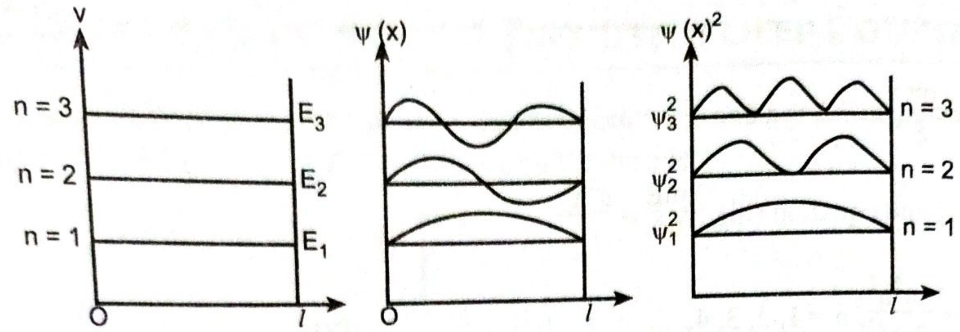

Figure: First three energy states, wave functions and probability densities for a particle in one dimensional box

Solved Example

Example 1: A ball of mass 10 gm has velocity 100 cm/sec. Calculate the wave length associated with it. Why does not this wave nature show up our daily observations. Given, $h = 6.62 \times 10^{-34}$ JS.

Solution:

Here, $m = 10 \text{ gm} = 10 \times 10^{-3}$ kg, $h = 6.62 \times 10^{-34}$ Js

We have,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.62 \times 10^{-34}}{10 \times 10^{-3} \times 1}$$

$$= 6.62 \times 10^{-32} \text{ m}$$

This wavelength is much smaller than the dimensions of the balls therefore in such cases wave-like properties of matter can not be observed in our daily observations.

Example 2: Calculate the wave length associated with an electron subjected to a potential difference of 1.25 KV.

Solution:

We have,

$$\frac{1}{2} mv^2 = eV$$

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

Now,

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1.25 \times 10^3}}$$

$$\lambda = 0.347 \text{Å}$$

Example 3: Show that wave velocity is greater than velocity of light

Solution:

We have, wave velocity, $u = f\lambda$

From Planck's law and Einstein's mass energy relation.

$$hf = mc^2$$

$$f = \frac{mc^2}{h}$$

Substituting f for u

$$u = \frac{mc^2}{h}.\lambda = \frac{mc^2}{h}.\frac{h}{mv} = \frac{c^2}{v}$$

$$c^2 = uv$$

Since particle velocity (v) must be less than velocity of light (c), so wave velocity (u) is greater than velocity of light.

Example 4: Find the energy of the neutron in units of electron - volt whose de - Broglie wave length is 1Å. Given mass of neutron = 1.674×10^{-27} kg, h = 6.62×10^{-34} joule-sec.

Solution:

Here, $\lambda = 1$ Å $= 1\times10^{-10}$ m, $m = 1.674\times10^{-27}$kg, $h = 6.62\times10^{-34}$ Joule sec.

We have, $E = \frac{1}{2}mv^2$

Also, $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

Therefore,

$$E = \frac{1}{2}m\frac{h^2}{m^2\lambda^2} = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62\times10^{-34})^2}{2\times1.674\times10^{-27}\times(1\times10^{-10})^2}$$

$$= 1.3\times10^{-20} \text{ joule}$$

$$= \frac{1.3\times10^{-20}}{1.6\times10^{-19}} = 8.13\times10^{-2} \text{ eV}$$

Example 5: Show that group velocity is equal to particle velocity.

Solution:

The group velocity is given by

$$v_g = \frac{d\omega}{dk} = \frac{d(\hbar\omega)}{d(\hbar k)} = \frac{dE}{dP} \quad \left[\text{Since } E = hf = \frac{h}{2\pi}.2\pi f = \hbar\omega \text{ and } p = \frac{h}{\lambda} = \frac{h}{2\pi}.\frac{2\pi}{\lambda} = \hbar k\right]$$

Since, $E = \frac{P^2}{2m}$ for free particle

Since, $v_g = \frac{d}{dp}\left(\frac{P^2}{2m}\right) = \frac{2P}{2m} = \frac{mv}{m} = v =$ particle velocity.

