In [1]:
```python
# Importing necessary libraries
import numpy as np
import pandas as pd
import yfinance as yf
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
from sklearn.metrics import r2_score
from pyts.image import GramianAngularField
import tensorflow as tf
from keras import Sequential
from keras.callbacks import EarlyStopping
from keras.layers import Dense, LSTM, Dropout, Conv2D, MaxPool2D, Flatten

sns.set(style="whitegrid")
```

Step 1

a. Gather information on time series of the prices

In [2]:
```python
# Define stock information
ticker = 'AMZN'
start = '2015-01-01'
end = '2023-01-01'

# Download stock data using yfinance
stock_data = yf.download(ticker, start=start, end=end)

# Calculate daily returns and drop missing values
stock_data['returns'] = stock_data['Close'].pct_change()
stock_data.dropna(inplace=True)

# Display the first few rows of the stock data
stock_data.head()
```

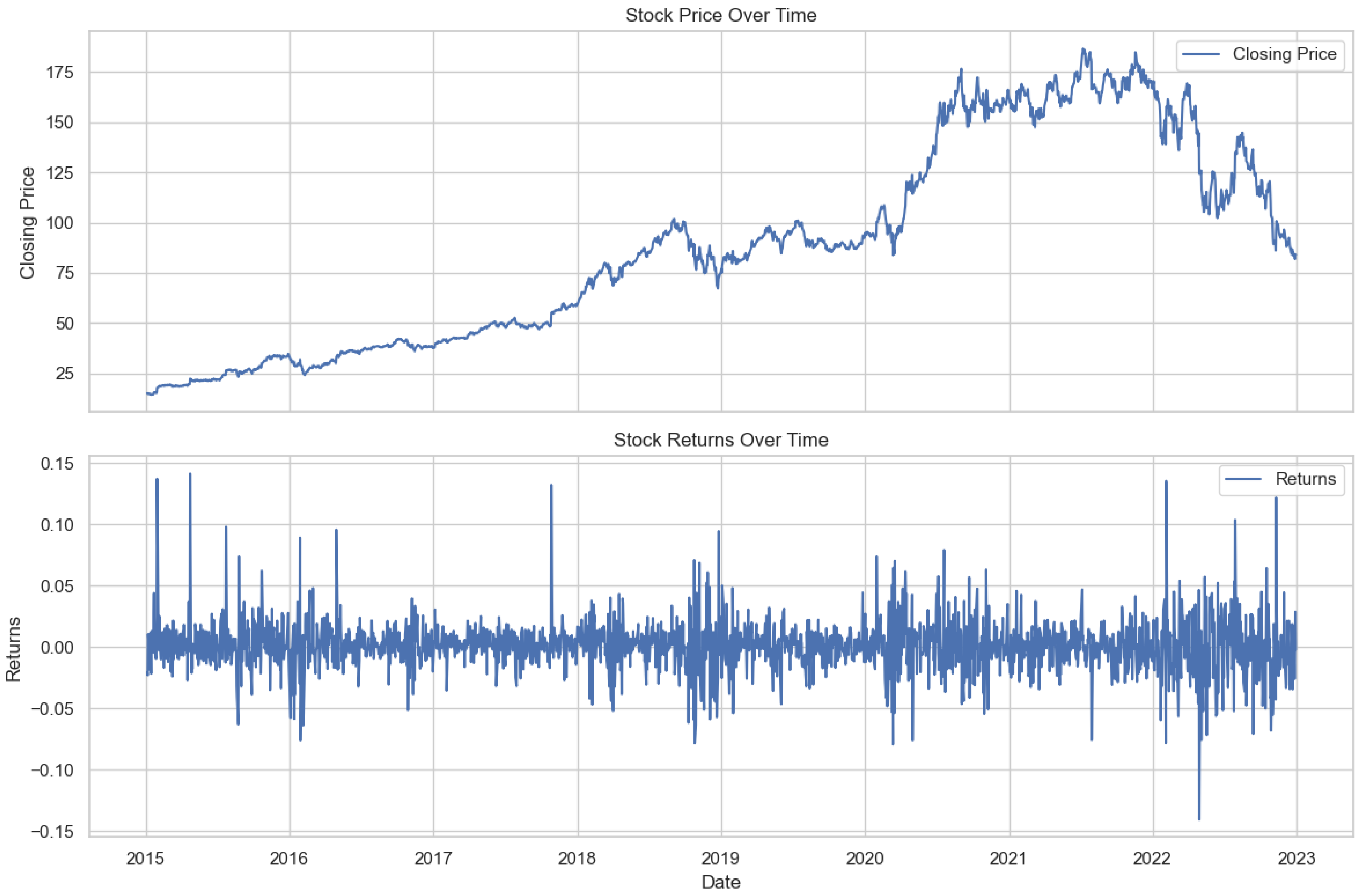
```
[*********************100%%**********************]  1 of 1 completed
```

Out[2]:

	Open	High	Low	Close	Adj Close	Volume	returns
Date							
2015-01-05	15.3505	15.4190	15.0425	15.1095	15.1095	55484000	-0.020517
2015-01-06	15.1120	15.1500	14.6190	14.7645	14.7645	70380000	-0.022833
2015-01-07	14.8750	15.0640	14.7665	14.9210	14.9210	52806000	0.010600
2015-01-08	15.0160	15.1570	14.8055	15.0230	15.0230	61768000	0.006836
2015-01-09	15.0740	15.1435	14.8340	14.8465	14.8465	51848000	-0.011749

In [3]:
```python
# Create a 2x1 subplot grid
fig, axs = plt.subplots(2, 1, figsize=(12, 8), sharex=True)

# First subplot for Closing Price
axs[0].plot(stock_data['Close'], label='Closing Price')
axs[0].set_ylabel('Closing Price')
axs[0].set_title('Stock Price Over Time')
axs[0].legend()

# Second subplot for Stock Returns
axs[1].plot(stock_data['returns'], label='Returns')
axs[1].set_xlabel('Date')
axs[1].set_ylabel('Returns')
axs[1].set_title('Stock Returns Over Time')
axs[1].legend()

# Adjust layout for better appearance
plt.tight_layout()

# Descriptive statistics for Closing Price
print("Descriptive Statistics for Closing Price:")
print(stock_data['Close'].describe())

# Descriptive statistics for Returns
print("\nDescriptive Statistics for Returns:")
print(stock_data['returns'].describe())

# Show the plot
plt.show()
```

```
Descriptive Statistics for Closing Price:
count    2013.000000
mean       88.334524
std        49.948132
min        14.347500
25%        40.949501
50%        86.982498
75%       127.898003
max       186.570496
Name: Close, dtype: float64

Descriptive Statistics for Returns:
count    2013.000000
mean        0.001061
std         0.020975
min        -0.140494
25%        -0.008544
50%         0.001168
75%         0.010892
max         0.141311
Name: returns, dtype: float64
```

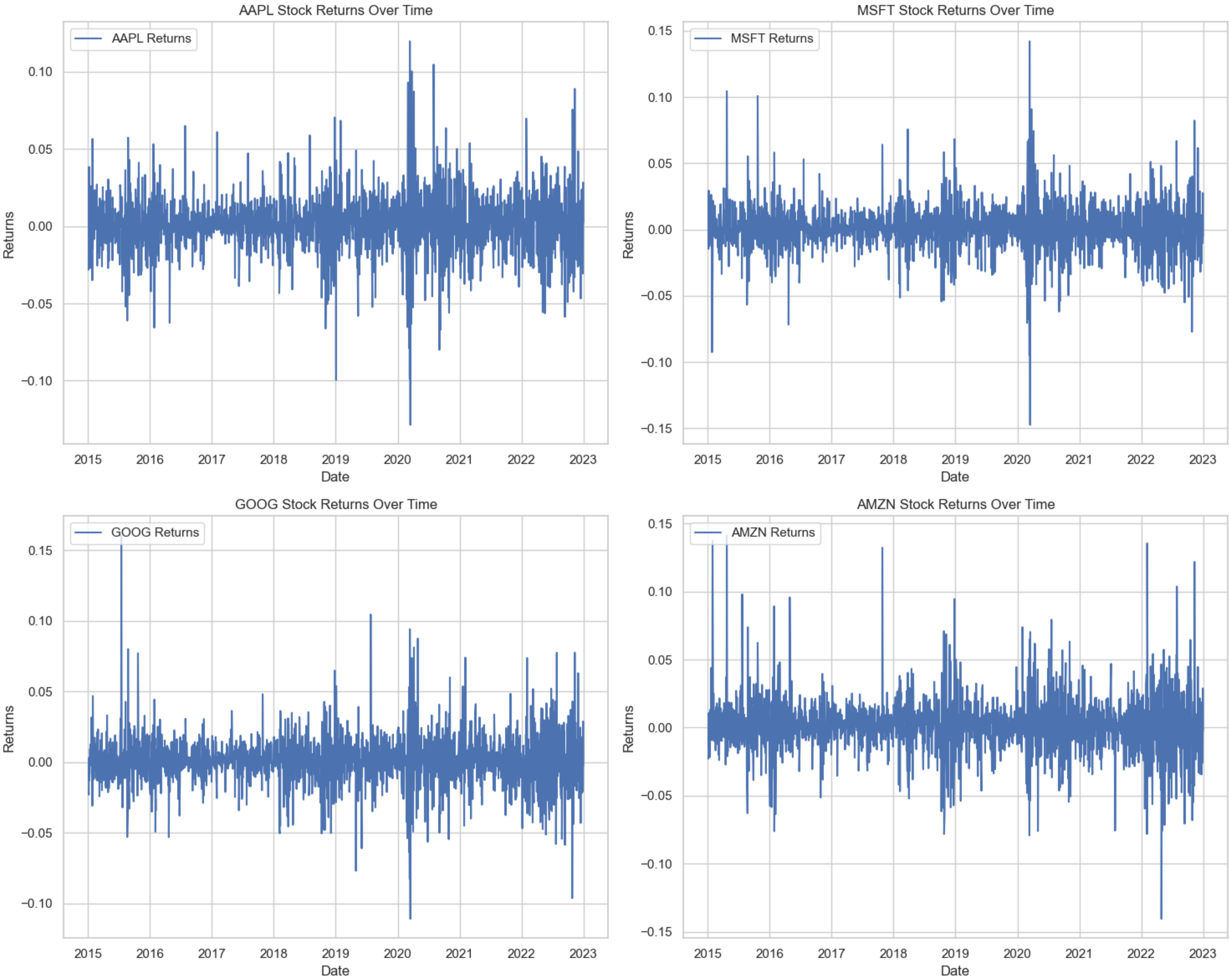


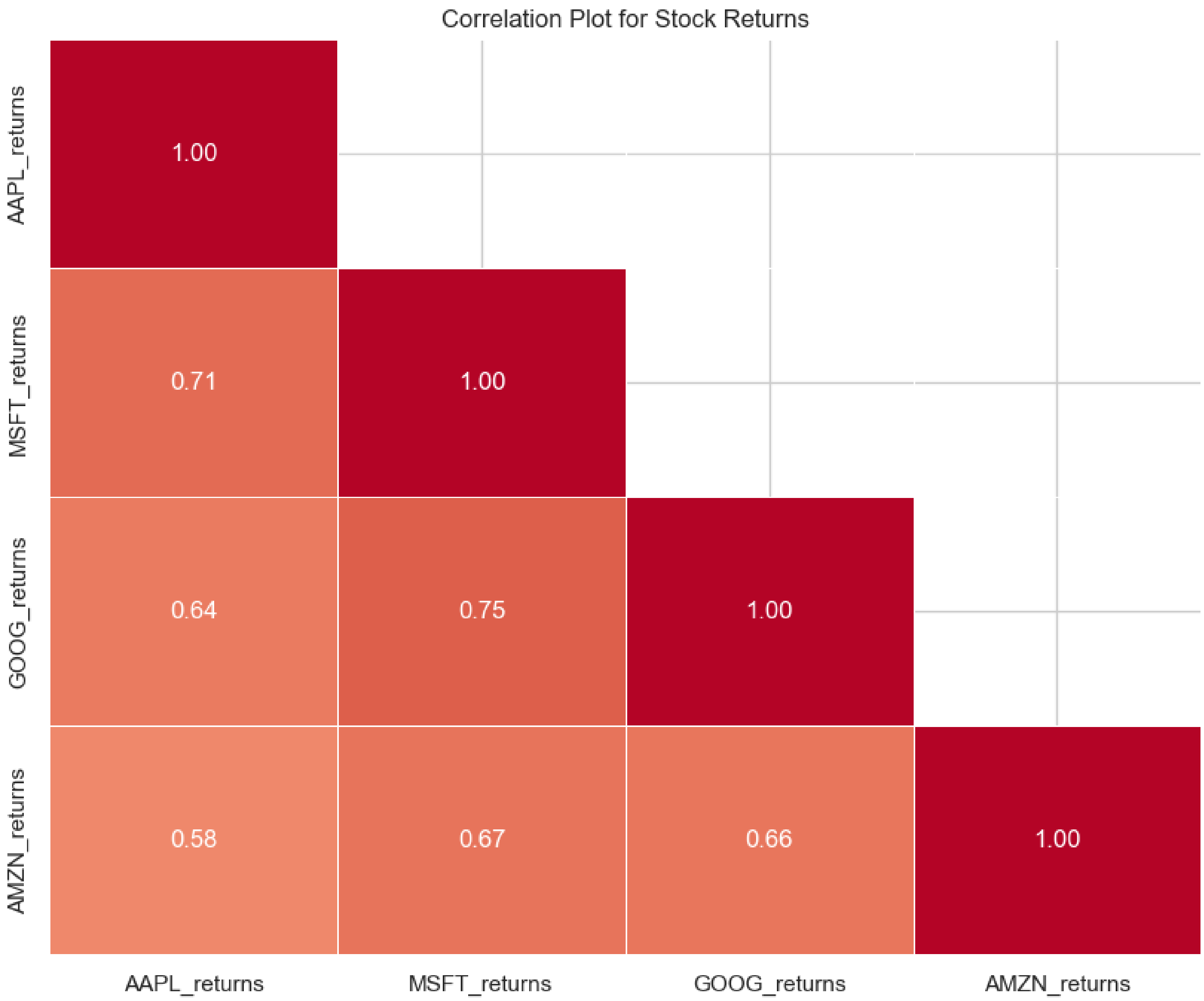
b. Build labels with leakage of information

