Artificial Intelligence and Neural Network

BE Software Engineering

[Fifth Semester]

Nepal College of Information Technology

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Axiom

An **axiom** is a sentence or proposition that is not proved or demonstrated and is considered as self-evident or as an initial necessary consensus for a theory building or acceptation. According as requirements, the new sentences are added to the knowledge base and then new sentences are also derived from old axiom & theorems, called **inference**.

Data → Information → Knowledge → Wisdom

Knowledge

- Knowledge is a theoretical or practical understanding of a subject or a domain and it is also the sum of what is currently known.

- Knowledge is the sum of what is known: the body of truth, information, and principles acquired by mankind.

- Knowledge is human proficiency stored in a person's mind, gained through experience, and interaction with the person's environment

Knowledge

Research literature classifies knowledge as follows:

- Classification-based Knowledge »Ability to classify information
- Decision-oriented Knowledge »Choosing the best option
- Descriptive knowledge » State of some world (heuristic)
- Procedural knowledge » How to do something
- Reasoning knowledge » What conclusion is valid in what situation?
- Assimilative knowledge »What its impact is?

Logic

Logic is method of reasoning process in which conclusions are drawn from premises using rules of inference. The logic is knowledge representation technique that involves:

- **Syntax:** defines well-formed sentences or legal expression in the language
- **Semantics:** defines the "meaning" of sentences
- **Inference rules:** for manipulating sentences in the language

Basically, the logic can be classified as:

- Proposition (or statements or calculus) logic
- Predicate [or First Order Predicate Logic (FOPL)] logic

Propositional Logic

A proposition is a declarative sentence to which only one of the "Truth value" (i.e. TRUE or FALSE) can be assigned (but not both). Hence, the propositional logic is also called Boolean logic. When a proposition is true, we say that its truth value is T, otherwise its truth value is F.

For example:

- The square of 4 is 16 $\rightarrow$T
- The square of 5 is 27 $\rightarrow$F

Atomic Sentences(Simple)

The atomic sentences consist of a single proposition symbol.

Each such symbol stands for a proposition that can be true or false.

We use symbols that start with an uppercase letter and may contain other letters or subscripts, for example: p, q, r, s etc.

For example:

p = Sun rises in West. (False sentence)

Complex Sentences (molecular or combined or compound)

The two or more statements connected together with some logical connectives such as AND ($\wedge$), OR ($\vee$), Implication ($\rightarrow$), etc.

Name	**Representation**	**Meaning**
Negation	$\neg p$	not p
Conjunction (true when both statement are true, otherwise false)	$p \wedge q$	p and q
Disjunction (false when both statement are false, otherwise true)	$p \vee q$	p or q (or both)
Exclusive Or (false when both statement are same)	$P \oplus q$	either p or q, but not both
Implication (false when p is true and q is false)	$p \rightarrow q$	if p then q
Bi-conditional or Bi-implication (true when both statement have same truth value)	$p \leftrightarrow q$	p if and only if q
The order of precedence in propositional logic is (from highest to lowest): Inverse, AND, OR, Implication and Double Implication.		

Converse:	If $p \rightarrow q$ is an implication, then its converse is $q \rightarrow p$
Inverse:	If $p \rightarrow q$ is an implication, then its inverse is $\neg p \rightarrow \neg q$
Contrapositive:	If $p \rightarrow q$ is an implication, then its contrapositive is $\neg q \rightarrow \neg p$

Q. Write implication, converse, inverse, negation, contra positive of the following integration.
"The program is readable only if it is well structured"

p = "the program is readable"

q = "the program is well structured"

- **Implication**: $(p \rightarrow q)$

If the program is readable, then it is well structured.

- **Converse**: $(q \rightarrow p)$

If the program is well structured, then it is readable.

- **Inverse**: $(\neg p \rightarrow \neg q)$

If the program is not readable, then it is not well structured.

- **Contrapositive**: $(\neg q \rightarrow \neg p)$

If the program is not well structured, then it is not readable

- **Negation** of p: $(\neg p)$

The program is not readable.

Q. Verify that $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Q. Construct the truth table of $\neg(p \wedge q) \vee (r \wedge \neg p)$

p	q	r	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$r \wedge \neg p$	$\neg(p \wedge q) \vee (r \wedge \neg p)$
T	T	T	T	F	F	F	F
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	F	T

Logical equivalence

Two proposition p and q are logically equivalent and written as if both p and q have identical truth values

$$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$$

The following logical equivalences apply to any statements; the p's, q's and r's can stand for atomic statements or compound statements.

i. Double Negative Law

$\neg(\neg p) \equiv p$

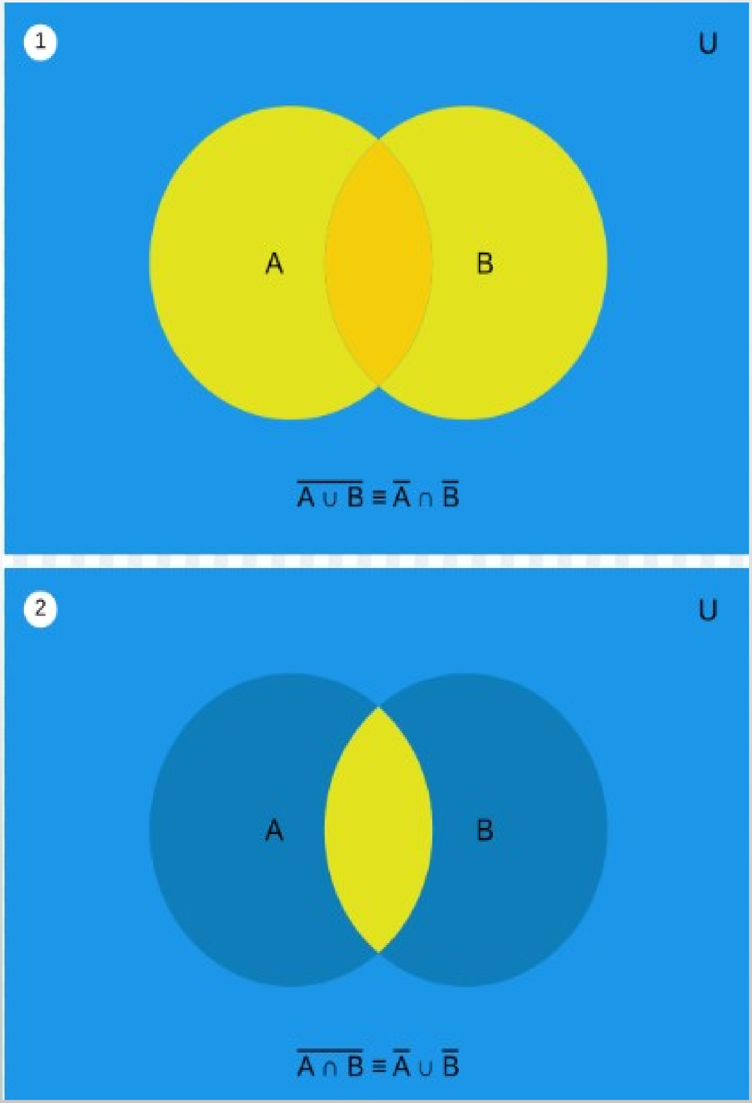
i. De Morgan's Laws

$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$

i. Distributive Laws

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$



Tautology	If a proposition have a truth value for every interpretation
	E.g.: i) $p \vee \neg p$ ii) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
Contradiction	If a proposition have a false value for every interpretation
	E.g: $p \wedge \neg p$
Contingent	If a proposition have both true and false value
	E.g.: $p \wedge q$

Argument, premises and conclusion

- An argument in propositional logic is sequence of propositions.
- All proposition are called premises and the final proposition is called the conclusion.
- An argument is valid if the truth of all its premises implies that the conclusion is true

Q. There are two restaurants next to each other.
One has a sign board as: "Good food is not cheap".
The other has a sign board as "Cheap food is not good".

Are both the sign board saying the same thing?