Example 6: An electron is confined to an infinite height box of size 0.1 nm. Calculate the ground state energy of the electron. How this electron can be put to the third energy level.

Solution:

The energy of the particle in one dimensional rigid box of side l is given by

$$E_n = \frac{n^2\pi^2\hbar^2}{2ml^2}$$

$$= \frac{n^2h^2}{8ml^2}$$

$$= \frac{(6.62\times10^{-34})^2\times n^2}{8\times9.1\times10^{-31}\times(10^{-10})^2}$$

$$= 6.03\times10^{-18}n^2 \text{ joule}$$

$$= 37.7\,n^2 \text{ eV}$$

In the ground state, $n = 1$, $E_1 = 37.7$ eV

For third energy level, $n = 3$, $E_3 = 37.7\times3^2 = 37.7\times9$ eV

$\therefore$ $E_3 - E_1 = (9-1)\times37.7 = 301.5$ eV

Hence to put the electron to third energy level an extra energy of 301.5 eV is to be given.

Example 7: What voltage must be applied to an electron microscope to produce electrons of wave length 0.50 Å? Given, $e = 1.6\times10^{-19}$ Coulomb, $m = 9.0\times10^{-31}$ kg, $h = 6.62\times10^{-34}$ Joule.sec.

Solution:

We have, $\frac{1}{2}mv^2 = eV$

$$V = \frac{mv^2}{2e}$$

The de-Broglie wavelength is given by, $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

Therefore,

$$V = \frac{m}{2e}\cdot\frac{h^2}{m^2\lambda^2} = \frac{h^2}{2me\lambda^2}$$

$$= \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times(0.5\times10^{-10})^2}$$

$$= 602 \text{ Volts.}$$

Example 8: The wave function of a particle confined in a box of length l is $\psi(x) = \sqrt{\frac{2}{l}}\sin\frac{\pi x}{l}$.

Calculate the probability of finding the particle in the region $0 < x < \frac{l}{2}$.

Solution:

The probability of finding the particle in the length 0 to $\frac{l}{2}$ is

$$P = \int_0^{l/2}(\psi)^2\,dx = \frac{2}{l}\int_0^{l/2}\sin^2\frac{\pi x}{l}\,dx = \frac{1}{l}\int_0^{l/2}\left(1-\cos\frac{2\pi x}{l}\right)dx = \frac{1}{l}\cdot\frac{l}{2} = \frac{1}{2} = 0.5$$

Example 9: Board **Normalize the one dimensional wave function**

$$\Psi = A \sin\left(\frac{\pi x}{a}\right), 0 < x < a$$

$$= 0 \qquad , \text{outside}$$

Solution:

We have normalizing condition

$$\int_{-\infty}^{\infty} |\Psi|^2 \, dx = 1$$

or $$\int_0^a A^2 \sin^2 \frac{\pi x}{a} \, dx = 1$$

or $$\frac{A^2}{2} \int_0^a \left(1 - \cos\frac{2\pi x}{a}\right) dx = 1$$

or $$\frac{A^2}{2} . a = 1$$

$\therefore$ $$A = \sqrt{\frac{2}{a}}$$

Hence the normalized wave function is $\sqrt{\frac{2}{a}} \sin\frac{\pi x}{a}$.

Example 10: Board **An electron moving is a wave has wave function $\Psi(x) = 2 \sin 2\pi x$. Find the probability of finding the electron in the region x = 0.25 to 0.5 m.**

Solution:

The probability of finding the electron in given region is,

$$P = \int_{0.25}^{0.5} \Psi \Psi^* \, dx = \int_{0.25}^{0.5} 4 \sin^2 2\pi x \, dx$$

$$P = 2 \int_{0.25}^{0.5} 2 \sin^2 2\pi x \, dx = 2 \int_{0.25}^{0.5} (1 - \cos 4\pi x) = 2\left[\int_{0.25}^{0.5} dx - \int_{0.25}^{0.5} \cos 4\pi x \, dx\right]$$

$$= 2\left[(x)\Big|_{0.25}^{0.5} - \frac{\sin 4\pi x}{4\pi}\Big|_{0.25}^{0.5}\right] = 2\,[(0.5 - 0.25) - 0] = 2\,[0.25 - 0] = 0.5$$

Example 11: Board **A particle is moving in 1–D box of infinite potential. Evaluate the probability of finding the particle within range 1 Å at the centre of box when it is in lowest energy state.**

Solution:

$L = 1$ Å $= 1 \times 10^{-10}$ m

We have, wave function for the particle in infinite potential well

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$$

At the centre, $x = \frac{L}{2} = 0.5$ Å and for lowest energy state, n = 1.

