Contents

Modeling non-stationarity

In [1]:

```
import numpy as np
import pandas as pd
import yfinance as yf
import matplotlib.pyplot as plt
import seaborn as sns
from arch.unitroot import ADF, KPSS
from arch import arch_model
import statsmodels.api as sm
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.api import ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.stats.diagnostic import acorr_ljungbox
import scipy.stats as stats
from pmdarima import auto_arima
import warnings
import sys

warnings.filterwarnings("ignore")
sns.set(style='whitegrid', rc={"grid.linewidth": 0.1})
sns.set_context("paper")
params = {
    'legend.fontsize': 'medium',
    'axes.labelsize': 'large',
    'xtick.labelsize':'medium',
    'ytick.labelsize':'medium',
    'axes.titlesize': 'large',
    'figure.figsize':(12,9)
}
plt.rcParams.update(params)
```

Selecting the dataset

Microsoft, Tesla and Bitcoin's closing prices with daily frequencies were chosen from 2020 to 2021.

In [2]:
```
dataset = yf.download(["MSFT", "TSLA", "BTC-USD"], start="2019-01-01", end="2022-01-01")["Adj Close"].dropna()
dataset.head()
```

```
[*********************100%***********************]  3 of 3 completed
```

Out[2]:

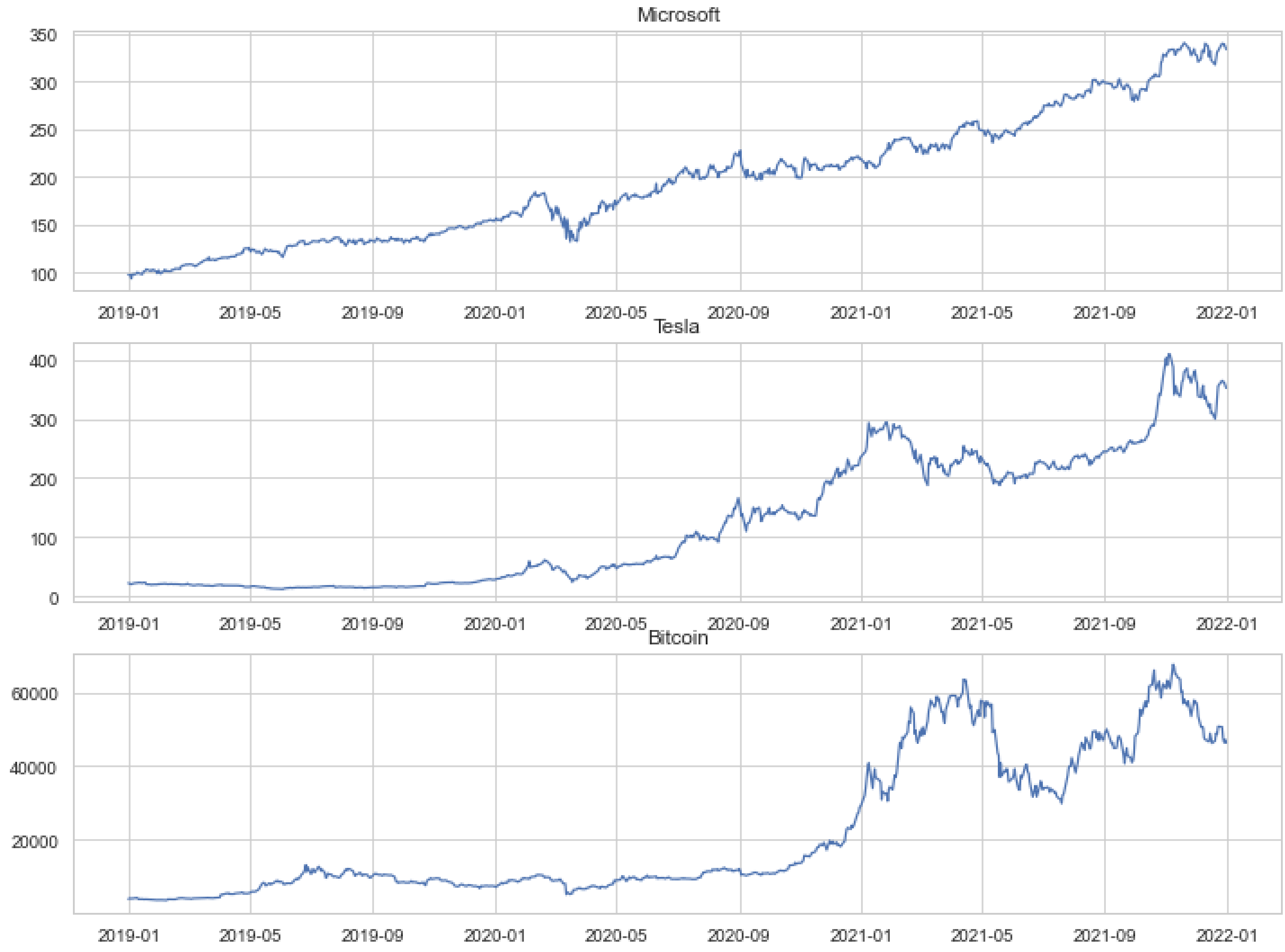
Date	BTC-USD	MSFT	TSLA
2018-12-31	3742.700439	97.580017	22.186666
2019-01-02	3943.409424	97.147713	20.674667
2019-01-03	3836.741211	93.573822	20.024000
2019-01-04	3857.717529	97.925896	21.179333
2019-01-07	4025.248291	98.050781	22.330667

