a01-fnn_solution

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1 Work done by pair of students

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[]: #

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```
# IE 678 Deep Learning, University of Mannheim
# Author: Rainer Gemulla
# Edited by:

[]: import math
  import matplotlib.pyplot as plt
  import torch
  import torch.nn as nn
  import torch.nn.functional as F

# import helper functions
  import sys, os
```

from a01helper import * # check out the helper functions there, if you like

2 Task 1: Implement an MLP

2.1 1a Logistic Regression

sys.path.append(os.getcwd())

```
[]: # nn.Module is the superclass of all PyTorch models.
class LogisticRegression(nn.Module):
    """A logistic regression model.

Parameters
-----
D number of inputs
C number of classes
"""

# the definition of all parameters the model uses happens here, i.e., during
# initialization
```

```
def __init__(self, D, C):
             super(LogisticRegression, self).__init__()
             # Create and initialize model parameters. For (multinomial) logistic
      ⇔regression,
             # we have a DxC-dimensional weight matrix W and a C-dimensional bias b.
             self.W = torch.randn(D, C) / math.sqrt(D)
             self.b = torch.randn(C) / math.sqrt(C)
             # Model parameters must be registered to PyTorch as follows. Here we \Box
      \hookrightarrowprovide
             # a useful name that helps to access/analyze the model later on.
             self.register_parameter("0_weight", nn.Parameter(self.W))
             self.register_parameter("0_bias", nn.Parameter(self.b))
         # the forward function computes the model output for the provided (for this
         # assignent: single) input
         def forward(self, x):
             eta = self.W.t() @ x + self.b
             logprob = F.log_softmax(eta, dim=-1)
             return logprob
[]: # let's test it
     logreg = LogisticRegression(3, 2)
     x = torch.rand(3) # input
     logreg(x) # output (log probabilities)
     logreg(x).exp() # output (probabilities)
[]: tensor([0.8802, 0.1198])
[]: # you can access individual parameters as follows
     logreg.get_parameter("0_bias")
[]: Parameter containing:
     tensor([-0.5327, -1.3804], requires_grad=True)
[]: # or all of them at once
     list(logreg.named_parameters())
[]: [('0_weight',
      Parameter containing:
      tensor([[ 0.6674, 0.1922],
               [ 0.1847, 0.3390],
               [-1.1552, -0.6920]], requires_grad=True)),
      ('0 bias',
      Parameter containing:
      tensor([0.2366, 0.1139], requires grad=True))]
```

2.2 1b MLP

```
[]: class MLP(nn.Module):
         """A fully-connected MLP.
         Parameters
         sizes Contains the layer sizes. The first entry is the number of inputs, \Box
      \hookrightarrow the last
         entry the number of outputs. All entries in between correspond to the ⊔
      ⇔number of
         units in the respective hidden layer. E.q., [2,5,7,1] means: 2 inputs -> 5D_{\sqcup}
         layer -> 7D hidden layer -> 1 output.
         phi Activation function used in every hidden layer (the output layer is \Box
      \hookrightarrow linear).
         11 11 11
         def __init__(self, sizes: list[int], phi=F.sigmoid):
             super().__init__()
              # let's remember the specification in this model
             self.sizes = sizes
             self.phi = phi
             for i in range(len(sizes) -1):
                  W = torch.randn(sizes[i], sizes[i+1]) / math.sqrt(sizes[i])
                  self.register_parameter(f"{i}_weight", nn.Parameter(W))
                  b = torch.randn(sizes[i+1])
                  self.register_parameter(f"{i}_bias", nn.Parameter(b))
         def num_layers(self):
              """Number of layers (excluding input layer)"""
```

```
return len(self.sizes) - 1
         def forward(self, x):
             out = x
             # iterate through each layer
             for i in range(self.num_layers()):
                 # extract weight matrix ad bias from current layer
                 W = getattr(self, f"{i}_weight")
                 b = getattr(self, f"{i}_bias")
                 # apply matrix multiplication
                 out = torch.matmul(out, W) + b
                 # apply activation function to every layer besides the output layer
                 if i < self.num_layers() - 1:</pre>
                     out = self.phi(out)
             return out
[]: # here you should see the correct parameter sizes
     mlp = MLP([2, 3, 4, 2], torch.relu)
     list(mlp.named_parameters())
[]: [('0_weight',
      Parameter containing:
      tensor([[ 0.0835, 0.3357, -0.1311],
               [ 0.2920, 0.3099, -0.4164]], requires_grad=True)),
      ('0_bias',
      Parameter containing:
      tensor([ 0.7342, -0.8138,  0.7079], requires_grad=True)),
      ('1_weight',
      Parameter containing:
      tensor([[-0.5385, 0.4991, 0.0204, -0.0382],
               [-0.3959, 0.0521, 0.1780, 0.6273],
               [ 0.4448, 0.4533, -0.1692, 0.7084]], requires_grad=True)),
      ('1_bias',
      Parameter containing:
      tensor([-0.1616, -1.4976, -1.5695, -0.4077], requires grad=True)),
      ('2_weight',
      Parameter containing:
      tensor([[ 0.0626, -0.9391],
               [0.2034, -0.2423],
               [-0.0426, -0.5341],
               [ 0.5613, -0.3643]], requires_grad=True)),
      ('2_bias',
      Parameter containing:
       tensor([-0.8349, -0.2435], requires_grad=True))]
```

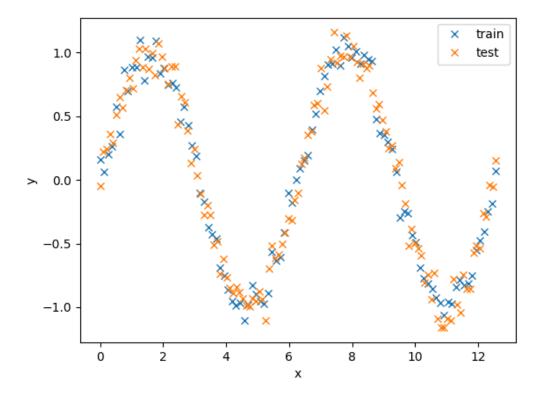
```
[]: # Test your code; we fix the parameters and check the result
    with torch.no_grad():
        torch.manual_seed(0)
        for 1 in range(mlp.num_layers()):
             W, b = mlp.get_parameter(f"{1}_weight"), mlp.get_parameter(f"{1}_bias")
            W[:] = torch.randn(W.shape)
            b[:] = torch.randn(b.shape)
    mlp(torch.tensor([-1.0, 2.0])) # must give: [ 0.8315, -3.6792]
[]: tensor([0.8315, -3.6792], grad_fn=<AddBackward0>)
[]: # You can also evaluate your model on multiple inputs at once. Here "torch.func.
     ⇔υmap"
     # produces a function that applies the provided function (mlp#forward) to each
    # its argument (torch.tensor...).
    # [[ 0.8315, -3.6792],
     # [ 4.8448, -6.8813]]
    torch.func.vmap(mlp)(torch.tensor([[-1.0, 2.0], [1.0, -2.0]]))
[]: tensor([[ 0.8315, -3.6792],
             [ 4.8448, -6.8813]], grad_fn=<AddBackward0>)
    2.3 1c Batching
[]: # After you adapted the MLP class, you should get the same results as above.
    mlp(torch.tensor([-1.0, 2.0])) # must give: [ 0.8315, -3.6792]
[]: tensor([0.8315, -3.6792], grad_fn=<AddBackward0>)
[]: # Now without umap. Only proceed to task 2 once this works correctly.
     # [[ 0.8315, -3.6792],
     # [ 4.8448, -6.8813]]
    mlp(torch.tensor([[-1.0, 2.0], [1.0, -2.0]]))
[]: tensor([[ 0.8315, -3.6792],
             [ 4.8448, -6.8813]], grad_fn=<AddBackward0>)
```

3 2 Multi-Layer Feed-Forward Neural Networks

3.1 2a Conjecture how an FNN fit will look like

```
[]: # here is the one-dimensional dataset that we will use
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plt.legend()
```

[]: <matplotlib.legend.Legend at 0x13f8c9b10>



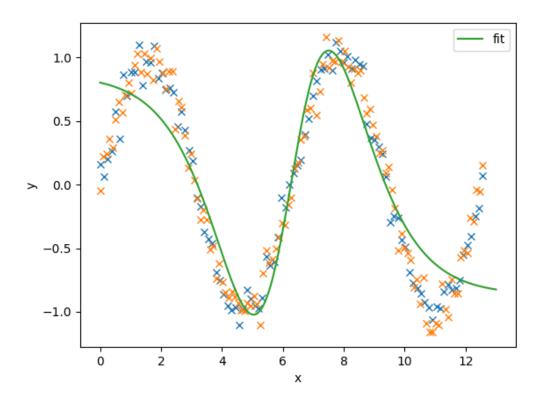
- zero: essentially, a linear regression function. Line Would go through the middle.
- one:
- two:
- three:

3.2 2b Train with 2 hidden units

