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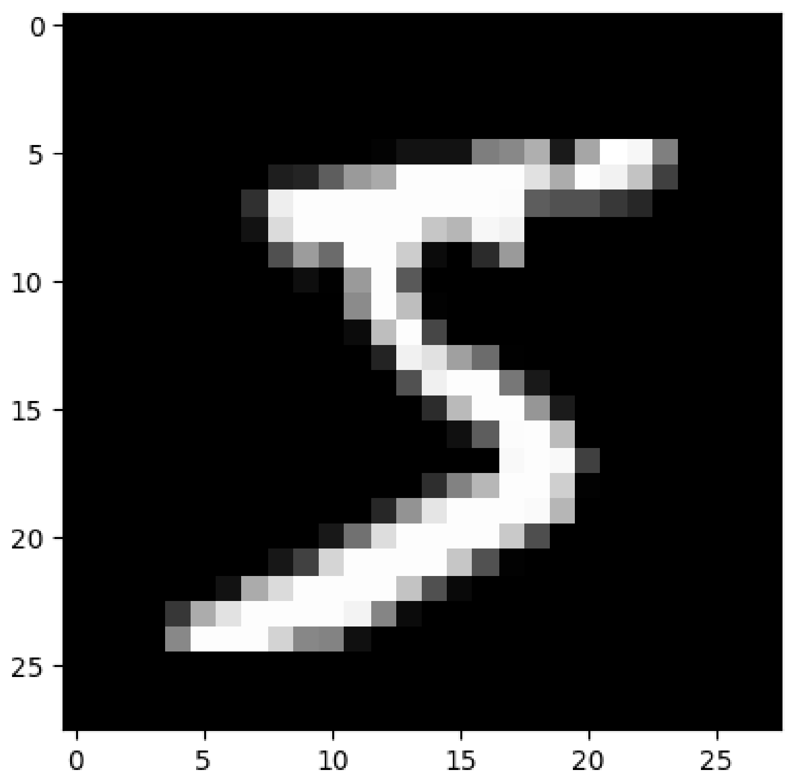
```
%matplotlib widget

import math
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np
import numpy.random
from mnist import MNIST  # run from Anaconda shell: pip install
python-mnist
import sklearn
import sklearn.metrics
from sklearn.model_selection import KFold
import scipy.special as sp

# setup plotting
from IPython import get_ipython
import psutil
inTerminal = not "IPKernelApp" in get_ipython().config
inJupyterNb = any(filter(lambda x: x.endswith("jupyter-notebook"),
psutil.Process().parent().cmdline()))
get_ipython().run_line_magic("matplotlib", "" if inTerminal else
"notebook" if inJupyterNb else "widget")
def nextplot():
    if inTerminal:
        plt.clf()     # this clears the current plot
    else:
        plt.figure()  # this creates a new plot
```

Load the data

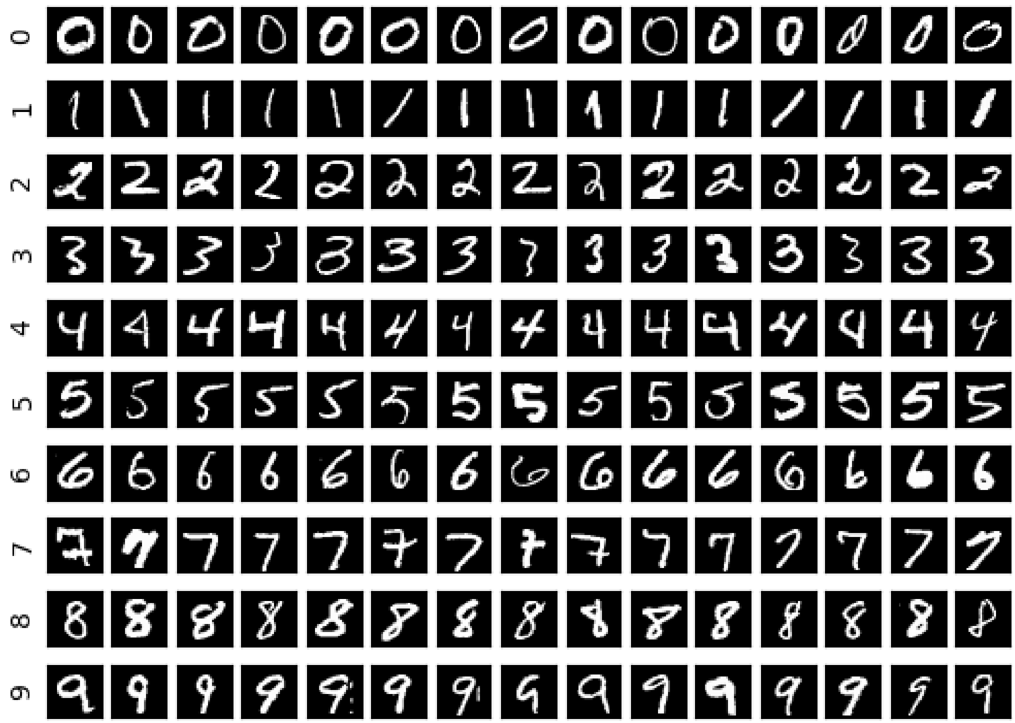
```
mndata = MNIST("data/")
X, y = mndata.load_training()
y = np.array(y, dtype="uint8")
X = np.array([np.array(x) for x in X], dtype="uint8")
N, D = X.shape
Xtest, ytest = mndata.load_testing()
ytest = np.array(ytest, dtype="uint8")
Xtest = np.array([np.array(x) for x in Xtest], dtype="uint8")
Ntest = Xtest.shape[0]

# Optional: use a smaller sample of the data
p = np.zeros(0, dtype="int")
for c in range(10):
    p = np.append(p, np.random.choice(np.where(y == c)[0], size=100,
replace=False))
X_s = X[p, :]
```

```
y_s = y[p]
N_s = X_s.shape[0]
p = np.zeros(0, dtype="int")
for c in range(10):
    p = np.append(p, np.random.choice(np.where(ytest == c)[0],
size=10, replace=False))
Xtest_s = Xtest[p, :]
ytest_s = ytest[p]
Ntest_s = Xtest_s.shape[0]

def showdigit(x):
    "Show one digit as a gray-scale image."
    plt.imshow(x.reshape(28, 28), norm=mpl.colors.Normalize(0, 255),
cmap="gray")

# Example: show first digit
nextplot()
showdigit(X[0,])
print(y[0])
```

