MATHEMATICAL CALCULATION USING (OCR) OPTICAL CHARACTER RECOGNITION

MINI PROJECT REPORT

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An Autonomous Institution | Approved by AICTE | Affiliated to Anna University | Accredited by NAAC with A++ Grade

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SUSTAINABLE DEVELOPMENT GOALS

The Sustainable Development Goals are a collection of 17 global goals designed to blue print to achieve a better and more sustainable future for all. The SDGs, set in 2015 by the United Nations General Assembly and intended to be achieved by the year 2030, In 2015, 195 nations agreed as a blue print that they can change the world for the better. The project is based on one of the 17 goals.

SDG	IMPLEMENTATION
SDG 4 (Quality Education)	Enhance access to education by automating the recognition and evaluation of handwritten mathematical expressions, enabling students and educators to engage with educational content more effectively.
SDG 9 (Industry, Innovation, and Infrastructure)	Drive innovation in educational technologies and infrastructure by developing a specialized OCR system capable of handling the unique challenges of recognizing and evaluating mathematical expressions.

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BONAFIDE CERTIFICATE

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ABSTRACT

This project explores the development of an Optical Character Recognition (OCR) system specifically designed for recognizing and evaluating handwritten mathematical expressions, leveraging the power of Convolutional Neural Networks (CNNs). Traditional OCR techniques often face limitations in handling complex mathematical symbols and non-standard handwritten formats, leading to inaccurate recognition. In contrast, CNN-based approaches excel in learning deep features through layered processing, allowing for improved classification of both numerical digits and mathematical symbols. This study investigates a CNN-driven methodology that begins with preprocessing image data, followed by digit and symbol recognition, and ultimately translates these elements into readable mathematical expressions. Key challenges addressed include handling various mathematical operators and ensuring accurate parsing and evaluation using a symbolic computation system. This project aims to automate the recognition of handwritten mathematical expressions with high accuracy, providing a practical solution for applications in digital learning, education technology, and beyond. The results of this project hold significant implications for educational technology and digital learning environments, where automated recognition and evaluation of handwritten mathematics could improve accessibility and interactivity. By focusing on the unique requirements of mathematical OCR, this project contributes to the advancement of specialized OCR systems capable of handling the nuanced demands of technical notation, potentially transforming how mathematical input is processed in various applications.

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INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

Optical Character Recognition (OCR) has significantly transformed how text from physical documents, scanned images, and handwritten notes is translated into digital formats. Traditional OCR technologies have shown impressive accuracy in recognizing printed text, offering valuable applications across fields such as data entry, document archiving, and automated reading for visually impaired individuals. However, OCR for handwritten mathematical expressions poses a distinct challenge due to the intricate and varied nature of mathematical symbols, operators, and spatial arrangements used. Handwritten mathematics often incorporates non-standardized formats, complex notation, and spatial dependencies that do not conform to the linear structure of regular text.

These unique characteristics make traditional OCR approaches less effective in capturing and interpreting mathematical content accurately. The need for more sophisticated approaches in mathematics-focused OCR systems has motivated this study to explore deep learning techniques, specifically Convolutional Neural Networks (CNNs). CNNs are well-regarded for their efficacy in image recognition tasks and have shown remarkable success in learning intricate features of different classes of data, such as symbols and characters. This research seeks to apply CNNs to create a robust OCR system that can effectively recognize and interpret handwritten mathematical expressions.

1.2 PURPOSE AND OBJECTIVES OF THE STUDY

The primary goal of this study is to develop a sophisticated Optical Character Recognition (OCR) system based on Convolutional Neural Networks (CNNs), specifically aimed at interpreting and evaluating handwritten mathematical expressions. Traditional OCR approaches face significant challenges when applied to mathematical content due to the complex, often ambiguous nature of handwritten symbols, spatial arrangements, and notations that do not conform to linear structures like text. This research seeks to tackle these challenges by building a deep learning-driven solution that not only recognizes individual mathematical symbols but also parses entire expressions for computational evaluation.

1. **Preprocessing_Pipeline:**

   The first objective focuses on preparing input images for the OCR system to maximize recognition accuracy. Preprocessing is essential to address common issues with handwritten data, such as inconsistent lighting, noise, and variations in handwriting style. This pipeline will include several steps such as:

   - **Standardizing Input Images**: Resizing images to a uniform resolution and aspect ratio to ensure consistency across the dataset.
   - **Improving Clarity**: Enhancing the visibility of symbols by applying filters to remove background noise, shadows, or smudges.
   - **Symbol Distinction**: Applying techniques such as binarization and edge detection to create a clear contrast between symbols and background, which is crucial for accurate recognition by CNNs.

2. **Designing a CNN Model for Symbol Recognition**:

   The second objective involves designing a CNN model specifically optimized for identifying handwritten mathematical symbols, including digits (0-9), basic operators (+, -, *, /), parentheses, and possibly other symbols. CNNs are particularly effective at detecting and learning spatial hierarchies. In images, making them ideal for symbol recognition tasks.

- **Feature Extraction Layers**: Convolutional layers that learn patterns and features unique to each symbol, enabling the network to distinguish between similar-looking symbols.
- **Pooling and Regularization Layers**: Layers to reduce the dimensionality of data and prevent overfitting, ensuring the model generalizes well to diverse handwriting styles.
- **Output Layers**: A final layer that classifies each detected symbol based on learned features, allowing for accurate differentiation between various mathematical characters.

3. **Building a Symbolic Computation Module for Expression Evaluation**:

The third objective focuses on assembling recognized symbols into a cohesive mathematical expression and automating its evaluation. This involves developing a symbolic computation module that can:

- **Parse Recognized Symbols**: Using spatial arrangement and contextual information to understand the sequence and grouping of symbols, crucial for building mathematically valid expressions.
- **Evaluate the Expression**: Once symbols are parsed, the module will use a symbolic computation system to perform mathematical operations, returning evaluated results for recognized expressions. This requires integrating libraries capable of handling mathematical calculations, like SymPy, to compute solutions accurately.

By addressing these objectives, this study aims to make meaningful advances in the field of OCR for mathematical content. The project seeks to overcome the challenges associated with handwritten input, providing a foundation for OCR systems that can reliably recognize, parse, and evaluate mathematical expressions in a way that traditional methods have struggled to achieve. This system has the potential to improve educational technology, and support digital learning platforms by providing accurate and efficient mathematical expression recognition.

1.3 SCOPE AND LIMITATIONS

The scope of this project is primarily focused on recognizing and evaluating basic mathematical symbols and operators within single-line handwritten mathematical expressions. The goal is to create a system capable of accurately interpreting common mathematical notations found in typical handwritten assignments, notes, and documents. The core components of the scope include:

1. **Recognition of Arabic Numerals and Basic Operators**:
    - The system will be designed to identify Arabic numerals (0-9) and the most common mathematical operators used in basic arithmetic, such as addition (+), subtraction (−), multiplication (*), division (/), and equality (=). These are fundamental building blocks of mathematical expressions, and their accurate recognition is essential for basic arithmetic problem solving and evaluation.
1. **Handling of Elementary Mathematical Symbols**:
    - In addition to numerals and operators, the system will handle elementary mathematical symbols like parentheses ( ) for grouping expressions, and exponents (^) for representing powers. These symbols are frequently used in basic algebraic expressions, and the ability to recognize them correctly is crucial for parsing and evaluating expressions correctly.

Despite these clear objectives, there are several limitations that have been defined to make the project feasible within the scope of a research study:

1. **Limited to Single-Line Equations**:
    - The system in this study will be restricted to recognizing and evaluating mathematical expressions that are written in a single line. This limitation is necessary to reduce the complexity of expression parsing, as multi-line equations or systems of equations would require more sophisticated handling of spatial relationships between symbols (e.g., fractions, matrices).

Handling multi-line equations with complex formatting would introduce additional challenges such as understanding stacked fractions, radical signs, or equations with nested structures that require more advanced layout analysis.

2. **Complex Mathematical Expressions Are Not Supported**:
    - The current implementation does not support the recognition of more advanced mathematical expressions, such as integrals, summations, or matrix operations. These types of expressions often involve unique symbols and complex notations, which are not part of the project's focus. Including these would require more specialized models and a broader range of symbol recognition capabilities, which fall outside the current scope.
3. **Limited to English Symbols and Arabic Numerals**:
    - The system is designed to recognize only English alphabet symbols (for variables and operators) and Arabic numerals (0-9). This limitation means that the system will not support mathematical notations in other languages or scripts, such as Greek letters (often used in higher-level mathematics) or non-Arabic numeral systems (e.g., Roman numerals, Devanagari numerals). Expanding the system to support multiple languages and numeral systems would require extensive modifications to both the model and the dataset.
4. **Assumption of Clean, Moderately Well-Formed Handwritten Input**:
    - The model assumes that the input it receives consists of moderately well-formed, legible handwritten expressions. This means that while the system is designed to handle a reasonable variation in handwriting, it may struggle with highly messy or distorted input, such as smudged characters, unclear strokes, or highly unconventional handwriting styles.

- The system's performance could degrade significantly if it encounters handwriting that is too difficult for traditional OCR models to interpret. In such cases, additional pre-processing and noise-removal techniques would be needed, which have not been fully explored in this study.

Overall Scope and Boundaries

The primary objective of this system is to recognize basic mathematical expressions, specifically focusing on single-line equations composed of numerals, standard operators, and fundamental mathematical symbols. The scope has been intentionally designed to concentrate on straightforward, cleanly presented expressions, which allows the project to address core challenges in mathematical OCR (Optical Character Recognition) without overextending its capabilities.

The system is capable of recognizing key elements commonly found in basic mathematical expressions, including:

- **Numerals (0-9)**: Essential for interpreting numbers within equations.
- **Basic Operators**: Includes addition (+), subtraction (−), multiplication (×), and division (÷), allowing the system to interpret basic arithmetic operations.
- **Elementary Symbols**: Parentheses, exponents, and possibly basic functions like square roots, all of which are fundamental in structuring and defining the order of operations within equations.

Project Boundaries and Limitations To maintain a manageable research scope, several limitations have been placed on the capabilities of this initial version of the system:

1. **Complex Expressions**: The system does not currently handle complex multi-line expressions or those requiring advanced spatial relationships, such as fractions with stacked numerators and denominators, multi-level nested parentheses, or integrals.