```
[]: # Training code. You do not need to modify this code.
           train_bfgs = lambda model, **kwargs: train_scipy(X1, y1, model, **kwargs)
           def train1(hidden_sizes, nreps=10, phi=F.sigmoid, train=train_bfgs, **kwargs):
                    """Train an FNN.
                    hidden_sizes is a (possibly empty) list containing the sizes of the hidden of the hid
             \hookrightarrow layer(s).
                    nreps refers to the number of repetitions.
                    best_model = None
                    best_cost = math.inf
                    for rep in range(nreps):
                             model = MLP([1] + hidden_sizes + [1], phi) # that's your model!
                             print(f"X1 shape: {X1.shape}")
                             print(f"Repetition {rep: 2d}: ", end="")
                             model = train(model, **kwargs)
                             mse = F.mse loss(y1, model(X1)).item()
                             if mse < best_cost:</pre>
                                      best model = model
                                      best_cost = mse
                             print(f"best_cost={best_cost:.3f}")
                    return best_model
[]: # Let's fit the model with one hidden layer consisting of 2 units.
           model = train1([2], nreps=10)
           print("Training error:", F.mse_loss(y1, model(X1)).item())
           print("Test error
                                                        :", F.mse_loss(y1test, model(X1test)).item())
         X1 shape: torch.Size([100, 1])
         Repetition 0:
                                                                Current function value: 0.079573
                              Iterations: 355
                             Function evaluations: 515
                             Gradient evaluations: 499
         best_cost=0.080
         X1 shape: torch.Size([100, 1])
         Repetition 1:
         /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
         OptimizeWarning: Desired error not necessarily achieved due to precision loss.
              res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
         OptimizeWarning: Desired error not necessarily achieved due to precision loss.
              res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
Current function value: 0.286909
         Iterations: 350
         Function evaluations: 470
         Gradient evaluations: 461
best cost=0.080
X1 shape: torch.Size([100, 1])
Repetition 2: Optimization terminated successfully.
         Current function value: 0.301865
         Iterations: 156
         Function evaluations: 196
         Gradient evaluations: 196
best_cost=0.080
X1 shape: torch.Size([100, 1])
                        Current function value: 0.079573
Repetition 3:
         Iterations: 340
         Function evaluations: 474
         Gradient evaluations: 462
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition 4:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.286909
         Iterations: 353
         Function evaluations: 582
         Gradient evaluations: 571
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition 5: Optimization terminated successfully.
         Current function value: 0.357250
         Iterations: 83
         Function evaluations: 89
         Gradient evaluations: 89
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition 6: Optimization terminated successfully.
         Current function value: 0.357250
         Iterations: 70
         Function evaluations: 78
         Gradient evaluations: 78
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition 7:
                      Current function value: 0.079572
```

```
Iterations: 432
             Function evaluations: 625
             Gradient evaluations: 609
    best_cost=0.080
    X1 shape: torch.Size([100, 1])
    Repetition 8:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.079573
             Iterations: 550
             Function evaluations: 867
             Gradient evaluations: 853
    best_cost=0.080
    X1 shape: torch.Size([100, 1])
    Repetition 9:
                            Current function value: 0.286909
             Iterations: 315
             Function evaluations: 458
             Gradient evaluations: 449
    best cost=0.080
    Training error: 0.0795721486210823
    Test error
                  : 0.08671437948942184
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: # plot the data and the fit
     nextplot()
     plot1(X1, y1, label="train")
     plot1(X1test, y1test, label="test")
     plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
```



```
[]: # The weight matrices and bias vectors can be read out as follows. If you want,
      чusе
     # these parameters to compute the output of the network (on X1) directly and
     ⇔compare to
     # vmap(model)(X1).
     for par, value in model.state_dict().items():
         print(f"{par:<15}= {value}")</pre>
    0_weight
                   = tensor([[1.0680, 1.0574]])
                   = tensor([-6.7078, -6.6409])
    0_bias
                   = tensor([[ 696.4487],
    1_weight
            [-698.1782]])
                   = tensor([0.8637])
    1_bias
[]: # now repeat this multiple times
     models = {
     # "model_1_1" : train1([0], nreps=1),
     # "model_1_1" : train1([0], nreps=10),
     # "model_1_1" : train1([1], nreps=1),
     # "model_1_10" : train1([1], nreps=10),
     # "model_2_1" : train1([2], nreps=1),
```

```
# "model_2_10" : train1([2], nreps=10),
"model_3_1" : train1([3], nreps=1),
"model_3_10" : train1([3], nreps=10)
X1 shape: torch.Size([100, 1])
Repetition 0:
                        Current function value: 0.049879
         Iterations: 445
         Function evaluations: 663
         Gradient evaluations: 652
best_cost=0.050
X1 shape: torch.Size([100, 1])
Repetition 0:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.049878
         Iterations: 527
         Function evaluations: 751
         Gradient evaluations: 740
best_cost=0.050
X1 shape: torch.Size([100, 1])
Repetition 1:
                        Current function value: 0.049877
         Iterations: 481
         Function evaluations: 650
         Gradient evaluations: 641
best_cost=0.050
X1 shape: torch.Size([100, 1])
Repetition 2:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.116200
         Iterations: 480
         Function evaluations: 603
         Gradient evaluations: 592
best_cost=0.050
```

```
X1 shape: torch.Size([100, 1])
                        Current function value: 0.007324
Repetition 3:
         Iterations: 338
         Function evaluations: 470
         Gradient evaluations: 458
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 4:
                        Current function value: 0.275464
         Iterations: 453
         Function evaluations: 604
         Gradient evaluations: 593
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 5:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.049878
         Iterations: 465
         Function evaluations: 614
         Gradient evaluations: 607
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 6:
                        Current function value: 0.049893
         Iterations: 300
         Function evaluations: 418
         Gradient evaluations: 407
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 7:
                        Current function value: 0.079572
         Iterations: 411
         Function evaluations: 554
         Gradient evaluations: 548
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 8:
                        Current function value: 0.051868
         Iterations: 269
         Function evaluations: 518
         Gradient evaluations: 504
best_cost=0.007
X1 shape: torch.Size([100, 1])
```

```
Repetition 9: Current function value: 0.079573
```

Iterations: 355

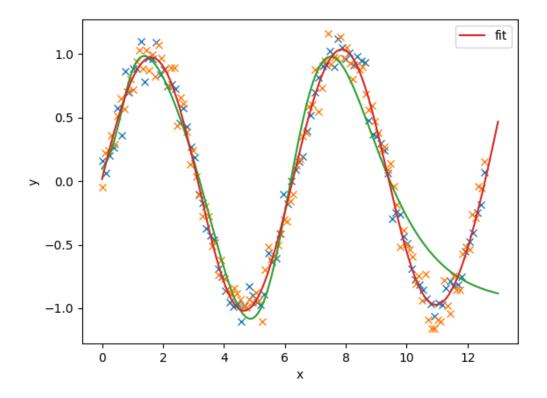
Function evaluations: 595 Gradient evaluations: 582

best_cost=0.007

/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

```
[]: nextplot()
  plot1(X1, y1, label="train")
  plot1(X1test, y1test, label="test")

for model in models.values():
    plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
```



3.3 2c Width

```
[]: # Experiment with different hidden layer sizes. To avoid recomputing
     # models, you may want to save your models using torch.save(model, filename) and
     # load them again using torch.load(filename).
     # Let's fit the model with one hidden layer consisting of 2 units.
     model1 = train1([1])
     print("Training error:", F.mse_loss(y1, model1(X1)).item())
     print("Test error :", F.mse_loss(y1test, model1(X1test)).item())
     torch.save(model1.state_dict(), "./models/model1.pt")
    X1 shape: torch.Size([100, 1])
    Repetition 0: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 41
             Function evaluations: 44
             Gradient evaluations: 44
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 1: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 37
             Function evaluations: 49
             Gradient evaluations: 49
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 2: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 40
             Function evaluations: 46
             Gradient evaluations: 46
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 3: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 44
             Function evaluations: 45
             Gradient evaluations: 45
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 4: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 42
             Function evaluations: 44
```

```
Gradient evaluations: 44
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 5: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 49
             Function evaluations: 53
             Gradient evaluations: 53
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 6: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 35
             Function evaluations: 40
             Gradient evaluations: 40
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 7: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 44
             Function evaluations: 47
             Gradient evaluations: 47
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 8: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 42
             Function evaluations: 47
             Gradient evaluations: 47
    best_cost=0.373
    X1 shape: torch.Size([100, 1])
    Repetition 9: Optimization terminated successfully.
             Current function value: 0.372919
             Iterations: 44
             Function evaluations: 52
             Gradient evaluations: 52
    best cost=0.373
    Training error: 0.37291884422302246
    Test error : 0.37431666254997253
[]: model2 = train1([2])
     print("Training error:", F.mse_loss(y1, model2(X1)).item())
     print("Test error :", F.mse_loss(y1test, model2(X1test)).item())
     torch.save(model2.state_dict(), "./models/model2.pt")
    X1 shape: torch.Size([100, 1])
```

Current function value: 0.079572

Repetition 0:

Iterations: 464 Function evaluations: 751 Gradient evaluations: 738 best cost=0.080 X1 shape: torch.Size([100, 1]) Repetition 1: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.079573 Iterations: 366 Function evaluations: 509 Gradient evaluations: 499 best cost=0.080 X1 shape: torch.Size([100, 1]) Repetition 2: Current function value: 0.079573 Iterations: 322 Function evaluations: 464 Gradient evaluations: 456 best_cost=0.080 X1 shape: torch.Size([100, 1]) Current function value: 0.079572 Repetition 3: Iterations: 390 Function evaluations: 601 Gradient evaluations: 590 best_cost=0.080 X1 shape: torch.Size([100, 1]) Repetition 4: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.079573 Iterations: 380 Function evaluations: 501