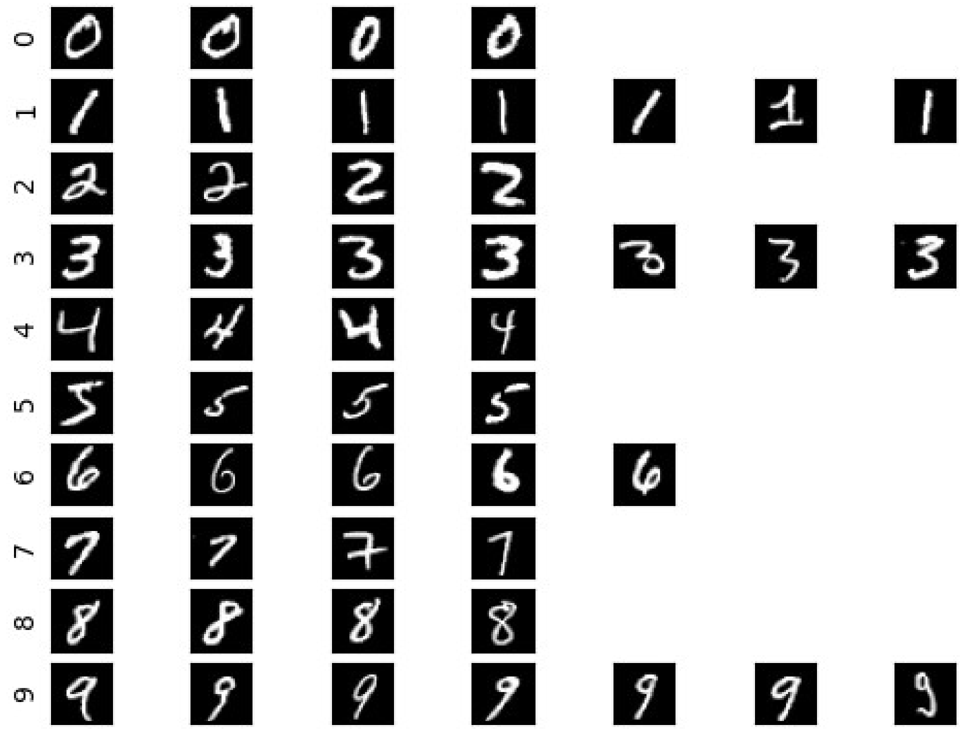
```
5
```



```python
def showdigits(X, y, max_digits=15):
    "Show up to max_digits random digits per class from X with class 
labels from y."
    num_cols = min(max_digits, max(np.bincount(y)))
    for c in range(10):
        ii = np.where(y == c)[0]
        if len(ii) > max_digits:
            ii = np.random.choice(ii, size=max_digits, replace=False)
        for j in range(num_cols):
            ax = plt.gcf().add_subplot(
                10, num_cols, c * num_cols + j + 1, aspect="equal"
            )
            ax.get_xaxis().set_visible(False)
            if j == 0:
                ax.set_ylabel(c)
                ax.set_yticks([])
            else:
                ax.get_yaxis().set_visible(False)
            if j < len(ii):
                ax.imshow(
                    X[ii[j],].reshape(28, 28),
                    norm=mpl.colors.Normalize(0, 255),
                    cmap="gray",
                )
            else:
                ax.axis("off")

# Example: show 15 random digits per class from training data
nextplot()
showdigits(X, y)
```




```
# Example: show a specific set of digits
nextplot()
showdigits(X[0:50,], y[0:50])
```




```python
# A simple example dataset that you can use for testing
Xex = np.array([1, 0, 0, 1, 1, 1, 2, 0]).reshape(4, 2)
yex = np.array([0, 1, 2, 0])

print(Xex, yex)
```

```
[[1 0]
 [0 1]
 [1 1]
 [2 0]] [0 1 2 0]
```

1 Training

```python
def nb_train(X, y, alpha=1, K=None, C=None):
    """Train a Naive Bayes model.

    We assume that all features are encoded as integers and have the 
same domain
    (set of possible values) from 0:(K-1). Similarly, class labels 
have domain
```

```
    0:(C-1).

    Parameters
    ----------
    X : ndarray of shape (N,D)
        Design matrix.
    y : ndarray of shape (N,)
        Class labels.
    alpha : int
        Parameter for symmetric Dirichlet prior (Laplace smoothing)
for all
        fitted distributions.
    K : int
        Each feature takes values in [0,K-1]. None means auto-detect.
    C : int
        Each class label takes values in [0,C-1]. None means auto-
detect.

    Returns
    -------
    A dictionary with the following keys and values:

    logpriors : ndarray of shape (C,)
        Log prior probabilities of each class such that logpriors[c]
contains
        the log prior probability of class c.

    logcls : ndarray of shape(C,D,K)
        A class-by-feature-by-value array of class-conditional log-
likelihoods
        such that logcls[c,j,v] contains the conditional log-
likelihood of value
        v in feature j given class c.
    """
    N, D = X.shape
    if K is None:
        K = int(np.max(X)) + 1
    if C is None:
        C = int(np.max(y)) + 1


    # Compute class priors and store them in priors
    priors = np.zeros(C)
    # for each class, we divide class count by total count, adding in
pseudocounts according to alpha value
    for c in range(C):
        priors[c] = (np.sum(y == c) + alpha - 1) / (N + (alpha - 1)*C)
```

```
    # Compute class-conditional densities in a class x feature x value
array
    # and store them in cls.
    cls = np.zeros((C, D, K))
    # for each class, we select all instances with corresponding class
label
    for c in range(C):
        Xc = X[y == c]
        # for each feature,we  get counts of its values in
corresponding class
        for j in range(D):
            feature_counts = np.bincount(Xc[:, j], minlength=K)
            # we divide feature count by total count, adding
pseudocounts according to alpha value, and store in cls
            cls[c, j, :] = (feature_counts + alpha-1) / (len(Xc) + K *
(alpha-1))


    # Output result
    return dict(logpriors=np.log(priors), logcls=np.log(cls))

# Test your code (there should be a warning when you run this)
model = nb_train(Xex, yex, alpha=1)
model
# This should produce:
# {'logcls': array([[[       -inf, -0.69314718, -0.69314718],
#          [ 0.        ,        -inf,        -inf]],
#
#         [[ 0.        ,        -inf,        -inf],
#          [       -inf,  0.        ,        -inf]],
#
#         [[       -inf,  0.        ,        -inf],
#          [       -inf,  0.        ,        -inf]]]),
#  'logpriors': array([-0.69314718, -1.38629436, -1.38629436])}
```

```
/var/folders/t3/h38q5w_d36ncdxty42rj79mr0000gn/T/
ipykernel_22893/1846586245.py:64: RuntimeWarning: divide by zero
encountered in log
  return dict(logpriors=np.log(priors), logcls=np.log(cls))
```

```
{'logpriors': array([-0.69314718, -1.38629436, -1.38629436]),
 'logcls': array([[[       -inf, -0.69314718, -0.69314718],
         [ 0.        ,        -inf,        -inf]],

        [[ 0.        ,        -inf,        -inf],
         [       -inf,  0.        ,        -inf]],

        [[       -inf,  0.        ,        -inf],
         [       -inf,  0.        ,        -inf]]])}
```

```
# Test your code (this time no warning)
model = nb_train(Xex, yex, alpha=2)  # here we use add-one smoothing
model
# This should produce:
# {'logcls': array([[[-1.60943791, -0.91629073, -0.91629073],
#          [-0.51082562, -1.60943791, -1.60943791]],
#
#         [[-0.69314718, -1.38629436, -1.38629436],
#          [-1.38629436, -0.69314718, -1.38629436]],
#
#         [[-1.38629436, -0.69314718, -1.38629436],
#          [-1.38629436, -0.69314718, -1.38629436]]]),
#  'logpriors': array([-0.84729786, -1.25276297, -1.25276297])}
```

```
{'logpriors': array([-0.84729786, -1.25276297, -1.25276297]),
 'logcls': array([[[-1.60943791, -0.91629073, -0.91629073],
         [-0.51082562, -1.60943791, -1.60943791]],

        [[-0.69314718, -1.38629436, -1.38629436],
         [-1.38629436, -0.69314718, -1.38629436]],

        [[-1.38629436, -0.69314718, -1.38629436],
         [-1.38629436, -0.69314718, -1.38629436]]])}
```

