Tumour Growth Model Proposal

Objective

The objective of this project is to simulate and analyze tumor growth under drug treatment using a system of ordinary differential equations

Ordinary Differential Equations

$$\begin{cases} f(t) = k_g\, T \left(1 - \frac{T}{T_{\max}}\right) \\ \dfrac{dS}{dt} = f(S) - k_d \,\text{Exposure}\, S \\ \dfrac{dD}{dt} = k_d \,\text{Exposure}\, S - d\, D \\ T = S + D \end{cases}$$

Variables:

- f(t): Natural growth function using logistic growth
- T(t): Total tumor size
- S(t): Viable (alive) tumor cells
- D(t): Cells that have been hit by the drug (converted from S) but not yet cleared from the tumor
- k_g: Tumor growth rate constant
- $T_{\max}$: Carrying capacity, maximum tumor size that can be sustained by the environment
- k_d: Drug kill coefficient, Controls how strongly the drug converts viable cells S into damaged cells D
- Exposure: Drug exposure or effect over time, can be changed to vary over time
- d: Rate at which damaged/dead cells D are removed from the tumor volume

Tumor growth and response to anti-cancer therapy is a well-studied system that can be effectively modeled using ordinary differential equations. These equations describe the relation between tumor cell growth, drug-induced cell death, and the clearance of dead/damaged cells.

Implementation

All calculations and visualizations for this project will be carried out in C++. The system of ODEs will be implemented as a set of functions representing the rate of change.

The program will be tested under several conditions to verify that the model behaves as expected. Under no drug exposure the tumor should follow pure logistic growth, increasing smoothly until it approaches the carrying capacity. Under constant drug exposure, viable cells (S) should decrease while damaged cells (D) initially rise and then decay as they clear, leading to a drop in total tumor size. Additionally we can add varying drug exposure to see how the model performs.

We can also modify our 2nd equation to account for drug resistance

$$\frac{dT}{dt} = f(T) - k_d \cdot e^{-\lambda t} \cdot \text{Exposure} \cdot T$$

Reference

Yin, Anyue. A Review of Mathematical Models for Tumor Dynamics and Treatment Resistance Evolution of Solid Tumors. 9 August 2019. National Library Of Medicine, https://pmc.ncbi.nlm.nih.gov/articles/PMC6813171/.