G= “Food is Good”

C= “Food is Cheap”

Sentence 1:- “Good food is not cheap” can be symbolically written as $G \rightarrow \neg C$

Sentence 2:- “Cheap food is not good” can be symbolically written as $C \rightarrow \neg G$

Now, The Truth Table is:

G	C	$\neg G$	$\neg C$	$G \rightarrow \neg C$	$C \rightarrow \neg G$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Since, $G \rightarrow \neg C$ and $C \rightarrow \neg G$ are logically equivalent. So both are saying same thing.

Inference

- The process of drawing conclusion from given premises in an argument is called inference.
- To draw the conclusion from the given statements, we must be able to apply some well-defined steps that helps reaching the conclusion.

Soundness and completeness

Let S be the set of all right answers.

- A **sound** algorithm never includes a wrong answer in S, but it might miss a few right answers. => not necessarily "complete".
- A **complete** algorithm should get every right answer in S: include the complete set of right answers. But it might include a few wrong answers. It might return a wrong answer for a single input. => not necessarily "sound".

Rules of inference for propositional logic:

- Addition rule

$$\frac{p}{\therefore p \vee q}$$

Example:

- Ram is a student of BE.

∴Ram is a student of BE or BCA.

Corresponding Tautology:
$p \to (p \vee q)$

p	q	p or q	p -> (p or q)
F	F	F	T
F	T	T	T
T	F	T	T
T	T	T	T

- Simplication rule

$$\frac{p \wedge q}{\therefore p}$$

Corresponding Tautology:
$(p \wedge q) \rightarrow p$

Example:

-Subodh and Shyam are the students of BE.

$\therefore$ Subodh is the student of BE

or

Shyam is the student of BE.

- Modus Ponens Rule

$$\begin{array}{l} p \rightarrow q \\ P \\ \hline \therefore q \end{array}$$

Corresponding Tautology:
$(p \wedge (p \rightarrow q)) \rightarrow q$

Example:

- If Ram is hard working, then he is intelligent.
- Ram is hard working.

$\therefore$ Ram is intelligent.

- Modus Tollens Rule

$$p \rightarrow q$$

$$\neg q$$

$$\therefore \neg p$$

Example:

- If it is sunny, then we will go to swimming.
- We will not go swimming.

∴ It is not Sunny.

Corresponding Tautology:

$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

- Hypothetical Syllogism Rule

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Corresponding Tautology:
$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Example:

- If Subodh is a BE student, then he loves programming.
- If Subodh loves programming, then he is expert in java.

∴ If Subodh is a BE student, then he is expert in java.

Disjunctive Syllogism Rule

$p \vee q$

$\neg p$

$\therefore q$

Example:

- Today is Wednesday or Thursday.
- Today is not Wednesday.

$\therefore$ Today is Thursday.

Corresponding Tautology:
$(\neg p \wedge (p \vee q)) \rightarrow q$

- Conjunction rule

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$

Example:

- Shyam is the student of BE.
- Hari is the student of BE.

∴ Shyam and Hari are the students of BE.

Corresponding Tautology:
$((p) \land (q)) \rightarrow (p \land q)$

- Resolution Rule

$$\begin{array}{l} p \lor q \\ \neg q \lor r \\ \hline \therefore\ p \lor r \end{array}$$

Corresponding Tautology:
$((p \lor q) \land (\neg q \lor r)) \to (p \lor r)$

"I will study discrete math or I will study English literature."
<u>"I will not study English literature</u> <u>or I will study database."</u>
_____I will study discrete math or I will study database."

Can we conclude that the conclusion is true if the premises are true?

- If Socrates is human, then Socrates is mortal.
- Socrates is human.

$\therefore$ Socrates is mortal.

Can we conclude that the conclusion is true if the premises are true?

- If George does not have eight legs, then he is not a spider.
- George is a spider.

∴ George has eight legs.

Which rule of inference is used in each argument below?

- Alice is a Math major. Therefore, Alice is either a Math major or a CSI major.
- Jerry is a Math major and a CSI major. Therefore, Jerry is a Math major.
- If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
- If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
- If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.
- I go swimming or eat an ice cream. I did not go swimming. Therefore, I eat an ice cream

If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Q. **"If you send me an e-mail message then I will finish writing the program", "If you do not send me an e-mail message then I will go to sleep early", and "If I go to sleep early then I will wake up feeling refreshed". Lead to the conclusion "If I do not finish writing the program then I will wake up feeling refreshed".**

Solution:

Let

p = "You send me an e-mail message"

q = "I will finish writing the program"

r = "I will go to sleep early"

s = "I will wake up feeling refreshed"

Hypothesis:

a. $p \to q$

b. $\neg p \to r$

c. $r \to s$

Conclusion: $\neg q \to s$

Steps	Operations	Reasons
1	$p \to q$	Given hypothesis
2	$\neg q \to \neg p$	Using contra positive on 1
3	$\neg p \to r$	Given hypothesis
4	$\neg q \to r$	Using hypothetical syllogism on 2 and 3
5	$r \to s$	Given hypothesis
6	$\neg q \to s$	Using hypothetical syllogism on 4 and 5

Hence the given hypotheses lead to the conclusion $\neg q \to s$

Q. **“ Hari is playing in garden”, “If he is playing in garden then he is not doing homework”, “If he is not doing homework, then he is not learning” leads to the conclusion “He is not learning”.**

Use rules of inference to show that the hypotheses "Ram works hard," "If Ram works hard, then he is a dull boy," and "If Ram is a dull boy, then he will not get the job" imply the conclusion "Ram will not get the job."

Q. Prove the conclusion based on the given hypothesis using the rule of resolution

Hypothesis

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

Conclusion:

$\neg q \rightarrow s$

Predicate

- Predicate is a part of declarative sentences describing the properties of an object or relation among objects. For example: “is a student” is a predicate as ‘A is a student’ and ‘B is a student’.
- A predicate logic is a formal system that uses objects/ variables and quantifiers ($\forall$, $\exists$) to formulate propositions.

Assignment :

Differentiate between Propositional Logic and Predicate Logic

Quantification

- A quantifier is a symbol that permits one to declare the range or scope of variables in a logical expression.
- The process of binding propositional variable over a given domain is called quantification.
- Two common quantifier are the existential quantifier (“there exists or for some or at least one”) and universal quantifier (“for all or for each or for any or for every and or for arbitrary”).

Universal Quantifier (∀: For All)

- It is denoted by ∀ and used for universal quantification.
- The universal quantification of p(x) denoted by ∀x p(x) is proposition that is true for all values in universal set
- The universal quantifier is read as:
  - For all x, p(x) holds
  - For each x, p(x) holds
  - For every x, p(x) holds

Existential Quantifier(∃ : For Some)

- It is denoted by ∃ and used for existential quantification.
- The existential quantification of p(x) denoted by ∃x p(x) is proposition that is true for some values in universal set.
- The existential quantifier is read as:
  - There is an x, such that p(x)
  - There is at least one x such that p(x)
  - For some x, p(x)

Q. Assume that

P (x) denotes "x is an accountant."

Q (x) denotes "x owns a maruti."

Now, represent the following statement symbols.

a) All accountants own maruti.