2 Prediction

```
def logsumexp(x):
    """Computes log(sum(exp(x)).

    Uses offset trick to reduce risk of numeric over- or underflow.
When x is a
    1D ndarray, computes logsumexp of its entries. When x is a 2D
ndarray,
    computes logsumexp of each column.

    Keyword arguments:
    x : a 1D or 2D ndarray
    """
    offset = np.max(x, axis=0)
    return offset + np.log(np.sum(np.exp(x - offset), axis=0))

def nb_predict(model, Xnew):
    """Predict using a Naive Bayes model.

    Parameters
    ----------
    model : dict
        A Naive Bayes model trained with nb_train.
```

```
    Xnew : nd_array of shape (Nnew,D)
        New data to predict.

    Returns
    -------
    A dictionary with the following keys and values:

    yhat : nd_array of shape (Nnew,)
        Predicted label for each new data point.

    logprob : nd_array of shape (Nnew,)
        Log-probability of the label predicted for each new data
point.
    """
    logpriors = model["logpriors"]
    logcls = model["logcls"]
    Nnew = Xnew.shape[0]
    C, D, K = logcls.shape

    # Compute the unnormalized log joint probabilities P(Y=c, x_i) of
each
    # test point (row i) and each class (column c); store in logjoint
    logjoint = np.zeros((Nnew, C))

    # for each data point we first consider the log-prior probability
of it belonging to each of the classes
    for i in range(Nnew):
        for c in range(C):
            logjoint[i,c] = logpriors[c]
            # and then increasy it by class-conditional log-
likelihoods for each feature's value
            for j in range(D):
                x_j = Xnew[i, j]  # The value of feature j for
instance i
                logjoint[i,c] += logcls[c, j, x_j]

    # Compute predicted labels (in "yhat") and their log probabilities
    # P(yhat_i | x_i) (in "logprob")

    # predicted label corresponds to one with the highest log joint
probability for each test point
    yhat = np.argmax(logjoint, axis=1)
    # normalized logprob (normalization is not mandatory to choose
label yhat itself, but helps to interpret how sure the classifier is
about label assignment)
    logprob = np.max(logjoint, axis=1)-logsumexp(logjoint.T).T

    return dict(yhat=yhat, logprob=logprob)
```

```
# Test your code
model = nb_train(Xex, yex, alpha=2)
nb_predict(model, Xex)
# This should produce:
# {'logprob': array([-0.41925843, -0.55388511, -0.68309684, -
0.29804486]),
#  'yhat': array([0, 1, 2, 0], dtype=int64)}
```

```
{'yhat': array([0, 1, 2, 0]),
 'logprob': array([-0.41925843, -0.55388511, -0.68309684, -
0.29804486])}
```

3 Experiments on MNIST Digits Data

```
# Let's train the model on the digits data and predict
model_nb2 = nb_train(X, y, alpha=2)
pred_nb2 = nb_predict(model_nb2, Xtest)
yhat = pred_nb2["yhat"]
logprob = pred_nb2["logprob"]

# Accuracy
sklearn.metrics.accuracy_score(ytest, yhat)
```

```
0.8363
```

```
# we will see the class distribution of the test data
unique_values, counts = np.unique(y, return_counts=True)
relative_frequency = counts / len(y)

print("Unique values:", unique_values)
print("Counts:", counts)
print("Relative frequency:", relative_frequency)
```

```
Unique values: [0 1 2 3 4 5 6 7 8 9]
Counts: [5923 6742 5958 6131 5842 5421 5918 6265 5851 5949]
Relative frequency: [0.09871667 0.11236667 0.0993     0.10218333
0.09736667 0.09035
 0.09863333 0.10441667 0.09751667 0.09915   ]
```

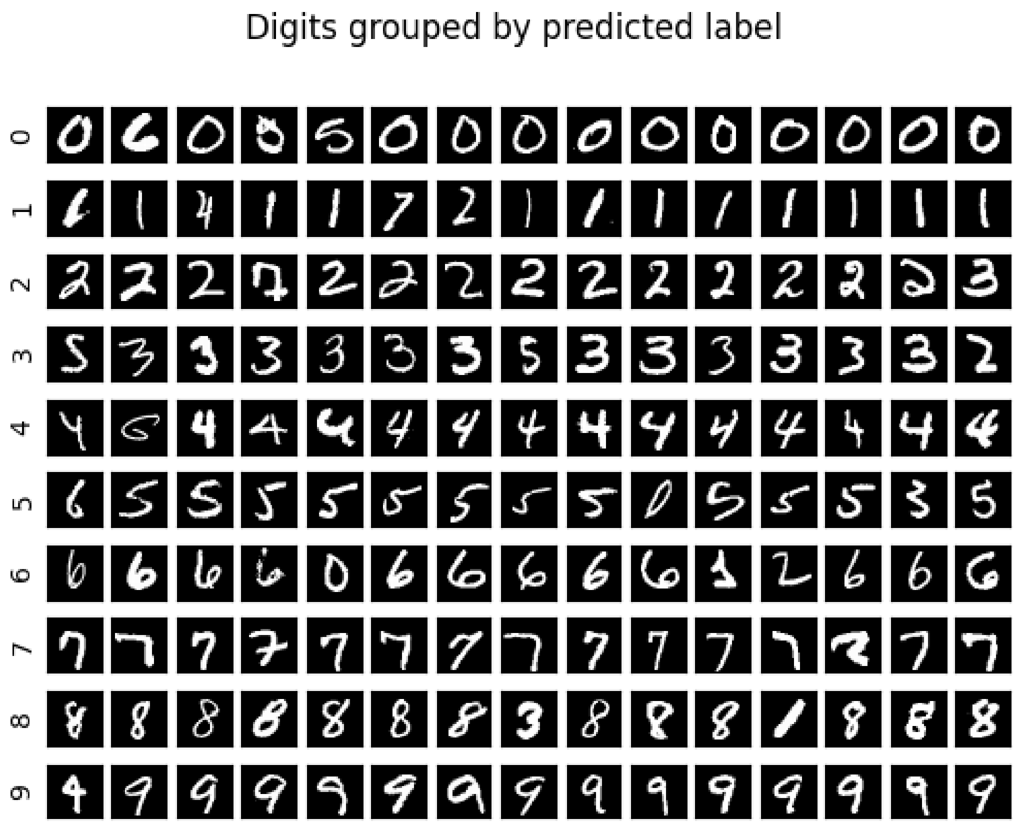
```
# we will define a baseline classifier that always predicts 1(the most
frequent class in training), basically producing yhat1 with 1 as class
for all entries
yhat1 = np.ones((Xtest.shape[0],1))
sklearn.metrics.accuracy_score(ytest, yhat1)
```

```
0.1135
```

```
# show some digits grouped by prediction; can you spot errors?
nextplot()
```

