

$|B| = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 \times 1$

$= 2^6$

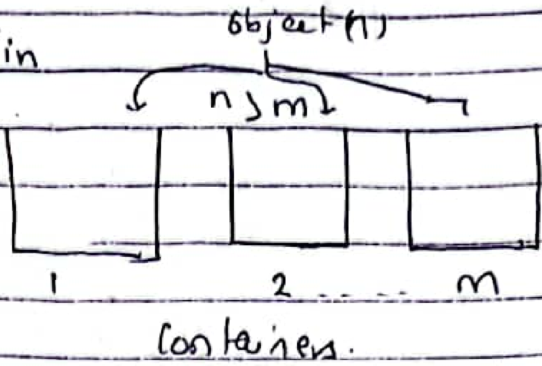
- Bit strings that starts with '1' and end with '00'

$|A \cap B| = 2 \times 2 \times 2 \times 2 \times 2 \times$

$= 2^5$

The pigeonhole principle

- If n objects are placed in m containers such that $(n > m)$ then atleast one of the container must contain more than one object.



* It states that "if there are more pigeons then pigeonhole then one of the pigeonhole must contain more than one pigeon"

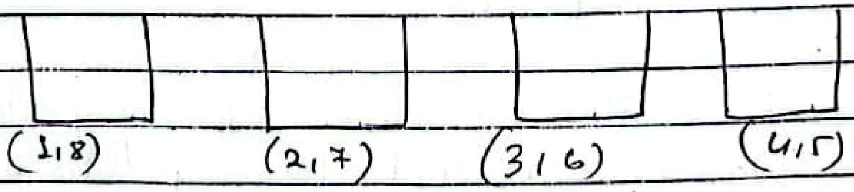
example

(*) In a group of '13' students at least two students must have birthday on same month.

(*) show that if any 5 numbers from 1 to 8 are choosen then two of them will add ~~to~~ to 9

solution

we divide the set of number into following pigeonholes



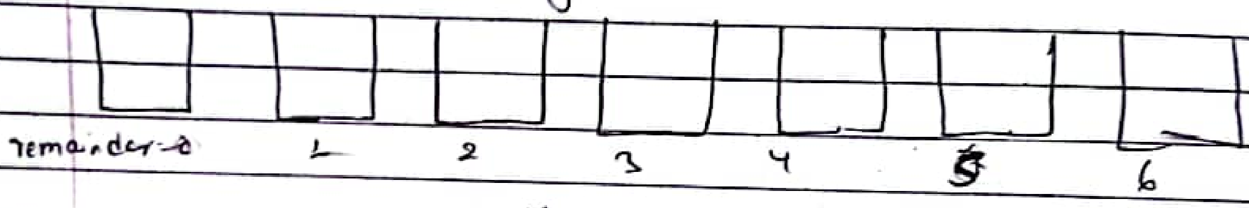
$m = 4$.

Since there are 4 pigeonhole and the object to be placed is 5. by the principle of pigeonhole at least two of the numbers must fall into same pigeon hole

* Show that if 8 positive integers are chosen two of them will have same remainder when divided by Seven.

Solution

we divide the set of positive integers into the following pigeonhole according to the remainder when divided by Seven



$m = 7$.

Since 8 positive integers were choosen and we have Seven pigeonhole then by principle of pigeonhole atleast two numbers must fall into Same ~~pigeonhole~~ pigeonhole i.e both number must have Same remainder when divided by 7.

Extended pigeonhole principle

If N objects are placed in k boxes then there is atleast one box containing atleast $\left\lceil \frac{N}{k} \right\rceil$ objects

example

* Find the minimum number of student in a class to be sure that Four of them are born in Same month.

Solution

Since there are 12 month in year, we can consider each month has pigeonhole i.e $k = 12$

we want to find the minimum of number of SOP student i.e. N such that atleast 4 of them will fall into same pigeon hole(month).

Then by generalize pigeonhole principle

$$\left\lceil \frac{N}{k} \right\rceil \geq 4$$

$$\left\lceil \frac{N}{12} \right\rceil \geq 4$$

The minimum value of N is,

$N = 12 \times 3 + 1$

$N = 37$.