Plot chosen dataset

In [3]:
```
plt.subplot(3, 1, 1)
plt.plot(dataset['MSFT'])
plt.title('Microsoft')

plt.subplot(3, 1, 2)
plt.plot(dataset['TSLA'])
plt.title('Tesla')

plt.subplot(3, 1, 3)
plt.plot(dataset['BTC-USD'])
plt.title('Bitcoin');
```



We can realize that all item have trend.
We can determine that ADF test will be executed with trend.

Stationary Test

ADF Test with trend

In [25]:
```
microsoft_adf_with_trend = ADF(dataset['MSFT'], trend="ct")
tesla_adf_with_trend = ADF(dataset['TSLA'], trend="ct")
bitcoin_adf_with_trend = ADF(dataset['BTC-USD'], trend="ct")
print(f"Test statistics \n{microsoft_adf_with_trend.summary()}")
print(f"Test statistics \n{tesla_adf_with_trend.summary()}")
print(f"Test statistics \n{bitcoin_adf_with_trend.summary()}")
```

```
Test statistics
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -2.655
P-value                         0.255
Lags                               16
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.97 (1%), -3.42 (5%), -3.13 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
Test statistics
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -2.296
P-value                         0.436
Lags                                0
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.97 (1%), -3.42 (5%), -3.13 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
Test statistics
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -1.989
P-value                         0.608
Lags                               12
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.97 (1%), -3.42 (5%), -3.13 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

KPSS test

In [5]:
```
microsoft_kpss = KPSS(dataset['MSFT'])
tesla_kpss = KPSS(dataset['TSLA'])
bitcoin_kpss = KPSS(dataset['BTC-USD'])
print(microsoft_kpss.summary())
print(tesla_kpss.summary())
print(bitcoin_kpss.summary())
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  4.134
P-value                         0.000
Lags                               17
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  3.902
P-value                         0.000
Lags                               17
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  3.343
P-value                         0.000
Lags                               17
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

We have conclude that all items cannot reject the null hypothesis that the process contains a unit root, which means all item are not stationary by ADF with trend and KPSS test.

Mitigation of non-stationarity

In [6]:
```
diff_data = dataset.diff().dropna()
diff_data.head()
```

Out[6]:

Date	BTC-USD	MSFT	TSLA
2019-01-02	200.708984	-0.432304	-1.511999
2019-01-03	-106.668213	-3.573891	-0.650667
2019-01-04	20.976318	4.352074	1.155333
2019-01-07	167.530762	0.124886	1.151335
2019-01-08	5.599609	0.710922	0.025999