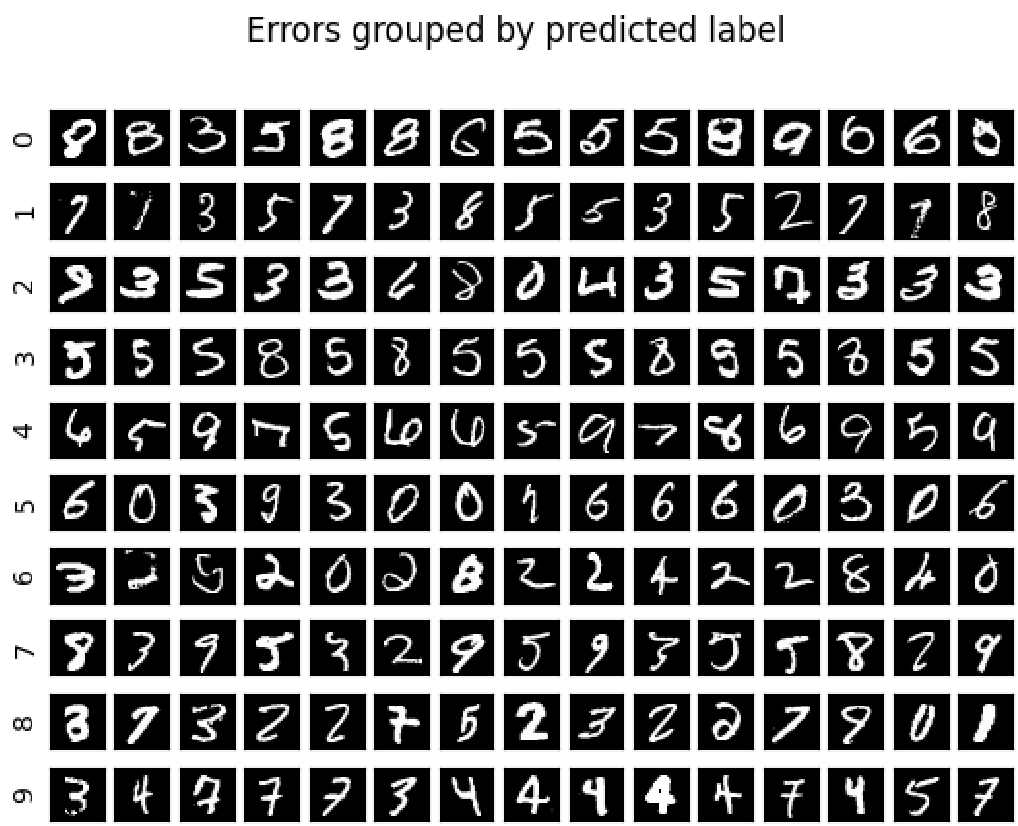
```
showdigits(Xtest, yhat)
plt.suptitle("Digits grouped by predicted label")
```

```
Text(0.5, 0.98, 'Digits grouped by predicted label')
```




```
# do the same, but this time show wrong predicitions only
perror = ytest != yhat
nextplot()
showdigits(Xtest[perror, :], yhat[perror])
plt.suptitle("Errors grouped by predicted label")
```

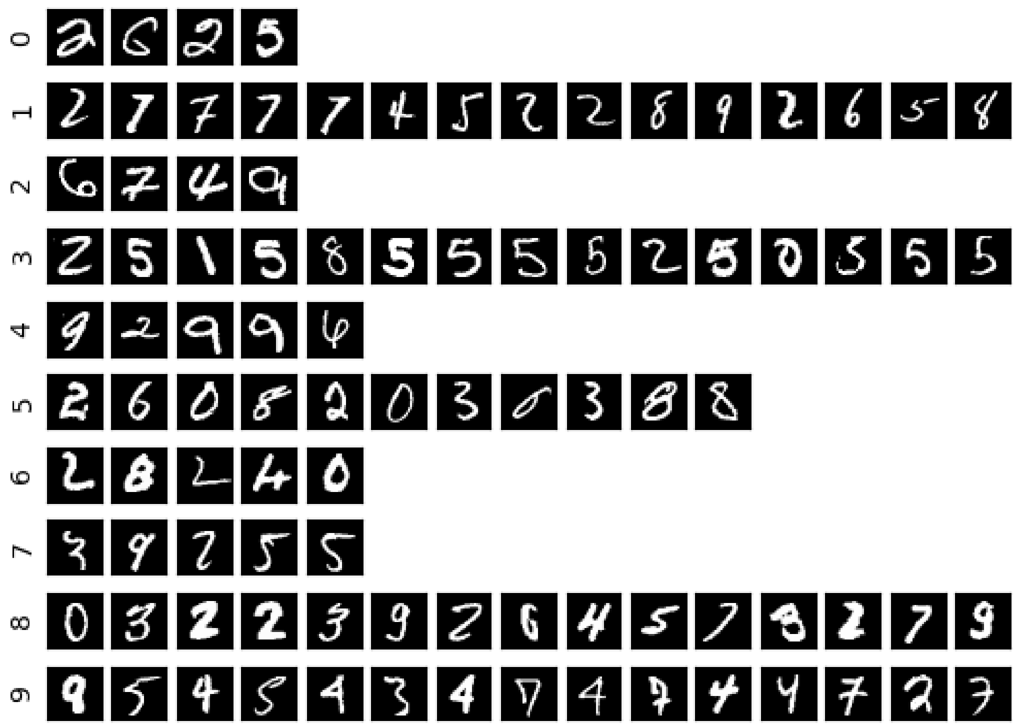
```
Text(0.5, 0.98, 'Errors grouped by predicted label')
```




```
# do the same, but this time on a sample of wrong preditions to see
# error proportions
ierror_s = np.random.choice(np.where(perror)[0], 100, replace=False)
nextplot()
showdigits(Xtest[ierror_s, :], yhat[ierror_s])
plt.suptitle("Errors grouped by predicted label")
```

```
Text(0.5, 0.98, 'Errors grouped by predicted label')
```