Then,

$$\text{Probability} = \left(\sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}\right)^2 = \left|\sqrt{\frac{2}{L}} \sin\frac{\pi}{2}\right|^2 = \frac{2}{L}$$

The probability in the interval Δx is, $P = |\Psi_1(x)|^2 \Delta x = \left(\frac{2}{L}\right) \Delta x = \frac{2 \times 0.5 \times 10^{-10}}{1 \times 10^{-10} \text{m}} = 1$

THEORETICAL ANSWER QUESTIONS

1. What is wave function? Describe it's significance. Derive Schrodinger time dependent wave equation for a free particle like electron.
2. Determine the total energy of a particle using Schrodinger equation, when the potential energy has value V = 0 for 0 < x < a and V = ∞ for x ≤ 0 and x ≥ a.
3. Prove that energy levels are quantized, when the electron is confined in an infinite potential well of width 'a'.
5. Derive an expression for the energy of a particle in an one dimensional infinite deep potential well.
6. What do you mean by operator in quantum mechanics? Explain energy and momentum operator.
7. An electron is trapped in an one dimensional infinite potential well of width 'a' such that.

   V = ∞ for x ≤ 0 and x ≥ a; V = 0 for 0 < x < a

   Using boundary condition, prove that the total potential energy of system is $E = \frac{n^2 \pi^2 \hbar^2}{2ml^2}$. where symbols carry their usual meaning.
8. Discuss the significance of the wave function and deduce the time independent schrodinger equation.
9. A particle is confined in a box of width L. Find an expression for energy eigen value to show that the particle can have only discrete energy and momentum.
10. Define tunneling effect and derive the expression for reflection and transmission coefficient for a barrier of width 'a' and potential of height V_0.
11. Why quantum mechanics is superior to the classical mechanics? Explain.

NUMERICAL PROBLEMS

1. Calculate the de-Broglie wavelength of neutron of energy 28.8eV, h = 6.62×10^{-34} Joule-sec, m = 1.67×10^{-27} kg. **[Ans: 5.34 × 10^{-12} m]**
2. Calculate the energy in electron volt of an electron wave of $\lambda = 3 \times 10^{-2}$ m, Given h = 6.62×10^{-34} JS. **[Ans: 1.68 × 10^{-15} eV]**
3. Calculate the de Broglie wave length of an α - particle accelerated through a potential difference of 2000 volts. Given mass of proton = 1.67×10^{-27} kg; Plancks constant = 6.62×10^{-34} Joule-sec. **[Ans: 6.41 × 10^{-13} m]**
4. Calculate the energy difference between the ground state (n = 1) and the first excited state for an electron in one dimensional rigid box of length 1 Å. **[Ans: 1.808 × 10^{-17} J]**
5. Calculate the velocity and kinetic energy of an electron of wavelength 1.66 Å. **[Ans: 4.39 × 10^{6} m/s, 54.7 eV]**
6. Calculate the velocity and kinetic energy of a neutron having de-broglie wavelength of 1 Å. **[Ans: 3.97 × 10^{3} m/s; 0.0825 eV]**
7. An electron has a wave length 0.2 nm. What are the momentum and energy of electron. **[Ans: 7.36 × 10^{-25} kgm/s; 2.976 × 10^{-16} J]**
8. Find the minimum energy of an electron in a box of width 2Å. **[Ans: 7.36 × 10^{-25} kgm/s]**
9. The energy of an electron constrained to move in one dimensional box of length 4.0 Å is 9.664×10^{-17} J. Find out the order of excited state and the momentum of electron in that state. **[Ans: 16; 1.32 × 10^{-23} kgm/s]**

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