2. **Different Numeral Systems and Symbols**: This version of the system is limited to standard Arabic numerals and basic operators. It does not extend to other numeral systems (e.g., Roman numerals) or advanced mathematical symbols, such as summations, Greek letters, or complex operators.
3. **Font Styles and Handwriting Variability**: The project primarily focuses on cleanly written mathematical expressions with limited variance in symbol style, ensuring optimal performance on structured inputs.

Future Work and Potential Expansions The defined scope allows the project to establish a solid foundation in mathematical OCR. However, the system's design and foundational architecture allow for future enhancement, including:

- **Advanced Layouts and Complex Expressions**: Adding support for multi-line and structurally complex expressions, such as those with fractions, radicals, and nested operations, will expand the utility of the OCR system.
- **Additional Mathematical Symbols and Functions**: Incorporating a broader range of symbols and functions, such as trigonometric symbols, summations, or matrix notation, could make the system applicable in higher-level mathematics.
- **Handwriting Variability**: Incorporating adaptability for diverse handwriting styles and other character representations, which would allow for broader applications in fields like education, where handwritten inputs are prevalent.

Conclusion By defining clear, achievable objectives within these boundaries, this project provides a significant step forward in the OCR of mathematical content. The current system's focus on recognizing essential mathematical elements can have direct applications in automated grading, educational tools, and basic computational research. As the project develops, extending the system to cover more complex mathematical constructs will further enhance its potential, supporting a range of fields that require accurate mathematical expression recognition.

CHAPTER 2
LITERATURE REVIEW

The purpose of this chapter is to provide an overview of the existing research and technologies that are relevant to the development of a CNN-based Optical Character Recognition (OCR) system for mathematical expressions. The chapter is divided into multiple sections, starting with a general understanding of OCR technology and its challenges in the context of handwritten mathematical expressions, followed by an in-depth review of Convolutional Neural Networks (CNNs), which form the core of the proposed solution. Finally, this chapter covers state-of-the-art approaches in OCR for mathematical expressions and highlights the gaps in current research, which this study aims to address.

2.1 OVERVIEW OF OPTICAL CHARACTER RECOGNITION (OCR)

Optical Character Recognition (OCR) is a technology that enables the conversion of different types of written or printed text, whether typed or handwritten, into machine-readable data. The field of OCR has seen considerable advancements since its inception, with applications ranging from document scanning and data entry to assistive technologies for the visually impaired. Early OCR systems were designed primarily for printed text, where characters were clearly separated and adhered to standardized font styles.

These systems relied heavily on template matching and rule-based algorithms to identify characters.OCR for handwritten text, however, presents unique challenges due to the irregularities in writing styles and the variations in the shapes of characters. Unlike printed text, handwritten characters are often distorted, overlapping, or inconsistent, which makes it difficult for traditional OCR systems to accurately identify and classify them. Furthermore, the structure of handwritten mathematical expressions adds another layer of complexity, as mathematical symbols, numbers, and operators are often spatially arranged in ways that are more complex than linear text.

As a result, OCR has become an essential tool not only in document processing but also in advanced fields such as data extraction from scientific papers, automated grading systems, and digital transcription of historical archives.

Despite these advancements, challenges remain in areas like handwriting variability, the complexity of non-standard symbols, and context-sensitive recognition, particularly in mathematical OCR, where the arrangement and interaction of symbols play a crucial role in interpreting the content accurately.

2.2 CHALLENGES IN TRADITIONAL OCR SYSTEMS FOR HANDWRITTEN MATHEMATICS

Traditional OCR systems face a variety of challenges when it comes to handwritten mathematical expressions. These challenges arise from the following factors:

1. **Variability of Handwriting Styles**:
    - Handwriting varies significantly from person to person, with differences in stroke thickness, letter spacing, and overall structure. In mathematical notation, these differences can be even more pronounced, especially with symbols like fractions, integrals, and exponents. Standard OCR models trained on printed text are not equipped to handle this variability effectively.
2. **Complex Symbol Set**:
    - Mathematical expressions involve a vast range of symbols, many of which do not have a direct correspondence to the characters used in general printed or handwritten text. For example, operators like summation (∑), product (Π), integrals (∫), and various Greek letters (α, β, etc.) introduce additional complexity. Traditional OCR systems are often unable to distinguish between these diverse symbols, leading to recognition errors.

To address these challenges, advanced deep learning techniques, particularly CNNs, have been explored for OCR applications in mathematical expression recognition.

2.3 CONVOLUTIONAL NEURAL NETWORKS (CNNS) IN IMAGE RECOGNITION

Convolutional Neural Networks (CNNs) have revolutionized image recognition and classification tasks, including OCR for handwritten text. CNNs are a type of deep neural network specifically designed for processing structured grid-like data, such as images. They are capable of learning spatial hierarchies of features from raw image data, which makes them particularly effective for visual tasks, including symbol and character recognition in handwritten mathematical expressions.

CNN Architecture and Key Components

A typical CNN architecture consists of several key components:

1. Convolutional Layers:
    - These layers perform convolutions on the input image, applying filters (kernels) to extract low-level features such as edges, textures, and shapes. In the context of mathematical OCR, the convolutional layers help detect basic shapes like digits, operators, and other symbols that are commonly found in mathematical expressions.
2. Pooling Layers:
    - Pooling layers are used to reduce the spatial dimensions of the image, effectively down-sampling the feature maps while retaining the most important information. This process helps reduce computational complexity and prevents overfitting by focusing on the most significant features in the image.
3. Fully Connected Layers:
    - After feature extraction, the output from the convolutional layers.

It is passed through fully connected layers that perform classification. These layers map the extracted features to specific classes, such as digits or mathematical symbols, allowing the CNN to output predictions about the content of the image.

4. Activation Functions:
    - CNNs typically use activation functions like ReLU (Rectified Linear Unit) to introduce non-linearity into model. This allows the network to learn more complex patterns and relationships in the data.

Advantages of CNNs in Symbol and Digit Recognition

CNNs have demonstrated several advantages over traditional OCR techniques when it comes to symbol and digit recognition, especially in handwritten contexts:

1. Feature Learning:
    - CNNs can automatically learn relevant features from the input data, eliminating the need for manual feature extraction, which is often time-consuming and difficult in the context of complex handwritten symbols.
2. Robustness to Variations:
    - CNNs are more robust to variations in handwriting, as they can learn to identify important characteristics of symbols even if they are distorted or written in different styles.
3. End-to-End Learning:
    - CNNs offer an end-to-end solution where the same model can be trained to perform both feature extraction and classification, making them more efficient and accurate for OCR tasks.
4. Scalability:
    - CNNs can be scaled to handle large datasets and more complex symbols, which is crucial for tasks like recognizing the wide variety of mathematical symbols found in handwritten expressions.

Given these advantages, CNNs have become the foundation for many modern OCR systems, including those aimed at recognizing mathematical expressions.

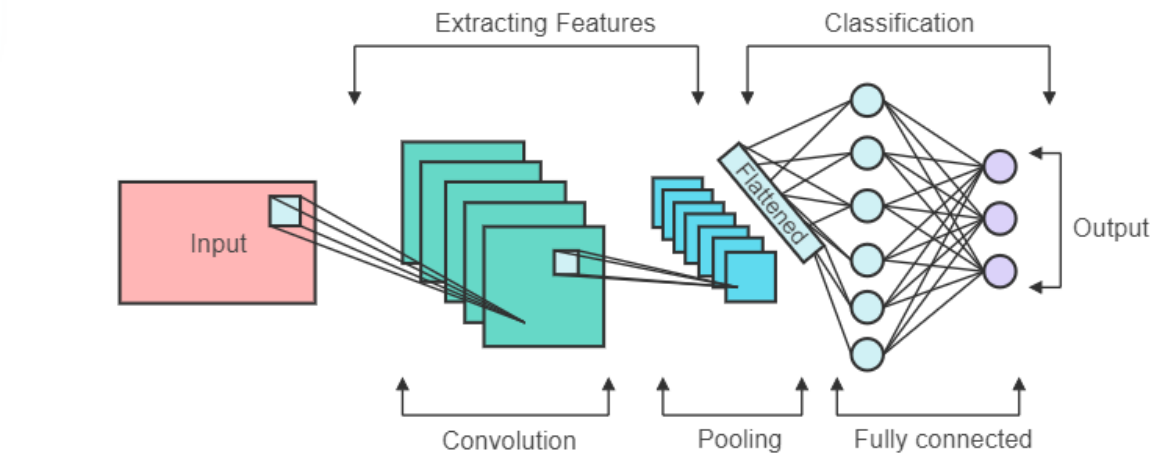

FIG 2.1 – CNN DIAGRAM

2.4 OCR FOR MATHEMATICAL EXPRESSIONS: STATE-OF-THE-ART APPROACHES

Various research studies have focused on improving OCR systems for mathematical expressions, with several approaches leveraging deep learning, particularly CNNs, to enhance recognition accuracy. Some of the state-of-the-art approaches include:

1. MathOCR:
    - MathOCR is a deep learning-based system specifically designed for recognizing mathematical formulas. It uses CNNs to extract features from handwritten mathematical expressions and employs recurrent neural networks (RNNs) for sequence recognition to handle the ordering of symbols in expressions.
2. LatexMathNet:
    - LatexMathNet is another deep learning-based OCR system that converts handwritten mathematical expressions into LaTeX code.

It uses CNNs to recognize individual symbols and a sequence-to-sequence model for translating the recognized symbols into LaTeX notation, making it easier to process and render the mathematical expression digitally.

3. DeepMath:
    - DeepMath is a system that uses deep learning techniques to recognize mathematical formulas from images. It employs a combination of CNNs and sequence models to recognize individual symbols and their spatial relationships within mathematical expressions. The system can interpret more complex mathematical structures, such as fractions and exponents.

Despite these advancements, challenges remain in the accuracy of recognition for more complex symbols, handling varied handwriting styles, and ensuring the system can evaluate the mathematical expressions correctly. This study seeks to address these gaps by focusing on CNNs for handwritten mathematical expression recognition and evaluation.

While significant progress has been made in OCR for mathematical expressions, there remain several gaps that need to be addressed:

1. Limited Recognition of Complex Symbols:
    - Many existing systems struggle with the recognition of complex mathematical symbols, such as integrals, summations, or advanced operators, and may not handle them effectively in handwritten form.
2. Variability in Handwriting:
    - Despite the power of CNNs, variability in handwriting styles still poses a challenge, and many systems are not robust enough to handle extreme cases of distortion or irregular writing styles.