Meaning: $\underbrace{\text{For all x,}}_{\forall x}\ \underbrace{\text{if x is an accountant,}}_{p(x)}\ \text{then}\ \underbrace{\text{x owns maruti}}_{q(x)}$

$\Rightarrow \forall x\ (\, p(x) \rightarrow q(x)\,)$

b) Some accountants own maruti

Meaning: $\underbrace{\text{For some}}_{\exists x}\ \text{x,}\ \underbrace{\text{x is an accountant}}_{p(x)}\ \text{and}\ \underbrace{\text{x owns maruti}}_{q(x)}$

$\Rightarrow \exists x\ (\, p(x) \wedge q(x)\,)$

c) All owners of maruti are accountants

Meaning: For all x, if x is a owner of maruti, then x is an accountant.

$$\Rightarrow \forall x \, (q(x) \rightarrow f(x))$$

d) Someone who owns a maruti, is an accountant

Meaning: For some x, who owns a maruti and x is an accountant

$$\Rightarrow \exists x \, (q(x) \wedge f(x))$$

Q. Convert into FOPL

a. All men are people.

b. Marcus was Pompeian.

c. All Pompeian were Roman.

d. Ram tries to assassinate Hari.

e. All Romans were either loyal to caser or hated him.

f. Socrates is a man. All men are mortal; therefore Socrates is mortal.

g. Some student in this class has studied mathematics.

a. **All men are people.**

$\Rightarrow \forall x$ MAN (x) $\rightarrow$ PEOPLE (x)

b. **Marcus was Pompeian.**

$\Rightarrow$POMPEIAN (Marcus)

c. **All Pompeian were Roman.**

$\Rightarrow \forall x$ POMPEIAN (x) $\rightarrow$ ROMAN (x)

d. **Ram tries to assassinate Hari.**

$\Rightarrow$ASSASSINATE (Ram, Hari)

e. All Romans were either loyal to caser or hated him.

$\Rightarrow \forall x$ ROMAN $(x) \rightarrow$ LOYAL $(x, \text{caser}) \vee$HATES (x, caser)

f. Socrates is a man. All men are mortal; therefore Socrates is mortal.

$\Rightarrow$ MAN (Socrates), $\forall$ x MAN $(x) \rightarrow$ MORTAL (x), MORTAL (Socrates)

g. Some student in this class has studied mathematics.

$\Rightarrow$ Let

- $S(x)$ = "x is a student in this class"
- $M(x)$ = "x has studied mathematics"

Hence, required expression is: $\exists x\ [S(x) \wedge M(x)]$

Rules of Inference for Quantified Statements

a. Universal Instantiation

$$\frac{\forall x\ p(x)}{\therefore p(d)}$$

Where d is in the domain of discourse D.

For example: If all balls in a box are red then any randomly drawn ball is also red .

b. Universal Generalization

$$\frac{p(d)}{\therefore \forall x\ p(x)}$$

Where, d is the domain of discourse D.

For example: If all the ball in a box are taken one by one in randomly manner and if all are red then we conclude that all balls in the box are red.

c. Existential Instantiations

$$\frac{\exists x\ p(x)}{\therefore p(d)}$$

For some d in the domain of discourse.

For example: If some balls in a box are red then resulting ball after experiment will also be red.

d. Existential Generalization

$$\frac{p(d)}{\therefore \exists x\ p(x)}$$

For some d in the domain of discourse.

For example: If we take only one random experiment for the ball drawn and apply the result of ball to the domain.

Combining Rules of Inference for Propositions and Quantified Statements

These inference rules are frequently used and combine propositions and quantified statements:

- **Universal Modus Ponens**

$$\begin{array}{rl} & \forall x(P(x) \to Q(x)) \\ & P(a), \text{ where } a \text{ is a particular element in the domain} \\ \hline \therefore & Q(a) \end{array}$$

- **Universal Modus Tollens**

$$\begin{array}{rl} & \forall x(P(x) \to Q(x)) \\ & \neg Q(a), \text{ where } a \text{ is a particular element in the domain} \\ \hline \therefore & \neg P(a) \end{array}$$

Q. Given Expression: All men are mortal. Einstein is a man. Prove that "Einstein is mortal" using FOPL.

Solution: Let

$M(x)$ = "x is a man"

$N(x)$ = "x is mortal"

Hypothesis: $\forall x\,[M(x) \rightarrow N(x)]$, M (Einstein)

Conclusion: N (Einstein)

Steps	Operations	Reasons
1.	$\forall x\,[M(x) \rightarrow N(x)]$	Given Hypothesis
2.	M(Einstein)→N (Einstein)	Using universal instantiation on 1
3.	M (Einstein)	Given Hypothesis
4.	N (Einstein)	Using modus pollens on 2 and 3

Hence the given hypotheses lead to the conclusion "Einstein is mortal".

Q. Given Expression: "Lions are dangerous animals", and "There are lions". Prove that "There are dangerous animals" using FOPL.

Solution: Let

$D(x)$ = "x is a dangerous animal"

$L(x)$ = "x is a lion"

Hypothesis: $\forall x\, [L(x) \rightarrow D(x)]$, $\exists x\, L(x)$

Conclusion: $\exists x\, D(x)$

Steps	**Operations**	**Reasons**
1.	$\forall x\, [L(x) \rightarrow D(x)]$	Given Hypothesis
2.	$L(a) \rightarrow D(a)$	Using universal instantiation on 1
3.	$\exists x\, L(x)$	Given Hypothesis
4.	$L(a)$	Using existential instantiation on 3
5.	$D(a)$	Using modus pollenson 2 and 4
6.	$\exists x\, D(x)$	Using existential generalization on 5

Hence the given hypotheses lead to the conclusion "There are dangerous animals"

Q. Show that the premises “Everyone in this AI class has taken a course in computer computer engineering” and “Janak is a student in this class” imply the conclusion “Janak has taken a course in computer engineering.”

Show that the premises “A student in this class has not read the book,” and “Everyone in this class passed the first exam” imply the conclusion “Someone who passed the first exam has not
read the book.”

Solution: Let $C(x)$ be "x is in this class," $B(x)$ be "x has read the book," and $P(x)$ be "x passed the first exam." The premises are $\exists x(C(x) \wedge \neg B(x))$ and $\forall x(C(x) \rightarrow P(x))$. The conclusion is $\exists x(P(x) \wedge \neg B(x))$. These steps can be used to establish the conclusion from the premises.

Step	Reason
1. $\exists x(C(x) \wedge \neg B(x))$	Premise
2. $C(a) \wedge \neg B(a)$	Existential instantiation from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x(C(x) \rightarrow P(x))$	Premise
5. $C(a) \rightarrow P(a)$	Universal instantiation from (4)
6. $P(a)$	Modus ponens from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \wedge \neg B(a)$	Conjunction from (6) and (7)
9. $\exists x(P(x) \wedge \neg B(x))$	Existential generalization from (8)

Nested Quantifiers

quantifiers are nested if one is within the scope of the other, such as

$$\forall x \exists y(x + y = 0).$$

Note that everything within the scope of a quantifier can be thought of as a propositional function. For example,

$$\forall x \exists y(x + y = 0)$$

is the same thing as $\forall x Q(x)$, where $Q(x)$ is $\exists y P(x, y)$, where $P(x, y)$ is $x + y = 0$.

Nested Quantifiers

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

Nested Quantifiers

- Brothers are siblings
  $\forall\, x;y \quad Brother(x;y) \Rightarrow Sibling(x;y)$

- "Sibling" is symmetric
  $\forall\, x;y \quad Sibling(x;y)\ , \quad Sibling(y;x)$

- One's mother is one's female parent
  $\forall\, x;y\ Mother(x;y)\ , \quad (Female(x) \wedge Parent(x;y))$

- All purple mushrooms are poisonous.
  $\forall\ x\ (mushroom(x)\ \wedge\ purple(x)) \Rightarrow poisonous(x)$

- Every gardener likes the sun.
  $\forall\ x\ gardener(x) \Rightarrow likes(x, Sun)$

Clausal Forms

- A clause is an expression formed from a finite collection of literals (variables or their negations)
- A clause that contains only $\vee$ is called a **disjunctive clause** and only $\wedge$ is called a **conjunctive clause.**
- Negation is allowed, but only directly on variables.

➢ $p \vee \neg q \vee r$: a disjunctive clause

➢ $\neg p \wedge q \wedge \neg r$: a conjunctive clause

➢ $\neg p \wedge \neg q \vee r$: neither

CNF and DNF

If we put a bunch of disjunctive clauses together with $\wedge$, it is called conjunctive normal form.

- For example: $(p \vee r) \wedge (\neg q \vee \neg r) \wedge q$ is in conjunctive normal form.

Similarly, putting conjunctive clauses together with $\vee$, it is called disjunctive normal form.

- For example: $(p \wedge \neg q \wedge r) \vee (\neg q \wedge \neg r)$ is in disjunctive normal form.