ADF Test

In [7]:
```
diff_microsoft_adf_with_trend = ADF(diff_data['MSFT'], trend="ct")
diff_tesla_adf_with_trend = ADF(diff_data['TSLA'], trend="ct")
diff_bitcoin_adf_with_trend = ADF(diff_data['BTC-USD'], trend="ct")
print(f"Test statistics \n{diff_microsoft_adf_with_trend.summary()}")
print(f"Test statistics \n{diff_tesla_adf_with_trend.summary()}")
print(f"Test statistics \n{diff_bitcoin_adf_with_trend.summary()}")
```

```
Test statistics
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -8.128
P-value                         0.000
Lags                               12
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.97 (1%), -3.42 (5%), -3.13 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
Test statistics
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                -27.683
P-value                         0.000
Lags                                0
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.97 (1%), -3.42 (5%), -3.13 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
Test statistics
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -8.025
P-value                         0.000
Lags                               11
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.97 (1%), -3.42 (5%), -3.13 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

KPSS test

In [8]:
```
diff_microsoft_kpss = KPSS(diff_data['MSFT'])
diff_tesla_kpss = KPSS(diff_data['TSLA'])
diff_bitcoin_kpss = KPSS(diff_data['BTC-USD'])
print(diff_microsoft_kpss.summary())
print(diff_tesla_kpss.summary())
print(diff_bitcoin_kpss.summary())
```

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.099
P-value                         0.589
Lags                                8
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.182
P-value                         0.306
Lags                                4
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.091
P-value                         0.633
Lags                                6
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

The null-hypothesis, that the process is stationary cannot be rejected.

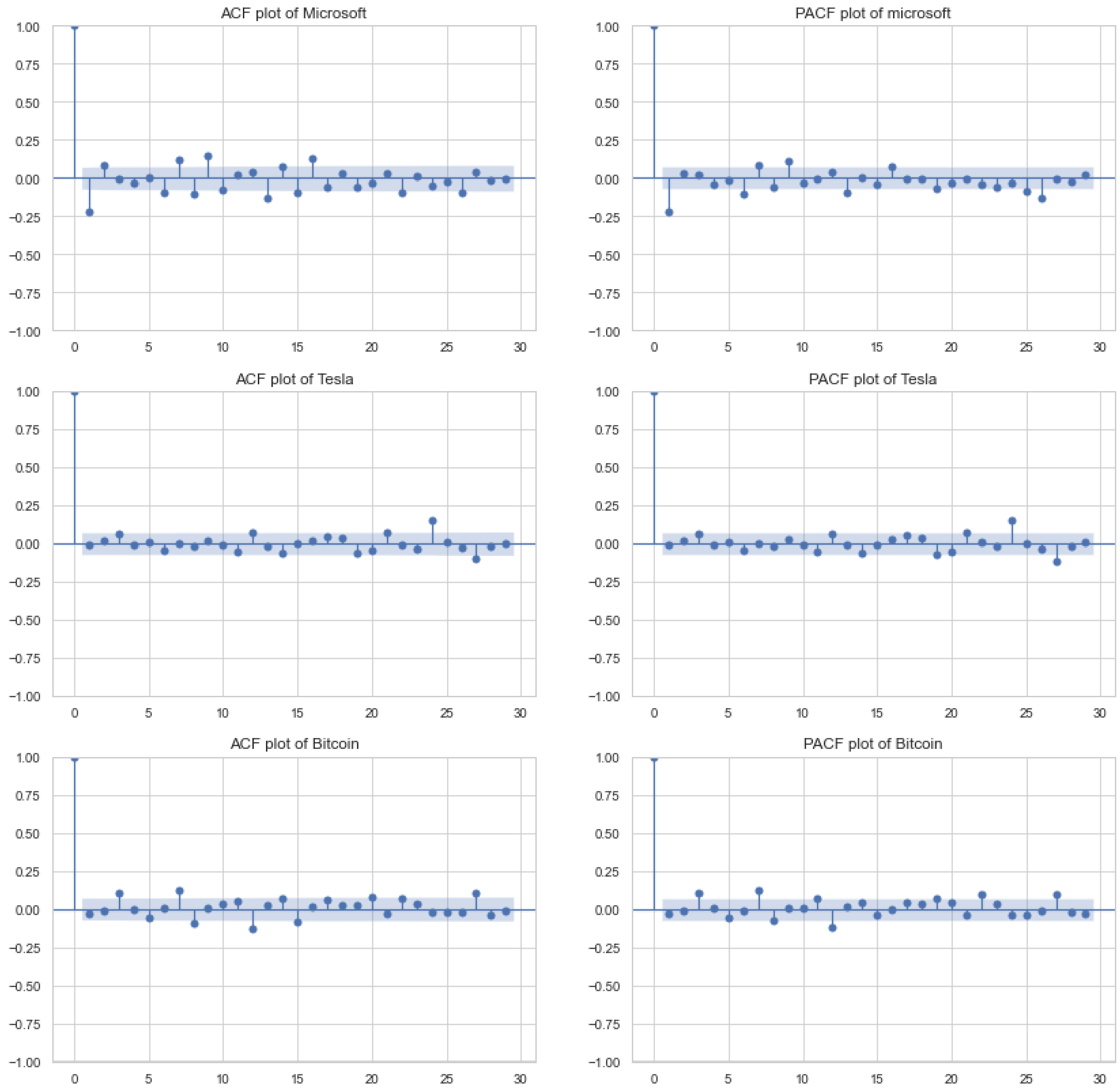
The above tests suggests that a difference component should be added to the model.

Parameter selection for time series modeling

ACF and PACF plots of the chosen datasets (First difference)

In [9]:
```
fig, ax = plt.subplots(3,2, figsize=(14,14))