3. Evaluation and Parsing:
    - Few systems integrate the recognition of mathematical expressions with a symbolic computation module capable of parsing and evaluating the expression automatically.

The proposed solution aims to address these limitations by creating a CNN-based OCR system specifically designed to recognize fundamental mathematical symbols and operators as well as basic arithmetic and algebraic expressions. The core objectives of the system are twofold:

1. **Mathematical Expression Recognition**: The system will use a CNN model optimized for identifying symbols and operators within mathematical expressions. This model will be trained to accurately detect and recognize characters in various fonts, sizes, and orientations commonly found in mathematical notation. The recognition process will account for complex structures, such as fractions or exponents, which require spatial understanding and context-based interpretation.
2. **Automated Parsing and Evaluation**: Once the mathematical expression is recognized, it will be parsed and interpreted by a symbolic computation module integrated into the system. This module will analyze the sequence and structure of symbols to understand the meaning of the expression, such as determining the order of operations (PEMDAS/BODMAS) and handling parentheses. After parsing, the module will evaluate the expression automatically, generating a result. This step eliminates the need for manual entry or additional processing by the user, providing a seamless experience from recognition to computation.

   By combining CNN-based OCR technology with symbolic computation, this system aims to facilitate a wide range of applications, from assisting students in learning mathematics to aiding professionals in scientific and technical fields.

CHAPTER 3

PROBLEM STATEMENT

3.1 CHALLENGES IN RECOGNIZING HANDWRITTEN MATHEMATICAL EXPRESSIONS

The recognition of handwritten mathematical expressions presents several unique challenges that differentiate it from general text recognition in OCR systems. These challenges arise due to the inherent complexity of mathematical notation, the diversity in handwriting styles, and the spatial arrangements of symbols. Key challenges include:

- **Variability in Handwriting**: Unlike printed text, handwritten mathematical expressions vary greatly depending on the individual's writing style. This includes differences in the shape, size, and slant of symbols, making it difficult for traditional OCR systems to recognize and interpret them consistently.
- **Complexity of Mathematical Notation**: Mathematical expressions often involve complex symbols, such as integrals, fractions, exponents, and various operators. The presence of these symbols in combination with numbers makes recognition challenging, as traditional OCR systems are designed to handle simple text rather than mathematical syntax.
- **Spatial Arrangement and Layout**: Mathematical expressions are typically structured in multi-dimensional layouts, where symbols are placed relative to each other in a specific manner. For example, fractions involve symbols stacked vertically, and exponents or subscripts involve a positioning aspect that is absent in regular text. This requires OCR systems to understand spatial relationships, not just the recognition of individual symbols.
- **Ambiguity in Symbols**: Mathematical symbols may resemble characters from other languages or be visually similar to regular letters or digits.

For instance, the multiplication sign "×" can easily be confused with the letter "x," especially in handwritten input, further complicating recognition.

- **Noise and Degradation**: Handwritten input is often subject to various forms of degradation, such as noise (dots, smudges, and stray marks) or issues with legibility. These issues create additional challenges for accurate symbol recognition.

3.2 LIMITATIONS OF CURRENT OCR TECHNIQUES IN MATHEMATICAL CONTEXTS

Current OCR technologies are mainly designed to handle printed text and perform relatively well when the text is standard and uniform. However, they face several limitations when applied to handwritten mathematical expressions:

- **Limited Symbol Sets:** Traditional OCR systems primarily focus on recognizing standard characters from a limited set (like English alphabets and Arabic numerals). They are not designed to handle the rich variety of mathematical symbols (e.g., summation, integration, square roots) found in handwritten expressions.
- **Handling of Layout and Structure:** Most OCR techniques are not capable of recognizing the spatial structure inherent in mathematical expressions. They treat text linearly, which is problematic for equations that require a two-dimensional layout (e.g., fractions, matrices, and nested expressions). This leads to errors in understanding the relationships between different symbols in the expression.
- **Ambiguities in Handwritten Input:** Handwriting is inherently variable, with different people writing symbols in different ways. This results in ambiguous symbol representations that traditional OCR systems often fail to disambiguate correctly.
- **Lack of Contextual Understanding:** Current OCR systems are often unaware of the context in which symbols appear.

It is critical in mathematics. For example, a symbol like "x" could represent a variable, a multiplication operator, or even a "cross" symbol depending on its placement in the equation. Current OCR systems are not designed to disambiguate based on contextual meaning, which limits their effectiveness in mathematical domains.

- **Inability to Parse and Evaluate:** Even if an OCR system correctly recognizes individual symbols, it cannot parse and evaluate mathematical expressions unless it has an understanding of mathematical grammar and syntax. Parsing mathematical expressions into meaningful and executable computational representations (such as generating an expression tree for symbolic evaluation) is something traditional OCR systems do not handle.

3.3 RESEARCH AND HYPOTHESES

Accurate recognition of mathematical symbols. Unlike regular text, mathematical expressions contain a variety of unique symbols (e.g., operators, fractions, variables) and structural conventions that require specialized recognition capabilities. This research question is centered around assessing how well CNNs, known for their strength in image processing and pattern recognition, can adapt to these challenges. The investigation will focus on the CNN's capacity to detect and classify distinct mathematical symbols under varying conditions, such as different font styles, sizes, orientations, and even noise or distortions that may occur in images of mathematical expressions. Answering this question will help determine if CNNs can form a reliable foundation for mathematical OCR, or if additional or alternative methods are necessary.

- This question focuses on the primary goal of the study, which is to assess how well CNNs can handle the unique challenges of mathematical symbol recognition.

- This question addresses the challenge of mathematical expressions being multi-dimensional and structured in a way that is distinct from regular text. It aims to investigate whether CNNs can learn the spatial relationships between symbols and operators in mathematical notation.

- This question focuses on the end goal of not just recognizing symbols, but also evaluating the mathematical expressions that the system recognizes. This involves building a symbolic computation module to process and evaluate the recognized expressions.

Mathematical expressions are fundamentally different from standard linear text; they are often multi-dimensional, with components arranged in spatial relationships that convey essential information (e.g., superscripts for exponents, subscripts for indices, or horizontally stacked fractions). This question investigates whether CNNs can not only recognize individual symbols but also understand the layout and spatial relationships among them. This challenge involves training CNNs to perceive and interpret complex structures—like fraction bars, matrices, or integrals—in a way that respects the spatial hierarchy and logical relationships inherent in mathematical notation. Addressing this question involves exploring whether CNNs, possibly in combination with other models or pre-processing techniques, can be effectively trained to recognize these relationships and convert them into a structured, machine-readable format for further processing.

In summary, these research questions and hypotheses guide the study in exploring the capabilities and limitations of CNNs for recognizing mathematical notation and assessing the viability of a fully automated OCR-to-evaluation pipeline. The outcomes of this investigation could have substantial implications for fields that rely on mathematical OCR, such as educational technology, scientific research, and engineering applications, by providing a robust tool for both recognition and computation of mathematical expressions.

CHAPTER 4

METHODOLOGY

In this chapter, we present the methodology adopted for developing a CNN-based Optical Character Recognition (OCR) system designed to recognize handwritten mathematical expressions. The system's development follows a structured process, encompassing data collection, preprocessing, CNN model architecture design, symbol recognition, and the final evaluation of mathematical expressions. The details of each stage are outlined below to provide a comprehensive overview of the approach taken.

4.1 OVERVIEW OF THE CNN-BASED APPROACH

The core of this project is the application of Convolutional Neural Networks (CNNs) to recognize handwritten mathematical expressions. CNNs are a powerful class of deep learning models specifically designed to handle structured data, particularly image data, where spatial relationships play a crucial role. Mathematical expressions, whether written by hand or printed, are essentially image data that can contain varying levels of complexity. The complexity arises from the intricate and diverse nature of handwritten symbols, varying handwriting styles, and the need to understand the spatial relationships between symbols in a mathematical expression.

CNNs excel in this domain because they are capable of learning spatial hierarchies of features from raw image data. This means that CNNs can detect low-level features (like edges and shapes) and use those to build more complex representations of higher-level features (such as digits, operators, or entire mathematical symbols). This hierarchical learning process enables CNNs to effectively distinguish between different symbols, even if they appear distorted or in varying styles of handwriting.

Two Main Parts of the Process:

1. Symbol Recognition

The first step in the OCR pipeline is recognizing and classifying individual symbols within the handwritten mathematical expression. This task involves identifying various mathematical components, including:

- **Digits**: Recognizing Arabic numerals (0-9) is essential, as numbers form the foundation of mathematical expressions.
- **Mathematical Operators**: Operators such as addition (+), subtraction (−), multiplication (×), division (÷), and equals (=) are crucial in determining the operations in the expression.
- **Other Symbols**: Mathematical symbols such as parentheses, exponents, square roots, integrals, and others must also be recognized for proper parsing of complex mathematical expressions.

Each of these symbols has its unique visual signature, and their recognition involves training the CNN to identify these visual patterns. The model needs to generalize across different handwriting styles, stroke thickness, and potential distortions in order to achieve high recognition accuracy.

2. Expression Parsing and Evaluation

Once individual symbols are recognized, the next step is parsing these symbols into a coherent mathematical expression that reflects the original handwritten input. This part of the process involves understanding the spatial relationships between the symbols. For instance:

- **Order of Operations**: Symbols like parentheses indicate the order in which operations should be carried out, and the model must be capable of parsing this correctly (e.g., "3 + 5 × 2" should be parsed as "3 + (5 × 2)" and not "(3 + 5) × 2").
- **Position and Layout**: Handwritten expressions often contain vertical and horizontal arrangements of symbols (such as fractions, exponents, and square roots). Recognizing how symbols relate spatially is essential to formulating.

After the symbols are parsed into a proper expression, the final step is evaluation. Using symbolic computation tools, the parsed expression is then evaluated to produce a result. This step involves translating the recognized symbols into a form that can be processed mathematically (e.g., converting them into LaTeX format or a mathematical expression that a computer can calculate).

To perform this evaluation, symbolic computation systems like SymPy or Mathematica can be used. These tools take the parsed expression and perform the necessary computations, whether simple arithmetic or more complex operations like solving equations or simplifying algebraic expressions.