Conversion Procedure for proposition into Normal Forms

- We can use the definitions to get rid of $\rightarrow$, $\leftrightarrow$, and $\oplus$
  - $B \leftrightarrow (P \vee Q)$ as $(B \rightarrow (P \vee Q)) \wedge ((P \vee Q) \rightarrow B)$
  - $(B \rightarrow (P \vee Q))$ as $(\neg B \vee P \vee Q)$
  - $(P \oplus Q)$ as $(P \wedge \neg Q) \vee (\neg P \wedge Q)$
- Use DE Morgan's laws.
  - $\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$ (De Morgan)
  - $\neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta)$ (De Morgan)
- Use double negation to get rid of any ¬¬ that showed up.
- Use the distributive rules to move things in/out of parens as we need to.
  - $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$

Converting to conjunctive normal form

a. $\neg((\neg p \rightarrow \neg q) \wedge \neg r)$

$$
\begin{aligned}
&\equiv \neg((\neg\neg p \vee \neg q) \wedge \neg r) && \text{[definition]} \\
&\equiv \neg((p \vee \neg q) \wedge \neg r) && \text{[double negation]} \\
&\equiv \neg(p \vee \neg q) \vee \neg\neg r && \text{[DeMorgan's]} \\
&\equiv \neg(p \vee \neg q) \vee r && \text{[double negation]} \\
&\equiv (\neg p \wedge \neg\neg q) \vee r && \text{[DeMorgan's]} \\
&\equiv (\neg p \wedge q) \vee r && \text{[double negation]} \\
&\equiv (\neg p \vee r) \wedge (q \vee r) && \text{[distributive]}
\end{aligned}
$$

b. $(p \rightarrow q) \rightarrow (\neg r \wedge q)$ into DNF

$$\equiv \neg(p \rightarrow q) \vee (\neg r \wedge q) \qquad \text{[definition]}$$
$$\equiv \neg(\neg p \vee q) \vee (\neg r \wedge q) \qquad \text{[definition]}$$
$$\equiv (\neg\neg p \wedge \neg q) \vee (\neg r \wedge q) \qquad \text{[DeMorgan's]}$$
$$\equiv (p \wedge \neg q) \vee (\neg r \wedge q) \qquad \text{[double negation]}$$

b. $(p \rightarrow q) \rightarrow (\neg r \wedge q)$ into DNF

$\equiv (p \wedge \neg q) \vee (\neg r \wedge q)$

$$\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee (\neg r \wedge q)) \quad \text{[distributive]}$$
$$\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \wedge (\neg q \vee q) \quad \text{[distributive]}$$
$$\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \wedge \mathbf{T} \quad \text{[negation]}$$
$$\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \quad \text{[identity]}$$
$$\equiv (p \vee \neg r) \wedge (p \vee q) \wedge (\neg q \vee \neg r) \quad \text{[distributive]}$$

Horn Clause

- A Horn clause is a clause (a disjunction of literals) with at most one positive, i.e. unnegated, literal

- Any Horn clause therefore belongs to one of four categories:
- 1 positive literal, at least 1 negative literal.

  A rule has the form " ¬ P1 V ¬ P2 V ... V ¬ Pk V Q". This is logically equivalent to "[P1^P2^ ... ^Pk] => Q"
- 1 positive literal, 0 negative literals.

  Examples: "man(socrates)", "parent(elizabeth,charles)", "ancestor(X,X)"
- 0 positive literals, at least 1 negative literal.

  $\neg p \vee \neg q \vee ... \vee \neg t$
- The null clause: 0 positive and 0 negative literals.

  Appears only as the end of a resolution proof.

Horn Clause

A Horn clause with exactly one positive literal is a definite clause; a definite clause with no negative literals is sometimes called a fact

Definite clause	$\neg p \vee \neg q \vee \ldots \vee \neg t \vee u$	$u \leftarrow p \wedge q \wedge \ldots \wedge t$
Fact	u	u

Horn Clause

In the non-propositional case, all variables in a clause are universally quantified with the scope being the entire clause.

$\forall X(\neg \text{human}(X) \vee \text{mortal}(X))$ is logically equivalent to:

$$\forall X(\text{human}(X) \rightarrow \text{mortal}(X))$$

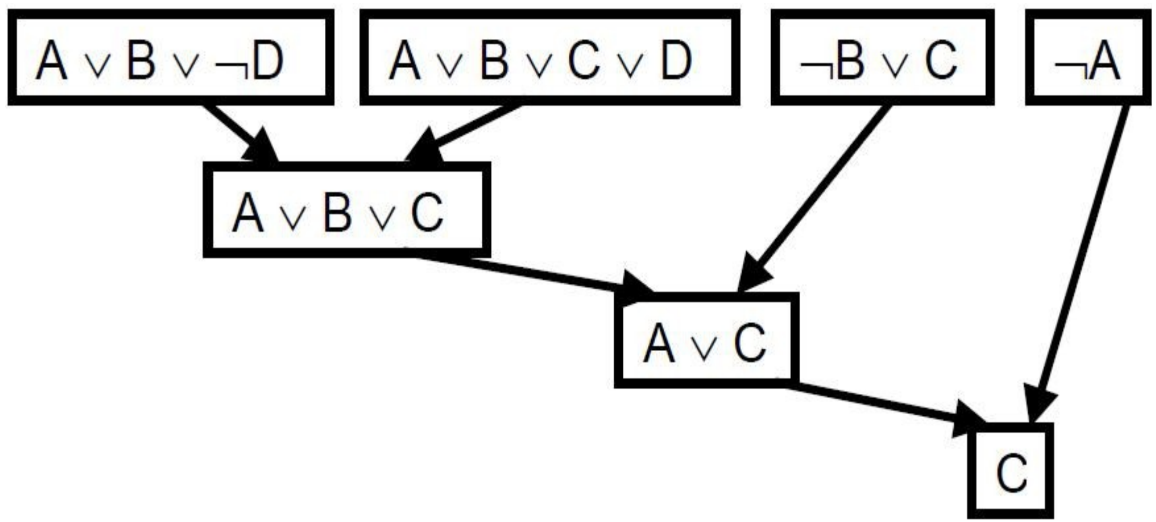
Resolution in Propositional Logic

- The resolution rule in propositional logic is a single valid inference rule that produces a new clause implied by two clauses
- Resolution principle was introduced by John Alan Robinson in 1965.
- The resolution technique can be applied for sentences in propositional logic and first-order logic.
- Resolution technique can be used only for disjunctions of literals to derive new conclusion.
- The resolution rule for the propositional calculus can be stated as following:

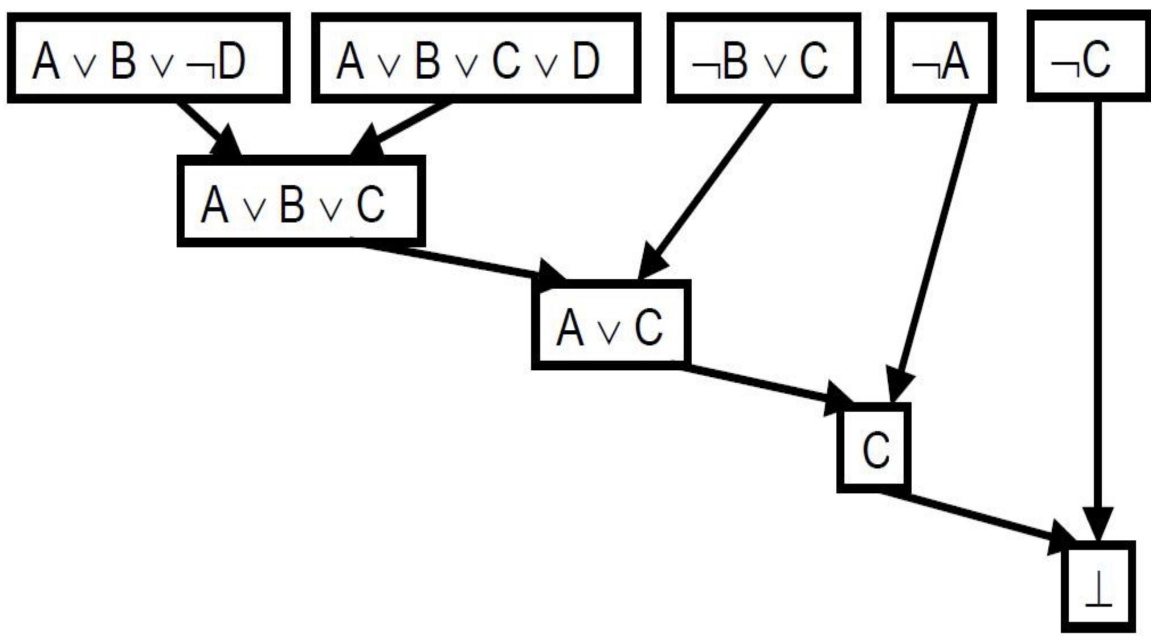
  $(P \vee Q)$ and $(\neg Q \vee R)$, gives $(P \vee R)$.