In [4]:
```
# Function to fetch stock data
def fetch_stock_data(tickers, start, end):
    stock_data = pd.DataFrame()

    # Fetch stock data for each ticker and calculate returns and log volume
    for tick in tickers:
        df = yf.download(tick, start=start, end=end)
        stock_data[f'{tick}_returns'] = df['Close'].pct_change()[1:]
        stock_data[f'{tick}_volume'] = np.log(df['Volume'][1:])

    stock_data.dropna(inplace=True)
    return stock_data

# Define a list of stock tickers
tickers = ['AAPL', 'MSFT', 'GOOG', 'AMZN']

# Fetch stock data using the defined function
stock_data = fetch_stock_data(tickers, start, end)

# Create a 2x2 subplot grid
fig, axs = plt.subplots(2, 2, figsize=(15, 12))

# Plot individual stock closing prices and returns
for i, tick in enumerate(tickers):
    row, col = divmod(i, 2)

    axs[row, col].plot(stock_data.index, stock_data[f'{tick}_returns'], label=f'{tick} Returns')
    axs[row, col].set_title(f'{tick} Stock Returns Over Time')
    axs[row, col].set_xlabel('Date')
```

```python
    axs[row, col].set_ylabel('Returns')
    axs[row, col].legend(loc='upper left')

plt.tight_layout()

# Descriptive statistics for returns
print("Descriptive Statistics for Returns:")
print(stock_data.filter(like='_returns').describe())

# Correlation plot for stock returns
correlation_matrix = stock_data.filter(like='_returns').corr()
plt.figure(figsize=(10, 8))
sns.heatmap(correlation_matrix,
            mask=np.triu(correlation_matrix, 1),
            annot=True,
            cmap='coolwarm',
            vmin=-1,
            vmax=1,
            fmt=".2f",
            linewidths=0.5,
            cbar=False)
plt.title('Correlation Plot for Stock Returns')
plt.show()
```

```
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
[*********************100%%**********************]  1 of 1 completed
Descriptive Statistics for Returns:
       AAPL_returns  MSFT_returns  GOOG_returns  AMZN_returns
count   2013.000000   2013.000000   2013.000000   2013.000000
mean       0.000953      0.000970      0.000765      0.001061
std        0.018875      0.017747      0.017852      0.020975
min       -0.128647     -0.147390     -0.111008     -0.140494
25%       -0.007630     -0.006849     -0.006885     -0.008544
50%        0.000749      0.000827      0.000866      0.001168
75%        0.010345      0.009845      0.009048      0.010892
max        0.119808      0.142169      0.160524      0.141311
```





In [5]:
```
# Set parameters for data preparation
window_size = 20
window_hp = 10

# Data preparation with leakage (deliberately ignoring the past signals from the rolling mean)
stock_data['target'] = (1 + stock_data[f'{ticker}_returns']).rolling(window=window_hp).apply(np.prod, raw=True) - 1
stock_data.dropna(inplace=True)

# Split data into features (X) and target variable (y)
X, y = stock_data.drop(columns=[f'{ticker}_returns', 'target']), stock_data['target']

# Split data into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.4, shuffle=False)

# Standardize the features using StandardScaler
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)
```

In [6]:
```
# Convert data to numpy arrays for MLP model
X_train_mlp = np.array(X_train_scaled)
X_test_mlp = np.array(X_test_scaled)
y_train_mlp = np.array(y_train)
y_test_mlp = np.array(y_test)

# Data preparation for LSTM model
X_train_lstm = []
y_train_lstm = []
X_test_lstm = []
y_test_lstm = []

# Create sequences for training data
for i in range(window_size, y_train.shape[0]):
    X_train_lstm.append(X_train_scaled[i - window_size: i, :])
    y_train_lstm.append(y_train.iloc[i])

X_train_lstm, y_train_lstm = np.array(X_train_lstm), np.array(y_train_lstm)

# Create sequences for testing data
for i in range(window_size, y_test.shape[0]):
    X_test_lstm.append(X_test_scaled[i - window_size: i, :])
    y_test_lstm.append(y_test.iloc[i])

X_test_lstm, y_test_lstm = np.array(X_test_lstm), np.array(y_test_lstm)

# Data preparation for CNN model
X_train_cnn = []
y_train_cnn = y_train_lstm.copy()

# Create Gramian Angular Field images for training data
```

```
for c in range(X_train_scaled.shape[1]):
    slice_train = []
    for i in range(window_size, y_train.shape[0]):
        transformer = GramianAngularField()
        slice_train.append(X_train_scaled[:, c][i - window_size: i])
    X_train_cnn.append(((transformer.transform(slice_train) + 1) / 2) * 255)

X_train_cnn = np.array(X_train_cnn).transpose(1, 2, 0, 3)
X_train_cnn = X_train_cnn.reshape(X_train_scaled.shape[0] - window_size, X_train_scaled.shape[1] * window_size, window_size)

# Data preparation for testing data
X_test_cnn = []
y_test_cnn = y_test_lstm.copy()
for c in range(X_test_scaled.shape[1]):
    slice_test = []
    for i in range(window_size, y_test.shape[0]):
        transformer = GramianAngularField()
        slice_test.append(X_test_scaled[:, c][i - window_size: i])
    X_test_cnn.append(((transformer.transform(slice_test) + 1) / 2) * 255)

X_test_cnn = np.array(X_test_cnn).transpose(1, 2, 0, 3)
X_test_cnn = X_test_cnn.reshape(X_test_scaled.shape[0] - window_size, X_test_scaled.shape[1] * window_size, window_size)

# Print shapes of the prepared data
print("X_train_mlp shape:", X_train_mlp.shape)
print("X_test_mlp shape:", X_test_mlp.shape)
print("y_train_mlp shape:", y_train_mlp.shape)
print("y_test_mlp shape:", y_test_mlp.shape)
print("X_train_lstm shape:", X_train_lstm.shape)
print("y_train_lstm shape:", y_train_lstm.shape)
print("X_test_lstm shape:", X_test_lstm.shape)
print("y_test_lstm shape:", y_test_lstm.shape)
print("X_train_cnn shape:", X_train_cnn.shape)
print("y_train_cnn shape:", y_train_cnn.shape)
print("X_test_cnn shape:", X_test_cnn.shape)
print("y_test_cnn shape:", y_test_cnn.shape)
```

```
X_train_mlp shape: (1202, 7)
X_test_mlp shape: (802, 7)
y_train_mlp shape: (1202,)
y_test_mlp shape: (802,)
X_train_lstm shape: (1182, 20, 7)
y_train_lstm shape: (1182,)
X_test_lstm shape: (782, 20, 7)
y_test_lstm shape: (782,)
X_train_cnn shape: (1182, 140, 20)
y_train_cnn shape: (1182,)
X_test_cnn shape: (782, 140, 20)
y_test_cnn shape: (782,)
```

c. Deep learning models

In [7]:

```
# Set random seed for reproducibility
seed = 42
np.random.seed(seed)
tf.random.set_seed(seed)
initializer = tf.keras.initializers.GlorotUniform(seed)
```

MLP model

In [8]:

```
# MLP Model
print("Training MLP model")
model_mlp = Sequential([
    Dense(units=64, activation='relu', kernel_initializer=initializer, bias_initializer='zeros', input_shape=(X_train_mlp.shape[1],)),
    Dense(units=128, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=64, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=32, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=1, activation='linear')
])

model_mlp.compile(optimizer='adam', loss='mean_absolute_error')
es = EarlyStopping(monitor='val_loss', mode='min', verbose=0, patience=5, restore_best_weights=True)

# MLP Model Training
model_mlp.fit(X_train_mlp, y_train_mlp, validation_split=0.3, epochs=30, batch_size=32, verbose=1, callbacks=[es])

# MLP Model Prediction
y_pred_mlp = model_mlp.predict(X_test_mlp)

# Calculate R2 score for MLP
r2_mlp = r2_score(y_test, y_pred_mlp.flatten())
print("R2 Score (MLP):", r2_mlp)
```

```
Training MLP model
Epoch 1/30
27/27 [==============================] - 1s 7ms/step - loss: 0.0572 - val_loss: 0.0473
Epoch 2/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0422 - val_loss: 0.0492
Epoch 3/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0384 - val_loss: 0.0466
Epoch 4/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0353 - val_loss: 0.0452
Epoch 5/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0352 - val_loss: 0.0489
Epoch 6/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0341 - val_loss: 0.0491
Epoch 7/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0349 - val_loss: 0.0489
Epoch 8/30
27/27 [==============================] - 0s 3ms/step - loss: 0.0348 - val_loss: 0.0466
Epoch 9/30
27/27 [==============================] - 0s 4ms/step - loss: 0.0326 - val_loss: 0.0513
26/26 [==============================] - 0s 800us/step
R2 Score (MLP): -0.1053166058982018
```

LSTM model

In [9]:

```
# LSTM Model
print("Training LSTM model")
model_lstm = Sequential([
    LSTM(units=64, return_sequences=True, activation='tanh', input_shape=(window_size, X_train_lstm.shape[2]), kernel_initializer=init
    LSTM(units=128, return_sequences=True, activation='tanh', kernel_initializer=initializer, bias_initializer='zeros'),
    LSTM(units=32, return_sequences=False, activation='tanh', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=64, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=128, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=64, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=32, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=1, activation='linear')
])

model_lstm.compile(optimizer='adam', loss='mean_absolute_error')

# LSTM Model Training
model_lstm.fit(X_train_lstm, y_train_lstm, validation_split=0.3, epochs=30, batch_size=32, verbose=1, callbacks=[es])

# LSTM Model Prediction
y_pred_lstm = model_lstm.predict(X_test_lstm)

# Calculate R2 score for LSTM
r2_lstm = r2_score(y_test[window_size:], y_pred_lstm.flatten())
print("R2 Score (LSTM):", r2_lstm)
```

```
Training LSTM model
Epoch 1/30
26/26 [==============================] - 4s 38ms/step - loss: 0.0358 - val_loss: 0.0392
Epoch 2/30
26/26 [==============================] - 0s 12ms/step - loss: 0.0301 - val_loss: 0.0380
Epoch 3/30
26/26 [==============================] - 0s 11ms/step - loss: 0.0285 - val_loss: 0.0419
Epoch 4/30
26/26 [==============================] - 0s 11ms/step - loss: 0.0250 - val_loss: 0.0451
Epoch 5/30
26/26 [==============================] - 0s 13ms/step - loss: 0.0252 - val_loss: 0.0494
Epoch 6/30
26/26 [==============================] - 0s 11ms/step - loss: 0.0227 - val_loss: 0.0398
Epoch 7/30
26/26 [==============================] - 0s 12ms/step - loss: 0.0212 - val_loss: 0.0384
25/25 [==============================] - 1s 4ms/step
R2 Score (LSTM): 0.21429252349232386
```

CNN model

In [10]:

```
# CNN Model
print("Training CNN model")
model_cnn = Sequential([
    Conv2D(filters=32, kernel_size=3, activation='relu', input_shape=(X_train_scaled.shape[1] * window_size, window_size, 1), kernel_i
    MaxPool2D(pool_size=2),
    Conv2D(filters=64, kernel_size=3, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    MaxPool2D(pool_size=2),
    Flatten(),
    Dense(units=128, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=32, activation='relu', kernel_initializer=initializer, bias_initializer='zeros'),
    Dense(units=1, activation='linear')
])

model_cnn.compile(optimizer='adam', loss='mean_absolute_error')

# CNN Model Training
model_cnn.fit(X_train_cnn, y_train_cnn, validation_split=0.3, epochs=30, batch_size=32, verbose=1, callbacks=[es])

# CNN Model Prediction
y_pred_cnn = model_cnn.predict(X_test_cnn)

# Calculate R2 score for CNN
```

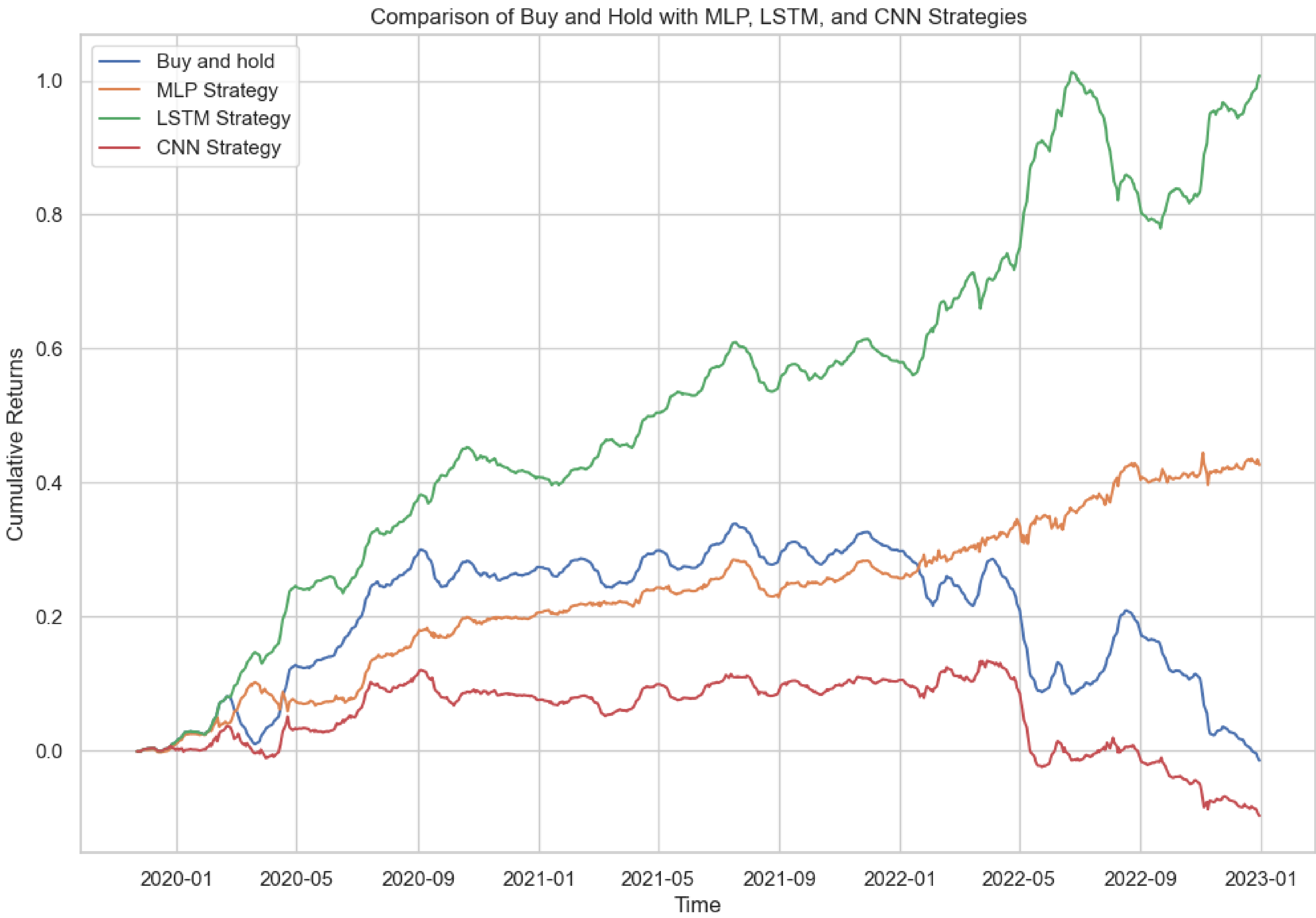
```
r2_cnn = r2_score(y_test[window_size:], y_pred_cnn.flatten())
print("R2 Score (CNN):", r2_cnn)
```

```
Training CNN model
Epoch 1/30
26/26 [==============================] - 2s 19ms/step - loss: 10.6622 - val_loss: 0.3835
Epoch 2/30
26/26 [==============================] - 0s 5ms/step - loss: 0.1627 - val_loss: 0.0815
Epoch 3/30
26/26 [==============================] - 0s 5ms/step - loss: 0.0510 - val_loss: 0.0590
Epoch 4/30
26/26 [==============================] - 0s 5ms/step - loss: 0.0366 - val_loss: 0.0561
Epoch 5/30
26/26 [==============================] - 0s 7ms/step - loss: 0.0310 - val_loss: 0.0579
Epoch 6/30
26/26 [==============================] - 0s 5ms/step - loss: 0.0279 - val_loss: 0.0586
Epoch 7/30
26/26 [==============================] - 0s 5ms/step - loss: 0.0257 - val_loss: 0.0595
Epoch 8/30
26/26 [==============================] - 0s 5ms/step - loss: 0.0231 - val_loss: 0.0600
Epoch 9/30
26/26 [==============================] - 0s 5ms/step - loss: 0.0224 - val_loss: 0.0617
25/25 [==============================] - 0s 4ms/step
R2 Score (CNN): -0.29289432516301694
```