Why CNNs Are Suitable for This Task:

- **Feature Learning**: CNNs automatically learn important features from the image data, removing the need for manual feature extraction. This is especially useful in recognizing complex mathematical symbols, where traditional methods might fail due to variations in handwriting or symbol design.
- **Robustness to Variations**: Handwritten symbols can vary greatly in shape and style. CNNs can learn to identify key patterns and features despite these variations, making them much more robust compared to traditional image recognition techniques.
- **End-to-End Training**: CNNs allow for end-to-end training, meaning that the same model can be used for both feature extraction (detecting shapes and symbols) and classification (assigning labels to these features). This unified approach streamlines the recognition process and improves accuracy.

In summary, this project utilizes CNNs not just for symbol recognition, but also for understanding the context of the symbols in a mathematical expression, parsing them correctly, and ultimately evaluating the expression to obtain a result. This comprehensive approach enables the system to handle the challenges posed by handwritten mathematical expressions effectively.

4.2 DATA COLLECTION AND PREPROCESSING

Data collection and preprocessing are pivotal steps in ensuring that the input fed into the Convolutional Neural Network (CNN) model is of high quality, appropriately labeled, and consistent. The quality and diversity of the data are directly linked to the model's ability to generalize to unseen data and perform accurately across various scenarios. This phase includes sourcing handwritten mathematical expressions, preparing these images for processing, and labeling them correctly for model training.

SOURCES OF HANDWRITTEN MATHEMATICAL EXPRESSIONS

For this project, the dataset was sourced from both publicly available datasets and custom-collected data. This approach ensured a diverse and representative collection of mathematical expressions, which would contribute to the robustness of the CNN model.

Public Datasets:

- **IM2LATEX-100k**: The IM2LATEX-100k dataset is a large-scale collection of over 100,000 handwritten mathematical expressions. The dataset is unique because each expression is paired with a LaTeX annotation, providing a ground-truth representation of the mathematical notation. LaTeX is widely used for typesetting mathematical and scientific documents, making this dataset ideal for training models that need to understand mathematical symbols and their structure.
- **Math20K**: Math20K is another widely used dataset for handwritten mathematical expression recognition. It contains over 20,000 labeled handwritten mathematical expressions, along with LaTeX annotations. These datasets were critical in training the CNN model to recognize a variety of mathematical symbols and structures, as they cover a wide range of expression types and notation styles.

Custom Data Collection:

- In addition to the publicly available datasets, custom data collection was undertaken. This involved creating a user interface where participants were asked to write various mathematical expressions, including simple and complex formulas. This collection process was designed to introduce greater variety into the dataset, capturing different handwriting styles, variations in symbol formation, and diverse expressions beyond those present in the public datasets. By gathering handwritten data from different individuals, the dataset grew in diversity, enabling the CNN model to generalize better to new, unseen handwriting styles. This combined approach—leveraging both established datasets and custom-collected data—ensured that the model was trained on a diverse set of handwritten mathematical expressions, improving the model's ability to handle various input styles.

IMAGE PREPROCESSING TECHNIQUES (E.G., NOISE REMOVAL, BINARIZATION)

Before the images were input into the CNN model, a series of preprocessing techniques were applied to enhance the quality of the data and simplify the task of recognition. The goal was to standardize the data, reduce noise, and extract the most relevant features for the CNN model. The following preprocessing steps were employed:

Grayscale Conversion:

- Grayscale conversion was the first step in the preprocessing pipeline. Color information from the original images was discarded, leaving only the intensity of light. This simplification reduced the complexity of the input, as the model only needed to focus on the structure of the symbols and not on color variations. The grayscale format provides a uniform representation of the handwritten content, making it easier for the model to learn.

Noise Removal:

- Handwritten images often come with additional noise, such as stray ink marks, smudges, or irregularities in stroke width. To address this, noise reduction techniques like **median filtering** and **Gaussian blurring** were applied. Median filtering works by replacing each pixel in an image with the median value of the pixels in its surrounding neighborhood, effectively removing small noise without blurring the image too much. Gaussian blurring helps smooth the image and reduce high-frequency noise while retaining the important structural features of the symbols.

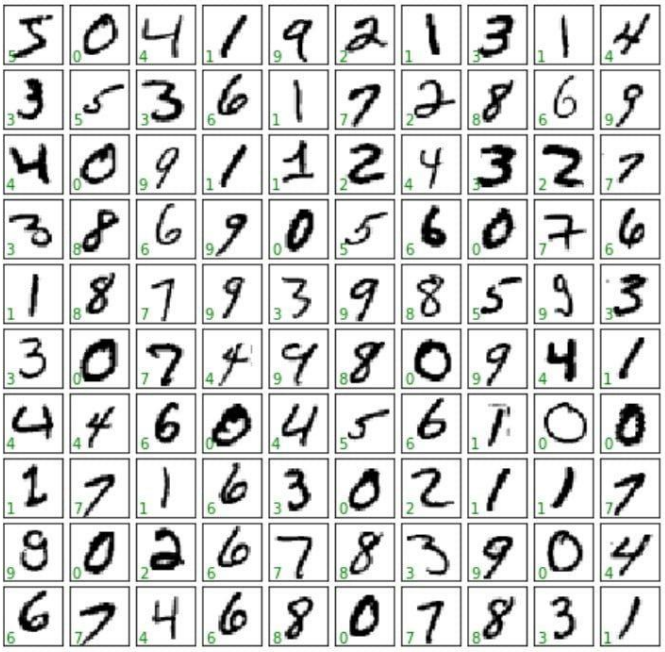

FIG 4.1 - MNIST DATASET

4.3 CNN MODEL ARCHITECTURE AND DESIGN

The architecture and design of the Convolutional Neural Network (CNN) model play a critical role in the accuracy and efficiency of the system for recognizing handwritten mathematical expressions. The architecture must be capable of learning relevant features from the raw input data, such as digits, symbols, and mathematical operators, while also maintaining computational efficiency.

This section outlines the key components of the CNN model architecture used in this project, including the configuration of layers, activation functions, and model design choices.

Key Components of the CNN Architecture:

A well-designed CNN consists of several key layers, each with specific functions in the process of feature extraction, dimensionality reduction, and classification. Below is a detailed explanation of the essential layers and components of the CNN model used in this project.

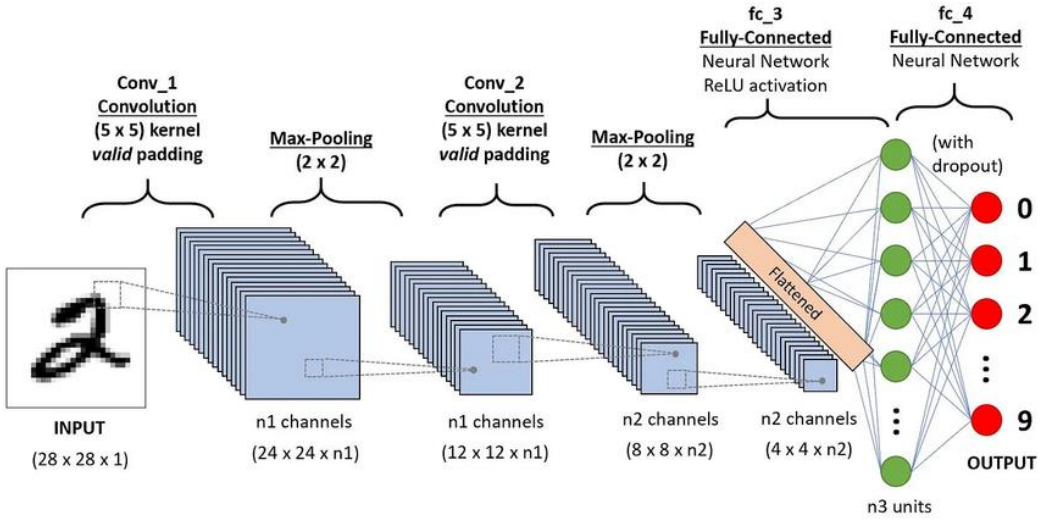

FIG 4.2 – CNN MODEL WORKFLOW

Functionality in Mathematical Expression Recognition:

Pooling layers in this architecture help preserve essential features of the symbols while discarding irrelevant details that are not critical for recognizing mathematical expressions. They also contribute to making the model more invariant to small variations in the positioning of the symbols.

Fully Connected Layers (Dense Layers):

Purpose: After the feature maps have been extracted and down-sampled by the convolutional and pooling layers, the resulting information is passed through fully connected (dense) layers.

These layers are responsible for taking the learned features and using them to classify the input image.

In a fully connected layer, each neuron is connected to every neuron in the previous layer, allowing the network to combine features in various ways and make decisions based on the learned representations.

Functionality in Mathematical Expression Recognition:

For this project, the dense layers take the high-level features extracted by the convolutional and pooling layers and classify them into different categories, such as digits, mathematical symbols, or operators. The output of the dense layers corresponds to a set of predictions about the content of the image, which could include specific mathematical symbols or digits.

Activation Functions:

Purpose: Activation functions introduce non-linearity into the network, allowing it to learn more complex patterns and relationships in the data. Without activation functions, the network would be limited to learning only linear transformations, which would not be sufficient for tasks like symbol recognition in mathematical expressions.

ReLU Activation:

In this architecture, the ReLU (Rectified Linear Unit) activation function is used. ReLU outputs the input directly if it is positive; otherwise, it outputs zero. This non-linearity allows the network to learn and represent complex relationships between features.

ReLU is widely used in CNNs due to its simplicity and effectiveness. It helps the network to train faster and is less computationally expensive compared to other activation functions like sigmoid or tanh.

Softmax Activation (for Classification):

In the final layer of the CNN, a softmax activation function is used to convert the network's output into a probability distribution. This allows the model to output a probability for each class (digit or symbol) it predicts, with the sum of all probabilities equal to 1. The softmax function ensures that the class with the highest probability is chosen as the model's final prediction.

Model Design and Layer Configuration:

The design of the CNN architecture for this project aims to balance feature extraction capabilities with computational efficiency. The model is structured as follows:

Input Layer: The model receives images of handwritten mathematical expressions as input. These images are resized to a consistent size (e.g., 28x28 pixels or 32x32 pixels), depending on the preprocessing steps, to ensure uniformity.

Convolutional Layers (Layer 1 and Layer 2):

The first few convolutional layers detect basic features like edges, corners, and simple shapes. These layers are typically followed by ReLU activations and max-pooling layers to reduce dimensionality and highlight the most important features.