Q. Let $P_1 = A \vee B \vee \neg D$, $P_2 = A \vee B \vee C \vee D$, $P_3 = \neg B \vee C$, $P_4 = \neg A$, $P_5 = C$ then Show that $\{P_1, P_2, P_3, P_4\} = P_5$

Solution:



Q. Let $P_1 = A \vee B \vee \neg D$, $P_2 = A \vee B \vee C \vee D$, $P_3 = \neg B \vee C$, $P_4 = \neg A$, $P_5 = C$ then Show that $\{P_1, P_2, P_3, P_4, \neg P_5\} = \perp$



Resolution in FOPL

- During resolution in propositional logic, it is easy to determine that two literals (e.g. p and ¬p) cannot both be true at the same time.
- In predicate logic this matching process is more complicated since the argument of the predicate must be considered.
- For example, MAN (John) and ¬MAN (John) is a contradiction, while MAN (John) and ¬MAN (Smith) is not. Thus, in order to determine contradictions, we need a matching procedure, called unification algorithm that compares two literals and discovers whether there exists a set of substitutions that makes them identical.

Resolution in FOPL

To unify two literals, the initial predicate symbol on both must be same; otherwise there is no way of unification.

For example, Q(x, y) and R(x, y) cannot unify but P(x, x) and P(y, z) can be unify by substituting z by x and y by x.

Q. Given Expression: John likes all kinds of foods. Apples are food. Chicken is food. Prove that John likes Peanuts using resolution.

Soln:

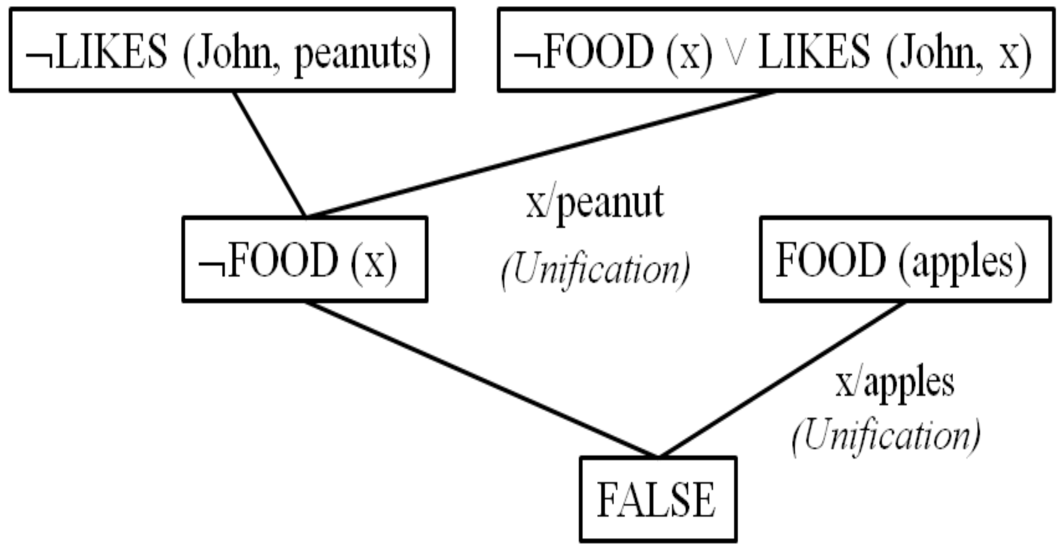
- FOPL
  - $\forall x$ FOOD (x) → LIKES (John, x)

  or
  - ¬FOOD (x) ∨LIKES (John, x)
  - FOOD (apples)
  - FOOD (chicken)

Now, we have to prove: LIKES (John, peanuts).

To prove the statement using resolution, let's take the negation of this as:

¬LIKES (John, peanuts)



Since, ¬LIKES (John, peanuts) is not possible and hence the: LIKES (John, peanuts) is proved.

Q. Given Expression:

Bhaskar is a physician. All physicians know surgery.

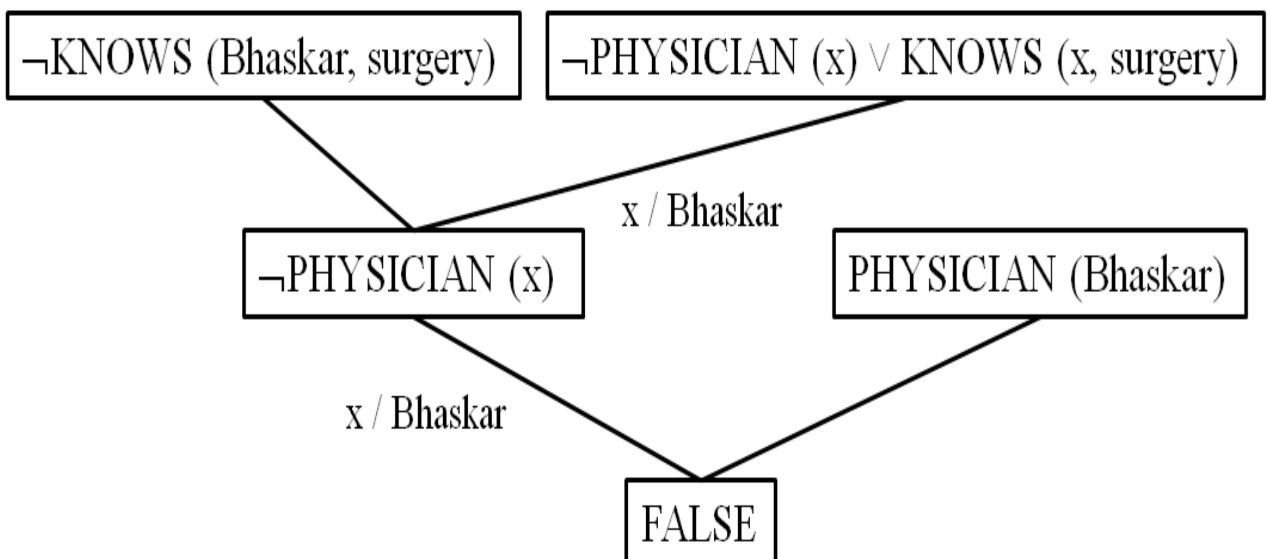
Prove that Bhaskar knows surgery using principle of resolution.

- PHYSICIAN (Bhaskar)
- ∀x PHYSICIAN (x) → KNOWS (x, surgery) **or**

¬PHYSICIAN (x) ∨ KNOWS (x, surgery)

Now, we have to prove that: KNOWS (Bhaskar, surgery). To prove the statement using resolution (proof by contradiction); let's take the negation of this as: ¬KNOWS (Bhaskar, surgery)

Now,



Q. Given Expression:

All carnivorous animals have sharp teeth. Tiger is carnivorous. Fox is carnivorous.

Prove that tiger has sharp teeth.