```
Gradient evaluations: 489
    best_cost=0.080
    X1 shape: torch.Size([100, 1])
    Repetition 5: Optimization terminated successfully.
             Current function value: 0.302737
             Iterations: 130
             Function evaluations: 145
             Gradient evaluations: 145
    best cost=0.080
    X1 shape: torch.Size([100, 1])
                            Current function value: 0.079573
    Repetition 6:
             Iterations: 365
             Function evaluations: 492
             Gradient evaluations: 480
    best_cost=0.080
    X1 shape: torch.Size([100, 1])
    Repetition 7:
                            Current function value: 0.079573
             Iterations: 388
             Function evaluations: 528
             Gradient evaluations: 519
    best cost=0.080
    X1 shape: torch.Size([100, 1])
    Repetition 8:
                            Current function value: 0.079573
             Iterations: 356
             Function evaluations: 528
             Gradient evaluations: 516
    best_cost=0.080
    X1 shape: torch.Size([100, 1])
    Repetition 9:
                            Current function value: 0.079573
             Iterations: 370
             Function evaluations: 494
             Gradient evaluations: 482
    best_cost=0.080
    Training error: 0.0795714259147644
    Test error
                  : 0.08671518415212631
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: model3= train1([3])
     print("Training error:", F.mse_loss(y1, model3(X1)).item())
```

```
print("Test error :", F.mse loss(y1test, model3(X1test)).item())
torch.save(model3.state_dict(), "./models/model3.pt")
X1 shape: torch.Size([100, 1])
Repetition 0:
                        Current function value: 0.049877
         Iterations: 484
         Function evaluations: 642
         Gradient evaluations: 632
best_cost=0.050
X1 shape: torch.Size([100, 1])
Repetition 1:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.049877
         Iterations: 584
         Function evaluations: 784
         Gradient evaluations: 772
best_cost=0.050
X1 shape: torch.Size([100, 1])
Repetition 2:
                        Current function value: 0.008496
         Iterations: 456
         Function evaluations: 685
         Gradient evaluations: 668
best_cost=0.008
X1 shape: torch.Size([100, 1])
Repetition 3:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.053382
         Iterations: 511
         Function evaluations: 759
         Gradient evaluations: 748
best_cost=0.008
X1 shape: torch.Size([100, 1])
                        Current function value: 0.007324
Repetition 4:
         Iterations: 595
         Function evaluations: 806
```

Gradient evaluations: 792 best_cost=0.007 X1 shape: torch.Size([100, 1]) Repetition 5: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.371771 Iterations: 327 Function evaluations: 470 Gradient evaluations: 458 best_cost=0.007 X1 shape: torch.Size([100, 1]) Repetition 6: Current function value: 0.007792 Iterations: 389 Function evaluations: 608 Gradient evaluations: 593 best_cost=0.007 X1 shape: torch.Size([100, 1]) Current function value: 0.051894 Repetition 7: Iterations: 419 Function evaluations: 539 Gradient evaluations: 527 best_cost=0.007 X1 shape: torch.Size([100, 1]) Current function value: 0.049877 Repetition 8: Iterations: 502 Function evaluations: 628 Gradient evaluations: 617 best cost=0.007 X1 shape: torch.Size([100, 1]) Current function value: 0.007324 Repetition 9: Iterations: 354 Function evaluations: 451 Gradient evaluations: 439 best_cost=0.007 Training error: 0.007324240170419216 Test error : 0.010335271246731281

/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss.

```
res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: model10= train1([10])
     print("Training error:", F.mse_loss(y1, model10(X1)).item())
     print("Test error :", F.mse_loss(y1test, model10(X1test)).item())
     torch.save(model10.state_dict(), "./models/model10.pt")
    X1 shape: torch.Size([100, 1])
    Repetition 0:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.006654
             Iterations: 1923
             Function evaluations: 2408
             Gradient evaluations: 2393
    best cost=0.007
    X1 shape: torch.Size([100, 1])
    Repetition 1:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.005709
             Iterations: 2753
             Function evaluations: 3149
             Gradient evaluations: 3136
    best cost=0.006
    X1 shape: torch.Size([100, 1])
    Repetition 2:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.006452
             Iterations: 3022
             Function evaluations: 3425
             Gradient evaluations: 3414
    best_cost=0.006
    X1 shape: torch.Size([100, 1])
```

```
Repetition 3:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.005936
         Iterations: 2293
         Function evaluations: 2601
         Gradient evaluations: 2590
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 4:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = minimize bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.006493
         Iterations: 2748
         Function evaluations: 3173
         Gradient evaluations: 3157
best cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 5:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.006827
         Iterations: 1563
         Function evaluations: 1819
         Gradient evaluations: 1807
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 6:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.006237
         Iterations: 3076
         Function evaluations: 3478
         Gradient evaluations: 3467
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 7:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
```

```
res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.006660
             Iterations: 1110
             Function evaluations: 1305
             Gradient evaluations: 1294
    best_cost=0.006
    X1 shape: torch.Size([100, 1])
    Repetition 8:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.006638
             Iterations: 1792
             Function evaluations: 2141
             Gradient evaluations: 2130
    best_cost=0.006
    X1 shape: torch.Size([100, 1])
    Repetition 9:
                            Current function value: 0.006652
             Iterations: 863
             Function evaluations: 1093
             Gradient evaluations: 1081
    best_cost=0.006
    Training error: 0.005708649288862944
    Test error
                  : 0.023016968742012978
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: model50= train1([50])
     print("Training error:", F.mse_loss(y1, model50(X1)).item())
     print("Test error :", F.mse_loss(y1test, model50(X1test)).item())
     torch.save(model50.state_dict(), "./models/model50.pt")
    X1 shape: torch.Size([100, 1])
    Repetition 0:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.004760
             Iterations: 3795
             Function evaluations: 4283
             Gradient evaluations: 4272
    best_cost=0.005
    X1 shape: torch.Size([100, 1])
```

```
Repetition 1:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.004108
         Iterations: 5131
         Function evaluations: 5687
         Gradient evaluations: 5676
best_cost=0.004
X1 shape: torch.Size([100, 1])
Repetition 2:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = minimize bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.004358
         Iterations: 4048
         Function evaluations: 4494
         Gradient evaluations: 4486
best cost=0.004
X1 shape: torch.Size([100, 1])
Repetition 3:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.002975
         Iterations: 6598
         Function evaluations: 7335
         Gradient evaluations: 7322
best_cost=0.003
X1 shape: torch.Size([100, 1])
Repetition 4:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.003348
         Iterations: 4949
         Function evaluations: 5502
         Gradient evaluations: 5492
best_cost=0.003
X1 shape: torch.Size([100, 1])
Repetition 5:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Maximum number of iterations has been exceeded.
```

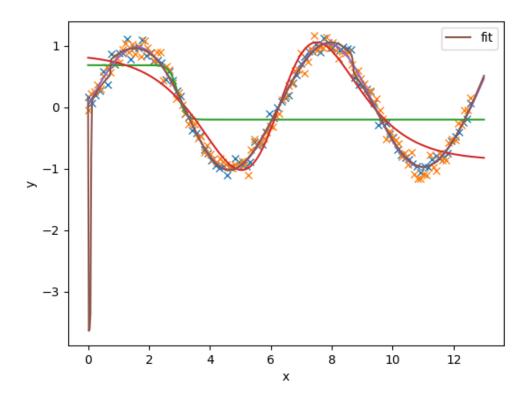
```
res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.004299
         Iterations: 10000
         Function evaluations: 10844
         Gradient evaluations: 10844
best_cost=0.003
X1 shape: torch.Size([100, 1])
Repetition 6:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.003333
         Iterations: 5263
         Function evaluations: 5736
         Gradient evaluations: 5726
best_cost=0.003
X1 shape: torch.Size([100, 1])
Repetition 7:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.004762
         Iterations: 2570
         Function evaluations: 2906
         Gradient evaluations: 2894
best_cost=0.003
X1 shape: torch.Size([100, 1])
Repetition 8:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.003223
         Iterations: 7202
         Function evaluations: 7949
         Gradient evaluations: 7930
best_cost=0.003
X1 shape: torch.Size([100, 1])
Repetition 9:
                        Current function value: 0.004972
         Iterations: 3782
         Function evaluations: 4249
         Gradient evaluations: 4238
best_cost=0.003
Training error: 0.002975039416924119
Test error : 0.068841353058815
```