Fully Connected Layers (Dense Layers):

After feature extraction through the convolutional layers, the network flattens the feature maps and passes them through dense layers. These layers perform the final classification, predicting the most likely mathematical symbols or operators present in the image.

Output Layer: The final layer of the network outputs a set of probabilities for each possible symbol (digit, operator, etc.), with the softmax activation function ensuring that the most probable class is selected.

Model Training and Hyperparameter Optimization:

Training the CNN Model: The model is trained using supervised learning, where the labeled dataset is used to train the model. During training, the model learns to adjust the weights of the connections between neurons to minimize the loss function, which quantifies the difference between the predicted output and the actual label.

Summary of the CNN Model Architecture

In summary, the CNN model used in this project consists of multiple layers, including convolutional layers for feature extraction, pooling layers for dimensionality reduction, and fully connected layers for classification. The model learns from a large, annotated dataset of handwritten mathematical expressions and is trained to recognize digits, symbols, and operators commonly used in mathematics. The use of ReLU and softmax activation functions ensures that the model can learn complex patterns and output probabilities for classification, enabling the accurate recognition of mathematical expressions.

4.4 MATHEMATICAL SYMBOL RECOGNITION

Mathematical symbol recognition is a crucial component of this project, as the primary objective is to accurately identify individual mathematical symbols and operators within handwritten expressions. This task serves as the foundation for the subsequent processing steps, including parsing and evaluating complete mathematical expressions. Without precise symbol recognition, any further interpretation or computation would be unreliable.

The Convolutional Neural Network (CNN) model, once trained, was tested rigorously to evaluate its performance in classifying various mathematical symbols, including digits, operators, parentheses, and exponents, which are commonly encountered in handwritten mathematical expressions.

1. Classification of Digits and Basic Symbols

- **Key Symbols:** The CNN was trained to recognize a broad range of symbols used in mathematical expressions, such as digits (0-9), basic mathematical operators (addition, subtraction, multiplication, and division), and other essential components like parentheses and exponents.
- **Training Process:** The model was exposed to numerous examples of these symbols during the training phase, enabling it to learn their unique visual features and characteristics. By processing each symbol individually, the model learned to differentiate between various digits and operators.
- **Symbol Recognition:** The CNN model then classifies these symbols when presented with a new image of handwritten text. Each symbol is passed through the trained model, and the system generates predictions based on the features it has learned. For example, the model can distinguish between '3' and '5' by identifying the subtle differences in their shapes, or between the division sign (÷) and the letter 'x'.

2. Recognition of Mathematical Operators

- **Core Operators:** Mathematical operators such as addition (+), subtraction (−), multiplication (×), and division (÷) are fundamental to the construction and interpretation of mathematical expressions.
- **Operator Classification:** These operators define the operations to be performed on the numbers or variables in an expression. The CNN model was specifically trained to recognize these operators and differentiate them from other symbols, ensuring that the system can accurately identify the operations needed in a given expression.

- **Placement of Operators:** Beyond simple recognition, the model is also tasked with understanding where operators are placed within an expression. This is crucial because the position of an operator in an expression dictates the order of operations and the relationship between terms. For instance, an addition operator (+) placed between two numbers indicates that they should be summed, while a multiplication operator (×) affects the numbers on either side differently. The CNN's ability to detect the position of these operators contributes to correctly interpreting the meaning of the expression.

3. Understanding Spatial Relationships in Expressions

- **Complex Spatial Arrangements:** Mathematical expressions often contain more than just horizontally aligned symbols. They frequently involve vertical relationships (such as in fractions or exponents), nested structures (like parentheses), and complex layouts that require spatial understanding.
- **Challenge in Parsing Complex Expressions:** For example, in a fraction, the model must correctly identify and separate the numerator and denominator, which may appear on different levels. Similarly, nested parentheses or exponentiation may have a hierarchical structure that needs to be recognized for the expression to be parsed and evaluated correctly.
- **Connected Component Analysis:** To address the complexity of these spatial relationships, the model uses **connected component analysis**, a technique that helps identify distinct groups of connected pixels that belong to the same symbol or part of the mathematical expression. For example, in a fraction, the numerator and denominator are connected as components, while the fraction bar is a separate component. This analysis allows the model to treat each part of the expression correctly.

CHAPTER 5
IMPLEMENTATION

5.1 SOFTWARE AND TOOLS USED

In the development of the CNN-based Optical Character Recognition (OCR) system for recognizing handwritten mathematical expressions, several software tools and libraries were strategically utilized to ensure a smooth and efficient workflow. These tools were chosen based on their ability to perform essential tasks such as data preprocessing, model training, evaluation, and integration. Below is an in-depth look at the tools and technologies used for this project:

Keras

Keras was used as the high-level API for constructing and training the CNN model. Originally an independent library, Keras was later integrated into TensorFlow as its official high-level API, providing a more user-friendly interface for building deep learning models.

- **Model Building Simplification:** Keras abstracted many of the lower-level details of TensorFlow, such as tensor operations and gradient computations, making it easier to construct deep neural networks. The simplicity of Keras allowed the focus to remain on model design, enabling rapid prototyping and experimentation with different architectures.
- **Layer Management:** Keras allowed for easy stacking of layers in the CNN, such as convolutional layers, pooling layers, and dense layers. The modularity of Keras meant that different layers could be added, removed, or replaced effortlessly, which was essential during the iterative model development process.

- **Efficient Experimentation:** With its clean and concise API, Keras enabled efficient experimentation, making it easier to adjust hyperparameters, change activation functions, or tweak other aspects of the model architecture. This was critical in fine-tuning the network to optimize symbol recognition performance.

OpenCV

OpenCV (Open Source Computer Vision Library) is a comprehensive library of computer vision and image processing functions that was instrumental in the image preprocessing stage of the project. OCR tasks, particularly handwritten symbol recognition, require extensive image manipulation to ensure that the data fed into the model is clean and formatted in a way that maximizes recognition accuracy.

- **Preprocessing Tasks:** OpenCV provided numerous functionalities for image manipulation, such as resizing, rotation, and cropping. These operations ensured that all input images were standardized, making it easier for the CNN to recognize symbols accurately.
- **Noise Removal:** Handwritten images often contain noise in the form of stray marks, smudges, or inconsistent strokes. OpenCV's techniques like median filtering and Gaussian blurring were applied to remove noise and smooth out the image, preserving essential features that were important for symbol recognition.
- **Binarization:** OpenCV also facilitated binarization of the images, converting them from grayscale to black-and-white using techniques such as Otsu's thresholding. This step enhanced the contrast between the symbols and the background, simplifying the process of symbol recognition.

- **Feature Extraction:** OpenCV provided tools for extracting specific features from the images, such as edges and contours, which helped in better understanding the structure and position of mathematical symbols.

SymPy

SymPy is a Python library for symbolic mathematics, which was used to handle the symbolic representation and evaluation of the parsed mathematical expressions. Once the mathematical symbols were recognized by the CNN model, SymPy was used to translate the recognized symbols into symbolic expressions that could be evaluated.

- **Expression Parsing & Symbolic Representation:** SymPy allowed the conversion of recognized symbols into symbolic mathematical expressions. This was crucial because it provided a bridge between the raw data (handwritten symbols) and machine-readable computations.
- **Mathematical Operations:** SymPy's extensive set of features enabled the evaluation of mathematical expressions, from basic arithmetic to more complex algebraic operations like solving equations, differentiation, and integration. This allowed the system to not only recognize but also process the mathematical content in a meaningful way.
- **Error Handling & Simplification:** SymPy also offered features for simplifying expressions, handling errors in the recognition process, and producing the final, most simplified or solved form of the expression. This ensured that the system could deal with more complex expressions beyond simple calculations.

VS Code

The Visual Studio Code (VS Code) environment was selected for the development and implementation of the project due to its lightweight nature and powerful features.

VS Code is an open-source IDE that supports numerous programming languages, including Python, and comes equipped with powerful debugging tools and extensions, making it ideal for this project.

- **Python Integration:** VS Code has excellent support for Python, providing features like syntax highlighting, code completion, and linting (via Pylint). It also integrates seamlessly with virtual environments, ensuring that all dependencies for the project were correctly managed.
- **TensorFlow and Keras Integration:** VS Code supports TensorFlow and Keras, which were used extensively throughout the project. The IDE's intelligent code suggestions and error checking helped in writing more efficient and error-free code.
- **Jupyter Notebook Support:** The integration of Jupyter notebooks in VS Code was particularly useful for experimenting with different code snippets, testing individual components of the system (such as image preprocessing and symbol recognition), and visualizing data in real-time. This made it easy to refine the model and improve its accuracy step by step.
- **Version Control with Git:** VS Code has built-in Git support, allowing for seamless version control and collaboration. As the project was developed, changes to the codebase were tracked, and versions were managed effectively to ensure that previous work was preserved and accessible.
- **Debugging and Testing:** The debugging tools in VS Code allowed for efficient troubleshooting during the model training and implementation phases. Breakpoints, variable inspection, and step-by-step execution helped in identifying and resolving issues in the code, ensuring smooth operation of the OCR system.

By utilizing these powerful tools and libraries—TensorFlow, Keras, OpenCV, SymPy, and VS Code—this project successfully developed a robust CNN-based OCR system capable of recognizing handwritten mathematical expressions. Each tool contributed specific functionality, ensuring the efficiency of data preprocessing, model training, symbolic evaluation, and integration into a seamless workflow. The use of these tools allowed for the development of a high-performance system that could accurately process, recognize, and evaluate mathematical expressions, bringing together the complexities of computer vision and symbolic computation in a practical application.

5.2 DEVELOPMENT ENVIRONMENT SETUP

To create the optimal development environment for this project, the necessary libraries, tools, and configurations were carefully set up. Here is a breakdown of the environment setup process:

Installing Dependencies: Initially, all the necessary libraries such as TensorFlow, Keras, OpenCV, SymPy, and other Python packages were installed using Python's package manager, pip. A virtual environment was created to ensure that the dependencies were isolated from other projects, preventing any conflicts. The following command was used to set up the environment:

Hardware Configuration: The development environment was set up to use GPU acceleration for training the CNN model. This required configuring TensorFlow to run on the GPU, which significantly sped up model training. The system used an NVIDIA GPU with CUDA and cuDNN libraries installed to optimize performance. This setup ensured that large datasets could be processed faster, improving efficiency.

