Explicit Laser Powder Bed Fusion (LPBF) MPM Time Step

Particle State

- x_p - position
- m_p - mass
- v_p - volume
- e_p - specific internal energy

Auxiliary particle variables

- $\hat{\phi}_p$ -regularized signed distance to zero isocontour (free surface)
- κ_p - thermal conductivity
- c_{vp} - isovolumetric specific heat
- $\mathbf{F}_p$ - heat flux
- ρ_p - density
- S_p^{laser} -laser specific energy

Primary grid variables

- m_i - node mass
- $\tilde{e}_i$ - node Favre averaged internal energy

Auxiliary grid variables

- $\tilde{e}_i^*$ - node updated Favre averaged internal energy
- $(m\tilde{e})_i$ - node mass-weighted Favre averaged internal energy
- ϕ_i - node signed distance to zero isocontour (free surface)
- $\hat{\phi}_i$ - node regularized signed distance to zero isocontour (free surface)

Material Properties

- ρ - density
- C_p - isobaric specific heat capacity
- κ - thermal conductivity

Shape Functions

- N_{ip}^2 - quadratic node-to-particle B-spline
- ∇N_{ip}^2 - gradient of quadratic node-to-particle B-spline

Algorithm

1. Compute signed distance function, ϕ_i, to the free surface from the particle positions, x_p
   - See signed distance algorithm on how to compute ϕ_i from x_p
   - All particles are used to compute the free surface. Only those that are liquid will have a non-zero surface tension coefficient
2. Compute a regularized signed distance function so the level set is now scaled to the range [-0.5,0.5] with a sharp gradient at the zero isocontour with negative values inside the volume (Grid)

   NOTE: The choice of ϵ here can vary (e.g. Δx could also be used)
   - $\epsilon = \frac{\Delta x}{2}$
   - $\hat{\phi}_i = \frac{1}{2} - \frac{1}{1+\exp(\frac{\phi_i}{\epsilon})}$
3. Update grid state (P2G)
   - $m_i = \sum_p N_{ip}^2 m_p$
   - $(m\tilde{e})_i = \sum_p N_{ip}^2 m_p e_p$
4. Compute Favre averaged internal energy (Grid)
   - $\tilde{e}_i = \frac{(m\tilde{e})_i}{m_i}$
5. Apply thermal diffusion operator and Neumann conditions (G2P2G)

   NOTE: Here the Neumann condition is either zero on the adiabatic boundaries or is equal to the deposition from laser energy, loss due to evaporation, loss due to radiative flux, or re-absorption of radiative losses emitted from elsewhere in the problem.

   NOTE: See the document on the laser source for its formulation
   - $\hat{\phi}_p = \sum_i N_{ip}^2 \hat{\phi}_i$
   - $$S_p^{laser} = \begin{cases} e_v^{laser}(x_p, t), & \text{if } |\hat{\phi}_p| \leq 0.45 \\ 0, & \text{otherwise} \end{cases}$$
   - $\rho_p = \frac{m_p}{v_p}$
   - $\kappa_p = EOS(e_p, \rho_p)$
   - $C_{vp} = EOS(e_p, \rho_p)$
   - $\mathbf{F}_p = -\frac{\kappa_p}{C_{vp}} \sum_i \nabla N_{ip}^2$
   - $\Delta(m\tilde{e})_i = \Delta t \sum_p v_p (S_p^{laser} - \mathbf{F}_p \cdot \nabla N_{ip}^2)$
6. Compute updated grid energy and apply Dirichlet boundary conditions (Grid)

   NOTE: Neumann conditions are applied directly to the particles in the previous algorithmic step
   - $\tilde{e}_i^* = \tilde{e}_i + \frac{\Delta(m\tilde{e})_i}{m_i}$
   - $\tilde{e}_i^* \leftarrow BC(\tilde{e}_i^*)$
7. Update particle internal energy from the grid energy increment (G2P)
   - $e_p \leftarrow e_p + \sum_i N_{ip}^2 (\tilde{e}_i^* - \tilde{e}_i)$