4a) Write down Maxwell's Equation in integral form. Convert them into differential form. Explain the physical significance of each equation. [10 marks]

Soln→

Maxwell's equations: There are four sets of Maxwell's equation in the integral and differential form which are listed below,

S.N.	Laws	Integral form	Differential form
1	Gauss's law in electrostatics	$\phi_E = \oint_s \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} q$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
2.	Gauss's law in magnetostatics	$\phi_B = \oint_s \vec{B} \cdot d\vec{s} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
3	Faraday's law of electromagnetic induction	$\varepsilon = \oint_l \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$	$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$
4	modified Ampere's law or Ampere-Maxwell law	$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$	$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right)$

Conversion of Maxwell's equation from integral form to differential form / Derivation of Maxwell's equation. [10 marks]

① Maxwell's 1st equation is Gauss's law in electrostatics which states that the total electric flux (ϕ_E) passing through a closed surface is equal to $1/\epsilon_0$ times the charge enclosed by that surface.

i.e. $\phi_E = \frac{1}{\epsilon_0} q$

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \quad \text{—①}$$ which is Maxwell's 1st eqn in integral form.

Using Gauss's divergence theorem,

$$\oint_v (\vec{\nabla}\cdot\vec{E})\, dv = \frac{1}{\epsilon_0}\oint \rho\, dV$$

$$= \oint_V \left(\frac{\rho}{\epsilon_0}\right) dV$$

On equating, $\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$

which is maxwell's 1st equation in differential form.

Physical significance

The 1st equation is, $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ which tells that divergence of electric field is non-zero. i.e. a single charge can act as the source of electric field or electric monopole exists in nature or electric charges can be isolated.

(ii) Maxwell's 2nd equation is Gauss's law in magnetostatics which tells that the total magnetic flux passing through a closed surface is ~~zero~~ equal to zero. i.e.

$$\phi_B = 0$$

or, $$\oint_s \vec{B}\cdot d\vec{s} = 0$$

Using Gauss divergence theorem in L.H.S,

$$\oint_V (\vec{\nabla}\cdot\vec{B})\, dV = 0$$

On equating,

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

which is maxwell's 2nd equation in differential form.

Physical significance

The 2nd equation is $\vec{\nabla}\cdot\vec{B} = 0$ which tells that divergence of ~~electric fiel~~ magnetic field is zero. i.e. it does not have source and sink. Alternatively magnetic poles always occur in pairs or magnetic monopoles does not exist in nature or magnetic lines of force always form closed loops.

(iii) maxwell's 3rd equation is Faraday's law of electromagnetic induction which tells that an induced emf is produced due to the change in magnetic flux. i.e.

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

or, $$\oint_l \vec{E}\cdot\vec{dl} = -\frac{d\phi_B}{dt}$$

which is maxwell's 3rd equation in integral form

Using stoke's theorem in L.H.S,

$$\oint_s (\vec{\nabla}\cdot\vec{E}).\vec{ds} = -\frac{d\phi_B}{dt}$$

$$= -\frac{d}{dt}\oint_s \vec{B}.\vec{ds}$$

$$= \oint_s \left(-\frac{d\vec{B}}{dt}\right).\vec{ds}$$

on equating,

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{dB}{dt}}$$

which is maxwell's 3rd equation in differential form.

Physical signification

The 3rd equation is $\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$ which tells that the time rate of change of the magnetic field produces the electric field.

(iv) maxwell's 4th equation is the modified Ampere's law. Ampere's law states that the line integral of magnetic field around a closed loop is equal to μ_0 times the current enclosed by that loop.

i.e. $\oint_l \vec{B} \cdot \vec{dl} = \mu_0 I$ ——— (1)

Note:

ϵ_0 = permittivity of free space or vacuum or air $\rightarrow 8.854 \times 10^{-12} \, fm^{-1}$

μ_0 = permeability of free space or vacuum

$= 4 \times 10^{-7} \, wbm^{-1}$

From faraday's law of electromagnetic induction,

$\mathcal{E} = \oint_l \vec{E} = -\frac{d\phi_B}{dt}$, we see that the change in the magnetic flux produces electric field. By symmetry, maxwell assumed that the converse must be also true. i.e. the change in the ~~a~~ elecric flux should also produce magnetic field.

such that,

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}}$$

which is called max-well's law of ~~equati~~ induction

Then the modified Ampere's law or Ampere maxwell law or maxwell's 4th equation in the ~~diffe~~ deferential form becomes,

$$\oint_l \vec{B} \cdot \vec{dl} = \mu_0 \left[I + \epsilon_0 \frac{d\phi_E}{dt} \right]$$

Using stokes theorem in L.H.S.,

$$\oint_S \vec{\nabla}(\vec{\nabla}\times\vec{B}).d\vec{s} = \mu_0\left[\oint_s \vec{J}\, ds + \epsilon_0 \frac{d}{dt}\oint \vec{E}.d\vec{s}\right]$$

or, on equating,

$$\vec{\nabla}\times\vec{B} = \mu_0\left[\vec{J} + \epsilon_0\frac{d\vec{E}}{dt}\right]$$

which is maxwell's fourth equation in differential form.

Physical significance

The 4th equation is, $\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{d\vec{E}}{dt}\right)$. It tells that the magnetic field is produced ~~to~~ due to;

a) conduction current density $\vec{J}$.

& b) Displacement current density

$$J_d = \epsilon_0\frac{d\vec{E}}{dt}$$ This J_d is produced due to the changes in the electric field.

Board

Q. what is displacement current? Explain its significance (5)

OR, what is displacement current density? Discuss its significance (5)

Q. why did maxwell modify Amperes law? explain with mathematical details?