plot_acf(diff_data["MSFT"], ax=ax[0][0], title="ACF plot of Microsoft")
plot_pacf(diff_data["MSFT"], ax=ax[0][1], method="ywm", title="PACF plot of microsoft");
plot_acf(diff_data["TSLA"], ax=ax[1][0], title="ACF plot of Tesla")
plot_pacf(diff_data["TSLA"], ax=ax[1][1], method="ywm", title="PACF plot of Tesla");
plot_acf(diff_data["BTC-USD"], ax=ax[2][0], title="ACF plot of Bitcoin")
plot_pacf(diff_data["BTC-USD"], ax=ax[2][1], method="ywm", title="PACF plot of Bitcoin");
```



We have concluded to choose Microsoft prices as the ACF and PACF plot clearly shows that Tesla and Bitcoin does not have any significant ARMA component that can be modeled.

Microsoft
Lag(1) is significant in both the ACF and PACF plots.

An AR(1) component and MA(1) component exists, which are both negative.

MA(2) is positive and significant.

There are some seasonal components on 7^{th} and 9^{th} lags

Modeling the series

From the above inferences, we can use an ARIMA(1,1,2) for Microsoft as initial parameters to model the time-series.

ARIMA(1,1,2)

In [10]:
```
arima_112 = SARIMAX(dataset['MSFT'], order=(1,1,2)).fit()
```

In [11]:
```
print(arima_112.summary())
```

```
                               SARIMAX Results                                
==============================================================================
Dep. Variable:                   MSFT   No. Observations:                  758
Model:               SARIMAX(1, 1, 2)   Log Likelihood               -2009.475
Date:                Tue, 04 Oct 2022   AIC                           4026.950
Time:                        15:44:56   BIC                           4045.467
Sample:                             0   HQIC                          4034.082
                                - 758                                         
Covariance Type:                  opg                                         
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.0363      0.287     -0.127      0.899      -0.598       0.526
ma.L1         -0.1692      0.283     -0.597      0.550      -0.724       0.386
ma.L2          0.0896      0.064      1.402      0.161      -0.036       0.215
sigma2        11.8352      0.385     30.703      0.000      11.080      12.591
===================================================================================
Ljung-Box (L1) (Q):                   0.09   Jarque-Bera (JB):               467.20
Prob(Q):                              0.76   Prob(JB):                         0.00
Heteroskedasticity (H):               5.61   Skew:                            -0.30
Prob(H) (two-sided):                  0.00   Kurtosis:                         6.80
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

From the summary, no coefficient rejects the null hypothesis. So, the initial ARIMA parameters needs to be modified for a better fit.

Auto-ARIMA

auto_arima module from pmdarima selects the parameters that minimizes the selected information criterion (AIC in our case).

In [12]:
```
auto_model = auto_arima(
    dataset["MSFT"],
    start_p=0,
    start_d=0,
    start_q=0,
    max_p=3,
    max_d=3,
    max_q=3,
    trace=True,
    with_intercept=False,
    return_valid_fits=True
)
```

```
Performing stepwise search to minimize aic
 ARIMA(0,1,0)(0,0,0)[0]             : AIC=4058.795, Time=0.02 sec
 ARIMA(1,1,0)(0,0,0)[0]             : AIC=4025.207, Time=0.02 sec
 ARIMA(0,1,1)(0,0,0)[0]             : AIC=4029.518, Time=0.03 sec
 ARIMA(2,1,0)(0,0,0)[0]             : AIC=4025.709, Time=0.04 sec
 ARIMA(1,1,1)(0,0,0)[0]             : AIC=4026.097, Time=0.05 sec
 ARIMA(2,1,1)(0,0,0)[0]             : AIC=4027.428, Time=0.11 sec
 ARIMA(1,1,0)(0,0,0)[0] intercept   : AIC=4017.841, Time=0.04 sec
 ARIMA(0,1,0)(0,0,0)[0] intercept   : AIC=4054.831, Time=0.02 sec
 ARIMA(2,1,0)(0,0,0)[0] intercept   : AIC=4019.060, Time=0.09 sec
 ARIMA(1,1,1)(0,0,0)[0] intercept   : AIC=4019.203, Time=0.10 sec
 ARIMA(0,1,1)(0,0,0)[0] intercept   : AIC=4021.803, Time=0.05 sec
 ARIMA(2,1,1)(0,0,0)[0] intercept   : AIC=4020.985, Time=0.19 sec

Best model:  ARIMA(1,1,0)(0,0,0)[0] intercept
Total fit time: 0.750 seconds
```