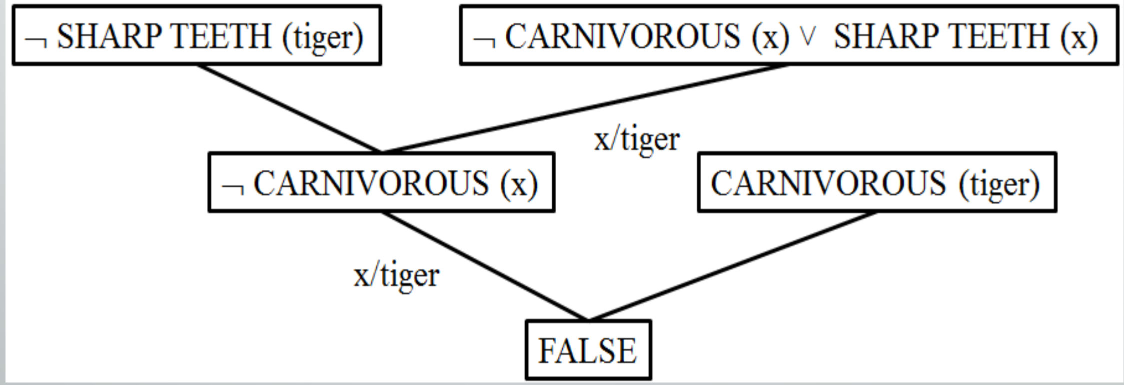
Soln:

FOPL is

- ∀x CARNIVOROUS (x) → SHARP TEETH (x) **or** ¬CARNIVOROUS (x) ∨SHARP TEETH (x)
- CARNIVOROUS (tiger)
- CARNIVOROUS (fox)

Now, we have to prove that: SHARP TEETH (tiger). To prove the statement using resolution (proof by contradiction); let's take the negation of this as: ¬SHARPTEETH (tiger)

Now



What is the difference between inference, reasoning and deduction?

Reason is the capacity for consciously making sense of things, applying logic, establishing and verifying facts, and changing or justifying practices, institutions, and beliefs based on new or existing information.

We use reasons or reasoning to form **inferences** which are basically conclusions drawn from propositions or assumptions that are supposed to be true.

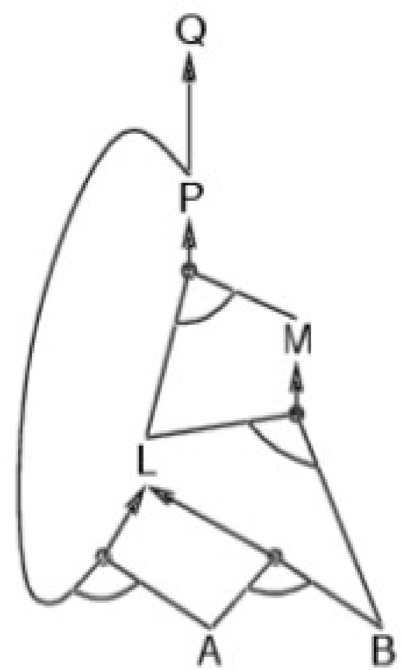
Deduction is a specific form of reasoning and starts with a hypothesis and examines the possibilities within that hypothesis to reach a conclusion.

Forward Chaining (What will Happen Next?)

- Forward chaining is one of the two main methods of reasoning when using inference rules
- Described logically as repeated application of modus ponens.
- An inference engine, using forward chaining, searches the inference rules until it finds one where the antecedent (If clause) is known to be true. When such a rule is found, the engine can conclude, or infer, the consequent (Then clause), resulting in the addition of new information to its data.
- Forward chaining is a popular implementation strategy for expert systems, business and production rule systems.
- Forward chaining starts with the available data and uses inference rules to extract more data until a goal is reached.

Idea: fire any rule whose premises are satisfied in the *KB*,

– add its conclusion to the *KB*, until query is found



Prove that Q can be inferred from above KB

Backward Chaining (Why This Happened?)

- Backward chaining (or backward reasoning) is an inference method that can be described as working backward from the goal(s).
- In game theory, its application to sub games in order to find a solution to the game is called backward induction.

Rule Based Deduction System

Rule-based systems are used as a way to store and manipulate knowledge to interpret information in a useful way.

A classic example of a rule-based system is the domain-specific expert system that uses rules to make deductions or choices. For example, an expert system might help a doctor choose the correct diagnosis based on a cluster of symptoms

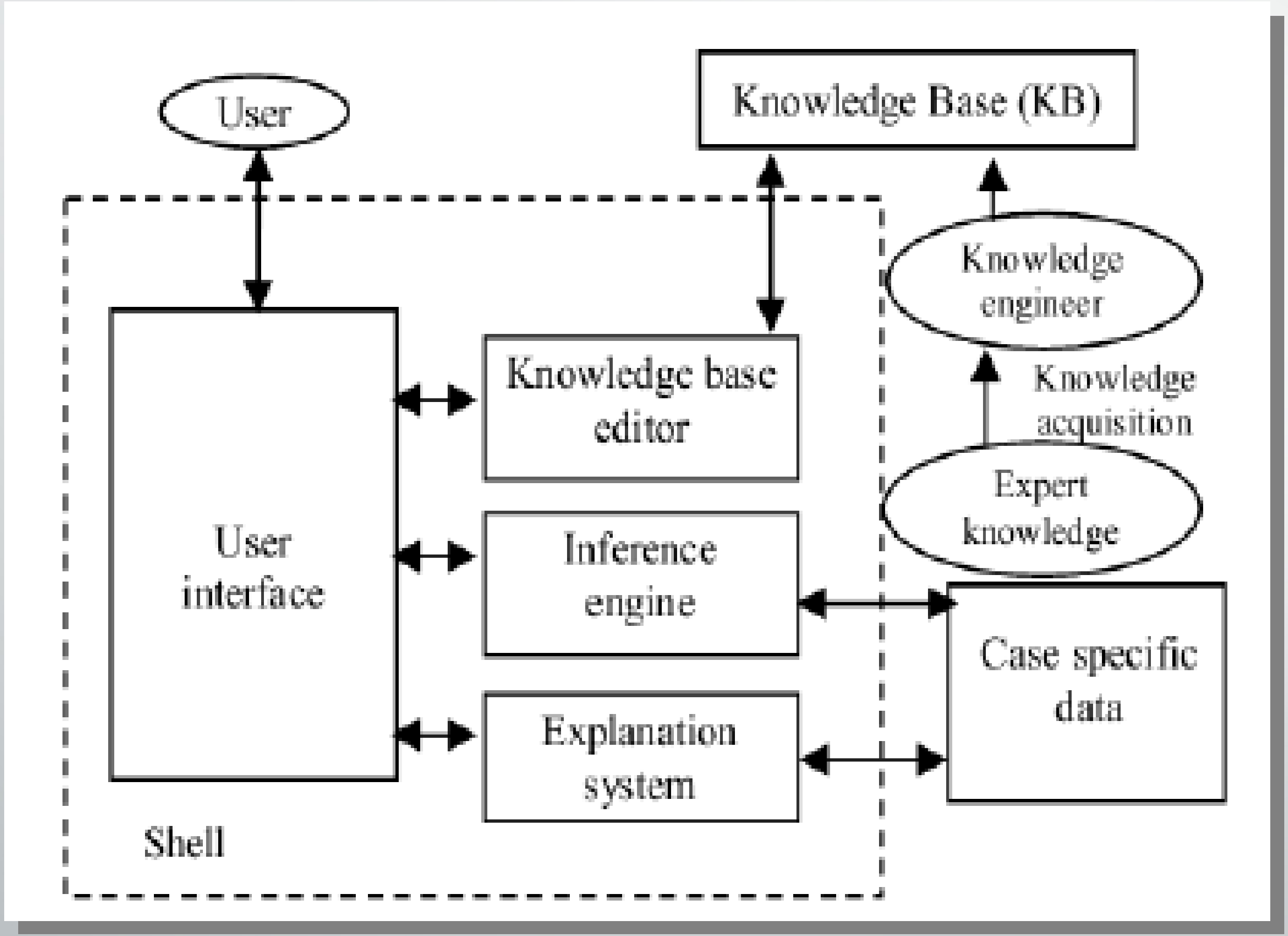
In this approach, idea is to use production rules, sometimes called IF-THEN rules. The syntax structure is

IF <premise> THEN <action>

- **<premise>**- is Boolean. The AND, and to a lesser degree OR and NOT, logical connectives are possible.
- **<action>**- a series of statements

A typical rule-based system has four basic components:

- A **list of rules** or **rule base**, which is a specific type of knowledge base.
- An **inference engine**, which infers information or takes action based on the interaction of input and the rule base.
- Temporary **working memory**.
- A **user interface** or other connection to the outside world through which input and output signals are received and sent.