In [11]:

```
# Create a DataFrame with predictions and strategy returns
df_pred = pd.DataFrame({
    'y_test': y_test[window_size:],
    'pred_mlp': y_pred_mlp.flatten()[window_size:],
    'pred_lstm': y_pred_lstm.flatten(),
    'pred_cnn': y_pred_cnn.flatten()
})

# Calculate buy and hold cumulative return
df_pred['cum_ret_bh'] = df_pred['y_test'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# MLP Strategy
df_pred['positions_mlp'] = df_pred['pred_mlp'].apply(np.sign)
df_pred['strat_ret_mlp'] = df_pred['positions_mlp'] * df_pred['y_test']
df_pred['cum_ret_mlp'] = df_pred['strat_ret_mlp'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# LSTM Strategy
df_pred['positions_lstm'] = df_pred['pred_lstm'].apply(np.sign)
df_pred['strat_ret_lstm'] = df_pred['positions_lstm'] * df_pred['y_test']
df_pred['cum_ret_lstm'] = df_pred['strat_ret_lstm'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# CNN Strategy
df_pred['positions_cnn'] = df_pred['pred_cnn'].apply(np.sign)
df_pred['strat_ret_cnn'] = df_pred['positions_cnn'] * df_pred['y_test']
df_pred['cum_ret_cnn'] = df_pred['strat_ret_cnn'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# Calculate final returns
buy_return = np.prod(1 + df_pred['y_test']) ** (window_hp/252) - 1
strat_return_mlp = np.prod(1 + df_pred['strat_ret_mlp']) ** (window_hp/252) - 1
strat_return_lstm = np.prod(1 + df_pred['strat_ret_lstm']) ** (window_hp/252) - 1
strat_return_cnn = np.prod(1 + df_pred['strat_ret_cnn']) ** (window_hp/252) - 1

# Print returns
print("Buy and Hold Return: {:.4%}".format(buy_return))
print("MLP Strategy Return: {:.4%}".format(strat_return_mlp))
print("LSTM Strategy Return: {:.4%}".format(strat_return_lstm))
print("CNN Strategy Return: {:.4%}".format(strat_return_cnn))

# Combined Plot
plt.figure(figsize=(12, 8))

# Plot cumulative returns with labels and legend
plt.plot(df_pred['cum_ret_bh'], label='Buy and hold')
plt.plot(df_pred['cum_ret_mlp'], label='MLP Strategy')
plt.plot(df_pred['cum_ret_lstm'], label='LSTM Strategy')
plt.plot(df_pred['cum_ret_cnn'], label='CNN Strategy')

# Add labels and title
plt.xlabel('Time')
plt.ylabel('Cumulative Returns')
plt.title('Comparison of Buy and Hold with MLP, LSTM, and CNN Strategies')
plt.legend()

# Display the plot
plt.show()
```

```
Buy and Hold Return: -1.4490%
MLP Strategy Return: 42.6781%
LSTM Strategy Return: 100.7310%
CNN Strategy Return: -9.6984%
```



Step 2

a. Non achored walk forward method with train/test split with 500 observations in each set.

In [12]:
```
def walk_forward_backtest(model, X, y, train_size, test_size):

    predictions = []

    # Calculate the number of steps for the walk-forward backtest
    num_steps = len(range(train_size, len(X), test_size))

    # Iterate through the data for walk-forward backtesting
    for step, i in enumerate(range(train_size, len(X), test_size)):
        X_train = X[i - train_size:i]
        X_test = X[i:i + test_size]
        y_train = y[i - train_size:i]
        y_test = y[i:i + test_size]

        # Print information about the current step
        print(f"Step {step + 1}/{num_steps}, Index: {i} \n")
        print(f"X_train shape: {X_train.shape}, X_test shape: {X_test.shape}, y_train shape: {y_train.shape}, y_test shape: {y_test.sl

        # Train the model on the current training set
        model.fit(X_train, y_train, validation_split=0.3, epochs=30, batch_size=32, verbose=1, callbacks=[es])

        # Extend the predictions with the model's predictions on the current test set
        predictions.extend(model.predict(X_test).flatten())

    r2 = r2_score(y[-len(predictions):], predictions)
    print("R2 Score:", r2)

    return np.array(predictions)
```

In [13]:
```
# Combine training and test sets for MLP
X_mlp = np.vstack((X_train_mlp, X_test_mlp))
y_mlp = np.hstack((y_train_mlp, y_test_mlp))

# Perform walk-forward backtest for MLP
pred_mlp = walk_forward_backtest(model_mlp, X_mlp, y_mlp, train_size=500, test_size=500)
```

```
Step 1/4, Index: 500

X_train shape: (500, 7), X_test shape: (500, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0429 - val_loss: 0.0248
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0410 - val_loss: 0.0290
Epoch 3/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0380 - val_loss: 0.0278
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0360 - val_loss: 0.0273
Epoch 5/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0359 - val_loss: 0.0270
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0350 - val_loss: 0.0291
16/16 [==============================] - 0s 800us/step
Step 2/4, Index: 1000

X_train shape: (500, 7), X_test shape: (500, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0301 - val_loss: 0.0575
Epoch 2/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0269 - val_loss: 0.0625
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0267 - val_loss: 0.0546
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0259 - val_loss: 0.0633
Epoch 5/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0240 - val_loss: 0.0596
Epoch 6/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0221 - val_loss: 0.0640
Epoch 7/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0233 - val_loss: 0.0578
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0222 - val_loss: 0.0596
16/16 [==============================] - 0s 800us/step
Step 3/4, Index: 1500

X_train shape: (500, 7), X_test shape: (500, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0466 - val_loss: 0.0496
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0438 - val_loss: 0.0509
Epoch 3/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0405 - val_loss: 0.0436
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0358 - val_loss: 0.0453
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0345 - val_loss: 0.0485
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0378 - val_loss: 0.0488
Epoch 7/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0364 - val_loss: 0.0472
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0340 - val_loss: 0.0460
16/16 [==============================] - 0s 800us/step
Step 4/4, Index: 2000

X_train shape: (500, 7), X_test shape: (4, 7), y_train shape: (500,), y_test shape: (4,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0521 - val_loss: 0.0717
Epoch 2/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0464 - val_loss: 0.0690
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0442 - val_loss: 0.0741
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0423 - val_loss: 0.0690
Epoch 5/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0414 - val_loss: 0.0690
Epoch 6/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0405 - val_loss: 0.0698
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0374 - val_loss: 0.0684
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0379 - val_loss: 0.0683
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0359 - val_loss: 0.0715
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0366 - val_loss: 0.0721
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0353 - val_loss: 0.0687
Epoch 12/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0351 - val_loss: 0.0746
Epoch 13/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0346 - val_loss: 0.0717
1/1 [==============================] - 0s 17ms/step
R2 Score: -0.04258133444236645
```

In [14]:

```python
# Combine training and test sets for LSTM
X_lstm = np.vstack((X_train_lstm, X_test_lstm))
y_lstm = np.hstack((y_train_lstm, y_test_lstm))
```

```python
# Perform walk-forward backtest for LSTM
pred_lstm = walk_forward_backtest(model_lstm, X_lstm, y_lstm, train_size=500, test_size=500)
```

```
Step 1/3, Index: 500

X_train shape: (500, 20, 7), X_test shape: (500, 20, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0312 - val_loss: 0.0261
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0308 - val_loss: 0.0303
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0297 - val_loss: 0.0320
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0245 - val_loss: 0.0405
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0211 - val_loss: 0.0432
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0227 - val_loss: 0.0325
16/16 [==============================] - 0s 4ms/step
Step 2/3, Index: 1000

X_train shape: (500, 20, 7), X_test shape: (500, 20, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0278 - val_loss: 0.0536
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0237 - val_loss: 0.0600
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0202 - val_loss: 0.0585
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0183 - val_loss: 0.0557
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0172 - val_loss: 0.0488
Epoch 6/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0168 - val_loss: 0.0528
Epoch 7/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0171 - val_loss: 0.0470
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0160 - val_loss: 0.0506
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0159 - val_loss: 0.0494
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0156 - val_loss: 0.0486
Epoch 11/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0157 - val_loss: 0.0477
Epoch 12/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0149 - val_loss: 0.0521
16/16 [==============================] - 0s 4ms/step
Step 3/3, Index: 1500

X_train shape: (500, 20, 7), X_test shape: (464, 20, 7), y_train shape: (500,), y_test shape: (464,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0344 - val_loss: 0.0461
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0309 - val_loss: 0.0438
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0283 - val_loss: 0.0458
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0260 - val_loss: 0.0375
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0219 - val_loss: 0.0461
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0211 - val_loss: 0.0443
Epoch 7/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0188 - val_loss: 0.0446
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0181 - val_loss: 0.0443
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0193 - val_loss: 0.0451
15/15 [==============================] - 0s 4ms/step
R2 Score: 0.2725306559477475
```

In [15]:
```python
# Combine training and test sets for CNN
X_cnn = np.vstack((X_train_cnn, X_test_cnn))
y_cnn = np.hstack((y_train_cnn, y_test_cnn))

# Perform walk-forward backtest for CNN
pred_cnn = walk_forward_backtest(model_cnn, X_cnn, y_cnn, train_size=500, test_size=500)
```

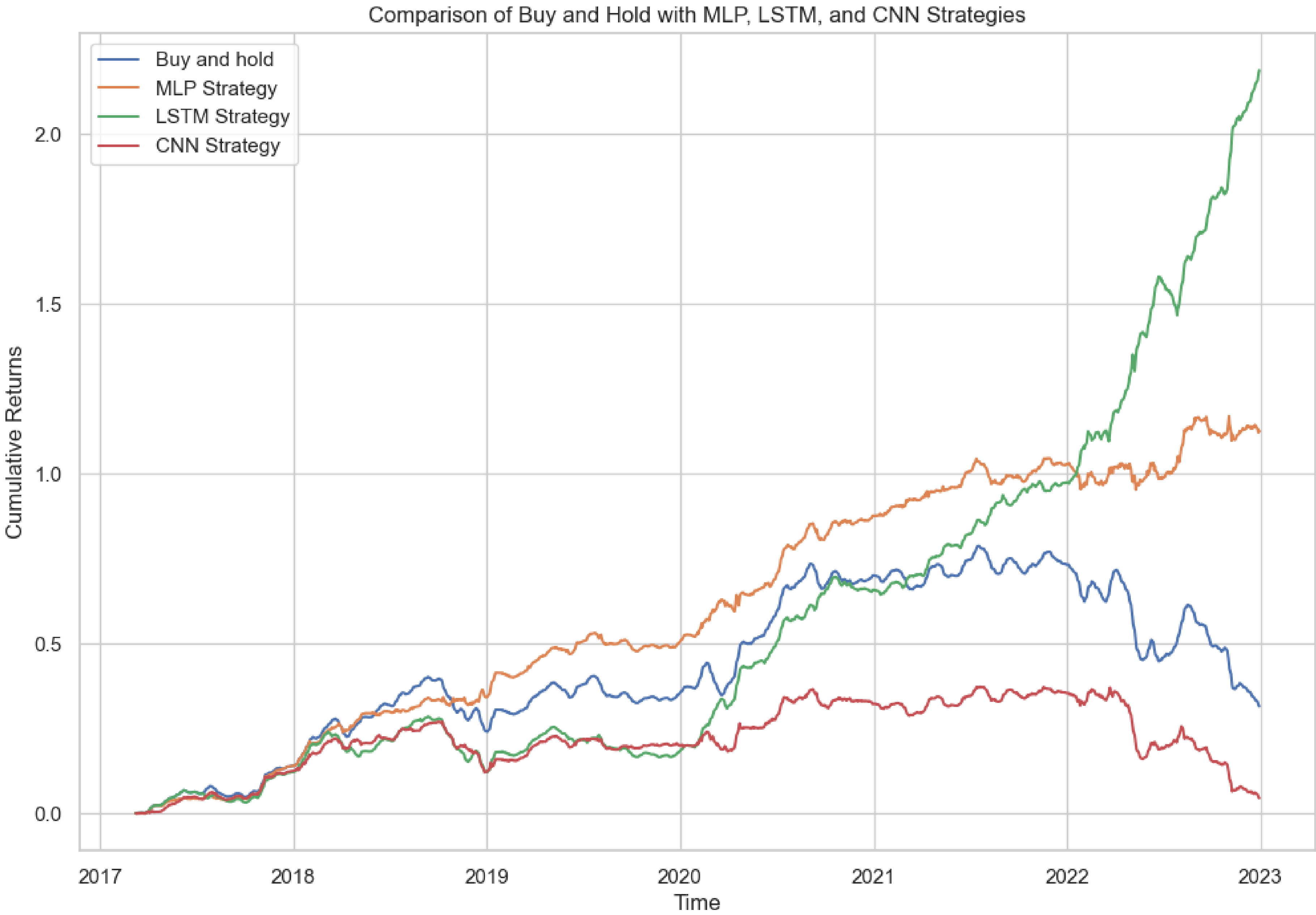
```
Step 1/3, Index: 500

X_train shape: (500, 140, 20), X_test shape: (500, 140, 20), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 40ms/step - loss: 0.0363 - val_loss: 0.0227
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0328 - val_loss: 0.0237
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0283 - val_loss: 0.0253
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0236 - val_loss: 0.0253
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0222 - val_loss: 0.0264
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0206 - val_loss: 0.0271
16/16 [==============================] - 0s 6ms/step
Step 2/3, Index: 1000

X_train shape: (500, 140, 20), X_test shape: (500, 140, 20), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0297 - val_loss: 0.0736
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0285 - val_loss: 0.0740
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0231 - val_loss: 0.0738
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0234 - val_loss: 0.0739
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0197 - val_loss: 0.0757
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0197 - val_loss: 0.0762
16/16 [==============================] - 0s 1ms/step
Step 3/3, Index: 1500

X_train shape: (500, 140, 20), X_test shape: (464, 140, 20), y_train shape: (500,), y_test shape: (464,)
Epoch 1/30
11/11 [==============================] - 0s 11ms/step - loss: 0.0557 - val_loss: 0.0525
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0436 - val_loss: 0.0500
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0380 - val_loss: 0.0501
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0344 - val_loss: 0.0503
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0309 - val_loss: 0.0510
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0297 - val_loss: 0.0509
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0289 - val_loss: 0.0503
15/15 [==============================] - 0s 6ms/step
R2 Score: -0.28222179667747116
```