```
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: model100= train1([100])
     print("Training error:", F.mse_loss(y1, model100(X1)).item())
     print("Test error :", F.mse_loss(y1test, model100(X1test)).item())
     torch.save(model100.state_dict(), "./models/model100.pt")
    X1 shape: torch.Size([100, 1])
    Repetition 0:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.004656
             Iterations: 3504
             Function evaluations: 3996
             Gradient evaluations: 3984
    best cost=0.005
    X1 shape: torch.Size([100, 1])
    Repetition 1:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.002520
             Iterations: 8639
             Function evaluations: 9477
             Gradient evaluations: 9464
    best_cost=0.003
    X1 shape: torch.Size([100, 1])
    Repetition 2:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.003220
             Iterations: 5656
             Function evaluations: 6289
             Gradient evaluations: 6275
    best_cost=0.003
    X1 shape: torch.Size([100, 1])
    Repetition 3:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Maximum number of iterations has been exceeded.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
Current function value: 0.001788
         Iterations: 10000
         Function evaluations: 10769
         Gradient evaluations: 10769
best cost=0.002
X1 shape: torch.Size([100, 1])
Repetition 4:
                        Current function value: 0.002305
         Iterations: 10000
         Function evaluations: 10653
         Gradient evaluations: 10653
best_cost=0.002
X1 shape: torch.Size([100, 1])
Repetition 5:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.003947
         Iterations: 3706
         Function evaluations: 4161
         Gradient evaluations: 4149
best_cost=0.002
X1 shape: torch.Size([100, 1])
Repetition 6:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.003632
         Iterations: 5133
         Function evaluations: 5907
         Gradient evaluations: 5894
best_cost=0.002
X1 shape: torch.Size([100, 1])
Repetition 7:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.003517
         Iterations: 9020
         Function evaluations: 9972
         Gradient evaluations: 9961
best cost=0.002
X1 shape: torch.Size([100, 1])
Repetition 8:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
```

```
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.001982
             Iterations: 9906
             Function evaluations: 10761
             Gradient evaluations: 10749
    best cost=0.002
    X1 shape: torch.Size([100, 1])
    Repetition 9:
                            Current function value: 0.002907
             Iterations: 7061
             Function evaluations: 7670
             Gradient evaluations: 7660
    best_cost=0.002
    Training error: 0.0017877757782116532
    Test error
                  : 0.7997748851776123
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: model1 = (MLP([1] + [1] + [1], F.sigmoid))
     model1.load_state_dict(torch.load('./models/model1.pt'))
     model2 = (MLP([1] + [2] + [1], F.sigmoid))
     model2.load_state_dict(torch.load('./models/model2.pt'))
     model3 = (MLP([1] + [3] + [1], F.sigmoid))
     model3.load_state_dict(torch.load('./models/model3.pt'))
     model10 = (MLP([1] + [10] + [1], F.sigmoid))
     model10.load_state_dict(torch.load('./models/model10.pt'))
     model50 = (MLP([1] + [50] + [1], F.sigmoid))
     model50.load_state_dict(torch.load('./models/model50.pt'))
     model100 = (MLP([1] + [100] + [1], F.sigmoid))
     model100.load_state_dict(torch.load('./models/model100.pt'))
[]: <All keys matched successfully>
[]: model1
[]: MLP()
[]: nextplot()
     plot1(X1, y1, label="train")
     plot1(X1test, y1test, label="test")
```

```
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model1)
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model2)
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model3)
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model10)
```



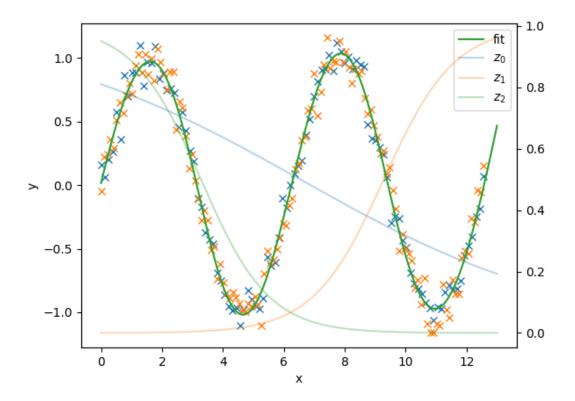
3.4 2d Distributed representations

OptimizeWarning: Desired error not necessarily achieved due to precision loss.

```
res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.051086
         Iterations: 474
         Function evaluations: 637
         Gradient evaluations: 627
best cost=0.050
X1 shape: torch.Size([100, 1])
Repetition 2:
                        Current function value: 0.007324
         Iterations: 559
         Function evaluations: 779
         Gradient evaluations: 766
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 3:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.049882
         Iterations: 283
         Function evaluations: 463
         Gradient evaluations: 446
best_cost=0.007
X1 shape: torch.Size([100, 1])
                        Current function value: 0.007324
Repetition 4:
         Iterations: 672
         Function evaluations: 875
         Gradient evaluations: 862
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition 5:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.007324
         Iterations: 554
         Function evaluations: 688
```

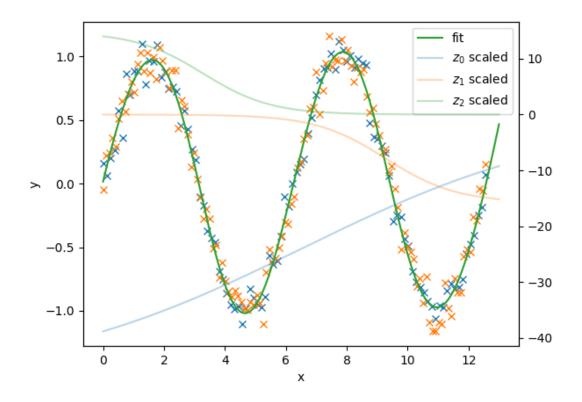
```
Gradient evaluations: 676
    best_cost=0.007
    X1 shape: torch.Size([100, 1])
    Repetition 6:
                            Current function value: 0.007324
             Iterations: 313
             Function evaluations: 530
             Gradient evaluations: 516
    best_cost=0.007
    X1 shape: torch.Size([100, 1])
                            Current function value: 0.007324
    Repetition 7:
             Iterations: 295
             Function evaluations: 407
             Gradient evaluations: 395
    best_cost=0.007
    X1 shape: torch.Size([100, 1])
    Repetition 8:
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
             Current function value: 0.007324
             Iterations: 394
             Function evaluations: 523
             Gradient evaluations: 510
    best_cost=0.007
    X1 shape: torch.Size([100, 1])
                            Current function value: 0.079573
    Repetition 9:
             Iterations: 406
             Function evaluations: 555
             Gradient evaluations: 543
    best_cost=0.007
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
[]: | # plot the fit as well as the outputs of each neuron in the hidden
     # layer (scale for the latter is shown on right y-axis)
     nextplot()
     plot1(X1, y1, label="train")
     plot1(X1test, y1test, label="test")
```

```
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model, hidden=True, uscale=False)
```



```
[]: # plot the fit as well as the outputs of each neuron in the hidden layer, scaled
    # by its weight for the output neuron (scale for the latter is shown on right
    # y-axis)

nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model, hidden=True, usecale=True)
```



3.5 2e Experiment with different optimizers (optional)

