Assignment

1. Consider the logistic map:

$$x_{n+1} = r\,x_n\,(1-\mathrm{x}_n) = \mathrm{f}\,(\mathrm{x})$$

   a) Obtain fixed points for period two. For this get $f^2(\mathrm{x})$ but do not go through the tedious work of solving the quartic polynomial. Instead, factor out the fixed points of $f(\mathrm{x})$ from $f^2(\mathrm{x})$. So, now all you need to do is solve this quadratic polynomial for solutions of period 2.

   b) Draw x_n vs n for period 2,4,8 & 16 where period doubling cycling occur at:

$r_1 = 3$	period 2 starts
r_2=3.449…	4
r_3=3.54409…	8
r_4=3.5644…	16

   Observe the initial transient and how the solution settles to the period doubling cycle.

   c) Find the range in which period 2 solution is stable. For this:

      i. Find derivative of $f^2(\mathrm{x})$ and check:

         $|f^2(\mathrm{x})'| < 1$

      ii. Also, Check for stability as was taught in class by:

         $|f'(\mathrm{a})\,\mathrm{f}'(\mathrm{b})| < 1$

         Where ‘a’ & ‘b’ are period two solutions.

         Do you get the same range of r from (i.) and (ii.)

   d) Draw the orbit diagram for logistic map i.e. x vs r. (computationally)

2. (Computational) Consider the Henon map:

$$x_{n+1} = 1 - a\,x_n^2 + y_n$$
$$y_{n+1} = b\,x_n$$

   a. Plot x_n vs y_n for a=1.4 and b=0.3
   b. Draw orbit diagram for Henon map x vs a, at b=0.3. Is this similar to that of Logistic map? Expalin