Now that we have found ARIMA(1,1,0) minimizes the AIC criterion, we can use the parameters to fit an ARIMA model and diagnose the coefficients

In [13]:
```
arima_110 = SARIMAX(dataset['MSFT'], order=(1,1,0)).fit()
```

In [14]:
```
print(arima_110.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                   MSFT   No. Observations:                  758
Model:               SARIMAX(1, 1, 0)   Log Likelihood               -2010.603
Date:                Tue, 04 Oct 2022   AIC                           4025.207
Time:                        15:44:57   BIC                           4034.465
Sample:                             0   HQIC                          4028.773
                                - 758
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.2143      0.021    -10.268      0.000      -0.255      -0.173
sigma2        11.8709      0.377     31.504      0.000      11.132      12.609
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):               505.17
Prob(Q):                              0.93   Prob(JB):                         0.00
Heteroskedasticity (H):               5.67   Skew:                            -0.36
Prob(H) (two-sided):                  0.00   Kurtosis:                         6.94
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

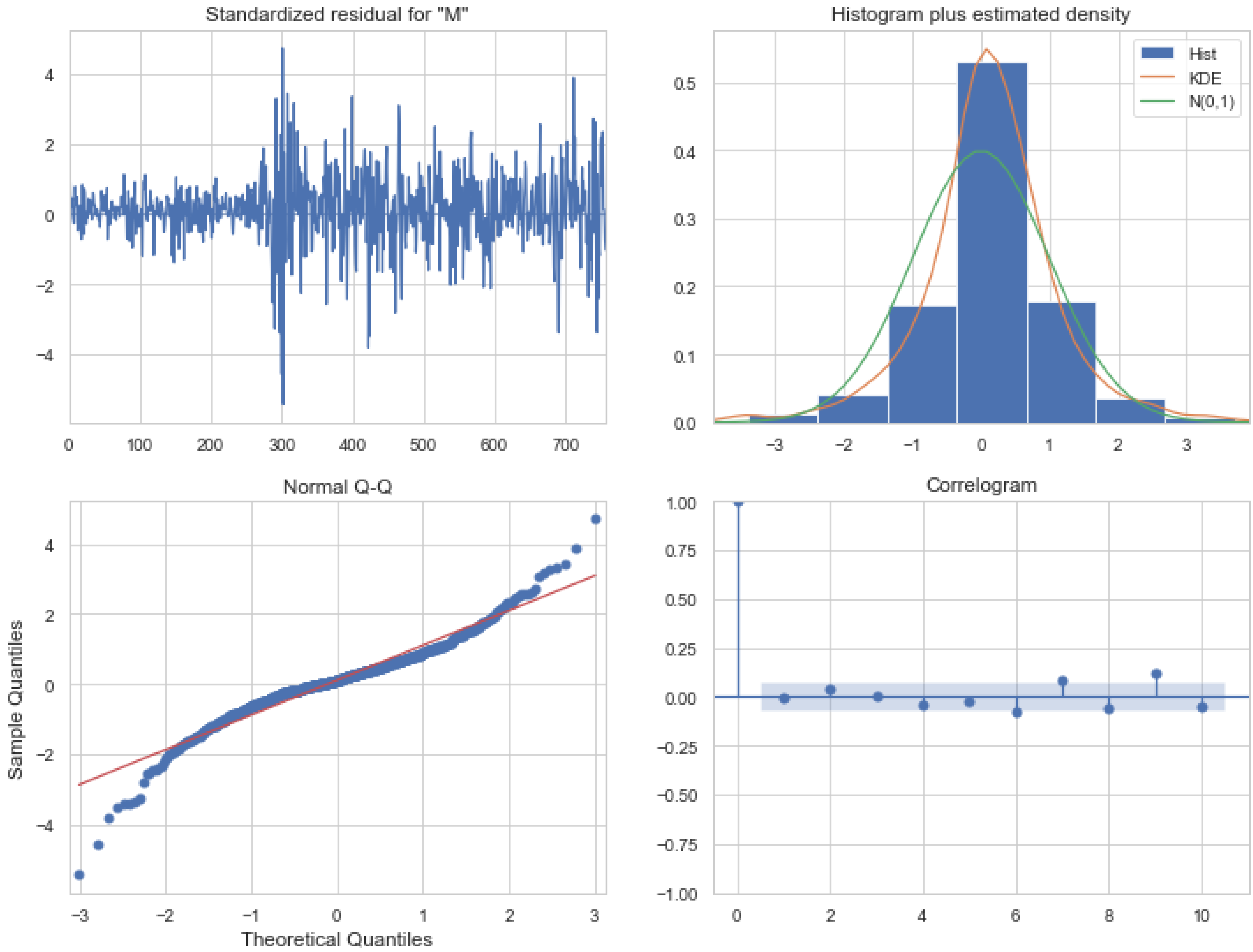
True value vs fitted value

In [15]:
```
plt.plot(dataset['MSFT'])
plt.plot(arima_110.fittedvalues[1:])
```

Out[15]: [<matplotlib.lines.Line2D at 0x21ef1043b20>]

Model diagnostics

In [16]:
```
arima_110.plot_diagnostics();
```



Modeling heteroskedasticity in residuals with GARCH

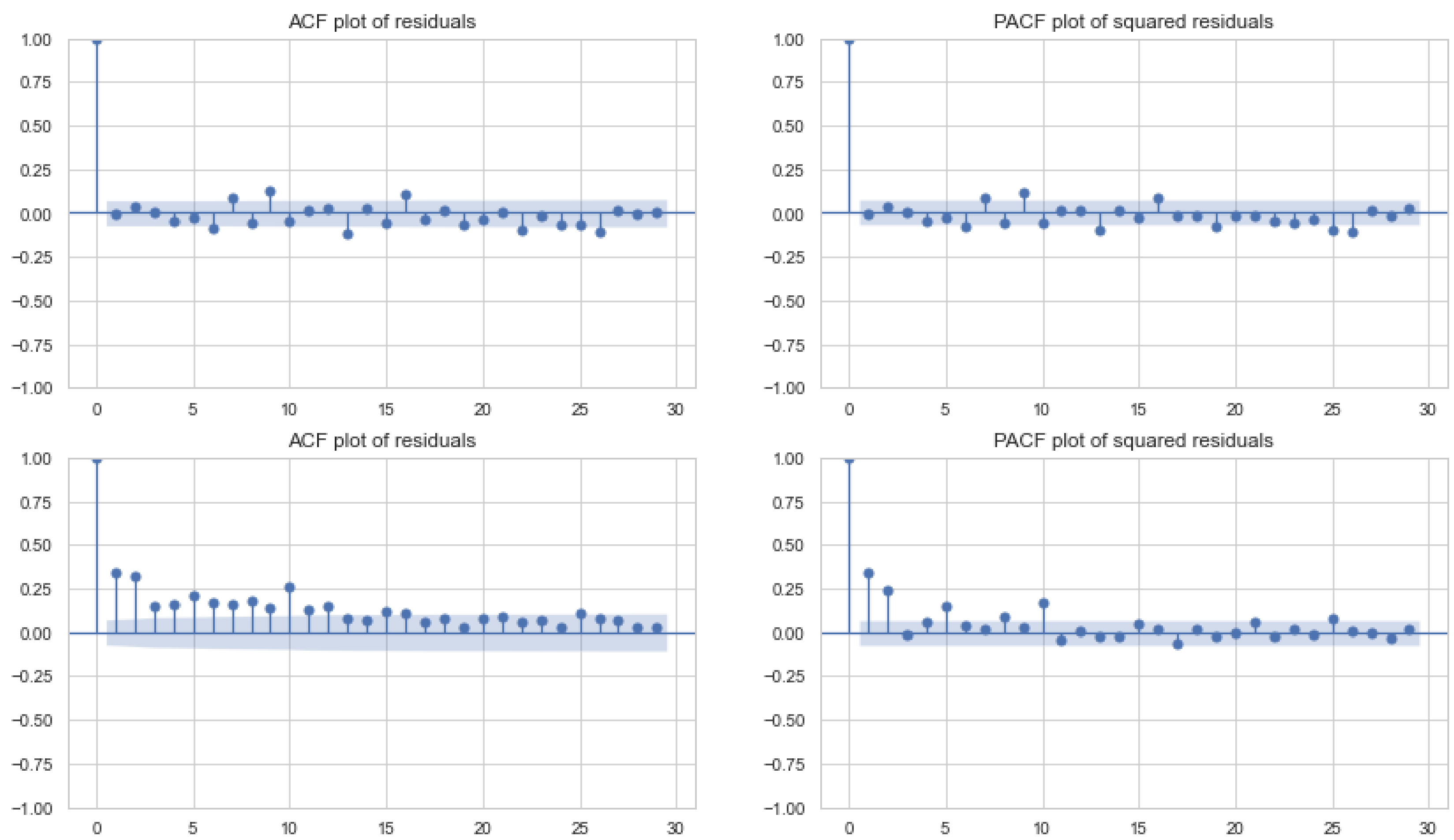
The residual plot shows clear heteroskedasticity, which suggests GARCH effects.

Also, the distribution is leptokurtic. So, a students-T distribution can be used for the GARCH model.

ACF and PACF of the residuals and the squares

In [17]:
```
fig, ax = plt.subplots(2,2, figsize=(14,8))