```
[]: # PyTorch provides many gradient-based optimizers; see
     # https://pytorch.org/docs/stable/optim.html. You can use a PyTorch optimizer
     # as follows.
     train_adam = lambda model, **kwargs: fnn_train(
         X1, y1, model, optimizer=torch.optim.Adam(model.parameters(), lr=0.01), u
      →**kwargs
    model = train1([50], nreps=1, train=train_adam, max_epochs=5000, tol=1e-8,__
      ⇔verbose=True)
    X1 shape: torch.Size([100, 1])
    Repetition 0: Epoch
                             0: cost=
                                         1.293
    Epoch
              1: cost=
                         0.855
    Epoch
                         0.581
              2: cost=
    Epoch
              3: cost=
                         0.463
    Epoch
              4: cost=
                         0.468
    Epoch
              5: cost=
                         0.539
    Epoch
              6: cost=
                         0.612
    Epoch
              7: cost=
                         0.646
```

Epoch	8:	cost=	0.633
Epoch	9:	cost=	0.589
Epoch	10:	cost=	0.533
Epoch	11:	cost=	0.483
Epoch	12:	cost=	0.451
Epoch	13:	cost=	0.440
Epoch	14:	cost=	0.448
Epoch	15:	cost=	0.467
Epoch	16:	cost=	0.486
Epoch	17:	cost=	0.498
Epoch	18:	cost=	0.499
Epoch	19:	cost=	0.489
Epoch	20:	cost=	0.473
Epoch	21:	cost=	0.456
Epoch	22:	cost=	0.442
Epoch	23:	cost=	0.435
Epoch	24:	cost=	0.435
Epoch	25:	cost=	0.440
Epoch	26:	cost=	0.447
Epoch	27:	cost=	0.452
Epoch	28:	cost=	0.454
Epoch	29:	cost=	0.451
Epoch	30:	cost=	0.445
Epoch	31:	cost=	0.438
Epoch	32:	cost=	0.432
Epoch	33:	cost=	0.429
Epoch	34:	cost=	0.428
Epoch	35:	cost=	0.430
Epoch	36:	cost=	0.432
Epoch	37:	cost=	0.433
Epoch	38:	cost=	0.434
Epoch	39:	cost=	0.432
Epoch	40:	cost=	0.429
Epoch	41:	cost=	0.426
Epoch	42:	cost=	0.424
Epoch	43:	cost=	0.422
Epoch	44:	cost=	0.422
Epoch	45:	cost=	0.423
Epoch	46:	cost=	0.423
Epoch	47:	cost=	0.424
Epoch	48:	cost=	0.423
Epoch	49:	cost=	0.421
Epoch	50:	cost=	0.420
Epoch	51:	cost=	0.418
Epoch	52:	cost=	0.417
Epoch	53:	cost=	0.417
Epoch	54:	cost=	0.417
Epoch	55:	cost=	0.417

Epoch	56:	cost=	0.416
Epoch	57:	cost=	0.416
Epoch	58:	cost=	0.415
Epoch	59:	cost=	0.414
Epoch	60:	cost=	0.413
Epoch	61:	cost=	0.412
Epoch	62:	cost=	0.412
Epoch	63:	cost=	0.411
Epoch	64:	cost=	0.411
Epoch	65:	cost=	0.410
Epoch	66:	cost=	0.410
Epoch	67:	cost=	0.409
Epoch	68:	cost=	0.408
Epoch	69:	cost=	0.408
Epoch	70:	cost=	0.407
Epoch	71:	cost=	0.407
Epoch	72:	cost=	0.406
Epoch	73:	cost=	0.405
Epoch	74:	cost=	0.405
Epoch	75:	cost=	0.404
Epoch	76:	cost=	0.403
Epoch	77:	cost=	0.403
Epoch	78:	cost=	0.402
Epoch	79:	cost=	0.402
Epoch	80:	cost=	0.401
Epoch	81:	cost=	0.401
Epoch	82:	cost=	0.400
Epoch	83:	cost=	0.399
Epoch	84:	cost=	0.399
Epoch	85:	cost=	0.398
Epoch	86:	cost=	0.397
Epoch	87:	cost=	0.397
Epoch	88:	cost=	0.396
Epoch	89:	cost=	0.396
Epoch	90:	cost=	0.395
Epoch	91:	cost=	0.395
Epoch	92:	cost=	0.394
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Epoch	94:	cost=	0.393
Epoch	95:	cost=	0.392
Epoch	96:	cost=	0.392
Epoch	97:	cost=	0.391
Epoch	98:	cost=	0.391
Epoch	99:	cost=	0.390
Epoch	100:	cost=	0.390
Epoch	101:	cost=	0.389
Epoch	102:	cost=	0.389
Epoch	103:	cost=	0.388

Epoch	104:	cost=	0.388
Epoch	105:	cost=	0.387
Epoch	106:	cost=	0.387
Epoch	107:	cost=	0.386
Epoch	108:	cost=	0.386
Epoch	109:	cost=	0.385
Epoch	110:	cost=	0.385
Epoch	111:	cost=	0.385
Epoch	112:	cost=	0.384
Epoch	113:	cost=	0.384
Epoch	114:	cost=	0.383
Epoch	115:	cost=	0.383
Epoch	116:	cost=	0.382
Epoch	117:	cost=	0.382
Epoch	118:	cost=	0.382
Epoch	119:	cost=	0.381
Epoch	120:	cost=	0.381
Epoch	121:	cost=	0.381
Epoch	122:	cost=	0.380
Epoch	123:	cost=	0.380
Epoch	124:	cost=	0.379
Epoch	125:	cost=	0.379
Epoch	126:	cost=	0.379
Epoch	127:	cost=	0.378
Epoch	128:	cost=	0.378
Epoch	129:	cost=	0.378
Epoch	130:	cost=	0.377
Epoch	131:	cost=	0.377
Epoch	132:	cost=	0.377
Epoch	133:	cost=	0.376
Epoch	134:	cost=	0.376
Epoch	135:	cost=	0.376
Epoch	136:	cost=	0.375
Epoch	137:	cost=	0.375
Epoch	138:	cost=	0.375
Epoch	139:	cost=	0.375
Epoch	140:	cost=	0.374
Epoch	141:	cost=	0.374
Epoch	142:	cost=	0.374
Epoch	143:	cost=	0.373
Epoch	144:	cost=	0.373
Epoch	145:	cost=	0.373
Epoch	146:	cost=	0.372
Epoch	147:	cost=	0.372
Epoch	148:	cost=	0.372
Epoch	149:	cost=	0.371
Epoch	150:	cost=	0.371
Epoch	151:	cost=	0.371

Epoch	152:	cost=	0.370
Epoch	153:	cost=	0.370
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Epoch	181:	cost=	0.362
Epoch	182:	cost=	0.361
Epoch	183:	cost=	0.361
Epoch	184:	cost=	0.361
Epoch	185:	cost=	0.361
Epoch	186:	cost=	0.360
Epoch	187:	cost=	0.360
Epoch	188:	cost=	0.360
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Epoch	192:	cost=	0.358
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Epoch	198:	cost=	0.357
Epoch	199:	cost=	0.356
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Epoch	200:	cost=	0.356
Epoch	201:	cost=	0.356
Epoch	202:	cost=	0.355
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Epoch	228:	cost=	0.346
Epoch	229:	cost=	0.346
Epoch	230:	cost=	0.345
Epoch	231:	cost=	0.345
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Epoch	233:	cost=	0.344
Epoch	234:	cost=	0.344
Epoch	235:	cost=	0.343
Epoch	236:	cost=	0.343
Epoch	237:	cost=	0.343
Epoch	238:	cost=	0.342
Epoch	239:	cost=	0.342
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Epoch		cost=	0.341
Epoch	241:	cost=	
Epoch	242:	cost=	0.341
Epoch	243:	cost=	0.340
Epoch	244:	cost=	0.340
Epoch	245:	cost=	0.339
Epoch	246:	cost=	0.339
Epoch	247:	cost=	0.338

Epoch	248:	cost=	0.338
Epoch	249:	cost=	0.338
Epoch	250:	cost=	0.337
Epoch	251:	cost=	0.337
Epoch	252:	cost=	0.336
Epoch	253:	cost=	0.336
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Epoch	260:	cost=	0.333
Epoch	261:	cost=	0.332
Epoch	262:	cost=	0.332
Epoch	263:	cost=	0.331
Epoch	264:	cost=	0.331
Epoch	265:	cost=	0.330
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Epoch	270:	cost=	0.327
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Epoch	296:	cost=	0.312
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Epoch	309:	cost=	0.303
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Epoch	312:	cost=	0.301
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Epoch	336:	cost=	0.282
Epoch	337:	cost=	0.281
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Epoch	342:	cost=	0.277
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Epoch	344:	cost=	0.275
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Epoch	361:	cost=	0.259
Epoch	362:	cost=	0.258
Epoch	363:	cost=	0.256
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Epoch	367:	cost=	0.252
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Epoch	372:	cost=	0.247
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Epoch	387:	cost=	0.231
Epoch	388:	cost=	0.230
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Epoch	390:	cost=	0.228
Epoch	391:	cost=	0.226
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Epoch
        392: cost=
                       0.225
Epoch
                       0.224
        393: cost=
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        394: cost=
                       0.223
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        395: cost=
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        396: cost=
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        397: cost=
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        398: cost=
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        418: cost=
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Epoch
        440: cost=
                       0.171
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        441: cost=
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        442: cost=
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        444: cost=
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Epoch	3656:	cost=	0.013
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Epoch 3706: cost=
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    Epoch 3707: cost=
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    Epoch 3708: cost=
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    Epoch 3709: cost=
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    Epoch 3710: cost=
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    Epoch 3736: cost=
                         0.008
    best_cost=0.008
[]: # Experiment with different number of layers and activation functions. Here is
     # an example with three hidden layers (of sizes 4, 5, and 6) and ReLU_{\sqcup}
     \hookrightarrow activations.
     # You can also plot the outputs of the hidden neurons in the first layer (using
     # the same code above).
     model = train1([4, 5, 6], nreps=50, phi=F.relu)
     nextplot()
     plot1(X1, y1, label="train")
     plot1(X1test, y1test, label="test")
     plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
     print("Training error:", F.mse_loss(y1, model(X1)).item())
```

Epoch 3704: cost=

Epoch 3705: cost=

0.008

0.008

```
print("Test error
                     :", F.mse_loss(y1test, model(X1test)).item())
X1 shape: torch.Size([100, 1])
Repetition 0:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.081754
         Iterations: 197
         Function evaluations: 330
         Gradient evaluations: 324
best_cost=0.082
X1 shape: torch.Size([100, 1])
                        Current function value: 0.309341
Repetition 1:
         Iterations: 47
         Function evaluations: 174
         Gradient evaluations: 166
best_cost=0.082
X1 shape: torch.Size([100, 1])
Repetition 2:
                        Current function value: 0.021929
         Iterations: 48
         Function evaluations: 133
         Gradient evaluations: 127
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 3:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.082625
         Iterations: 94
         Function evaluations: 176
         Gradient evaluations: 169
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 4:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.089863
         Iterations: 120
         Function evaluations: 226
         Gradient evaluations: 220
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 5:
                        Current function value: 0.082474
         Iterations: 39
         Function evaluations: 129
         Gradient evaluations: 126
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 6:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.098277
         Iterations: 132
         Function evaluations: 207
         Gradient evaluations: 203
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 7:
                        Current function value: 0.098335
         Iterations: 99
         Function evaluations: 203
         Gradient evaluations: 198
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 8:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.084660
         Iterations: 63
         Function evaluations: 150
         Gradient evaluations: 143
best_cost=0.022
X1 shape: torch.Size([100, 1])
Repetition 9:
```

```
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.008690
         Iterations: 159
         Function evaluations: 265
         Gradient evaluations: 258
best_cost=0.009
X1 shape: torch.Size([100, 1])
Repetition 10:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.011249
         Iterations: 201
         Function evaluations: 311
         Gradient evaluations: 309
best cost=0.009
X1 shape: torch.Size([100, 1])
Repetition 11:
                         Current function value: 0.356422
         Iterations: 75
         Function evaluations: 170
         Gradient evaluations: 166
best_cost=0.009
X1 shape: torch.Size([100, 1])
Repetition 12:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.356366
         Iterations: 63
         Function evaluations: 157
         Gradient evaluations: 152
best_cost=0.009
X1 shape: torch.Size([100, 1])
Repetition 13:
                         Current function value: 0.082331
         Iterations: 81
         Function evaluations: 196
         Gradient evaluations: 190
```

```
best_cost=0.009
X1 shape: torch.Size([100, 1])
Repetition 14:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.008332
         Iterations: 214
         Function evaluations: 393
         Gradient evaluations: 381
best_cost=0.008
X1 shape: torch.Size([100, 1])
Repetition 15:
                         Current function value: 0.356475
         Iterations: 82
         Function evaluations: 181
         Gradient evaluations: 178
best cost=0.008
X1 shape: torch.Size([100, 1])
Repetition 16:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.011342
         Iterations: 188
         Function evaluations: 319
         Gradient evaluations: 307
best_cost=0.008
X1 shape: torch.Size([100, 1])
                         Current function value: 0.355779
Repetition 17:
         Iterations: 67
         Function evaluations: 164
         Gradient evaluations: 159
best_cost=0.008
X1 shape: torch.Size([100, 1])
Repetition 18:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.084557
```

