1 Monte Hall Problem

- 3 doors and they contain 2 goats, 1 sports car
- You pick a door at random. It stays closed
- Monte opens one the other two doors and shows you a goat (always).
- You can have what's behind your current door or the other (unopened) door.
- $S = \{Car, Goat\#1, Goat\#2\}$
- $Pr(w) = 1/3$ for each w $\in S$.
- A = "You picked the door with the car"
- $Pr(A) = \frac{1}{3}$
- $Pr(A^{\complement}) = 1 - Pr(A) = 2/3$

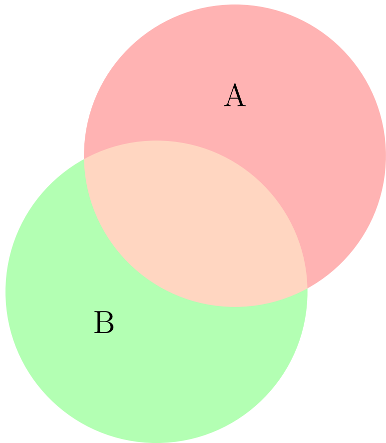
1.1 example

- Anil has two children, $S = \{bb, gg, bg, gb\}$
- At least one of Anil's children is a boy
- $B = \{bb, bg, gb\}$
- What is the probability that both of Anil's children are boys
- $A = \{bb\}$
- $Pr(A) = \frac{1}{3}$

Definition: Let (S, Pr) be a probability space are $A, B \subseteq S$ be events with $Pr(B) > 0$ then

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

This is telling the *"probability of A given B."*



$$Pr(A) = \frac{Pr(A)}{Pr(S)}$$

In the universial set S

Now we can compute the probability of Anil's children

$$\begin{aligned} Pr(A|B) &= \frac{Pr(A \cap B)}{Pr(B)} \\ &= \frac{Pr(bb)}{3/4} = \frac{1/4}{3/4} \\ &= 1/3 \end{aligned}$$

Roll a die $S = \{1, 2, 3, 4, 5, 6\} Pr(w) = 1/6$ for each $w \in S$
A = "the result of the roll is 3" $= \{3\}$. $PrA = 1/6$
B = "the result is an odd integer" $= \{1, 3, 5\}$ $PrB = \frac{B}{S}$

$$\begin{aligned} \Pr(A|B) &= \frac{Pr(A \cap B)}{Pr(B)} \\ &= \frac{Pr(\{3\})}{1/2} = \frac{1/6}{1/2} = 2/3 \\ &= 1/3 \end{aligned}$$

$$\begin{aligned} Pr((B|A)^{\complement}) &= \frac{Pr(A \cap B)}{Pr(A)} \\ &= \frac{Pr(\{3\})}{Pr(\{3\})} = 1 \end{aligned}$$

C = "the result is a prime number" $= 2, 3, 5$

$$Pr(C|B) = \frac{Pr(B \cap C)}{Pr(B)}$$
$$= \frac{Pr(\{3,5\})}{3/6} = \frac{2/6}{3/6} = 2/3$$

$B^{\complement}$ = "The result is an even integer {2,4,6}.

$$Pr(C|B^{\complement}) = \frac{Pr(C \cap B^{\complement})}{Pr(B)}$$
$$= \frac{Pr(\{2\})}{3/6} = \frac{1/6}{3/6} = 1/3$$

Lemma 1.

$$Pr(A|B) + Pr(A^{\complement}|B) = 1$$

Def.

$$Pr(A|B) + Pr(A^{\complement}|B) = \frac{Pr(A \cap B) + Pr(A^{\complement} \cap B)}{Pr(B)}$$

(missed some)
- Anil has two children
- At least one of Anil's children is a boy who was born on a Sunday.
- What is the probability that both of Anil children are boys

$S = \{(s_1, d_1, s_2, d_2) : s_1, s_2 in\{b, g\} d_1, d_2 \in W\}$
$W = \{Su, Mo, Tu, Thur, Fri, Sat\}$
$|S| = 14^2$
$|W| = 7$
B = "At least one of Anil's children is a boy who was born on a Sunday."
A = "both Anil's children are boys."
$B = \{(b, Su, s_2, d_2) : s_2 \in \{b, g\}, d_2 \in W\} \cup \{(s_1, d_1, b, Su) : s_1 \in \{b, g\}, d_1 \in W\}$
$A = \{(b_1, d_1, b_2, d_2) : d_1, d_2 \in W\}$
$A \cap B = \{(b, Su, b, d_2) : d_2 \in W\} \cup \{(b, d_1, b, Su) : d_1 \in W\}$
$|A \cap B| = |X \cup Y|$
$|A \cap B| = |X| + |Y| - |X \cap Y| = 7 + 7 - 1 = 13$
$|B| = |X' \cup Y'|$
$|B| = |X'| + |Y'| - |X'\ capY'| = 14 + 14 - 1 = 27$

$$
\begin{aligned}
\Pr(A|B) &= \frac{Pr(A \cap B)}{Pr(B)} \\
&= \frac{Pr(|A \cup B| \setminus |S|)}{Pr(|A \cup B| \setminus |S|)} \\
&= \frac{13}{27} \\
&= 48.15\%???!?!
\end{aligned}
$$