plot_acf(arima_110.resid[1:], ax=ax[0][0], title="ACF plot of residuals")
plot_pacf(arima_110.resid[1:], ax=ax[0][1], method="ywm", title="PACF plot of squared residuals");
plot_acf(arima_110.resid[1:] ** 2, ax=ax[1][0], title="ACF plot of residuals")
plot_pacf(arima_110.resid[1:] ** 2, ax=ax[1][1], method="ywm", title="PACF plot of squared residuals");
```



GARCH fit

Let's begin with a GARCH(2,1) model.

In [18]:
```
garch_21 = arch_model(arima_110.resid[1:], mean="constant", p=2, q=1, dist="studentsT").fit(update_freq=5)
```

```
Iteration:      5,   Func. Count:     41,   Neg. LLF: 1844.998766359014
Iteration:     10,   Func. Count:     80,   Neg. LLF: 1844.580363372783
Optimization terminated successfully    (Exit mode 0)
            Current function value: 1844.5802009055478
            Iterations: 12
            Function evaluations: 94
            Gradient evaluations: 12
```

In [19]:
```
print(garch_21.summary())
```

```
                        Constant Mean - GARCH Model Results
==================================================================================
Dep. Variable:                          None   R-squared:                     0.000
Mean Model:                    Constant Mean   Adj. R-squared:                0.000
Vol Model:                             GARCH   Log-Likelihood:             -1844.58
Distribution:      Standardized Student's t   AIC:                         3701.16
Method:                   Maximum Likelihood   BIC:                         3728.94
                                               No. Observations:                757
Date:                       Tue, Oct 04 2022   Df Residuals:                    756
Time:                               15:44:58   Df Model:                          1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu             0.4101  7.097e-02      5.778  7.574e-09 [  0.271,  0.549]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega          0.0980  7.398e-02      1.325      0.185 [-4.696e-02,  0.243]
alpha[1]       0.1293  5.597e-02      2.310  2.091e-02  [1.957e-02,  0.239]
alpha[2]   7.9480e-03  6.578e-02      0.121      0.904   [ -0.121,  0.137]
beta[1]        0.8628  4.438e-02     19.442  3.384e-84   [  0.776,  0.950]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             6.6877      1.244      5.375  7.648e-08 [  4.249,  9.126]
========================================================================