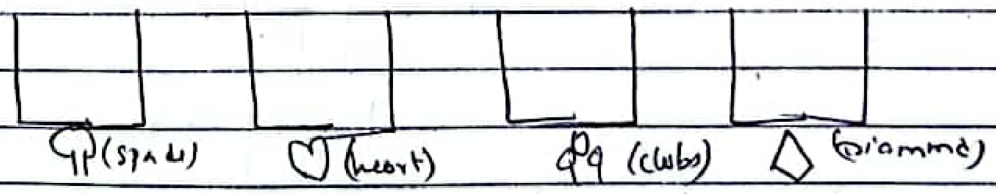
* How many carts must be selected from a standard deck of 52 cards to gurantee that:

a) atleast three cards of same suit are chosen

b) atleast three cards of same value are selected.

Solution

(a)



A standard deck of card has 4 suit which is divided into following pigeonne

Here, $k = 4$

$N =$

we have to find minimum number of cards

(N) such that atleast three of them will fall into same pigeonhole (suit). Then by generalized pigeonhole principle

$$\left\lceil \frac{N}{k} \right\rceil \geq 3$$

$$\left\lceil \frac{N}{4} \right\rceil \geq 3.$$

$$N = 4 \times 2 + 1$$
$$= 9.$$

b)

There are 13 value in a deck of cards, 2, 3, 4, 5, 6, 7, 8, 9, 10, A, J, K, Q

let us consider each value has pigenhole ie $K = 13$.

now we have to find minimum number of cards (N) such that at least 3 of them fall into same pigeonhole

Then by generalized pigeonhole principle

$$\left\lceil \frac{N}{k} \right\rceil \geq 3$$

$$\left\lceil \frac{N}{13} \right\rceil \geq 3$$

The minimum value of N is.

$$N = 13 \times 2 + 1$$
$$= 27.$$

Permutation and combination.

A permutation is a way of arranging a set of object in a specific order. Each permutation is ~~a way of~~ unique arrangement of objects. The order in which the objects are arranged matters (AB is different from BA)

The permutation of n objects taken 'r' at a time is given by

$$P(n,r) = \frac{n!}{(n-r)!}$$

* How many permutation of letter "ABCDEFG" contain.

a) The string BCD
b) The string CFGA
c) The string ABC and DE

Solution.

a) The string BCD has three distinct letters which we can consider as a single letter. There are then 4 distinct letter to be arrange. Therefore the total permutation of letter "ABCDEFG" that contain string BCD = P(5,5)

Combination

A combination is a way of selecting items from a group without considering the order in which they are chosen.

For example

We have three set of letter {A, B, C}

The possible combination of two letter are {AB, AC, BC}

The combination of n object taken r at a time is given by $C(n,r) = \frac{n!}{(n-r)!\,r!}$

* In how many ways can a community of three people be selected from a group of 10 people.

* How many different seven person community can be formed each containing three women from a available set of 20 women and 4 men from a available of 30 men.

Solution

total no of women $= n = 20$

$r = 3$

$C(n,r) = C(20,3)$

total no of men $= n = 30$

$r = 4$

$C(n,r) = C(30,4)$

14(B)

Q 1, 2, 1, 2

(*) How many bit strings of length 10 contains

a) Exactly four ~~ones~~ 1's

b) Atmost four 1's

c) Atleast four 1's

d) equal number of 0's and 1's.

Solution

(a) $n = 10$

$r = 4$

$$C(10,4) = \frac{10!}{(10-4)!\,4!}$$

$$= 210$$

(b) $\boxed{1\text{'s} \leq 4}$

$C(10,0) + C(10,1) + C(10,2) + C(10,3) + C(10,4)$

$\Rightarrow$

(c) (1's ≥ 4)

$C(10,4) + C(10,5) + C(10,6) + C(10,7) + C(10,8) + C(10,9) + C(10,10)$

$=$

(d) 1's $\to 5$

0's $\to 5$

$n = 10$

$r = 5$

$C(10,5) =$

(4) A coin is flip 10 times when each flip can be either head or tail

(i) how many possible outcomes are there in total

(ii) contain exactly two head

(iii) contain almost three tails

(iv) contain same number of head and tails.

Solution

a(i)

$n = 10$

$r = 2$

C