Iterations: 96 Function evaluations: 230 Gradient evaluations: 223 best cost=0.008 X1 shape: torch.Size([100, 1]) Repetition 19: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.013663 Iterations: 89 Function evaluations: 205 Gradient evaluations: 193 best_cost=0.008 X1 shape: torch.Size([100, 1]) Repetition 20: Current function value: 0.355771 Iterations: 97 Function evaluations: 192 Gradient evaluations: 185 best_cost=0.008 X1 shape: torch.Size([100, 1]) Repetition 21: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.008507 Iterations: 199 Function evaluations: 414 Gradient evaluations: 402 best_cost=0.008 X1 shape: torch.Size([100, 1]) Current function value: 0.084680 Repetition 22: Iterations: 65 Function evaluations: 149 Gradient evaluations: 143 best cost=0.008 X1 shape: torch.Size([100, 1]) Repetition 23: Optimization terminated successfully. Current function value: 0.506238 Iterations: 2

```
Function evaluations: 4
        Gradient evaluations: 4
best_cost=0.008
X1 shape: torch.Size([100, 1])
Repetition 24:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.355875
         Iterations: 83
        Function evaluations: 189
        Gradient evaluations: 182
best_cost=0.008
X1 shape: torch.Size([100, 1])
Repetition 25:
                         Current function value: 0.006384
         Iterations: 112
        Function evaluations: 216
        Gradient evaluations: 209
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 26:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.356134
         Iterations: 58
        Function evaluations: 164
        Gradient evaluations: 156
best cost=0.006
X1 shape: torch.Size([100, 1])
                         Current function value: 0.357622
Repetition 27:
         Iterations: 36
        Function evaluations: 116
         Gradient evaluations: 110
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 28:
                        Current function value: 0.398953
```

Iterations: 20 Function evaluations: 115 Gradient evaluations: 109 best cost=0.006 X1 shape: torch.Size([100, 1]) Repetition 29: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.356162 Iterations: 45 Function evaluations: 156 Gradient evaluations: 154 best_cost=0.006 X1 shape: torch.Size([100, 1]) Repetition 30: Current function value: 0.355884 Iterations: 79 Function evaluations: 185 Gradient evaluations: 181 best_cost=0.006 X1 shape: torch.Size([100, 1]) Repetition 31: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss. res = _minimize_bfgs(fun, x0, args, jac, callback, **options) Current function value: 0.098431 Iterations: 51 Function evaluations: 134 Gradient evaluations: 128 best_cost=0.006 X1 shape: torch.Size([100, 1]) Current function value: 0.355989 Repetition 32: Iterations: 86 Function evaluations: 181 Gradient evaluations: 177 best cost=0.006 X1 shape: torch.Size([100, 1]) Repetition 33: /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:

```
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.006600
         Iterations: 307
         Function evaluations: 433
         Gradient evaluations: 421
best cost=0.006
X1 shape: torch.Size([100, 1])
                         Current function value: 0.357715
Repetition 34:
         Iterations: 28
         Function evaluations: 138
         Gradient evaluations: 131
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 35:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.355809
         Iterations: 94
         Function evaluations: 180
         Gradient evaluations: 176
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 36:
                         Current function value: 0.023239
         Iterations: 108
         Function evaluations: 223
         Gradient evaluations: 212
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 37:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.355985
         Iterations: 62
```

```
Function evaluations: 169
         Gradient evaluations: 166
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 38:
                         Current function value: 0.082410
         Iterations: 46
         Function evaluations: 133
         Gradient evaluations: 123
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 39:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.008140
         Iterations: 86
         Function evaluations: 172
         Gradient evaluations: 165
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 40: Optimization terminated successfully.
         Current function value: 0.506238
         Iterations: 4
         Function evaluations: 6
         Gradient evaluations: 6
best_cost=0.006
X1 shape: torch.Size([100, 1])
                         Current function value: 0.351567
Repetition 41:
         Iterations: 14
         Function evaluations: 112
         Gradient evaluations: 101
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 42:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/ minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.011727
         Iterations: 96
         Function evaluations: 177
         Gradient evaluations: 172
best_cost=0.006
X1 shape: torch.Size([100, 1])
```

```
Repetition 43:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.008865
         Iterations: 152
         Function evaluations: 273
         Gradient evaluations: 266
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 44:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.355520
         Iterations: 96
         Function evaluations: 201
         Gradient evaluations: 196
best cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 45:
                         Current function value: 0.356199
         Iterations: 74
         Function evaluations: 172
         Gradient evaluations: 166
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 46:
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
 res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.084039
         Iterations: 66
         Function evaluations: 130
         Gradient evaluations: 125
best cost=0.006
X1 shape: torch.Size([100, 1])
Repetition 47:
                         Current function value: 0.311066
         Iterations: 28
         Function evaluations: 134
```

Gradient evaluations: 130

best_cost=0.006

X1 shape: torch.Size([100, 1])

Repetition 48:

/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705: OptimizeWarning: Desired error not necessarily achieved due to precision loss.

res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

/opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
OptimizeWarning: Desired error not necessarily achieved due to precision loss.

res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

Current function value: 0.013557

Iterations: 162

Function evaluations: 311 Gradient evaluations: 302

best_cost=0.006

X1 shape: torch.Size([100, 1])

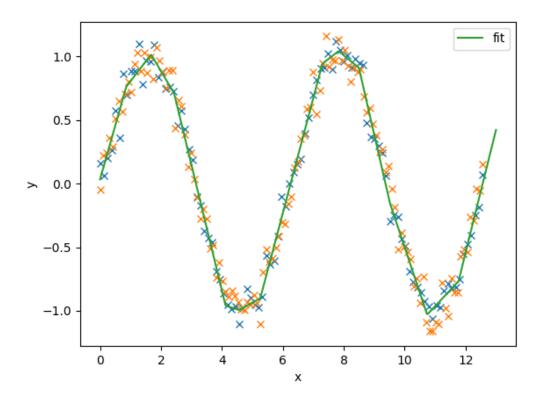
Repetition 49: Current function value: 0.356307