User
Knowledge Base (KB)
Knowledge engineer
Knowledge acquisition
Expert knowledge
User interface
Knowledge base editor
Inference engine
Explanation system
Case specific data
Shell

Example:

“If the patient has stiff neck, high fever and a headache, check for Brain Meningitis”. Then it can be represented in rule based approach as:

IF

<fever, over, 39> and <neck, stiff, yes> and <head, pain, yes>

THEN

Add(<PATIENT,DIAGNOSE, MENINGITIS>)

Monotonic Reasoning

- A reasoning system is monotonic if the truthfulness of a conclusion does not change when new information is added to the system
- Adding knowledge base does not reduce the set of propositions that can be derived.
- Eg: We are A//B and B//C. We can conclude A//C.

  Now adding a fact that A//D, does not change the result A//C

Non-Monotonic Reasoning

- A logic is non-monotonic if some conclusions can be invalidated by adding more knowledge in the knowledge base.
- Non monotonic system are harder to deal with than monotonic systems
- Non – monotonic systems require more storage space as well as more processing time than monotonic systems.
- Eg: Given “Birds Fly”. Simply means all birds fly.

  But a fact “Ostrich is bird” is added, results the conclusion is false.

Probabilistic Reasoning

Probability theory is used to discuss events, categories, and hypotheses about which there is not 100% certainty.

We might write $A \rightarrow B$, which means that if A is true, then B is true. If we are unsure whether A is true, then we cannot make use of this expression.

In many real-world situations, it is very useful to be able to talk about things that lack certainty. For example, what will the weather be like tomorrow? We might formulate a very simple hypothesis based on general observation, such as “it is sunny only 10% of the time, and rainy 70% of the time”.
Probabilistic Reasoning plays important role in these situations.

Conditional Probability and Bayes Theorem

Let A and B are two dependent events then the probability of the event A when the event B has already happened is called the conditional probability.

It is denoted by P (A|B) and is given by:

$P(A|B) = P(A \cap B)/P(B)$, where $P(B) \neq 0$, $\Rightarrow P(A \cap B) = P(A|B).\ P(B)$ ---- (i)

Similarly,

$P(B|A) = P(A \cap B)/P(A)$, where $P(A) \neq 0$, $\Rightarrow P(A \cap B) = P(B|A).\ P(A)$ ---- (ii)

From equation (i) and (ii), we have

$P(B|A).\ P(A) = P(A|B).\ P(B)$

Bayes rule is useful for those cases where P (A|B) can be estimated but P (B|A) is hard to find experimentally

Conditional Probability and Bayes Theorem

- In a task such as medical diagnosis, we often have conditional probabilities on causal relationships and want to derive a diagnosis.

- A doctor knows the probability of symptoms condition to disease P (S|D), and the patient knows his own feeling or symptoms P(S). Also the doctor knows about probability of disease P (D), then the probability of disease condition to symptoms can be defined.

For example:

Let,

- S = Symptoms on patients such as stiff neck whose probability P(S) is =1/20
- D = Disease known by doctor whose probability P (D) is = 1/50000
- Given P (S|D) = 0.5 or 50%

Now, the probability of disease condition to symptoms,

P (D|S)

= [P (S|D). P (D)] / P(S)

= 0.0002

Example 1

In a group of 100 sports car buyers, 40 bought alarm systems, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random bought an alarm system, what is the probability they also bought bucket seats?

Step 1:

Figure out P(A). It’s given in the question as 40%, or 0.4.

Step 2:

Figure out P(A∩B). This is the intersection of A and B: both happening together. It’s given in the question 20 out of 100 buyers, or 0.2.

Step 3:

$$P(B|A) = P(A \cap B) / P(A) = 0.2 / 0.4 = 0.5.$$

Q. At a certain University, 4% of men are over 6 feet tall and 1% of women are 6 Feet tall. The total student population is divided in the ratio 3:2. If a student is selected at random from among all those over six feet tall, what is the probability that the selected student is woman.

Let

m= men student

w= women student

t= student over 6 feet tall

We know

probability of male student, p(m) = 2/5 = 0.4

probability of women student, p(w) = 3/5 = 0.6

Also, p(t/m)= 4%= 0.04

p(t/w)= 1% = 0.01

Using Bayes theorem

p(w/t) = (p(t/w).p(w))/ (p(t/w).p(w)+ p(t/m).p(m))

= 3/11

Application of Bayes Theorem

- In Manufacturing Process, Selecting best products among two manufacturers

- In bio-chemistry, deciding the diseases based on various blood sample tests. In fact those results are based on probability, so it never be 100% true.

- For project managers : All project managers want to know whether the projects they're working on will finish on time. So, as our example, we'll assume that a project manager asks the question: what's the probability that my project will finish on time? There are only two possibilities here: either the project finishes on (or before) time or it doesn't on the basis of various factors like tools, number of workers, efficiency of workers..

Bayesian Network

- **Bayesian Network** are also known as **Bayes Network, Belief Networks and Probabilistic Networks**

- **Bayesian Network** is a probabilistic graphical model that represents a set of variables and their conditional dependencies via a directed acyclic graph (DAG).

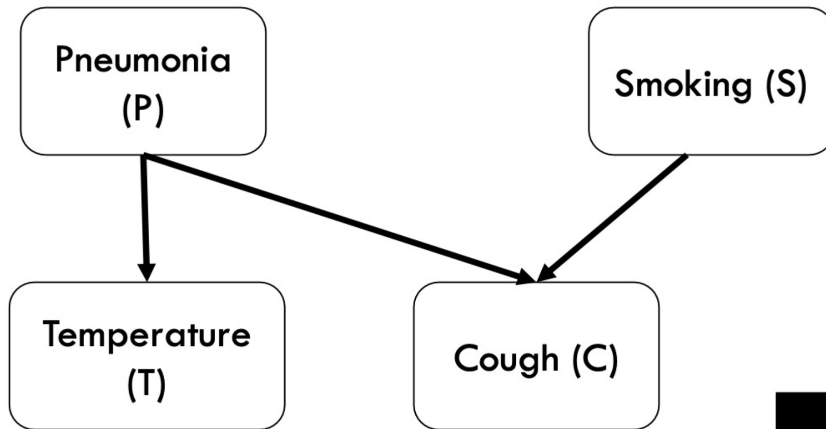
- A BN is defined by two parts, **Directed Acyclic Graph** (DAG) and **Conditional Probability Tables** (CPT)

- For example, a Bayesian network could represent the probabilistic relationships between diseases and symptoms.

- Given symptoms, the network can be used to compute the probabilities of the presence of various diseases.