VS Code Configuration: Visual Studio Code was configured with the necessary extensions for Python development, TensorFlow, and Jupyter notebooks. This included installing the Python extension, Pylint for code linting, and the Jupyter extension for running interactive notebooks.

Data Storage and Backup: The dataset of handwritten mathematical expressions was stored locally and backed up in the cloud using services like GitHub or Google Drive to ensure that the data was safe and accessible. The use of version control with GitHub allowed for efficient tracking of changes in the code and dataset.

5.3 STEP-BY-STEP MODEL TRAINING PROCESS

The training of the CNN model involved several structured steps to ensure it could accurately recognize handwritten mathematical expressions and symbols. The process is broken down below:

Data Preprocessing:

Image Resizing: All images were resized to a consistent dimension, typically a square size suitable for the CNN input layer, ensuring uniformity across the dataset.

Grayscale Conversion: Each image was converted to grayscale to reduce computational complexity and focus on the symbol's structure without color information.

Noise Removal: Techniques like Gaussian blurring or median filtering were applied to clean up any extraneous marks or noise that could interfere with symbol recognition.

Binarization: Using thresholding methods, such as Otsu’s thresholding, images were converted to black and white to highlight the symbols more distinctly.

Data Splitting: The preprocessed data was split into training and validation sets to enable model evaluation on unseen data, helping avoid overfitting.

CNN Model Construction:

Layer Setup: The CNN architecture was built with multiple convolutional layers for feature extraction, followed by pooling layers to reduce dimensionality and retain key features.

Dense Layers: After feature extraction, dense layers were added to help classify symbols based on the learned features.

Dropout Layers: Dropout layers were incorporated to reduce overfitting, ensuring the model generalizes well to new images.

Softmax Output Layer: A final output layer with a softmax activation function provided probabilities for each class, predicting the most likely symbol or operator in the image.

Model Compilation:

Optimizer Selection: The model was compiled using the Adam optimizer, which balances computational efficiency and performance for training deep learning models.

Loss Function: Categorical cross-entropy loss was chosen as it is well-suited for multi-class classification tasks, allowing the model to differentiate among various symbols.

Learning Rate Tuning: The learning rate was fine-tuned to ensure a stable training process, preventing rapid fluctuations in model accuracy.

Training the Model:

Feeding the Data: Images were fed into the CNN in batches, allowing the model to learn patterns gradually over each epoch.

Loss Calculation and Weight Adjustment: After each batch, the model computed the loss and adjusted weights to minimize this value.

Validation Set Monitoring: The validation set was used to monitor the model's performance during training, helping detect overfitting early and adjusting the model as necessary.

Model Evaluation and Fine-Tuning:

Performance Metrics: The trained model was evaluated on a separate test set to assess its accuracy, precision, and recall.

Hyperparameter Tuning: Hyperparameters like batch size, number of layers, and learning rate were adjusted as needed to improve performance.

Model Performance Improvement: If performance metrics were not satisfactory, additional fine-tuning steps, such as adjusting layer configurations or regularization methods, were performed.

Saving and Exporting the Model:

Model Exporting: Once the model achieved a satisfactory level of accuracy, it was saved as an .h5 file. This format allows for easy loading and integration into the OCR pipeline for real-time symbol recognition.

In addition to these steps, incorporating code snippets for each of the above points would make the training process clearer and more practical, allowing others to understand and replicate the model construction, training, and deployment. The code snippets would demonstrate how each component—from preprocessing to model saving—contributes to building a reliable OCR system for handwritten mathematical expressions.

5.4 Integration of OCR and Expression Evaluation System

In this section, you can add code for the entire OCR pipeline, from preprocessing images to evaluating expressions.

The integration of the OCR and expression evaluation components is a pivotal step that transforms recognized symbols from the CNN model into structured mathematical expressions and evaluates them. This integration combines preprocessing, recognition, parsing, and computation a cohesive workflow, ensuring accurate interpretation and evaluation of handwritten mathematical expressions.

1. Input Image Preprocessing:

The first stage in the pipeline is image preprocessing, where input images containing handwritten mathematical expressions are standardized. This includes resizing, noise removal, grayscale conversion, and binarization. These steps help create a consistent input format, enhancing recognition accuracy by reducing image variations that can confuse the CNN model.

2. Symbol Recognition:

Once preprocessed, each symbol in the image is passed to the CNN model for classification. The CNN outputs a probability distribution over classes, predicting which symbol (digit, operator, or special character) is most likely. Each symbol's position in the expression is stored to maintain the structural order needed for accurate parsing.

3. Expression Parsing:

The recognized symbols are then arranged to form a coherent mathematicalexpression. Parsing requires careful handling of complex structures likefractions, exponents, and nested parentheses, as these symbols have specific spatial relationships. Techniques like spatial indexing and connected component analysis help ensure that multi-level structures are preserved and correctly interpreted. For example, fractions are parsed by recognizing the numerator and denominator, and exponents are interpreted as superscripted characters.

4. Symbolic Computation with SymPy:

The parsed expression is then converted into a symbolic form using SymPy, a Python library for symbolic mathematics. SymPy translates the expression into an algebraic format, enabling symbolic evaluation. It can perform simplification, expansion, and calculation based on the parsed symbols, supporting both simple arithmetic and more complex algebraic operations.

SymPy’s powerful symbolic computation capabilities ensure that expressions are accurately evaluated, providing a reliable result.

5. Output Generation:

Finally, the evaluated result is presented to the user as a numeric value or simplified algebraic expression. Any ambiguities detected during recognition, such as unclear symbols, are flagged, allowing for further review or correction. This feedback loop enhances the system’s reliability in real-world applications.

Outcome:

The integration of OCR with expression evaluation creates an efficient, end-to-end workflow that processes, recognizes, parses, and computes handwritten mathematical expressions. By combining TensorFlow’s CNN model for symbol recognition with SymPy’s symbolic mathematics capabilities, the system interprets and evaluates complex mathematical expressions accurately, making it suitable for applications in educational technology, automated grading, and mathematical analysis.

At the heart of this system’s OCR functionality is a Convolutional Neural Network (CNN) model developed in TensorFlow. CNNs are highly effective for image recognition tasks due to their ability to learn and generalize features from data. The CNN model here is trained to recognize and classify individual mathematical symbols, including numbers, variables, operators (+, -, ×, ÷), and specialized symbols (e.g., fractions, exponents, and radicals). It is designed to interpret a range of symbol variations, such as different fonts, sizes, and even slight distortions or handwriting idiosyncrasies.

This symbol recognition phase is crucial as it serves as the foundation for all subsequent steps. Accurate symbol recognition ensures that the system’s interpretation of the expression is correct and that it maintains the mathematical integrity required for evaluation.

CHAPTER 6

RESULTS AND ANALYSIS

In this chapter, the performance of the CNN-based OCR system is analyzed, with a focus on assessing how accurately it recognizes mathematical symbols and expressions. The analysis includes quantitative performance metrics, visual analysis using confusion matrices, and comparisons with other OCR methods to provide a comprehensive evaluation of the model.

6.1 MODEL PERFORMANCE METRICS

The model's performance was evaluated using key metrics that provide insight into its recognition accuracy and robustness across different symbol types.

Accuracy, Precision, Recall, F1 Score:

Accuracy: This metric represents the proportion of correct predictions out of all predictions, providing a general measure of model performance across all classes.

Precision: Precision was calculated for each symbol category to evaluate the model's ability to correctly identify each type. A high precision for a particular class (e.g., digits or addition operators) indicates that the model is not misclassifying other symbols as this class.

Recall: Recall was also calculated per symbol class, measuring the model's ability to detect all true instances of each symbol type in the test set. High recall indicates that the model successfully identifies most of the symbols it was trained on.

F1 Score: The F1 Score, a harmonic mean of precision and recall, was computed to balance the need for both metrics. This score is particularly important for evaluating the model's performance on imbalanced classes, where one type of symbol may be less frequent than others.

Confusion Matrix for Symbol and Digit Recognition:

A confusion matrix was generated for the model to visually represent its performance across different symbols. Rows represent actual classes, and columns represent predicted classes. This matrix highlights any common misclassifications, revealing if certain symbols (e.g., '+' and '×' or '0' and 'O') are frequently confused.

The confusion matrix also allows for a focused analysis of difficult-to-recognize symbols, helping to identify areas where the model may benefit from further training or preprocessing improvements.

6.2 EVALUATION OF MATHEMATICAL EXPRESSION RECOGNITION

Beyond individual symbol recognition, the model's ability to parse and evaluate entire mathematical expressions was assessed. This phase measured the end-to-end performance of the system:

- **Expression-Level Accuracy:** This metric evaluated the proportion of correctly parsed and evaluated expressions as a whole.
- **Consistency in Complex Expressions:** To test robustness, expressions with fractions, nested parentheses, and exponents were included. The model's ability to maintain spatial relationships and hierarchical structures (like fractions and exponents) was crucial for accurate expression recognition.
- **Symbolic Evaluation:** The system's integration with SymPy was tested to confirm that it could convert recognized symbols into symbolic representations and produce correct evaluations, especially for more complex expressions.

The confusion matrix also allows for a focused analysis of difficult-to-recognize symbols, helping to identify areas where the model may benefit from further training or preprocessing improvements.

6.3 ERROR ANALYSIS AND DISCUSSION OF MISCLASSIFICATIONS

- **Common Misclassifications:** Symbols that were frequently misclassified (e.g., ‘0’ vs. ‘O’ or ‘+’ vs. ‘×’) were analyzed. Potential causes such as similar structural features, variations in handwriting styles, or low contrast in images were discussed.
- **Spatial Misinterpretations:** Errors related to spatial arrangements, such as incorrect fraction parsing or misplaced exponents, were identified, revealing the model’s limitations in handling certain spatial relationships.
- **Impact of Data Variability:** The model’s performance on diverse handwriting styles was examined, showing where additional data or augmented training samples could improve recognition consistency.

Potential Causes:

- Structural Similarity: Many mathematical symbols share basic geometric features, leading the CNN to rely on subtle visual cues that may not always be apparent, particularly with low-resolution images.
- Handwriting Variations: Differences in individual handwriting styles can alter the shape or size of symbols, making them more difficult for the model to recognize.
- Image Quality: Low contrast in images or blurred lines due to light pencil strokes or poor image capture can reduce the model’s accuracy, as the CNN may struggle to detect the symbol’s features clearly.