Covariance estimator: robust
```

All the coefficients are statistically significant.

Diagnostics

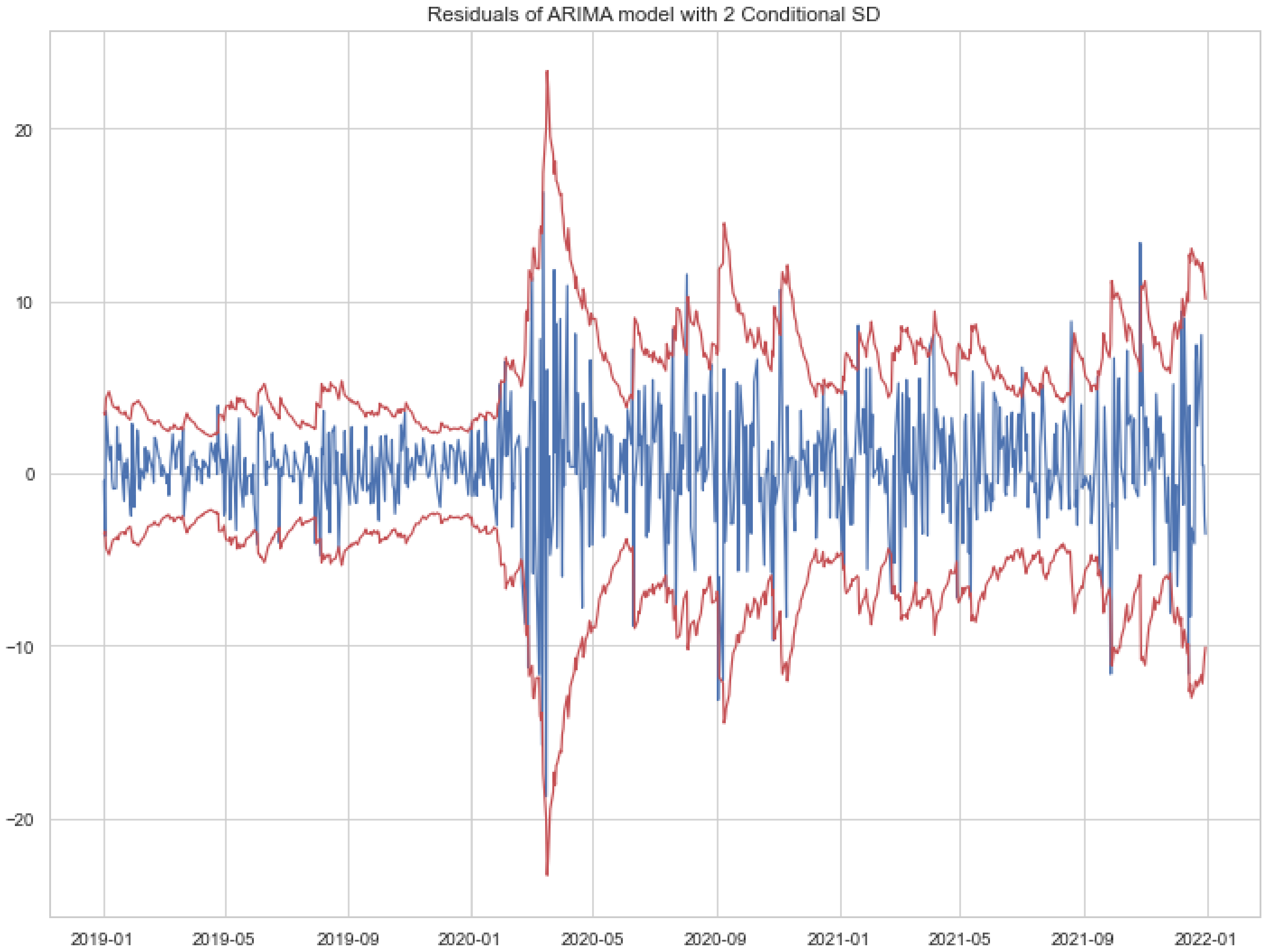
Legrange Multiplier test

In [20]:
```
# ARCH LM test for conditional heteroskedasticity
print("\nARCH LM test for conditional heteroskedasticity")
print(garch_21.arch_lm_test(standardized=True))
```

```
ARCH LM test for conditional heteroskedasticity
ARCH-LM Test
H0: Standardized residuals are homoskedastic.
ARCH-LM Test
H1: Standardized residuals are conditionally heteroskedastic.
Statistic: 9.6122
P-value: 0.9747
Distributed: chi2(20)
```

Conditional SD plot

In [21]:
```
plt.plot(arima_110.resid[1:])
plt.plot(2*garch_21.conditional_volatility, c="r")
plt.plot(-2*garch_21.conditional_volatility, c="r")
plt.title("Residuals of ARIMA model with 2 Conditional SD");
```



Students-T QQ Plot

In [22]:
```
sm.qqplot(garch_21.resid, stats.t, fit=True, line="q")
plt.title("Students-T QQ Plot");
```