1 Independent sets

Def
Two events A and B are independent if $Pr(A \cap B) = Pr(A) \cdot Pr(B)$.
Consquence
If A and B are independent and $Pr(B) > 0$, then

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A) \cdot Pr(B)}{Pr(B)}$$

Example 1. *Rolling Dice* $S = \{(D_1, D_2) : D_1, D_2 \in \{1, 2, 3, 4, 5, 6\}\}$
$Pr(w) = \frac{1}{|S|} = \frac{1}{36}$
$A =$ "$D_1 = 4$" $= \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$
$B =$ "$D_2 = 3$" $= \{(1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)\}$
$A \cap B = \{(4, 3)\}$
$Pr(A \cap B) = 1/36$
$Pr(A) = |A|/36 = 6/36 = 1/6$
$Pr(B) = |B|/36 = 6/36 = 1/6$
$Pr(A) \cdot Pr(B) = 1/6 \cdot 1/6 = 1/36$
Thus, A and B are independent.

Note: Do not use the word dependent when refering the sets that are not independent.

Now assume that $B =$ "$D_1 + D_2 = 7$" $= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$
$A \cap B = \{(4, 3)\}$
$Pr(A \cap B) = 1/36$
$Pr(A) = |A|/36 = 6/36 = 1/6$
$Pr(B) = |B|/36 = 6/36 = 1/6$
$Pr(A) \cdot Pr(B) = 1/6 \cdot 1/6 = 1/36$
Thus, A and B are independent.

Lemma 1. *If A and B are independent. then A and* $B^{\complement}$ *are independent.*

. Need to show

$$\begin{aligned} Pr(A \cap B^{\complement}) &= Pr(A) \cdot Pr(B^{\complement}) \\ &= Pr(A)(-Pr(B)) \end{aligned}$$

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\begin{aligned}
Pr(A) &= Pr(A \cap B) + Pr(A \cap B^{\complement}) \\
&= Pr(A) \cdot Pr(B) + Pr(A \cap B^{\complement}) \\
\leftrightarrow Pr(A \cap B^{\complement}) &= Pr(A) - Pr(A) \cdot Pr(B) \\
&= Pr(A)(1 - Pr(B))
\end{aligned}
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□