In [16]:

```python
# Create a DataFrame for walk-forward backtest results
df_pred_wf = pd.DataFrame(index=y.iloc[-len(pred_lstm):].index)

# Populate DataFrame with actual and predicted values
df_pred_wf['y_test'] = y.iloc[-len(pred_lstm):]
df_pred_wf['pred_mlp'] = pred_mlp[-len(pred_lstm):]
df_pred_wf['pred_lstm'] = pred_lstm
df_pred_wf['pred_cnn'] = pred_cnn

# Calculate buy and hold cumulative return
df_pred_wf['cum_ret_bh'] = df_pred_wf['y_test'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# MLP Strategy
df_pred_wf['positions_mlp'] = df_pred_wf['pred_mlp'].apply(np.sign)
df_pred_wf['strat_ret_mlp'] = df_pred_wf['positions_mlp'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_mlp'] = df_pred_wf['strat_ret_mlp'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# LSTM Strategy
df_pred_wf['positions_lstm'] = df_pred_wf['pred_lstm'].apply(np.sign)
df_pred_wf['strat_ret_lstm'] = df_pred_wf['positions_lstm'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_lstm'] = df_pred_wf['strat_ret_lstm'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# CNN Strategy
df_pred_wf['positions_cnn'] = df_pred_wf['pred_cnn'].apply(np.sign)
df_pred_wf['strat_ret_cnn'] = df_pred_wf['positions_cnn'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_cnn'] = df_pred_wf['strat_ret_cnn'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# Calculate final returns
buy_return = np.prod(1 + df_pred_wf['y_test'])**(window_hp/252) - 1
strat_return_mlp = np.prod(1 + df_pred_wf['strat_ret_mlp'])**(window_hp/252) - 1
strat_return_lstm = np.prod(1 + df_pred_wf['strat_ret_lstm'])**(window_hp/252) - 1
strat_return_cnn = np.prod(1 + df_pred_wf['strat_ret_cnn'])**(window_hp/252) - 1

# Print returns
print("Buy and Hold Return: {:.4%}".format(buy_return))
print("MLP Strategy Return: {:.4%}".format(strat_return_mlp))
print("LSTM Strategy Return: {:.4%}".format(strat_return_lstm))
print("CNN Strategy Return: {:.4%}".format(strat_return_cnn))

# Combined Plot
```

```
plt.figure(figsize=(12, 8))

# Plot cumulative returns with labels and legend
plt.plot(df_pred_wf['cum_ret_bh'], label='Buy and hold')
plt.plot(df_pred_wf['cum_ret_mlp'], label='MLP Strategy')
plt.plot(df_pred_wf['cum_ret_lstm'], label='LSTM Strategy')
plt.plot(df_pred_wf['cum_ret_cnn'], label='CNN Strategy')

# Add labels and title
plt.xlabel('Time')
plt.ylabel('Cumulative Returns')
plt.title('Comparison of Buy and Hold with MLP, LSTM, and CNN Strategies')
plt.legend()

# Display the plot
plt.show()
```

```
Buy and Hold Return: 31.5357%
MLP Strategy Return: 112.4213%
LSTM Strategy Return: 218.7272%
CNN Strategy Return: 4.4786%
```



b. Non achored walk forward method with train/test split with 500 observations in training set and 100 observations in test set.

In [17]:
```
# Perform walk-forward backtest for MLP
pred_mlp = walk_forward_backtest(model_mlp, X_mlp, y_mlp, train_size=500, test_size=100)
```

```
Step 1/16, Index: 500

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0528 - val_loss: 0.0361
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0470 - val_loss: 0.0285
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0428 - val_loss: 0.0323
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0400 - val_loss: 0.0286
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0370 - val_loss: 0.0306
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0357 - val_loss: 0.0292
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0345 - val_loss: 0.0312
4/4 [==============================] - 0s 1ms/step
Step 2/16, Index: 600

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0372 - val_loss: 0.0310
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0344 - val_loss: 0.0295
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0327 - val_loss: 0.0306
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0301 - val_loss: 0.0277
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0297 - val_loss: 0.0281
Epoch 6/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0296 - val_loss: 0.0341
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0274 - val_loss: 0.0303
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0257 - val_loss: 0.0306
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0248 - val_loss: 0.0321
4/4 [==============================] - 0s 1ms/step
Step 3/16, Index: 700

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0292 - val_loss: 0.0327
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0280 - val_loss: 0.0327
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0270 - val_loss: 0.0320
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0254 - val_loss: 0.0306
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0244 - val_loss: 0.0303
Epoch 6/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0233 - val_loss: 0.0334
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0231 - val_loss: 0.0319
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0224 - val_loss: 0.0328
Epoch 9/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0222 - val_loss: 0.0314
Epoch 10/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0229 - val_loss: 0.0315
4/4 [==============================] - 0s 1ms/step
Step 4/16, Index: 800

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0218 - val_loss: 0.0377
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0211 - val_loss: 0.0384
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0215 - val_loss: 0.0391
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0198 - val_loss: 0.0383
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0185 - val_loss: 0.0407
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0188 - val_loss: 0.0396
4/4 [==============================] - 0s 1ms/step
Step 5/16, Index: 900

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0253 - val_loss: 0.0435
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0235 - val_loss: 0.0419
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0215 - val_loss: 0.0430
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0207 - val_loss: 0.0419
Epoch 5/30
```

```
11/11 [==============================] - 0s 5ms/step - loss: 0.0197 - val_loss: 0.0417
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0196 - val_loss: 0.0425
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0194 - val_loss: 0.0407
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0200 - val_loss: 0.0405
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0196 - val_loss: 0.0427
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0187 - val_loss: 0.0448
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0181 - val_loss: 0.0456
Epoch 12/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0188 - val_loss: 0.0429
Epoch 13/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0179 - val_loss: 0.0436
4/4 [==============================] - 0s 1ms/step
Step 6/16, Index: 1000

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0274 - val_loss: 0.0628
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0273 - val_loss: 0.0553
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0262 - val_loss: 0.0568
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0237 - val_loss: 0.0562
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0220 - val_loss: 0.0602
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0215 - val_loss: 0.0620
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0206 - val_loss: 0.0599
4/4 [==============================] - 0s 1ms/step
Step 7/16, Index: 1100

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0322 - val_loss: 0.0572
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0323 - val_loss: 0.0510
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0286 - val_loss: 0.0549
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0275 - val_loss: 0.0537
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0260 - val_loss: 0.0568
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0250 - val_loss: 0.0595
Epoch 7/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0239 - val_loss: 0.0590
4/4 [==============================] - 0s 1ms/step
Step 8/16, Index: 1200

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0409 - val_loss: 0.0349
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0390 - val_loss: 0.0402
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0381 - val_loss: 0.0391
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0361 - val_loss: 0.0392
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0356 - val_loss: 0.0395
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0340 - val_loss: 0.0381
4/4 [==============================] - 0s 1ms/step
Step 9/16, Index: 1300

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0412 - val_loss: 0.0357
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0382 - val_loss: 0.0382
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0362 - val_loss: 0.0351
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0345 - val_loss: 0.0358
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0332 - val_loss: 0.0349
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0319 - val_loss: 0.0353
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0319 - val_loss: 0.0368
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0318 - val_loss: 0.0357
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0307 - val_loss: 0.0346
Epoch 10/30
```

```
11/11 [==============================] - 0s 6ms/step - loss: 0.0276 - val_loss: 0.0358
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0276 - val_loss: 0.0387
Epoch 12/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0269 - val_loss: 0.0355
Epoch 13/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0254 - val_loss: 0.0371
Epoch 14/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0269 - val_loss: 0.0379
4/4 [==============================] - 0s 1ms/step
Step 10/16, Index: 1400

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0303 - val_loss: 0.0624
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0279 - val_loss: 0.0692
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0273 - val_loss: 0.0673
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0266 - val_loss: 0.0652
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0241 - val_loss: 0.0679
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0228 - val_loss: 0.0685
4/4 [==============================] - 0s 1ms/step
Step 11/16, Index: 1500

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0373 - val_loss: 0.0458
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0347 - val_loss: 0.0461
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0325 - val_loss: 0.0457
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0312 - val_loss: 0.0470
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0300 - val_loss: 0.0465
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0293 - val_loss: 0.0492
Epoch 7/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0291 - val_loss: 0.0489
Epoch 8/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0297 - val_loss: 0.0457
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0271 - val_loss: 0.0470
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0254 - val_loss: 0.0469
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0258 - val_loss: 0.0486
Epoch 12/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0236 - val_loss: 0.0483
Epoch 13/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0233 - val_loss: 0.0489
4/4 [==============================] - 0s 1ms/step
Step 12/16, Index: 1600

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0371 - val_loss: 0.0436
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0342 - val_loss: 0.0425
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0314 - val_loss: 0.0383
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0326 - val_loss: 0.0441
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0332 - val_loss: 0.0450
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0320 - val_loss: 0.0401
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0305 - val_loss: 0.0442
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0278 - val_loss: 0.0411
4/4 [==============================] - 0s 1ms/step
Step 13/16, Index: 1700

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0351 - val_loss: 0.0452
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0325 - val_loss: 0.0446
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0315 - val_loss: 0.0464
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0303 - val_loss: 0.0434
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0293 - val_loss: 0.0435
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0280 - val_loss: 0.0444
Epoch 7/30
```

```
11/11 [==============================] - 0s 5ms/step - loss: 0.0274 - val_loss: 0.0451
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0287 - val_loss: 0.0464
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0272 - val_loss: 0.0433
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0286 - val_loss: 0.0428
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0255 - val_loss: 0.0469
Epoch 12/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0250 - val_loss: 0.0451
Epoch 13/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0238 - val_loss: 0.0434
Epoch 14/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0239 - val_loss: 0.0435
Epoch 15/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0224 - val_loss: 0.0455
4/4 [==============================] - 0s 1ms/step
Step 14/16, Index: 1800

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0324 - val_loss: 0.0509
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0311 - val_loss: 0.0627
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0301 - val_loss: 0.0512
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0277 - val_loss: 0.0521
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0266 - val_loss: 0.0515
Epoch 6/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0258 - val_loss: 0.0518
4/4 [==============================] - 0s 1ms/step
Step 15/16, Index: 1900

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0339 - val_loss: 0.0817
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0301 - val_loss: 0.0826
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0291 - val_loss: 0.0774
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0276 - val_loss: 0.0769
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0268 - val_loss: 0.0786
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0260 - val_loss: 0.0792
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0248 - val_loss: 0.0748
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0239 - val_loss: 0.0781
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0239 - val_loss: 0.0785
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0239 - val_loss: 0.0774
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0240 - val_loss: 0.0785
Epoch 12/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0238 - val_loss: 0.0766
4/4 [==============================] - 0s 1ms/step
Step 16/16, Index: 2000

X_train shape: (500, 7), X_test shape: (4, 7), y_train shape: (500,), y_test shape: (4,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0409 - val_loss: 0.0676
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0377 - val_loss: 0.0677
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0368 - val_loss: 0.0662
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0345 - val_loss: 0.0683
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0340 - val_loss: 0.0685
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0318 - val_loss: 0.0664
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0317 - val_loss: 0.0689
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0337 - val_loss: 0.0686
1/1 [==============================] - 0s 17ms/step
R2 Score: -0.08242593962143041
```

In [18]:

```
# Perform walk-forward backtest for LSTM
pred_lstm = walk_forward_backtest(model_lstm, X_lstm, y_lstm, train_size=500, test_size=100)
```

```
Step 1/15, Index: 500

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 25ms/step - loss: 0.0372 - val_loss: 0.0270
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0309 - val_loss: 0.0365
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0284 - val_loss: 0.0339
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0276 - val_loss: 0.0325
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0224 - val_loss: 0.0367
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0215 - val_loss: 0.0353
4/4 [==============================] - 0s 4ms/step
Step 2/15, Index: 600

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0369 - val_loss: 0.0181
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0334 - val_loss: 0.0181
Epoch 3/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0290 - val_loss: 0.0193
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0284 - val_loss: 0.0184
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0272 - val_loss: 0.0208
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0261 - val_loss: 0.0251
4/4 [==============================] - 0s 4ms/step
Step 3/15, Index: 700

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0285 - val_loss: 0.0307
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0239 - val_loss: 0.0312
Epoch 3/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0213 - val_loss: 0.0291
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0195 - val_loss: 0.0344
Epoch 5/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0209 - val_loss: 0.0287
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0177 - val_loss: 0.0307
Epoch 7/30
11/11 [==============================] - 0s 12ms/step - loss: 0.0179 - val_loss: 0.0324
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0164 - val_loss: 0.0310
Epoch 9/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0179 - val_loss: 0.0356
Epoch 10/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0177 - val_loss: 0.0343
4/4 [==============================] - 0s 4ms/step
Step 4/15, Index: 800

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0162 - val_loss: 0.0413
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0160 - val_loss: 0.0403
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0154 - val_loss: 0.0420
Epoch 4/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0140 - val_loss: 0.0423
Epoch 5/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0128 - val_loss: 0.0419
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0123 - val_loss: 0.0436
Epoch 7/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0121 - val_loss: 0.0444
4/4 [==============================] - 0s 5ms/step
Step 5/15, Index: 900

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0187 - val_loss: 0.0351
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0167 - val_loss: 0.0428
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0161 - val_loss: 0.0362
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0165 - val_loss: 0.0306
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0152 - val_loss: 0.0353
Epoch 6/30
11/11 [==============================] - 0s 17ms/step - loss: 0.0139 - val_loss: 0.0322
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0132 - val_loss: 0.0312
Epoch 8/30
```

```
11/11 [==============================] - 0s 13ms/step - loss: 0.0130 - val_loss: 0.0345
Epoch 9/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0121 - val_loss: 0.0329
4/4 [==============================] - 0s 4ms/step
Step 6/15, Index: 1000

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 17ms/step - loss: 0.0234 - val_loss: 0.0529
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0202 - val_loss: 0.0461
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0170 - val_loss: 0.0516
Epoch 4/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0154 - val_loss: 0.0561
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0156 - val_loss: 0.0510
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0148 - val_loss: 0.0437
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0144 - val_loss: 0.0488
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0135 - val_loss: 0.0499
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0148 - val_loss: 0.0477
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0133 - val_loss: 0.0526
Epoch 11/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0125 - val_loss: 0.0527
4/4 [==============================] - 0s 4ms/step
Step 7/15, Index: 1100

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0210 - val_loss: 0.0420
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0193 - val_loss: 0.0423
Epoch 3/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0179 - val_loss: 0.0393
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0176 - val_loss: 0.0362
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0191 - val_loss: 0.0413
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0189 - val_loss: 0.0377
Epoch 7/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0179 - val_loss: 0.0401
Epoch 8/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0164 - val_loss: 0.0416
Epoch 9/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0151 - val_loss: 0.0456
4/4 [==============================] - 0s 4ms/step
Step 8/15, Index: 1200

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0258 - val_loss: 0.0312
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0250 - val_loss: 0.0284
Epoch 3/30
11/11 [==============================] - 0s 18ms/step - loss: 0.0246 - val_loss: 0.0273
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0234 - val_loss: 0.0322
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0212 - val_loss: 0.0325
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0207 - val_loss: 0.0336
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0197 - val_loss: 0.0334
Epoch 8/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0213 - val_loss: 0.0374
4/4 [==============================] - 0s 4ms/step
Step 9/15, Index: 1300

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0259 - val_loss: 0.0405
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0250 - val_loss: 0.0392
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0230 - val_loss: 0.0358
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0211 - val_loss: 0.0386
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0200 - val_loss: 0.0378
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0193 - val_loss: 0.0394
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0182 - val_loss: 0.0405
Epoch 8/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0167 - val_loss: 0.0395
4/4 [==============================] - 0s 4ms/step
```

```
Step 10/15, Index: 1400

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0238 - val_loss: 0.0456
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0209 - val_loss: 0.0469
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0205 - val_loss: 0.0434
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0194 - val_loss: 0.0431
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0191 - val_loss: 0.0494
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0182 - val_loss: 0.0450
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0171 - val_loss: 0.0553
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0181 - val_loss: 0.0421
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0168 - val_loss: 0.0475
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0163 - val_loss: 0.0450
Epoch 11/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0154 - val_loss: 0.0428
Epoch 12/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0164 - val_loss: 0.0427
Epoch 13/30
11/11 [==============================] - 0s 18ms/step - loss: 0.0142 - val_loss: 0.0440
4/4 [==============================] - 0s 4ms/step
Step 11/15, Index: 1500

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0225 - val_loss: 0.0394
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0186 - val_loss: 0.0393
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0163 - val_loss: 0.0378
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0163 - val_loss: 0.0350
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0164 - val_loss: 0.0408
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0152 - val_loss: 0.0387
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0155 - val_loss: 0.0396
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0151 - val_loss: 0.0369
Epoch 9/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0155 - val_loss: 0.0399
4/4 [==============================] - 0s 4ms/step
Step 12/15, Index: 1600

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0247 - val_loss: 0.0343
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0233 - val_loss: 0.0305
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0209 - val_loss: 0.0297
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0200 - val_loss: 0.0354
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0185 - val_loss: 0.0290
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0187 - val_loss: 0.0294
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0173 - val_loss: 0.0326
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0175 - val_loss: 0.0306
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0163 - val_loss: 0.0321
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0165 - val_loss: 0.0314
4/4 [==============================] - 0s 4ms/step
Step 13/15, Index: 1700

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0226 - val_loss: 0.0330
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0227 - val_loss: 0.0334
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0196 - val_loss: 0.0331
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0185 - val_loss: 0.0337
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0182 - val_loss: 0.0358
Epoch 6/30
11/11 [==============================] - 0s 17ms/step - loss: 0.0181 - val_loss: 0.0321
Epoch 7/30
```

```
11/11 [==============================] - 0s 13ms/step - loss: 0.0183 - val_loss: 0.0327
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0177 - val_loss: 0.0343
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0169 - val_loss: 0.0345
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0164 - val_loss: 0.0348
Epoch 11/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0184 - val_loss: 0.0334
4/4 [==============================] - 0s 4ms/step
Step 14/15, Index: 1800

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0225 - val_loss: 0.0537
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0213 - val_loss: 0.0489
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0207 - val_loss: 0.0491
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0191 - val_loss: 0.0502
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0177 - val_loss: 0.0508
Epoch 6/30
11/11 [==============================] - 0s 12ms/step - loss: 0.0164 - val_loss: 0.0581
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0164 - val_loss: 0.0516
4/4 [==============================] - 0s 4ms/step
Step 15/15, Index: 1900

X_train shape: (500, 20, 7), X_test shape: (64, 20, 7), y_train shape: (500,), y_test shape: (64,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0257 - val_loss: 0.0632
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0229 - val_loss: 0.0654
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0237 - val_loss: 0.0696
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0193 - val_loss: 0.0680
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0179 - val_loss: 0.0687
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0192 - val_loss: 0.0685
2/2 [==============================] - 0s 4ms/step
R2 Score: 0.2836281185037809
```