Errors grouped by predicted label



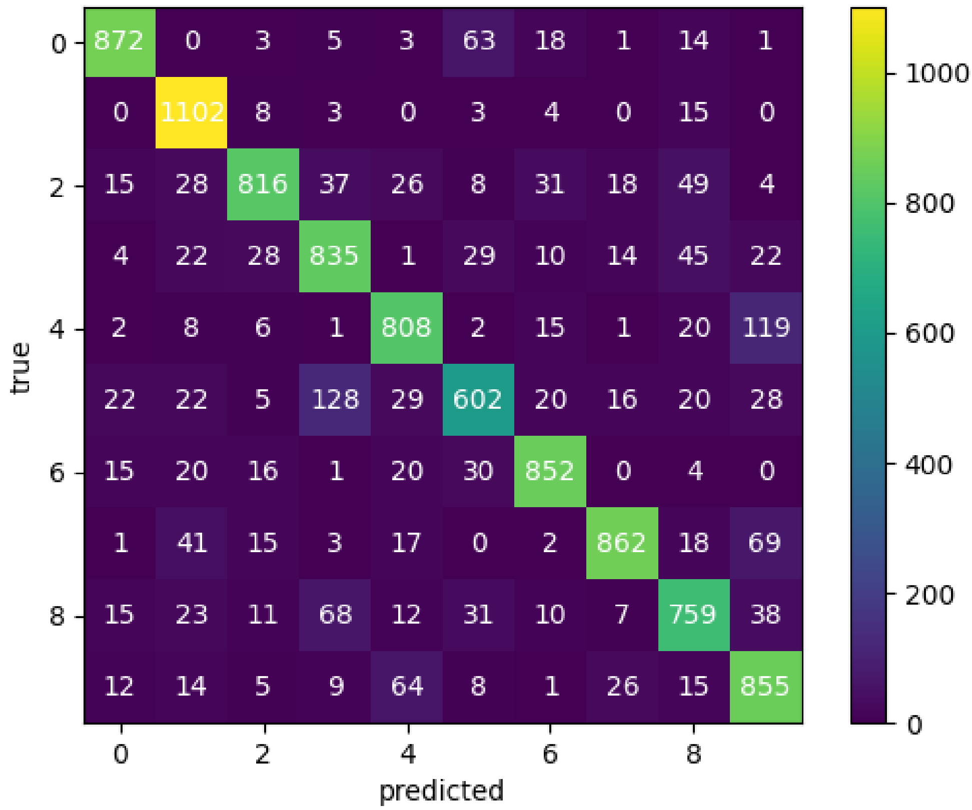

```python
# now let's look at this in more detail
print(sklearn.metrics.classification_report(ytest, yhat))
print(sklearn.metrics.confusion_matrix(ytest, yhat))  # true x predicted
```

```
              precision    recall  f1-score   support

           0       0.91      0.89      0.90       980
           1       0.86      0.97      0.91      1135
           2       0.89      0.79      0.84      1032
           3       0.77      0.83      0.80      1010
           4       0.82      0.82      0.82       982
           5       0.78      0.67      0.72       892
           6       0.88      0.89      0.89       958
           7       0.91      0.84      0.87      1028
           8       0.79      0.78      0.79       974
           9       0.75      0.85      0.80      1009

    accuracy                           0.84     10000
   macro avg       0.84      0.83      0.83     10000
weighted avg       0.84      0.84      0.84     10000
```

```
[[ 872    0    3    5    3   63   18    1   14    1]
 [   0 1102    8    3    0    3    4    0   15    0]
 [  15   28  816   37   26    8   31   18   49    4]
 [   4   22   28  835    1   29   10   14   45   22]
 [   2    8    6    1  808    2   15    1   20  119]
 [  22   22    5  128   29  602   20   16   20   28]
 [  15   20   16    1   20   30  852    0    4    0]
 [   1   41   15    3   17    0    2  862   18   69]
 [  15   23   11   68   12   31   10    7  759   38]
 [  12   14    5    9   64    8    1   26   15  855]]
```

```
# plot the confusion matrix
nextplot()
M = sklearn.metrics.confusion_matrix(ytest, yhat)
plt.imshow(M, origin="upper")
for ij, v in np.ndenumerate(M):
    i, j = ij
    plt.text(j, i, str(v), color="white", ha="center", va="center")
plt.xlabel("predicted")
plt.ylabel("true")
plt.colorbar()
```

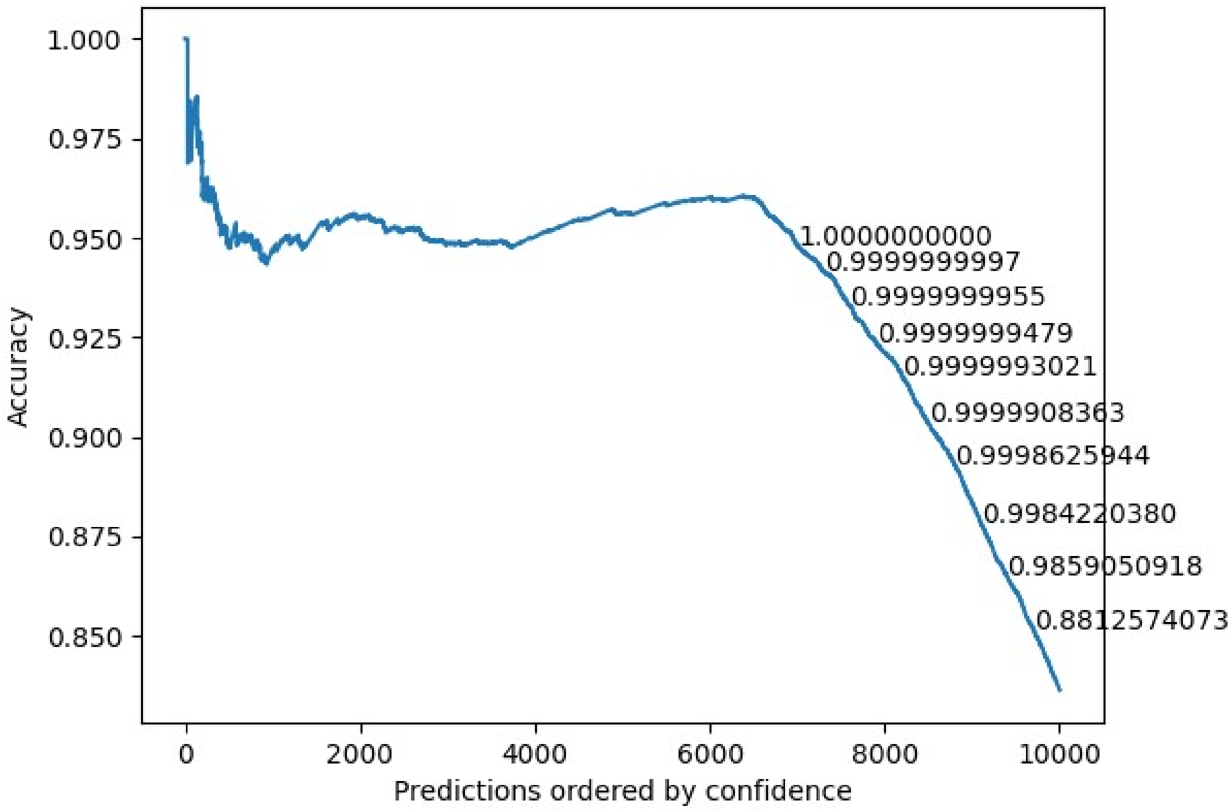
```
<matplotlib.colorbar.Colorbar at 0x352b9acd0>
```




```
# cumulative accuracy for predictions ordered by confidence (labels
show predicted
# confidence)
order = np.argsort(logprob)[::-1]
accuracies = np.cumsum(ytest[order] == yhat[order]) /
(np.arange(len(yhat)) + 1)
nextplot()
plt.plot(accuracies)
plt.xlabel("Predictions ordered by confidence")
plt.ylabel("Accuracy")
for x in np.linspace(0.7, 1, 10, endpoint=False):
    index = int(x * (accuracies.size - 1))
    print(np.exp(logprob[order][index]))
    plt.text(index, accuracies[index],
"{:.10f}".format(np.exp(logprob[order][index])))
```

```
0.9999999999822649
0.9999999996949782
0.9999999955447265
0.9999999478873192
0.999999302093004
0.9999908362580441
```