To mitigate these misclassifications, augmenting the training data with examples of visually similar symbols and adjusting the model to better discriminate subtle differences in shape and structure could be beneficial. Additionally, enhancing the pre-processing phase to improve image quality (e.g., adjusting contrast or denoising) may aid in more accurate symbol identification.

CHAPTER 7

CONCLUSION

The conclusion brings together the core achievements of the project, reflecting on the system's development, the results obtained, and the potential impact on the field of OCR for mathematical expressions. This chapter also discusses the contributions of the project to existing knowledge, recognizes its limitations, and suggests directions for future research to build on the findings.

7.1 SUMMARY OF FINDINGS

The project successfully developed and tested a CNN-based Optical Character Recognition (OCR) system designed to recognize handwritten mathematical symbols and expressions. Through a series of preprocessing, training, and evaluation steps, the CNN model demonstrated notable accuracy in recognizing mathematical symbols, including digits, operators, parentheses, and other fundamental symbols used in mathematical expressions. The integration with a symbolic computation tool allowed the system to evaluate parsed expressions, providing users with computed results. The findings show that the model effectively translates handwritten mathematical input into accurate symbolic output, validating the viability of CNNs for complex OCR tasks in the mathematical domain.

7.2 LIMITATIONS OF THE MODEL

Despite its achievements, the model has several limitations that must be acknowledged. One significant limitation lies in its sensitivity to varied handwriting styles. While the model was trained on a diverse dataset, certain handwriting styles—particularly those with unusual symbol variations or non-standard structures—may still challenge its accuracy.

Additionally, while the system performs well on basic mathematical symbols and operations, it may struggle with more complex expressions or densely populated mathematical notation. The current architecture also has limited capability for handling multi-line expressions or interpreting multi-dimensional relationships, such as intricate subscripts and superscripts that often appear in higher-level mathematics.

7.3 FUTURE WORK

To enhance the system's accuracy and applicability, future research could explore several directions. One area of focus is expanding the dataset to include a broader range of handwriting styles and more complex mathematical symbols, which could improve the model's robustness. Experimenting with advanced neural network architectures, such as Transformer-based models or recurrent neural networks (RNNs), may also help the system handle sequences and hierarchical structures more effectively. Furthermore, integrating additional preprocessing techniques, such as advanced segmentation for multi-line equations, would make the system more versatile for real-world applications. Incorporating more robust error-handling mechanisms and introducing semi-supervised learning techniques could also improve performance on challenging or ambiguous symbols.

7.4 CONCLUSION

The CNN-based Optical Character Recognition (OCR) system developed in this project achieved significant progress in recognizing and evaluating handwritten mathematical expressions. Through a structured approach involving data preprocessing, model training, and integration with a symbolic computation tool, the system proved capable of accurately identifying a range of mathematical symbols, including digits, operators, and complex constructs like fractions and exponents.

This achievement demonstrates the potential of convolutional neural networks (CNNs) in handling specialized OCR tasks, particularly within the field of mathematical expression recognition.

This project contributes to the field of OCR by tackling the unique challenges of mathematical symbol recognition and parsing, By allowing for symbolic evaluation using tools like SymPy, the system not only converts handwritten input into digital form but also computes results, making it a valuable tool for educational software, automated grading, and accessibility aids.

However, the project also revealed limitations, particularly in handling diverse handwriting styles and interpreting more complex, multi-dimensional expressions. These challenges underscore areas for future improvement, such as expanding the dataset to include a wider variety of handwriting styles, employing advanced model architectures like Transformers, and enhancing the system's ability to interpret multi-line and nested mathematical notation.

In conclusion, this CNN-based OCR system represents a meaningful step forward in automated mathematical expression recognition and evaluation. While there are opportunities for refinement, the current model serves as a strong foundation for future advancements, with potential applications in academic, educational, and accessibility settings. The insights gained from this project pave the way for further research, aiming to enhance the accuracy, versatility, and real-world applicability of OCR systems in specialized fields

Furthermore, integrating additional preprocessing techniques, such as advanced segmentation for multi-line equations, would make the system more versatile for real-world applications. Incorporating more robust error-handling mechanisms and introducing semi-supervised learning techniques could also improve performance on challenging or ambiguous symbols.

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APPENDICES

APPENDIX – I

Data Preprocessing - dataset_loadind.ipynb:

```
import numpy as np
import pandas as pd
import cv2
import os
from os import listdir
from os.path import join
import matplotlib.pyplot as plt
def get_index(directory):
    return index_dir[directory]
def load_images(folder):
    train_data = []
    for filename in os.listdir(folder):
        img = cv2.imread(os.path.join(folder,filename), cv2.IMREAD_GRAYSCALE)
        #img = ~img
        if img is not None:
            img = ~img
            _, img = cv2.threshold(img, 127, 255, cv2.THRESH_BINARY)
            ctrs, _ = cv2.findContours(img, cv2.RETR_EXTERNAL, cv2.CHAIN_APPROX_NONE)
```

```
        cnt = sorted(ctrs, key=lambda ctr:
cv2.boundingRect(ctr)[0])
        m = 0
        for c in cnt:
            x, y, w, h = cv2.boundingRect(c)
            m = max(w*h, m)
            if m == w*h:
                x_max,y_max,w_max,h_max=x,y,w,h
        im_crop = img[y_max:y_max+h_max+10,
x_max:x_max+w_max+10]
        im_resize = cv2.resize(im_crop, (28, 28))
        im_resize = np.reshape(im_resize, (784, 1))
        train_data.append(im_resize)
    return train_data
dataset_dir = './dataset/'
directory_list = listdir(dataset_dir)
#print(directory_list)
first = True
data = []
print('Imporitng...')
for directory in directory_list:
    print(directory)
    if first:
        first = False
        data = load_images(dataset_dir + directory)
        for i in range(0, len(data)):
```

```
            data[i] = np.append(data[i],
[str(get_index(directory))])
            continue
        auxillary_data = load_images(dataset_dir + directory)
        for i in range(0, len(auxillary_data)):
            auxillary_data[i] = np.append(auxillary_data[i],
[str(get_index(directory))])
        data = np.concatenate((data, auxillary_data))

df=pd.DataFrame(data,index=None)
df.to_csv('model/train_data.csv',index=False)
```

CNN Model Construction : - cnn_model.ipynb

```
import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split
df_train = pd.read_csv('model/train_data.csv', index_col =
False)
labels = df_train[['784']]
df_train.head()
df_train.drop(df_train.columns[[784]], axis=1,
inplace=True)
np.random.seed(1212)
from tensorflow import keras
from tensorflow.keras import layers
```

```
from tensorflow.keras.utils import to_categorical
import tensorflow.keras.backend as K
labels = np.array(labels)
from tensorflow.keras.utils import to_categorical
categorical_data = to_categorical(labels, num_classes = 13)
l = []
for i in range(df_train.shape[0]):
    l.append(np.array(df_train[i:i+1]).reshape(28, 28, 1))
print(len(l))
train_X, test_X, train_y, test_y = train_test_split(np.array(l),
categorical_data, test_size=0.20, random_state=42)
np.random.seed(7)
model = keras.Sequential([
    layers.Conv2D(30, (5, 5), input_shape=(28, 28, 1),
activation='relu'),
    layers.MaxPool2D(pool_size=2),
    layers.Conv2D(15, (3, 3), activation='relu'),
    layers.MaxPool2D(pool_size=2),
    layers.Dropout(0.2),
    layers.Flatten(),
    layers.Dense(128, activation='relu'),
    layers.Dense(50, activation='relu'),
    layers.Dense(13, activation='softmax'),
])
model.compile(loss='categorical_crossentropy',
optimizer='adam', metrics=['accuracy'])
```

```
#from tensorflow.keras.models import model_from_json
model.fit(train_X, train_y, validation_split=0.25, epochs=10, batch_size=200, shuffle=True, verbose=1)
model.evaluate(test_X, test_y)
model.summary()
from tensorflow.keras.models import model_from_json
model_json = model.to_json()
with open('model/model.json', 'w') as json_file:
    json_file.write(model_json)
model.save_weights('model/model_weights.weights.h5')
```

Model Loading :

```
from tensorflow.keras.models import model_from_json
print('Loading Model...')
model_json = open('model/model.json', 'r')
loaded_model_json = model_json.read()
model_json.close()
model = model_from_json(loaded_model_json)
print('Loading weights...')
model.load_weights("model/model_weights.h5")
```

Symbol Recognition:

Code for recognizing symbols using the trained CNN model.

```
from sympy import *
class Solver:
```

```
    def __init__(self, equation):
        self.equation = str(equation)
        self.leftEqu = []
    def convertEquationIntoGeneralForm(self):
        print(self.equation)
        leftSide, rightSide = '', ''
        equalIndx = self.equation.index('=')
        leftSide = self.equation[0:equalIndx]
        rightSide =
self.equation[equalIndx+1:len(self.equation)]
        if rightSide[0].isalpha() or rightSide[0].isdigit():
            rightSide = '+' + rightSide
        for i in range(0, len(rightSide)):
            if rightSide[i] == '+':
                rightSide = rightSide[0:i] + '-' +
rightSide[i+1:len(rightSide)]
            elif rightSide[i] == '-':
                rightSide = rightSide[0:i] + '+' +
rightSide[i+1:len(rightSide)]
            leftSide += rightSide[i]
        self.equation = leftSide + '=' + '0'
        self.leftEqu = leftSide
    def solveEquation(self):
        self.convertEquationIntoGeneralForm()
        sympy_eq = sympify("Eq(" +
self.equation.replace("=", ",") + ")")
        roots = solve(sympy_eq)
```

```
    return roots
def solution():
                                    img                    =
cv2.imread('canvas.jpg',cv2.IMREAD_GRAYSCALE)
  img = ~img
            ret,thresh      =      cv2.threshold(img,127,
255,cv2.THRESH_BINARY)
             ctrs,_      =      cv2.findContours(thresh,
cv2.RETR_EXTERNAL,
cv2.CHAIN_APPROX_SIMPLE)
          cnt     =     sorted(ctrs,     key=lambda     ctr:
cv2.boundingRect(ctr)[0])
  img_data = []
  rects = []
  for c in cnt :
    x, y, w, h = cv2.boundingRect(c)
    rect = [x, y, w, h]
    rects.append(rect)
  final_rect = [i for i in rects]
  for r in final_rect:
    x,y,w,h = r[0],r[1],r[2],r[3]
    img = thresh[y:y+h+10, x:x+w+10]
    img = cv2.resize(img, (28, 28))
    img = np.reshape(img, (1, 28, 28))
    img_data.append(img)

  mainEquation=[]
```

```
    operation = ''
    for i in range(len(img_data)):
            img_data[i] = np.array(img_data[i])
            img_data[i] = img_data[i].reshape(-1, 28, 28, 1)
            result=predictFromArray(img_data[i])
            i=result[0]
            mainEquation.append(labels[i])
    StringEquation=""
    for i in range(len(mainEquation)):
        a=mainEquation[i]
            if(a.isdigit()==False and a.isalpha()==False and
i<len(mainEquation)-1):
            if(a==mainEquation[i+1]=='-'):
                StringEquation+='='
            else:
                StringEquation+=a
        if(a.isalpha()==True):
            if(i>0):
                if(mainEquation[i-1].isdigit()):
                    StringEquation+="*"+a
                else:
                    StringEquation+=a
            else:
                StringEquation+=a
        if(a.isdigit()==True):
            if(i>0):
```

```
            if(mainEquation[i-1].isdigit()):
                StringEquation+=a
            elif(mainEquation[i-1].isalpha()):
                 StringEquation+="^"+a
            else:
                StringEquation+=a
        else:
            StringEquation+=a
newStr=""
l=list(StringEquation)
for i in range(len(l)):
    if(l[i]=="="):
        newStr=l[:i+1]+l[i+2:]
print(newStr)
equ=""
for i in newStr:
    equ+=i
solution=Solver(equ)
str1="
roots=solution.solveEquation()
st=[]
for i in roots:
   i=str(i)
   st.append(i)
str1=', '.join(st)
solving(equ,str1)
```