In [19]:

```python
# Perform walk-forward backtest for CNN
pred_cnn = walk_forward_backtest(model_cnn, X_cnn, y_cnn, train_size=500, test_size=100)
```

```
Step 1/15, Index: 500

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0398 - val_loss: 0.0263
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0345 - val_loss: 0.0264
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0297 - val_loss: 0.0262
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0258 - val_loss: 0.0265
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0231 - val_loss: 0.0280
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0209 - val_loss: 0.0280
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0205 - val_loss: 0.0280
Epoch 8/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0180 - val_loss: 0.0286
4/4 [==============================] - 0s 20ms/step
Step 2/15, Index: 600

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0279 - val_loss: 0.0248
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0255 - val_loss: 0.0257
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0228 - val_loss: 0.0248
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0192 - val_loss: 0.0264
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0187 - val_loss: 0.0268
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0171 - val_loss: 0.0270
4/4 [==============================] - 0s 1ms/step
Step 3/15, Index: 700

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0243 - val_loss: 0.0299
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0221 - val_loss: 0.0299
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0179 - val_loss: 0.0302
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0177 - val_loss: 0.0306
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0166 - val_loss: 0.0311
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0142 - val_loss: 0.0315
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0130 - val_loss: 0.0332
4/4 [==============================] - 0s 1ms/step
Step 4/15, Index: 800

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0218 - val_loss: 0.0384
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0205 - val_loss: 0.0389
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0173 - val_loss: 0.0394
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0166 - val_loss: 0.0399
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0144 - val_loss: 0.0393
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0156 - val_loss: 0.0407
4/4 [==============================] - 0s 1ms/step
Step 5/15, Index: 900

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0255 - val_loss: 0.0371
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0237 - val_loss: 0.0373
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0198 - val_loss: 0.0391
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0197 - val_loss: 0.0385
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0176 - val_loss: 0.0385
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0159 - val_loss: 0.0382
4/4 [==============================] - 0s 1ms/step
Step 6/15, Index: 1000

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0268 - val_loss: 0.0703
Epoch 2/30
```

```
11/11 [==============================] - 0s 6ms/step - loss: 0.0246 - val_loss: 0.0730
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0203 - val_loss: 0.0706
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0192 - val_loss: 0.0736
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0184 - val_loss: 0.0698
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0170 - val_loss: 0.0757
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0160 - val_loss: 0.0737
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0146 - val_loss: 0.0749
Epoch 9/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0136 - val_loss: 0.0737
Epoch 10/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0119 - val_loss: 0.0751
4/4 [==============================] - 0s 1ms/step
Step 7/15, Index: 1100

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0284 - val_loss: 0.0582
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0255 - val_loss: 0.0565
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0222 - val_loss: 0.0568
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0191 - val_loss: 0.0572
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0178 - val_loss: 0.0589
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0177 - val_loss: 0.0570
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0158 - val_loss: 0.0583
4/4 [==============================] - 0s 1ms/step
Step 8/15, Index: 1200

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0361 - val_loss: 0.0366
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0317 - val_loss: 0.0374
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0273 - val_loss: 0.0375
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0243 - val_loss: 0.0386
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0225 - val_loss: 0.0400
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0195 - val_loss: 0.0380
4/4 [==============================] - 0s 1ms/step
Step 9/15, Index: 1300

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0369 - val_loss: 0.0493
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0322 - val_loss: 0.0490
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0284 - val_loss: 0.0504
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0261 - val_loss: 0.0510
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0249 - val_loss: 0.0514
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0224 - val_loss: 0.0510
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0205 - val_loss: 0.0512
4/4 [==============================] - 0s 1ms/step
Step 10/15, Index: 1400

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0340 - val_loss: 0.0646
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0303 - val_loss: 0.0628
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0257 - val_loss: 0.0643
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0238 - val_loss: 0.0629
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0241 - val_loss: 0.0661
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0206 - val_loss: 0.0644
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0198 - val_loss: 0.0670
4/4 [==============================] - 0s 1ms/step
Step 11/15, Index: 1500

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
```

```
11/11 [==============================] - 0s 9ms/step - loss: 0.0346 - val_loss: 0.0503
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0316 - val_loss: 0.0501
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0287 - val_loss: 0.0500
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0281 - val_loss: 0.0509
Epoch 5/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0261 - val_loss: 0.0515
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0237 - val_loss: 0.0507
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0235 - val_loss: 0.0515
Epoch 8/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0222 - val_loss: 0.0516
4/4 [==============================] - 0s 1ms/step
Step 12/15, Index: 1600

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0364 - val_loss: 0.0458
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0324 - val_loss: 0.0453
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0278 - val_loss: 0.0478
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0264 - val_loss: 0.0455
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0235 - val_loss: 0.0468
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0224 - val_loss: 0.0467
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0226 - val_loss: 0.0465
4/4 [==============================] - 0s 1ms/step
Step 13/15, Index: 1700

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0376 - val_loss: 0.0450
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0333 - val_loss: 0.0454
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0272 - val_loss: 0.0447
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0253 - val_loss: 0.0433
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0241 - val_loss: 0.0439
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0231 - val_loss: 0.0442
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0220 - val_loss: 0.0487
Epoch 8/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0219 - val_loss: 0.0430
Epoch 9/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0203 - val_loss: 0.0457
Epoch 10/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0187 - val_loss: 0.0443
Epoch 11/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0171 - val_loss: 0.0429
Epoch 12/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0166 - val_loss: 0.0445
Epoch 13/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0155 - val_loss: 0.0436
Epoch 14/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0165 - val_loss: 0.0456
Epoch 15/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0161 - val_loss: 0.0445
Epoch 16/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0160 - val_loss: 0.0465
4/4 [==============================] - 0s 1ms/step
Step 14/15, Index: 1800

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0240 - val_loss: 0.0723
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0213 - val_loss: 0.0686
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0170 - val_loss: 0.0719
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0157 - val_loss: 0.0714
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0133 - val_loss: 0.0715
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0133 - val_loss: 0.0695
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0120 - val_loss: 0.0702
4/4 [==============================] - 0s 1ms/step
Step 15/15, Index: 1900

X_train shape: (500, 140, 20), X_test shape: (64, 140, 20), y_train shape: (500,), y_test shape: (64,)
Epoch 1/30
```

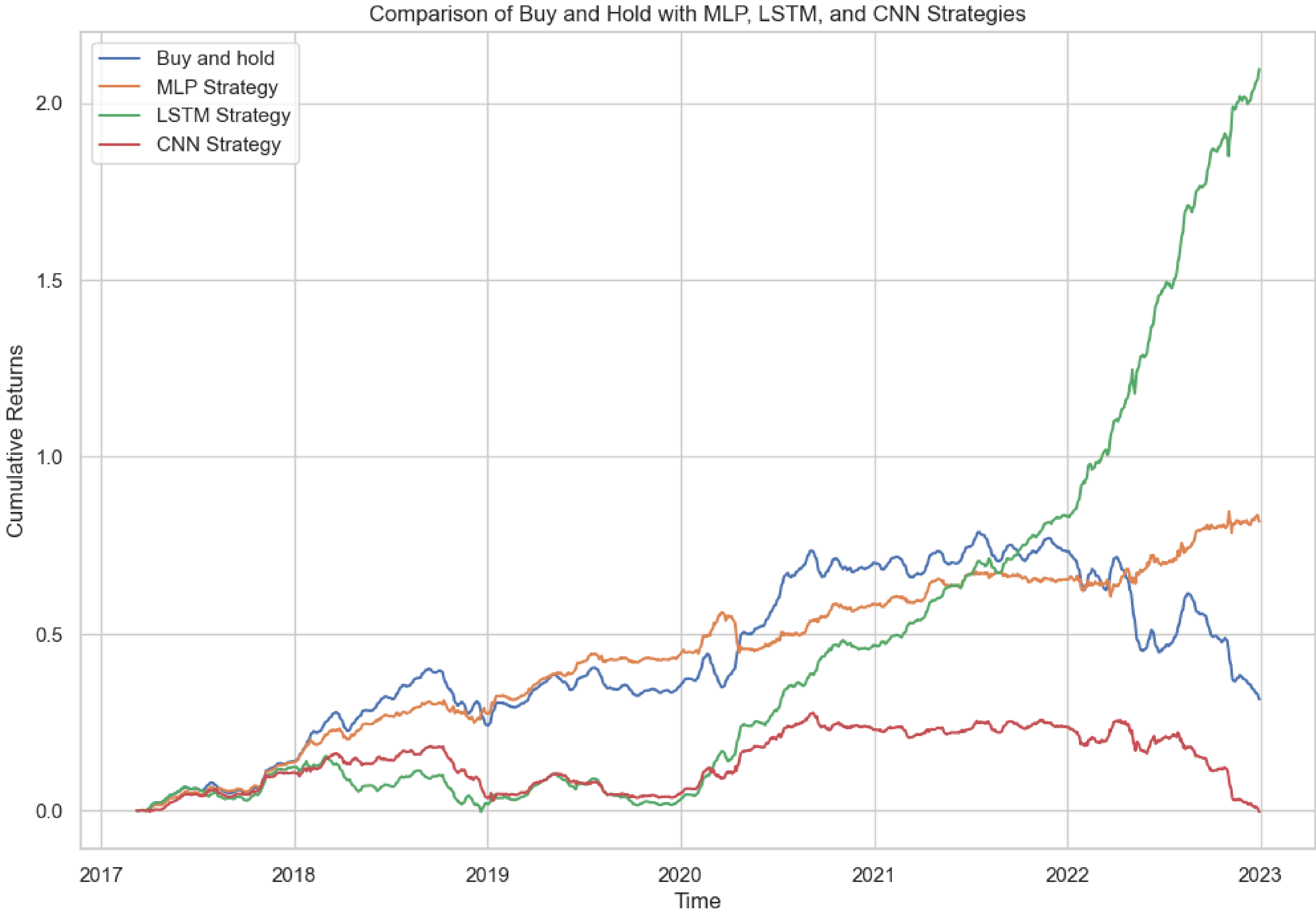
```
11/11 [==============================] - 0s 11ms/step - loss: 0.0296 - val_loss: 0.0924
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0249 - val_loss: 0.0921
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0214 - val_loss: 0.0941
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0196 - val_loss: 0.0926
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0188 - val_loss: 0.0931
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0180 - val_loss: 0.0928
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0165 - val_loss: 0.0916
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0161 - val_loss: 0.0922
Epoch 9/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0149 - val_loss: 0.0924
Epoch 10/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0139 - val_loss: 0.0923
Epoch 11/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0130 - val_loss: 0.0923
Epoch 12/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0132 - val_loss: 0.0912
Epoch 13/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0139 - val_loss: 0.0906
Epoch 14/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0120 - val_loss: 0.0921
Epoch 15/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0117 - val_loss: 0.0918
Epoch 16/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0114 - val_loss: 0.0922
Epoch 17/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0115 - val_loss: 0.0907
Epoch 18/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0106 - val_loss: 0.0902
Epoch 19/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0109 - val_loss: 0.0908
Epoch 20/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0104 - val_loss: 0.0902
Epoch 21/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0101 - val_loss: 0.0908
Epoch 22/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0099 - val_loss: 0.0909
Epoch 23/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0113 - val_loss: 0.0893
Epoch 24/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0102 - val_loss: 0.0892
Epoch 25/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0092 - val_loss: 0.0902
Epoch 26/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0083 - val_loss: 0.0889
Epoch 27/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0085 - val_loss: 0.0902
Epoch 28/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0089 - val_loss: 0.0891
Epoch 29/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0084 - val_loss: 0.0897
Epoch 30/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0087 - val_loss: 0.0919
2/2 [==============================] - 0s 2ms/step
R2 Score: -0.1733671050747274
```