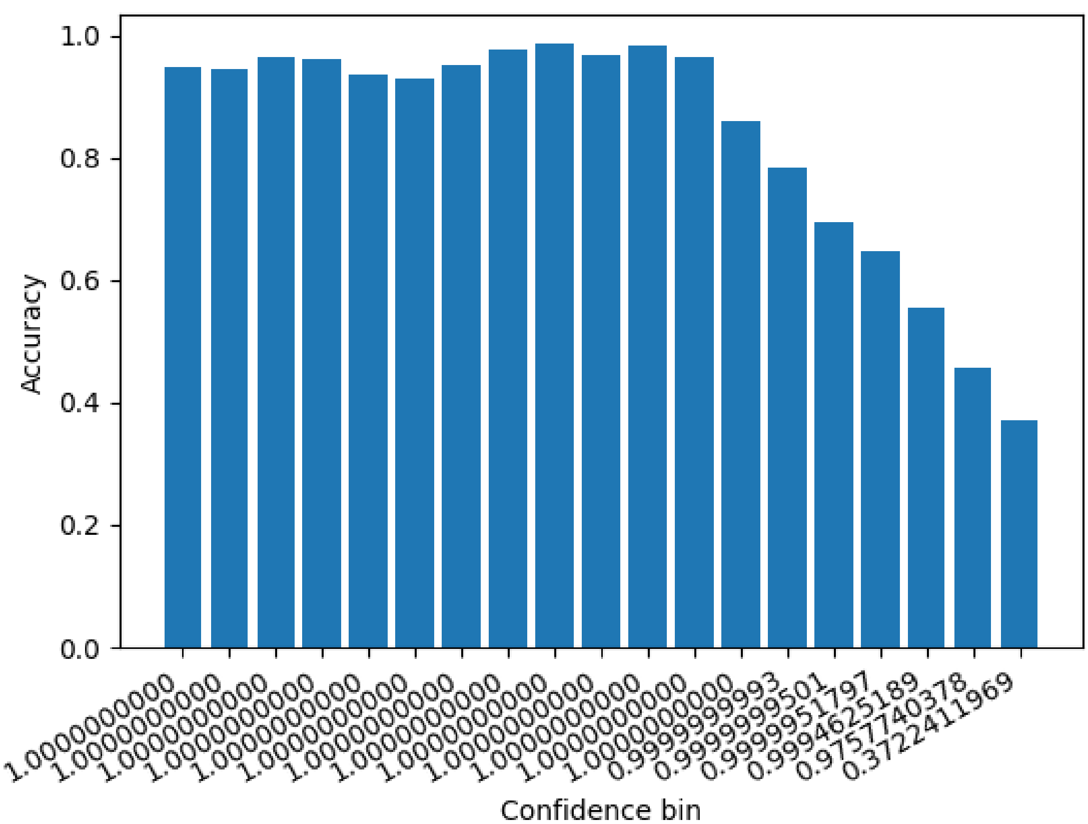
```
0.9998625944161882
0.9984220379937704
0.9859050917808865
0.8812574072791101
```




```
# Accuracy for predictions grouped by confidence (labels show
# predicted confidence). Make the plot large (or reduce number of
bins) to see
# the labels.
bins = (np.linspace(0, 1, 20) * len(yhat)).astype(int)
mean_accuracy = [
    np.mean(ytest[order][bins[i] : bins[i + 1]] == yhat[order][bins[i]
: bins[i + 1]])
    for i in range(len(bins) - 1)
]
nextplot()
plt.bar(np.arange(len(mean_accuracy)), mean_accuracy)
plt.xticks(
    np.arange(len(mean_accuracy)),
    [
        "{:.10f}".format(x)
```

```
        for x in np.exp(logprob[order][np.append(bins[1:-1], len(yhat)
- 1)])
    ],
)
plt.gcf().autofmt_xdate()
plt.xlabel("Confidence bin")
plt.ylabel("Accuracy")
```

```
/var/folders/t3/h38q5w_d36ncdxty42rj79mr0000gn/T/
ipykernel_22893/1544979684.py:22: RuntimeWarning: More than 20 figures
have been opened. Figures created through the pyplot interface
(`matplotlib.pyplot.figure`) are retained until explicitly closed and
may consume too much memory. (To control this warning, see the rcParam
`figure.max_open_warning`). Consider using
`matplotlib.pyplot.close()`.
  plt.figure()  # this creates a new plot

Text(0, 0.5, 'Accuracy')
```



4 Model Selection (optional)

```
# To create folds, you can use:
K = 5
Kf = KFold(n_splits=K, shuffle=True, random_state=42)
for i_train, i_test in Kf.split(X):
    # code here is executed K times, once per test fold
    # i_train has the row indexes of X to be used for training
    # i_test has the row indexes of X to be used for testing
    print(
        "Fold has {:d} training points and {:d} test points".format(
            len(i_train), len(i_test)
        )
    )
```