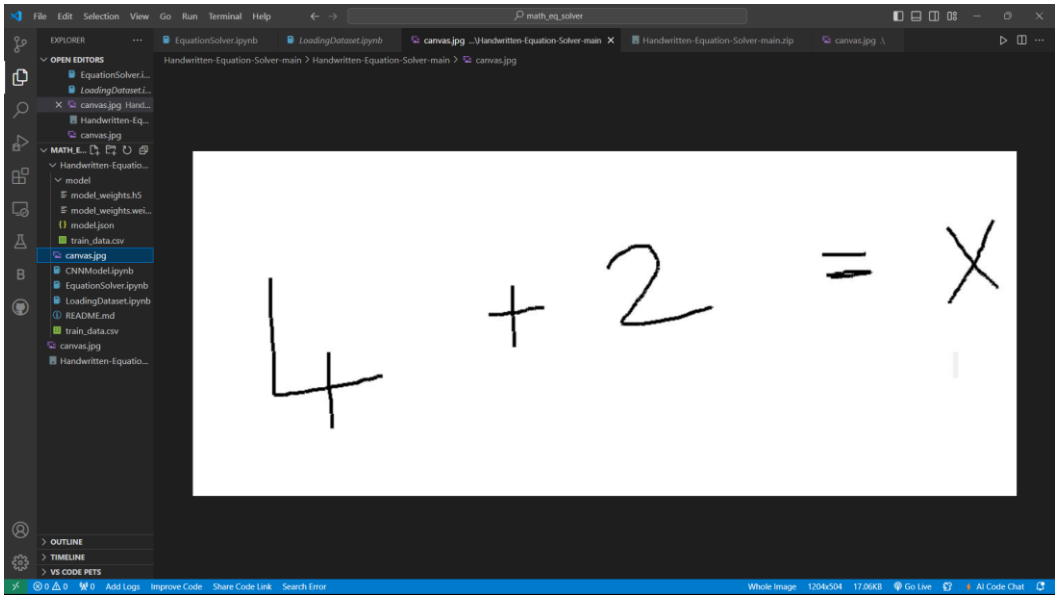
Compiling Code:

```
from tkinter import*
import cv2
import numpy as np
root=Tk()
root.resizable(0,0)
root.title('Equation Solver')
lasx,lasy=None,None
cv=Canvas(root,width=1200,height=500,bg='white')
#cv=Canvas(root,width=1600,height=500,bg='white')
cv.grid(row=0,column=0,pady=2,sticky=W,columnspan=2)
cve2=Label(root)
cve=Label(root,font=("Helvetica",16))
cve.grid(row=0, column=1,pady=1, padx=1)
cve2.grid(row=1, column=1,pady=1, padx=1)
cv.bind('<Button-1>',activate_event)
cv.bind('<B1-Motion>',draw_smth)
btn_save=Button(text="Save",command=save,bg='#6495E
D',fg='White')
btn_save.grid(row=2,column=0,pady=1,padx=1)
btn_predict=Button(text="Predict",command=solution,bg='
#6495ED',fg='White')
btn_predict.grid(row=2,column=1,pady=1,padx=1)
def solving(equ,roots):
```

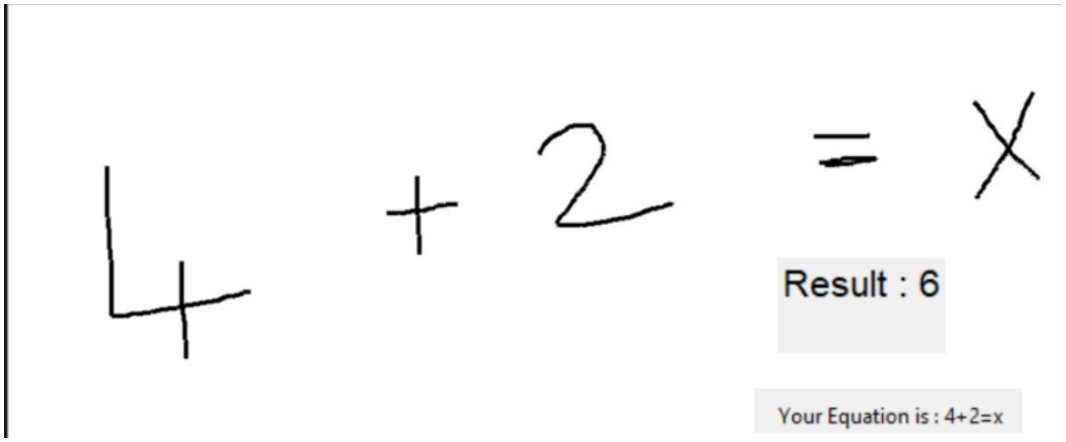
```
    print(equ)
    cve2.configure(text='Your Equation is : '+equ)
    cve.configure(text='Result : '+(roots)+'\n')
root.mainloop()
```

APPENDIX – II

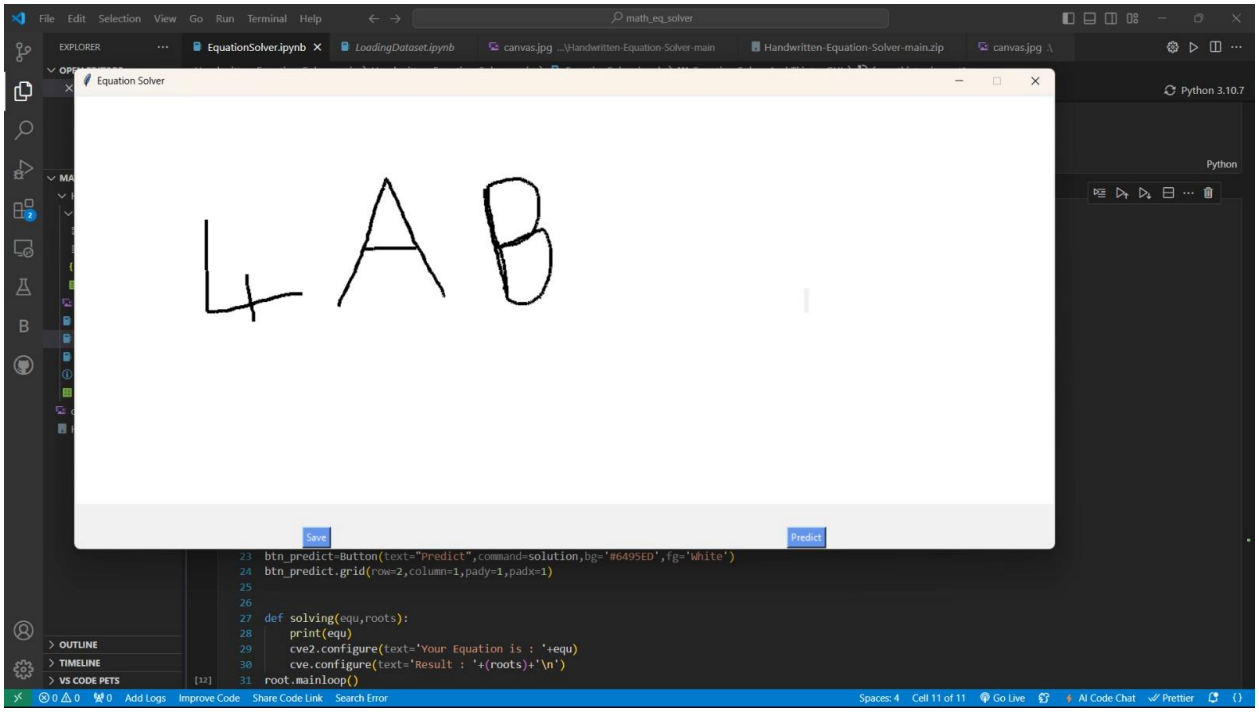
INPUT 1:



OUTPUT 1:



INPUT 2:



OUTPUT 2:

```
Exception in Tkinter callback
Traceback (most recent call last):
  File "c:\Users\abish\AppData\Local\Programs\Python\Python310\lib\tkinter\__init__.py", line 1921, in __call__
    return self.func(*args)
  File "C:\Users\abish\AppData\Local\Temp\ipykernel_14676\2361275096.py", line 69, in solution
    roots=solution.solveEquation()
  File "C:\Users\abish\AppData\Local\Temp\ipykernel_14676\3947809542.py", line 32, in solveEquation
    self.convertEquationIntoGeneralForm()
  File "C:\Users\abish\AppData\Local\Temp\ipykernel_14676\3947809542.py", line 13, in convertEquationIntoGeneralForm
    equalIndx = self.equation.index('=')
ValueError: substring not found
```

```
ValueError: substring not found
```