In [20]:

```python
# Create a DataFrame for walk-forward backtest results
df_pred_wf = pd.DataFrame(index=y.iloc[-len(pred_lstm):].index)

# Populate DataFrame with actual and predicted values
df_pred_wf['y_test'] = y.iloc[-len(pred_lstm):]
df_pred_wf['pred_mlp'] = pred_mlp[-len(pred_lstm):]
df_pred_wf['pred_lstm'] = pred_lstm
df_pred_wf['pred_cnn'] = pred_cnn

# Calculate buy and hold cumulative return
df_pred_wf['cum_ret_bh'] = df_pred_wf['y_test'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# MLP Strategy
df_pred_wf['positions_mlp'] = df_pred_wf['pred_mlp'].apply(np.sign)
df_pred_wf['strat_ret_mlp'] = df_pred_wf['positions_mlp'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_mlp'] = df_pred_wf['strat_ret_mlp'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# LSTM Strategy
df_pred_wf['positions_lstm'] = df_pred_wf['pred_lstm'].apply(np.sign)
df_pred_wf['strat_ret_lstm'] = df_pred_wf['positions_lstm'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_lstm'] = df_pred_wf['strat_ret_lstm'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# CNN Strategy
df_pred_wf['positions_cnn'] = df_pred_wf['pred_cnn'].apply(np.sign)
df_pred_wf['strat_ret_cnn'] = df_pred_wf['positions_cnn'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_cnn'] = df_pred_wf['strat_ret_cnn'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# Calculate final returns
buy_return = np.prod(1 + df_pred_wf['y_test'])**(window_hp/252) - 1
strat_return_mlp = np.prod(1 + df_pred_wf['strat_ret_mlp'])**(window_hp/252) - 1
```

```
strat_return_lstm = np.prod(1 + df_pred_wf['strat_ret_lstm'])**(window_hp/252) - 1
strat_return_cnn = np.prod(1 + df_pred_wf['strat_ret_cnn'])**(window_hp/252) - 1

# Print returns
print("Buy and Hold Return: {:.4%}".format(buy_return))
print("MLP Strategy Return: {:.4%}".format(strat_return_mlp))
print("LSTM Strategy Return: {:.4%}".format(strat_return_lstm))
print("CNN Strategy Return: {:.4%}".format(strat_return_cnn))

# Combined Plot
plt.figure(figsize=(12, 8))

# Plot cumulative returns with labels and legend
plt.plot(df_pred_wf['cum_ret_bh'], label='Buy and hold')
plt.plot(df_pred_wf['cum_ret_mlp'], label='MLP Strategy')
plt.plot(df_pred_wf['cum_ret_lstm'], label='LSTM Strategy')
plt.plot(df_pred_wf['cum_ret_cnn'], label='CNN Strategy')

# Add labels and title
plt.xlabel('Time')
plt.ylabel('Cumulative Returns')
plt.title('Comparison of Buy and Hold with MLP, LSTM, and CNN Strategies')
plt.legend()

# Display the plot
plt.show()
```

```
Buy and Hold Return: 31.5357%
MLP Strategy Return: 81.7002%
LSTM Strategy Return: 209.4659%
CNN Strategy Return: -0.2701%
```



Step 3

a. Set up and describe a method to reduce the extent of leakage between training and test samples

In [21]:
```
# Alleviate leakage by shifting the labels which carry the information
shifted_X_mlp = X_mlp[window_hp:]
shifted_y_mlp = y_mlp[:-window_hp]

shifted_X_cnn = X_cnn[window_hp:]
shifted_y_cnn = y_cnn[:-window_hp]

shifted_X_lstm = X_lstm[window_hp:]
shifted_y_lstm = y_lstm[:-window_hp]
```

b. Non achored walk forward method with train/test split with 500 observations in each set.

In [22]:
```
# Perform walk-forward backtest for MLP
pred_mlp = walk_forward_backtest(model_mlp, shifted_X_mlp, shifted_y_mlp, train_size=500, test_size=500)
```

```
Step 1/3, Index: 500

X_train shape: (500, 7), X_test shape: (500, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0572 - val_loss: 0.0370
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0479 - val_loss: 0.0344
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0447 - val_loss: 0.0360
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0424 - val_loss: 0.0338
Epoch 5/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0392 - val_loss: 0.0345
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0375 - val_loss: 0.0352
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0373 - val_loss: 0.0344
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0350 - val_loss: 0.0358
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0344 - val_loss: 0.0363
16/16 [==============================] - 0s 800us/step
Step 2/3, Index: 1000

X_train shape: (500, 7), X_test shape: (500, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0367 - val_loss: 0.0640
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0332 - val_loss: 0.0647
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0305 - val_loss: 0.0644
Epoch 4/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0275 - val_loss: 0.0640
Epoch 5/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0262 - val_loss: 0.0642
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0249 - val_loss: 0.0646
16/16 [==============================] - 0s 800us/step
Step 3/3, Index: 1500

X_train shape: (500, 7), X_test shape: (494, 7), y_train shape: (500,), y_test shape: (494,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0488 - val_loss: 0.0490
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0423 - val_loss: 0.0493
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0396 - val_loss: 0.0465
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0370 - val_loss: 0.0475
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0357 - val_loss: 0.0491
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0339 - val_loss: 0.0497
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0331 - val_loss: 0.0477
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0325 - val_loss: 0.0498
16/16 [==============================] - 0s 733us/step
R2 Score: -0.17727516257561438
```

In [23]:
```
# Perform walk-forward backtest for LSTM
pred_lstm = walk_forward_backtest(model_lstm, shifted_X_lstm, shifted_y_lstm, train_size=500, test_size=500)
```

```
Step 1/3, Index: 500

X_train shape: (500, 20, 7), X_test shape: (500, 20, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 18ms/step - loss: 0.0506 - val_loss: 0.0255
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0406 - val_loss: 0.0261
Epoch 3/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0340 - val_loss: 0.0275
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0307 - val_loss: 0.0266
Epoch 5/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0290 - val_loss: 0.0261
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0255 - val_loss: 0.0227
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0250 - val_loss: 0.0257
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0227 - val_loss: 0.0277
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0217 - val_loss: 0.0261
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0229 - val_loss: 0.0355
Epoch 11/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0203 - val_loss: 0.0213
Epoch 12/30
11/11 [==============================] - 0s 17ms/step - loss: 0.0190 - val_loss: 0.0279
Epoch 13/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0184 - val_loss: 0.0237
Epoch 14/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0185 - val_loss: 0.0276
Epoch 15/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0190 - val_loss: 0.0242
Epoch 16/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0164 - val_loss: 0.0277
16/16 [==============================] - 0s 4ms/step
Step 2/3, Index: 1000

X_train shape: (500, 20, 7), X_test shape: (500, 20, 7), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0329 - val_loss: 0.0484
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0234 - val_loss: 0.0502
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0192 - val_loss: 0.0479
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0166 - val_loss: 0.0460
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0144 - val_loss: 0.0438
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0135 - val_loss: 0.0478
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0135 - val_loss: 0.0453
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0128 - val_loss: 0.0464
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0122 - val_loss: 0.0464
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0123 - val_loss: 0.0449
16/16 [==============================] - 0s 4ms/step
Step 3/3, Index: 1500

X_train shape: (500, 20, 7), X_test shape: (454, 20, 7), y_train shape: (500,), y_test shape: (454,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0354 - val_loss: 0.0329
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0281 - val_loss: 0.0310
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0231 - val_loss: 0.0259
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0201 - val_loss: 0.0296
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0200 - val_loss: 0.0293
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0192 - val_loss: 0.0292
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0181 - val_loss: 0.0287
Epoch 8/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0169 - val_loss: 0.0269
15/15 [==============================] - 0s 4ms/step
R2 Score: 0.2610945087871973
```

In [24]:

```python
# Perform walk-forward backtest for CNN
pred_cnn = walk_forward_backtest(model_cnn, shifted_X_cnn, shifted_y_cnn, train_size=500, test_size=500)
```

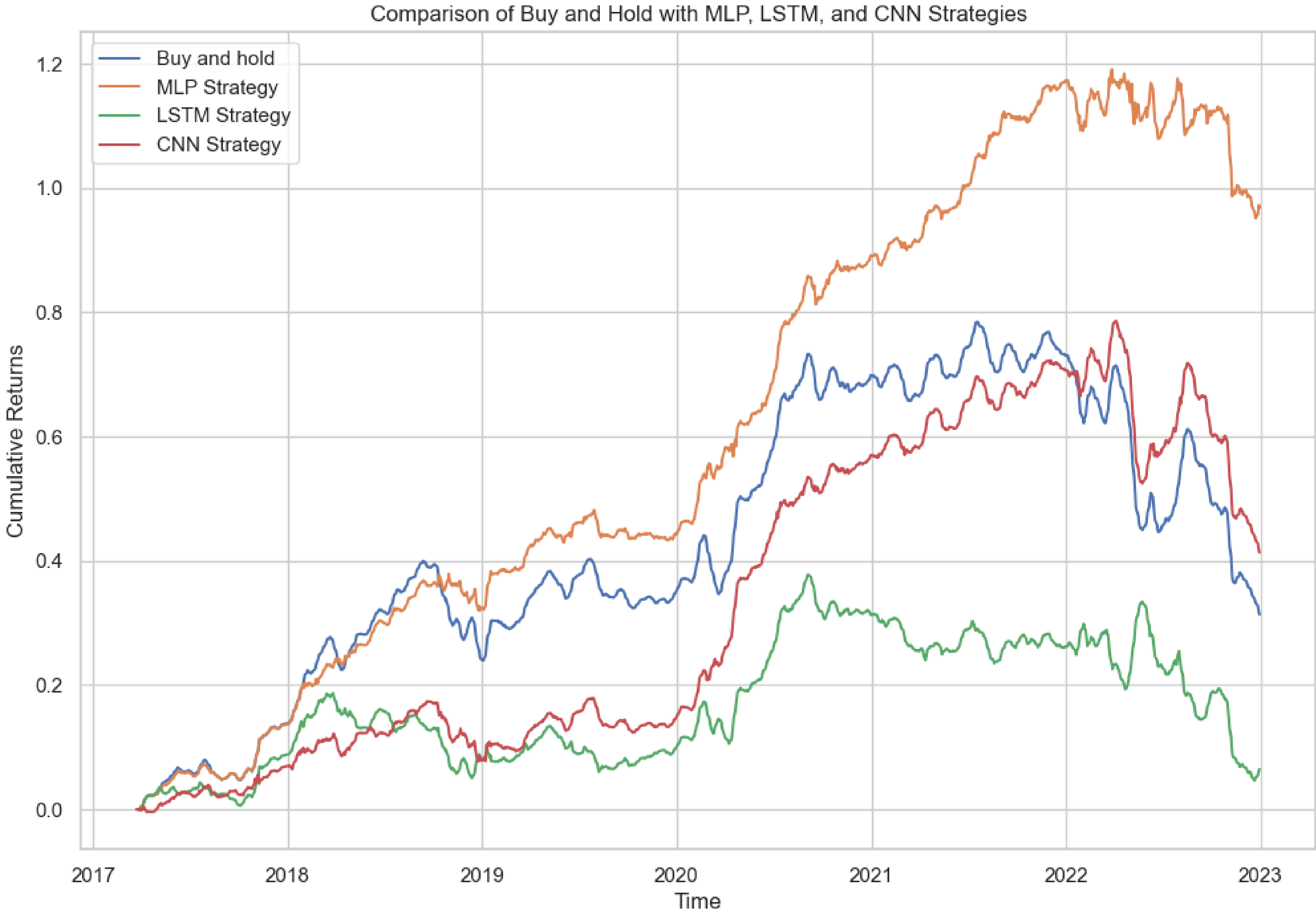
```
Step 1/3, Index: 500

X_train shape: (500, 140, 20), X_test shape: (500, 140, 20), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0538 - val_loss: 0.0355
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0445 - val_loss: 0.0292
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0406 - val_loss: 0.0327
Epoch 4/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0360 - val_loss: 0.0318
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0331 - val_loss: 0.0321
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0308 - val_loss: 0.0349
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0285 - val_loss: 0.0338
16/16 [==============================] - 0s 1ms/step
Step 2/3, Index: 1000

X_train shape: (500, 140, 20), X_test shape: (500, 140, 20), y_train shape: (500,), y_test shape: (500,)
Epoch 1/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0372 - val_loss: 0.0599
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0327 - val_loss: 0.0588
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0296 - val_loss: 0.0594
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0271 - val_loss: 0.0593
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0259 - val_loss: 0.0599
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0250 - val_loss: 0.0593
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0236 - val_loss: 0.0606
16/16 [==============================] - 0s 1ms/step
Step 3/3, Index: 1500

X_train shape: (500, 140, 20), X_test shape: (454, 140, 20), y_train shape: (500,), y_test shape: (454,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0471 - val_loss: 0.0397
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0418 - val_loss: 0.0405
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0373 - val_loss: 0.0422
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0368 - val_loss: 0.0411
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0342 - val_loss: 0.0418
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0328 - val_loss: 0.0422
15/15 [==============================] - 0s 5ms/step
R2 Score: -0.11343733139140966
```

In [25]:

```python
# Create a DataFrame for walk-forward backtest results
df_pred_wf = pd.DataFrame(index=y.iloc[-len(pred_lstm):].index)

# Populate DataFrame with actual and predicted values
df_pred_wf['y_test'] = y.iloc[-len(pred_lstm):]
df_pred_wf['pred_mlp'] = pred_mlp[-len(pred_lstm):]
df_pred_wf['pred_lstm'] = pred_lstm
df_pred_wf['pred_cnn'] = pred_cnn

# Calculate buy and hold cumulative return
df_pred_wf['cum_ret_bh'] = df_pred_wf['y_test'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# MLP Strategy
df_pred_wf['positions_mlp'] = df_pred_wf['pred_mlp'].apply(np.sign)
df_pred_wf['strat_ret_mlp'] = df_pred_wf['positions_mlp'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_mlp'] = df_pred_wf['strat_ret_mlp'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# LSTM Strategy
df_pred_wf['positions_lstm'] = df_pred_wf['pred_lstm'].apply(np.sign)
df_pred_wf['strat_ret_lstm'] = df_pred_wf['positions_lstm'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_lstm'] = df_pred_wf['strat_ret_lstm'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# CNN Strategy
df_pred_wf['positions_cnn'] = df_pred_wf['pred_cnn'].apply(np.sign)
df_pred_wf['strat_ret_cnn'] = df_pred_wf['positions_cnn'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_cnn'] = df_pred_wf['strat_ret_cnn'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# Calculate final returns
buy_return = np.prod(1 + df_pred_wf['y_test'])**(window_hp/252) - 1
strat_return_mlp = np.prod(1 + df_pred_wf['strat_ret_mlp'])**(window_hp/252) - 1
strat_return_lstm = np.prod(1 + df_pred_wf['strat_ret_lstm'])**(window_hp/252) - 1
strat_return_cnn = np.prod(1 + df_pred_wf['strat_ret_cnn'])**(window_hp/252) - 1

# Print returns
print("Buy and Hold Return: {:.4%}".format(buy_return))
print("MLP Strategy Return: {:.4%}".format(strat_return_mlp))
print("LSTM Strategy Return: {:.4%}".format(strat_return_lstm))
print("CNN Strategy Return: {:.4%}".format(strat_return_cnn))
```

```python
# Combined Plot
plt.figure(figsize=(12, 8))

# Plot cumulative returns with labels and legend
plt.plot(df_pred_wf['cum_ret_bh'], label='Buy and hold')
plt.plot(df_pred_wf['cum_ret_mlp'], label='MLP Strategy')
plt.plot(df_pred_wf['cum_ret_lstm'], label='LSTM Strategy')
plt.plot(df_pred_wf['cum_ret_cnn'], label='CNN Strategy')

# Add labels and title
plt.xlabel('Time')
plt.ylabel('Cumulative Returns')
plt.title('Comparison of Buy and Hold with MLP, LSTM, and CNN Strategies')
plt.legend()

# Display the plot
plt.show()
```

```
Buy and Hold Return: 31.3705%
MLP Strategy Return: 96.8299%
LSTM Strategy Return: 6.4004%
CNN Strategy Return: 41.3245%
```



c. Non achored walk forward method with train/test split with 500 observations in training set and 100 observations in test set.