Iterations: 91

Function evaluations: 212 Gradient evaluations: 207

best_cost=0.006

Training error: 0.006383862346410751 Test error : 0.010709108784794807



4 3 Backpropagation

```
[]: # Let's fit the model with one hidden layer consisting of 50 units.
    model = train1([50], nreps=1)
    print("Training error:", F.mse_loss(y1, model(X1)).item())
                       :", F.mse_loss(y1test, model(X1test)).item())
    print("Test error
    # Extract parameters
    pars = dict(model.named_parameters())
    W1 = pars["0_weight"].data # 1x50
    b1 = pars["0_bias"].data # 50
    W2 = pars["1_weight"].data # 50x1
    b2 = pars["1_bias"].data # 1
    X1 shape: torch.Size([100, 1])
    Repetition 0:
                            Current function value: 0.004123
             Iterations: 4749
             Function evaluations: 5216
             Gradient evaluations: 5208
    best_cost=0.004
```

```
Training error: 0.004123299848288298
                  : 0.5570471286773682
    Test error
    /opt/homebrew/lib/python3.11/site-packages/scipy/optimize/_minimize.py:705:
    OptimizeWarning: Desired error not necessarily achieved due to precision loss.
      res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
    4.1 3a Forward pass
[]: # Compute results of forward pass on an example x (i.e., z1, z2, z3, z4, yhat, \Box
     →l) using Pytorch
    x = X1test[1, :]
    y = y1test[1, :]
    print(f"x={x}, y={y}, yhat={model(x).detach()}, l={torch.nn.}
      x=tensor([0.1030]), y=tensor([0.2253]), yhat=tensor([0.0573]),
    1=0.02819862589240074
[]: # Now do this by hand (including all intermediate values). You should get the
     # results as above.
    def compute loss(y, y hat):
        return torch.square(y - y_hat)
    # Compute the forward pass
    def forward_pass(x, y, W1, b1, W2, b2):
        z1 = torch.matmul(x, W1)
        z2 = z1 + b1
        z3 = torch.sigmoid(z2)
        z4 = torch.matmul(z3, W2)
        y_hat = z4 + b2
        loss = compute_loss(y, y_hat)
        return z1, z2, z3, z4, y_hat, loss
[]: z1, z2, z3, z4, y_hat, loss = forward_pass(x, y, W1, b1, W2, b2)
    print(f"x={x}, y={y}, yhat={y_hat}, l={loss}")
    x=tensor([0.1030]), y=tensor([0.2253]), yhat=tensor([0.0573]),
    l=tensor([0.0282])
    4.2 3b Backward pass
[]: # Define the sigmoid derivative function
    def sigmoid_derivative(z):
        return torch.sigmoid(z) * (1 - torch.sigmoid(z))
```

```
# Compute the backward pass
def backward_pass(x, y, y_hat, z1, z2, z3, z4, W1, W2):
    delta l = 1
    delta_y = 2 * (y - y_hat)
    delta_yhat = -delta_y
    delta_b2 = delta_yhat
    delta z4 = delta yhat
    delta_W2 = torch.matmul(z3.view(-1, 1), delta_z4.view(1, -1))
    delta z3 = torch.matmul(delta z4, W2.t())
    delta_z2 = delta_z3 * sigmoid_derivative(z2)
    delta_b1 = delta_z2
    delta_z1 = delta_z2
    delta_W1 = torch.matmul(x.view(-1, 1), delta_z1.view(1, -1))
    delta_x = torch.matmul(delta_z1, W1.t())
    return delta x, delta W1, delta z1, delta b1, delta z2, delta z3, delta W2, u
 →delta_z4, delta_b2, delta_yhat, delta_l, delta_y
# Compute the forward pass
z1, z2, z3, z4, y_hat, loss = forward_pass(x, y, W1, b1, W2, b2)
# Compute the backward pass
delta_x, delta_W1, delta_z1, delta_b1, delta_z2, delta_z3, delta_W2, delta_z4,_
 \Rightarrowz3, z4, W1, W2)
# Print the results
print(f"delta x={delta x}, delta W1={delta W1}, delta z1={delta z1},...
 delta_b1={delta_b1}, delta_z2={delta_z2}, delta_z3={delta_z3},__

delta_W2={delta_W2}, delta_z4={delta_z4}, delta_b2={delta_b2},

□

delta_yhat={delta_yhat}, delta_l={delta_l}, delta_y={delta_y}")

delta_x=tensor([-0.0546], grad_fn=<SqueezeBackward4>),
delta_W1=tensor([[-1.2498e-20, -3.3407e-11, -7.5456e-05, -3.1627e-03,
6.6566e-07,
         0.0000e+00, -6.9054e-09, -1.8054e-02, 5.9882e-04, -1.1690e-04,
        -2.6779e-22, -9.5940e-25, -1.5367e-04, 9.9927e-05, 0.0000e+00,
         2.3231e-25, 2.5774e-05, 1.9602e-04, -3.6733e-03, -1.2149e-03,
        -4.7099e-03, 6.3735e-05, 0.0000e+00, -8.7016e-03, 6.0340e-05,
         0.0000e+00, 0.0000e+00, -8.7417e-08, -2.9660e-05, -4.9669e-03,
        -8.8448e-07, -2.5948e-06, 0.0000e+00, 0.0000e+00, -3.1340e-02,
         4.9667e-03, -3.2181e-04, -2.4263e-02, 8.8048e-08, 8.4567e-06,
         1.8478e-11, -9.2940e-06, -3.9415e-02, -2.5835e-15, 0.0000e+00,
        -9.7880e-06, -6.3194e-03, 0.0000e+00, -1.5671e-05, -1.5880e-03]],
      grad_fn=<MmBackward0>), delta_z1=tensor([-1.2134e-19, -3.2433e-10,
-7.3256e-04, -3.0705e-02, 6.4626e-06,
       -0.0000e+00, -6.7041e-08, -1.7528e-01, 5.8137e-03, -1.1349e-03,
       -2.5998e-21, -9.3143e-24, -1.4919e-03, 9.7014e-04, 0.0000e+00,
```

```
2.2554e-24, 2.5023e-04, 1.9031e-03, -3.5662e-02, -1.1795e-02,
       -4.5726e-02, 6.1877e-04, -0.0000e+00, -8.4479e-02, 5.8581e-04,
       -0.0000e+00, 0.0000e+00, -8.4869e-07, -2.8796e-04, -4.8221e-02,
       -8.5869e-06, -2.5191e-05, -0.0000e+00, 0.0000e+00, -3.0426e-01,
        4.8219e-02, -3.1243e-03, -2.3556e-01, 8.5481e-07, 8.2101e-05,
        1.7939e-10, -9.0230e-05, -3.8265e-01, -2.5082e-14, -0.0000e+00,
       -9.5027e-05, -6.1352e-02, 0.0000e+00, -1.5214e-04, -1.5417e-02],
      grad_fn=<MulBackward0>), delta_b1=tensor([-1.2134e-19, -3.2433e-10,
-7.3256e-04, -3.0705e-02, 6.4626e-06,
       -0.0000e+00, -6.7041e-08, -1.7528e-01, 5.8137e-03, -1.1349e-03,
       -2.5998e-21, -9.3143e-24, -1.4919e-03, 9.7014e-04, 0.0000e+00,
        2.2554e-24, 2.5023e-04, 1.9031e-03, -3.5662e-02, -1.1795e-02,
       -4.5726e-02, 6.1877e-04, -0.0000e+00, -8.4479e-02, 5.8581e-04,
       -0.0000e+00, 0.0000e+00, -8.4869e-07, -2.8796e-04, -4.8221e-02,
       -8.5869e-06, -2.5191e-05, -0.0000e+00, 0.0000e+00, -3.0426e-01,
        4.8219e-02, -3.1243e-03, -2.3556e-01, 8.5481e-07, 8.2101e-05,
        1.7939e-10, -9.0230e-05, -3.8265e-01, -2.5082e-14, -0.0000e+00,
       -9.5027e-05, -6.1352e-02, 0.0000e+00, -1.5214e-04, -1.5417e-02],
      grad_fn=<MulBackward0>), delta_z2=tensor([-1.2134e-19, -3.2433e-10,
-7.3256e-04, -3.0705e-02, 6.4626e-06,
       -0.0000e+00, -6.7041e-08, -1.7528e-01, 5.8137e-03, -1.1349e-03,
       -2.5998e-21, -9.3143e-24, -1.4919e-03, 9.7014e-04, 0.0000e+00,
        2.2554e-24, 2.5023e-04, 1.9031e-03, -3.5662e-02, -1.1795e-02,
       -4.5726e-02, 6.1877e-04, -0.0000e+00, -8.4479e-02, 5.8581e-04,
       -0.0000e+00, 0.0000e+00, -8.4869e-07, -2.8796e-04, -4.8221e-02,
       -8.5869e-06, -2.5191e-05, -0.0000e+00, 0.0000e+00, -3.0426e-01,
        4.8219e-02, -3.1243e-03, -2.3556e-01, 8.5481e-07, 8.2101e-05,
        1.7939e-10, -9.0230e-05, -3.8265e-01, -2.5082e-14, -0.0000e+00,
       -9.5027e-05, -6.1352e-02, 0.0000e+00, -1.5214e-04, -1.5417e-02],
      grad_fn=<MulBackward0>), delta_z3=tensor([ -5.7655, -9.7092, -0.2775,
          3.5219, -4.6906, -3.8438,
-0.7629,
        -6.9920, 10.6412, -2.2282, -11.9009, -17.5671, -5.8399,
                                                                    8.3463,
        40.9113,
                 8.0653, 0.7241, 0.1432, -0.9021, -2.3441, -1.0778,
         7.0700, -0.3174, -1.1393, 0.5834, -1.9495, 3.8695, -7.1193,
        -3.5422, -2.3220, -11.8693, -2.1132, -5.8690, 3.8300, -2.1714,
         0.5442, -1.1922, -1.4804,
                                               0.0427, 11.6495, -0.6808,
                                      4.8671,
        -3.1774, -4.1564, -13.2173, -3.6154, -1.5305, 1.9429, -0.9462,
        -1.7004], grad_fn=<SqueezeBackward4>), delta_W2=tensor([[-7.0680e-21],
        [-1.1219e-11],
        [-3.3496e-01],
        [-1.4111e-02],
        [-6.1627e-07],
        [-3.3585e-01],
        [-5.8577e-09],
        [-8.6417e-03],
        [-3.3567e-01],
        [-3.3568e-01],
        [-7.3368e-23],
```

```
[-3.3576e-01],
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             [-1.3848e-02],
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             [-3.3585e-01],
             [-2.7087e-02],
             [-3.3760e-04],
             [-3.3585e-01],
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             [-3.3582e-01],
             [-3.2872e-01],
             [-2.4297e-07],
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             [-3.3585e-01],
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             [-5.6596e-02],
             [-3.0285e-01],
             [-3.3497e-01],
             [-6.6676e-02],
             [-5.8985e-08],
             [-6.4641e-04],
             [-5.1718e-12],
             [-3.3580e-01],
             [-4.7032e-02],
             [-2.0267e-15],
             [-3.3585e-01],
             [-8.8276e-06],
             [-3.2180e-01],
             [-3.3585e-01],
             [-3.3579e-01],
             [-3.3278e-01]], grad_fn=<MmBackward0>), delta_z4=tensor([-0.3358],
    grad_fn=<NegBackward0>), delta_b2=tensor([-0.3358], grad_fn=<NegBackward0>),
    delta_yhat=tensor([-0.3358], grad_fn=<NegBackward0>), delta_l=1,
    delta_y=tensor([0.3358], grad_fn=<MulBackward0>)
[]: # Use PyTorch's backprop
     x.requires_grad = True
     y.requires_grad = True
     if x.grad is not None:
```