Bayesian Network

Pneumonia	
True	0.1
False	0.9



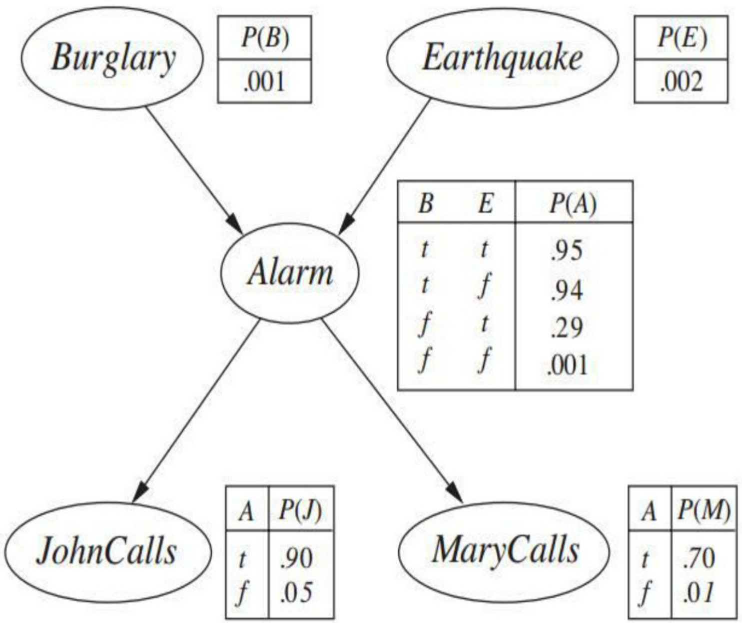
smoking	
True	0.2
False	0.8

	Temperature	
Pneumonia	Yes	No
Yes	0.9	0.1
No	0.2	0.8

		Cough	
Pneumonia	Smoking	True	False
True	Yes	0.95	0.05
True	No	0.8	0.2
False	Yes	0.6	0.4
False	No	0.05	0.95

$$P(C \mid S,P) = P(C \mid \{S=Yes, P=True\}) = P(C=True \mid \{S=Yes, P=True\}) = 0.95$$

Bayesian Network



Inference from Effect to cause; given Burglary, what is P(J|B)?

P(J | B) = ?

first calculate probability of Alarm ringing on burglary:

P(A | B) = P(B)P(¬E)P(B∩ ¬E) + P(B)P(E)P(B∩E)

P(A | B) = 1*(0.998)*(0.94) + 1*(0.002)*(0.95)

P(A | B) = 0.94

Now, Let us calculate P(J | B)

P(J | B) = P(A | B)*P(J) + P(¬(A | B))*P(¬J)

P(J | B) = (0.94) * (0.9) + (0.06) * (0.05) = 0.85

Also calculate P(M|B) = ?

Bayesian Network

Benefits of BN:

- It can readily handle incomplete data sets
- It allows one to learn about causal relationships
- It readily facilitate use of prior knowledge
- It Provide a natural representation for conditional independence

Case-Base Reasoning(CBR):

- AI programs solve problems by reasoning from first principles. They can explain their reasoning by reporting the string of deductions that led from the input data to the conclusion, with the Human Experts.
- An expert encountering a new problem is usually reminded of similar cases seen in the past, remembering the result of those cases and perhaps the reasoning behind those results.

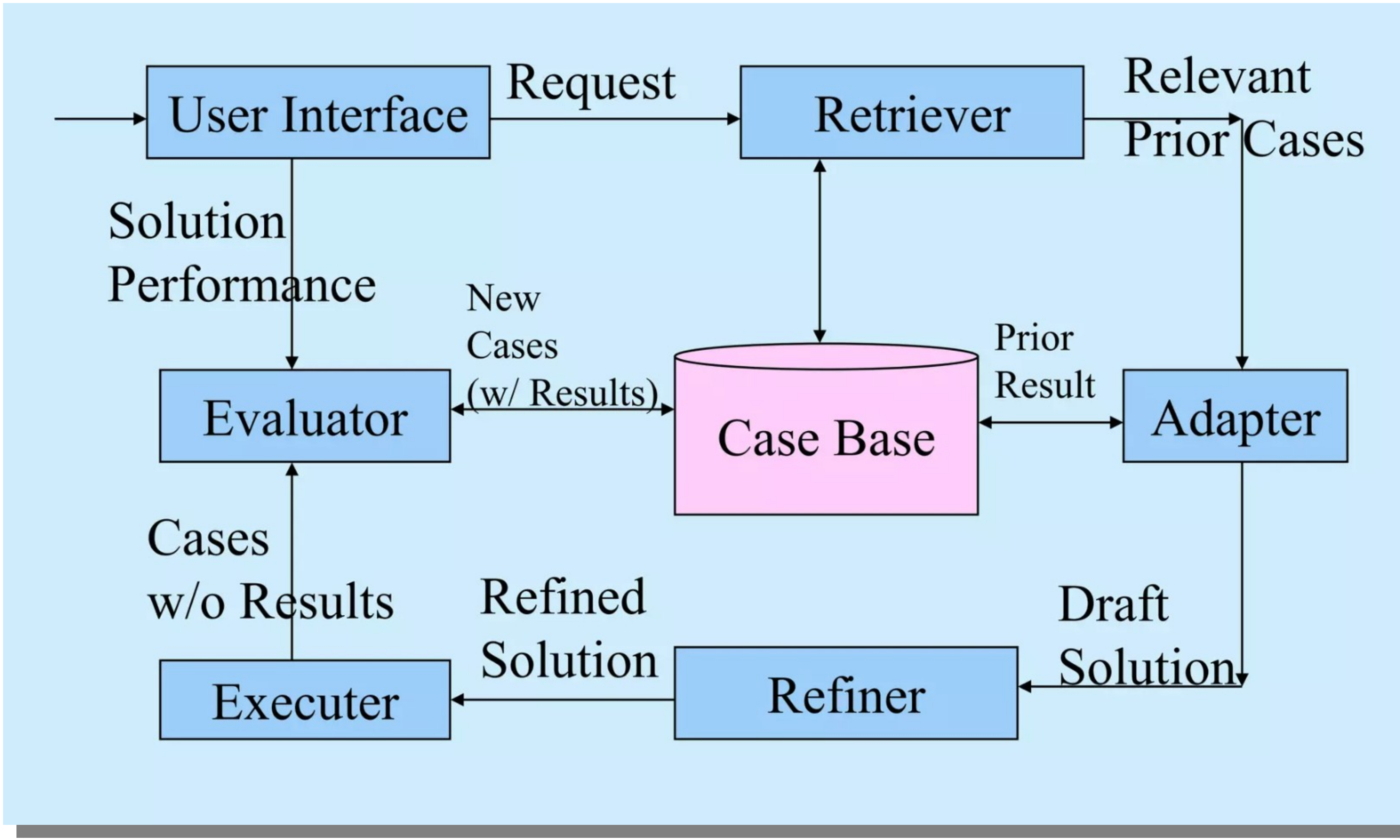
Example: Medical expertise follow this pattern.

- Computer systems that solve new problems by analogy with old ones are often called Case Base Reasoning (CBR).

A successful CBR systems must answer the following questions.

1. How are cases organized in memory ?
2. How are relevant cases retrieved from the memory ?
3. How can previous cases be adopted to new problems ?
4. How are cases originally acquired ?

CBR Components



Reasoning with Uncertainty

- Though there are various types of uncertainty in various aspects of a reasoning system, the "reasoning with uncertainty" research in AI has been focused on the uncertainty of truth value, that is, to allow and process truth values other than "true" and "false".
- To develop a system that reasons with uncertainty means to provide the following:

- a semantic explanation about the origin and nature of the uncertainty
- a way to represent uncertainty in a formal language
- a set of inference rules that derive uncertain (though well-justified) conclusions
- an efficient memory-control mechanism for uncertainty management

Causal Networks

A causal network is an acyclic (not cyclic) directed graph arising from an evolution of a substitution system. The substitution system is a map which uses a set of rules to transform elements of a sequence into anew sequence using a set of rules which "translate" from the original sequence to its transformation.

For example, the substitution system $1 \to 0$, $0 \to 11$ would take $10 \to 011 \to 1100 \to 001111 \to 11110000 \to$ …. ……..

A causal network is a Bayesian network with an explicit requirement that the relationships be causal.

3. Knowledge Representation

3.1 Logic

3.2 Semantic Networks

3.3 Predicate Calculus

3.4 Frames

4. Inference and Reasoning

4.1 Inference Theorems

4.2 Deduction and truth maintenance

4.3 Heuristic search state-space representation

4.4 Game Playing

4.5 Reasoning about uncertainty probability

4.6 Bayesian Networks

4.7 Case-based Reasoning