In [26]:
```python
# Perform walk-forward backtest for MLP
pred_mlp = walk_forward_backtest(model_mlp, shifted_X_mlp, shifted_y_mlp, train_size=500, test_size=100)
```

```
Step 1/15, Index: 500

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0452 - val_loss: 0.0298
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0415 - val_loss: 0.0326
Epoch 3/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0385 - val_loss: 0.0342
Epoch 4/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0371 - val_loss: 0.0342
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0364 - val_loss: 0.0330
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0342 - val_loss: 0.0352
4/4 [==============================] - 0s 1000us/step
Step 2/15, Index: 600

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0382 - val_loss: 0.0293
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0356 - val_loss: 0.0282
Epoch 3/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0339 - val_loss: 0.0298
Epoch 4/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0326 - val_loss: 0.0306
Epoch 5/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0309 - val_loss: 0.0304
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0308 - val_loss: 0.0305
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0291 - val_loss: 0.0307
4/4 [==============================] - 0s 1ms/step
Step 3/15, Index: 700

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0322 - val_loss: 0.0298
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0305 - val_loss: 0.0292
Epoch 3/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0281 - val_loss: 0.0299
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0269 - val_loss: 0.0304
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0260 - val_loss: 0.0296
Epoch 6/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0246 - val_loss: 0.0305
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0235 - val_loss: 0.0317
4/4 [==============================] - 0s 1ms/step
Step 4/15, Index: 800

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0259 - val_loss: 0.0391
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0241 - val_loss: 0.0357
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0223 - val_loss: 0.0367
Epoch 4/30
11/11 [==============================] - 0s 4ms/step - loss: 0.0216 - val_loss: 0.0374
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0209 - val_loss: 0.0350
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0190 - val_loss: 0.0380
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0196 - val_loss: 0.0354
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0184 - val_loss: 0.0352
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0169 - val_loss: 0.0358
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0165 - val_loss: 0.0349
Epoch 11/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0162 - val_loss: 0.0355
Epoch 12/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0158 - val_loss: 0.0351
Epoch 13/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0158 - val_loss: 0.0368
Epoch 14/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0151 - val_loss: 0.0340
Epoch 15/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0154 - val_loss: 0.0367
Epoch 16/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0149 - val_loss: 0.0352
Epoch 17/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0146 - val_loss: 0.0369
Epoch 18/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0137 - val_loss: 0.0357
Epoch 19/30
```

```
11/11 [==============================] - 0s 5ms/step - loss: 0.0136 - val_loss: 0.0355
4/4 [==============================] - 0s 1ms/step
Step 5/15, Index: 900

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0200 - val_loss: 0.0469
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0189 - val_loss: 0.0454
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0183 - val_loss: 0.0448
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0176 - val_loss: 0.0444
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0165 - val_loss: 0.0446
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0155 - val_loss: 0.0462
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0156 - val_loss: 0.0446
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0163 - val_loss: 0.0463
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0160 - val_loss: 0.0448
4/4 [==============================] - 0s 1ms/step
Step 6/15, Index: 1000

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0257 - val_loss: 0.0635
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0242 - val_loss: 0.0654
Epoch 3/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0236 - val_loss: 0.0652
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0228 - val_loss: 0.0656
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0221 - val_loss: 0.0649
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0217 - val_loss: 0.0661
4/4 [==============================] - 0s 1ms/step
Step 7/15, Index: 1100

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0343 - val_loss: 0.0571
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0325 - val_loss: 0.0569
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0300 - val_loss: 0.0575
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0286 - val_loss: 0.0573
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0275 - val_loss: 0.0572
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0265 - val_loss: 0.0558
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0247 - val_loss: 0.0573
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0237 - val_loss: 0.0580
Epoch 9/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0231 - val_loss: 0.0599
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0221 - val_loss: 0.0581
Epoch 11/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0217 - val_loss: 0.0576
4/4 [==============================] - 0s 1ms/step
Step 8/15, Index: 1200

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0403 - val_loss: 0.0409
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0377 - val_loss: 0.0437
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0360 - val_loss: 0.0409
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0336 - val_loss: 0.0401
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0336 - val_loss: 0.0448
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0312 - val_loss: 0.0443
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0308 - val_loss: 0.0454
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0303 - val_loss: 0.0451
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0294 - val_loss: 0.0420
4/4 [==============================] - 0s 1ms/step
Step 9/15, Index: 1300

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
```

```
11/11 [==============================] - 0s 8ms/step - loss: 0.0382 - val_loss: 0.0424
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0370 - val_loss: 0.0426
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0359 - val_loss: 0.0425
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0341 - val_loss: 0.0424
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0341 - val_loss: 0.0480
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0310 - val_loss: 0.0454
4/4 [==============================] - 0s 1ms/step
Step 10/15, Index: 1400

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 32ms/step - loss: 0.0370 - val_loss: 0.0653
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0360 - val_loss: 0.0699
Epoch 3/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0323 - val_loss: 0.0719
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0330 - val_loss: 0.0697
Epoch 5/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0295 - val_loss: 0.0680
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0285 - val_loss: 0.0673
4/4 [==============================] - 0s 1ms/step
Step 11/15, Index: 1500

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0423 - val_loss: 0.0521
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0386 - val_loss: 0.0545
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0373 - val_loss: 0.0478
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0352 - val_loss: 0.0511
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0333 - val_loss: 0.0513
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0328 - val_loss: 0.0485
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0314 - val_loss: 0.0501
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0320 - val_loss: 0.0510
4/4 [==============================] - 0s 1ms/step
Step 12/15, Index: 1600

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0433 - val_loss: 0.0374
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0416 - val_loss: 0.0386
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0389 - val_loss: 0.0404
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0376 - val_loss: 0.0386
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0369 - val_loss: 0.0400
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0366 - val_loss: 0.0388
4/4 [==============================] - 0s 1ms/step
Step 13/15, Index: 1700

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0429 - val_loss: 0.0418
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0418 - val_loss: 0.0405
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0385 - val_loss: 0.0402
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0386 - val_loss: 0.0403
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0359 - val_loss: 0.0397
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0349 - val_loss: 0.0399
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0326 - val_loss: 0.0408
Epoch 8/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0328 - val_loss: 0.0418
Epoch 9/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0317 - val_loss: 0.0406
Epoch 10/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0290 - val_loss: 0.0414
4/4 [==============================] - 0s 1ms/step
Step 14/15, Index: 1800

X_train shape: (500, 7), X_test shape: (100, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
```

```
11/11 [==============================] - 0s 8ms/step - loss: 0.0377 - val_loss: 0.0529
Epoch 2/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0363 - val_loss: 0.0580
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0347 - val_loss: 0.0548
Epoch 4/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0323 - val_loss: 0.0576
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0306 - val_loss: 0.0547
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0298 - val_loss: 0.0547
4/4 [==============================] - 0s 1ms/step
Step 15/15, Index: 1900

X_train shape: (500, 7), X_test shape: (94, 7), y_train shape: (500,), y_test shape: (94,)
Epoch 1/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0355 - val_loss: 0.0994
Epoch 2/30
11/11 [==============================] - 0s 8ms/step - loss: 0.0332 - val_loss: 0.0956
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0312 - val_loss: 0.0950
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0295 - val_loss: 0.0930
Epoch 5/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0285 - val_loss: 0.0948
Epoch 6/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0276 - val_loss: 0.0953
Epoch 7/30
11/11 [==============================] - 0s 5ms/step - loss: 0.0266 - val_loss: 0.0948
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0253 - val_loss: 0.0933
Epoch 9/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0246 - val_loss: 0.0931
3/3 [==============================] - 0s 1ms/step
R2 Score: -0.23800973403602144
```

In [27]:

```python
# Perform walk-forward backtest for LSTM
pred_lstm = walk_forward_backtest(model_lstm, shifted_X_lstm, shifted_y_lstm, train_size=500, test_size=100)
```

```
Step 1/15, Index: 500

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 24ms/step - loss: 0.0426 - val_loss: 0.0246
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0314 - val_loss: 0.0202
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0257 - val_loss: 0.0221
Epoch 4/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0245 - val_loss: 0.0215
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0216 - val_loss: 0.0223
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0185 - val_loss: 0.0221
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0173 - val_loss: 0.0246
4/4 [==============================] - 0s 4ms/step
Step 2/15, Index: 600

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0257 - val_loss: 0.0203
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0226 - val_loss: 0.0195
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0212 - val_loss: 0.0211
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0189 - val_loss: 0.0201
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0177 - val_loss: 0.0189
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0175 - val_loss: 0.0196
Epoch 7/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0176 - val_loss: 0.0195
Epoch 8/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0171 - val_loss: 0.0206
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0151 - val_loss: 0.0219
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0145 - val_loss: 0.0217
4/4 [==============================] - 0s 5ms/step
Step 3/15, Index: 700

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0160 - val_loss: 0.0284
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0154 - val_loss: 0.0294
Epoch 3/30
11/11 [==============================] - 0s 17ms/step - loss: 0.0137 - val_loss: 0.0254
Epoch 4/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0125 - val_loss: 0.0263
Epoch 5/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0122 - val_loss: 0.0250
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0121 - val_loss: 0.0274
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0125 - val_loss: 0.0257
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0116 - val_loss: 0.0257
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0119 - val_loss: 0.0275
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0118 - val_loss: 0.0260
4/4 [==============================] - 0s 4ms/step
Step 4/15, Index: 800

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0127 - val_loss: 0.0359
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0119 - val_loss: 0.0355
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0110 - val_loss: 0.0368
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0102 - val_loss: 0.0381
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0100 - val_loss: 0.0365
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0096 - val_loss: 0.0377
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0091 - val_loss: 0.0360
4/4 [==============================] - 0s 4ms/step
Step 5/15, Index: 900

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0170 - val_loss: 0.0245
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0141 - val_loss: 0.0249
Epoch 3/30
```

```
11/11 [==============================] - 0s 13ms/step - loss: 0.0136 - val_loss: 0.0255
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0124 - val_loss: 0.0246
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0118 - val_loss: 0.0267
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0103 - val_loss: 0.0233
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0104 - val_loss: 0.0269
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0103 - val_loss: 0.0280
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0096 - val_loss: 0.0260
Epoch 10/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0107 - val_loss: 0.0282
Epoch 11/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0096 - val_loss: 0.0239
4/4 [==============================] - 0s 4ms/step
Step 6/15, Index: 1000

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 20ms/step - loss: 0.0154 - val_loss: 0.0501
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0131 - val_loss: 0.0475
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0120 - val_loss: 0.0483
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0133 - val_loss: 0.0565
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0126 - val_loss: 0.0416
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0114 - val_loss: 0.0471
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0104 - val_loss: 0.0524
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0105 - val_loss: 0.0443
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0100 - val_loss: 0.0455
Epoch 10/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0101 - val_loss: 0.0467
4/4 [==============================] - 0s 4ms/step
Step 7/15, Index: 1100

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0196 - val_loss: 0.0345
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0164 - val_loss: 0.0343
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0152 - val_loss: 0.0346
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0137 - val_loss: 0.0333
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0141 - val_loss: 0.0313
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0144 - val_loss: 0.0342
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0158 - val_loss: 0.0371
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0158 - val_loss: 0.0340
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0140 - val_loss: 0.0328
Epoch 10/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0134 - val_loss: 0.0341
4/4 [==============================] - 0s 4ms/step
Step 8/15, Index: 1200

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0202 - val_loss: 0.0311
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0209 - val_loss: 0.0314
Epoch 3/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0202 - val_loss: 0.0307
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0164 - val_loss: 0.0326
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0169 - val_loss: 0.0321
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0175 - val_loss: 0.0331
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0170 - val_loss: 0.0319
Epoch 8/30
11/11 [==============================] - 0s 17ms/step - loss: 0.0148 - val_loss: 0.0328
4/4 [==============================] - 0s 4ms/step
Step 9/15, Index: 1300

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0224 - val_loss: 0.0434
Epoch 2/30
```

```
11/11 [==============================] - 0s 14ms/step - loss: 0.0191 - val_loss: 0.0426
Epoch 3/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0165 - val_loss: 0.0452
Epoch 4/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0158 - val_loss: 0.0421
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0159 - val_loss: 0.0434
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0149 - val_loss: 0.0453
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0137 - val_loss: 0.0459
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0129 - val_loss: 0.0428
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0139 - val_loss: 0.0434
4/4 [==============================] - 0s 4ms/step
Step 10/15, Index: 1400

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0198 - val_loss: 0.0502
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0183 - val_loss: 0.0442
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0156 - val_loss: 0.0496
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0151 - val_loss: 0.0496
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0143 - val_loss: 0.0503
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0149 - val_loss: 0.0492
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0127 - val_loss: 0.0445
4/4 [==============================] - 0s 4ms/step
Step 11/15, Index: 1500

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0233 - val_loss: 0.0344
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0222 - val_loss: 0.0291
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0198 - val_loss: 0.0339
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0175 - val_loss: 0.0305
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0156 - val_loss: 0.0312
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0152 - val_loss: 0.0323
Epoch 7/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0162 - val_loss: 0.0295
4/4 [==============================] - 0s 4ms/step
Step 12/15, Index: 1600

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0242 - val_loss: 0.0259
Epoch 2/30
11/11 [==============================] - 0s 18ms/step - loss: 0.0227 - val_loss: 0.0265
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0198 - val_loss: 0.0328
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0185 - val_loss: 0.0293
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0179 - val_loss: 0.0254
Epoch 6/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0159 - val_loss: 0.0259
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0152 - val_loss: 0.0294
Epoch 8/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0151 - val_loss: 0.0274
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0157 - val_loss: 0.0282
Epoch 10/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0146 - val_loss: 0.0291
4/4 [==============================] - 0s 5ms/step
Step 13/15, Index: 1700

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0203 - val_loss: 0.0362
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0217 - val_loss: 0.0387
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0188 - val_loss: 0.0388
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0180 - val_loss: 0.0380
Epoch 5/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0172 - val_loss: 0.0437
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0169 - val_loss: 0.0416
4/4 [==============================] - 0s 5ms/step
```

```
Step 14/15, Index: 1800

X_train shape: (500, 20, 7), X_test shape: (100, 20, 7), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0243 - val_loss: 0.0494
Epoch 2/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0204 - val_loss: 0.0546
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0189 - val_loss: 0.0545
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0181 - val_loss: 0.0538
Epoch 5/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0172 - val_loss: 0.0532
Epoch 6/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0169 - val_loss: 0.0555
4/4 [==============================] - 0s 4ms/step
Step 15/15, Index: 1900

X_train shape: (500, 20, 7), X_test shape: (54, 20, 7), y_train shape: (500,), y_test shape: (54,)
Epoch 1/30
11/11 [==============================] - 0s 16ms/step - loss: 0.0273 - val_loss: 0.0663
Epoch 2/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0224 - val_loss: 0.0590
Epoch 3/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0201 - val_loss: 0.0595
Epoch 4/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0183 - val_loss: 0.0610
Epoch 5/30
11/11 [==============================] - 0s 15ms/step - loss: 0.0180 - val_loss: 0.0547
Epoch 6/30
11/11 [==============================] - 0s 19ms/step - loss: 0.0182 - val_loss: 0.0647
Epoch 7/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0160 - val_loss: 0.0582
Epoch 8/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0144 - val_loss: 0.0577
Epoch 9/30
11/11 [==============================] - 0s 13ms/step - loss: 0.0157 - val_loss: 0.0568
Epoch 10/30
11/11 [==============================] - 0s 14ms/step - loss: 0.0159 - val_loss: 0.0621
2/2 [==============================] - 0s 5ms/step
R2 Score: 0.3308382308098812
```