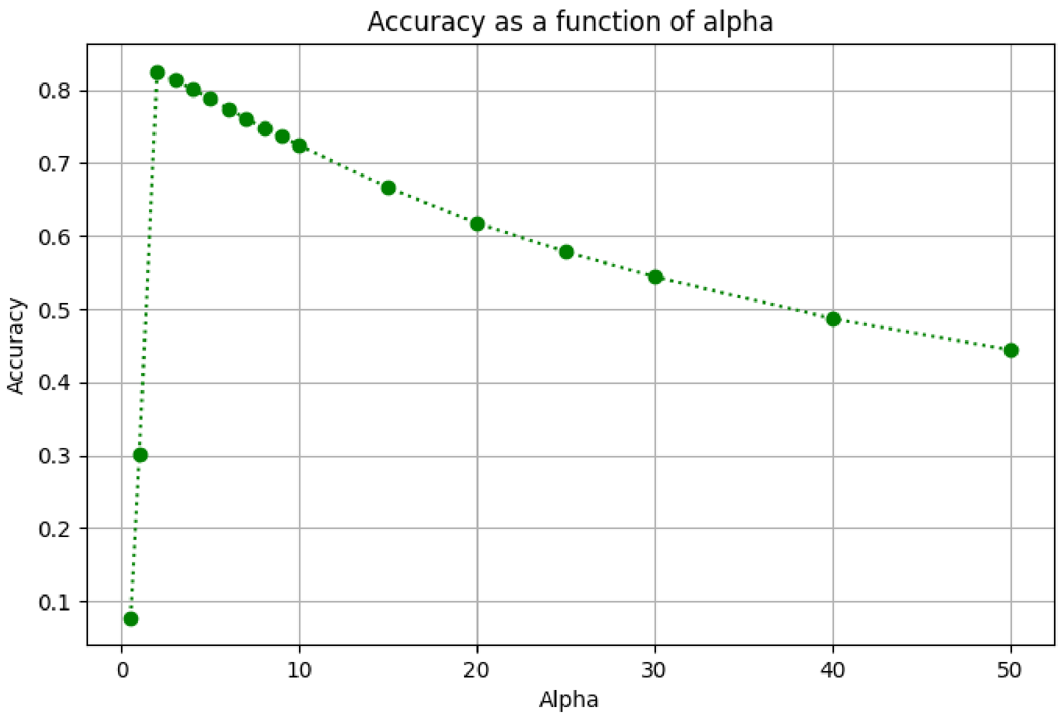
```
Fold has 48000 training points and 12000 test points
Fold has 48000 training points and 12000 test points
Fold has 48000 training points and 12000 test points
Fold has 48000 training points and 12000 test points
Fold has 48000 training points and 12000 test points
```

```
# Use cross-validation to find a good value of alpha. Also plot the
obtained
# accuracy estimate (estimated from CV, i.e., without touching test
data) as a
# function of alpha.

# in order to uppress warnings
import warnings
warnings.filterwarnings("ignore")

# we initialize list of alpha values from 0 to 7
#alphas = range(0, 7)
alphas = [0.5] + list(range(1, 11)) + list(range(15, 31, 5)) + [40,
50]
accuracy_scores = []


for a in alphas:
    fold_accuracy = []
    # for each value of alpha, perform K-fold cross-validation
    for i_train, i_test in Kf.split(X):
        X_train, y_train = X[i_train], y[i_train]
        X_test, y_test = X[i_test], y[i_test]

        model = nb_train(X_train, y_train, alpha=a)
        # predict labels for the test fold
        pred = nb_predict(model, X_test)
        y_hat = pred["yhat"]
        # calculate accuracy for the fold
```

```python
        accuracy = sklearn.metrics.accuracy_score(y_test, y_hat)
        fold_accuracy.append(accuracy)
    # store the mean accuracy over the folds for this alpha
    mean_accuracy = np.mean(fold_accuracy)
    accuracy_scores.append(mean_accuracy)
    print("Finished for alpha =", a, "with accuracy", mean_accuracy)

# find the best alpha based on highest mean accuracy
best_alpha_idx = np.argmax(accuracy_scores)
best_alpha = alphas[best_alpha_idx]
best_accuracy = accuracy_scores[best_alpha_idx]

print("The best parameter choice is alpha = {:.2f} with an accuracy
score of {:.2f}.".format(
    best_alpha, best_accuracy
))
```

```
Finished for alpha = 0.5 with accuracy 0.0778
Finished for alpha = 1 with accuracy 0.3022
Finished for alpha = 2 with accuracy 0.8261333333333333
Finished for alpha = 3 with accuracy 0.8135833333333332
Finished for alpha = 4 with accuracy 0.8007500000000001
Finished for alpha = 5 with accuracy 0.7879833333333334
Finished for alpha = 6 with accuracy 0.7744333333333333
Finished for alpha = 7 with accuracy 0.7610666666666667
Finished for alpha = 8 with accuracy 0.7481500000000001
Finished for alpha = 9 with accuracy 0.7364499999999999
Finished for alpha = 10 with accuracy 0.7240333333333334
Finished for alpha = 15 with accuracy 0.66615
Finished for alpha = 20 with accuracy 0.6175666666666666
Finished for alpha = 25 with accuracy 0.5785166666666667
Finished for alpha = 30 with accuracy 0.5442666666666667
Finished for alpha = 40 with accuracy 0.4871
Finished for alpha = 50 with accuracy 0.44386666666666663
The best parameter choice is alpha = 2.00 with an accuracy score of
0.83.
```

```python
# we plot accuracy as function of alpha

plt.figure(figsize=(8, 5))
plt.plot(alphas, accuracy_scores, linestyle='dotted', marker='o',
color='green')
plt.title('Accuracy as a function of alpha')
plt.xlabel('Alpha')
plt.ylabel('Accuracy')
plt.grid(True)
plt.show()
```



5 Generating Data

```
import numpy as np
import matplotlib.pyplot as plt

def nb_generate(model, ygen):
    """Given a Naive Bayes model, generate some data.

    Parameters
    ----------
    model : dict
        A Naive Bayes model trained with nb_train.
    ygen : nd_array of shape (n,)
        Vector of class labels for which to generate data.

    Returns
    -------
    nd_array of shape (n,D)

    Generated data. The i-th row is a sampled data point for the i-th
label in
    ygen.
    """
    logcls = model["logcls"]  # Shape: (C, D, K)
    n = len(ygen)
```

```
    C, D, K = logcls.shape
    Xgen = np.zeros((n, D), dtype=int)
    for i in range(n):
        c = ygen[i]
        for d in range(D):
            # Extract log probabilities for feature d, class c
            log_probs = logcls[c, d, :]
            # Convert log probabilities to probabilities
            probs = np.exp(log_probs)
            # Normalize to ensure probabilities sum to 1
            probs /= np.sum(probs)
            # Sample from the categorical distribution
            Xgen[i, d] = np.random.choice(range(K), p=probs)
    return Xgen
```

