CS545–Introduction to Robotics

Homework Assignment 4 (Due May 5)

In the following problems, you should use MATLAB to compute numerical results and visualize the data, and Simulink for simulations. A handout about getting started with MATLAB is in

http://www-clmc.usc.edu/~cs545/homework.php

This web page also contains all the files needed below. IMPORTANT: In your solutions of the homework, also provide intermediate steps how you derived the solution to a problem.

1. (100 Points) This problem is to familiarize you with data filtering, by comparing a 2nd Order Butterworth Filter with a Kalman Filter. The data in the file noisy.data was generated from the following difference equations:

$$\begin{aligned} x^{n+1} &= 0.9x^n + 2u^n + \varepsilon_x \\ y^n &= x^n + \varepsilon_y \end{aligned} \tag{1}$$

where $\varepsilon_x \sim N\left(0,\sigma^2{}_x\right)$, $\varepsilon_y \sim N\left(0,\sigma^2{}_y\right)$ are Gaussian noise variables. The file noisy.data contains 1000 observations of $\left(y^n, x^n, u^n\right)$ from 1000 time steps of the system sampled at $\Delta t = 0.01$ seconds.

a) A second order Butterworth filter has the following mathematical form:

$$x_f^n = b_1 y^n + b_2^{n-1} + b_3 y^{n-2} - a_2 x_f^{n-1} - a_3 x_f^{n-2}$$

where the subscript "f" denotes the filtered variable. The "butter" function in Matlab allows you to generate the filter coefficients for a discrete Butterworth filter (try "help butter" in Matlab). The function takes two arguments, the filter order and the cutoff frequency, expressed as a fraction of half of the sampling frequency, also called the Nyquist frequency. The filter order will be "2" for a second order filter as described above, and our sampling frequency is 100Hz. Calculate the filter coefficients b_1, b_2, b_3, a_2, a_3 for a low pass filter with cutoff frequency 5Hz. Provide a print-out of the coefficients.

b) Use the Matlab "filter" function and apply your filter to the y^n data from noisy.data. Plot the filtered, i.e., x_f^n and the true, i.e., x^n data on top of each other, provide the print-out, and comment on the quality of the filter.

c) Assume $\sigma_x^2 = 0.001$ and $\sigma_y^2 = 0.01$. Implement a Kalman filter for the system in Matlab, using your knowledge of $\left(y^n, u^n\right)$ (but not x^n -- this is only used for comparisons.) Provide a print-out of your Matlab program, and a plot of the filtered data, i.e., $\hat{x}^n$, and the true data x^n. Comment on the quality of the filter in comparison to the Butterworth filter.