In [28]:

```python
# Perform walk-forward backtest for CNN
pred_cnn = walk_forward_backtest(model_cnn, shifted_X_cnn, shifted_y_cnn, train_size=500, test_size=100)
```

```
Step 1/15, Index: 500

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 11ms/step - loss: 0.0427 - val_loss: 0.0300
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0403 - val_loss: 0.0298
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0371 - val_loss: 0.0312
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0349 - val_loss: 0.0330
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0325 - val_loss: 0.0311
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0323 - val_loss: 0.0335
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0310 - val_loss: 0.0312
4/4 [==============================] - 0s 2ms/step
Step 2/15, Index: 600

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0395 - val_loss: 0.0248
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0363 - val_loss: 0.0240
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0337 - val_loss: 0.0258
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0310 - val_loss: 0.0262
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0285 - val_loss: 0.0270
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0271 - val_loss: 0.0280
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0260 - val_loss: 0.0270
4/4 [==============================] - 0s 2ms/step
Step 3/15, Index: 700

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0301 - val_loss: 0.0314
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0278 - val_loss: 0.0308
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0251 - val_loss: 0.0306
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0229 - val_loss: 0.0319
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0215 - val_loss: 0.0316
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0192 - val_loss: 0.0308
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0196 - val_loss: 0.0322
Epoch 8/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0206 - val_loss: 0.0316
4/4 [==============================] - 0s 2ms/step
Step 4/15, Index: 800

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0214 - val_loss: 0.0420
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0214 - val_loss: 0.0411
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0198 - val_loss: 0.0429
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0187 - val_loss: 0.0409
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0175 - val_loss: 0.0418
Epoch 6/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0169 - val_loss: 0.0420
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0165 - val_loss: 0.0422
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0159 - val_loss: 0.0433
Epoch 9/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0154 - val_loss: 0.0421
4/4 [==============================] - 0s 1ms/step
Step 5/15, Index: 900

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0241 - val_loss: 0.0393
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0230 - val_loss: 0.0435
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0212 - val_loss: 0.0421
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0199 - val_loss: 0.0425
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0195 - val_loss: 0.0423
Epoch 6/30
```

```
11/11 [==============================] - 0s 7ms/step - loss: 0.0194 - val_loss: 0.0426
4/4 [==============================] - 0s 1ms/step
Step 6/15, Index: 1000

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0288 - val_loss: 0.0591
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0264 - val_loss: 0.0593
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0237 - val_loss: 0.0593
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0222 - val_loss: 0.0597
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0208 - val_loss: 0.0594
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0201 - val_loss: 0.0617
4/4 [==============================] - 0s 1ms/step
Step 7/15, Index: 1100

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0369 - val_loss: 0.0500
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0335 - val_loss: 0.0496
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0290 - val_loss: 0.0500
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0244 - val_loss: 0.0516
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0217 - val_loss: 0.0504
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0206 - val_loss: 0.0525
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0210 - val_loss: 0.0514
4/4 [==============================] - 0s 1ms/step
Step 8/15, Index: 1200

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0374 - val_loss: 0.0363
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0362 - val_loss: 0.0354
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0298 - val_loss: 0.0360
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0280 - val_loss: 0.0359
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0268 - val_loss: 0.0381
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0255 - val_loss: 0.0359
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0230 - val_loss: 0.0369
4/4 [==============================] - 0s 2ms/step
Step 9/15, Index: 1300

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0347 - val_loss: 0.0547
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0318 - val_loss: 0.0544
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0274 - val_loss: 0.0548
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0270 - val_loss: 0.0557
Epoch 5/30
11/11 [==============================] - 0s 10ms/step - loss: 0.0229 - val_loss: 0.0541
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0232 - val_loss: 0.0550
Epoch 7/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0216 - val_loss: 0.0563
Epoch 8/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0208 - val_loss: 0.0541
Epoch 9/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0205 - val_loss: 0.0565
Epoch 10/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0207 - val_loss: 0.0588
4/4 [==============================] - 0s 1ms/step
Step 10/15, Index: 1400

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0300 - val_loss: 0.0577
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0283 - val_loss: 0.0580
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0247 - val_loss: 0.0612
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0248 - val_loss: 0.0593
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0246 - val_loss: 0.0599
Epoch 6/30
```

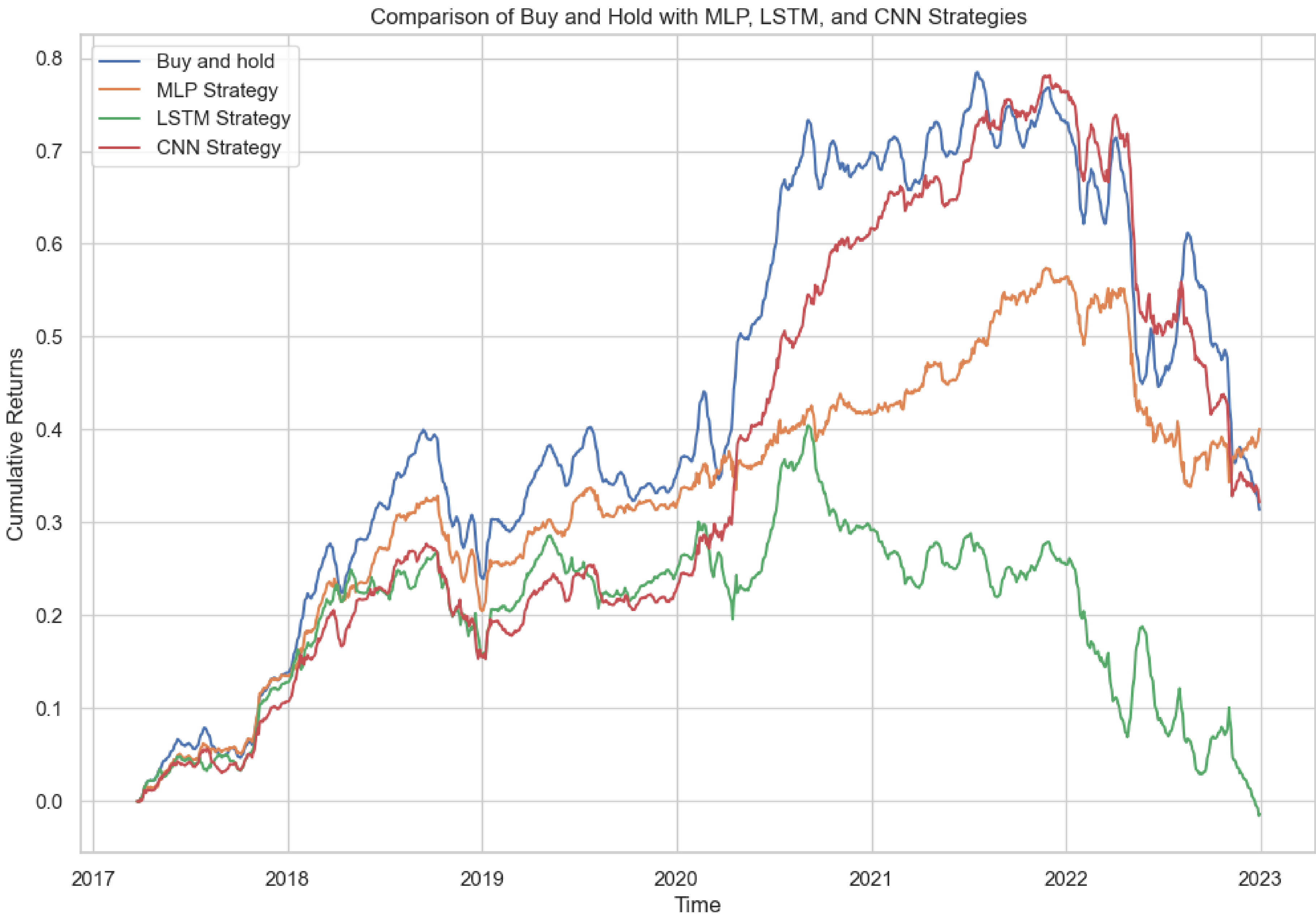
```
11/11 [==============================] - 0s 7ms/step - loss: 0.0213 - val_loss: 0.0595
4/4 [==============================] - 0s 1ms/step
Step 11/15, Index: 1500

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0369 - val_loss: 0.0449
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0326 - val_loss: 0.0462
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0308 - val_loss: 0.0463
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0292 - val_loss: 0.0478
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0271 - val_loss: 0.0464
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0252 - val_loss: 0.0486
4/4 [==============================] - 0s 1ms/step
Step 12/15, Index: 1600

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0427 - val_loss: 0.0400
Epoch 2/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0387 - val_loss: 0.0411
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0326 - val_loss: 0.0423
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0322 - val_loss: 0.0402
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0278 - val_loss: 0.0417
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0256 - val_loss: 0.0429
4/4 [==============================] - 0s 1ms/step
Step 13/15, Index: 1700

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0461 - val_loss: 0.0459
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0404 - val_loss: 0.0432
Epoch 3/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0383 - val_loss: 0.0457
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0323 - val_loss: 0.0439
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0329 - val_loss: 0.0487
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0292 - val_loss: 0.0459
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0314 - val_loss: 0.0461
4/4 [==============================] - 0s 1ms/step
Step 14/15, Index: 1800

X_train shape: (500, 140, 20), X_test shape: (100, 140, 20), y_train shape: (500,), y_test shape: (100,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0377 - val_loss: 0.0666
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0324 - val_loss: 0.0677
Epoch 3/30
11/11 [==============================] - 0s 11ms/step - loss: 0.0308 - val_loss: 0.0671
Epoch 4/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0280 - val_loss: 0.0689
Epoch 5/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0280 - val_loss: 0.0680
Epoch 6/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0289 - val_loss: 0.0680
4/4 [==============================] - 0s 1ms/step
Step 15/15, Index: 1900

X_train shape: (500, 140, 20), X_test shape: (54, 140, 20), y_train shape: (500,), y_test shape: (54,)
Epoch 1/30
11/11 [==============================] - 0s 9ms/step - loss: 0.0404 - val_loss: 0.0892
Epoch 2/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0360 - val_loss: 0.0876
Epoch 3/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0352 - val_loss: 0.0904
Epoch 4/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0313 - val_loss: 0.0906
Epoch 5/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0299 - val_loss: 0.0889
Epoch 6/30
11/11 [==============================] - 0s 6ms/step - loss: 0.0278 - val_loss: 0.0888
Epoch 7/30
11/11 [==============================] - 0s 7ms/step - loss: 0.0256 - val_loss: 0.0892
2/2 [==============================] - 0s 2ms/step
R2 Score: -0.0976623232553473
```

In [29]:

```
# Create a DataFrame for walk-forward backtest results
df_pred_wf = pd.DataFrame(index=y.iloc[-len(pred_lstm):].index)

# Populate DataFrame with actual and predicted values
```

```python
df_pred_wf['y_test'] = y.iloc[-len(pred_lstm):]
df_pred_wf['pred_mlp'] = pred_mlp[-len(pred_lstm):]
df_pred_wf['pred_lstm'] = pred_lstm
df_pred_wf['pred_cnn'] = pred_cnn

# Calculate buy and hold cumulative return
df_pred_wf['cum_ret_bh'] = df_pred_wf['y_test'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# MLP Strategy
df_pred_wf['positions_mlp'] = df_pred_wf['pred_mlp'].apply(np.sign)
df_pred_wf['strat_ret_mlp'] = df_pred_wf['positions_mlp'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_mlp'] = df_pred_wf['strat_ret_mlp'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# LSTM Strategy
df_pred_wf['positions_lstm'] = df_pred_wf['pred_lstm'].apply(np.sign)
df_pred_wf['strat_ret_lstm'] = df_pred_wf['positions_lstm'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_lstm'] = df_pred_wf['strat_ret_lstm'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# CNN Strategy
df_pred_wf['positions_cnn'] = df_pred_wf['pred_cnn'].apply(np.sign)
df_pred_wf['strat_ret_cnn'] = df_pred_wf['positions_cnn'] * df_pred_wf['y_test']
df_pred_wf['cum_ret_cnn'] = df_pred_wf['strat_ret_cnn'].expanding().apply(lambda x: np.prod(1 + x)**(window_hp/252) - 1)

# Calculate final returns
buy_return = np.prod(1 + df_pred_wf['y_test'])**(window_hp/252) - 1
strat_return_mlp = np.prod(1 + df_pred_wf['strat_ret_mlp'])**(window_hp/252) - 1
strat_return_lstm = np.prod(1 + df_pred_wf['strat_ret_lstm'])**(window_hp/252) - 1
strat_return_cnn = np.prod(1 + df_pred_wf['strat_ret_cnn'])**(window_hp/252) - 1

# Print returns
print("Buy and Hold Return: {:.4%}".format(buy_return))
print("MLP Strategy Return: {:.4%}".format(strat_return_mlp))
print("LSTM Strategy Return: {:.4%}".format(strat_return_lstm))
print("CNN Strategy Return: {:.4%}".format(strat_return_cnn))

# Combined Plot
plt.figure(figsize=(12, 8))

# Plot cumulative returns with labels and legend
plt.plot(df_pred_wf['cum_ret_bh'], label='Buy and hold')
plt.plot(df_pred_wf['cum_ret_mlp'], label='MLP Strategy')
plt.plot(df_pred_wf['cum_ret_lstm'], label='LSTM Strategy')
plt.plot(df_pred_wf['cum_ret_cnn'], label='CNN Strategy')

# Add labels and title
plt.xlabel('Time')
plt.ylabel('Cumulative Returns')
plt.title('Comparison of Buy and Hold with MLP, LSTM, and CNN Strategies')
plt.legend()

# Display the plot
plt.show()
```

```
Buy and Hold Return: 31.3705%
MLP Strategy Return: 40.0560%
LSTM Strategy Return: -1.3837%
CNN Strategy Return: 32.1789%
```


Comparison of Buy and Hold with MLP, LSTM, and CNN Strategies
Buy and hold
MLP Strategy
LSTM Strategy
CNN Strategy
Cumulative Returns
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
2017
2018
2019
2020
2021
2022
2023
Time