[-1.7807e-25],

```
x.grad.zero_()
     if y.grad is not None:
         y.grad.zero_()
     model.zero_grad()
     t_yhat = model(x)
     t_yhat.retain_grad()
     t_l = torch.nn.MSELoss()(t_yhat, y)
     t_l.backward()
     t delta l = 1
     t_delta_y = y.grad
     t_delta_yhat = t_yhat.grad
     t_delta_b2 = model.get_parameter("1_bias").grad
     t_delta_W2 = model.get_parameter("1_weight").grad
     t_delta_b1 = model.get_parameter("0_bias").grad
     t_delta_W1 = model.get_parameter("0_weight").grad
     t_delta_x = x.grad
[]: # Check if equal (show squared error)
     for v in ["y", "yhat", "b2", "W2", "b1", "W1", "x"]:
         print(f'{v}, squared error={torch.

sum((eval("t_delta_"+v)-eval("delta_"+v))**2)}')
    y, squared error=0.0
    yhat, squared error=0.0
    b2, squared error=0.0
    W2, squared error=0.0
    b1, squared error=2.542219572231237e-16
    W1, squared error=3.809587755738482e-18
    x, squared error=1.4210854715202004e-14
[]: # Check if equal (show actual values)
     for v in ["1", "y", "yhat", "b2", "W2", "b1", "W1", "x"]:
         print(f'{v}, pytorch={eval("t_delta_"+v)}, you={eval("delta_"+v)}')
    1, pytorch=1, you=1
    y, pytorch=tensor([0.3358]), you=tensor([0.3358])
    yhat, pytorch=tensor([-0.3358]), you=tensor([-0.3358])
    b2, pytorch=tensor([-0.3358]), you=tensor([-0.3358])
    W2, pytorch=tensor([[-7.0680e-21],
            [-1.1219e-11],
            [-3.3496e-01],
            [-1.4111e-02],
            [-6.1627e-07],
            [-3.3585e-01],
            [-5.8577e-09],
            [-8.6417e-03],
            [-3.3567e-01],
            [-3.3568e-01],
```

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[-7.3368e-23],
[-1.7807e-25],
[-3.3576e-01],
[-3.9042e-05],
[-3.3585e-01],
[-9.3917e-26],
[-3.3573e-01],
[-4.5245e-03],
[-1.3848e-02],
[-3.3415e-01],
[-3.2094e-01],
[-2.9396e-05],
[-3.3585e-01],
[-2.7087e-02],
[-3.3760e-04],
[-3.3585e-01],
[-3.3585e-01],
[-3.3585e-01],
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[-3.3584e-01],
[-3.3585e-01],
[-3.3585e-01],
[-5.6596e-02],
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[-3.3497e-01],
[-6.6676e-02],
[-5.8985e-08],
[-6.4641e-04],
[-5.1718e-12],
[-3.3580e-01],
[-4.7032e-02],
[-2.0267e-15],
[-3.3585e-01],
[-8.8276e-06],
[-3.2180e-01],
[-3.3585e-01],
[-3.3579e-01],
[-3.3278e-01]]), you=tensor([[-7.0680e-21],
[-1.1219e-11],
[-3.3496e-01],
[-1.4111e-02],
[-6.1627e-07],
[-3.3585e-01],
[-5.8577e-09],
[-8.6417e-03],
[-3.3567e-01],
```

```
[-3.3568e-01],
        [-7.3368e-23],
        [-1.7807e-25],
        [-3.3576e-01],
        [-3.9042e-05],
        [-3.3585e-01],
        [-9.3917e-26],
        [-3.3573e-01],
        [-4.5245e-03],
        [-1.3848e-02],
        [-3.3415e-01],
        [-3.2094e-01],
        [-2.9396e-05],
        [-3.3585e-01],
        [-2.7087e-02],
        [-3.3760e-04],
        [-3.3585e-01],
        [-3.3585e-01],
        [-3.3585e-01],
        [-3.3582e-01],
        [-3.2872e-01],
        [-2.4297e-07],
        [-3.3584e-01],
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        [-3.3497e-01],
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        [-6.4641e-04],
        [-5.1718e-12],
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        [-4.7032e-02],
        [-2.0267e-15],
        [-3.3585e-01],
        [-8.8276e-06],
        [-3.2180e-01],
        [-3.3585e-01],
        [-3.3579e-01],
        [-3.3278e-01]])
b1, pytorch=tensor([-1.2134e-19, -3.2433e-10, -7.3256e-04, -3.0705e-02,
6.4626e-06,
        -0.0000e+00, -6.7041e-08, -1.7528e-01, 5.8137e-03, -1.1349e-03,
        -2.5998e-21, -9.3143e-24, -1.4919e-03, 9.7014e-04, 0.0000e+00,
         2.2554e-24, 2.5023e-04, 1.9031e-03, -3.5662e-02, -1.1795e-02,
        -4.5726e-02, 6.1877e-04, -0.0000e+00, -8.4479e-02, 5.8581e-04,
        -0.0000e+00, 0.0000e+00, -8.4869e-07, -2.8796e-04, -4.8221e-02,
```

```
-8.5869e-06, -2.5191e-05, -0.0000e+00, 0.0000e+00, -3.0426e-01,
        4.8219e-02, -3.1243e-03, -2.3556e-01, 8.5481e-07, 8.2101e-05,
        1.7939e-10, -9.0230e-05, -3.8265e-01, -2.5082e-14, -0.0000e+00,
       -9.5027e-05, -6.1352e-02, 0.0000e+00, -1.5214e-04, -1.5417e-02]),
you=tensor([-1.2134e-19, -3.2433e-10, -7.3256e-04, -3.0705e-02, 6.4626e-06,
        -0.0000e+00, -6.7041e-08, -1.7528e-01, 5.8137e-03, -1.1349e-03,
       -2.5998e-21, -9.3143e-24, -1.4919e-03, 9.7014e-04, 0.0000e+00,
        2.2554e-24, 2.5023e-04, 1.9031e-03, -3.5662e-02, -1.1795e-02,
        -4.5726e-02, 6.1877e-04, -0.0000e+00, -8.4479e-02, 5.8581e-04,
        -0.0000e+00, 0.0000e+00, -8.4869e-07, -2.8796e-04, -4.8221e-02,
        -8.5869e-06, -2.5191e-05, -0.0000e+00, 0.0000e+00, -3.0426e-01,
        4.8219e-02, -3.1243e-03, -2.3556e-01, 8.5481e-07, 8.2101e-05,
        1.7939e-10, -9.0230e-05, -3.8265e-01, -2.5082e-14, -0.0000e+00,
        -9.5027e-05, -6.1352e-02, 0.0000e+00, -1.5214e-04, -1.5417e-02]
W1, pytorch=tensor([[-1.2498e-20, -3.3407e-11, -7.5456e-05, -3.1627e-03,
6.6566e-07,
         0.0000e+00, -6.9054e-09, -1.8054e-02, 5.9882e-04, -1.1690e-04,
        -2.6779e-22, -9.5940e-25, -1.5367e-04, 9.9927e-05, 0.0000e+00,
         2.3231e-25, 2.5774e-05, 1.9602e-04, -3.6733e-03, -1.2149e-03,
        -4.7099e-03, 6.3735e-05, 0.0000e+00, -8.7016e-03, 6.0340e-05,
         0.0000e+00, 0.0000e+00, -8.7417e-08, -2.9660e-05, -4.9669e-03,
        -8.8448e-07, -2.5948e-06, 0.0000e+00, 0.0000e+00, -3.1340e-02,
         4.9667e-03, -3.2181e-04, -2.4263e-02, 8.8048e-08, 8.4567e-06,
         1.8478e-11, -9.2940e-06, -3.9415e-02, -2.5835e-15, 0.0000e+00,
        -9.7880e-06, -6.3194e-03, 0.0000e+00, -1.5671e-05, -1.5880e-03]),
you=tensor([[-1.2498e-20, -3.3407e-11, -7.5456e-05, -3.1627e-03,
                                                                 6.6566e-07,
         0.0000e+00, -6.9054e-09, -1.8054e-02, 5.9882e-04, -1.1690e-04,
        -2.6779e-22, -9.5940e-25, -1.5367e-04, 9.9927e-05, 0.0000e+00,
         2.3231e-25, 2.5774e-05, 1.9602e-04, -3.6733e-03, -1.2149e-03,
        -4.7099e-03, 6.3735e-05, 0.0000e+00, -8.7016e-03, 6.0340e-05,
         0.0000e+00, 0.0000e+00, -8.7417e-08, -2.9660e-05, -4.9669e-03,
        -8.8448e-07, -2.5948e-06, 0.0000e+00, 0.0000e+00, -3.1340e-02,
         4.9667e-03, -3.2181e-04, -2.4263e-02, 8.8048e-08, 8.4567e-06,
         1.8478e-11, -9.2940e-06, -3.9415e-02, -2.5835e-15, 0.0000e+00,
        -9.7880e-06, -6.3194e-03, 0.0000e+00, -1.5671e-05, -1.5880e-03]])
x, pytorch=tensor([-0.0546]), you=tensor([-0.0546])
```