Generation and visualization data of the different models

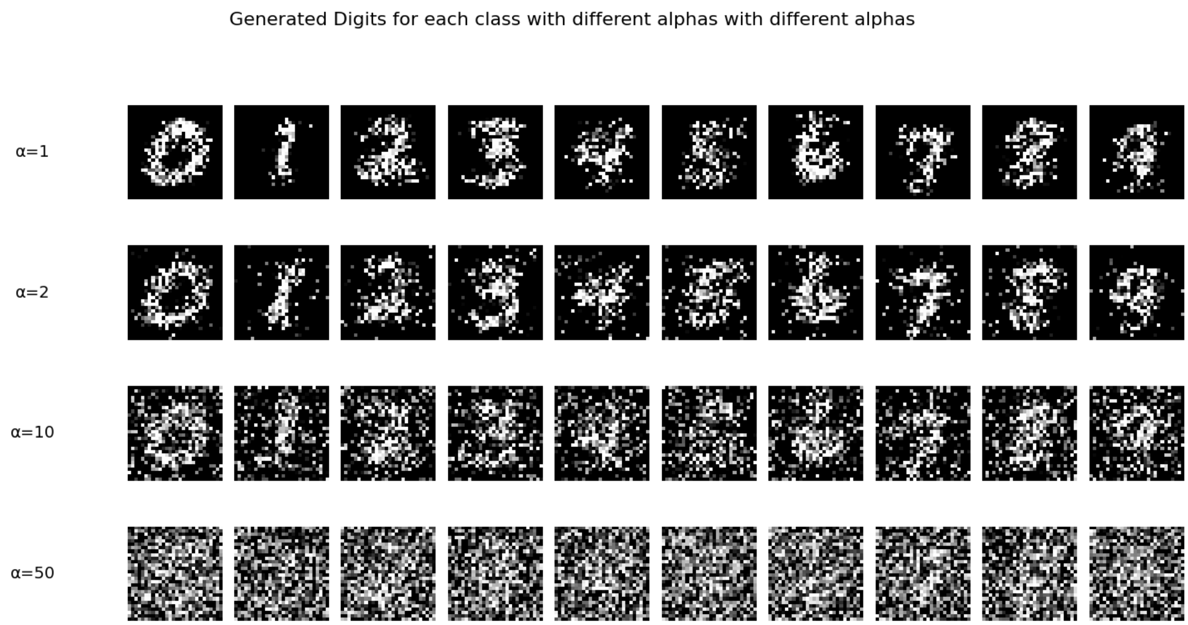
`alphas` = [1, 2, 10, 50]

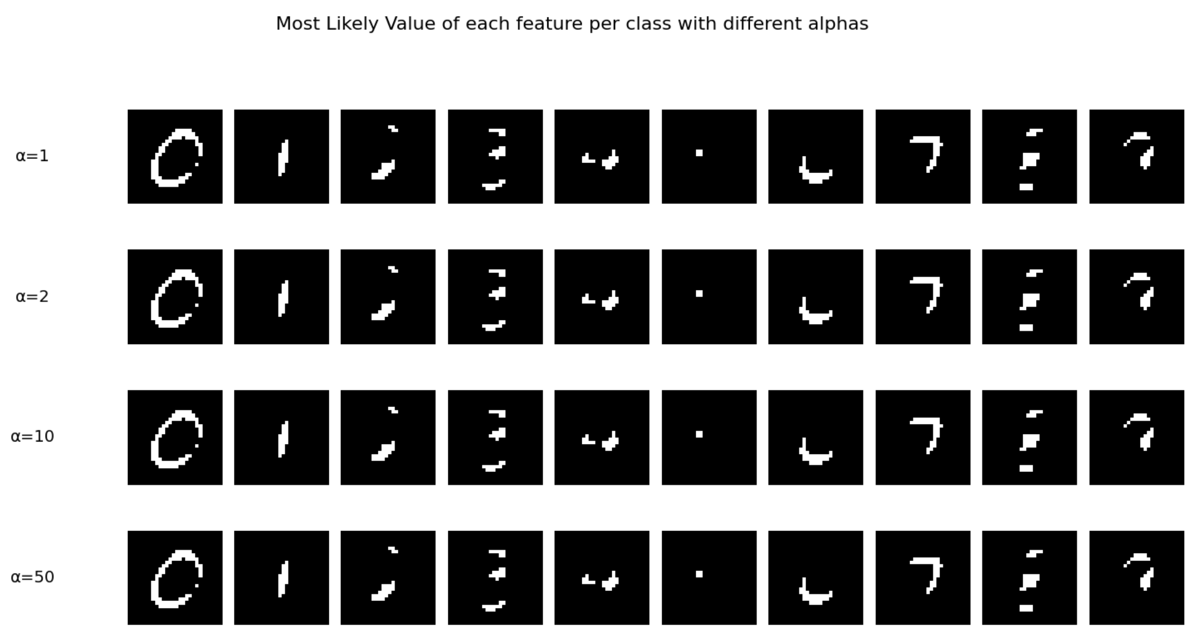
```
import numpy as np
import matplotlib.pyplot as plt

alphas = [1, 2, 10, 50]  # Different alpha values to test
num_digits_per_class = 1  # Number of digits to generate for each
class
num_classes = 10  # For MNIST, we have 10 classes (0-9)

# Function to show digits in a grid and display alpha value for each
row
def show_comparison(Xs, alphas, num_classes, num_per_class, title):
    fig, axes = plt.subplots(len(alphas), num_classes, figsize=(15,
8))

    for i, alpha in enumerate(alphas):
        for j in range(num_classes):
            ax = axes[i, j]
            ax.imshow(Xs[alpha][j].reshape(28, 28), cmap='gray')
            ax.axis('off')

        # Add alpha label for each row
        axes[i, 0].annotate(f'α={alpha}', xy=(-1, 0.5), xycoords="axes
fraction",
                            fontsize=14, ha='center', va='center',
bbox=dict(boxstyle="round,pad=0.3", edgecolor='none',
facecolor='white'))
```

```
    plt.suptitle(title, fontsize=16)
    plt.tight_layout(rect=[0.05, 0, 1, 0.95])
    plt.show()

# Store generated data for each alpha
Xgen_dict = {}
Xmax_dict = {}
Xmean_dict = {}

# Loop through the different alpha values and generate/plot digits for
each
for alpha in alphas:
    # Train the Naive Bayes model with the current alpha
    model_nb = nb_train(X, y, alpha=alpha)

    # Generate digits for each class
    ygen = np.repeat(np.arange(num_classes), num_digits_per_class)
    Xgen = nb_generate(model_nb, ygen)
    Xgen_dict[alpha] = Xgen[:num_classes]  # Store only one digit per
class for comparison

    # Most likely value of each feature per class
    ymax = np.arange(num_classes)
    Xmax = np.zeros((num_classes, X.shape[1]))
    for c in range(num_classes):
        Xmax[c,] = np.apply_along_axis(np.argmax, 1,
model_nb["logcls"][c, :, :])
    Xmax_dict[alpha] = Xmax

    # Expected value of each feature per class
    ymean = np.arange(num_classes)
    Xmean = np.zeros((num_classes, X.shape[1]))
    for c in range(num_classes):
        Xmean[c,] = np.apply_along_axis(
            np.sum, 1, np.exp(model_nb["logcls"][c, :, :]) *
np.arange(256)
        )
    Xmean_dict[alpha] = Xmean

# Display results Generated Digits
show_comparison(Xgen_dict, alphas, num_classes, num_digits_per_class,
"Generated Digits for each class with different alphas with different
alphas")

# Display results Most Likely Value of each feature per class
show_comparison(Xmax_dict, alphas, num_classes, num_digits_per_class,
"Most Likely Value of each feature per class with different alphas")

# Display results for Expected Value of each feature per class
```

```
show_comparison(Xmean_dict, alphas, num_classes, num_digits_per_class,
"Expected Value of each feature per class with different alphas")
```

```
/var/folders/t3/h38q5w_d36ncdxty42rj79mr0000gn/T/
ipykernel_22893/1846586245.py:64: RuntimeWarning: divide by zero
encountered in log
  return dict(logpriors=np.log(priors), logcls=np.log(cls))
```





Expected Value of each feature per class with different alphas